and the nonrelativistic retarded propagator is

$$G_0^+(x'-x) = \int \frac{\mathrm{d}^3p}{(2\pi)^3} \exp\left[\mathrm{i}\,\boldsymbol{p}\cdot(x'-x)\right] \times \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{2\pi} \exp\left[-\mathrm{i}\omega\,(t'-t)\right] G_0^+(\boldsymbol{p},\omega). \tag{1}$$

From the previous discussion of the Feynman propagator we have learnt that the appropriate boundary conditions correspond to shifting the poles by adding an infinitesimal imaginary constant, such that

$$S_F(p) = \frac{\not{p}+m_0}{p^2-m_0^2+\mathrm{i}\varepsilon}. \tag{2}$$

This form implies positive-energy solution propagating forward in time and negative-energy solutions backward in time. In order to find the nonrelativistic limit of S_F we consider (2) in the approximation $|\boldsymbol{p}|/m_0 \ll 1$ and investigate the vicinity of the poles. We write

$$\frac{\not{p}+m_0}{p_0^2-\boldsymbol{p}^2-m_0^2+\mathrm{i}\varepsilon} = \frac{p_0\gamma_0-\boldsymbol{p}\cdot\boldsymbol{\gamma}+m_0}{\left(p_0-\sqrt{\boldsymbol{p}^2+m_0^2}\right)\left(p_0+\sqrt{\boldsymbol{p}^2+m_0^2}\right)+\mathrm{i}\varepsilon}. \tag{3}$$

and obtain using the approximation $\sqrt{\boldsymbol{p}^2+m_0^2} = m_0+\boldsymbol{p}^2/2m_0+O(\boldsymbol{p}^4/m_0^4)$,

$$S_F(p) \approx \frac{p_0\gamma_0-\boldsymbol{p}\cdot\boldsymbol{\gamma}+m_0}{\left(p_0-m_0-\frac{\boldsymbol{p}^2}{2m_o}\right)\left(\omega+2m_0+\frac{\boldsymbol{p}^2}{2m_o}\right)+\mathrm{i}\varepsilon}. \tag{4}$$

Now we study the behaviour of the propagator in the vicinity of its positive-frequency pole. Introducing $\omega = p_0-m_0$ we can reduce (4) to

$$S_F(p) \approx \frac{(\omega+m_0)\gamma_0-\boldsymbol{p}\cdot\boldsymbol{\gamma}+m_0}{\left(\omega-\frac{\boldsymbol{p}^2}{2m_0}\right)\left(\omega+2m_0+\frac{\boldsymbol{p}^2}{2m_0}\right)+\mathrm{i}\varepsilon}. \tag{5}$$

For the positive-frequenccy pole, ω lies in the vicinity of $\boldsymbol{p}^2/2m_0$. Therefore we have $\omega > 0$ and $(\omega+2m_0+\boldsymbol{p}^2/2m_0) \approx 2m_0 > 0$. Thus, within the approximation of small momenta, (4) can be transformed into

$$\begin{aligned} S_F(p) &\approx \frac{1}{2m_0}\frac{m_0(\gamma_0+1)-\boldsymbol{p}\cdot\boldsymbol{\gamma}}{\left(\omega-\frac{\boldsymbol{p}^2}{2m_0}\right)+\frac{\mathrm{i}\varepsilon}{2m_0}} \\ &= \frac{\frac{1}{2}(\gamma_0+1)-\frac{\boldsymbol{p}\cdot\boldsymbol{\gamma}}{2m_0}}{\left(\omega-\frac{\boldsymbol{p}^2}{2m_0}\right)+\mathrm{i}\varepsilon'}, \end{aligned} \tag{6}$$

where also ε' is a small imaginary constant. The first term

$$\frac{1}{2}(\gamma_0+1) = \begin{pmatrix} 1 & & & 0 \\ & 1 & & \\ & & 0 & \\ 0 & & & 0 \end{pmatrix}$$