What's Up With Big *G*?

S. Cavens, D. M. Caro, L. G. Fleming, B. Fulton, H. J. Leggetter, T. W. Mitchell, D. A. Tirado, and W. S. Schmahl
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We have analysed a variety of papers on measurements of Newton's gravitational constant in order to determine reasons for the apparent discrepancies between experimental measurements. There seems to be too little evidence to conclude that the constant is fluctuating with sinusoidal behaviour; the best current explanation is that there are systemic errors, in part worsened due to the weak strength of the gravitational interaction. Our suggested course of action is to frequently repeat the experiments in order to best narrow down the value of G and best analyse what might be causing potential fluctuations in results - new technology for measuring G on milligram scales or using atom interferometry might help allow for such experimentation.

I. HISTORICAL OVERVIEW

In the early 17th century, astronomer Johannes Kepler had just spearheaded a huge leap in our understanding of cosmology by publishing his three laws of planetary motion. Many scientists and astronomers were left striving to gain an understanding as to why these laws worked, amongst them mathematician and physicist Isaac Newton, who had previously developed infinitesimal calculus and the first reflecting telescope. He, alongside others, had recognized that Kepler's laws could be explained by a force of attraction between the Sun and the planets which is inversely proportional to the square of their separation. Newton released his laws of motion and law of universal gravitation in his book *Philosophiæ Naturalis Principia Mathematica* (first published in 1687). His law of gravitation states every body exerts an attractional force on every other, this attractive force is proportional to the masses of the body and inversely proportional to the square of the distance between them (Newton, 1687). This led to the equation for the gravitational force:

$$F_g = \frac{Gm_1m_2}{r^2} \tag{1}$$

, where m_1 and m_2 are the mass of the bodies respectively, r is the separation between them and G is the gravitational constant. However, the gravitational constant G was left only implied in Newton's original publication of the law of universal gravitation as it did not have its current notation or a known value. Thus began the 'hunt' for the numerical value of the gravitational constant.

It was quickly realised that measuring G is exceedingly difficult due to the experiment requirements and in the 18th century the technology was very limited, thus, they needed to be clever and bring forth innovative ideas to find G.

The first successful attempt was actually an attempt to measure the density of the Earth and thus indirectly the gravitational constant. This experiment was undertaken in 1774 next to the Scottish mountain Schiehallion. The general idea was to measure the change in the angle of a plumbline bob from the vertical as the bob is pulled away from its central position due to the gravitational attraction of a nearby mountain. The change in the angle on opposite sides of the mountain can be used alongside the volume and density of the mountain, to calculate the mass of the Earth (Maskelyne, 1775). The results from the experiment gave a value of G of around $8 * 10^{-11}\text{m}^3\ \text{kg}^{-1}\ \text{s}^{-2}$, an impressively close value given the equipment being used as only varied from the modern value by around 20% (Hutton, 1778).

It was later, in 1798, that the most famous attempt to measure the gravitational constant was performed by Henry Cavendish. He used a torsion balance with two lead spheres on the beam in proximity to two much larger fixed lead spheres. This allowed him to measure the period of oscillation of the torsion balance and the angle of rotation of the torsion balance due to the small gravitational attraction hypothesised by Newton. His measurements were actually attempting to "weigh the world" which again indirectly finds the gravitational constant and were impressively accurate in doing so given the technology available at the time. His value of the mass of the Earth gives the value of $G = 6.75 \pm 0.04 * 10^{-11}\text{m}^3\ \text{kg}^{-1}\ \text{s}^{-2}$ which is just 1% off the modern value (Cavendish, 1798).

Few repeats of the Cavendish experiment gave a value of G as accurate as Cavendish himself; most experiments until later in the 19th century failed to reach Cavendish's low uncertainty, and none managed to lower it notably. Inl the 20th century when Paul R. Heyl and Peter Chrzanowski in 1942 produced a value of the gravitational constant of $6.670 \pm 0.005 * 10^{-11}\text{m}^3\ \text{kg}^{-1}\ \text{s}^{-2}$ which has a relatively low uncertainty and includes the modern accepted value in its uncertainty.

II. PROBLEMS IN MEASURING *G*

Proportionally the gravitational force is much weaker than any of the other forces. It is more than 10^{20} times smaller than the electromagnetic force, which is known to behave similarly otherwise. An approximate visual of the forces on the scale of a proton is given below, showing that the weak nuclear force still manages to outscale the electromagnetic force on small enough scales, despite being a 'weak' force.

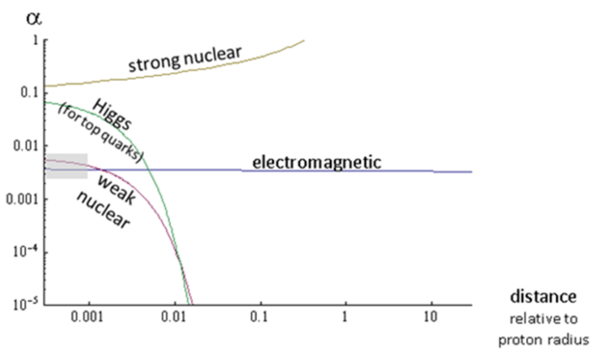

FIG. 1: Relative strength of fundamental forces on proton scales, taken from (Strassler, 2013)

Here the value of alpha for gravity, 10^{-38} (Strassler, 2013), is clearly not possible to plot even on a logarithmic graph with the other forces - this clearly shows how weak the gravitational interaction is comparatively. There is also very little link between gravity and the rest of theoretical physics and the fundamental forces; unlike, for example, the speed of light constant which can be found precisely and theoretically from the electric permittivity and magnetic permeability of free space, $\frac{1}{\sqrt{\epsilon_0 * \mu_0}}$.

Ultimately our theoretical understanding of physics requires that the Newtonian gravitational constant be found experimentally. Experimental setups frequently involve some configuration of large masses causing minute changes in movement. To do this accurately the test masses of any objects must be known incredibly precisely; by knowing the dimensions and density of the objects. If instead very small objects such as thin wafers or even atoms are used to measure the force, then the gravitational force from the experimenters and setup may influence the experiment and produce errors. The experiments must be free from any effect from the other major force, electromagnetism, and have the effects of gravity from the Earth, sun and moon all accounted for. The laboratory must also be mechanically stable so it is not affected by any seismic activity in the area, and the temperature of the apparatus stabilised precisely.

Still, even with all these harsh requirements, physicists have managed to design experiments with high precision. The LIGO experiment in 2014, in order to detect gravitational waves, had to measure changes in separation between test masses of about $\frac{1}{200}$th of the width of a proton ($4 * 10^{-18}$ m). Researchers have also been able to measure the gravitational force of a gold sphere weighing less than 100mg (Westphal *et al.*, 2021). Clearly then the ability to do highly sensitive measurements is not preventing progress in the field. An initial predominant experiment to formalise the value was the Gundlach and Merkowitz torsion balance experiment (Gundlach *et al.*, 1998) that tried to isolate the experiment to the fewest possible errors. It negates the uncertainties in the pendulum mass due to its shape, mass distribution and total mass if the mass is a thin flat plate. It has 8 large spherical masses on a rotating plate that are configured to average out the effects of any local gravity gradients. They will rotate on the plate towards the pendulum during the experiment so the torsion wire is twisted less allowing the measurement of G to be independent of the anelasticity of the wire. However even this experiment with an innovative design to reduce the uncertainty to 0.001% ultimately contradicted the earlier 1998 CODATA value of G by 0.024%.

This leads many to believe that it may not simply be human and random error preventing the precise measurement of the gravitational constant, but instead an unknown systematic error affecting the results. This is reinforced by the fact that many experiments performed using the same method, even with slightly adjusted laboratory setups, produce very similar results when their experiment is repeated many years later; but that results are inconsistent with experiments performed using different methods. It is still very difficult to determine what the case may be as many experiments use a variation on the Cavendish experiment or a new method entirely, which would suggest that the different lab setups would be affected by different sources of error, one that is not systematic.

III. POTENTIAL SINUSOIDAL FLUCTUATIONS

It has been suggested that perhaps the error originates from an innate fluctuation of the gravitational constant (Anderson *et al.*, 2015). Anderson's 2015 paper considers data since 1980. There appears to be a sinusoidal relationship between the measured value for G and time (when the experiment was conducted). This sinusoid appears to have a time period of 5.9 years. Current scientific research is unable to confidently explain the reason for this value fluctuation, however, comparisons with other scientific values have been made, such as the "Length of Day" period on Earth. (Anderson *et al.*, 2015)

It is particularly interesting that the value for the "Length of Day" also seems to sinusoidally fluctuate with a time period of 5.9 years. In fact, the two seemingly independent sinusoidal time periods have a strong correlation significance of 0.99764 (Anderson *et al.*, 2015). The "Length of Day" fluctuation is likely caused by changes in the Earth's core-mantle boundary, which cause a decadal episodic jerk in the Earth's rotation (Holme, 2013). These geomagnetic jerks can be linked with torsional flow in the Earth's core. Small changes in the conditions of measurement during the decadal sinusoidal fluctuation might influence the results obtained for the value of G during experimentation – but this relation has yet to be proved or scientifically theorised. The graph below shows the LOD fluctuation with a decadal time period of 5.9 years (Holme, 2013).

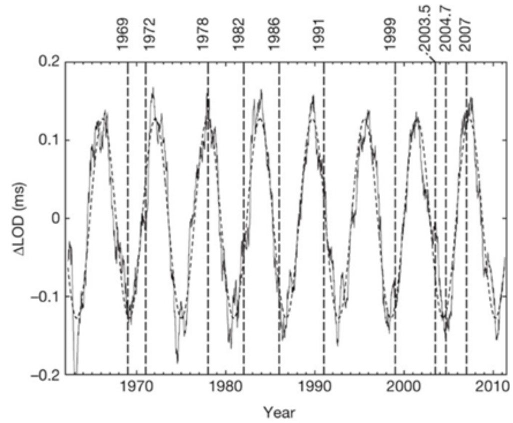

FIG. 2: The "Length Of Day" fluctuations, taken from (Holme, 2013)

However, other scientific research has suggested that the sinusoidal correlation for measurements of G may be far weaker than previously theorised by Anderson. Schlamminger *et al.* states in their paper (2015), that systematic error is underestimated in measurements for G and hence provides an alternative interpretation for the apparent fluctuation of value measurements. He also points out that many experimental data points were missing from the Anderson paper and some points were not plotted at the right time. Plotting these results at the correct time and adding the missing ones causes there to be a much looser fit to the sinusoidal pattern. The data is not as conclusive as originally expressed, as can be seen in the graph below.

It seems we can consider four potential fluctuations for the value of the Newtonian gravitational constant: $G(t)$, it varies with time; $G(x, y, z)$, it varies with position; $G(r)$, it varies with distance; or some combination of the three. $G(t)$ can only be explained helpfully through a potential

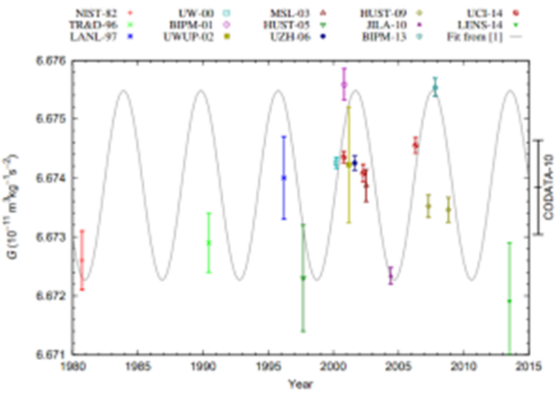

FIG. 3: Corrected Sinusoidal Correlation, taken from (Schlamminger, 2015)

sinusoidal relationship as discussed previously - the relationship might be random, or otherwise unpredictable, in which case further data will not help narrow down any relationship for G. However, it seems not to vary with time at all: an experiment by Moud, and Uddin (2014) that investigated the value of G by measuring the constant emission from standard candles in the form of type Ia supernovae put a constraint on the variation of G. They determined G, as it is related to the Chandrehaskar mass and their validity as standard candles. Their validity can be verified through use of them to measure redshift of distant galaxies and find values that agree with other independent methods, putting a constraint on the variation of G. The paper concluded finding that G has varied less than one part per billion per year over the last 9 billion years (Moud *et al.*, 2014).

$G(x, y, z)$ means that the gravitational constant could be a scalar field that changes with each position in space. There is little research or evidence to support this, however, as behaviour of gravitational interactions on a macroscopic level seem consistent. Furthermore, the Moud, and Uddin experiment mentioned before measured type Ia supernovae at significantly different locations and thus seemed to similarly provide evidence against fluctuations of the gravitational interaction as a function of space.

Finally there is potential for $G(r)$, that the value of G varies between the scale of two sheets of paper to a galaxy. This has been suggested as a reason for the inconsistent motion of galaxies besides an effect from dark matter, though the presence of dark matter has been well demonstrated. However, at the opposite scale, theoretical theorists trying to search for a theory of quantum gravity that is consistent with quantum electrodynamics through superstring theory suggest there may be a variation of G on the scale of micrometers. Here it is theorised that there are extra folded spatial dimensions in which only the forces of gravity can act. It can describe why the force of gravity is so much weaker than the other forces, ordinary particles can only exist in our current spacetime (or brane) whereas gravitons may pass into and through these extra dimensions, thereby ‘diluting’ themselves. This may mean that the force of gravity may vary slightly on this scale, which could be a cause of error in these fine tuned experiments. However, much more research must be done into these theories to allow any credence in their predictions.

As a further question, we wondered whether there is benefit to G not being constant. The literature we have looked at is divided on this issue. A paper by Sakr *et al.*, which investigated whether allowing for variable G could resolve the discrepancies in results from different methods in the Hubble constant and the matter fluctuation parameter, found no significant difference between using a constant or parametrised G. Meanwhile a paper by Hanimeli *et al.* suggests that a time-variable G could be the source of dark energy and allow us to substitute out dark energy in our models of cosmological expansion. Their claims that a time-variable G is consistent with local observations is perhaps dubious in the light of the general consensus and the characteristics of the observational data. A paper by Desmond *et al.* in which a value for G in the LMC is reported at $0.93 + 0.05 / - 0.04 G_{\text{Solar system}}$ invites further testing of variable G hypotheses using similar methods in to test the validity of variable-G cosmological models, which seems to be needed before any certainty on the variability of G can be reached.

IV. OUR ANALYSIS

In order to test the time variability of G as a reason for the discrepancy in measurements, we can look at measurements of G over a given period of time or we can model existing phenomena using a dynamical G and see if this results in more accurate models which rely on fewer assumptions, or alleviates existing ‘tensions’ in other results. We will first establish as best we can from as many existing measurements of G as possible, whether or not the G measurements oscillate. We plotted a graph of the data in the adjusted graph with additional data. In total we have an average of 1.45 measurements a year in the combined graph which may allow us to see any periodic variation if it exists. We used least squares regression to fit a sinusoidal function onto the data with free amplitude, phase, period, and amplitude offset. There remains the problem that the data represents a combination of 9 different methods, each of which will have their own systematic uncertainties. The opinion of many is that unconsidered systematic errors likely explain a lot of the variation in G.(De Salvo, 2015) (Schlamminger *et al.*, 2015) (Li *et al.*, 2018) (Kawasaki, 2020) (Gillies, 1997).

When the measurements from the TR&D study are represented individually, it becomes harder to see how to fit an oscillation over the whole data set, as was done by Anderson *et al.* The sinusoidal line of best fit does not have the same parameters as the oscillation reported by Anderson *et al.* (2015): our period is 0.990 ± 0.005 yrs instead of 5.899 ± 0.062 yrs, with the period equivalence being the most significant piece of evidence for a correlation between G and the Length Of Day cycles (Anderson *et al.*, 2015). The quality of the fit is poor and is evident from the amount of interpolation over the TR&D series - we did not feel the need to quantify it, given how visually obvious it seems. This, alongside the aforementioned problems with the Anderson *et al.* analysis, makes the case for an oscillation in the measurements of G with characteristics similar to the LOD cycles difficult to justify with our expanded data set, at least without further, more rigorous analysis.

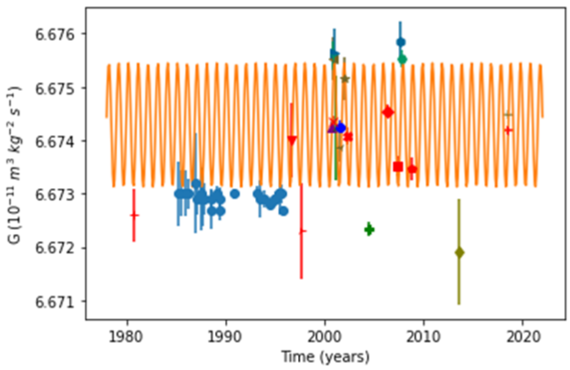

FIG. 4: Our graph of the combined G data. The data was collected by TR&D in their attempt to measure G over ten years, which consisted of 26 individual measurements of G which were then averaged, we plotted individually. We have also included two very precise measurements of G, made by HUST in 2018 after the adjusted graph was produced. CODATA values were not included as these are synthesised from various different measurements. In the Schlamminger et al. analysis, when a data point represented an average of many measurements over time, the time was taken to be the midpoint of the investigation. We have included a time uncertainty for these. (Schlamminger *et al.*,2015) (Q. Li *et al.*,2018)

V. RECENT LITERATURE AND NEW METHODS

In more recent years, new methodology has been designed in order to attempt to narrow down the exact value of G more exactly. The first example of a new method used is in a 2014 experiment by Rosi *et al.*, where, rather than a typical torsion balance experiment, G was determined through measuring the behaviour of cold atoms using atom interferometry. Though the result found by this paper is comparatively low even after considering its uncertainty when matched to previous measurements primarily obtained through torsion balances, the experiment was the first of its kind, measuring G through behaviour on a smaller scale, in this case through the behaviour of Rubidium atoms. More recent mechanical set-ups for atom-interferometry based measurements of G have been shown to be able to narrow down the uncertainty in G to as little as 59 ppm (Mao *et al.*, 2021).

Further experiments similarly seek to measure gravity on a far smaller scale. Recent development in 2019 (Matsumoto *et al.*) involves a set-up to measure gravity on a scale of milligrams, hoping to narrow down motion and effects of gravity in small scale objects to further measurements of gravity. A further paper from 2020 (Cataño-Lopez *et al.*) continues the development of this set-up and suggests that these new techniques can be used “not only for testing dark after via quantum-limited force sensors, but also Newtonian interaction in quantum regimes, namely, between two milligram-scale oscillators in quantum states, as well as improving the sensitivity of gravitational-wave detectors” - that is, this technique can be used to measure gravitational effects on quantum objects and scales.

Since the gravitational constant is so hard to measure on Earth, however, another major suggestion for experimentation has been to send an experimental set-up into deep space (¿25 AUs) and to take measurements there, in order to minimize the noise created by mass in the nearby vicinity (Feldman *et al.*, 2016) (Benish, 2016). The suggested approach for this has been taking a traditional torsion balance experiment with mild modifications or a Cavendish experiment, and sending an object out into deep space to take these measurements. Benish (2016) further lists as a reason for such an experiment that gravitational behaviour within massive objects have yet to be properly measured. By using retroreflectors on the pendulum, one can measure the time of swing by shining a laser against the reflector and measuring its time to move, and thus take measurements of the motion of the pendulum, which relate to and correspond with the behaviour of gravity, allowing for high-precision measurements of the gravitational constant unaffected by potential disrupting factors from the Earth.

An alternative, more theoretical approach to this issue is through the cosmic microwave background radiation (CMB) radiation. A recent proposition (Cui *et al.*, 2022) proposes that it is possible to find the gravitational force through the temperature of the CMB radiation. By initially considering the average expected energy of the CMB we can derive a relationship between the gravitational constant, G, and temperature, amongst other better-known constants. However, since the widely accepted experimental value of temperature of the CMB, $T = 2.73$ K (uncertainty not given) is thought to display some anisotropies, there is a limit on the precision with which we can calculate the gravitational constant using this method. This method currently gives $G = 6.474792 * 10^{-11}\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$ which differs from the currently accepted value (Tiesinga *et al.*, 2021). However, due to the current lack of accuracy and precision of the temperature of the CMB, this is understandable: there is a distinct limit to the precision and accuracy of this method. Furthermore, it has been reported that the temperature of the CMB might be decreasing with time, potentially implying G was higher in the past, if this relationship holds. If this holds true, then it may offer another explanation for the variation in G.

VI. CONCLUSION

Our findings suggest that, although a possibility, there is currently insufficient evidence to support the hypothesis of a sinusoidal variation in the gravitational constant. It is thought that the discrepancies in the measurement arise through the intrinsic difficulty in measuring such a weak force at a high precision and importantly the difficulty in shielding experiments from external interaction leading to potential systematic errors that are unaccounted for. New methods offer potential reduction in external influence such as deep space experiments with the intention of a reduction in noise from nearby bodies (Benish, 2016), or microscopic scale experimentation to measure the potential differing behaviour of gravitational interactions on a quantum level (Matsumoto *et al.*, 2019). Additionally, given the array of values measured on historical experiments, past experiments should be repeated with more care taken to increase precision on the accepted value, or to better uncover any patterns in result fluctuations. Taking these actions should help locate, define, and understand the fluctuations and variations in measurements of G, to ideally produce independent experiments performed through varying methods that are in agreement with each other to hopefully further narrow down its exact value.

BIBLIOGRAPHY

[1] J. D. Anderson, G. Schubert, V. Trimble, and M. R. Feldman, “Measurements of Newton’s gravitational constant and the length of day”, EPL **110** 10002 (2015).

[2] R. Benish, “Gravitational Clock: Near Space Proof-of-Concept Prior to Deep Space Measurement of G-Part I”, available at: http://vixra.org/abs/1612.0341 (Accessed June 16, 2022) (2016).

[3] S. B. Cataño-Lopez, J. G. Santiago-Condori, K. Edamatsu, and N. Matsumoto, “High-Q Milligram-Scale Monolithic Pendulum for Quantum-Limited Gravity Measurements”, Phys. Rev. Lett. 124, 221102 (2020).

[4] H. Cavendish, ”Experiments to determine the density of the Earth”, Philosophical Transactions of the Royal Society of London, **17**, 469 (1798).

[5] A. M. Cetto, L. Luis, and E. Santos, “Dirac’s large-number hypothesis revised”, Astronomy and Astrophysics. 164. 1-5. (1986).

[6] Q. Cui, “Revisiting gravity, fine-structure constant, speed of light, and mass-energy equation through cosmic microwave background” (2022).

[7] R. DeSalvo, “Unaccounted source of systematic errors in measurements of the Newtonian gravitational constant G”, Physics Letters A, Volume 379, Issues 18-19, Pages 1202-1205 (2015).

[8] H. Desmond, J. Sakstein, and B. Jain, “A 5% measurement of the gravitational constant in the Large Magellanic Cloud”, arXiv:2012.05028v2 [astro-ph.CO] (2021).

[9] M. R. Feldman, J. D. Anderson, G. Schubert, V. Trimble, S. M. Kopeikin, and C. Lämmerzahl, “Deep space experiment to measure G”, *Class. Quantum Grav.* **33** 125013 (2016).

[10] G. T. Gillies, “The Newtonian gravitational constant: recent measurements and related studies”, *Rep. Prog. Phys.* **60** 151 (1997).

[11] J. H. Gundlach, and S. M. Merkowitz, ”Measurement of Newton’s Constant Using a Torsion Balance with Angular Acceleration Feedback”, Phys. Rev. Lett. **85**, 2869-2872 (2000).

[12] E. T. Hanimeli, I. Tutusaus, B. Lamine, and A. Blanchard, “Can Dark Energy Emerge from a Varying G and Spacetime Geometry?”, arXiv:2201.04629v2 [gr-qc] (2022).

[13] P. R. Heyl, and P. Chrzanowski, “A redetermination of the constant of gravitation”, Bureau of Standards, Physics Department, (pp. 1243-1290) (1930).

[14] R. Holme, and O. de Viron, ”Characterization and implications of intradecadal variations in length of day.”, *Nature* **499**, 202-204 (2013).

[15] K. Horstman, and V. Trimble, “A citation history of measurements of Newton’s constant of Gravity”, arXiv:1811.10556 [physics.hist-ph] (2018).

[16] C. Hutton, ”An account of the calculations made from the survey and measures taken at Schiehallion, in order to ascertain the mean density of the Earth”, Philosophical transactions of the Royal Society of London, **68**, 689-788 (1778).

[17] A. Kawasaki, “Measurement of the Newtonian constant of gravitation G by precision displacement sensors”, *Class. Quantum Grav.* **37** 075002 (2020).

[18] Q. Li, C. Xue, JP. Liu, JF. Wu, SQ. Yang, CG. Shao, LD. Quan, WH. Tan, LC. Tu, Q. Liu, H. Xu, LX. Liu, QL. Wang, ZK. Hu, ZB. Zhou, PS. Luo, SC. Wu, V. Milyukov, and J. Luo, “Measurements of the gravitational constant using two independent methods”. *Nature* **560**, 582–588 (2018).

[19] ZT. Lu, “New determination of the gravitational constant G”, National Science Review, Volume 7, Issue 12, December 2020, Pages 1796–1797 (2020).

[20] DK. Mao, XB. Deng, HQ. Luo, YY. Xu, MK. Zhou, XC. Duan, and ZK. Hu, “A dual-magneto-optical-trap atom gravity gradiometer for determining the Newtonian gravitational constant”, Review of Scientific Instruments 92, 053202 (2021).

[21] N. Maskelyne, ”A Proposal for measuring the Attraction of some Hill in this Kingdom by Astronomical Observations”, Philosophical Transactions of the Royal Society of London, **65**, 495-499 (1775).

[22] N. Matsumoto, S. B. Cataño-Lopez, M. Sugawara, S. Suzuki, N. Abe, K. Komori, Y. Michimura, Y. Aso, and K. Edamatsu, “Demonstration of Displacement Sensing of a mg-Scale Pendulum for mm- and mg-Scale Gravity Measurements”, Phys. Rev. Lett. **122**, 071101 (2019).

[23] P. J. Mohr, B. N. Taylor, and D. B. Newell, “CODATA recommended values of the fundamental physical constants: 2010”, Review of Modern Physics 84, (2012).

[24] J. Mould, and S. Uddin, “Constraining a Possible Variation of G with Type Ia Supernovae”, Publications of the Astronomical Society of Australia, 31, E015 (2014).

[25] G. Rosi, F. Sorrentino, L. Cacciapuoti, M. Prevedelli, and G. M. Tino, “Precision measurement of the Newtonian gravitational constant using cold atoms”, *Nature* **510**, 518-521 (2014).

[26] I. Newton, ”Philosophiae Naturalis Principia Mathematica”, Vol. 1, G. Brookman (1833).

[27] T. Quinn, ”Measuring big G.”, *Nature* **408**, 919-920 (2020).

[28] J. L. Russell, “Kepler’s laws of planetary motion: 1609-1666”, The British journal for the history of science, 2(1), pp. 1-24 (1964).

[29] Z. Sakr, and D. Sapone, “Can varying the gravitational constant alleviate some tensions?”, Journal of Cosmology and Astroparticle Physics, Volume 2022 (2022).

[30] S. Schlamminger, “A cool way to measure big G”, *Nature* **510**, 478-480 (2014).

[31] S. Schlaminger, J. H. Gundlach, and R. D. Newman, “Recent measurements of the gravitational constant as a function of time”, arXiv:1505.01774v2 (2015).

[32] J. Schmöle, M. Dragosits, H. Hepach, and M. Aspelmeyer, “A micromechanical proof-of-principle experiment for measuring the gravitational force of milligram masses”, *Class. Quantum Grav.* **33** 125031 (2016).

[33] J. R. Smallwood, “Maskelyne’s 1774 Schiehallion experiment revisited”, Scottish Journal of Geology, 43(1), pp.15-31 (2007).

[34] M. Strassler, “The Strengths of the Known Forces”, Of Particular Significance, available at: https://profmattstrassler.com/articles-and-posts/particle-physics-basics/the-known-forces-of-nature/the-strength-of-the-known-forces/ (Accessed June 16, 2022) (2013).

[35] The LIGO Scientific Collaboration and The Virgo Collaboration, ”GW150914: The Advanced LIGO Detectors in the Era of First Discoveries”, American Physical Society **116**,13, (2016).

[36] E. Tiesinga, P. J. Mohr, B. N. Taylor, and D. B. Newell, “CODATA recommended values of the fundamental physical constants: 2018”, Reviews of Modern Physics 93 (2021).

[37] A. Unzicker, “Why do we Still Believe in Newton’s Law? Facts, Myths and Methods in Gravitational Physics”, arXiv:gr-qc/0702009 (2007).

[38] T. Westphal, H. Hepach, J. Pfaff, and M. Aspelmeyer, ”Measurement of gravitational coupling between millimeter-sized masses”, *Nature* **591**, 7849 (2021).

[39] C. Xue, JP. Liu, Q. Li, JF. Wu, SQ. Yang, CG. Shao, LC. Tu, ZK. Hu, J. Luo, “Precision measurement of the Newtonian gravitational constant”, Natl. Sci. Rev. 7(12): 1803-1817 (2020).