

# Random Variable Reference

## Discrete Random Variables

### Bernoulli Random Variable

**Notation:**  $X \sim \text{Bern}(p)$

**Description:** A boolean variable that is 1 with probability  $p$

**Parameters:**  $p$ , the probability that  $X = 1$ .

**Support:**  $x$  is either 0 or 1

**PMF equation:** 
$$P(X = x) = \begin{cases} p & \text{if } x = 1 \\ 1 - p & \text{if } x = 0 \end{cases}$$

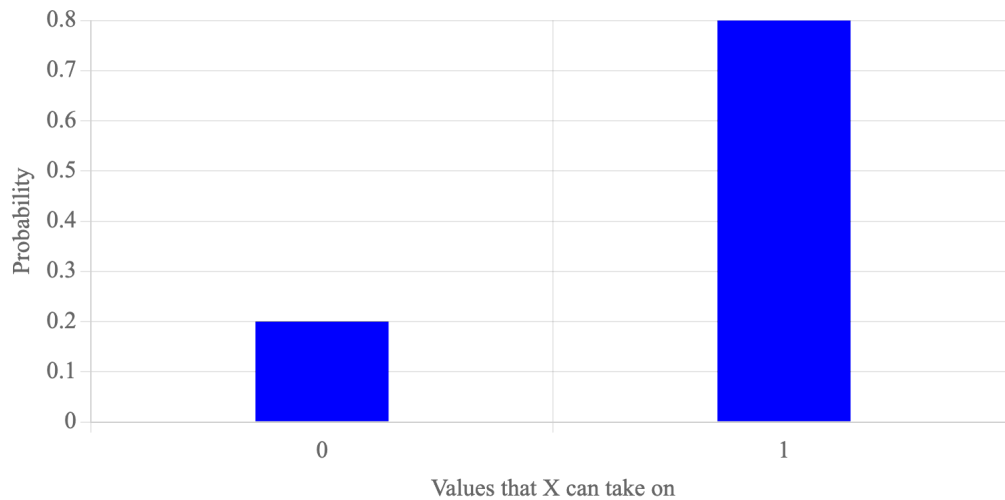
**PMF (smooth):**  $P(X = x) = p^x(1 - p)^{1-x}$

**Expectation:**  $E[X] = p$

**Variance:**  $\text{Var}(X) = p(1 - p)$

#### PMF graph:

Parameter  $p$ :



## Binomial Random Variable

**Notation:**  $X \sim \text{Bin}(n, p)$

**Description:** Number of "successes" in  $n$  identical, independent experiments each with probability of success  $p$ .

**Parameters:**  $n \in \{0, 1, \dots\}$ , the number of experiments.

$p \in [0, 1]$ , the probability that a single experiment gives a "success".

**Support:**  $x \in \{0, 1, \dots, n\}$

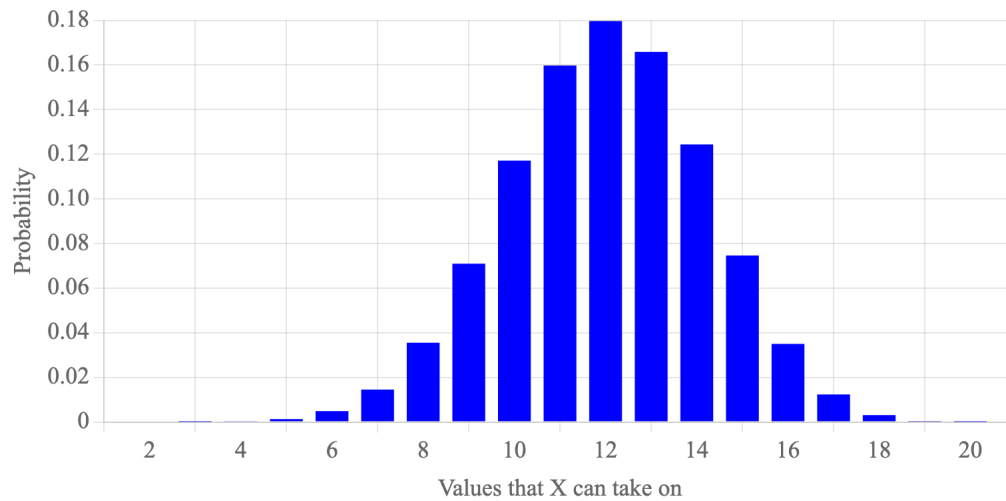
**PMF equation:**  $P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$

**Expectation:**  $E[X] = n \cdot p$

**Variance:**  $\text{Var}(X) = n \cdot p \cdot (1 - p)$

**PMF graph:**

Parameter  $n$ :  Parameter  $p$ :



## Poisson Random Variable

**Notation:**  $X \sim \text{Poi}(\lambda)$

**Description:** Number of events in a fixed time frame if (a) the events occur with a constant mean rate and (b) they occur independently of time since last event.

**Parameters:**  $\lambda \in \{0, 1, \dots\}$ , the constant average rate.

**Support:**  $x \in \{0, 1, \dots\}$

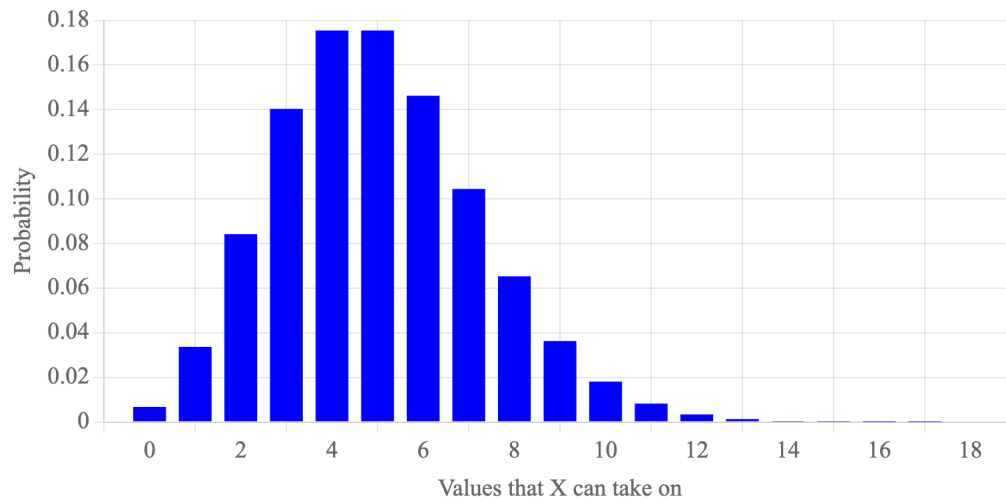
**PMF equation:**  $P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$

**Expectation:**  $E[X] = \lambda$

**Variance:**  $\text{Var}(X) = \lambda$

**PMF graph:**

Parameter  $\lambda$ :



## Geometric Random Variable

**Notation:**  $X \sim \text{Geo}(p)$

**Description:** Number of experiments until a success. Assumes independent experiments each with probability of success  $p$ .

**Parameters:**  $p \in [0, 1]$ , the probability that a single experiment gives a "success".

**Support:**  $x \in \{1, \dots, \infty\}$

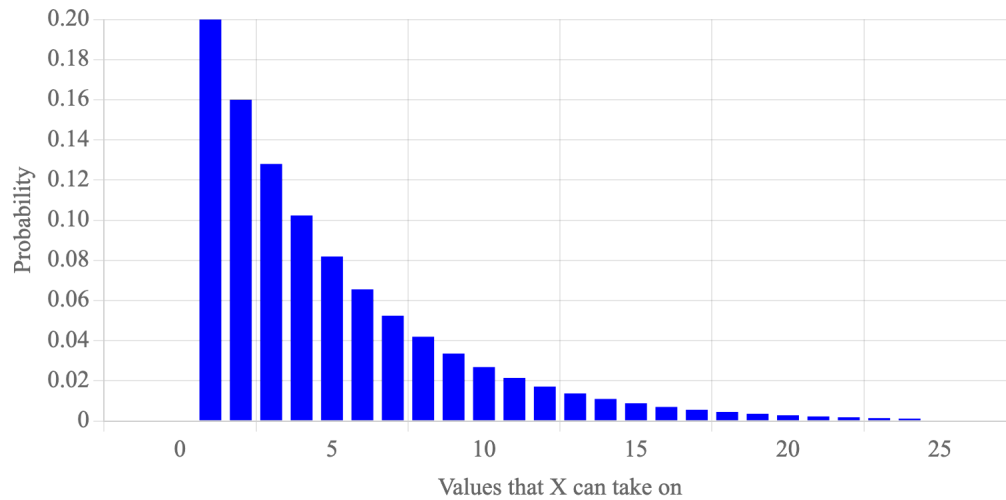
**PMF equation:**  $P(X = x) = (1 - p)^{x-1}p$

**Expectation:**  $E[X] = \frac{1}{p}$

**Variance:**  $\text{Var}(X) = \frac{1-p}{p^2}$

**PMF graph:**

Parameter  $p$ :



## Negative Binomial Random Variable

**Notation:**  $X \sim \text{NegBin}(r, p)$

**Description:** Number of experiments until  $r$  successes. Assumes each experiment is independent with probability of success  $p$ .

**Parameters:**  $r > 0$ , the number of success we are waiting for.

$p \in [0, 1]$ , the probability that a single experiment gives a "success".

**Support:**  $x \in \{r, \dots, \infty\}$

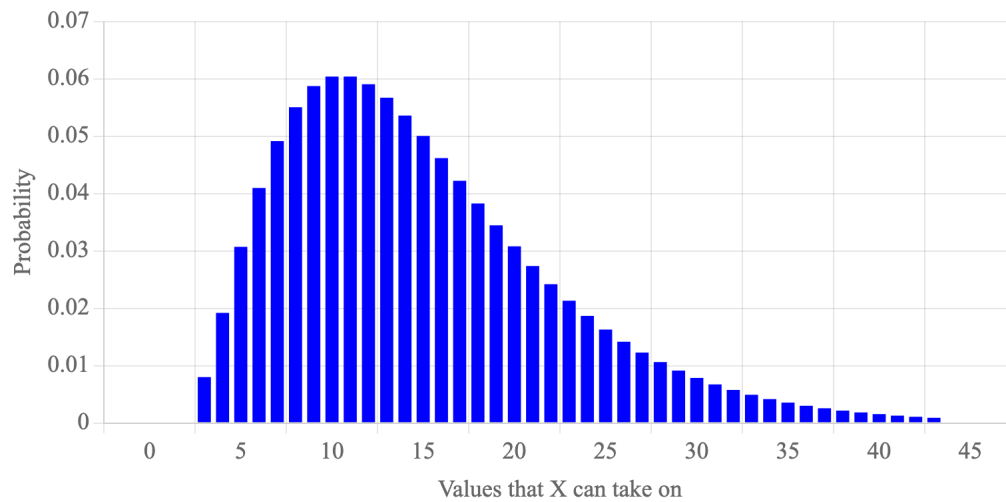
**PMF equation:**  $P(X = x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$

**Expectation:**  $E[X] = \frac{r}{p}$

**Variance:**  $\text{Var}(X) = \frac{r(1-p)}{p^2}$

**PMF graph:**

Parameter  $r$ :  Parameter  $p$ :



# Continuous Random Variables

## Uniform Random Variable

**Notation:**  $X \sim \text{Uni}(\alpha, \beta)$

**Description:** A continuous random variable that takes on values, with equal likelihood, between  $\alpha$  and  $\beta$

**Parameters:**  $\alpha \in \mathbb{R}$ , the minimum value of the variable.  
 $\beta \in \mathbb{R}$ ,  $\beta > \alpha$ , the maximum value of the variable.

**Support:**  $x \in [\alpha, \beta]$

**PDF equation:**  $f(x) = \begin{cases} \frac{1}{\beta - \alpha} & \text{for } x \in [\alpha, \beta] \\ 0 & \text{else} \end{cases}$

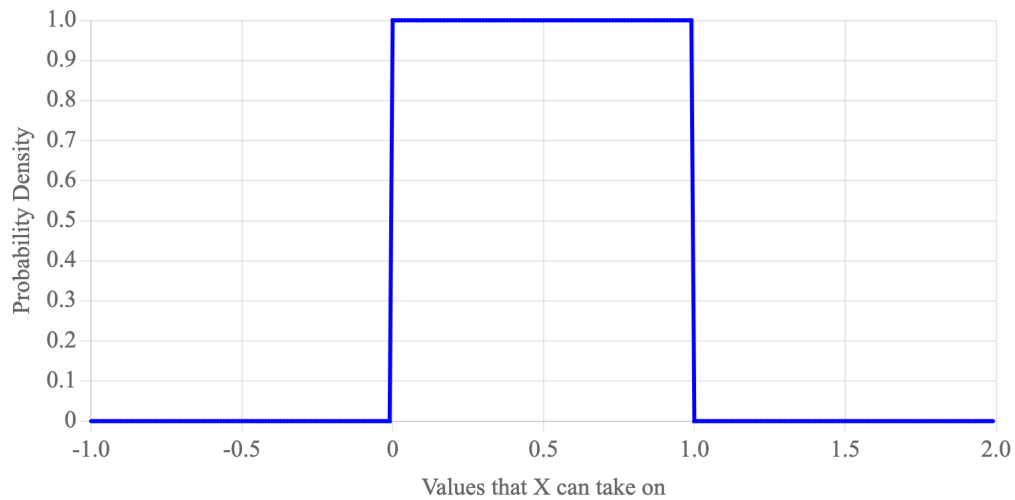
**CDF equation:**  $F(x) = \begin{cases} \frac{x - \alpha}{\beta - \alpha} & \text{for } x \in [\alpha, \beta] \\ 0 & \text{for } x < \alpha \\ 1 & \text{for } x > \beta \end{cases}$

**Expectation:**  $E[X] = \frac{1}{2}(\alpha + \beta)$

**Variance:**  $\text{Var}(X) = \frac{1}{12}(\beta - \alpha)^2$

**PDF graph:**

Parameter  $\alpha$ :  Parameter  $\beta$ :



## Exponential Random Variable

**Notation:**  $X \sim \text{Exp}(\lambda)$

**Description:** Time until next events if (a) the events occur with a constant mean rate and (b) they occur independently of time since last event.

**Parameters:**  $\lambda \in \{0, 1, \dots\}$ , the constant average rate.

**Support:**  $x \in \mathbb{R}^+$

**PDF equation:**  $f(x) = \lambda e^{-\lambda x}$

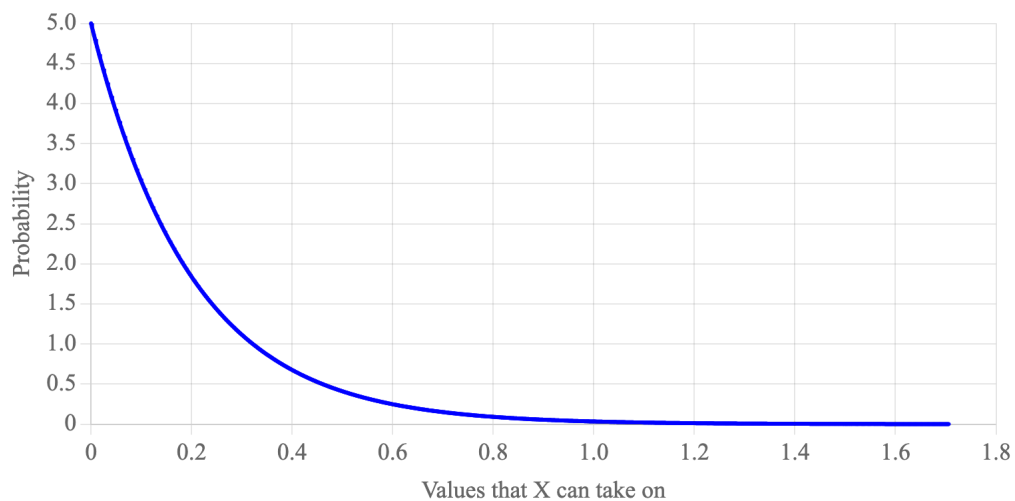
**CDF equation:**  $F(x) = 1 - e^{-\lambda x}$

**Expectation:**  $E[X] = 1/\lambda$

**Variance:**  $\text{Var}(X) = 1/\lambda^2$

**PDF graph:**

Parameter  $\lambda$ :



## Normal (aka Gaussian) Random Variable

**Notation:**  $X \sim N(\mu, \sigma^2)$

**Description:** A common, naturally occurring distribution.

**Parameters:**  $\mu \in \mathbb{R}$ , the mean.  
 $\sigma^2 \in \mathbb{R}$ , the variance.

**Support:**  $x \in \mathbb{R}$

**PDF equation:**  $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

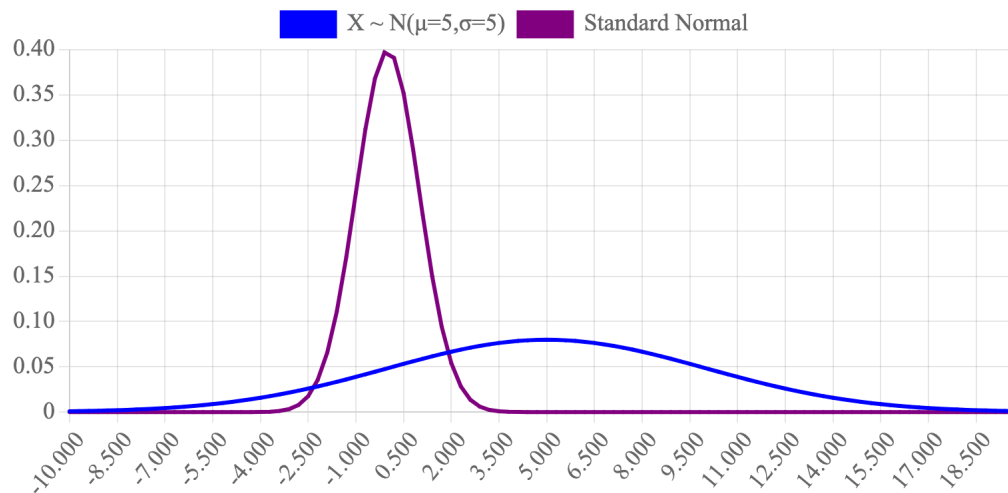
**CDF equation:**  $F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$  Where  $\Phi$  is the CDF of the standard normal

**Expectation:**  $E[X] = \mu$

**Variance:**  $\text{Var}(X) = \sigma^2$

**PDF graph:**

Parameter  $\mu$ :  Parameter  $\sigma$ :





## Beta Random Variable

**Notation:**  $X \sim \text{Beta}(a, b)$

**Description:** A belief distribution over the value of a probability  $p$  from a Binomial distribution after observing  $a - 1$  successes and  $b - 1$  fails.

**Parameters:**  $a > 0$ , the number successes + 1  
 $b > 0$ , the number of fails + 1

**Support:**  $x \in [0, 1]$

**PDF equation:**  $f(x) = B \cdot x^{a-1} \cdot (1-x)^{b-1}$

**CDF equation:** No closed form

**Expectation:**  $E[X] = \frac{a}{a+b}$

**Variance:**  $\text{Var}(X) = \frac{ab}{(a+b)^2(a+b+1)}$

**PDF graph:**

Parameter  $a$ :  Parameter  $b$ :

