Homework 12

November 29, 2019

Deadline

Due: December 6, 2019, 23:59. Good luck!

Problem 1

Let $A = \{1, 2, 3, 4, 5\}$, $B = \{a, b, c, d\}$, and $f_1 : A \rightarrow B = \{\langle 1, c \rangle, \langle 2, c \rangle, \langle 3, b \rangle, \langle 4, a \rangle, \langle 5, d \rangle\}$, $f_2 : B \rightarrow A = \{\langle a, 2 \rangle, \langle b, 5 \rangle, \langle c, 1 \rangle, \langle d, 3 \rangle\}$. Determine whether f_1, f_2 have left or right inverse. If so, find the left or right inverse for each function.

Problem 2

Let $h \in A_A$, prove that the state "for any $f, g \in A_A$, if $h \circ f = h \circ g$ then we have $f = g$" is true if and only if h is injective, or:

$$(\forall f)(\forall g)((f \in A_A \wedge g \in A_A \wedge h \circ f = h \circ g) \rightarrow f = g) \Leftrightarrow h \text{ is injective}$$

Problem 3

Design a DFA accepting the language $(a|b)^* c^+$ over the alphabeta $\{a, b, c\}$. (Transition table, transition diagram or giving the transition functions are all acceptable). And show how it accepts the string "$abaacc$" by showing all the changes of states in whole process.

Problem 4

Design a Turing Machine for the language $\{w | w$ has an equal nurnber of 0's and 1's$\}$ over input alphabeta $\Sigma = \{0, 1\}$. (Transition table, transition diagram or giving the transition functions are all acceptable) And show how it accepts the string 100011 by instantaneous descriptions.