Determination of The Viscosity of Water

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Viscosity is a measurement of a fluid's resistance to deformation or flow and is due to the internal friction between the layers of flow. The instantaneous flow rates of water at different heights using different radii tubes were measured. Chi-squared analysis can be used to find the linear model parameters between flow rate and height; by using the Hagen-Poiseuille equation (3) it is, in theory, possible to determine a value of viscosity. Due to affects of turbulence the value of viscosities shown in table. 1 varied between the tubes used, the most accurate value of viscosity measured was $\eta = 1.55 \pm 0.04$ mPas. This value disagreed with literature.

1. Introduction

Understanding the idea of viscosity of a fluid had its first break through by Sir Isaac Newton [1] who formulated a description of viscosity through a proportional differential relationship. Later in the 19th century Poiseuille [3] investigated the subject to understand the circulation of blood in the human body. Since then, measurements have been taken of various fluids to get increasingly precise values of the viscosity [2]; with further research into its variation due to state changes (temperature and pressure).

Viscosity is a measure of a fluids resistance to deformation at a given rate [1]. It quantifies the internal friction between adjacent layers (laminae) of a fluid in relative motion and is dependant on the state of the fluid. The layers move at different velocities and the viscosity arises from the shear stress between the layers that opposes any applied force. For a liquid between two large plates of area A, separated by distance y, with one moving at velocity u due to applied force F and the other a stationary surface; there will be a velocity gradient in the layers between the two plates. The force required to move the place will be proportional to the area of the place and velocity gradient.

$$\frac{F}{A} = \eta \frac{\partial u}{\partial y} = \tau \qquad (1)$$

This gives us Newton's equation for viscosity [1], where the factor of proportionality is the (dynamic) viscosity η, and can be expressed using τ, the shear stress. Different types of fluids have a different relationship that may not be linear. Whereas water is a Newtonian fluid with constant viscosity, non-Newtonian fluids may have shear thickening where their viscosity increases with rate of shear or shear thinning where the inverse occurs.

The flow rate of a fluid Q from high to low pressure is related to the pressure difference ΔP and its resistance to motion R [3].

$$Q = \frac{P_2 - P_1}{R} = \frac{\Delta P}{R} \qquad (2)$$

The resistance to motion is a constant of proportionality that affect the flow rate. This constant was first understood by Poiseuille and then defined by Hagen Bach through the Navier-Stokes equations to give the Hagen-Poiseuille equation known today [4].

$$Q = \frac{KD^4 \Delta P}{L} = \frac{\pi r^4 \Delta \mathrm{P}}{8\eta L} \qquad (3)$$

Where K is the constant found by Poiseuille, D is the diameter, L the length and r the radius of the tube. The pressure difference in this case is due to the force from the water on the tube given by mg. Where substituting $V \cdot \rho$ for the mass and simplifying due to the geometry of a cuboid gives us the hydrostatic pressure, with ρ being the density of water, g acceleration from gravity and h height.

$$\Delta P = \frac{F}{A} = \rho g h \qquad (4)$$

Substituting this into the Poiseuille equation (3) gives use the relation to viscosity required for this experiment

$$Q = \frac{\pi r^4 \rho g}{8\eta L} h = \kappa h \qquad (5)$$

2. Method

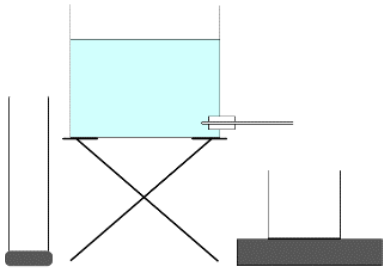


Fig. 1 The experiment setup. The apparatus (from left to right) are: 1L measuring cylinder; jack stand, 8L perspex box and tube; 500 ml beaker and electronic mass balance

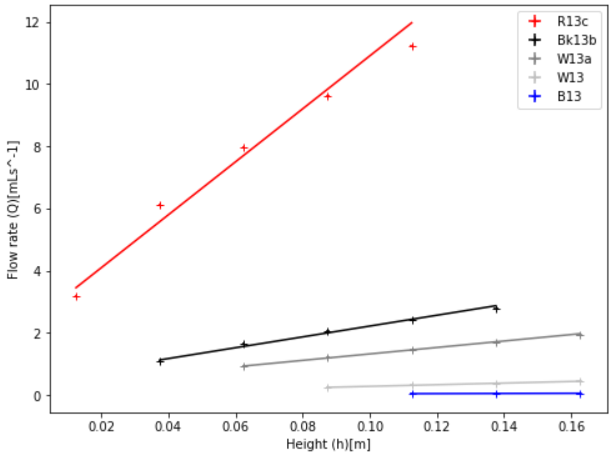
A perspex box is placed on a laboratory jack stand and risen to a suitable height. There is a hole near the bottom of the box for the tubes to be placed in. Positioned below the tube is a 0.5l beaker on top an electronic mass balance.

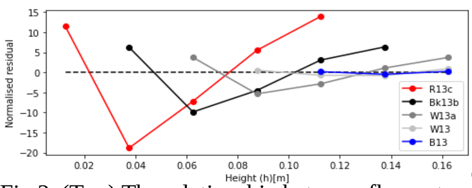
Measurements are taken of the apparatus; a travelling microscope is used to measure the radii r of the tubes. With a ruler to measure the length L of the tubes and the inside width w of the box. The volume of water below the tube v is found by pouring a litre of water into the box and measuring the volume that flows out into the beaker. An electronic balance is connected to the laptop and a python mass measurement script is configured to record the data when necessary. To start collecting data the largest radii tube is chosen and placed in the hole with the tip covered in blue-tac. The box is filled using a litre beaker up to the required volume V, to allow for laminar flow from the tube, no measurements of droplets taken. The blue-tac is removed from the tube and the script is started to record the mass for 15 seconds. Three repeats at a given volume are taken and then the volume is increased by a litre. Measurements are taken from the tube increasing the volume up to five times to a maximum of 7 litres. The subsequent tubes are then measured in order of decreasing radius.

3. Results

The volume of water in the beaker must be found from the mass measurements, this is done as $V = m/\rho$. The flow rates are calculated from the gradient of the volume-time measurements using linear regression and then averaged over the three repeats. Additionally, the height of the water above the bottom of the tubes must be found for each possible volume of water in the box. This requires the volume of water above the bottom of the tubes, real volume R, to be found with heights given by the equation

$$h = \frac{R}{w^2}, \qquad R = V - v \tag{6}$$





Tube	Reduced χ^2
R13c	254.581235
Bk13b	68.740555
W13a	21.947202
W13	1.105096
B13	0.323126

Fig 2. (Top) The relationship between flow rate and height for the various tubes. (Bottom) Normalised residual plot for the error in the heights

Table 1. reduced χ^2_{min} values of data fit

Each data set is run through χ^2 minimum analysis, which provided the linear model parameters (gradient and intercept). Due to the relatively high precision of the flow rate values and greater uncertainty in height, the heights and height error were entered through the script against flow rates. Thus, the equation had to be altered to find the values of viscosity as the constant kappa (κ) is equal to the inverse of the gradient b, so the viscosity is found by

$$\eta = \frac{\pi r^4 \rho g b}{8L} \tag{7}$$

	η [mPas]	error
R13c	5.79	0.05
Bk13b	2.49	0.04
W13a	2.45	0.04
W13	1.55	0.04
B13	2.34	0.10

Table 2. The values and errors of the viscosity of water as measured by the different tubes, ordered by decreasing radius. R, Bk, W, B standing for red. black, white, blue respectively

The values of viscosity measured vary greatly between the tubes (table 1.). The true value of the viscosity of water over 12-13-14 °C is 1.234-1.2005-1.1683 mPas [4]

4. Discussion

It was a generally inaccurate experiment with the true value of viscosity lying well outside the error ranges of the calculated values. The theory of a proportional relationship between height and flow rate ($Q \propto h$) is contradicted by the data, where there is a weak linear relationship as seen in the normalised residual plot (Fig 2.) and the reduced χ^2 values (Table 1.). The residuals show the data sets following a similar pattern, which approximates to a linear relationship upon smaller tube radii. The intercepts a of the data however, agree with the theory, with the average value being $6.407e-07$, where it is predicted to go through the origin.

Due to errors in the experiment the data didn't accurately follow Poiseuille's equation. The radius of the tubes were assumed to be circular; which was a fair assumption by eye, but not necessarily accurate. There were also visible air bubbles in the tube which would impede the flow of water and increase its effective viscosity. Most important is maintaining laminar flow, every recorded flow was determined by eye to be laminar. This was also affirmed using the Navier-Stokes equation of turbulence, the largest tube with highest velocity had a value 1450; being under 2100, it is within the expected range of laminar flow. However, the water still became more turbulent in the larger tubes at great flow rates, so Poiseuille equation became more inaccurate at higher flow rates. Which is why the model begins to agree with the data as the radii of the tubes become smaller, with the best fit and result attributed to the W13 data.

To improve upon the experiment, a larger variety of tubes can be used, especially those of a smaller radius. As well as using a larger tank which will provide a better approximation of a negligibly changing height and allow more volumes of laminar flow to be measured. Measuring any eccentricity in the tubes and adjusting the equation appropriately for ellipses. As well as a closed-loop control system can be used to regulate the temperature of the water to keep the viscosity constant, as the temperature ranged between 12-14°C during the experiment will also improve the accuracy. Ultimately however the theory of this experiment will never provide a reliable value of viscosity due to the Poiseuille equation being an approximation when the flows become less laminar.

5. Conclusion

I believe that this was a well performed experiment, with the largest fractional uncertainty being 4.09% in B13. However due to the method and approximation of the Poiseuille equation from increasingly turbulent flow, all values, including error range, lied outside of the true range of viscosity. The accuracy and linear relation of the data is seen to change with different sized tubes. Work with a larger variety of smaller radii tubes, with lower flow rates and larger range of volumes would be needed to increase the accuracy of the experiment. Fundamentally however, performing a new method would be required to measure a more accurate value of the viscosity of water.

References

[1] *Viscosity of Liquids and Gases*, 2011, Saylor Foundation Resources, [online] Available at: Saylor Foundation: Viscosity
[2] Coe, J. R. Jr. and Godfrey, T. B. (1944), Viscosity of Water, Journal of Applied Physic, 15(8), pp. 625-626, doi:/10.1063/1.1707481
[3] *Viscosity and Laminar Flow; Poiseuille's Law*, 2016, OpenStax, [online] Available at: 12.4 Viscosity and Laminar Flow; Poiseuille's Law
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[5] *Viscosity Tables*, hyperphysics hosted by Georgia State University, [online] Available at: Tables of Viscosities of Liquids and Gases , Accessed November 2022
[6] I.G. Hughes and T. P. A Hase, *Measurements and their Uncertainties*, Oxford University Press, Oxford (2010), p. 28-30, 37-40

Error Appendix

The error in the heights were calculated through propagating the error through the equations using the relevant error functional for (6). The error in R must be found first by adding the errors of V, with error associated with the measuring cylinder, and v, with error associated with the beaker, in quadrature

$$\alpha_R = \sqrt{\alpha_V^2 + \alpha_v^2} \tag{8}$$

The width of the box was measured using a ruler, with error associated with its resolution. The standard error in the height is therefore given by the error functional as provided by [6]

$$\alpha_h = h\sqrt{\frac{\alpha_R}{R}^2 + 4\frac{\alpha_w}{w}^2} \tag{9}$$

The error in the flow rates Q was calculated by finding the standard deviation in the flow rates of the three trials performed and dividing by the square root of the number of trials n as given through the standard error formula [6]

$$\alpha_Q = \frac{\sigma_Q}{\sqrt{n}} \tag{10}$$

Understanding the fit of a linear model to the data set was seen through the reduced χ^2 values and residuals (Fig 2. and Table 1.), which quantify the goodness-of-fit of a linear model to the data. Contour plots were used to find the error in the parameter values of the linear models.

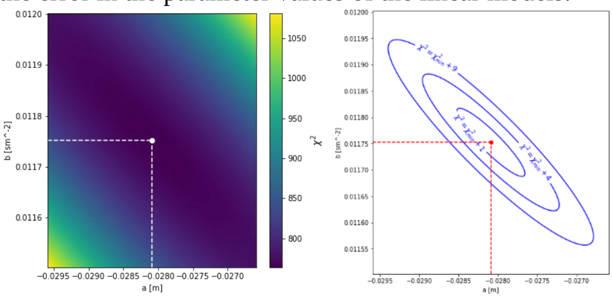

Fig 3. (Left) R13c spectral $\chi^2_{\min}$ contour plot of the parameters. (Right) line $\chi^2_{\min}$ contour plots showing the $n = 1,2,3$ levels of uncertainty

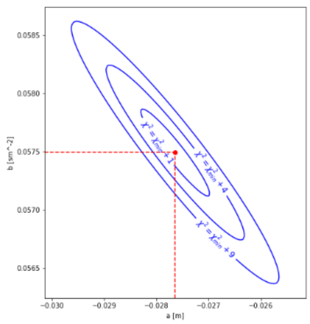

Fig 4. Bk13b line contour

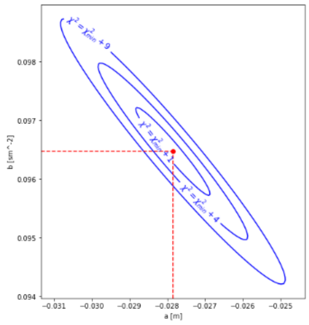

Fig 5. W13a line contour

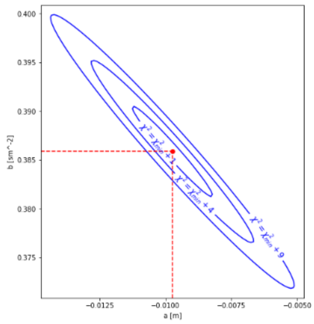

Fig 6. W13 line contour

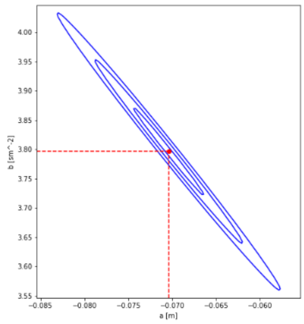

Fig 7. B13 line contour (line annotations removed for clarity)

	intercept error (a)[mm]	gradient error (b)[scm^-2]
R13c	0.430594	0.646949
Bk13b	0.655143	3.740904
W13a	0.982868	7.483141
W13	1.517118	46.257403
B13	3.990438	735.987963

Table 2. The error of the intercepts and gradients of the model fit to the tube data sets

The error in the values of gradient and intercept can be seen to increase through each of the tubes, potentially due to limited data point available. However, as the contour plots show, the correlation between the data increases with the decreasing tube radii.

The error in the values of viscosity were found using the gradient error in the linear models and the error associated with the resolution of the travelling microscope and ruler. Following the form of Poiseuille's equation (3) the error functional used was provided by [6]

$$\alpha_\eta = \eta\sqrt{16\frac{\alpha_r}{r}^2 + \frac{\alpha_b}{b}^2 + \frac{\alpha_L}{L}^2} \tag{11}$$

Scientific Summary for a General Audience

Viscosity is a property of a fluid (liquids and gases) that tries to quantify the question: what is a fluids resistance to deformation or flow? Intuitively this can be understood by how thick a liquid is, honey for example is very thick and viscous and so flows slowly when it moves, whereas water is thin and can flow easily. The reason for this difference is due to the intermolecular forces within the liquid, these are the forces of attraction between the molecules that make up the liquid. Understanding how strong these are and how they occur can be a complicated subject, but physicists look to quantify the overall (macro) effect of these forces into property that can help describe how a liquid will move.

The experiment is designed so that we can find a value of the viscosity of water. To do this we measure the flow rate of water at different heights out of different sized tubes, the box we use is quite large so the height of the water changes only very slightly so we can find an accurate value of the flow rate at that height. These values of flow rates against height are then all plotted on a graph, they are all straight lines (a linear relationship) and we measure the gradient of each line. Using an equation first formulated by Poiseuille we can calculate values of viscosity using these gradients. This is a very sensitive experiment as the viscosity of a fluid is highly dependent on temperature, and the flow rates changes a lot when the radius of the tube changes.