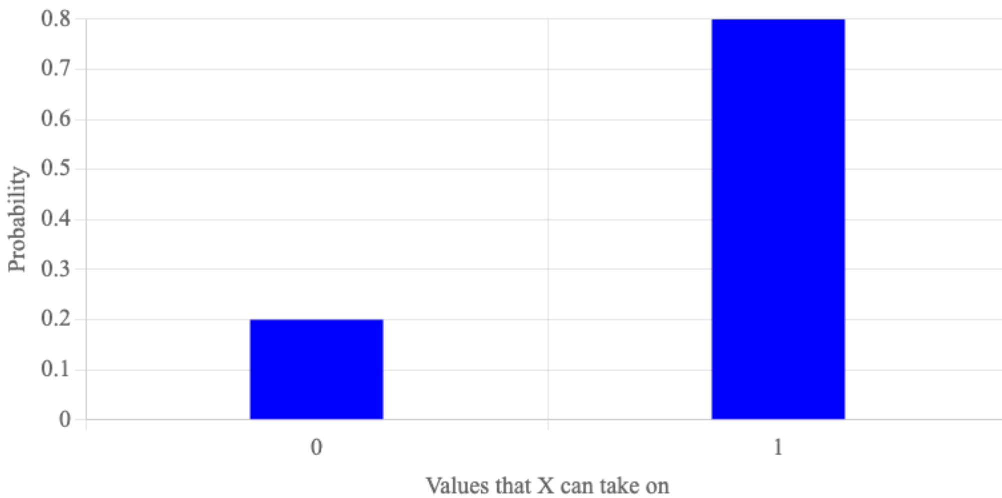
Random Variable Reference

Discrete Random Variables

Bernoulli Random Variable

Notation:	$X \sim \text{Bern}(p)$
Description:	A boolean variable that is 1 with probability p
Parameters:	p, the probability that $X = 1$.
Support:	x is either 0 or 1
PMF equation:	$\text{P}(X = x) = \begin{cases} p & \text{if } x = 1 \\ 1 - p & \text{if } x = 0 \end{cases}$
PMF (smooth):	$\text{P}(X = x) = p^x(1-p)^{1-x}$
Expectation:	$\text{E}[X] = p$
Variance:	$\text{Var}(X) = p(1-p)$
PMF graph:	

Parameter p: 0.80



Binomial Random Variable

Notation: $X \sim \text{Bin}(n, p)$

Description: Number of "successes" in n identical, independent experiments each with probability of success p.

Parameters: $n \in \{0, 1, \ldots\}$, the number of experiments.
$p \in [0, 1]$, the probability that a single experiment gives a "success".

Support: $x \in \{0, 1, \ldots, n\}$

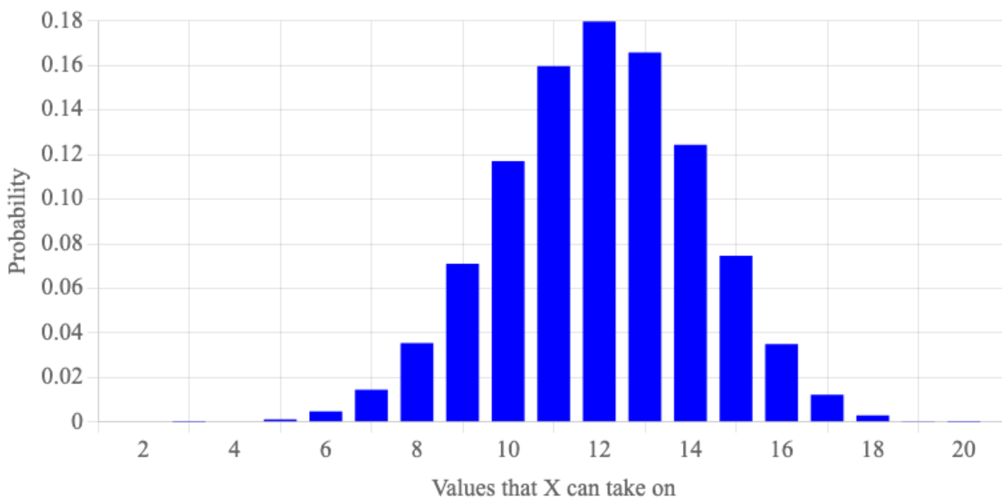
PMF equation: $\text{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$

Expectation: $\text{E}[X] = n \cdot p$

Variance: $\text{Var}(X) = n \cdot p \cdot (1 - p)$

PMF graph:

Parameter n: 20 Parameter p: 0.60



Poisson Random Variable

Notation: $X \sim \mathrm{Poi}(\lambda)$

Description: Number of events in a fixed time frame if (a) the events occur with a constant mean rate and (b) they occur independently of time since last event.

Parameters: $\lambda \in \mathbb{R}^+$, the constant average rate.

Support: $x \in \{0, 1, \ldots\}$

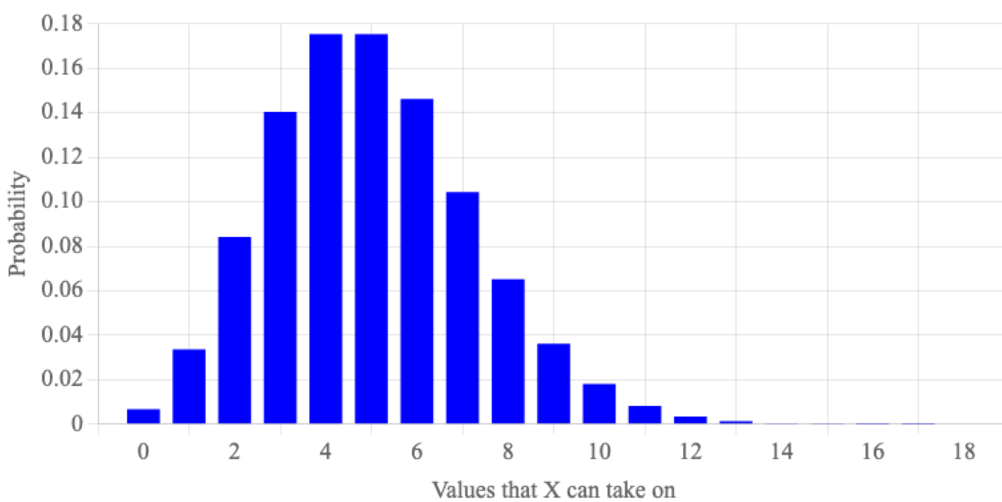
PMF equation: $\mathrm{P}(X = x) = \dfrac{\lambda^x e^{-\lambda}}{x!}$

Expectation: $\mathrm{E}[X] = \lambda$

Variance: $\mathrm{Var}(X) = \lambda$

PMF graph:

Parameter λ: 5



Geometric Random Variable

Notation: $X \sim \text{Geo}(p)$

Description: Number of experiments until a success. Assumes independent experiments each with probability of success p.

Parameters: $p \in [0, 1]$, the probability that a single experiment gives a "success".

Support: $x \in \{1, \ldots, \infty\}$

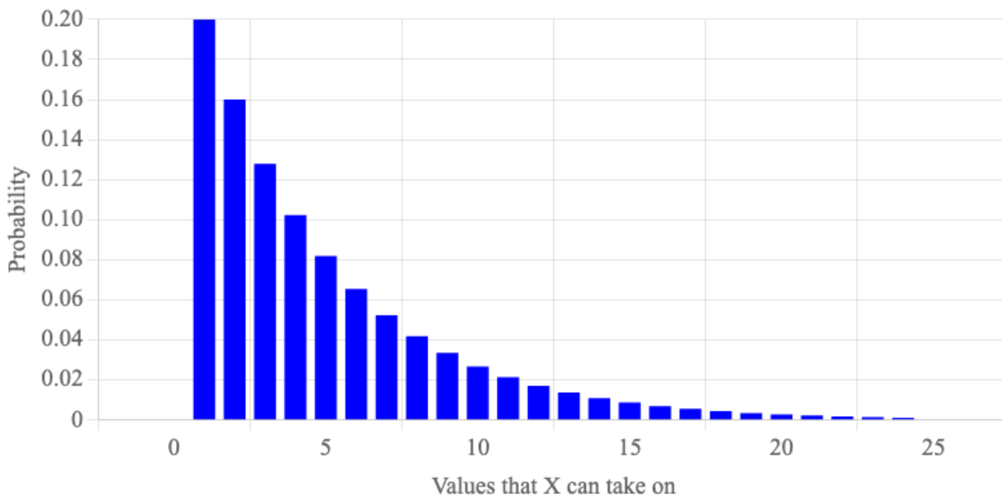
PMF equation: $\text{P}(X = x) = (1 - p)^{x-1}p$

Expectation: $\text{E}[X] = \frac{1}{p}$

Variance: $\text{Var}(X) = \frac{1-p}{p^2}$

PMF graph:

Parameter p: 0.20



Negative Binomial Random Variable

Notation: $X \sim \text{NegBin}(r, p)$

Description: Number of experiments until r successes. Assumes each experiment is independent with probability of success p.

Parameters: $r > 0$, the number of success we are waiting for.
$p \in [0, 1]$, the probability that a single experiment gives a "success".

Support: $x \in \{r, \ldots, \infty\}$

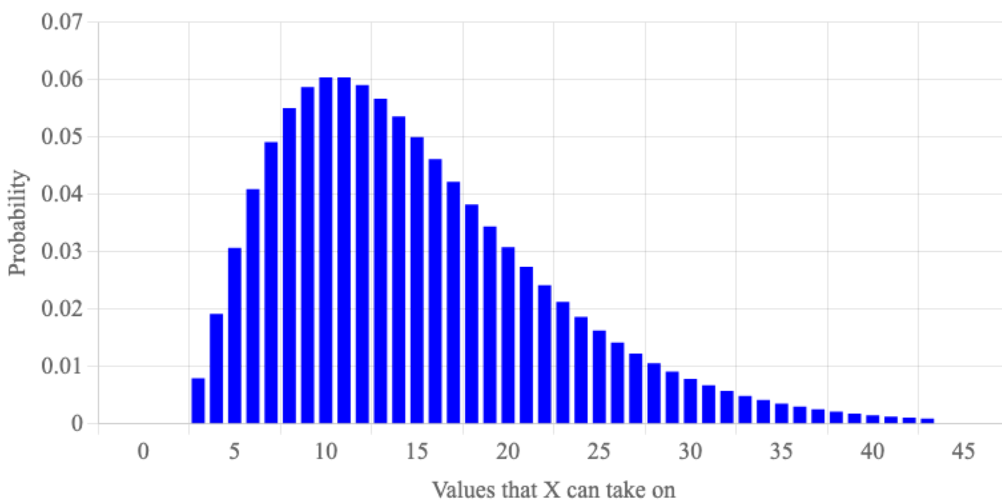
PMF equation: $\text{P}(X = x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$

Expectation: $\text{E}[X] = \frac{r}{p}$

Variance: $\text{Var}(X) = \frac{r \cdot (1-p)}{p^2}$

PMF graph:

Parameter r: 3 Parameter p: 0.20



Continuous Random Variables

Uniform Random Variable

Notation: $X \sim \text{Uni}(\alpha, \beta)$

Description: A continuous random variable that takes on values, with equal likelihood, between α and β

Parameters: $\alpha \in \mathbb{R}$, the minimum value of the variable.
$\beta \in \mathbb{R}$, $\beta > \alpha$, the maximum value of the variable.

Support: $x \in [\alpha, \beta]$

PDF equation:
$$f(x) = \begin{cases} \frac{1}{\beta-\alpha} & \text{for } x \in [\alpha, \beta] \\ 0 & \text{else} \end{cases}$$

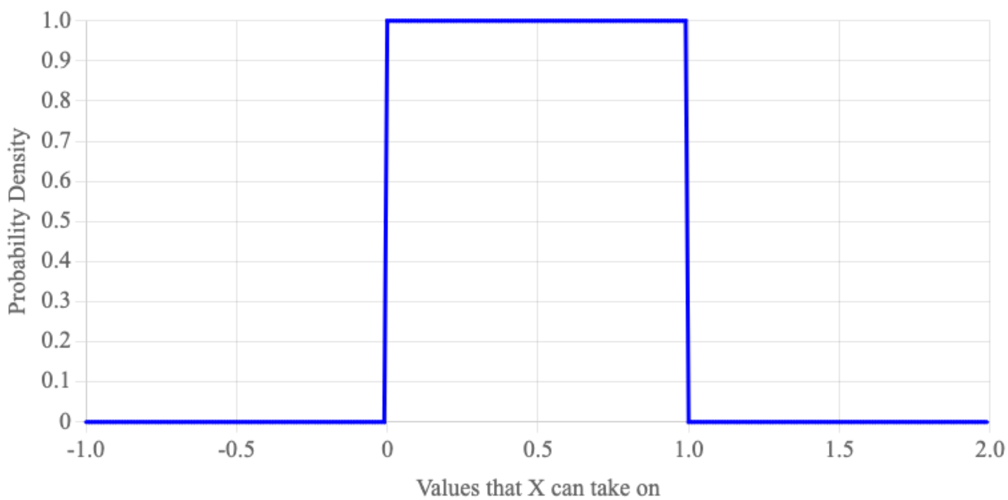
CDF equation:
$$F(x) = \begin{cases} \frac{x-\alpha}{\beta-\alpha} & \text{for } x \in [\alpha, \beta] \\ 0 & \text{for } x < \alpha \\ 1 & \text{for } x > \beta \end{cases}$$

Expectation: $\mathrm{E}[X] = \frac{1}{2}(\alpha + \beta)$

Variance: $\mathrm{Var}(X) = \frac{1}{12}(\beta - \alpha)^2$

PDF graph:

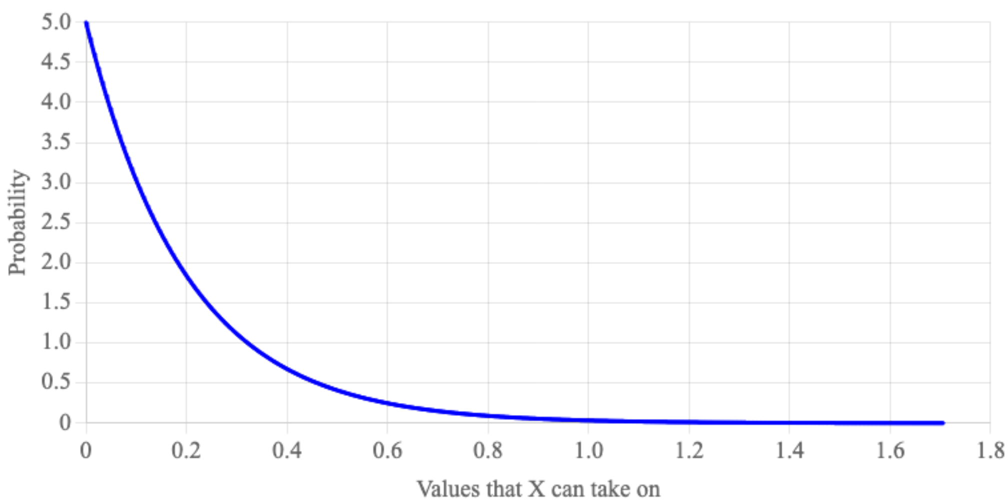
Parameter α: 0 Parameter β: 1



Exponential Random Variable

Notation:	$X \sim \text{Exp}(\lambda)$
Description:	Time until next events if (a) the events occur with a constant mean rate and (b) they occur independently of time since last event.
Parameters:	$\lambda \in \mathbb{R}^+$, the constant average rate.
Support:	$x \in \mathbb{R}^+$
PDF equation:	$f(x) = \lambda e^{-\lambda x}$
CDF equation:	$F(x) = 1 - e^{-\lambda x}$
Expectation:	$\text{E}[X] = 1/\lambda$
Variance:	$\text{Var}(X) = 1/\lambda^2$
PDF graph:	

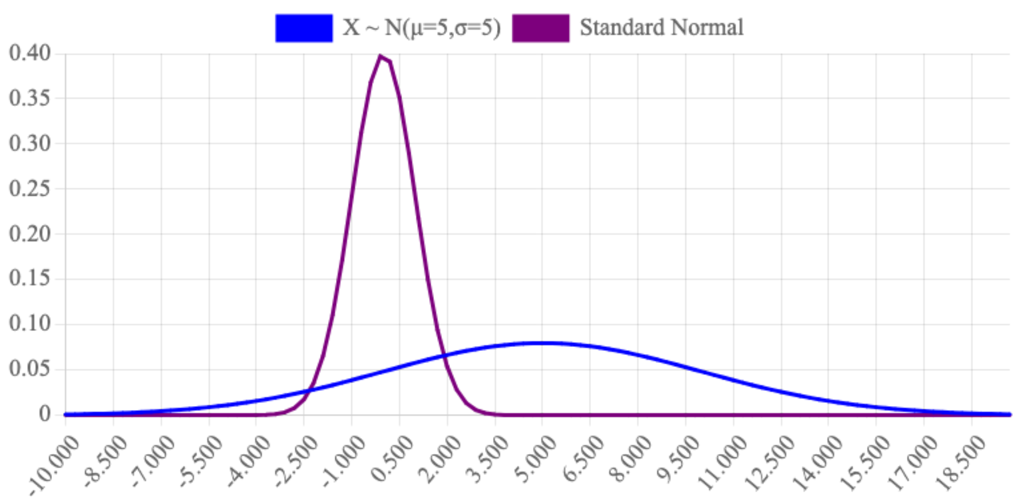
Parameter λ: 5



Normal (aka Gaussian) Random Variable

Notation:	$X \sim \mathrm{N}(\mu, \sigma^2)$
Description:	A common, naturally occurring distribution.
Parameters:	$\mu \in \mathbb{R}$, the mean. $\sigma^2 \in \mathbb{R}$, the variance.
Support:	$x \in \mathbb{R}$
PDF equation:	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$
CDF equation:	$F(x) = \phi(\frac{x-\mu}{\sigma})$ Where ϕ is the CDF of the standard normal
Expectation:	$\mathrm{E}[X] = \mu$
Variance:	$\mathrm{Var}(X) = \sigma^2$
PDF graph:	

Parameter μ: 5 Parameter σ: 5



Beta Random Variable

Notation: $X \sim \text{Beta}(a, b)$

Description: A belief distribution over the value of a probability p from a Binomial distribution after observing $a - 1$ successes and $b - 1$ fails.

Parameters: $a > 0$, the number successes + 1
$b > 0$, the number of fails + 1

Support: $x \in [0, 1]$

PDF equation: $f(x) = B(a, b) \cdot x^{a-1} \cdot (1 - x)^{b-1}$ where $B(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a,b)}$

CDF equation: No closed form

Expectation: $\text{E}[X] = \frac{a}{a+b}$

Variance: $\text{Var}(X) = \frac{ab}{(a+b)^2(a+b+1)}$

PDF graph:

Parameter a: 2 Parameter b: 4