Bacteria Evolution

A wonderful property of modern life is that we have anti-biotics to kill bacterial infections. However, we only have a fixed number of anti-biotic medicines, and bacteria are evolving to become resistent to our anti-biotics. In this example we are going to use probability to understand evolution of anti-biotic resistence in bacteria.

Imagine you have a population of 1 million infectious bacteria in your gut, 10% of which have a mutation that makes them slightly more resistant to anti-biotics. You take a course of anti-biotics. The probability that bacteria with the mutation survives is 20%. The probability that bacteria without the mutation survives is 1%.

What is the probability that a randomly chosen bacterium survives the anti-biotics?

Let E be the event that our bacterium survives. Let M be the event that a bacteria has the mutation. By the Law of Total Probability (LOTP):

$$\begin{aligned}
\mathrm{P}(E) &= \mathrm{P}(E \text{ and } M) + \mathrm{P}(E \text{ and } M^{\mathrm{C}}) && \text{LOTP} \\
&= \mathrm{P}(E|M)\,\mathrm{P}(M) + \mathrm{P}(E|M^{\mathrm{C}})\,\mathrm{P}(M^{\mathrm{C}}) && \text{Chain Rule} \\
&= 0.20 \cdot 0.10 + 0.01 \cdot 0.90 && \text{Substituting} \\
&= 0.029
\end{aligned}$$

What is the probability that a surviving bacterium has the mutation?

Using the same events in the last section, this question is asking for $\mathrm{P}(M|E)$. We aren't giving the conditional probability in that direction, instead we know $P(E|M)$. Such situations call for Bayes' Theorem:

$$\begin{aligned}
\mathrm{P}(M|E) &= \frac{\mathrm{P}(E|M)\,\mathrm{P}(M)}{\mathrm{P}(E)} && \text{Bayes} \\
&= \frac{0.20 \cdot 0.10}{\mathrm{P}(E)} && \text{Given} \\
&= \frac{0.20 \cdot 0.10}{0.029} && \text{Calculated} \\
&\approx 0.69
\end{aligned}$$

After the course of anti-biotics, 69% of bacteria have the mutation, up from 10% before. If this population is allowed to reproduce you will have a much more resistent set of bacteria!