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1. Programming "Algorithm and Complexity (DA 4005)"

This exercise counts as a part of the Practical Exercises

Exercise 1: Depth-First-Search (DFS)
Read Chapter *22.3 Depth-first search* and *22.4 Topological sort* in "Introduction to Algorithms", Cormen et al., to get familiar with the following algorithm:

DFS(G)
1: **for** each vertex v of G **do**
2: $v.color \leftarrow$ white
3: $v.pred \leftarrow NULL$
4: $t \leftarrow 0$
5: **for** each vertex v of G **do**
6: **if** $v.color =$ white **then**
7: DFS-Visit(v)

DFS-Visit(v)
1: $v.discoverTime \leftarrow t \leftarrow t+1$
2: $v.color \leftarrow$ gray
3: **for** each w in the adjacency list of v **do**
4: **if** $w.color =$ white **then**
5: $w.pred \leftarrow v$
6: DFS-Visit(w)
7: $v.finishTime \leftarrow t \leftarrow t+1$
8: $v.color \leftarrow$ black

In particular get familiar with back edges and the characterization of acyclic graph (Lemma 22.11).

Implement the depth-first-search (DFS) algorithm for directed graphs in `C++`, `Java`, or `Python`. Use this algorithm to decide, whether a given directed graph $G = (V, E)$ is cyclic.

Answer the following question doing simulations:
What is the approximate probability, that a random directed graph with $n = 10$ nodes is cyclic, when for any two vertices nodes u, v $(u \neq v)$ there is an edge from u to v with probability $p = 0.15$, independent of all other edges? Self-loops $(u, u) \in E$ are not allowed. Simulate at least $m = 1000$ graphs and report the relative frequency of cyclic graphs.
Turn in your code and the output of your program (incl. answer for the latter question).

When handing in programming exercises, always document how to compile and run your program. Comment your source code and do not copy from WWW!

Deadline: Monday - Sept 18, 2023