Probability of **and**

The probability of the ***and*** of two events, say E and F, written $\text{P}(E \text{ and } F)$, is the probability of both events happening. You might see equivalent notations $\text{P}(EF)$, $\text{P}(E \cap F)$ and $\text{P}(E, F)$ to mean the probability of and. How you calculate the probability of event E and event F happening depends on whether or not the events are "independent". In the same way that mutual exclusion makes it easy to calculate the probability of the ***or*** of events, independence is a property that makes it easy to calculate the probability of the ***and*** of events.

And with Independent Events

If events are **independent** then calculating the probability of ***and*** becomes simple multiplication:

Definition: Probability of ***and*** for independent events.
If two events: E, F are independent then the probability of E ***and*** F occurring is:

$$\text{P}(E \text{ and } F) = \text{P}(E) \cdot \text{P}(F)$$

This property applies regardless of how the probabilities of E and F were calculated and whether or not the events are mutually exclusive.

The independence principle extends to more than two events. For n events $E_1, E_2, \ldots E_n$ that are ***mutually*** independent of one another -- the independence equation also holds for all subsets of the events.

$$\text{P}(E_1 \text{ and } E_2 \text{ and } \ldots \text{ and } E_n) = \prod_{i=1}^{n} \text{P}(E_i)$$

We can prove this equation by combining the definition of conditional probability and the definition of independence.

Proof: If E is independent of F then $\text{P}(E \text{ and } F) = \text{P}(E) \cdot \text{P}(F)$

$$\begin{aligned}
\text{P}(E|F) &= \frac{\text{P}(E \text{ and } F)}{\text{P}(F)} && \text{Definition of conditional probability} \\
\text{P}(E) &= \frac{\text{P}(E \text{ and } F)}{\text{P}(F)} && \text{Definition of independence} \\
\text{P}(E \text{ and } F) &= \text{P}(E) \cdot \text{P}(F) && \text{Rearranging terms}
\end{aligned}$$

See the chapter on independence to learn about when you can assume that two events are independent

And with Dependent Events

Events which are not independent are called ***dependent*** events. How can you calculate the probability of the **and** of dependent events? If your events are mutually exclusive you might be able to use a technique called DeMorgan's law, which we cover in a later chapter. For the probability of and in dependent events there is a direct formula called the chain rule which can be directly derived from the definition of conditional probability:

Definition: The chain rule.

The formula in the definition of conditional probability can be re-arranged to derive a general way of calculating the probability of the ***and*** of any two events:

$$\mathrm{P}(E \text{ and } F) = \mathrm{P}(E|F) \cdot \mathrm{P}(F)$$

Of course there is nothing special about E that says it should go first. Equivalently:

$$\mathrm{P}(E \text{ and } F) = \mathrm{P}(F \text{ and } E) = \mathrm{P}(F|E) \cdot \mathrm{P}(E)$$

We call this formula the "chain rule." Intuitively it states that the probability of observing events E ***and*** F is the probability of observing F, multiplied by the probability of observing E, given that you have observed F. It generalizes to more than two events:

$$\begin{aligned}\mathrm{P}(E_1 \text{ and } E_2 \text{ and} \ldots \text{and } E_n) = &\mathrm{P}(E_1) \cdot \mathrm{P}(E_2|E_1) \cdot \mathrm{P}(E_3|E_1 \text{ and } E_2) \ldots \\ &\mathrm{P}(E_n|E_1 \ldots E_{n-1})\end{aligned}$$