Supporting Document for

A Novel Fuzzy Large Margin Distribution Machine with Unified Pinball Loss

This PDF file includes:

Recursive algorithm of FUPLDM.

The scoring strategy of FUPLDM on UCI dataset is shown in Table S1.

Algorithm accuracy on UCI dataset with Gaussian noise (including Tables S2 and S3).

The complete experimental results of parameter sensitivity analysis are shown in Figure S1.

Supporting Document

Recursive Algorithm of FUPLDM

Theorem 1. *The final dual QPP of FUPLDM can be written as:*

$$\begin{aligned} \min_{\eta} \quad & \frac{1}{2}\eta^T A\eta + (\frac{\lambda_2 Ae - me}{m})^T\eta, \qquad (\tau \in [-1,0) \cup (0,1]) \\ s.t. \quad & -v\left|\tau\right| C_i s_i \le \eta_i \le C_i s_i, \ i = 1, ..., m. \end{aligned} \tag{1}$$

Since (1) is a simple decoupled frame constraint and convex quadratic objective function, it can be solved efficiently by the two-coordinate descent method [1]. In the two-coordinate descent method [2], one variable is chosen to be minimized while the other variables are kept constant in each iteration, and a closed form solution is obtained in each iteration. Specifically, the following issues need to be addressed

$$\begin{aligned} \min_{t} \quad & f(\eta + te_i), \\ \text{s.t.} \quad & -v\left|\tau\right| C_i s_i \le \eta + te_i \le C_i s_i, \end{aligned} \tag{2}$$

where e_i represents a one-hot vector whose i-th component is 1 and the rest are 0. Let $A = [a_{ij}]_{i,j=1,...,m}$, we can get

$$f(\eta + te_i) = \frac{1}{2}a_{ii}t^2 + [\nabla f(\eta)]_i t + f(\eta), \tag{3}$$

where $[\nabla f(\eta)]_i$ is the i-th component of the gradient $\nabla f(\eta)$. Since $f(\eta)$ is a constant with respect to t, it can be removed. We can easily find that Eq. (2) has an optimal value at $t = 0$ if and only if $[\nabla^o f(\eta)]_i = 0$, which is as follows

$$[\nabla^o f(\eta)]_i = \begin{cases} [\nabla f(\eta)]_i, \ -v\left|\tau\right| C_i s_i \le \eta_i \le C_i s_i, \\ \min\left(-v\left|\tau\right| C_i s_i, [\nabla[f(\eta)]_i\right), \ \eta_i = -v\left|\tau\right| C_i s_i, \\ \max\left(-v\left|\tau\right| C_i s_i, [\nabla f(\eta)]_i\right), \ \eta_i = C_i s_i. \end{cases} \tag{4}$$

A closed-form solution to Eq. (1) is expressed as

$$\eta_i^{new} = \min(\max(\eta_i - \frac{[\nabla f(\eta)]_i}{a_{ii}}, -v\left|\tau\right| C_i s_i), C_i s_i), \tag{5}$$

where $f(\eta)$ is the objective function of Eq. (1). Meanwhile, η also can be solved by using the programming function $quadprog$ in the MATLAB toolbox.

The Scoring Strategy of FUPLDM on UCI datasset

Assuming $N^{\bullet}_{\text{algi}}, N^{\Omega}_{\text{algi}}, N^{\circ}_{\text{algi}}$ denotes the number of wins, draws, and losses for algorithm i in the j-th dataset, respectively. Calculate the $\sum N^{\diamondsuit}_{\text{algi}} (\diamondsuit \in \{\bullet, \Omega, \circ\})$ of each algorithm, which represents the 'WIN/DRAW/LOSS' project score of the algorithm, respectively. The final score for each algorithm is indicated with $\sum (N^{\bullet}_{\text{algi}} + N^{\Omega}_{\text{algi}}/2 - N^{\circ}_{\text{algi}}) + 80$. $(0 \leq FinalScore \leq 160)$. The scoring strategy on 6 algorithms are shown in Table S1.

TABLE S1: Scoring Strategy Sheet (#Style).

	SVM	Pin-SVM	UPSVM	LDM	F-LDM	FUPLDM	Total Score
WIN	$\sum N^{\bullet}_{\mathrm{SVM}}$	$\sum N^{\bullet}_{\mathrm{Pin-SVM}}$	$\sum N^{\bullet}_{\mathrm{UPSVM}}$	$\sum N^{\bullet}_{\mathrm{LDM}}$	$\sum N^{\bullet}_{\mathrm{F-LDM}}$	$\sum N^{\bullet}_{\mathrm{FUPLDM}}$	
DRAW	$\sum N^{\Omega}_{\mathrm{SVM}}$	$\sum N^{\Omega}_{\mathrm{Pin-SVM}}$	$\sum N^{\Omega}_{\mathrm{UPSVM}}$	$\sum N^{\Omega}_{\mathrm{LDM}}$	$\sum N^{\Omega}_{\mathrm{F-LDM}}$	$\sum N^{\Omega}_{\mathrm{FUPLDM}}$	
LOSS	$\sum N^{\circ}_{\mathrm{SVM}}$	$\sum N^{\circ}_{\mathrm{Pin-SVM}}$	$\sum N^{\circ}_{\mathrm{UPSVM}}$	$\sum N^{\circ}_{\mathrm{LDM}}$	$\sum N^{\circ}_{\mathrm{F-LDM}}$	$\sum N^{\circ}_{\mathrm{FUPLDM}}$	$\sum (N^{\bullet}_{\text{algi}} + N^{\Omega}_{\text{algi}}/2 - N^{\circ}_{\text{algi}}) + 80$
Final Score	-	-	-	-	-	-	

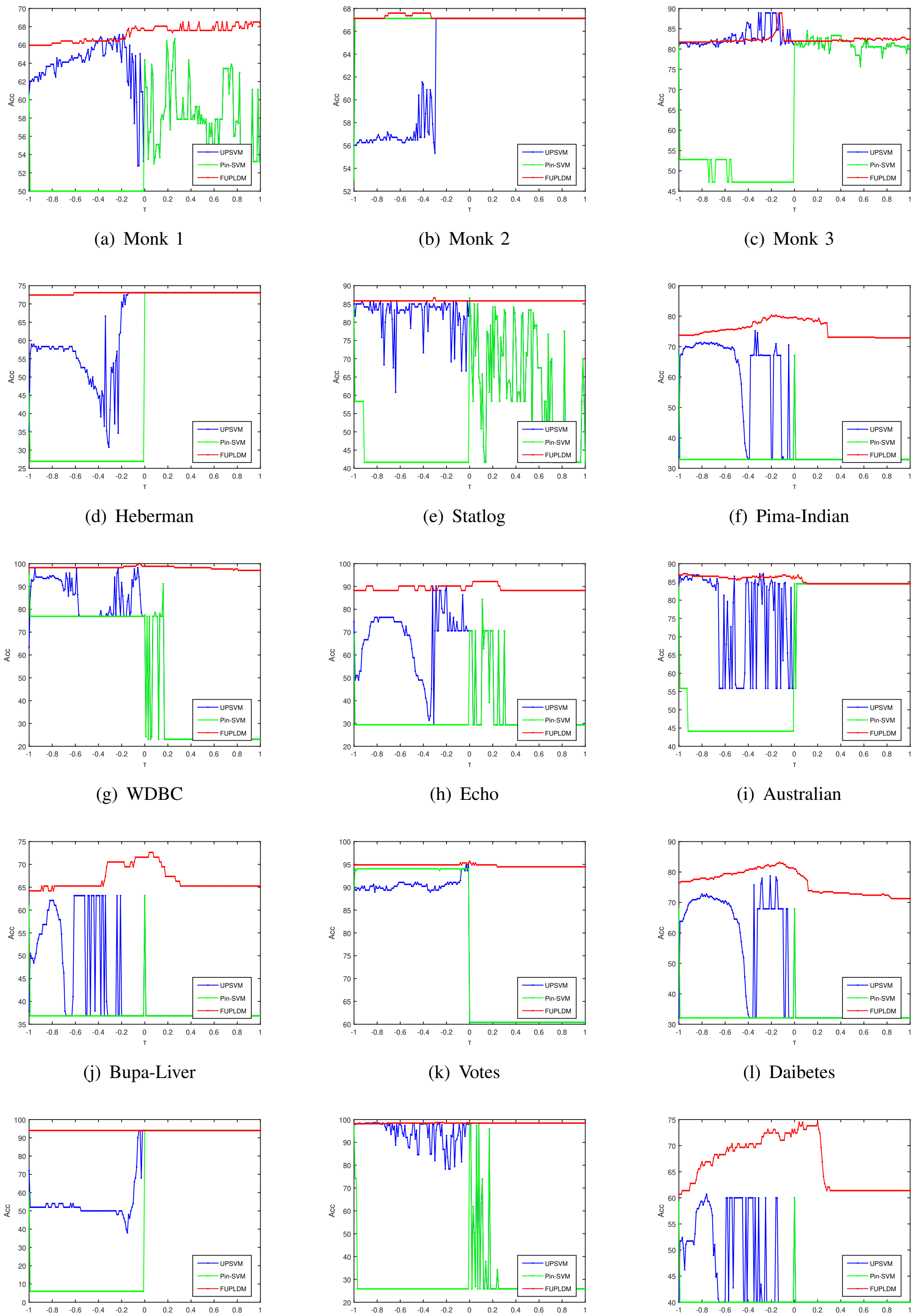
TABLE S2: Accuracy Comparison of SVM, Pin-SVM, UPSVM, LDM, F-LDM and FUPLDM on the Noisy UCI Dataset.

Dataset	SVM	Pin-SVM		UPSVM		LDM		F-LDM		FUPLDM	
	Accuracy/tau	Accuracy/tau	ΔAcc	Acc/tau	ΔAcc	Accuracy/tau	ΔAcc	Accuracy/tau	ΔAcc	Accuracy/tau	ΔAcc
	Time/C	Time/C	(Vs. SVM)	Time/C	(Vs. SVM)	Time/C	(Vs. SVM)	Time/C	(Vs. SVM)	Time/C	(Vs. SVM)
Monk 1(r = 0.05)	57.87/0	62.04/0.07	4.17	60.65/-0.75	2.78	61.11/0	3.24	63.43/0	5.56	**64.12/-0.37**	**6.25**
	1.16/0.25	1.31/0.06		0.15/0.25		0.04/4.00		0.04/2.00		**0.04/4.00**	
r = 0.1	57.64/0	59.95/-0.29	2.31	**65.05/-0.34**	**7.41**	61.11/0	3.47	63.89/0	6.25	64.58/-0.29	6.94
	0.87/0.25	0.98/0.06		0.98/4.00		0.89/4.00		0.98/8.00		**0.87/0.50**	
r = 0.5	57.64/0	58.33/0.06	0.69	**65.28/-0.33**	**7.64**	61.34/0	3.70	64.35/0	6.71	64.58/-0.24	6.94
	0.99/0.06	1.09/0.06		1.09/4.00		0.04/2.00		**0.02/2.00**		0.03/4.00	
Monk 2(r = 0.05)	**67.13/0**	**67.13/-0.99**	0.00	**67.13/-0.71**	0.00	**67.13/0**	0.00	**67.13/0**	0.00	**67.13/-1**	0.00
	1.60/0.02	1.81/0.02		0.23/0.02		**0.05/0.02**		**0.05/0.02**		**0.05/0.02**	
r = 0.1	**67.13/0**	**67.13/-1**	0.00	**67.13/-0.49**	0.00	**67.13/0**	0.00	**67.13/0**	0.00	**67.13/-1**	0.00
	1.27/0.03	1.45/0.03		1.40/0.03		1.28/0.03		1.28/0.03		**1.26/0.03**	
r = 0.5	**67.13/0**	**67.13/-0.99**	0.00	**67.13/-0.49**	0.00	**67.13/0**	0.00	**67.13/0**	0.00	**67.13/-1**	0.00
	1.23/0.03	1.39/0.03		1.34/0.03		**0.04/0.03**		0.05/0.03		**0.04/0.03**	
Monk 3(r = 0.05)	72.45/0	**74.07/0.61**	**1.62**	**74.07/0.61**	**1.62**	72.45/0	0.00	72.45/0	0.00	72.92/0.96	0.47
	1.12/0.50	1.28/0.50		0.16/0.50		0.04/0.50		**0.03/0.50**		**0.03/4.00**	
r = 0.1	71.76/0	**73.61/0.91**	**1.85**	**73.61/0.78**	**1.85**	72.45/0	0.69	72.45/0	0.69	72.92/0.91	1.16
	0.87/0.25	0.97/0.50		0.93/0.50		0.69/0.25		**0.68/0.25**		**0.68/4.00**	
r = 0.5	72.69/0	73.61/0.37	0.92	73.61/0.37	0.92	72.69/0	0.00	72.92/0	0.23	**73.84/0.9**	**1.15**
	0.88/0.25	0.98/0.25		0.98/0.25		**0.02/0.13**		0.04/1.00		**0.02/8.00**	
Heberman(r = 0.05)	73.08/0	73.08/0	0.00	**76.28/-0.43**	**3.20**	73.08/0	0.00	73.08/0	0.00	73.08/-1	0.00
	0.55/0.02	0.74/0.02		0.12/0.02		0.05/0.02		**0.04/0.02**		0.05/0.02	
r = 0.1	**73.08/0**	**73.08/-1**	0.00	**73.08/-0.17**	0.00	**73.08/0**	0.00	**73.08/0**	0.00	**73.08/-1**	0.00
	0.43/0.03	0.56/0.03		0.51/0.03		0.45/0.03		0.45/0.03		**0.42/0.03**	
r = 0.5	73.08/0	**76.28/-1**	**3.20**	73.08/-0.16	0.00	73.08/0	0.00	73.08/0	0.00	73.08/-1	0.00
	0.46/0.03	0.55/0.06		0.54/0.03		0.04/0.03		0.04/0.03		**0.03/0.03**	
Statlog(r = 0.05)	84.17/0	84.17/0	0.00	84.17/0	0.00	**85/0**	**0.83**	84.17/0	0.00	**85/-0.25**	**0.83**
	0.46/0.06	0.73/0.06		0.12/0.06		0.05/0.03		**0.04/0.03**		0.05/0.03	
r = 0.1	84.17/0	**85.00/-0.46**	**0.83**	**85.00/-0.23**	**0.83**	**85.00/0**	**0.83**	84.17/0	0.00	**85.00/-0.46**	**0.83**
	0.35/0.06	0.53/0.25		0.52/0.25		**0.22/0.03**		**0.22/0.06**		**0.22/0.03**	
r = 0.5	83.33/0	83.33/0	0.00	**85.83/-0.78**	**2.50**	85/0	1.67	84.17/0	0.84	85/-0.62	1.67
	0.35/0.13	0.52/0.13		**0.52/0.50**		0.05/0.03		**0.03/0.03**		0.04/0.03	
Pima-Indian(r = 0.05)	67.09/0	69.87/-1	2.78	67.09/-0.37	0.00	78.85/0	11.76	78.85/0	11.76	**79.06/-0.14**	**11.97**
	3.13/0.02	3.97/0.13		0.58/0.02		0.15/4.00		**0.13/2.00**		**0.13/1.00**	
r = 0.1	67.09/0	67.09/-0.21	0.00	76.71/-0.28	9.62	78.85/0	11.76	**79.06/0**	**11.97**	**79.06/-0.21**	**11.97**
	2.54/0.03	3.27/0.03		3.30/2.00		**2.49/8.00**		2.50/4.00		2.50/1.00	
r = 0.5	67.09/0	69.87/-1	2.78	74.79/-0.2	7.70	79.06/0	11.97	79.06/0	11.97	**79.27/0.01**	**12.18**
	2.53/0.03	3.09/0.03		3.08/4.00		0.08/4.00		0.09/4.00		**0.07/4.00**	
WDBC(r = 0.05)	91.72/0	91.72/-1	0.00	95.86/-0.62	4.14	96.45/0	4.73	95.86/0	4.14	**97.04/0.32**	**5.32**
	2.17/0.02	4.64/0.03		0.85/0.02		**0.30/8.00**		**0.30/0.02**		0.33/8.00	
r = 0.1	90.53/0	95.86/0.32	5.33	96.45/-0.79	5.92	96.45/0	5.92	95.86/0	5.33	**97.04/0.32**	**6.51**
	1.67/0.03	4.19/1.00		4.09/2.00		3.69/8.00		3.69/0.03		3.69/8.00	
r = 0.5	89.94/0	94.08/-1	4.14	96.45/-0.78	6.51	96.45/0	6.51	95.86/0	5.92	**97.04/0.39**	**7.10**
	1.55/0.03	3.55/0.13		3.59/4.00		0.19/8.00		**0.16/0.03**		0.19/8.00	
Echo(r = 0.05)	76.47/0	90.2/0.05	13.73	90.2/-0.29	13.73	92.16/0	15.69	94.12/0	17.65	**96.08/-0.11**	**19.61**
	0.12/0.02	0.18/0.02		0.03/0.02		**0.02/0.02**		**0.02/0.50**		**0.02/2.00**	
r = 0.1	70.59/0	90.2/-0.11	19.61	92.16/-0.32	21.57	94.12/0	23.53	94.12/0	23.53	**96.08/-0.11**	**25.49**
	0.09/0.03	0.15/2.00		0.14/0.50		0.20/0.06		0.20/0.13		0.20/4.00	
r = 0.5	70.59/0	94.12/0.09	23.53	94.12/0.09	23.53	94.12/0	23.53	94.12/0	23.53	**96.08/-0.11**	**25.49**
	0.09/0.03	0.13/2.00		0.13/2.00		**0.01/0.06**		**0.01/0.13**		**0.01/4.00**	
Australian(r = 0.05)	55.86/0	75.17/0.18	19.31	67.93/-0.83	12.07	74.48/0	18.62	74.48/0	18.62	**75.86/-0.09**	**20.00**
	3.06/0.02	5.58/0.25		0.89/0.02		0.29/1.00		**0.28/2.00**		0.30/4.00	
r = 0.1	55.86/0	75.86/-0.1	20.00	**77.24/-0.97**	**21.38**	74.48/0	18.62	74.48/0	18.62	76.21/-0.1	20.35
	2.27/0.03	4.27/0.25		4.10/4.00		**2.26/1.00**		**2.26/2.00**		**2.26/4.00**	
r = 0.5	55.86/0	74.83/0.41	18.97	**77.24/-0.97**	**21.38**	74.83/0	18.97	74.48/0	18.62	76.21/-0.09	20.35
	2.36/0.03	4.28/0.06		4.35/0.50		**0.15/1.00**		0.18/2.00		0.18/4.00	

TABLE S3: (*Continued*) Accuracy Comparison of SVM, Pin-SVM, UPSVM, LDM, F-LDM and FUPLDM on the Noisy UCI Datasets.

Dataset	SVM	Pin-SVM		UPSVM		LDM		F-LDM		FUPLDM	
	Accuracy/τ	Accuracy/τ	$\Delta\tau$	Accuracy/τ	ΔAcc	Accuracy/τ	ΔAcc	Accuracy/τ	ΔAcc	Accuracy/τ	ΔAcc
	Time/C	Time/C	(Vs. SVM)	Time/C	(Vs. SVM)	Time/C	(Vs. SVM)	Time/C	(Vs. SVM)	Time/C	(Vs. SVM)
Bupa-Liver(r = 0.05)	63.16/0	63.16/-1	0.00	63.16/0	0.00	**73.68/0**	**10.52**	72.63/0	9.47	**73.68/-0.09**	**10.52**
	0.75/0.03	1.49/0.13		0.27/0.03		0.11/4.00		**0.09/8.00**		0.11/8.00	
r = 0.1	63.16/0	63.16/-0.1	0.00	63.16/-1	0.00	**73.68/0**	**10.52**	72.63/0	9.47	**73.68/-0.1**	**10.52**
	0.55/0.03	1.09/0.03		1.01/8.00		0.46/4.00		0.46/8.00		**0.40/8.00**	
r = 0.5	63.16/0	63.16/-1	0.00	63.16/-0.61	0.00	**72.63/0**	**9.47**	71.58/0	8.42	**72.63/-0.14**	**9.47**
	0.57/0.03	1.07/0.06		1.05/2.00		**0.06/8.00**		**0.06/8.00**		**0.06/8.00**	
Votes(r = 0.05)	80/0	82.13/-1	2.13	**83.83/-1**	**3.83**	80.43/0	0.43	80.43/0	0.43	81.28/0.11	1.28
	1.09/0.02	1.55/0.02		0.23/0.02		**0.06/0.50**		0.07/0.50		**0.06/2.00**	
r = 0.1	77.87/0	80.85/-0.07	2.98	**84.26/-1**	**6.39**	80.85/0	2.98	80.43/0	2.56	81.28/-0.07	3.41
	0.86/0.03	1.22/0.50		1.20/0.03		1.15/0.25		1.15/0.50		1.15/0.50	
r = 0.5	78.72/0	82.13/0.13	3.41	**85.11/-1**	**6.39**	81.28/0	2.56	81.7/0	2.98	82.13/0.08	3.41
	0.89/0.03	1.22/0.50		1.19/0.50		**0.04/0.25**		0.05/0.50		0.05/2.00	
Daibetes(r = 0.05)	67.91/0	70.15/-1	2.24	77.99/-0.33	10.08	80.22/0	12.31	78.73/0	10.82	**81.34/-0.35**	**13.43**
	3.66/0.02	6.28/2.00		1.17/0.02		0.39/2.00		0.41/1.00		**0.37/1.00**	
r = 0.1	67.91/0	68.28/-0.35	0.37	80.22/-0.23	12.31	80.22/0	12.31	78.73/0	10.82	**81.34/-0.35**	**13.43**
	2.81/0.03	5.02/0.50		4.88/0.25		3.46/2.00		3.46/8.00		**3.46/1.00**	
r = 0.5	67.91/0	70.15/-1	2.24	80.22/-0.22	12.31	80.22/0	12.31	78.73/0	10.82	**81.72/-0.35**	**13.81**
	2.91/0.03	4.99/0.13		4.99/8.00		0.31/2.00		0.27/8.00		**0.21/1.00**	
Fertility(r = 0.05)	94/0	94/0	0.00	94/-1	0.00	94/0	0.00	94/0	0.00	**96/-0.03**	**2.00**
	0.07/0.02	0.09/0.02		**0.01/0.02**		**0.01/0.02**		**0.01/0.02**		**0.01/2.00**	
r = 0.1	94.00/0	94/-0.04	0.00	94/-1	0.00	94/0	0.00	94/0	0.00	**96/-0.04**	**2.00**
	0.05/0.03	0.06/0.03		0.06/0.03		0.10/0.03		0.12/0.03		0.08/1.00	
r = 0.5	94/0	94/0	0.00	94/-1	0.00	94/0	0.00	94/0	0.00	**96/-0.04**	**2.00**
	0.05/0.03	0.06/0.03		0.06/0.03		**0.01/0.03**		**0.01/0.03**		**0.01/1.00**	
Breast-cancer(r = 0.05)	98.43/0	98.43/-1	0.00	98.43/-0.88	0.00	**98.69/0**	0.26	**98.69/0**	0.26	**98.69/-0.3**	0.26
	2.09/0.03	3.06/0.03		3.03/0.25		0.09/0.06		0.09/0.06		**0.08/0.06**	
r = 0.1	**98.43/0**	**98.43/-0.14**	0.00	**98.43/-1**	0.00	**98.43/0**	0.00	**98.43/0**	0.00	**98.43/-0.14**	0.00
	2.06/0.03	2.99/0.03		2.95/0.03		1.58/0.13		1.58/0.13		**1.57/0.13**	
r = 0.5	98.43/0	98.43/-1	0.00	98.43/-1	0.00	98.43/0	0.00	98.43/0	0.00	**98.69/-0.05**	**0.26**
	2.05/0.03	2.92/0.03		2.96/0.03		**0.08/0.13**		**0.08/0.13**		**0.08/0.25**	
BUPA(r = 0.05)	60/0	60.69/-1	0.69	**73.79/-0.41**	13.79	**73.79/0**	13.79	68.97/0	8.97	73.1/-0.19	13.10
	0.56/0.25	0.84/0.25		0.80/2.00		0.05/4.00		0.05/4.00		**0.04/2.00**	
r = 0.1	60.00/0	60/-0.19	0.00	60/-0.59	0.00	72.41/0	12.41	**73.79/0**	**13.79**	**73.79/-0.19**	**13.79**
	0.62/0.06	0.80/0.06		0.79/2.00		0.78/8.00		**0.78/8.00**		**0.62/4.00**	
r = 0.5	60/0	60/0	0.00	60/-0.59	0.00	73.1/0	13.10	73.79/0	13.79	**74.48/0.08**	**14.48**
	0.57/0.03	0.80/0.03		0.81/2.00		**0.05/8.00**		**0.05/8.00**		**0.05/8.00**	
Wine(r = 0.05)	77.27/0	77.27/0	0.00	**80.68/-0.01**	3.41	77.27/0	0.00	77.27/0	0.00	79.55/-0.09	2.28
	0.17/0.50	0.23/0.50		0.23/4.00		0.02/0.03		0.02/0.03		**0.01/4.00**	
r = 0.1	79.55/0	81.82/0.72	2.27	**84.09/-0.18**	**4.54**	81.82/0	2.27	81.82/0	2.27	**84.09/0.72**	**4.54**
	0.23/0.06	0.29/0.25		0.29/8.00		0.52/0.50		0.52/0.50		0.50/0.13	
r = 0.5	79.55/0	81.82/0.21	2.27	84.09/-0.24	4.54	81.82/0	2.27	82.95/0	3.40	**86.36/0.7**	**6.81**
	0.15/0.06	0.23/0.13		0.20/2.00		**0.01/0.06**		**0.01/0.50**		**0.01/0.25**	
Average of ΔAcc	-		3.42		5.29		6.32		6.25		**7.36**

* The traversal range of τ is [-1:0.01:1]. 'Time' denotes the CPU time consumption of the algorithm. ΔAcc is the accuracy improvement of the algorithms compared with the conventional SVM.



(a) Monk 1 (b) Monk 2 (c) Monk 3

(d) Heberman (e) Statlog (f) Pima-Indian

(g) WDBC (h) Echo (i) Australian

(j) Bupa-Liver (k) Votes (l) Daibetes

(m) Fertility (n) Breast-cancer (o) BUPA

Fig. S1: The influence of τ on the accuracy of Pin-SVM, UPSVM and FUPLDM algorithms respectively. (In different datasets)

REFERENCES

[1] T. Zhang and Z. H. Zhou, "Large margin distribution machine," in *Proceedings of the 20th ACM SIGKDD international conference on Knowledge discovery and data mining*, 2014, pp. 313–322.

[2] C. J. Hsieh, K. W. Chang, C.-J. Lin, S. S. Keerthi, and S. Sundararajan, "A dual coordinate descent method for large-scale linear svm," in *Proceedings of the 25th International Conference on Machine Learning*, 2008, pp. 408–415.