

Homework 7

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Problem 1

1. $a > 3$
2. $a \neq 0$
3. $b - c = 5$
4. $a = 10$

Problem 2

Prove $\{a = v0 \wedge b = v1\} c := a; a := b; b := c \{a = v1 \wedge b = v0\}$

- (1) $\{a = v1 \wedge c = v0\} b := c \{a = v1 \wedge b = v0\}$
- (2) $\{b = v1 \wedge c = v0\} a := b \{a = v1 \wedge c = v0\}$
- (3) $\{b = v1 \wedge a = v0\} c := a \{b = v1 \wedge c = v0\}$
- (4) $\{a = v0 \wedge b = v1\} c := a; a := b; b := c \{a = v1 \wedge b = v0\} \quad (1)(2)(3)$

Problem 3

Prove $\{T\} \text{if}(a == 1) \text{ then } b := 3 \text{ else } b := 1 \{b = 1 \vee b = 3\}$

- (1) $\{3 = 1 \vee 3 = 3\} b := 3 \{b = 1 \vee b = 3\}$
- (2) $\{1 = 1 \vee 1 = 3\} b := 1 \{b = 1 \vee b = 3\}$
- (3) $T \wedge a == 1 \rightarrow 3 = 1 \vee 3 = 3$
- (4) $T \wedge a \neq 1 \rightarrow 3 = 1 \vee 3 = 3$
- (5) $\{T \wedge a == 1\} b := 3 \{b = 1 \vee b = 3\} \quad (1)(3)$
- (6) $\{T \wedge \neg(a == 1)\} b := 1 \{b = 1 \vee b = 3\} \quad (2)(4)$
- (7) $\{T\} \text{if}(a == 1) \text{ then } b := 3 \text{ else } b := 1 \{b = 1 \vee b = 3\} \quad (5)(6)$

Problem 4

Prove $\{a = 3\} \text{while}(a \neq 10) a := a + 1 \{a = 10\}$. The loop invariant is $a \leq 10$

- (1) $\{a + 1 \leq 10\} a := a + 1 \{a \leq 10\}$
- (2) $a = 3 \wedge a \neq 10 \rightarrow a + 1 \leq 10$
- (3) $\{a = 3 \wedge a \neq 10\} a := a + 1 \{a \leq 10\} \quad (1)(2)$
- (4) $\{a = 3\} \text{while}(a \neq 10) a := a + 1 \{a \leq 10 \wedge a \neq 10\}$
- (5) $a \leq 10 \wedge a \neq 10 \rightarrow a = 10$
- (6) $\{a = 3\} \text{while}(a \neq 10) a := a + 1 \{a = 10\} \quad (4)(5)$

Problem 5

Write a weaker precondition and a stronger postcondition for $\{b = 7\} a := b + 3 \{a > 5\}$

Weakest: $b > 2$

Stronger: $a = 10$

Problem 6

Compute $\text{wlp}(a := b-1; \text{if}(a == 10) \text{ then } c := a + 1 \text{ else } c := a + 2, c > 4)$

$\{P\} a := b-1; \text{if}(a == 10) \text{ then } c := a + 1 \text{ else } c := a + 2 \{c > 4\}$

$= \text{wlp}(a := b-1, (a == 10) \rightarrow \text{wlp}(c := a+1, c > 4) \wedge (a \neq 10) \rightarrow \text{wlp}(c := a+2, c > 4))$

$= \text{wlp}(a := b-1, a == 10 \rightarrow a > 3 \wedge a \neq 10 \rightarrow a > 2)$

$= b == 11 \rightarrow b > 4 \wedge b \neq 11 \rightarrow b > 3$