

COMP9517

Computer Vision

2025 Term 3 Week 1

Professor Erik Meijering



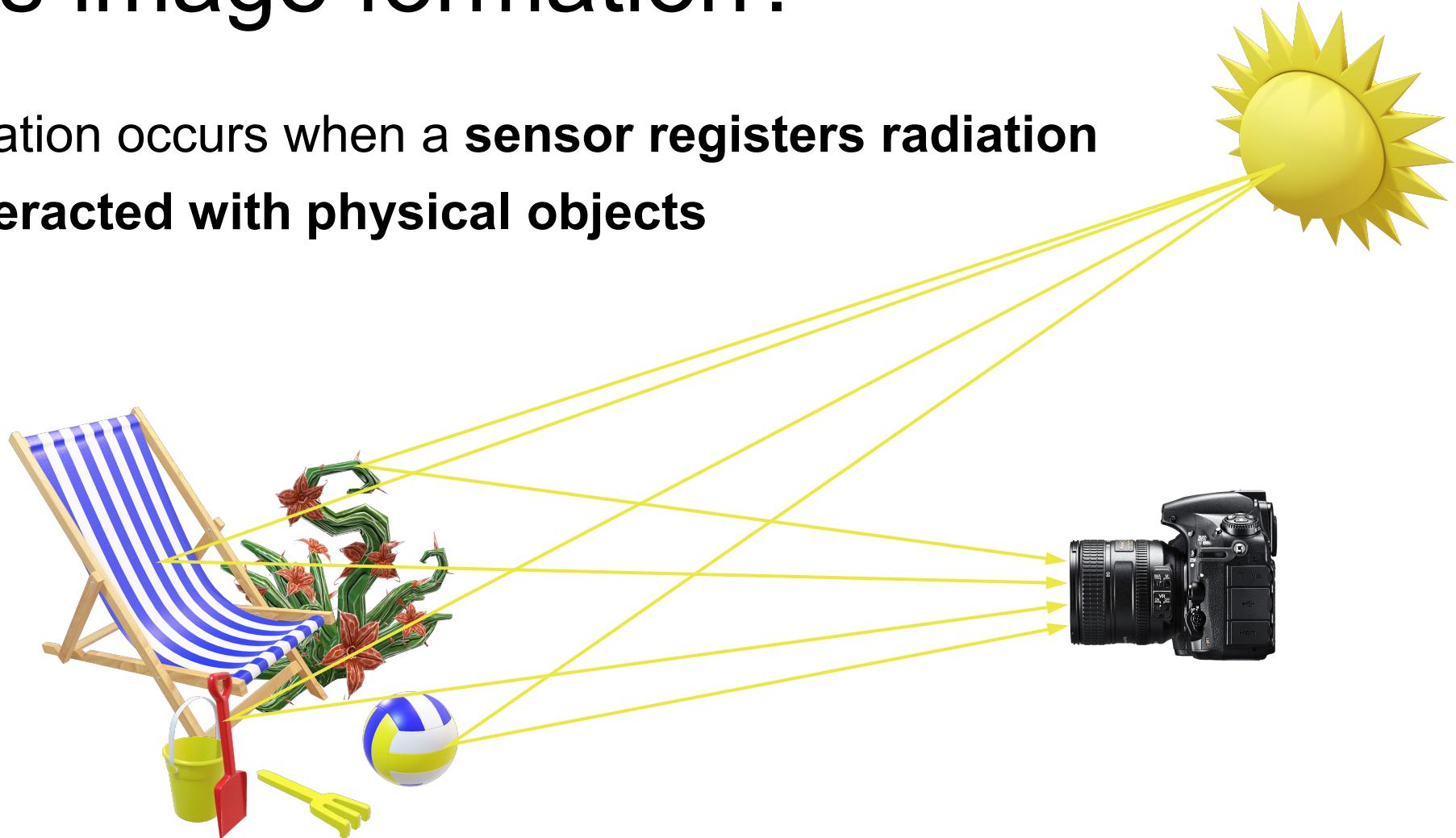
UNSW
SYDNEY



Image Formation

What is image formation?

Image formation occurs when a **sensor registers radiation** that has **interacted with physical objects**



Topics and learning goals

- Understand the **geometry of image formation**

Pinhole camera model, projective geometry, mathematics, distortions

- Describe the **principles of different colour models**

Human vision, spectrum, RGB, HSV, YCbCr, CIELAB

- Explain the **properties of digital image formation**

Spatial sampling, resolution, intensity quantisation, bits per pixel

Image formation

Idea 1: Put a piece of film in front of an object



Do we get a reasonable image?

Image formation

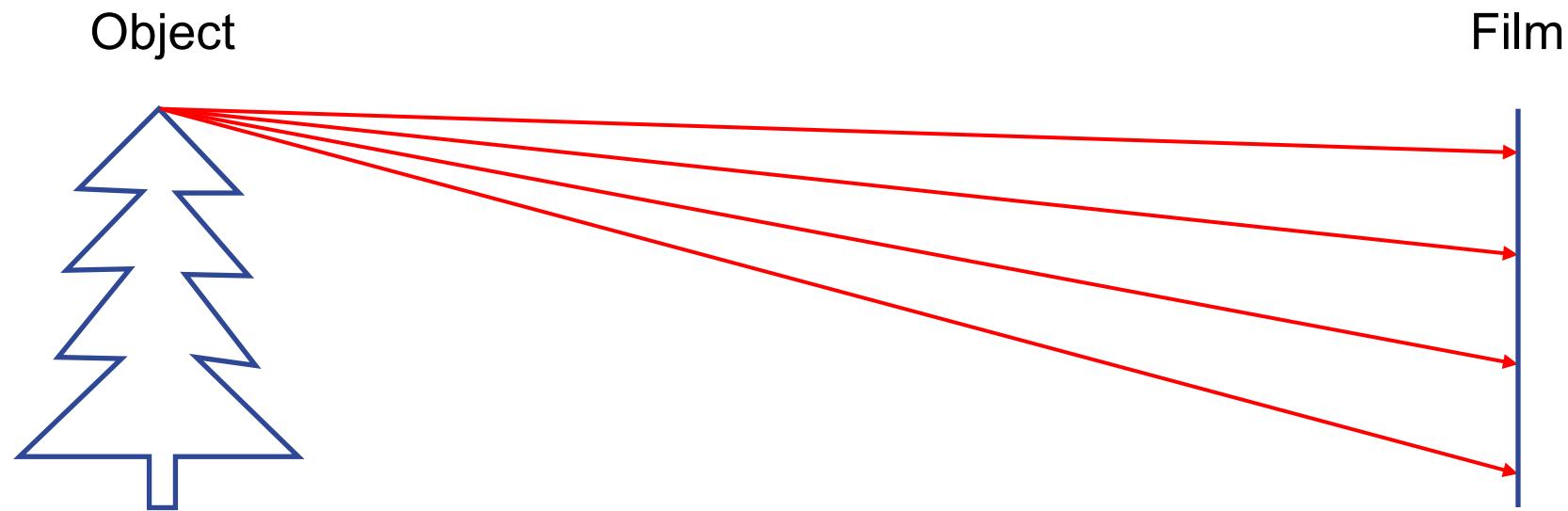
Idea 1: Put a piece of film in front of an object



Do we get a reasonable image?

Image formation

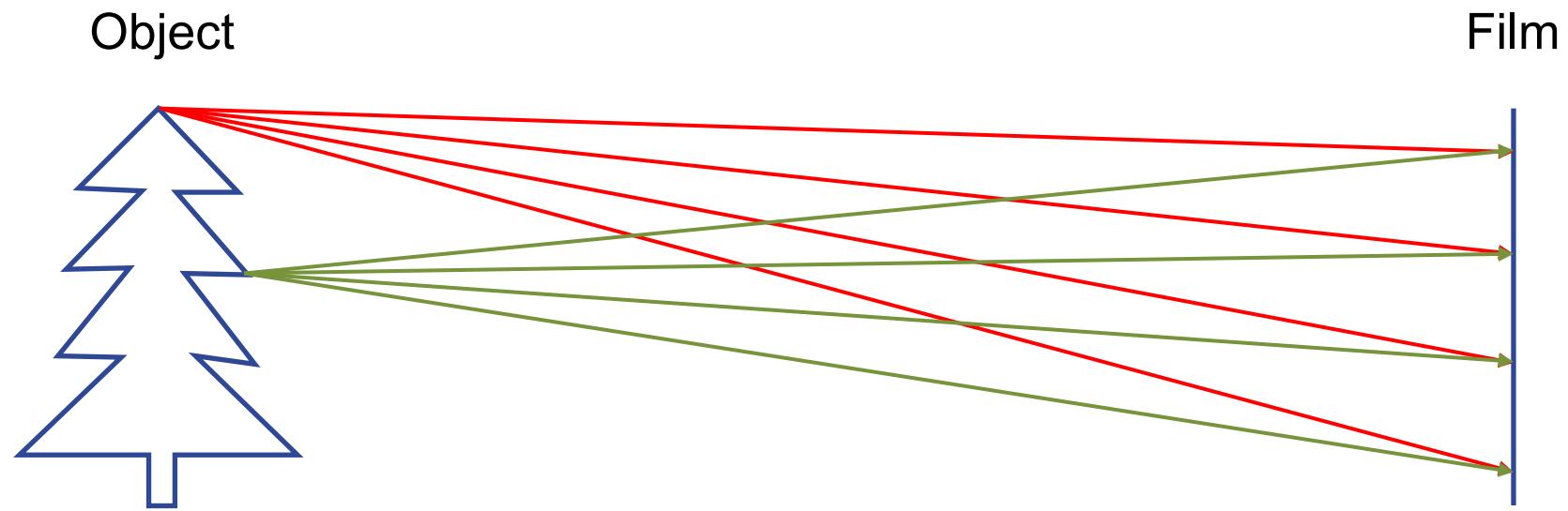
Idea 1: Put a piece of film in front of an object



Do we get a reasonable image?

Image formation

Idea 1: Put a piece of film in front of an object

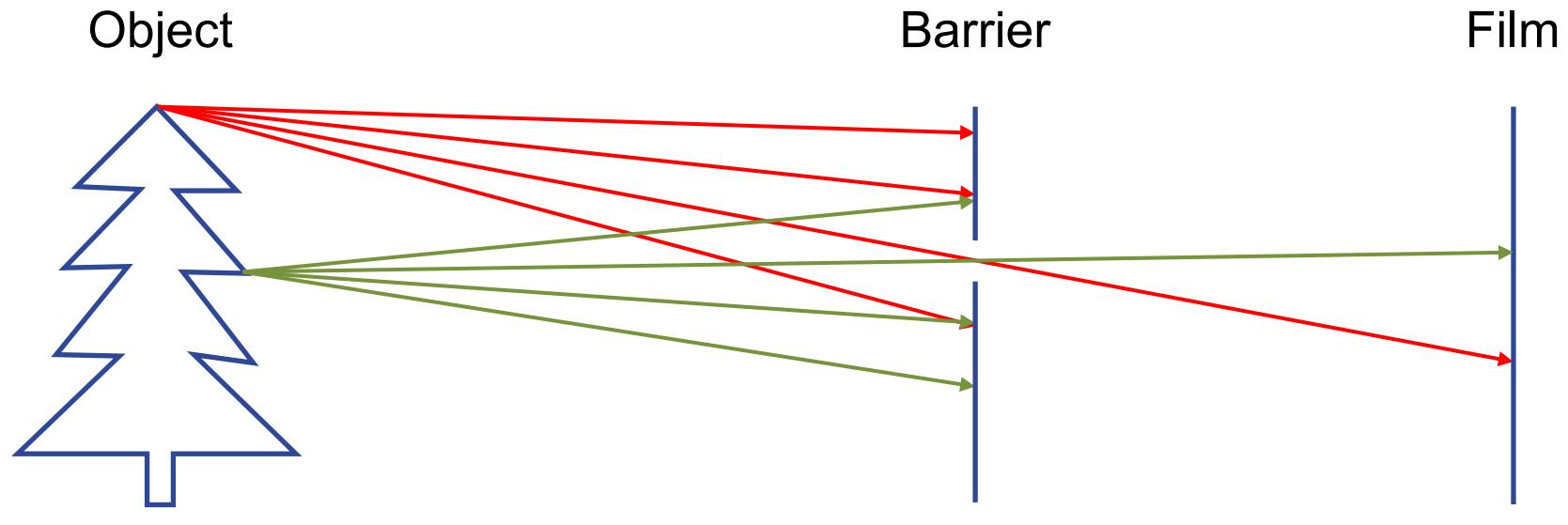


The resulting image is completely blurred

All object points are projected
to all points on the film

Image formation

Idea 2: Add a barrier to block off most of the rays



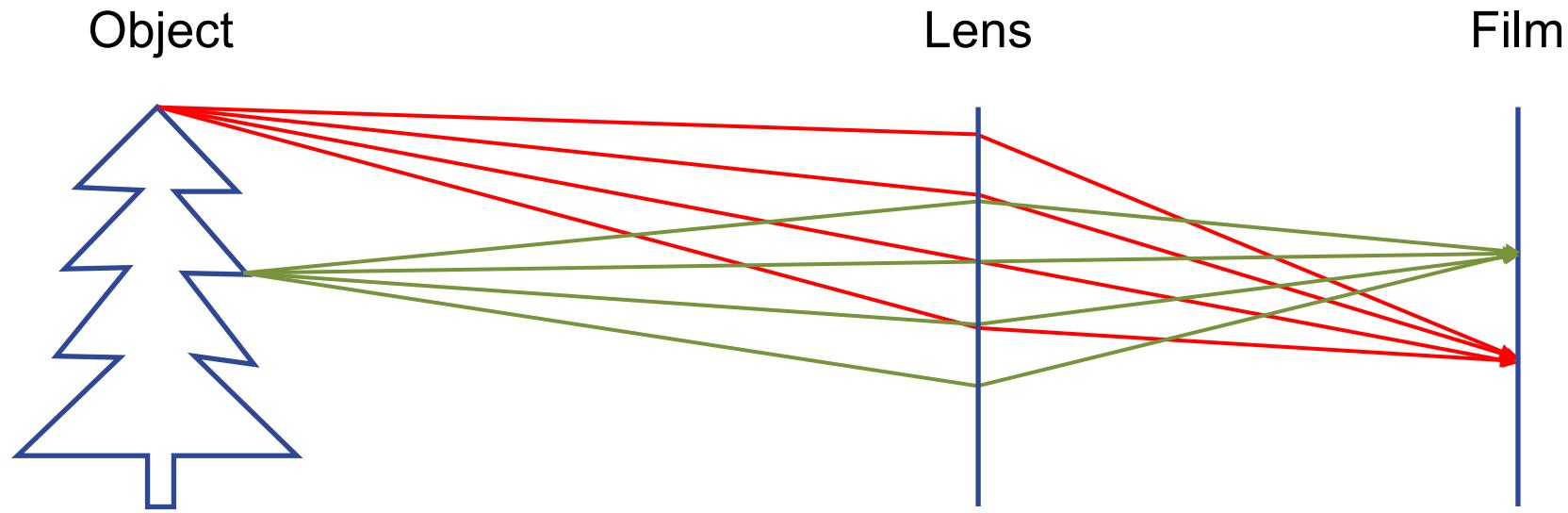
This reduces blurring

Opening known as the
pinhole or aperture

Object points are projected to
unique points on the film

Image formation

Idea 3: Add a lens to refract all the rays

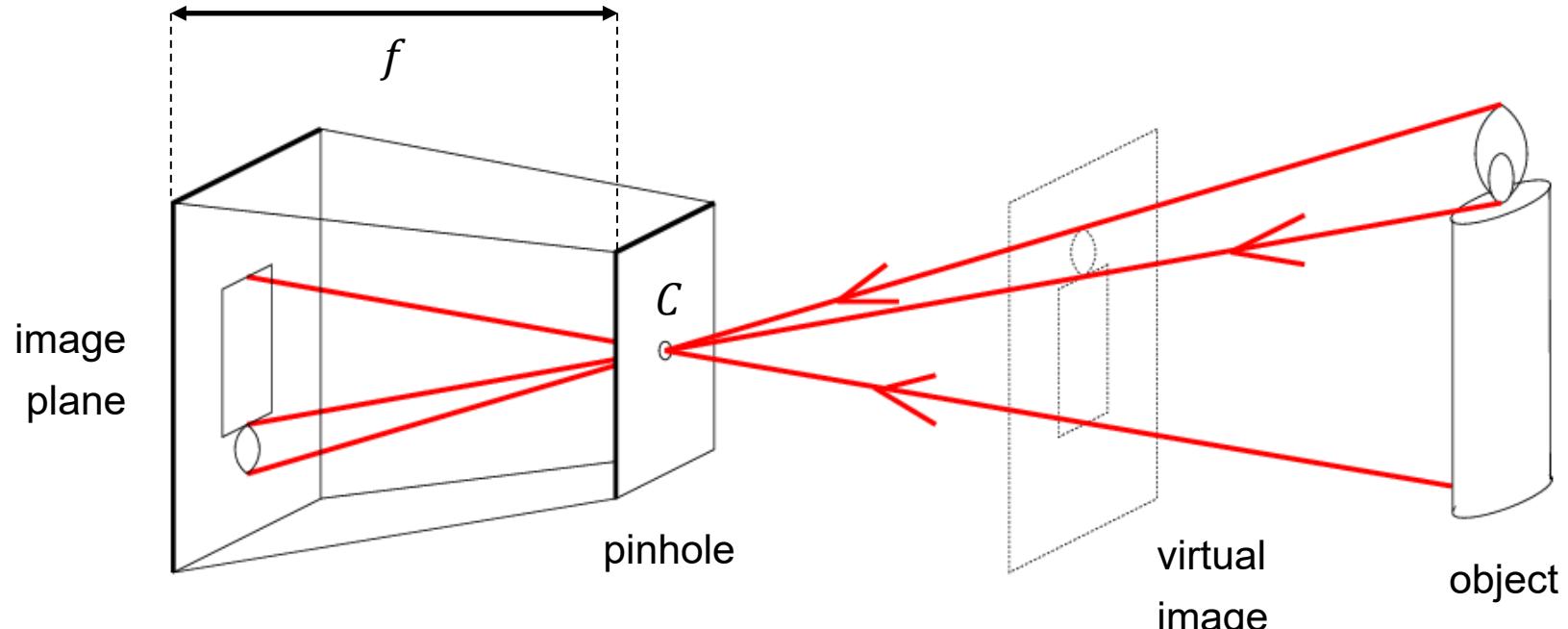


This reduces blurring

Acts like a pinhole but
avoids losing light

Object points are projected to
unique points on the film

Pinhole camera model



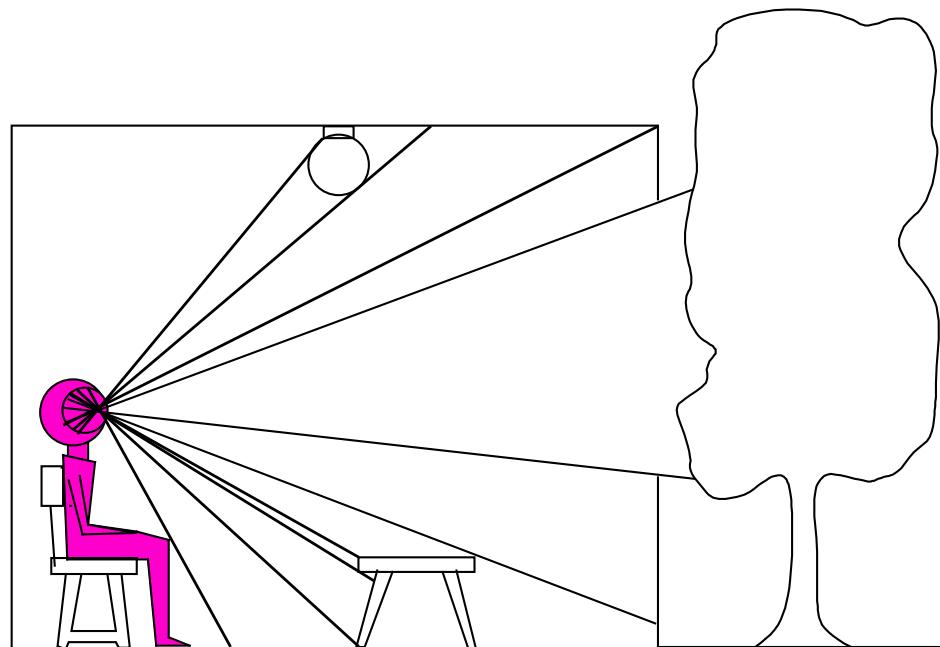
f = focal length

C = camera centre

Figure from Forsyth

Dimensionality reduction machine

3D world

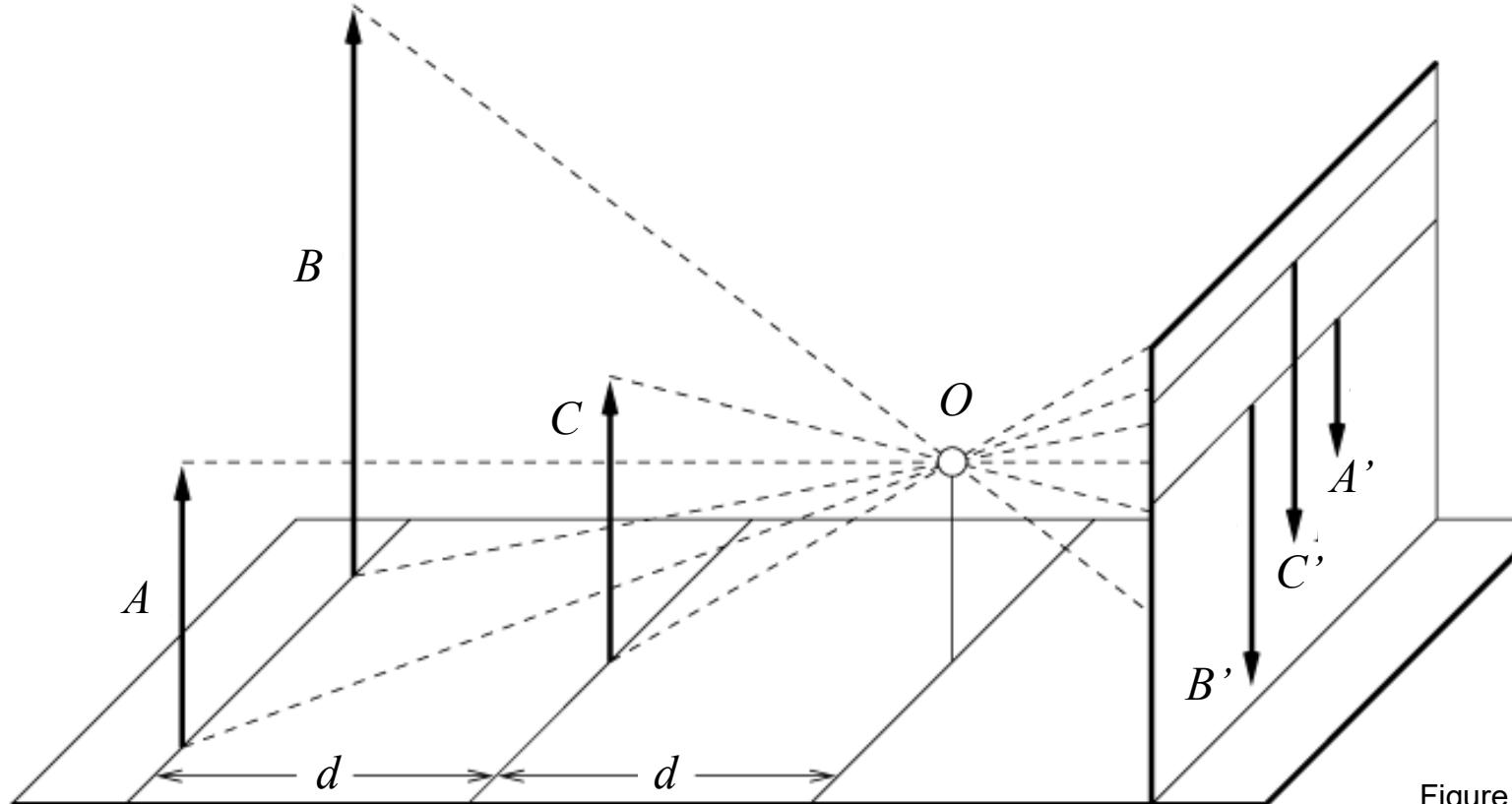


2D image

Projection can be tricky...



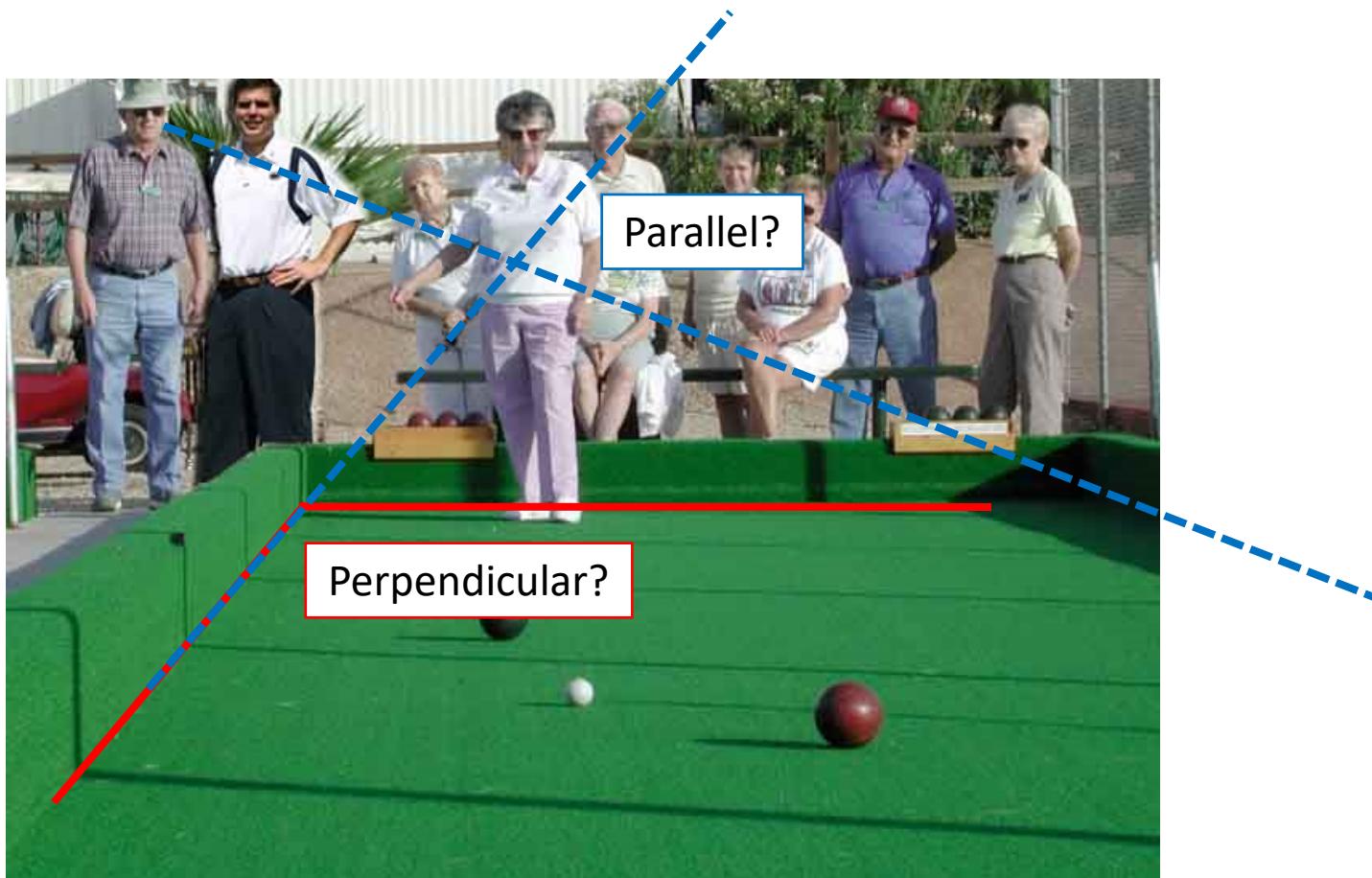
Projective geometry



Length and area are not preserved

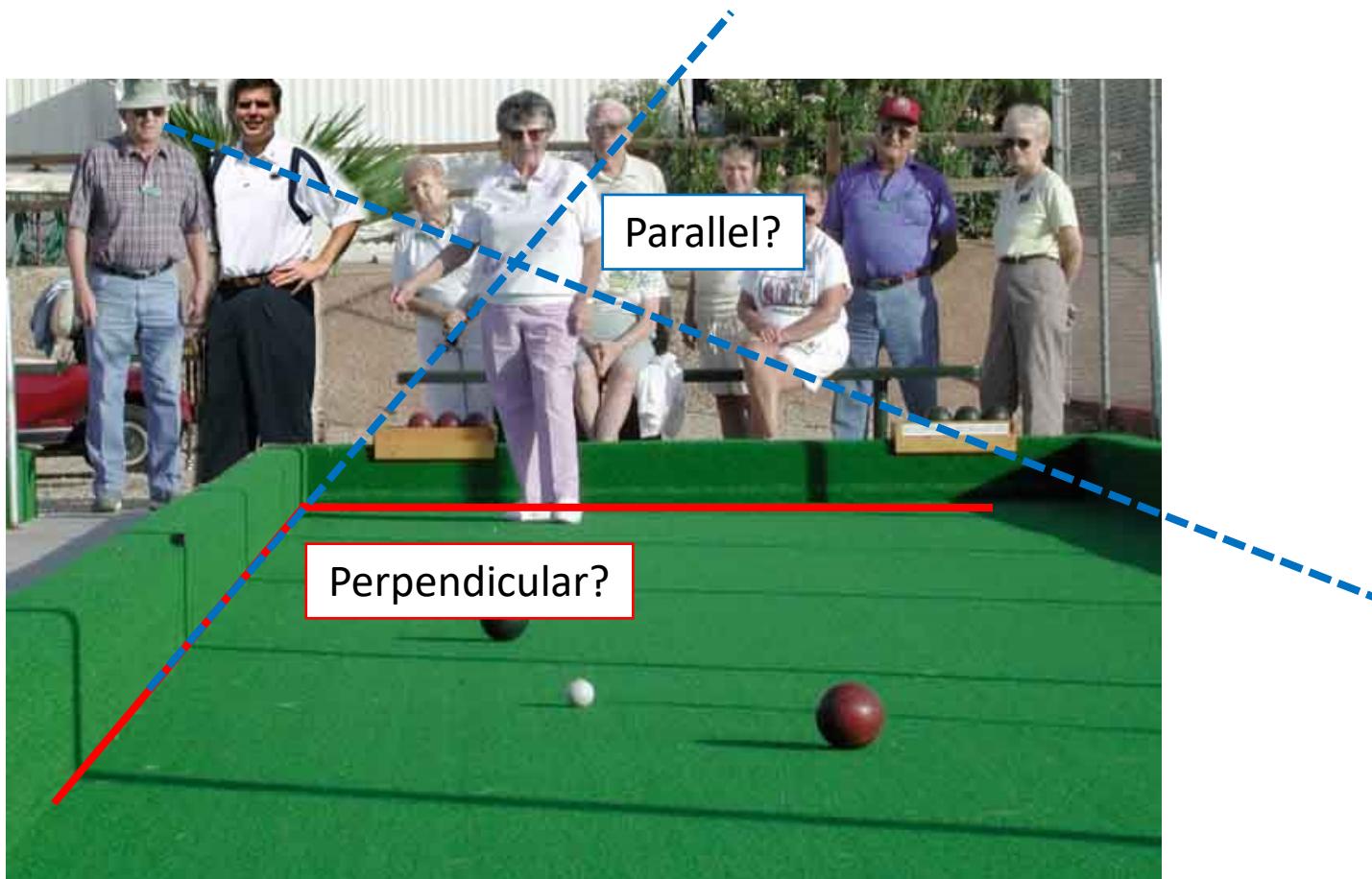
Figure from Forsyth

Projective geometry



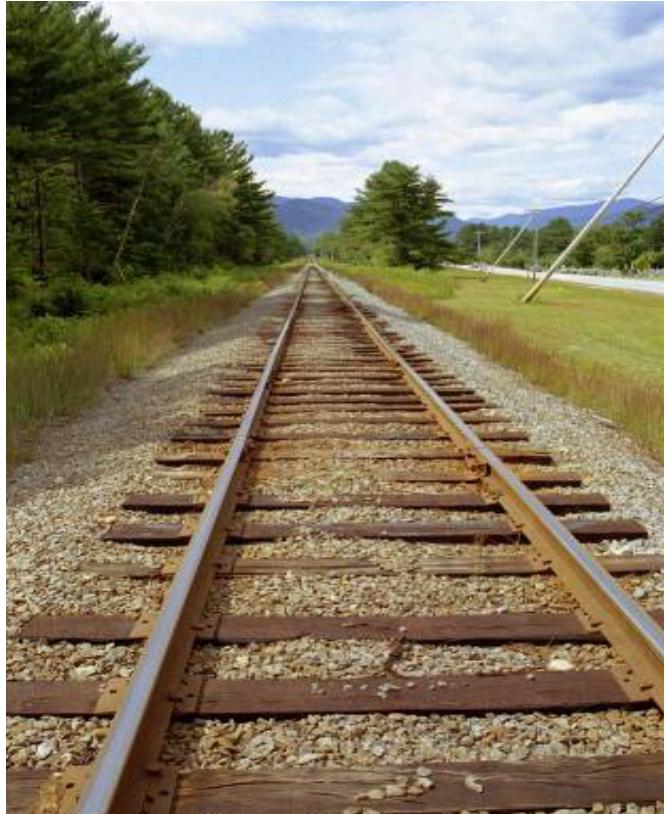
What is lost?
Length and angles
are not preserved

Projective geometry



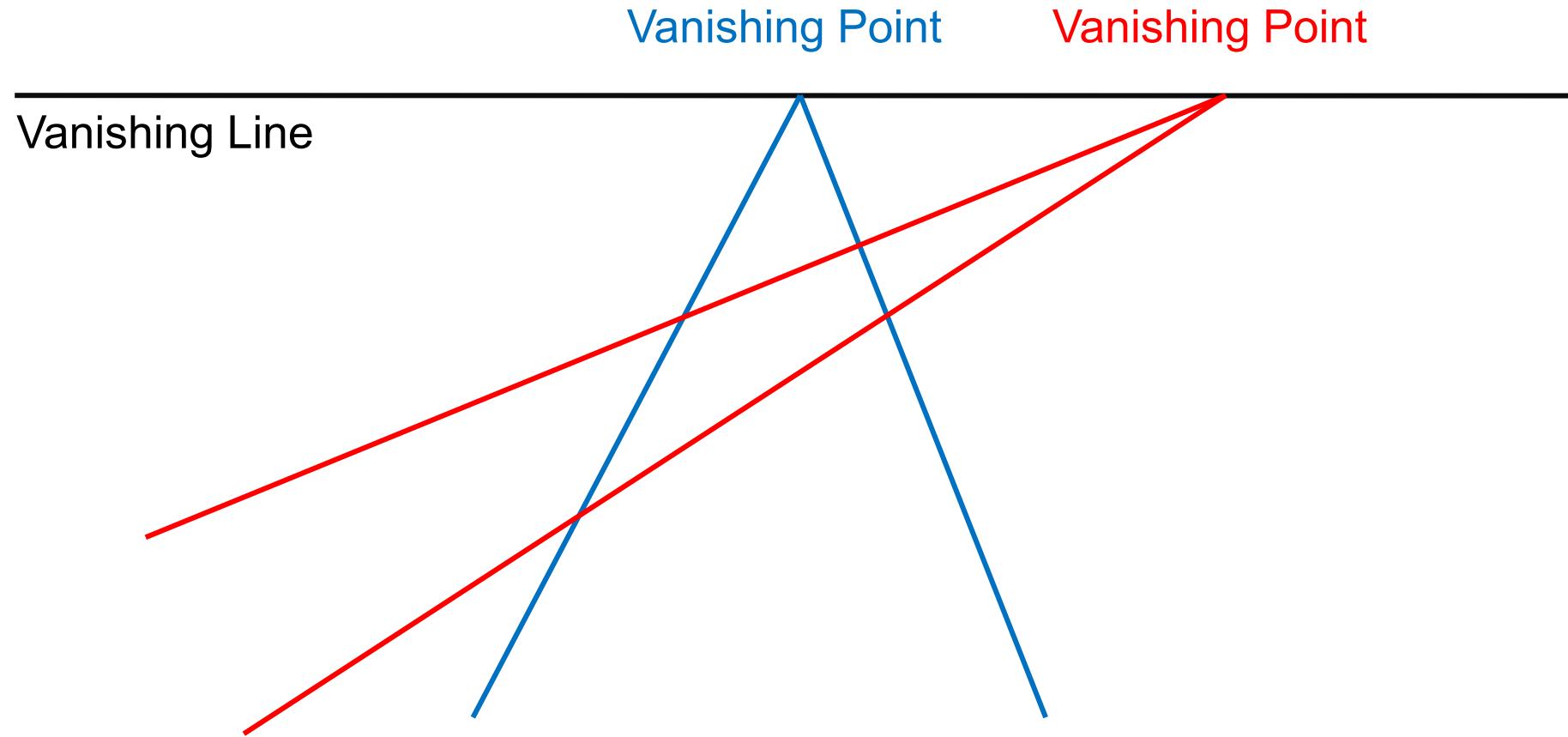
What is preserved?
Straight lines are
still straight

Vanishing points and lines



Parallel lines in the 3D world
intersect in the 2D image
at a “vanishing point”

Vanishing points and lines



Vanishing points and lines



Photo from Criminisi

Vanishing points and lines

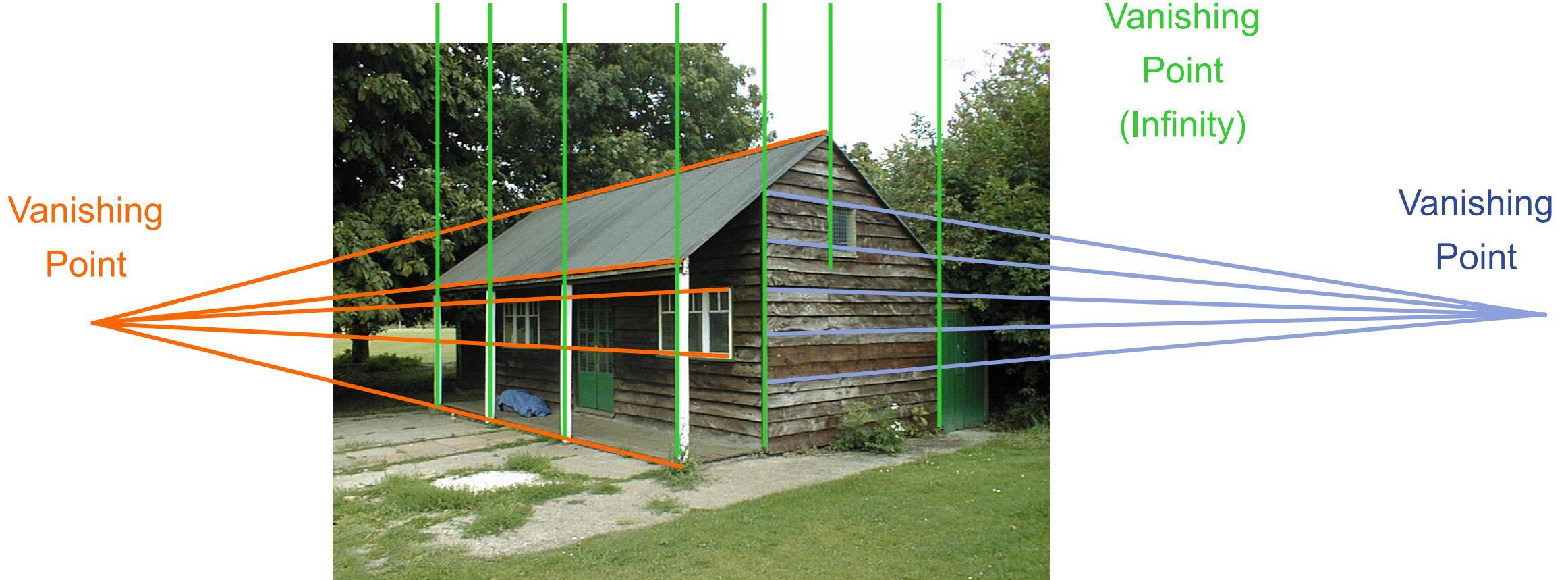
Vanishing
Point



Vanishing points and lines

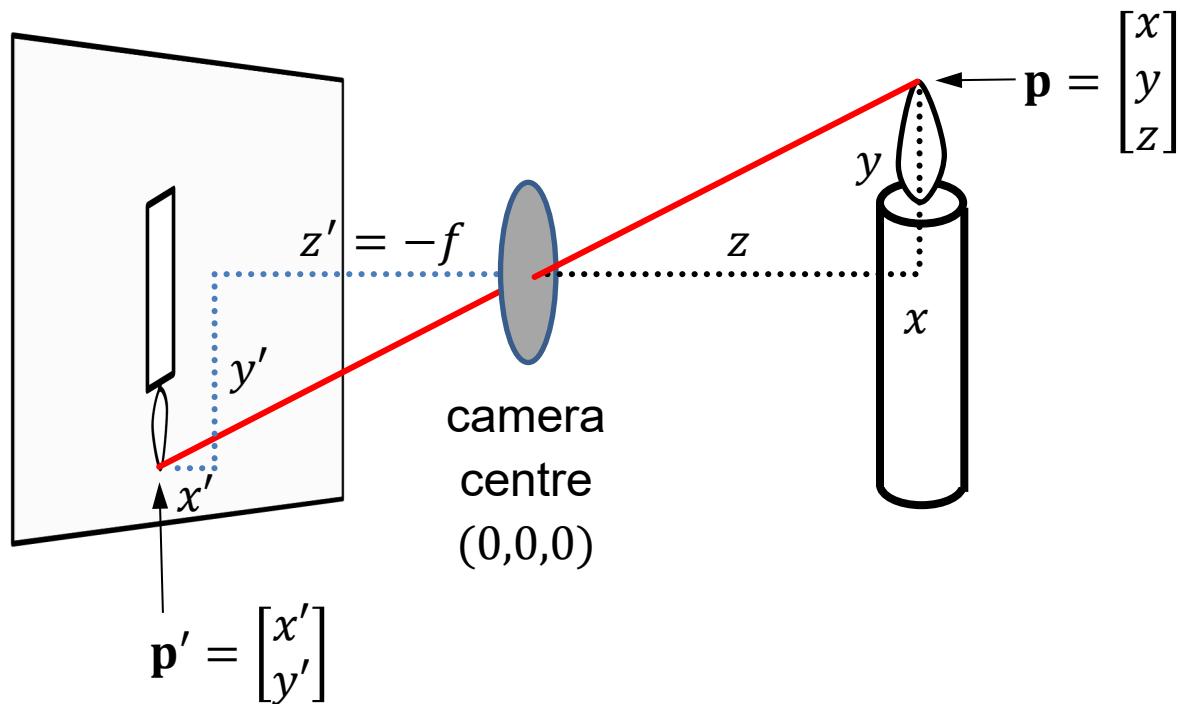


Vanishing points and lines



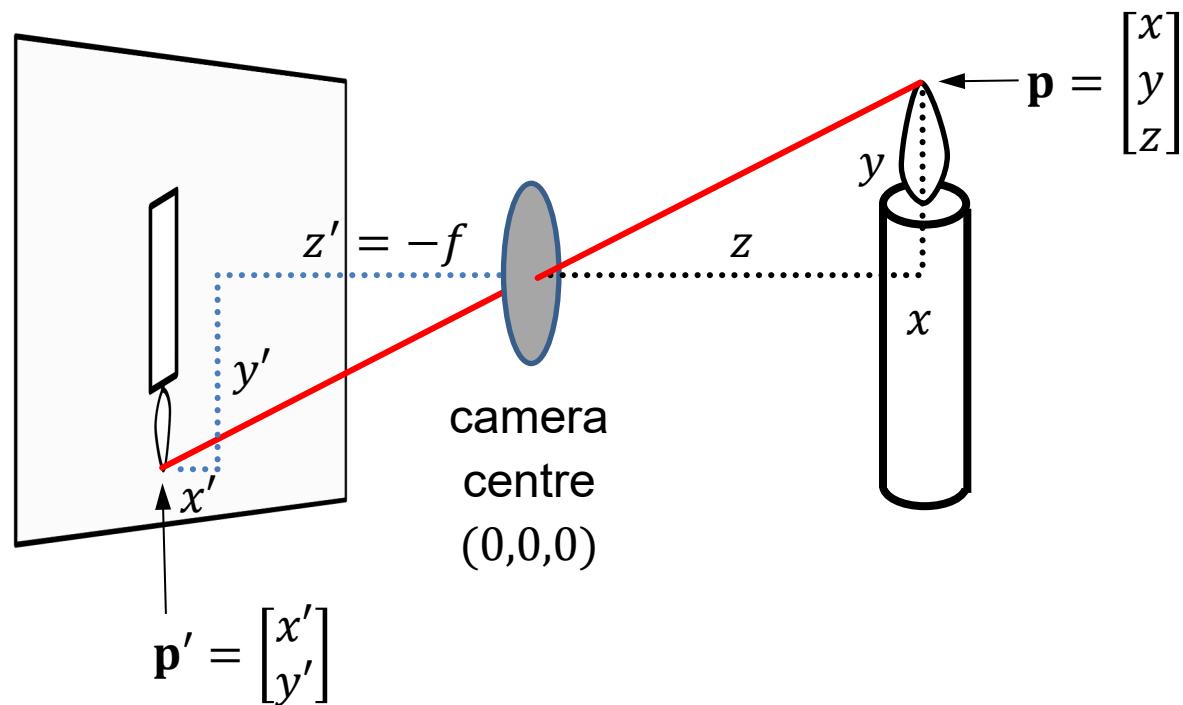
Projection mathematics

From **world coordinates** to **image coordinates**



Projection mathematics

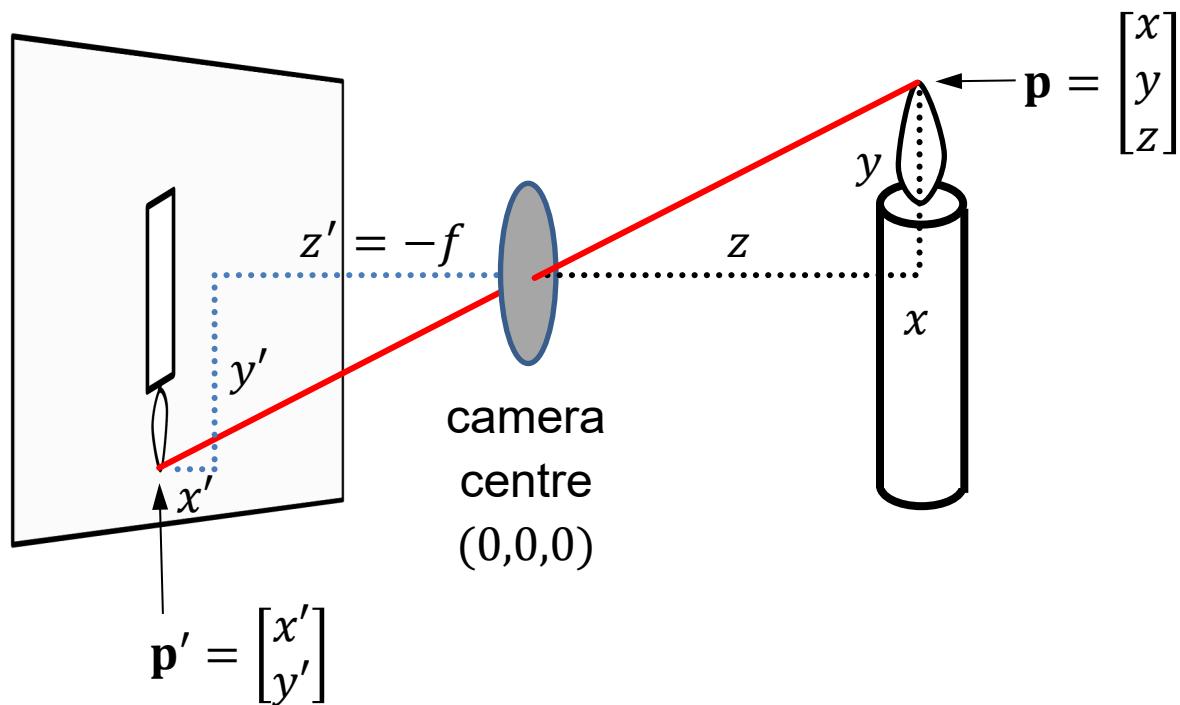
From **world coordinates** to **image coordinates**



If $x = 2, y = 3, z = 5$, and $f = 2$,
what are x' and y' ?

Projection mathematics

From **world coordinates** to **image coordinates**



If $x = 2, y = 3, z = 5$, and $f = 2$,
what are x' and y' ?

$$x' = -x \cdot \frac{f}{z} \quad \rightarrow \quad x' = -2 \cdot \frac{2}{5}$$
$$y' = -y \cdot \frac{f}{z} \quad \quad \quad y' = -3 \cdot \frac{2}{5}$$

Perspective projection

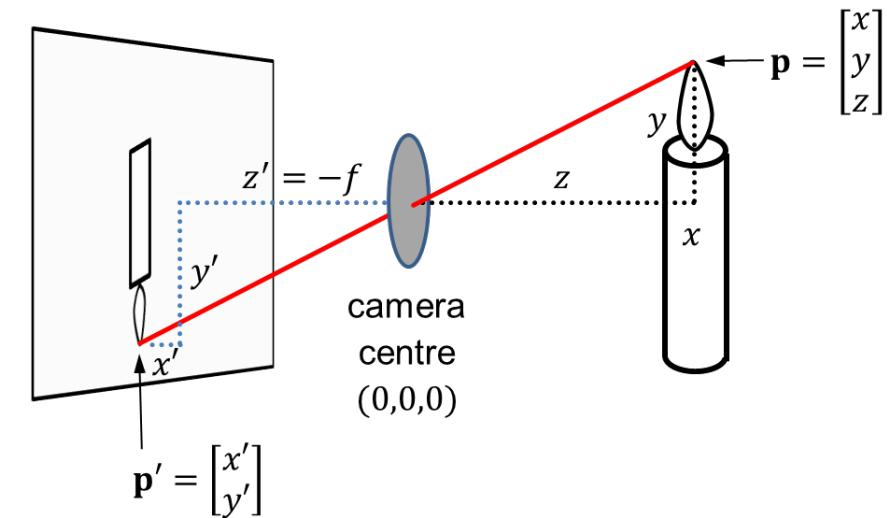
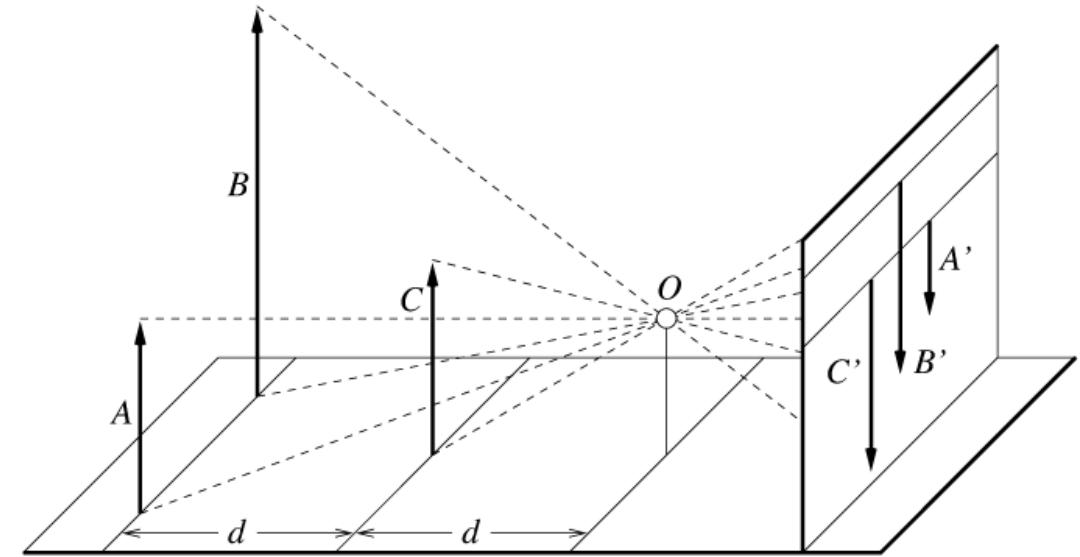
- Apparent size of object depends on its distance:
far objects appear smaller

- By similar triangles:

$$(x', y', z') = \left(-f \frac{x}{z}, -f \frac{y}{z}, -f \right)$$

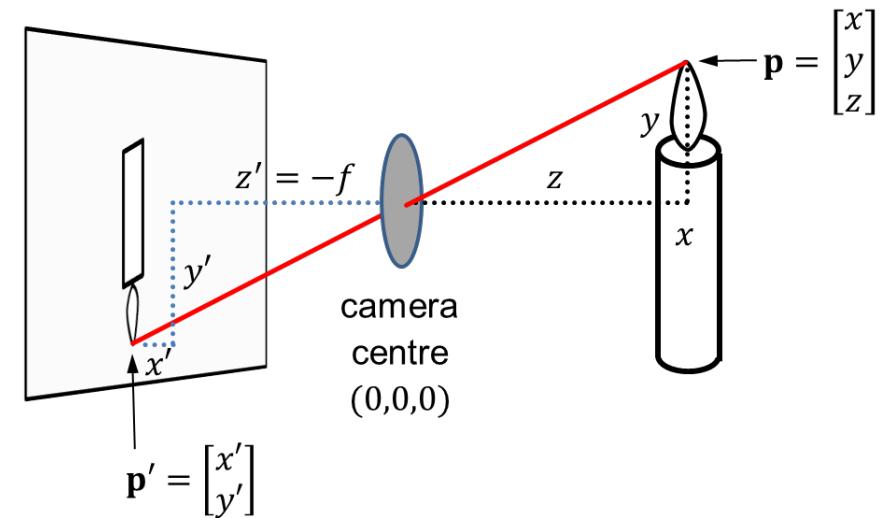
- Ignore third coordinate and mirror:

$$(x', y') = \left(f \frac{x}{z}, f \frac{y}{z} \right)$$



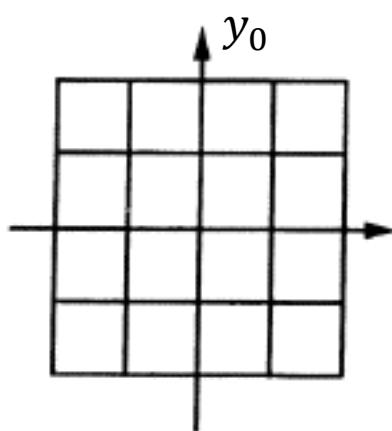
Affine projection

- Suitable when scene depth is small relative to average distance from camera
- Let magnification $m = f/z_0$ be a positive constant and all points in the scene have approximately constant distance z_0 to the camera
- Leads to weak perspective projection:
$$(x', y') = (mx, my)$$
- Orthographic projection when $m = 1$:
$$(x', y') = (x, y)$$

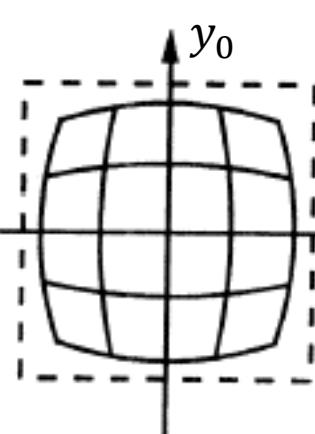


Beyond pinholes: radial distortions

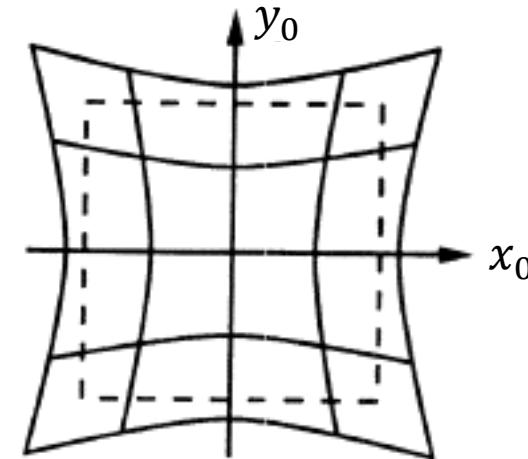
- Modern cameras use lenses instead of pinholes



No distortion



Barrel distortion



Pincushion distortion

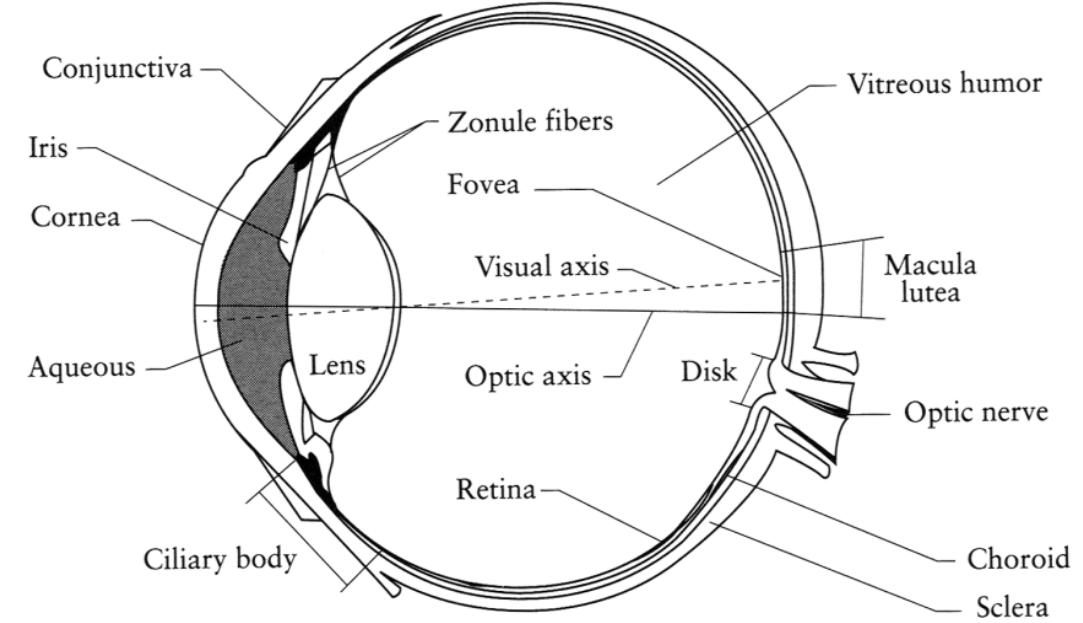


Image from Martin Habbecke

Barrel distortion corrected

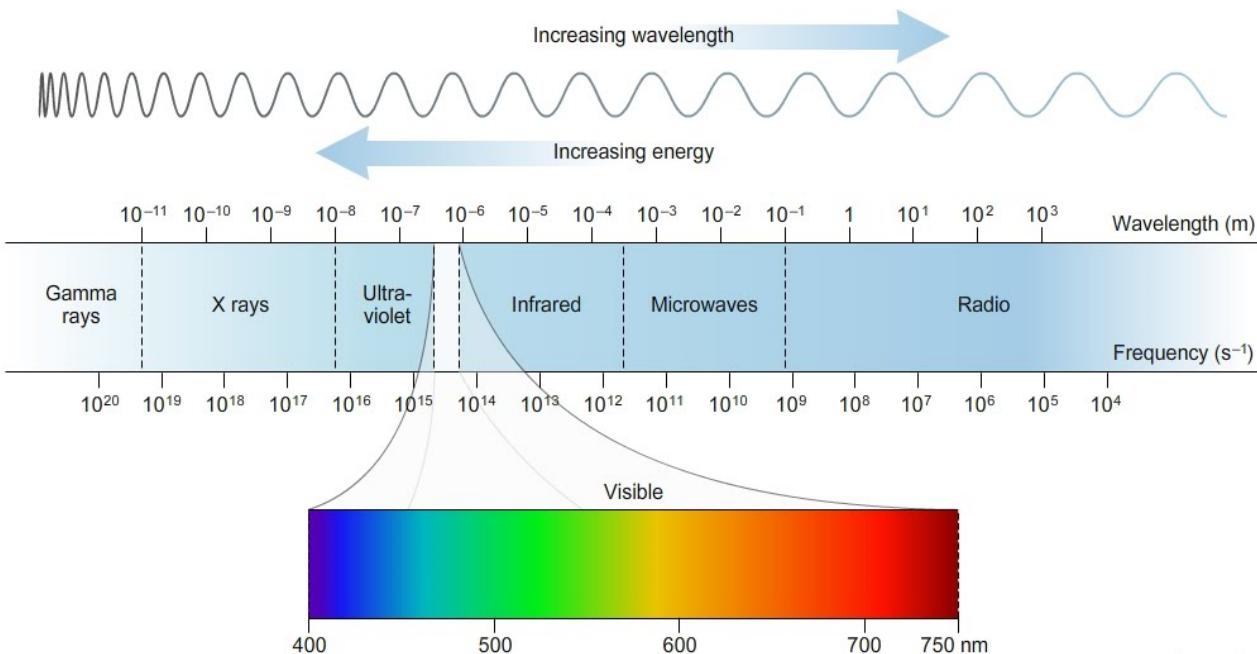
Comparing with human vision

- Cameras imitate the frequency response of the human eye so it is good to know something about it
- Computer vision probably would not get as much attention if biological vision (especially human vision) had not proven that it is possible to make important judgements from 2D images

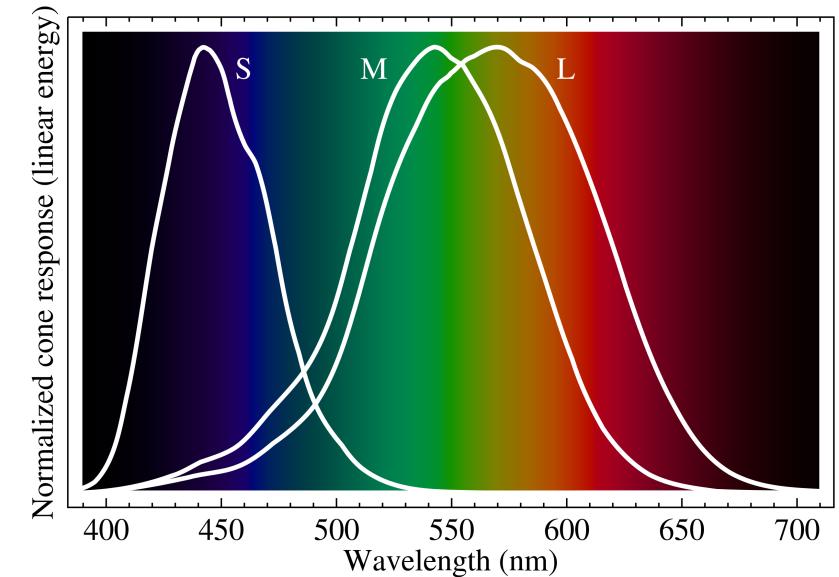


The Eye

Electromagnetic spectrum

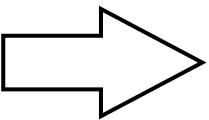


<https://sites.google.com/site/chempendix/em-spectrum>

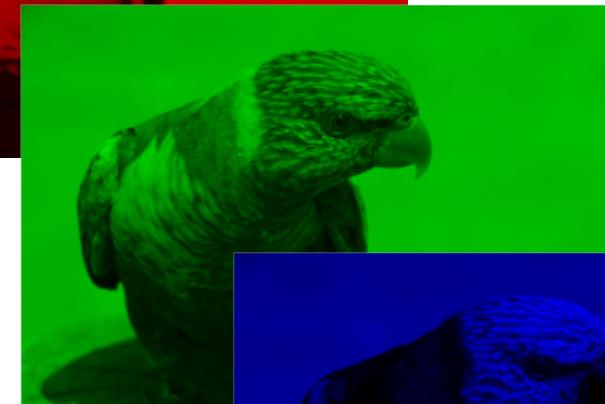


Normalized responsivity spectra of human cone cells (S, M, L types)

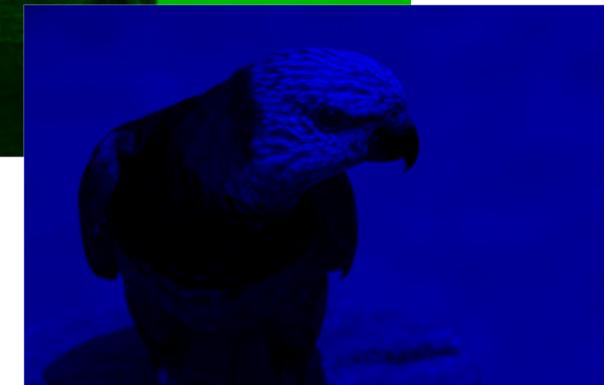
Colour represented by RGB images



Red



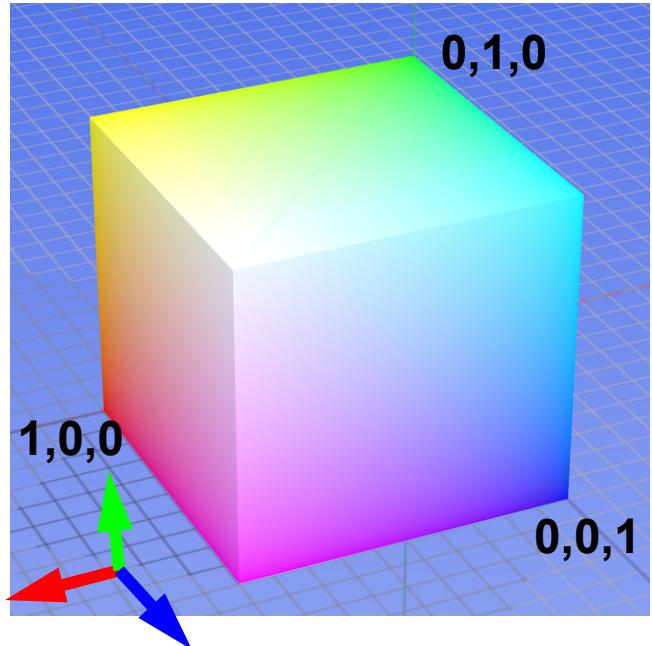
Green



Blue

Colour spaces: RGB

Default colour space in vision



[Source: Wikipedia](#)



R
(G=0,B=0)



G
(R=0,B=0)

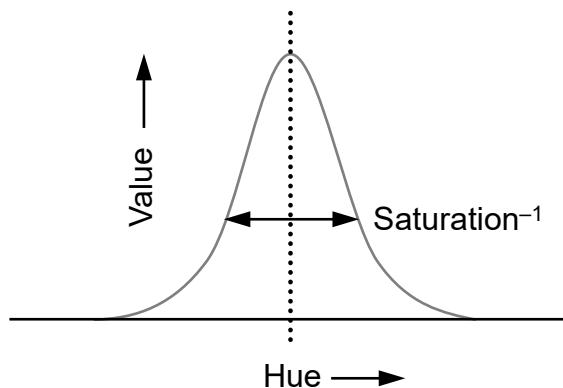
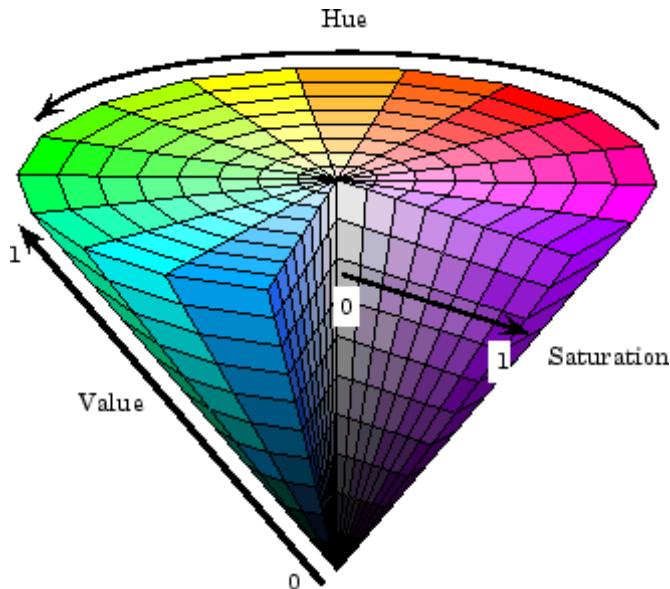


B
(R=0,G=0)

Drawback: strongly correlated channels

Colour spaces: HSV

Intuitive colour space, often used in colour selection and manipulation



H
(S=1,V=1)



S
(H=1,V=1)

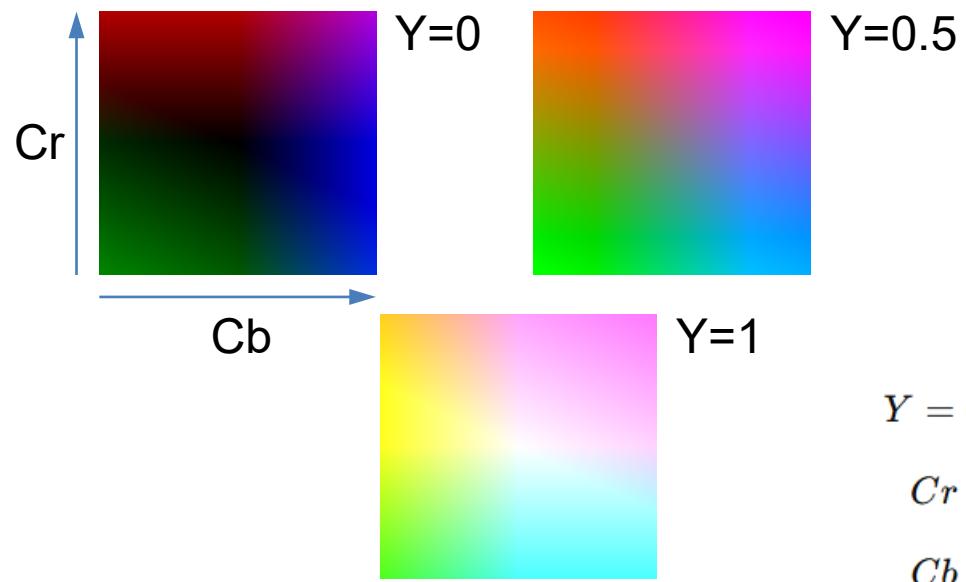


V
(H=1,S=0)

Drawback: confounded channels

Colour spaces: YCbCr

Fast to compute, good for compression (reduce the resolution for Cb/Cr), used by TV, JPEG



$$Y = 0.299R + 0.587G + 0.114B$$

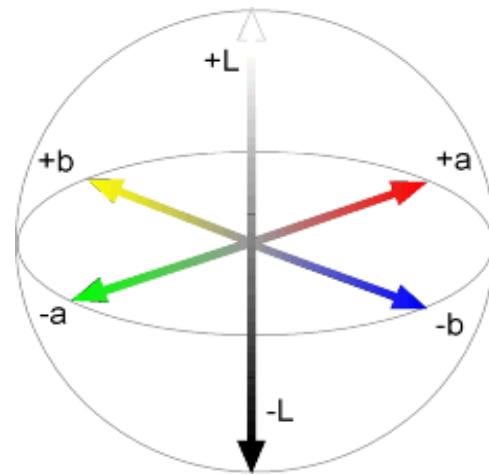
$$Cr = (R - Y) \times 0.713 + 128$$

$$Cb = (B - Y) \times 0.564 + 128$$



Colour spaces: L*a*b*

“Perceptually uniform” colour space, often used in Photoshop for image editing, and computer vision



a.k.a. CIELAB

Any numerical change corresponds to similar perceived change in color: Euclidean distances make sense



L
($a=0, b=0$)

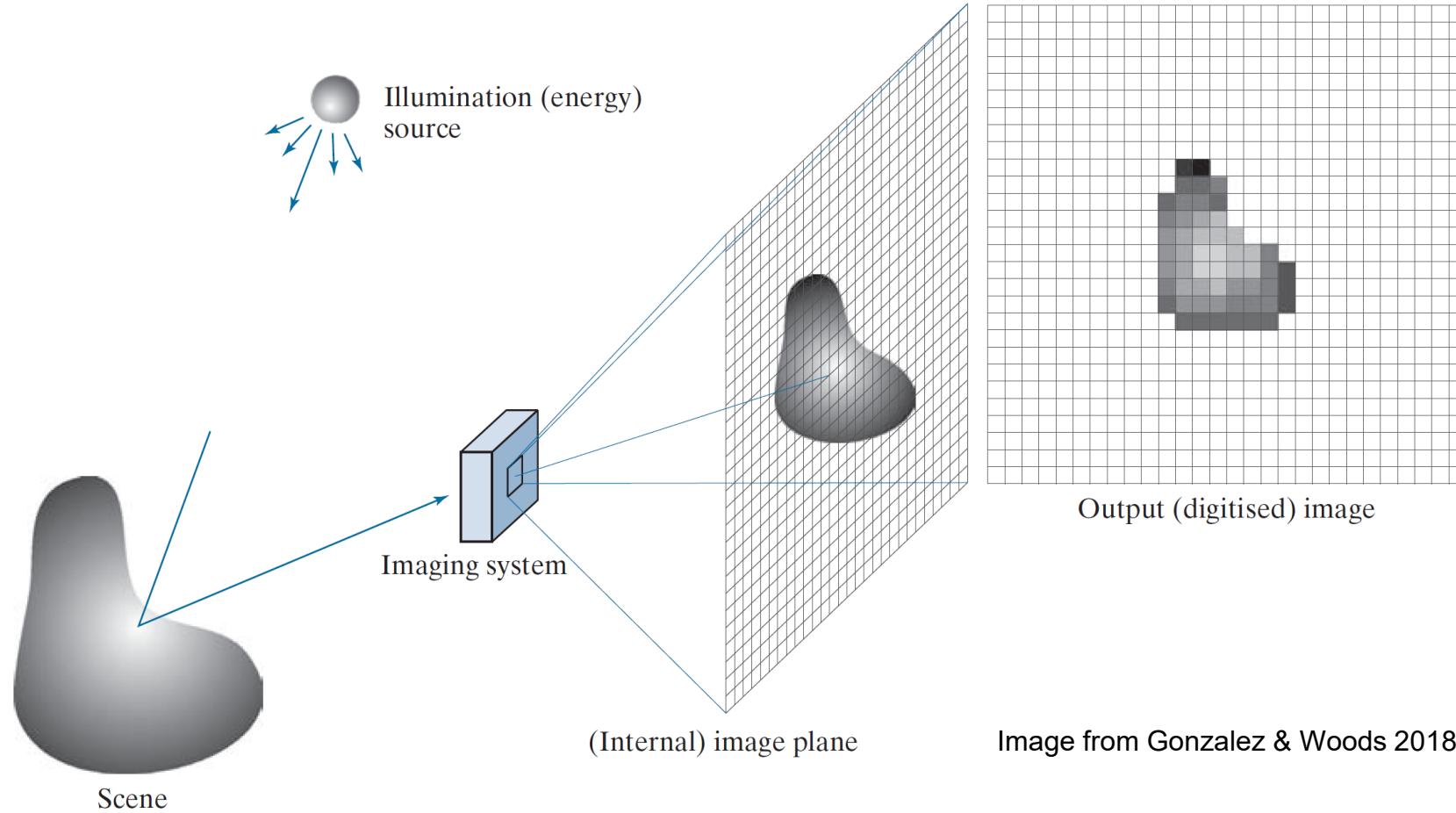


a
($L=65, b=0$)

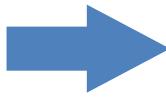
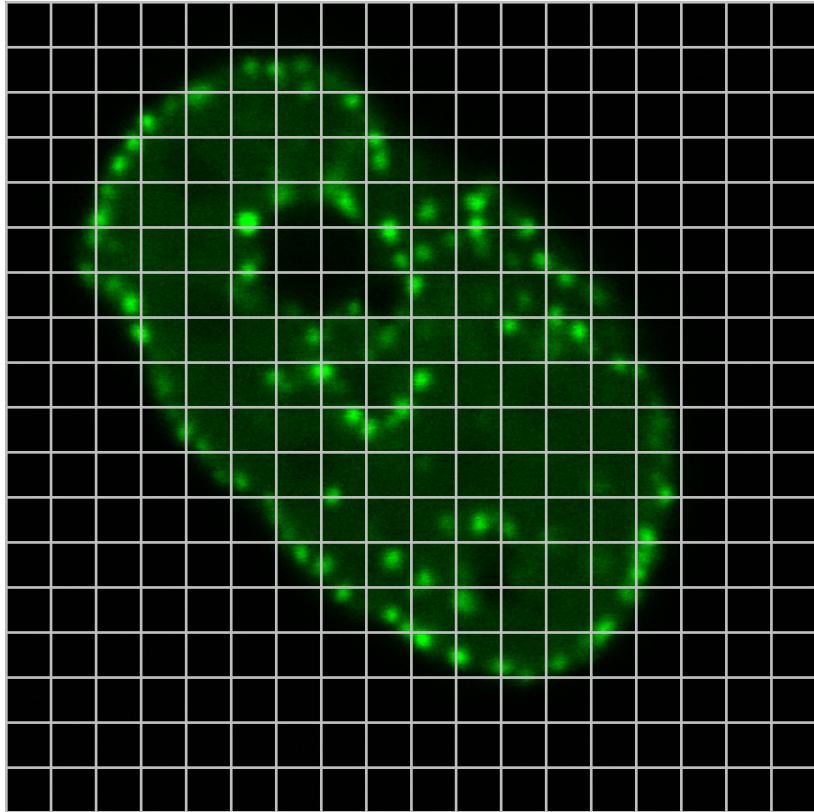


b
($L=65, a=0$)

Digital image formation



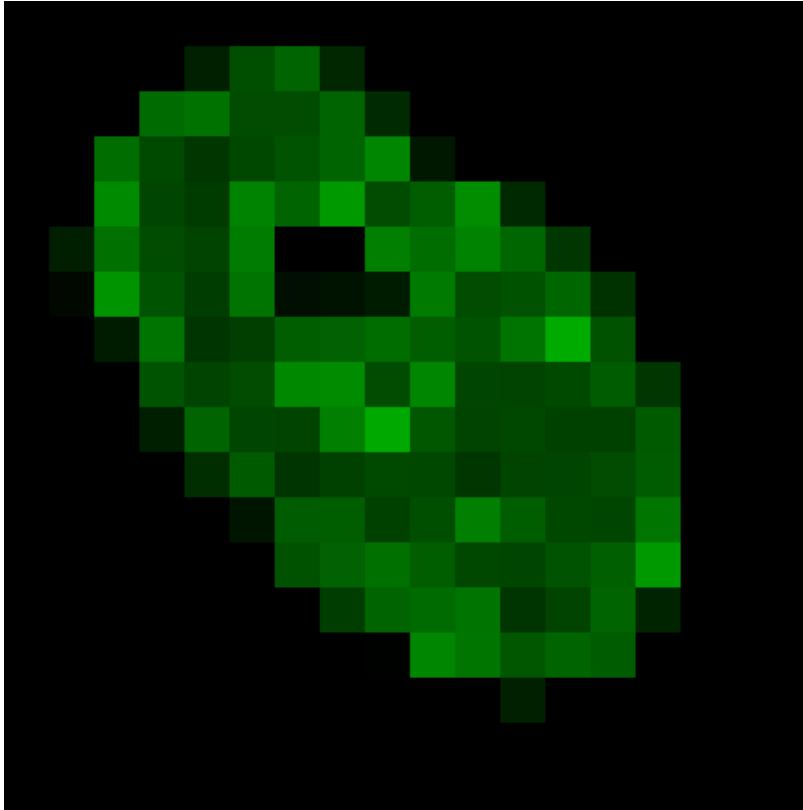
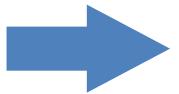
Digital image formation



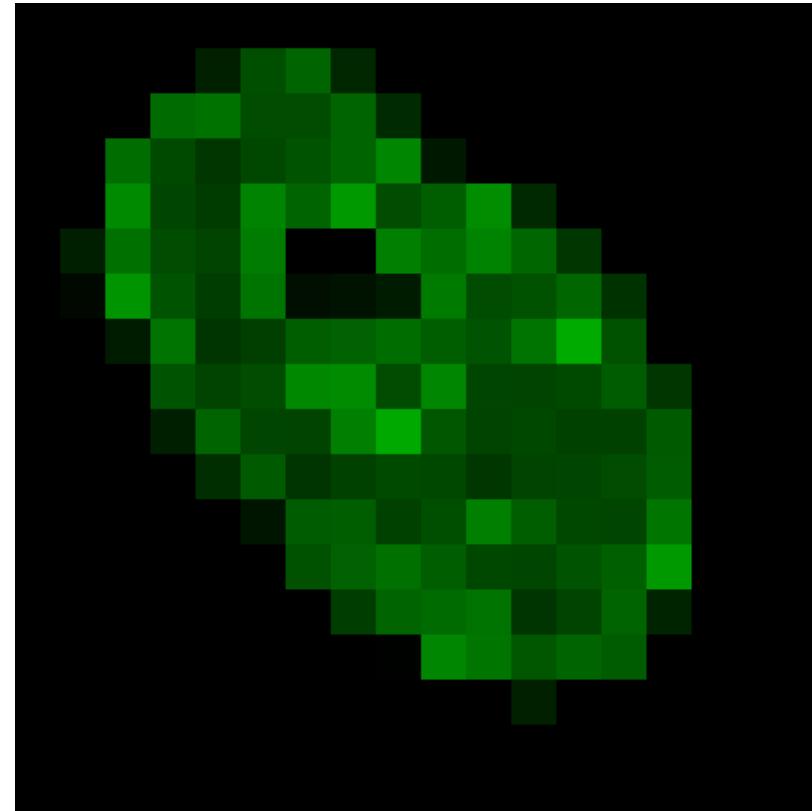
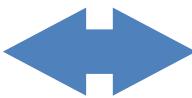
0	2	2	2	5	8	11	8	2	2	0	0	0	0	0	0	0	0	0	0
0	0	2	11	76	136	164	85	11	5	2	2	0	0	0	0	0	0	0	0
0	2	25	172	181	133	133	164	90	14	5	2	2	0	0	0	0	0	0	0
2	5	175	130	104	127	141	164	206	65	31	11	2	2	0	0	0	0	0	0
2	28	212	124	110	204	164	232	133	155	218	87	14	2	2	0	0	0	0	0
2	73	178	133	121	195	34	31	198	175	204	167	104	14	5	0	0	0	0	0
2	45	226	141	113	184	53	59	70	192	133	138	167	99	11	2	0	0	0	0
0	2	70	184	102	116	155	161	175	155	141	184	255	138	34	5	2	0	0	0
0	0	5	141	121	133	209	215	133	206	124	121	130	153	104	8	2	0	0	0
0	0	2	73	164	124	121	198	252	147	121	127	119	119	150	19	2	0	0	0
0	0	0	5	93	150	102	119	130	127	104	121	124	133	153	25	2	0	0	0
0	0	0	0	5	62	153	155	119	136	198	155	127	124	141	158	232	5	2	2
0	0	0	0	0	5	138	161	178	155	127	124	141	158	232	5	2	2	0	0
0	0	0	0	0	0	11	113	164	172	184	102	121	164	79	2	2	2	0	0
0	0	0	0	0	0	2	5	36	206	187	147	164	153	5	2	2	0	0	0
0	0	0	0	0	0	0	0	2	5	25	76	31	2	2	2	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Displaying a digital image

0	2	2	2	5	8	11	8	2	2	0	0	0	0	0	0	0	0	0	
0	0	2	11	76	136	164	85	11	5	2	2	0	0	0	0	0	0	0	
0	2	25	172	181	133	133	164	90	14	5	2	2	0	0	0	0	0	0	
2	5	175	130	104	127	141	164	206	65	31	11	2	2	0	0	0	0	0	
2	28	212	124	110	204	164	232	133	155	218	87	14	2	2	0	0	0	0	
2	73	178	133	121	195	34	31	198	175	204	167	104	14	5	0	0	0	0	
2	45	226	141	113	184	53	59	70	192	133	138	167	99	11	2	0	0	0	
0	2	70	184	102	116	155	161	175	155	141	184	255	138	34	5	2	0	0	
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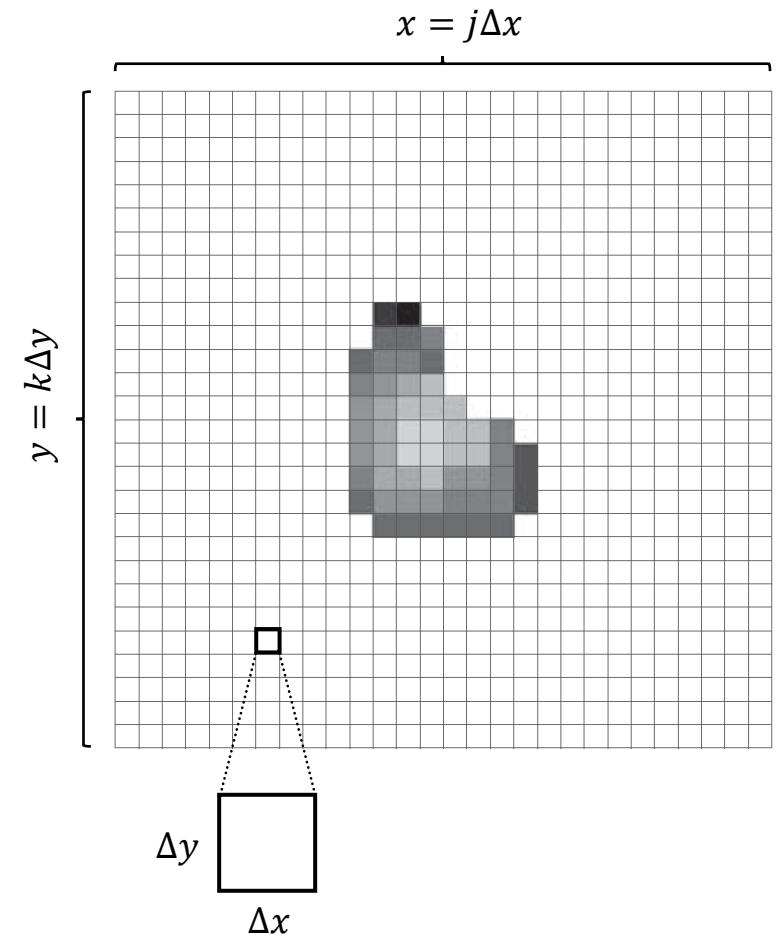


Comparing the original and digital image



Digitisation by spatial sampling

- Digitisation converts an analog image to a digital image by sampling the image space
- Sampling discretises the coordinates x and y
 - Spatial discretisation of a picture function $f(x, y)$
 - Typically a rectangular grid of sampling points is used
 - $x = j\Delta x, y = k\Delta y$ for $j = 0 \dots M - 1, k = 0 \dots N - 1$
 - The Δx and Δy are called the sampling intervals



Digital colour images

Each channel is a digital image with the same number of rows and columns

row	column →												R						G						B										
↓																																			
0.92	0.93	0.94	0.97	0.62	0.37	0.85	0.97	0.93	0.92	0.99	0.37	0.85	0.97	0.93	0.92	0.99	0.37	0.80	0.88	0.89	0.79	0.85	0.37	0.85	0.97	0.93	0.92	0.99							
0.95	0.89	0.82	0.89	0.56	0.31	0.75	0.92	0.81	0.95	0.91	0.31	0.75	0.92	0.81	0.95	0.91	0.31	0.58	0.60	0.50	0.60	0.58	0.50	0.61	0.45	0.33	0.31	0.75	0.92	0.81	0.95	0.91			
0.89	0.72	0.51	0.55	0.51	0.42	0.57	0.41	0.49	0.91	0.92	0.42	0.57	0.41	0.49	0.91	0.92	0.42	0.57	0.41	0.49	0.91	0.92	0.42	0.57	0.41	0.49	0.91	0.92							
0.96	0.95	0.88	0.94	0.56	0.46	0.91	0.87	0.90	0.97	0.95	0.46	0.91	0.87	0.90	0.97	0.95	0.46	0.91	0.87	0.90	0.97	0.95	0.46	0.91	0.87	0.90	0.97	0.95							
0.71	0.81	0.81	0.87	0.57	0.37	0.80	0.88	0.89	0.79	0.85	0.37	0.85	0.97	0.93	0.92	0.99	0.37	0.80	0.88	0.89	0.79	0.85	0.37	0.85	0.97	0.93	0.92	0.99							
0.49	0.62	0.60	0.58	0.50	0.60	0.58	0.50	0.61	0.45	0.33	0.31	0.75	0.92	0.81	0.95	0.91	0.31	0.58	0.60	0.50	0.61	0.45	0.33	0.31	0.75	0.92	0.81	0.95	0.91						
0.86	0.84	0.74	0.58	0.51	0.39	0.73	0.92	0.91	0.49	0.74	0.42	0.57	0.41	0.49	0.91	0.92	0.42	0.57	0.41	0.49	0.91	0.92	0.42	0.57	0.41	0.49	0.91	0.92							
0.96	0.67	0.54	0.85	0.48	0.37	0.88	0.90	0.94	0.82	0.93	0.46	0.91	0.87	0.90	0.97	0.95	0.46	0.91	0.87	0.90	0.97	0.95	0.46	0.91	0.87	0.90	0.97	0.95							
0.69	0.49	0.56	0.66	0.43	0.42	0.77	0.73	0.71	0.90	0.99	0.37	0.80	0.88	0.89	0.79	0.85	0.37	0.80	0.88	0.89	0.79	0.85	0.37	0.85	0.97	0.93	0.92	0.99							
0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97	0.60	0.58	0.50	0.61	0.45	0.33	0.31	0.75	0.92	0.81	0.95	0.91	0.60	0.58	0.50	0.61	0.45	0.33							
0.91	0.94	0.89	0.49	0.41	0.78	0.78	0.77	0.89	0.99	0.93	0.39	0.73	0.92	0.91	0.49	0.74	0.42	0.57	0.41	0.49	0.91	0.92	0.42	0.57	0.41	0.49	0.91	0.92							
						0.96	0.67	0.54	0.85	0.48	0.37	0.88	0.90	0.94	0.82	0.93	0.46	0.91	0.87	0.90	0.97	0.95	0.46	0.91	0.87	0.90	0.97	0.95							
						0.69	0.49	0.56	0.66	0.43	0.42	0.77	0.73	0.71	0.90	0.99	0.37	0.80	0.88	0.89	0.79	0.85	0.37	0.85	0.97	0.93	0.92	0.99							
						0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97	0.60	0.58	0.50	0.61	0.45	0.33	0.60	0.58	0.50	0.61	0.45	0.33							
						0.91	0.94	0.89	0.49	0.41	0.78	0.78	0.77	0.89	0.99	0.93	0.39	0.73	0.92	0.91	0.49	0.74	0.42	0.57	0.41	0.49	0.91	0.92							
							0.96	0.67	0.54	0.85	0.48	0.37	0.88	0.90	0.94	0.82	0.93	0.46	0.91	0.87	0.90	0.97	0.95	0.46	0.91	0.87	0.90	0.97	0.95						
							0.69	0.49	0.56	0.66	0.43	0.42	0.77	0.73	0.71	0.90	0.99	0.37	0.80	0.88	0.89	0.79	0.85	0.37	0.85	0.97	0.93	0.92	0.99						
							0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97	0.60	0.58	0.50	0.61	0.45	0.33	0.60	0.58	0.50	0.61	0.45	0.33						
							0.91	0.94	0.89	0.49	0.41	0.78	0.78	0.77	0.89	0.99	0.93	0.39	0.73	0.92	0.91	0.49	0.74	0.42	0.57	0.41	0.49	0.91	0.92						
								0.96	0.67	0.54	0.85	0.48	0.37	0.88	0.90	0.94	0.82	0.93	0.46	0.91	0.87	0.90	0.97	0.95	0.46	0.91	0.87	0.90	0.97	0.95					
								0.69	0.49	0.56	0.66	0.43	0.42	0.77	0.73	0.71	0.90	0.99	0.37	0.80	0.88	0.89	0.79	0.85	0.37	0.85	0.97	0.93	0.92	0.99					
								0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97	0.60	0.58	0.50	0.61	0.45	0.33	0.60	0.58	0.50	0.61	0.45	0.33					
								0.91	0.94	0.89	0.49	0.41	0.78	0.78	0.77	0.89	0.99	0.93	0.39	0.73	0.92	0.91	0.49	0.74	0.42	0.57	0.41	0.49	0.91	0.92					
									0.96	0.67	0.54	0.85	0.48	0.37	0.88	0.88	0.90	0.94	0.82	0.93	0.46	0.91	0.87	0.90	0.97	0.95	0.46	0.91	0.87	0.90	0.97	0.95			
									0.69	0.49	0.56	0.66	0.43	0.42	0.77	0.77	0.73	0.71	0.90	0.99	0.37	0.80	0.88	0.89	0.79	0.85	0.37	0.85	0.97	0.93	0.92	0.99			
									0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97	0.60	0.58	0.50	0.61	0.45	0.33	0.60	0.58	0.50	0.61	0.45	0.33				
									0.91	0.94	0.89	0.49	0.41	0.78	0.78	0.77	0.89	0.99	0.93	0.39	0.73	0.92	0.91	0.49	0.74	0.42	0.57	0.41	0.49	0.91	0.92				
										0.96	0.67	0.54	0.85	0.48	0.37	0.88	0.88	0.88	0.90	0.94	0.82	0.93	0.46	0.91	0.87	0.88	0.90	0.97	0.95	0.46	0.91	0.87	0.88	0.90	0.97
										0.69	0.49	0.56	0.66	0.43	0.42	0.77	0.77	0.73	0.71	0.71	0.90	0.99	0.37	0.80	0.88	0.89	0.79	0.85	0.37	0.85	0.97	0.93	0.92	0.99	
										0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.73	0.73	0.90	0.99	0.37	0.80	0.88	0.89	0.79	0.85	0.37	0.85	0.97	0.93	0.92	0.99	
										0.91	0.94	0.89	0.49	0.41	0.78	0.78	0.77	0.78	0.78	0.78	0.78	0.90	0.99	0.37	0.80	0.88	0.89	0.79	0.85	0.37	0.85	0.97	0.93	0.92	0.99

Spatial resolution

- Spatial resolution: number of pixels per unit of length
- Example: resolution decreases by one half each time (see right)
- Human faces can be recognized at 64×64 pixels per face
- Appropriate resolution is essential:
 - Too little resolution yields poor recognition
 - Too much resolution is slow and wastes memory

1/2



1/1



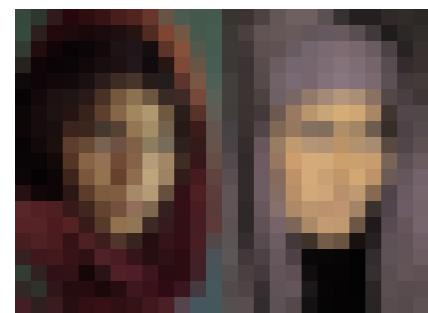
1/8



1/4



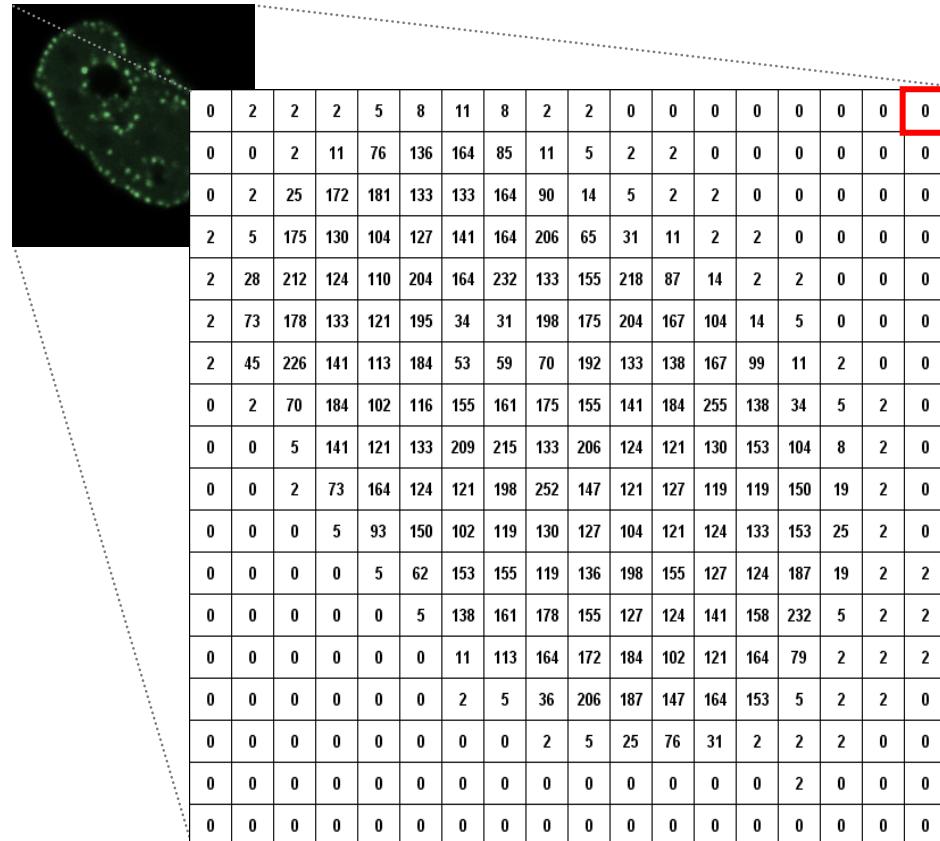
1/16



Quantisation

- Quantisation digitises the image intensity or amplitude values $f(x, y)$
 - Called intensity or gray-level quantisation
 - Gray-level resolution to be chosen per application
 - ❖ For example, 16, 32, 64,, 128, 256 levels
 - ❖ Should be high enough for human perception of shading details
 - ❖ The latter requires about 100 levels for a realistic image
 - ❖ Should not be higher than necessary to avoid wasting storage

Quantisation and bits per pixel



→ **Pixel** (picture element)

Levels per pixel:

$$8 \text{ bits} = 2^8 = 256$$

$$12 \text{ bits} = 2^{12} = 4,096$$

$$16 \text{ bits} = 2^{16} = 65,536$$

$$24 \text{ bits} = 2^{24} = 16,777,216$$

Further reading on discussed topics

- Chapter 2 of Szeliski
- Chapter 2 of Shapiro and Stockman

Acknowledgements

- Several slides from Derek Hoiem, Alexei Efros, Steve Seitz, and David Forsyth
- Some material drawn from referenced and associated online sources
- Image sources credited where possible

Example exam question

Which one of the following statements about colour spaces is incorrect?

- A. The R, G, and B channels of the RGB colour space are often correlated.
- B. The H and the S channel of the HSV colour space are confounded.
- C. The Y channel of the YCbCr colour space represents the brightness.
- D. The a^* channel of the $L^*a^*b^*$ colour space is the green-blue component.

Please note that in the final exam, no materials may be taken into the room, and it is not allowed (and in fact not possible) to consult the internet. So make sure you know the course materials well.