Bacteria Evolution

A wonderful property of modern life is that we have anti-biotics to kill bacterial infections. However, we only have a fixed number of anti-biotic medicines, and bacteria are evolving to become resistent to our anti-biotics. In this example we are going to use probability to understand evolution of anti-biotic resistence in bacteria.

Imagine you have a population of 1 million infectious bacteria in your gut, 10% of which have a mutation that makes them slightly more resistant to anti-biotics. You take a course of anti-biotics. The probability that bacteria with the mutation survives is 20%. The probability that bacteria without the mutation survives is 1%.

What is the probability that a randomly chosen bacterium survives the anti-biotics?

Let E be the event that our bacterium survives. Let M be the event that a bacteria has the mutation. By the Law of Total Probability (LOTP):

$$egin{aligned} \mathrm{P}(E) &= \mathrm{P}(E \, \mathrm{and} \, M) + \mathrm{P}(E \, \mathrm{and} \, M^{\,\mathrm{C}}) & \mathrm{LOTP} \ &= \mathrm{P}(E|M) \, \mathrm{P}(M) + \mathrm{P}(E|M^{\,\mathrm{C}}) \, \mathrm{P}(M^{\,\mathrm{C}}) & \mathrm{Chain \, Rule} \ &= 0.20 \cdot 0.10 + 0.01 \cdot 0.90 & \mathrm{Substituting} \ &= 0.029 \end{aligned}$$

What is the probability that a surviving bacterium has the mutation?

Using the same events in the last section, this question is asking for P(M|E). We aren't giving the conditional probability in that direction, instead we know P(E|M). Such situations call for <u>Bayes'</u> Theorem:

$$ext{P}(M|E) = rac{ ext{P}(E|M) ext{P}(M)}{ ext{P}(E)} \hspace{1cm} ext{Bayes}$$
 $= rac{0.20 \cdot 0.10}{ ext{P}(E)} \hspace{1cm} ext{Given}$ $= rac{0.20 \cdot 0.10}{0.029} \hspace{1cm} ext{Calculated}$ $pprox 0.69$

After the course of anti-biotics, 69% of bacteria have the mutation, up from 10% before. If this population is allowed to reproduce you will have a much more resistent set of bacteria!