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1. Introduction

This report explores hedging strategies for call options using real data and a combination of Python and R for computational tasks. The primary focus is on delta hedging and delta-vega hedging, applied to a single At-the-Money (ATM) call option of Microsoft (MSFT) as the underlying with varying maturities and re-hedging frequencies. The objective is to evaluate the accuracy of these strategies under different conditions, including the introduction of transaction costs. Delta hedging minimizes the portfolio's sensitivity to small changes in the underlying asset price, while delta-vega hedging extends this approach to address volatility risks. To account for variability in hedge performance, I will repeat the experiment 10 times for both hedging strategies, calculating the averages and standard deviations of their accuracy. From this, I would use it to compare the hedging efficiency between the two hedging strategies.

2. Data Retrieve

The data on options prices, both Call and Put and the underlying prices are fetched using Python. The underlying is Microsoft whose ISIN is 'US5949181045'. The price data are collected in one year from the 18th of March to the 18th of March 2022 – the maturity date. By looking at the price of Microsoft 45 days before the chosen maturity (Yahoo Finance, 2024), I obtained a list of strike prices for options prices fetching {280, 290, 300, 310, 320, 330, 340, 350, 360} units of dollars. All the price data is exported to the price_data.csv

3. Hedging

In this project, the hedged portfolio consists of the call option of MSFT as underlying and with 45 days to maturity. The option's strike is 300 dollars and will expire on the 18th of March 2022. I will hedge the portfolio using two hedging strategies Delta hedging and Delta-Vega hedging. For each hedging strategy, two rehedging frequencies, 1-day, and 5-day frequencies, will be utilized.

3.1 Delta hedging

Delta hedging is a fundamental strategy for managing risk in options trading, particularly in incomplete markets where perfect replication is not achievable. The original portfolio OP consists of a long position in the call option, priced using the Black-Scholes model, while the replicating portfolio RE includes a short amount of the underlying asset MSFT.

The key to delta hedging lies in the Delta ($\Delta = \frac{\partial C}{\partial S}$), which quantifies the sensitivity of the option's value to changes in the underlying asset price. The strategy involves holding an amount of the underlying equal to this Delta in the replicating portfolio RE, ensuring that the combined portfolio OP +RE is hedged against fluctuations in the underlying asset price. The portfolio is rebalanced every t days by updating the Delta based on the current underlying asset price and the reduced time to maturity. At each step, the observed change in the call option price is compared to the portfolio's value change, and the hedging error is computed as follows:

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$A_i = (C_{i+t} - C_i) - \Delta_i(S_{i+t} - S_i)$, where t is the rehedging frequency

The squared errors are accumulated over all time steps, and the mean squared error (MSE) is calculated as a measure of the hedge's effectiveness. Following the above steps, I calculated the total mean squared error of Delta-vega hedging using 2 different rehedging frequencies holding the strike, every day and every five days, and got the results:

Rehedge every day	**Rehedge every three days**
E = 1.111	E = 0.791

3.2 Delta-vega hedging

Delta-Vega hedging incorporates both the sensitivity of the option price to changes in the underlying asset's price (delta) and to changes in volatility (vega). This methodology was applied in the analysis to minimize the hedging error by constructing a portfolio that combines the underlying asset and a replicating option.

The process begins with the initialization of the portfolio. The strike price, risk-free rate, time to maturity, and observed market prices for the underlying asset and the hedged option are defined. Implied volatility is estimated using the observed option price, which is then used to calculate the Greeks for the hedged option ($Delta\ \Delta^{BS}$ and vega κ^{BS}):

The Delta is given by $\Delta = \frac{\partial C^{B5}}{\partial S} = N(d_1)$

The Vega is given by $\kappa = \frac{\partial C^{B5}}{\partial \sigma}$

Additionally, a replicating option with an additional 3 months to maturity is introduced, and its Greeks ($Delta\ \Delta^{Rep}$ and vega κ^{Rep}) are also computed.

The key hedging parameters, alpha α and eta η, are then calculated. Here, alpha α represents the portion of the portfolio allocated to the underlying asset, and eta η represents the portion allocated to the replicating option. These are determined using the relationships:

$$\alpha(\sigma) = \Delta^{BS}(\sigma) - \frac{\kappa^{BS}(\sigma)}{\kappa^{Rep}(\sigma)}\Delta^{Rep}(\sigma)$$

And

$$\eta(\sigma) = \frac{\kappa^{BS}(\sigma)}{\kappa^{Rep}(\sigma)}$$

In delta-vega hedging, we hold alpha α, not Delta Δ, of the underlying because the goal is to hedge both price (delta) and volatility (vega) risks. Adding a vega-neutralizing instrument, such as another option, alters the portfolio’s overall delta. The alpha α adjustment ensures the portfolio stays neutral to both delta and vega changes.

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The hedging strategy is implemented iteratively over the observed time steps. At each step, the portfolio's time to maturity is reduced, and the Greeks for both the hedged and replicating options are recalculated based on the updated market prices and implied volatility. The implied volatility and all the Greeks are calculated using R. [Appendix 1]

As observed from the graph [Appendix 2], we can see a downward trend in alpha and an upward trend in eta as the time to maturity decreases. This behavior is because, with less time until expiration, the option becomes less sensitive to small changes in the underlying asset's price, leading to a decrease in alpha. At the same time, the option's sensitivity to volatility increases, especially for options that are near the money, as there is less time for the underlying asset to realize its expected movements, making volatility a more critical factor. This results in an increase in eta as maturity approaches.

The hedging error for each step is computed as the difference between the observed change in the hedged option price and the combined effect of the changes in the underlying asset and the replicating option prices, scaled by alpha α and eta η, respectively. This error is squared and accumulated across all time steps. In mathematical form:

$$A_i = (C_{i+t} - C_i) - (a_i(S_{i+t} - S_i) + \eta_i(C_{i+t} - C_i)), \text{ where t is the rehedging frequency}$$

Finally, the total mean squared error (MSE) is calculated, providing a quantitative measure of the hedging performance. The Delta-Vega hedging strategy, by incorporating vega sensitivity through a replicating option, aims to mitigate volatility risk in addition to price risk, offering a more robust hedging framework compared to delta hedging alone.

Following the above steps, the total mean squared error of Delta-vega hedging is calculated using 2 different rehedging frequencies holding the strike, every day and every 5 days, and the results:

Rehedge every day	**Rehedge every three days**
E = 0.354	E = 0.306

3.3 Statistical Insights

That a single experiment might not capture the variability in hedge performance, I will repeat the hedge 10 times to capture the averages and standard deviations of the accuracy of different hedging strategies. The repetition is conducted on both hedging strategies, holding the same underlying, same strike price (E = 300), same time to maturity (T=45) but different expiration dates. To be specific, the expiration dates would be on date 15th of every month from June 2021 to March 2022. The calculations for the MSE of 2 hedging strategies on 10 different options are done using R in which the variable maturity is adjusted manually [Appendix 3]. The obtained MSE can be viewed in the below table.

Maturity	**Delta hedging**		**Delta-vega Hedging**	
	Every day	**Every 5-day**	**Every day**	**Every 5-day**

15.06.2021	0.128	0.027	0.085	0.044
15.07.2021	0.229	0.038	0.125	0.051
15.08.2021	0.240	0.034	0.087	0.029
15.09.2021	0.208	0.077	0.059	0.032
15.10.2021	0.318	0.069	0.064	0.016
15.11.2021	0.301	0.221	0.095	0.060
15.12.2021	3.067	0.975	1.768	0.389
15.01.2022	1.199	0.819	0.543	0.443
15.02.2022	1.843	1.216	0.698	0.341
15.03.2022	0.99	0.269	0.311	0.043
Average	**0.933**	**0.375**	**0.384**	**0.145**
Standard Deviation	**0.983**	**0.46**	**0.557**	**0.179**

4. Hedging accuracy compared

The above table reveals that less frequent hedging outperforms more frequent adjustments in terms of accuracy and consistency. For both Delta hedging and Delta-Vega hedging, errors are significantly lower when hedging every 5 days compared to daily adjustments. The average hedging error for Delta hedging drops from 0.933 (daily) to 0.375 (every 5 days), while for Delta-Vega hedging, it decreases from 0.384 to 0.145. Moreover, less frequent hedging exhibits lower variability, as reflected in the standard deviation of errors. This demonstrates that rebalancing less frequently reduces the impact of transaction costs, noise from short-term market fluctuations, and overreaction to minor price changes, making it a more efficient strategy.

Additionally, Delta-Vega hedging consistently outperforms Delta hedging, achieving both lower average errors and reduced variability across all frequencies. By accounting for vega (sensitivity to volatility), Delta-Vega hedging adapts better to market conditions, particularly during periods of high volatility, such as December 2021 and February 2022. While daily adjustments in Delta-Vega hedging improve accuracy compared to Delta hedging, they still underperform 5-day adjustments due to the cumulative costs of frequent rebalancing. Overall, Delta-Vega hedging with 5-day intervals offers the most effective balance between accuracy, consistency, and cost efficiency, making it the superior approach to managing hedging positions.

5. References

Yahoo Finance. (2024). Historical data for Microsoft (MSFT). Retrieved December 19, 2024, from https://finance.yahoo.com/quote/MSFT/history?p=MSFT

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Appendix

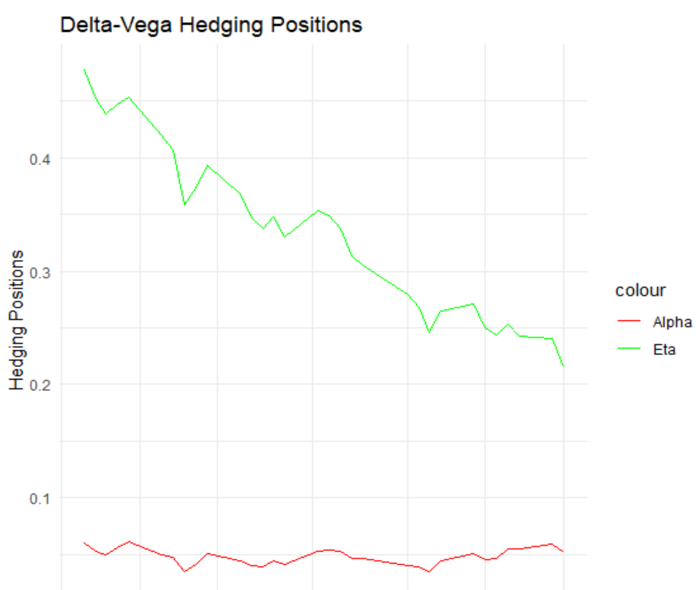
1.

```r
vola <- compute_volatility(S, Cobs, T)
greeks <- calculate_greeks(S, T, vola)
greeks_rep <- calculate_greeks(S, T_rep, vola)

alpha <- greeks$delta - (greeks$vega / greeks_rep$vega) * greeks_rep$delta
eta <- greeks$vega / greeks_rep$vega

```

2.



3.

2022.03.15

```
> cat(sprintf("Expiry date: %s\n", formatted_date))
Expiry date: 2022.03.15
> cat(sprintf("Delta Hedging - 1-Day Frequency: %.10f\n", delta_error_1d))
Delta Hedging - 1-Day Frequency: 0.9905504608
> cat(sprintf("Delta Hedging - 5-Day Frequency: %.10f\n", delta_error_5d))
Delta Hedging - 5-Day Frequency: 0.2693787157
> cat(sprintf("Delta-Vega Hedging - 1-Day Frequency: %.10f\n", delta_vega_error_1
d))
Delta-Vega Hedging - 1-Day Frequency: 0.3112851908
> cat(sprintf("Delta-Vega Hedging - 5-Day Frequency: %.10f\n", delta_vega_error_5
d))
Delta-Vega Hedging - 5-Day Frequency: 0.0432661047
```

2022.02.15

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```
> cat(sprintf("Expiry date: %s\n", formatted_date))
Expiry date: 2022.02.15
> cat(sprintf("Delta Hedging - 1-Day Frequency: %.10f\n", delta_error_1d))
Delta Hedging - 1-Day Frequency: 1.8431461177
> cat(sprintf("Delta Hedging - 5-Day Frequency: %.10f\n", delta_error_5d))
Delta Hedging - 5-Day Frequency: 1.2162009923
> cat(sprintf("Delta-Vega Hedging - 1-Day Frequency: %.10f\n", delta_vega_error_1
d))
Delta-Vega Hedging - 1-Day Frequency: 0.6983686000
> cat(sprintf("Delta-Vega Hedging - 5-Day Frequency: %.10f\n", delta_vega_error_5
d))
Delta-Vega Hedging - 5-Day Frequency: 0.3408896607
```

2022.01.15

```
> cat(sprintf("Expiry date: %s\n", formatted_date))
Expiry date: 2022.01.15
> cat(sprintf("Delta Hedging - 1-Day Frequency: %.10f\n", delta_error_1d))
Delta Hedging - 1-Day Frequency: 1.1995843211
> cat(sprintf("Delta Hedging - 3-Day Frequency: %.10f\n", delta_error_3d))
Delta Hedging - 3-Day Frequency: 0.8196654992
> cat(sprintf("Delta-Vega Hedging - 1-Day Frequency: %.10f\n", delta_vega_error_1
d))
Delta-Vega Hedging - 1-Day Frequency: 0.5429770550
> cat(sprintf("Delta-Vega Hedging - 3-Day Frequency: %.10f\n", delta_vega_error_3
d))
Delta-Vega Hedging - 3-Day Frequency: 0.4426545905
```

2021.12.15

```
> cat(sprintf("Expiry date: %s\n", formatted_date))
Expiry date: 2021.12.15
> cat(sprintf("Delta Hedging - 1-Day Frequency: %.10f\n", delta_error_1d))
Delta Hedging - 1-Day Frequency: 3.0671968602
> cat(sprintf("Delta Hedging - 5-Day Frequency: %.10f\n", delta_error_5d))
Delta Hedging - 5-Day Frequency: 0.9753602265
> cat(sprintf("Delta-Vega Hedging - 1-Day Frequency: %.10f\n", delta_vega_error_1
d))
Delta-Vega Hedging - 1-Day Frequency: 1.7686681223
> cat(sprintf("Delta-Vega Hedging - 5-Day Frequency: %.10f\n", delta_vega_error_5
d))
Delta-Vega Hedging - 5-Day Frequency: 0.3890821007
```

2021.11.15

```
> cat(sprintf("Expiry date: %s\n", formatted_date))
Expiry date: 2021.11.15
> cat(sprintf("Delta Hedging - 1-Day Frequency: %.10f\n", delta_error_1d))
Delta Hedging - 1-Day Frequency: 0.3014760015
> cat(sprintf("Delta Hedging - 5-Day Frequency: %.10f\n", delta_error_5d))
Delta Hedging - 5-Day Frequency: 0.2206423742
> cat(sprintf("Delta-Vega Hedging - 1-Day Frequency: %.10f\n", delta_vega_error_1
d))
Delta-Vega Hedging - 1-Day Frequency: 0.0954907166
> cat(sprintf("Delta-Vega Hedging - 5-Day Frequency: %.10f\n", delta_vega_error_5
d))
Delta-Vega Hedging - 5-Day Frequency: 0.0603555755
```

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2021.10.15

```
> cat(sprintf("Expiry date: %s\n", formatted_date))
Expiry date: 2021.10.15
> cat(sprintf("Delta Hedging - 1-Day Frequency: %.10f\n", delta_error_1d))
Delta Hedging - 1-Day Frequency: 0.3183057803
> cat(sprintf("Delta Hedging - 5-Day Frequency: %.10f\n", delta_error_5d))
Delta Hedging - 5-Day Frequency: 0.0690998771
> cat(sprintf("Delta-Vega Hedging - 1-Day Frequency: %.10f\n", delta_vega_error_1
d))
Delta-Vega Hedging - 1-Day Frequency: 0.0645023490
> cat(sprintf("Delta-Vega Hedging - 5-Day Frequency: %.10f\n", delta_vega_error_5
d))
Delta-Vega Hedging - 5-Day Frequency: 0.0162956342
```

2021.09.15

```
> cat(sprintf("Expiry date: %s\n", formatted_date))
Expiry date: 2021.09.15
> cat(sprintf("Delta Hedging - 1-Day Frequency: %.10f\n", delta_error_1d))
Delta Hedging - 1-Day Frequency: 0.2081930873
> cat(sprintf("Delta Hedging - 5-Day Frequency: %.10f\n", delta_error_5d))
Delta Hedging - 5-Day Frequency: 0.0770847584
> cat(sprintf("Delta-Vega Hedging - 1-Day Frequency: %.10f\n", delta_vega_error_1
d))
Delta-Vega Hedging - 1-Day Frequency: 0.0590276177
> cat(sprintf("Delta-Vega Hedging - 5-Day Frequency: %.10f\n", delta_vega_error_5
d))
Delta-Vega Hedging - 5-Day Frequency: 0.0320677628
```

2021.08.15

```
> cat(sprintf("Expiry date: %s\n", formatted_date))
Expiry date: 2021.08.15
> cat(sprintf("Delta Hedging - 1-Day Frequency: %.10f\n", delta_error_1d))
Delta Hedging - 1-Day Frequency: 0.2406083392
> cat(sprintf("Delta Hedging - 5-Day Frequency: %.10f\n", delta_error_5d))
Delta Hedging - 5-Day Frequency: 0.0341813878
> cat(sprintf("Delta-Vega Hedging - 1-Day Frequency: %.10f\n", delta_vega_error_1
d))
Delta-Vega Hedging - 1-Day Frequency: 0.0876006893
> cat(sprintf("Delta-Vega Hedging - 5-Day Frequency: %.10f\n", delta_vega_error_5
d))
Delta-Vega Hedging - 5-Day Frequency: 0.0289771274
```

2021.07.15

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```
> cat(sprintf("Expiry date: %s\n", formatted_date))
Expiry date: 2021.07.15
> cat(sprintf("Delta Hedging - 1-Day Frequency: %.10f\n", delta_error_1d))
Delta Hedging - 1-Day Frequency: 0.2292759322
> cat(sprintf("Delta Hedging - 5-Day Frequency: %.10f\n", delta_error_5d))
Delta Hedging - 5-Day Frequency: 0.0387121580
> cat(sprintf("Delta-Vega Hedging - 1-Day Frequency: %.10f\n", delta_vega_error_1
d))
Delta-Vega Hedging - 1-Day Frequency: 0.1251505136
> cat(sprintf("Delta-Vega Hedging - 5-Day Frequency: %.10f\n", delta_vega_error_5
d))
Delta-Vega Hedging - 5-Day Frequency: 0.0507854600
```

2021.06.15

```
Expiry date: 2021.06.15
> cat(sprintf("Delta Hedging - 1-Day Frequency: %.10f\n", delta_error_1d))
Delta Hedging - 1-Day Frequency: 0.1284425343
> cat(sprintf("Delta Hedging - 5-Day Frequency: %.10f\n", delta_error_5d))
Delta Hedging - 5-Day Frequency: 0.0267733233
> cat(sprintf("Delta-Vega Hedging - 1-Day Frequency: %.10f\n", delta_vega_error_1
d))
Delta-Vega Hedging - 1-Day Frequency: 0.0846403004
> cat(sprintf("Delta-Vega Hedging - 5-Day Frequency: %.10f\n", delta_vega_error_5
d))
Delta-Vega Hedging - 5-Day Frequency: 0.0442679929
```