Multinomial

The multinomial is an example of a *parametric* distribution for multiple random variables.

Say you perform n independent trials of an experiment where each trial results in one of m outcomes, with respective probabilities: $p_1, p_2, \dots, p_m$ (constrained so that $\sum_i p_i = 1$). Define X_i to be the number of trials with outcome i. A multinomial distribution is a closed form function that answers the question: What is the probability that there are c_i trials with outcome i. Mathematically:

$$P(X_1 = c_1, X_2 = c_2, \dots, X_m = c_m) = \binom{n}{c_1, c_2, \dots, c_m} \cdot p_1^{c_1} \cdot p_2^{c_2} \dots p_m^{c_m}$$

Often people will use the product notation to write the exact same equation:

$$P(X_1 = c_1, X_2 = c_2, \dots, X_m = c_m) = \binom{n}{c_1, c_2, \dots, c_m} \cdot \prod_i p_i^{c_i}$$

Example: A 6-sided die is rolled 7 times. What is the probability that you roll: 1 one, 1 two, 0 threes, 2 fours, 0 fives, 3 sixes (disregarding order).

$$\begin{aligned}
P(X_1 = 1, X_2 = 1, X_3 = 0, X_4 = 2, X_5 = 0, X_6 = 3) \\
&= \frac{7!}{2!3!}\left(\frac{1}{6}\right)^1\left(\frac{1}{6}\right)^1\left(\frac{1}{6}\right)^0\left(\frac{1}{6}\right)^2\left(\frac{1}{6}\right)^0\left(\frac{1}{6}\right)^3 \\
&= 420\left(\frac{1}{6}\right)^7
\end{aligned}$$

The multinomial is especially popular because of its use as a model of language. For a full example see the Federalist Paper Authorship example.