



# Probability and Babies

This demo used to be live. We now know that the delivery happened on Jan 23rd. Lets go back in time to Jan 1st and see what the probability looked like at that point.

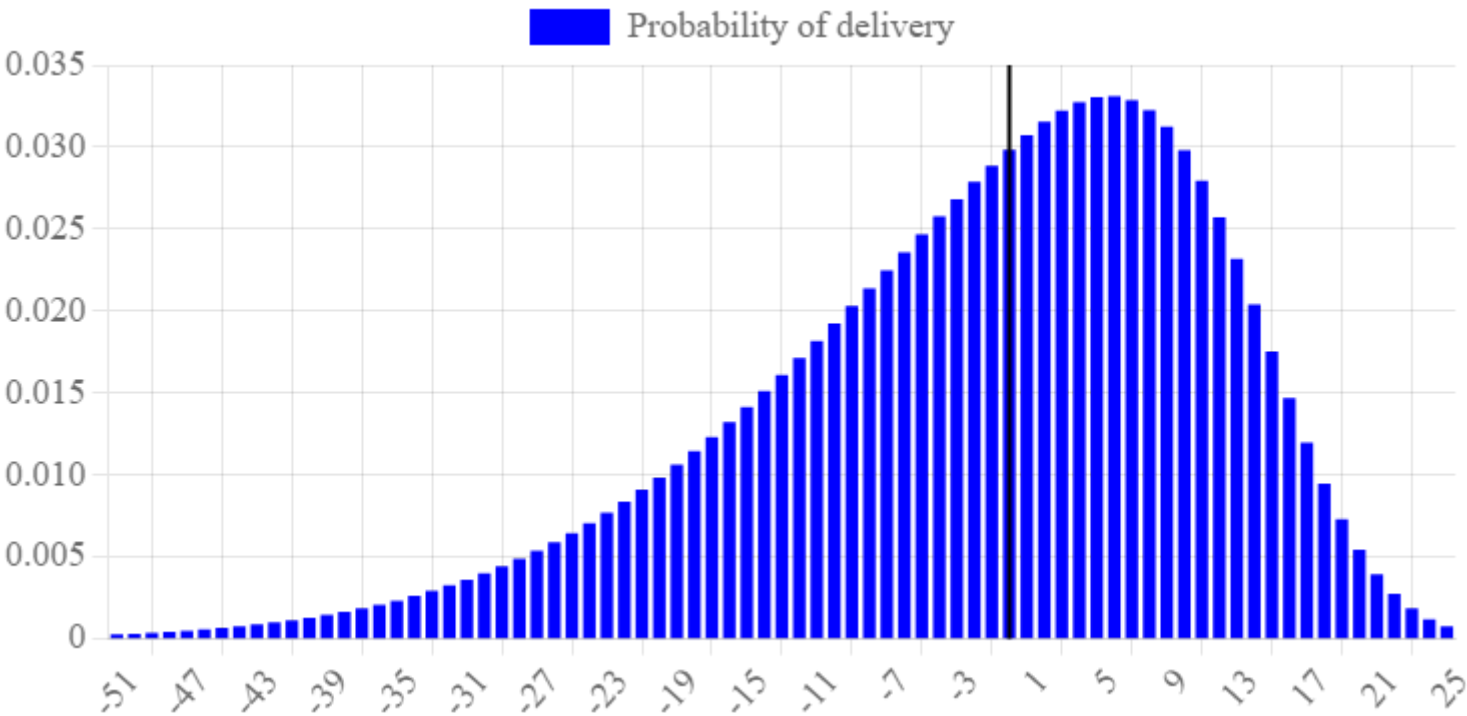
What is the probability that Laura gives birth today (given that she hasn't given birth up until today)?

Today's Date	1/Jan/2021
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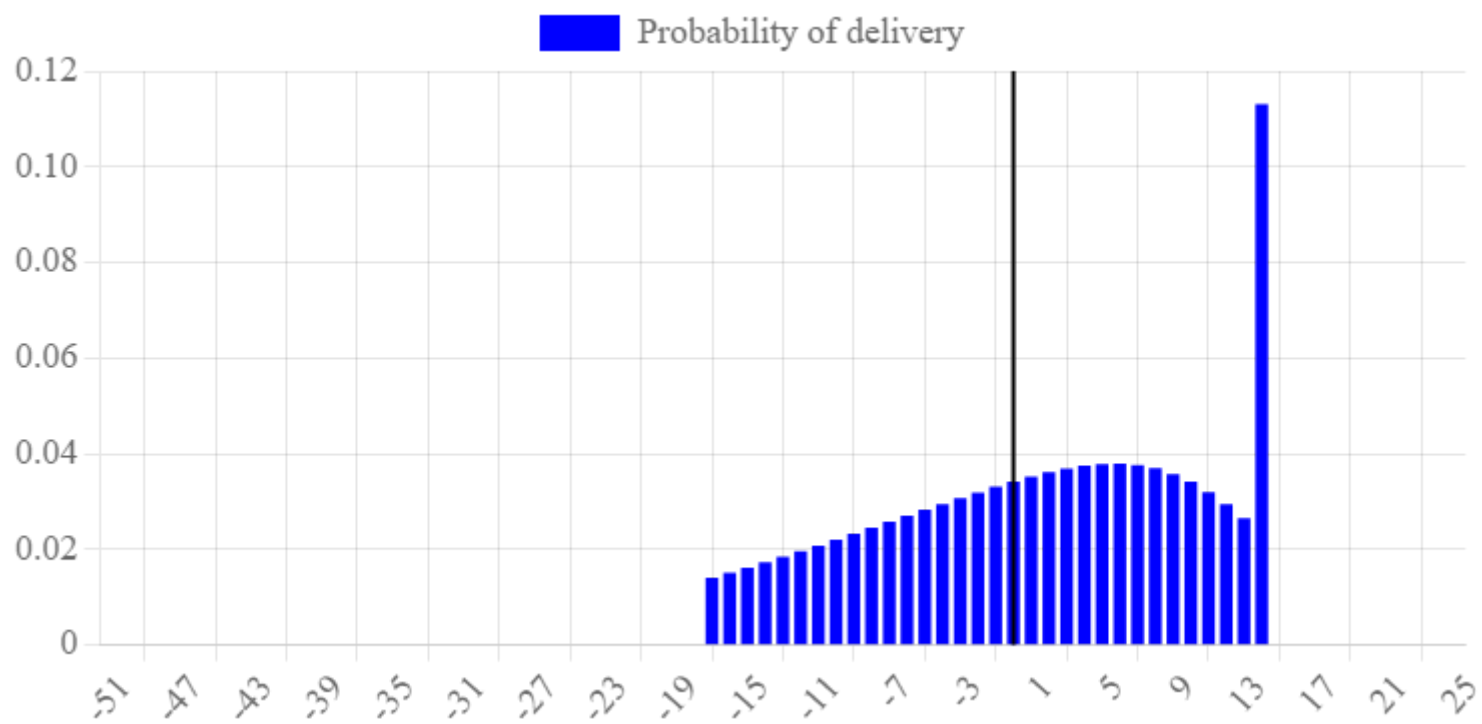
Due Date	18/Jan/2021
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Probability of delivery today: **0.014**  
Probability of delivery in next 7 days: **0.144**  
Current days past due date: **-17** days  
Unconditioned probability mass before today: **0.128**

How likely is delivery, in humans, relative to the due date? There have been millions of births which gives us a relatively good picture [1]. The length of human pregnancy varies by quite a lot! Have you heard that it is 9 months? That is a rough, point estimate. The mean duration of pregnancy is 278.6 days, and pregnancy length has a standard deviation (SD) of 12.5 days. This distribution is not normal, but roughly matches a "skewed normal". This is a general probability mass function for the first pregnancy collected from hundreds of thousands of women (this PMF is very similar across demographics, but changes based on whether the woman has given birth before):



Of course, we have more information. Specifically, we know that Laura **hasn't** given birth up until today (we will update this example when that changes). We also know that babies which are over 14 days late are "induced" on day 14. How likely is delivery given that we haven't delivered up until today? Note that the y-axis is scaled differently:



Implementation notes: this calculation was performed by storing the PDF as a list of (day, probability) points. These values are sometimes called weighted samples, or "particles" and are the key component to a ["particle filtering"](#) approach. After we observe no-delivery, we set the probability of every point which has a day before today to be 0, and then re-normalize the remaining points (aka we "filter" the "particles"). This is convenient because the "posterior" belief doesn't follow a simple equation -- using particles means we never have to write that equation down in our code.

Three friends have the exact same due date (Really! this isn't a hypothetical) What is the probability that all three couples deliver on the exact same day?

Probability of three couples on the same day: **0.002**

How did we get that number? Let  $p_i$  be the probability that one baby is delivered on day  $i$  -- this number can be read off the probability mass function. Let  $D_i$  be the event that all three babies are delivered on day  $i$ . Note that the event  $D_i$  is [mutually exclusive](#) with the event that all three babies are born on another day (So for example,  $D_1$  is mutually exclusive with  $D_2, D_3$  etc). Let  $N = 3$  be the event that all babies are born on the same day:

$$\begin{aligned}
 P(N = 3) &= \sum_i P(D_i) && \text{Since days are mutually exclusive} \\
 &= \sum_i p_i^3 && \text{Since the three couples are independent}
 \end{aligned}$$

[1] [Predicting delivery date by ultrasound and last menstrual period in early gestation](#)

Acknowledgements: This problem was first posed to me by Chris Gregg.