14 a

$$P_- = \langle h\nu \rangle \frac{dN}{dt} = c\sigma_T u \tag{14.21}$$

$$P_- = \langle h\nu \rangle \frac{dN}{dt} = c\sigma_T u \tag{14.7}$$

$$P_- = \langle h\nu \rangle \frac{dN}{dt} = c\sigma_T u \tag{14.19}$$

Now Thomson scattering is independent of frequency of photon, so it follows that the rate at which energy is scattered out of radiation field is given by multiplying dN/dt by the mean energy per photon, to give

$$P_{-} = \langle h\nu \rangle \frac{dN}{dt} = c\sigma_T u \tag{14.24}$$

Note that this is precisely the rate at which energy is scattered for a stationary electron. This is something of a coincidence since we can see from (14.21) that the radiation field by moving electron do *not* have an isotrophic distribution in the lab-frame, rather they have a $(1 - \beta \cos\theta)$ distribution, so the photons propagating in the direction opposite to the electron are more likely to be scattered.

Combining P_+ from (14.19) and P_- form (14.24) gives the net *inverse Compton power* for 1 electron of $P = P_+ - P_-$ or

$$P = \frac{4}{3}\beta^2\gamma^2 c\sigma_T u \tag{14.25}$$

and the total energy transfer rate per unit volume is given by mulitiplying this by the elctron density, or more generally by the distrobution function $n(\mathbf{r}, E)$ and intergrating over energy.

Equation (14.25) is remarkably simple, and also ramerkably similar to the synchrotron power and bremsstrahlung power, for reasons already disscussed.

Interstingly, for low velocities, the Compton power is quadratic in the velocity. There is no first order-effect, since a scatterings may increase or decrease the photon energy.

14.4 Compton vs Inverse Compton Scattering

Equation (14.25) is supposedly valid for all electron eneregies, and is clealry always positive. However, this does not make sense. For cold electrons, Compton scattering result in a loss of energy for the elsctrons vis the recoil, which was ignored in deriving (14.25).

For the low energy electrons (with $v/c = \beta \ll 1$), the radiation in the electron frame is very nearly isotropic ($\delta\nu/\nu \sim \beta \ll 1$, so consequently the variation of intensity $\delta I/I \sim \beta \ll 1$ and is also small, so we can incorpotate the effect of recoil by simply subtracting the meran photon energy loss givern by (14.7).

For $v \ll c$ mean rate of energy transfer is given by $dE/dt \backsimeq (4/3)c\sigma_T u v^2/c^2$ while the rate of scatterings is $dN/dt = c\sigma_T n = c\sigma_T u/\langle\epsilon\rangle$ so the mean photon eneregy gain per collision (neglecting recoil) is $\langle\Delta\epsilon\rangle/\langle\epsilon\rangle = (4/3)(v/c)^2$, and if $v \ll c$ that is approximately equal to the mean *frractional* energy gain $\langle\Delta\epsilon/\epsilon\rangle = (4/3)(v/c)^2$ For a thermal distribution of electrons this becomes $\langle\Delta\epsilon/\epsilon\rangle = 4kT/mc^2$. The mean fractional eneregy loss due to recoil if from (14.7) $\Delta\epsilon/\epsilon = \epsilon/Mc^2$ so combining these gives

$$\left\langle \frac{\Delta\epsilon}{\epsilon} \right\rangle = \frac{4kT - h\omega}{mc^2} \tag{14.26}$$

It $\epsilon > 4kT$ then there is net transfer of energy to the electrons and *vice versa*.

14.5 The Compton y- Parameter

The *Compton y-parameter* is defined as

$$y \equiv \left\langle \frac{\Delta\epsilon}{\epsilon} \right\rangle \times \langle\text{number of scatterings}\rangle$$

- In a system with y much less (greater) then unity spactrum will be little (strongly) affected by scattering(s).
- In computing y it is usual to either use the non relativistic expression (14.26) or the higlhy relativistic limit $\langle\Delta\epsilon/\epsilon\rangle \simeq 4\gamma^2/3$.
- The mean number of scatterings is given by max(τ, τ^2).