

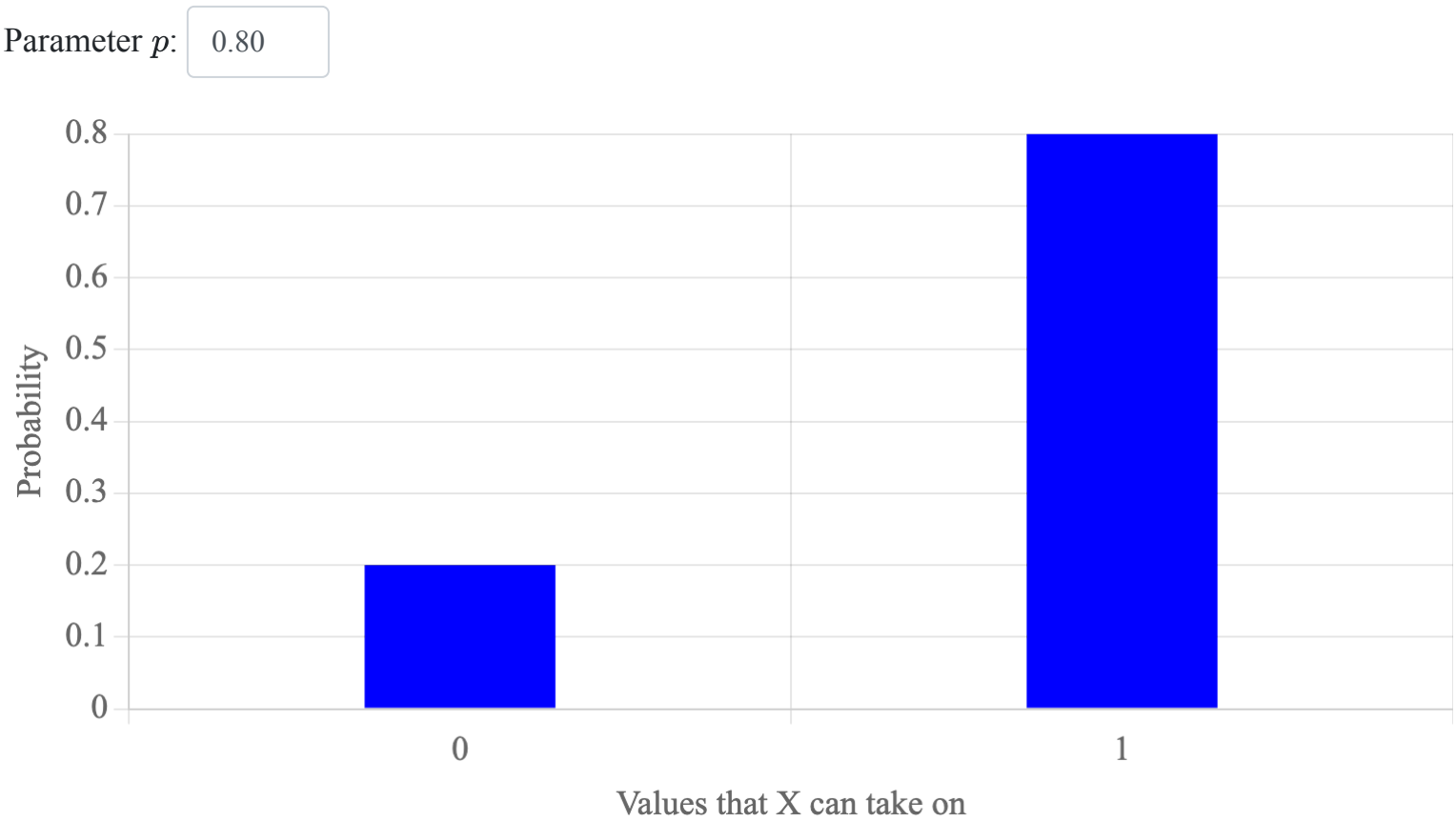
# Random Variable Reference

## Discrete Random Variables

### Bernoulli Random Variable

- Notation:**  $X \sim \text{Bern}(p)$
- Description:** A boolean variable that is 1 with probability  $p$
- Parameters:**  $p$ , the probability that  $X = 1$ .
- Support:**  $x$  is either 0 or 1
- PMF equation:**  $P(X = x) = \begin{cases} p & \text{if } x = 1 \\ 1 - p & \text{if } x = 0 \end{cases}$
- PMF (smooth):**  $P(X = x) = p^x(1 - p)^{1-x}$
- Expectation:**  $E[X] = p$
- Variance:**  $\text{Var}(X) = p(1 - p)$

**PMF graph:**

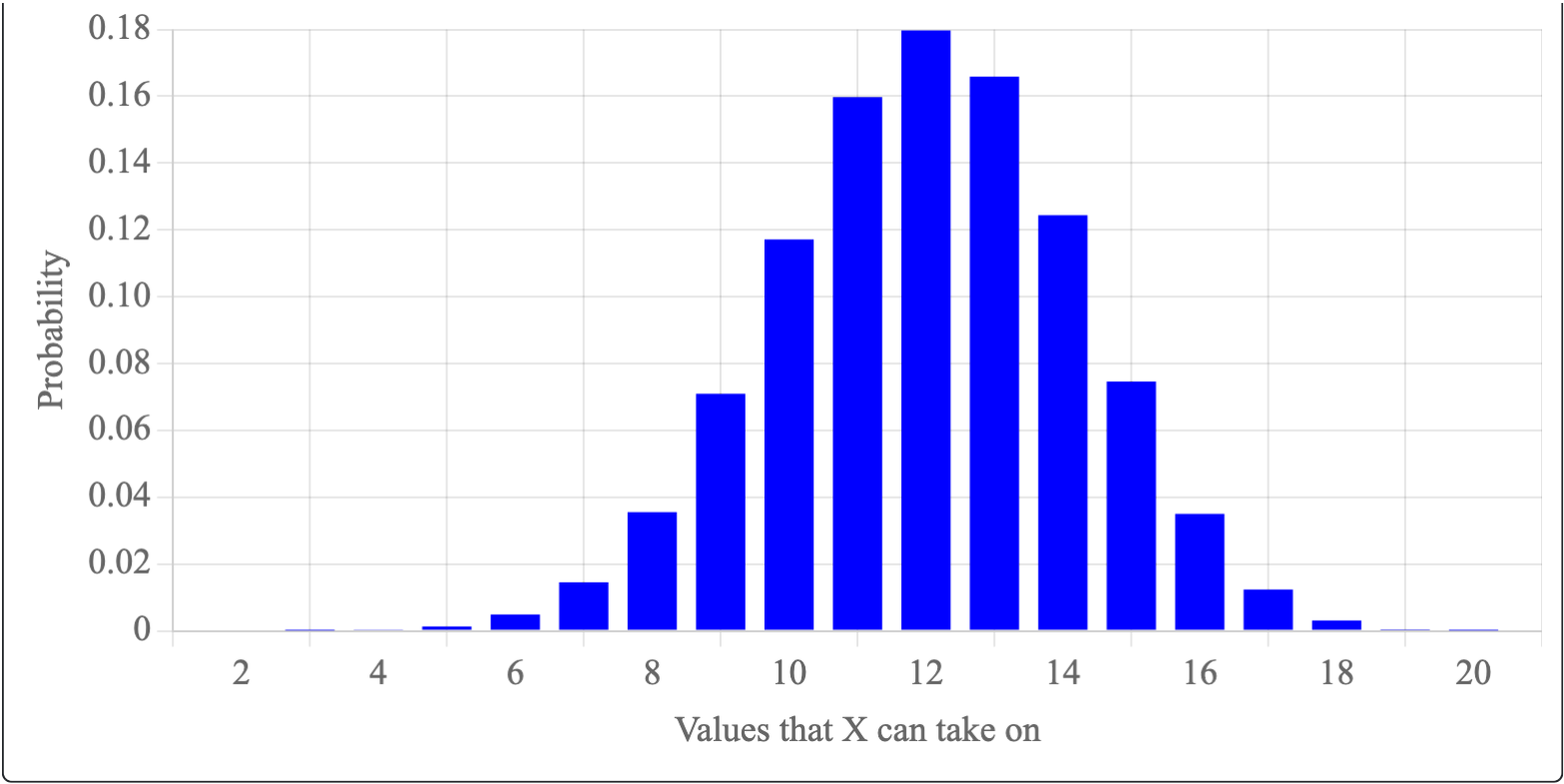


### Binomial Random Variable

- Notation:**  $X \sim \text{Bin}(n, p)$
- Description:** Number of "successes" in  $n$  identical, independent experiments each with probability of success  $p$ .
- Parameters:**  $n \in \{0, 1, \dots\}$ , the number of experiments.  
 $p \in [0, 1]$ , the probability that a single experiment gives a "success".
- Support:**  $x \in \{0, 1, \dots, n\}$
- PMF equation:**  $P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$
- Expectation:**  $E[X] = n \cdot p$
- Variance:**  $\text{Var}(X) = n \cdot p \cdot (1 - p)$

**PMF graph:**

Parameter  $n$ :  Parameter  $p$ :



**Poisson Random Variable**

**Notation:**

$X \sim \text{Poi}(\lambda)$

**Description:**

Number of events in a fixed time frame if (a) the events occur with a constant mean rate and (b) they occur independently of time since last event.

**Parameters:**

$\lambda \in \{0, 1, \dots\}$ , the constant average rate.

**Support:**

$x \in \{0, 1, \dots\}$

**PMF equation:**

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

**Expectation:**

$E[X] = \lambda$

**Variance:**

$\text{Var}(X) = \lambda$

**PMF graph:**

Parameter  $\lambda$ :

Values that X can take on	Probability
0	0.007
1	0.034
2	0.084
3	0.140
4	0.176
5	0.176
6	0.147
7	0.105
8	0.065
9	0.035
10	0.018
11	0.008
12	0.003
13	0.001
14	0.000
15	0.000
16	0.000
17	0.000
18	0.000

**Geometric Random Variable**

**Notation:**

$X \sim \text{Geo}(p)$

**Description:**

Number of experiments until a success. Assumes independent experiments each with probability of success  $p$ .

**Parameters:**

$p \in [0, 1]$ , the probability that a single experiment gives a "success".

**Support:**

$x \in \{1, \dots, \infty\}$

**PMF equation:**

$$P(X = x) = (1 - p)^{x-1} p$$

**Expectation:**

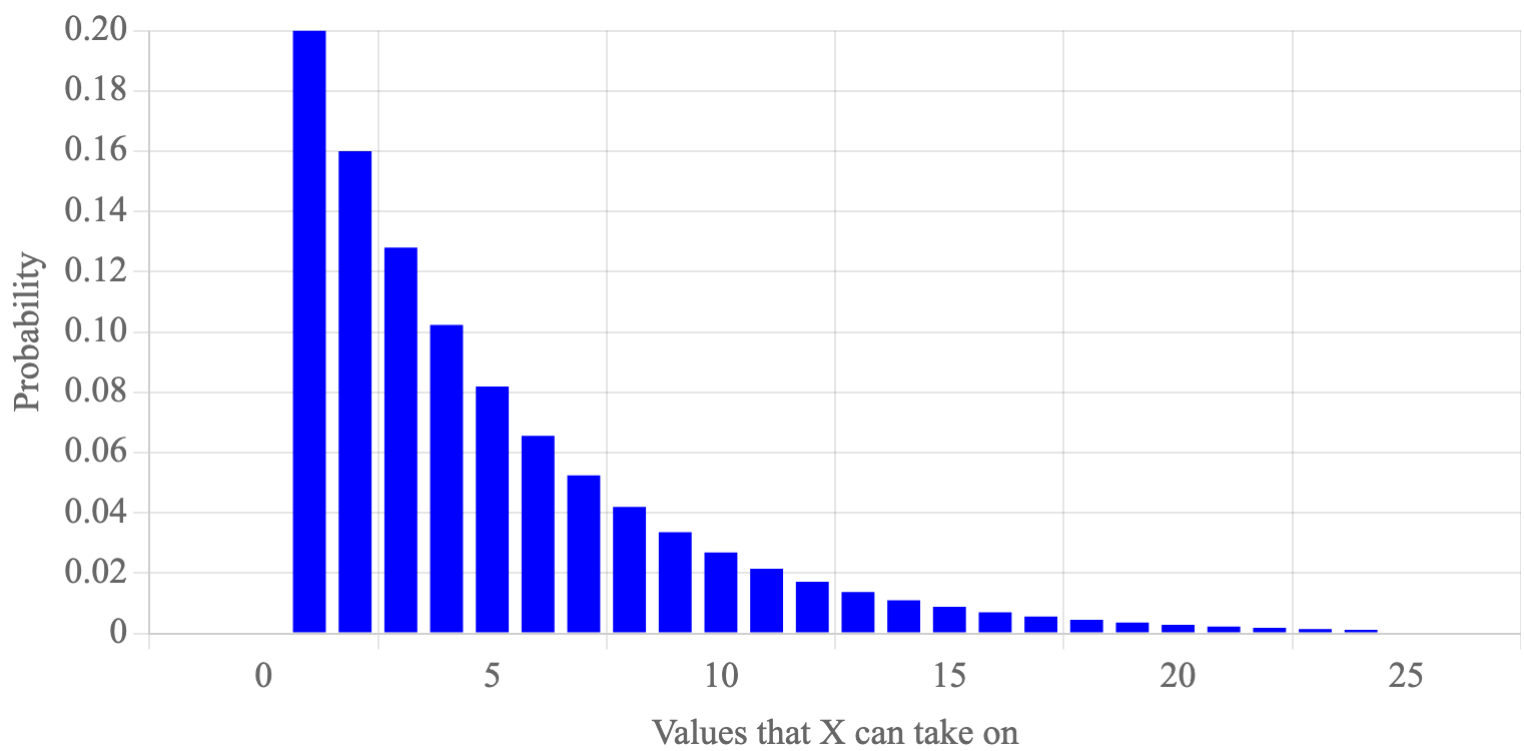
$E[X] = \frac{1}{p}$

**Variance:**

$\text{Var}(X) = \frac{1-p}{p^2}$

PMF graph:

Parameter  $p$ :



Negative Binomial Random Variable

**Notation:**  $X \sim \text{NegBin}(r, p)$

**Description:** Number of experiments until  $r$  successes. Assumes each experiment is independent with probability of success  $p$ .

**Parameters:**  $r > 0$ , the number of success we are waiting for.  
 $p \in [0, 1]$ , the probability that a single experiment gives a "success".

**Support:**  $x \in \{r, \dots, \infty\}$

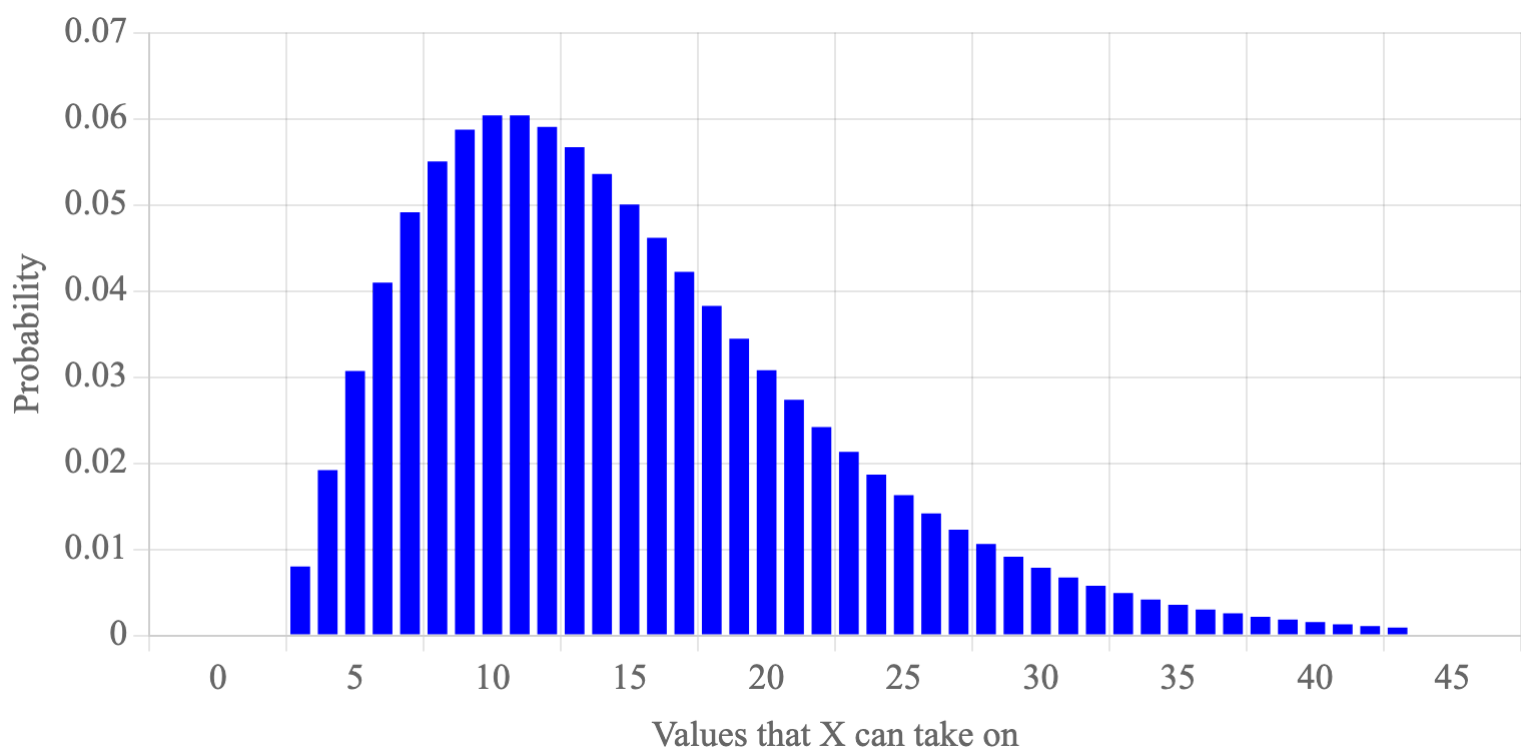
**PMF equation:**  $P(X = x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$

**Expectation:**  $E[X] = \frac{r}{p}$

**Variance:**  $\text{Var}(X) = \frac{r \cdot (1-p)}{p^2}$

PMF graph:

Parameter  $r$ :  Parameter  $p$ :



Continuous Random Variables

Uniform Random Variable

**Notation:**  $X \sim \text{Uni}(\alpha, \beta)$

**Description:**
A continuous random variable that takes on values, with equal likelihood, between  $\alpha$  and  $\beta$

**Parameters:**
 $\alpha \in \mathbb{R}$ , the minimum value of the variable.  
 $\beta \in \mathbb{R}, \beta > \alpha$ , the maximum value of the variable.

**Support:**
 $x \in [\alpha, \beta]$

**PDF equation:**

$$f(x) = \begin{cases} \frac{1}{\beta - \alpha} & \text{for } x \in [\alpha, \beta] \\ 0 & \text{else} \end{cases}$$

**CDF equation:**

$$F(x) = \begin{cases} \frac{x - \alpha}{\beta - \alpha} & \text{for } x \in [\alpha, \beta] \\ 0 & \text{for } x < \alpha \\ 1 & \text{for } x > \beta \end{cases}$$

**Expectation:**
 $E[X] = \frac{1}{2}(\alpha + \beta)$

**Variance:**
 $\text{Var}(X) = \frac{1}{12}(\beta - \alpha)^2$

**PDF graph:**

Parameter  $\alpha$ : 
Parameter  $\beta$ :

**Exponential Random Variable**

**Notation:**
 $X \sim \text{Exp}(\lambda)$

**Description:**
Time until next events if (a) the events occur with a constant mean rate and (b) they occur independently of time since last event.

**Parameters:**
 $\lambda \in \{0, 1, \dots\}$ , the constant average rate.

**Support:**
 $x \in \mathbb{R}^+$

**PDF equation:**

$$f(x) = \lambda e^{-\lambda x}$$

**CDF equation:**

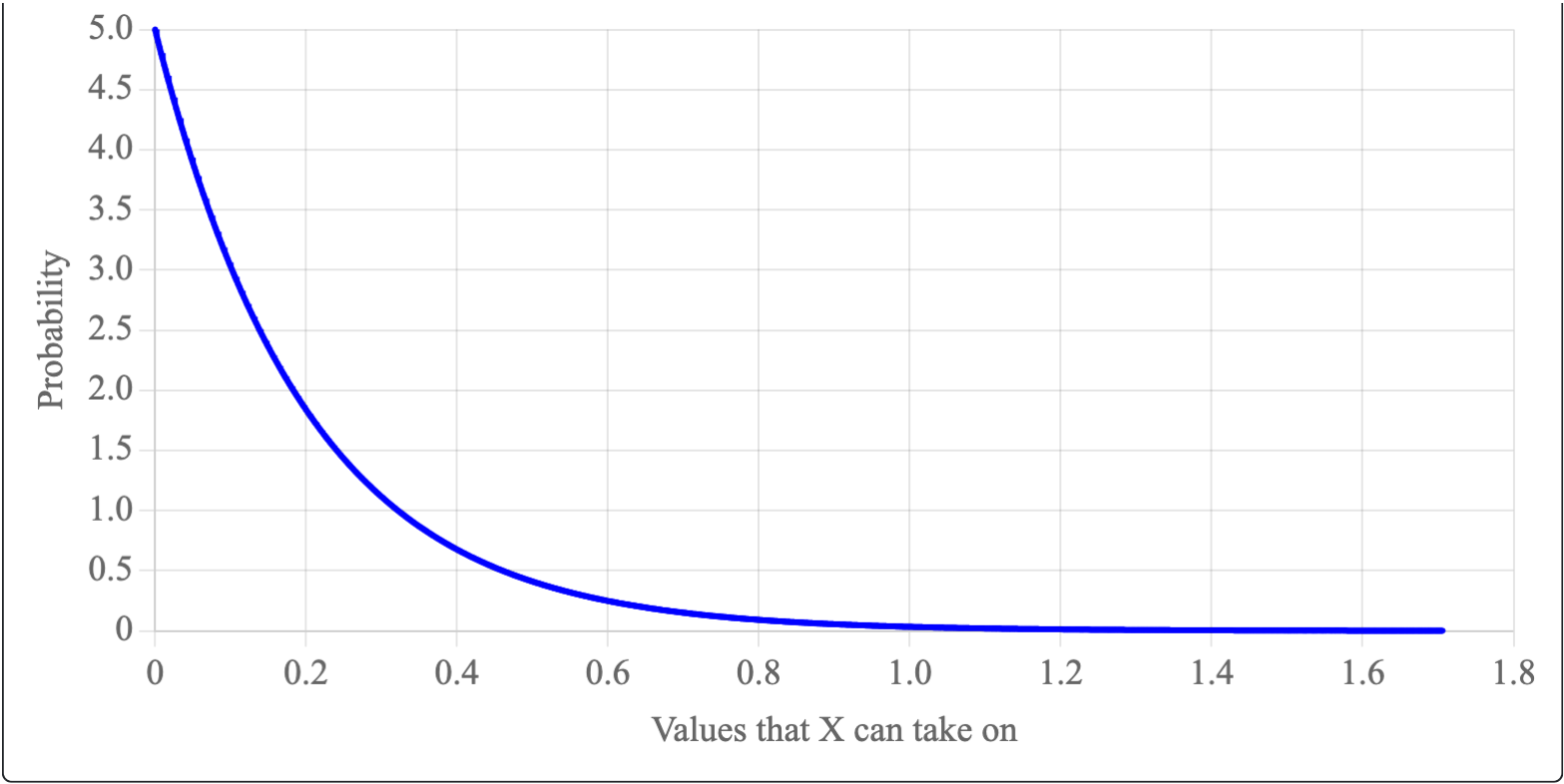
$$F(x) = 1 - e^{-\lambda x}$$

**Expectation:**
 $E[X] = 1/\lambda$

**Variance:**
 $\text{Var}(X) = 1/\lambda^2$

**PDF graph:**

Parameter  $\lambda$ :



**Normal (aka Gaussian) Random Variable**

**Notation:**

$X \sim N(\mu, \sigma^2)$

**Description:**

A common, naturally occurring distribution.

**Parameters:**

$\mu \in \mathbb{R}$ , the mean.  
 $\sigma^2 \in \mathbb{R}$ , the variance.

**Support:**

$x \in \mathbb{R}$

**PDF equation:**

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

**CDF equation:**

$$F(x) = \phi\left(\frac{x-\mu}{\sigma}\right)$$
 Where  $\phi$  is the CDF of the standard normal

**Expectation:**

$E[X] = \mu$

**Variance:**

$\text{Var}(X) = \sigma^2$

**PDF graph:**

Parameter  $\mu$ :

Parameter  $\sigma$ :

X ~ N( $\mu=5, \sigma=5$ )

Standard Normal

A line graph comparing two normal distributions. The x-axis ranges from -10.000 to 18.500 with major ticks every 1.500. The y-axis ranges from 0 to 0.40 with major ticks every 0.05. A blue curve, representing X ~ N(μ=5, σ=5), is broad and centered at 5.000, with a peak height of approximately 0.08. A purple curve, representing the Standard Normal distribution, is narrow and centered at 0.000, with a peak height of approximately 0.40. A legend at the top shows a blue square for 'X ~ N(μ=5, σ=5)' and a purple square for 'Standard Normal'.

**Beta Random Variable**

**Notation:**

$X \sim \text{Beta}(a, b)$

**Description:**

A belief distribution over the value of a probability  $p$  from a Binomial distribution after observing  $a - 1$  successes and  $b - 1$  fails.

**Parameters:**

$a > 0$ , the number successes + 1  
 $b > 0$ , the number of fails + 1

**Support:**

$x \in [0, 1]$

**PDF equation:**

$$f(x) = B \cdot x^{a-1} \cdot (1-x)^{b-1}$$

CDF equation: No closed form

Expectation:  $E[X] = \frac{a}{a+b}$

Variance:  $Var(X) = \frac{ab}{(a+b)^2(a+b+1)}$

PDF graph:

Parameter  $a$ :

Parameter  $b$ :

