# Random Variable Reference

## Discrete Random Variables

**Bernoulli Random Variable**

**Notation:** $X \sim \mathrm{Bern}(p)$

**Description:** A boolean variable that is 1 with probability $p$

**Parameters:** $p$, the probability that $X = 1$.

**Support:** $x$ is either 0 or 1

**PMF equation:** $\mathrm{P}(X = x) = \begin{cases} p & \text{if } x = 1 \\ 1 - p & \text{if } x = 0 \end{cases}$

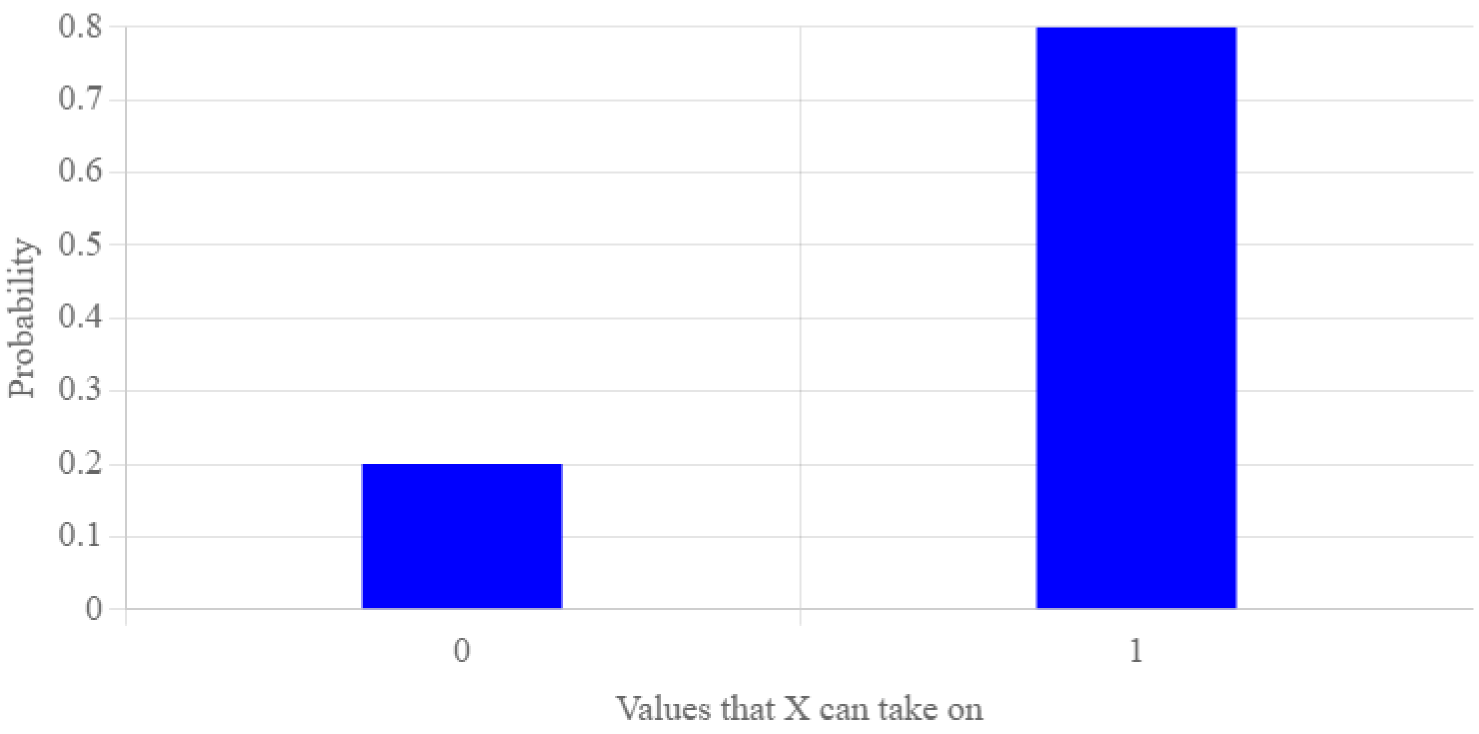
**PMF (smooth):** $\mathrm{P}(X = x) = p^x(1 - p)^{1-x}$

**Expectation:** $\mathrm{E}[X] = p$

**Variance:** $\mathrm{Var}(X) = p(1 - p)$

**PMF graph:**

Parameter $p$: 0.80




**Binomial Random Variable**

**Notation:** $X \sim \mathrm{Bin}(n, p)$

**Description:** Number of "successes" in $n$ identical, independent experiments each with probability of success $p$.

**Parameters:** $n \in \{0, 1, \ldots\}$, the number of experiments.
$p \in [0, 1]$, the probability that a single experiment gives a "success".

**Support:** $x \in \{0, 1, \ldots, n\}$

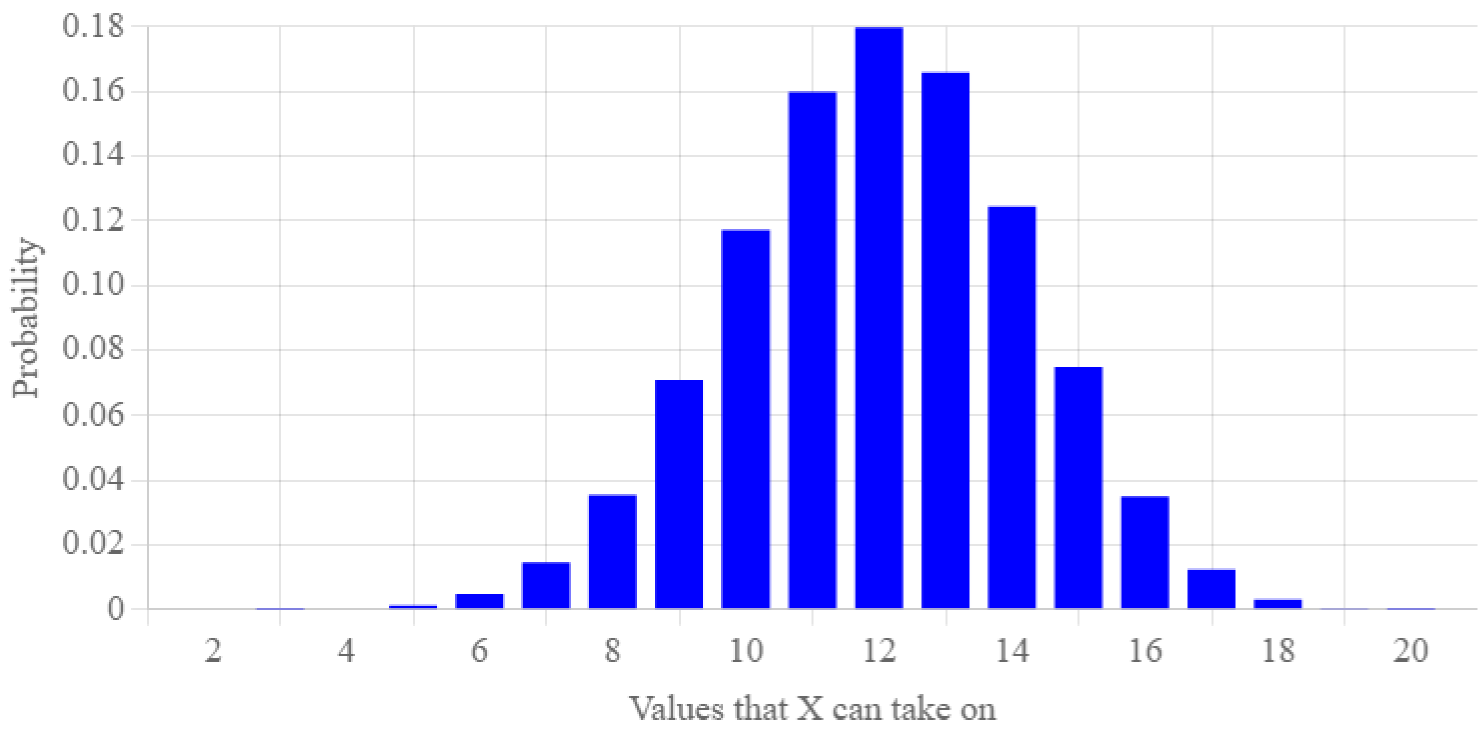
**PMF equation:** $\mathrm{P}(X = x) = \binom{n}{x} p^x(1 - p)^{n-x}$

**Expectation:** $\mathrm{E}[X] = n \cdot p$

**Variance:** $\mathrm{Var}(X) = n \cdot p \cdot (1 - p)$

**PMF graph:**

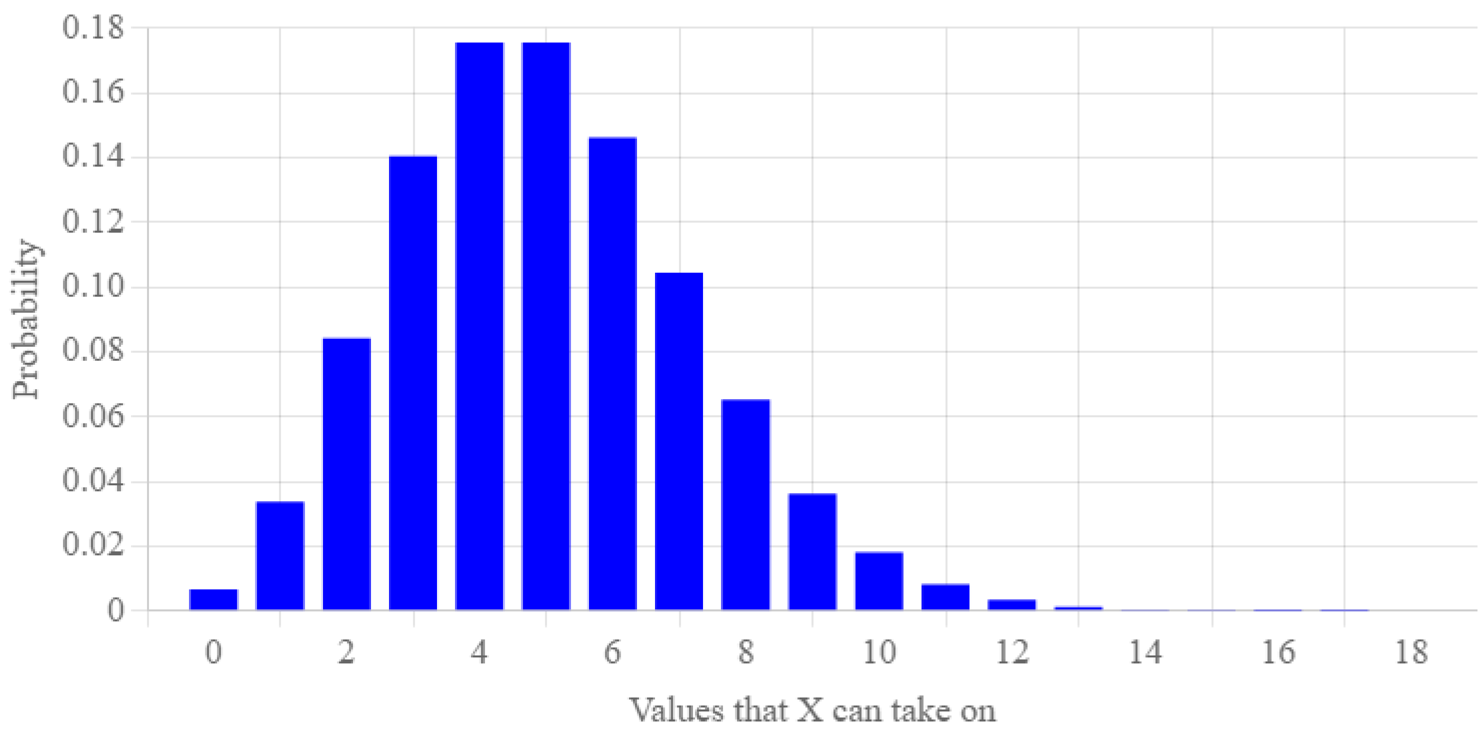
Parameter $n$: 20 Parameter $p$: 0.60




**Poisson Random Variable**

| | |
|---|---|
| **Notation:** | $X \sim \mathrm{Poi}(\lambda)$ |
| **Description:** | Number of events in a fixed time frame if (a) the events occur with a constant mean rate and (b) they occur independently of time since last event. |
| **Parameters:** | $\lambda \in \{0, 1, \ldots\}$, the constant average rate. |
| **Support:** | $x \in \{0, 1, \ldots\}$ |
| **PMF equation:** | $\mathrm{P}(X = x) = \dfrac{\lambda^x e^{-\lambda}}{x!}$ |
| **Expectation:** | $\mathrm{E}[X] = \lambda$ |
| **Variance:** | $\mathrm{Var}(X) = \lambda$ |
| **PMF graph:** | |

Parameter $\lambda$: 5




**Geometric Random Variable**

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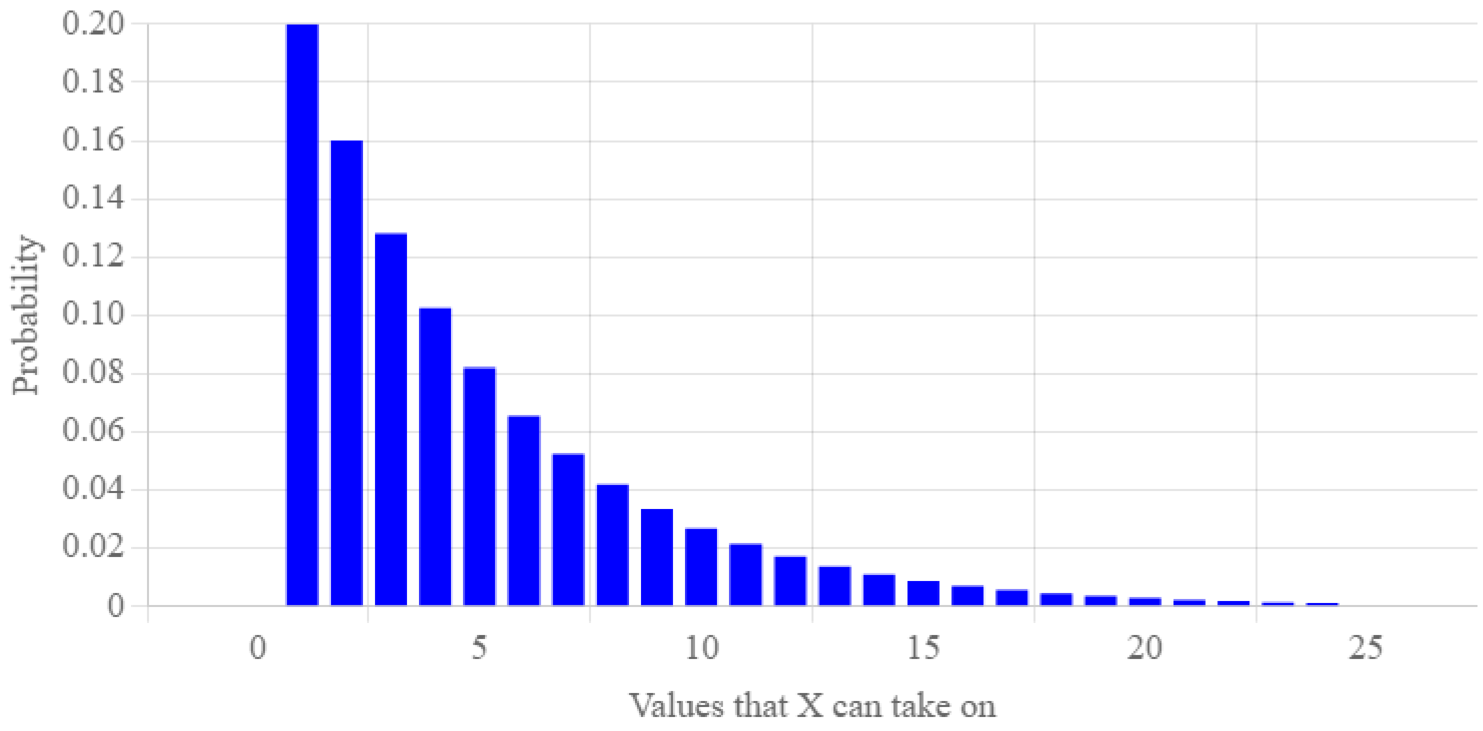
|---|---|
| **Notation:** | $X \sim \mathrm{Geo}(p)$ |
| **Description:** | Number of experiments until a success. Assumes independent experiments each with probability of success $p$. |
| **Parameters:** | $p \in [0, 1]$, the probability that a single experiment gives a "success". |
| **Support:** | $x \in \{1, \ldots, \infty\}$ |
| **PMF equation:** | $\mathrm{P}(X = x) = (1-p)^{x-1}p$ |
| **Expectation:** | $\mathrm{E}[X] = \frac{1}{p}$ |
| **Variance:** | $\mathrm{Var}(X) = \frac{1-p}{p^2}$ |
| **PMF graph:** | |

Parameter $p$: 0.20




**Negative Binomial Random Variable**

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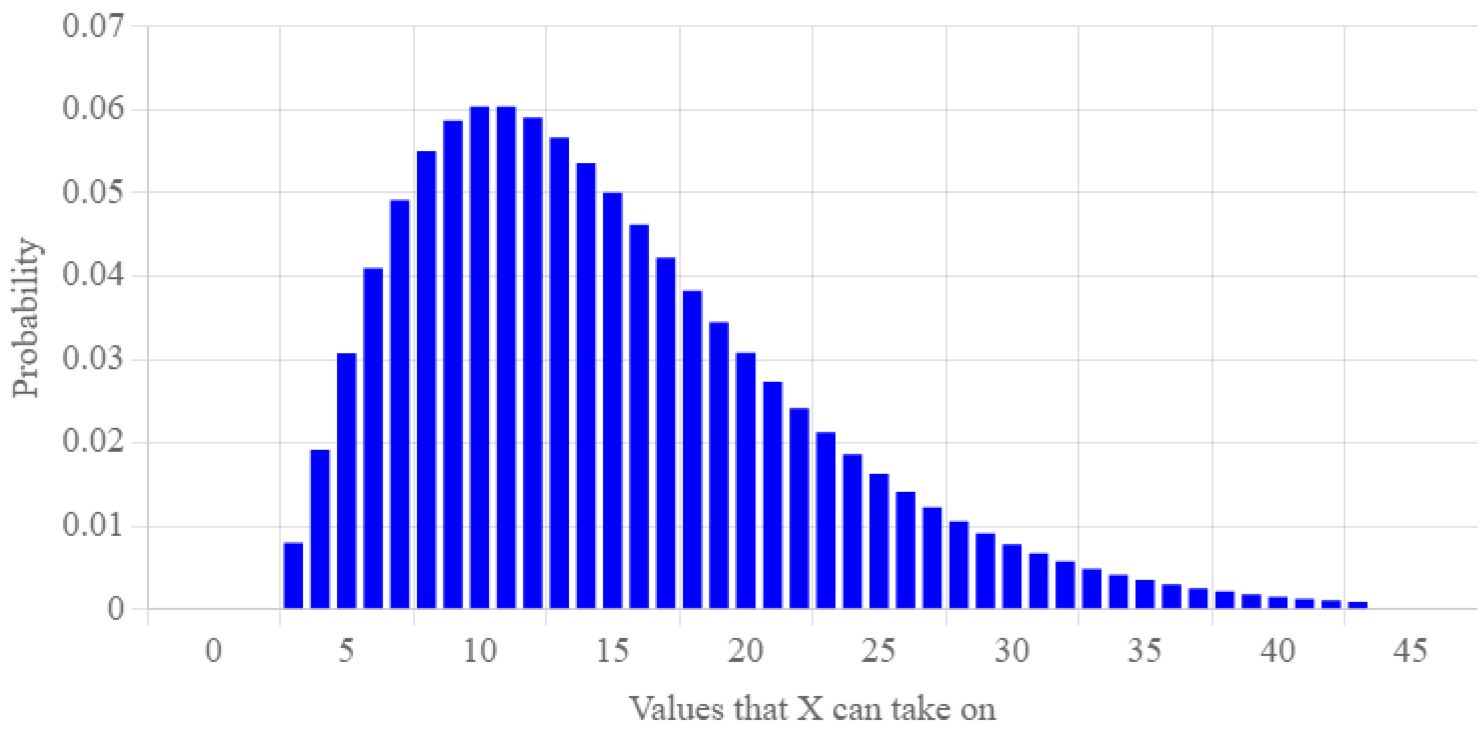
|---|---|
| **Notation:** | $X \sim \text{NegBin}(r, p)$ |
| **Description:** | Number of experiments until $r$ successes. Assumes each experiment is independent with probability of success $p$. |
| **Parameters:** | $r > 0$, the number of success we are waiting for.<br>$p \in [0, 1]$, the probability that a single experiment gives a "success". |
| **Support:** | $x \in \{r, \ldots, \infty\}$ |
| **PMF equation:** | $\text{P}(X = x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$ |
| **Expectation:** | $\text{E}[X] = \frac{r}{p}$ |
| **Variance:** | $\text{Var}(X) = \frac{r \cdot (1-p)}{p^2}$ |
| **PMF graph:** | |

Parameter $r$: 3 Parameter $p$: 0.20




## Continuous Random Variables

**Uniform Random Variable**

| | |
|---|---|
| **Notation:** | $X \sim \text{Uni}(\alpha, \beta)$ |
| **Description:** | A continuous random variable that takes on values, with equal likelihood, between $\alpha$ and $\beta$ |

**Parameters:** $\alpha \in \mathbb{R}$, the minimum value of the variable.
$\beta \in \mathbb{R}, \beta > \alpha$, the maximum value of the variable.

**Support:** $x \in [\alpha, \beta]$

**PDF equation:** $f(x) = \begin{cases} \frac{1}{\beta - \alpha} & \text{for } x \in [\alpha, \beta] \\ 0 & \text{else} \end{cases}$

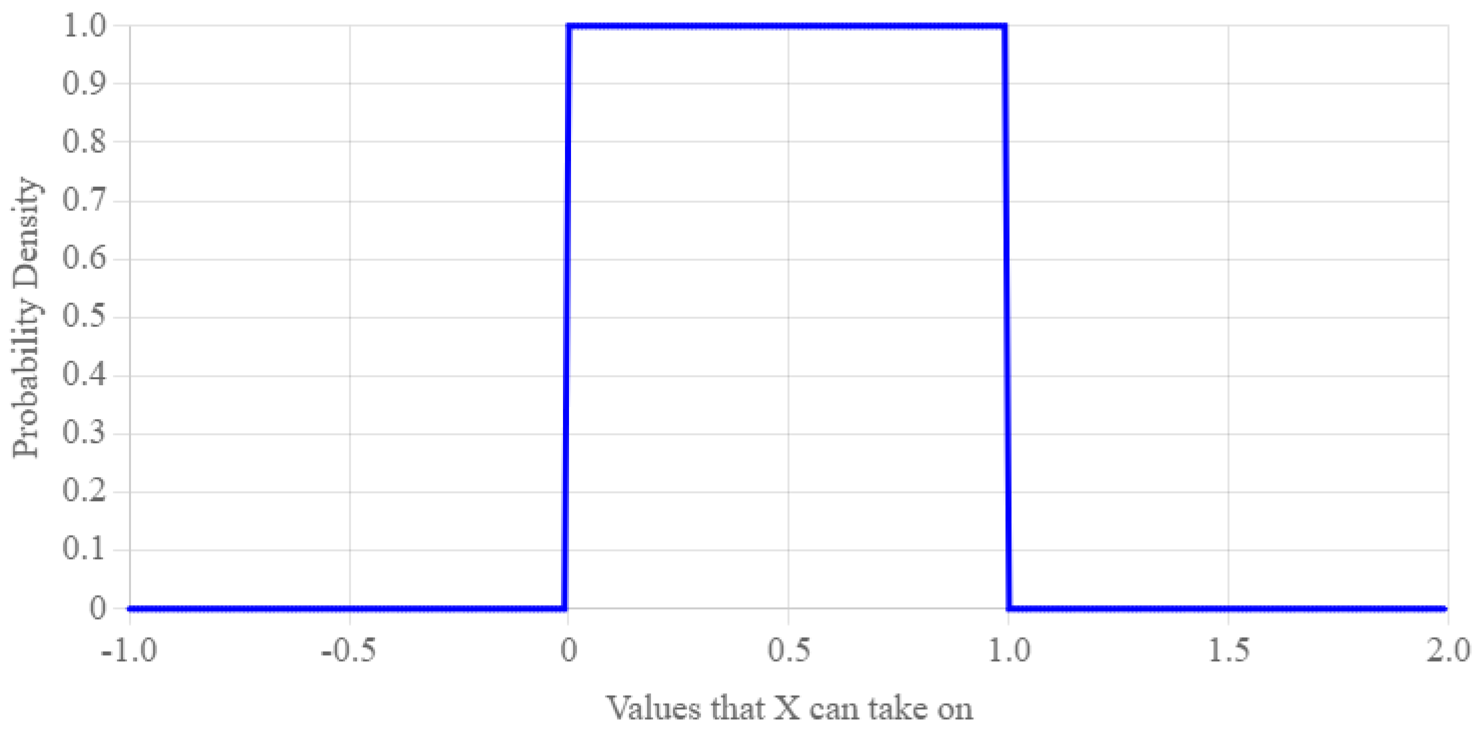
**CDF equation:** $F(x) = \begin{cases} \frac{x - \alpha}{\beta - \alpha} & \text{for } x \in [\alpha, \beta] \\ 0 & \text{for } x < \alpha \\ 1 & \text{for } x > \beta \end{cases}$

**Expectation:** $\mathrm{E}[X] = \frac{1}{2}(\alpha + \beta)$

**Variance:** $\mathrm{Var}(X) = \frac{1}{12}(\beta - \alpha)^2$

**PDF graph:**

Parameter $\alpha$: 0 Parameter $\beta$: 1




**Exponential Random Variable**

**Notation:** $X \sim \mathrm{Exp}(\lambda)$

**Description:** Time until next events if (a) the events occur with a constant mean rate and (b) they occur independently of time since last event.

**Parameters:** $\lambda \in \{0, 1, \ldots\}$, the constant average rate.

**Support:** $x \in \mathbb{R}^+$

**PDF equation:** $f(x) = \lambda e^{-\lambda x}$

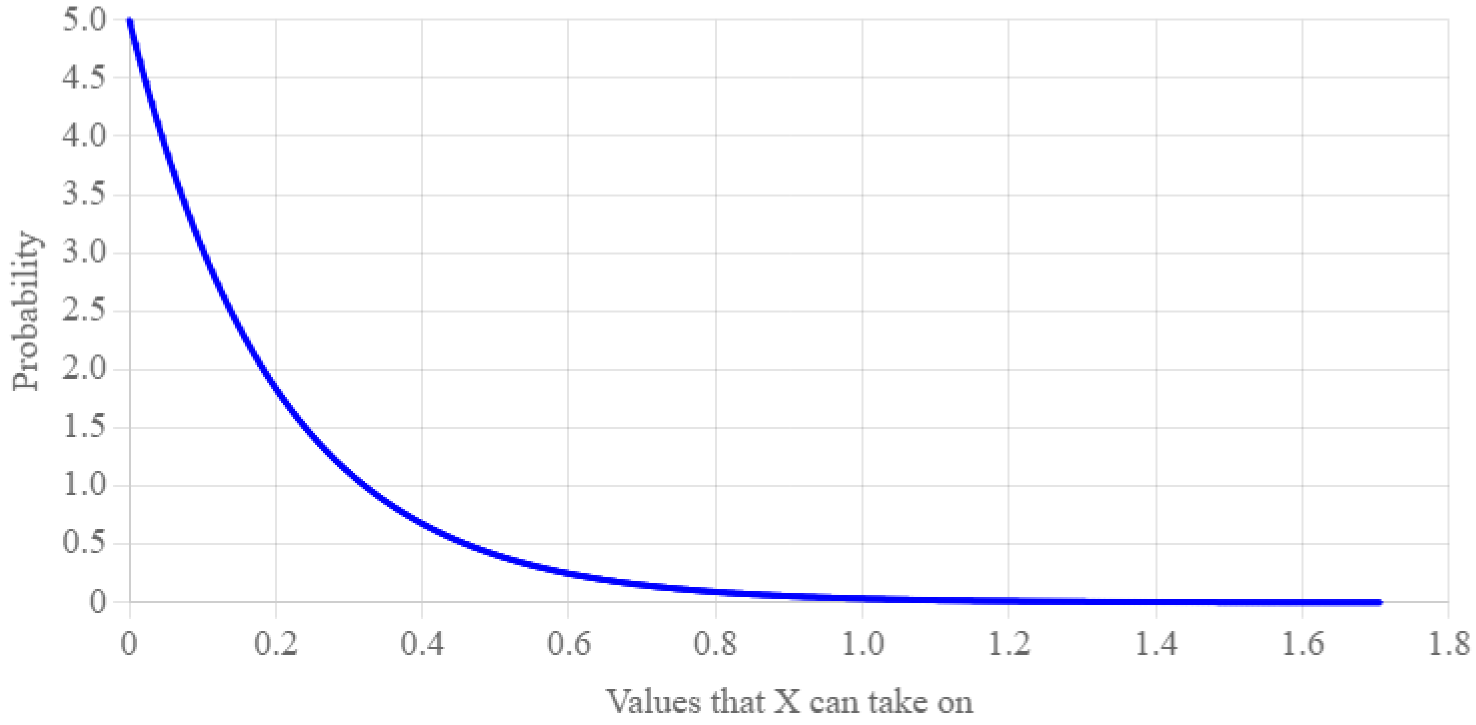
**CDF equation:** $F(x) = 1 - e^{-\lambda x}$

**Expectation:** $\mathrm{E}[X] = 1/\lambda$

**Variance:** $\mathrm{Var}(X) = 1/\lambda^2$

**PDF graph:**

Parameter $\lambda$: 5




**Normal (aka Gaussian) Random Variable**

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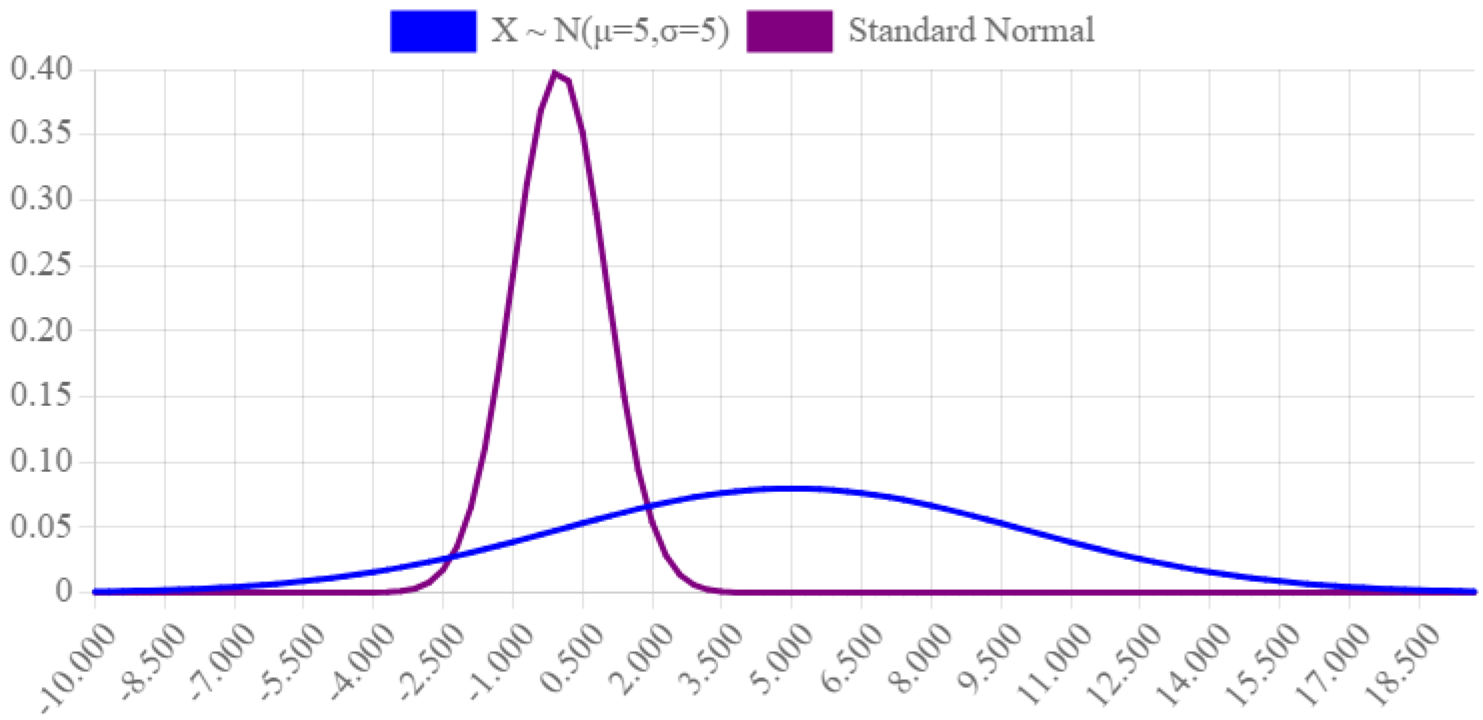
|---|---|
| **Notation:** | $X \sim \mathrm{N}(\mu, \sigma^2)$ |
| **Description:** | A common, naturally occurring distribution. |
| **Parameters:** | $\mu \in \mathbb{R}$, the mean.<br>$\sigma^2 \in \mathbb{R}$, the variance. |
| **Support:** | $x \in \mathbb{R}$ |
| **PDF equation:** | $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ |
| **CDF equation:** | $F(x) = \phi(\frac{x-\mu}{\sigma})$ Where $\phi$ is the CDF of the standard normal |
| **Expectation:** | $\mathrm{E}[X] = \mu$ |
| **Variance:** | $\mathrm{Var}(X) = \sigma^2$ |
| **PDF graph:** | |

Parameter $\mu$: 5 Parameter $\sigma$: 5




**Beta Random Variable**

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|---|---|
| **Notation:** | $X \sim \mathrm{Beta}(a, b)$ |
| **Description:** | A belief distribution over the value of a probability $p$ from a Binomial distribution after observing $a-1$ successes and $b-1$ fails. |
| **Parameters:** | $a > 0$, the number successes + 1<br>$b > 0$, the number of fails + 1 |
| **Support:** | $x \in [0, 1]$ |
| **PDF equation:** | $f(x) = B \cdot x^{a-1} \cdot (1-x)^{b-1}$ |
| **CDF equation:** | No closed form |

**Expectation:** $\mathrm{E}[X] = \frac{a}{a+b}$

**Variance:** $\mathrm{Var}(X) = \frac{ab}{(a+b)^2(a+b+1)}$

**PDF graph:**

Parameter $a$: 2 Parameter $b$: 4