Core Probability Reference

Definition: Empirical Definition of Probability

The probability of any event E can be defined as:

$$\mathrm{P}(E) = \lim_{n \to \infty} \frac{\text{count}(E)}{n}$$

Where $\text{count}(E)$ is the number of times that E occured in n experiments.

Definition: Core Identities

For an event E and a sample space S

$0 \leq \mathrm{P}(E) \leq 1$	All probabilities are numbers between 0 and 1.
$\mathrm{P}(S) = 1$	All outcomes must be from the Sample Space.
$\mathrm{P}(E) = 1 - \mathrm{P}(E^{\mathrm{C}})$	The probability of an event from its complement.

Definition: Probability of Equally Likely Outcomes

If S is a sample space with equally likely outcomes, for an event E that is a subset of the outcomes in S:

$$\mathrm{P}(E) = \frac{\text{number of outcomes in } E}{\text{number of outcomes in } S} = \frac{|E|}{|S|}$$

Definition: Conditional Probability.
The probability of E given that (aka conditioned on) event F already happened:

$$\mathrm{P}(E|F) = \frac{\mathrm{P}(E \text{ and } F)}{\mathrm{P}(F)}$$

Definition: Probability of **or** with Mututally Exclusive Events
If two events E and F are mutually exclusive then the probability of E ***or*** F occurring is:

$$\mathrm{P}(E \text{ or } F) = \mathrm{P}(E) + \mathrm{P}(F)$$

For n events $E_1, E_2, \ldots E_n$ where each event is mutually exclusive of one another (in other words, no outcome is in more than one event). Then:

$$\mathrm{P}(E_1 \text{ or } E_2 \text{ or } \ldots \text{ or } E_n) = \mathrm{P}(E_1) + \mathrm{P}(E_2) + \cdots + \mathrm{P}(E_n) = \sum_{i=1}^{n} \mathrm{P}(E_i)$$

Definition: General Probability of **or** (Inclusion-Exclusion)
For any two events E and F:

$$\mathrm{P}(E \text{ or } F) = \mathrm{P}(E) + \mathrm{P}(F) - \mathrm{P}(E \text{ and } F)$$

For three events, E, F, and G the formula is:

$$\begin{aligned}P(E \text{ or } F \text{ or } G) = \; & P(E) + P(F) + P(G) \\ & - P(E \text{ and } F) - P(E \text{ and } G) - P(F \text{ and } G) \\ & + P(E \text{ and } F \text{ and } G)\end{aligned}$$

For more than three events see the chapter of probability of or.

Definition: Probability of **and** for Independent Events.

If two events: E, F are independent then the probability of E ***and*** F occurring is:

$$P(E \text{ and } F) = P(E) \cdot P(F)$$

For n events $E_1, E_2, \ldots E_n$ that are independent of one another:

$$P(E_1 \text{ and } E_2 \text{ and} \ldots \text{and } E_n) = \prod_{i=1}^{n} P(E_i)$$

Definition: General Probability of **and** (The Chain Rule)

For any two events E and F:

$$P(E \text{ and } F) = P(E|F) \cdot P(F)$$

For n events $E_1, E_2, \ldots E_n$:

$$\begin{aligned}P(E_1 \text{ and } E_2 \text{ and} \ldots \text{and } E_n) = & P(E_1) \cdot P(E_2|E_1) \cdot P(E_3|E_1 \text{ and } E_2) \ldots \\ & P(E_n|E_1 \ldots E_{n-1})\end{aligned}$$

Definition: The Law of Total Probability

For any two events E and F:

$$\begin{aligned}P(E) &= P(E \text{ and } F) + P(E \text{ and } F^{C}) \\ &= P(E|F)\,P(F) + P(E|F^{C})\,P(F^{C})\end{aligned}$$

For mutually exclusive events: $B_1, B_2, \ldots B_n$ such that every outcome in the sample space falls into one of those events:

$$\begin{aligned}P(E) &= \sum_{i=1}^{n} P(E \text{ and } B_i) && \text{Extension of our observation} \\ &= \sum_{i=1}^{n} P(E|B_i)\,P(B_i) && \text{Using chain rule on each term}\end{aligned}$$

Definition: Bayes' Theorem

The most common form of Bayes' Theorem is **Bayes' Theorem Classic**:

$$P(B|E) = \frac{P(E|B) \cdot P(B)}{P(E)}$$

Bayes' Theorem combined with the Law of Total Probability:

$$P(B|E) = \frac{P(E|B) \cdot P(B)}{P(E|B) \cdot P(B) + P(E|B^{C}) \cdot P(B^{C})}$$