Homework 11

November 25, 2019

Deadline

Due: December 2, 2019, 23:59. Good luck!

Problem 1

Find $t(R)$ using warshall algorithm for the relation given blow.

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Problem 2

Theorem 10.5.11: For a relation R on a non-empty set, prove that:

1. If R is reflexive, then $s(R)$ and $t(R)$ are both reflexive.
2. If R is transitive, then $r(R)$ is transitive. And find a counterexample to show that $s(R)$ is not transitive.

Problem 3

An equivalence closure $e(R)$ for releation R is defined by:

1. $e(R)$ is an equivalence relation.
2. For any equivalence R', if $R \subseteq R'$, then $e(R) \subseteq R'$

For a relation R on a non-empty set, prove that $tsr(R)$ (defined in theorem 10.5.12) is the equivalence closure of R.

Problem 4

Determine wether $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is surjective if:

1. $f(m, n) = m + n$
2. $f(m, n) = m - n$
3. $f(m, n) = |m| - |n|$
4. $f(m, n) = m^2 + n^2$
5. $f(m, n) = m^2 - n^2$

Problem 5

For every function below, answer the questions:

1. Whether the function is injective, surjective or bijective. If it is bijective, write down the expression of f^{-1}
2. Write down the image of the function and the inverse image of a given set S.
3. The relation $R = \{\langle x, y\rangle | x, y \in dom(f) \land f(x) = f(y)\}$ is an equivalence relation on $dom(f)$, find this relation for the function.

All the functions:

1. $f : \mathbb{R} \to (0, \infty), f(x) = 2^x, S = [1, 2]$
2. $f : \mathbb{N} \to \mathbb{N}, f(n) = 2n + 1, S = \{2, 3\}$
3. $f : \mathbb{Z} \to \mathbb{N}, f(x) = |x|, S = \{0, 2\}$
4. $f : \mathbb{N} \to \mathbb{N} \times \mathbb{N}, f(n) = \langle n, n + 1\rangle, S = \langle 2, 2\rangle$
5. $f : [0, 1] \to [0, 1], f(x) = \frac{2x+1}{4}, S = [0, \frac{1}{2}]$

Problem 6

Let $f, g \in A_B$, and $f \cap g \neq \varnothing$, are $f \cap g$ and $f \cup g$ are functions? If so, prove it. If not, show the counterexample.