Homework11
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Problem1
Find t(R) using warshall algorithm for the relation given blow.
MR=

0 1 0 0
0 0 1 0
0 0 0 1
0 1 0 0

Warshall:
i=1,j=1~4
0 1 0 0
0 0 1 0
0 0 0 1
0 1 0 0

i=2,j=1	i=2,j=2	i=2,j=3	i=2,j=4
0 1 1 0			0 1 1 0
0 0 1 0			0 0 1 0
0 0 0 1			0 0 0 1
0 1 0 0			0 1 1 0
i=3,j=1	i=3,j=2	i=3,j=3	i=3,j=4
0 1 1 1	0 1 1 1	0 1 1 1	0 1 1 1
0 0 1 0	0 0 1 1	0 0 1 1	0 0 1 1
0 0 0 1	0 0 0 1	0 0 0 1	0 0 0 1
0 1 1 0	0 1 1 0	0 1 1 0	0 1 1 1
i=4,j=1	i=4,j=2	i=4,j=3	i=4,j=4
0 1 1 1	0 1 1 1	0 1 1 1	0 1 1 1
0 0 1 1	0 1 1 1	0 1 1 1	0 1 1 1
0 0 0 1	0 0 0 1	0 1 1 1	0 1 1 1
0 1 1 1	0 1 1 1	0 1 1 1	0 1 1 1

T(R)
0 1 1 1
0 1 1 1
0 1 1 1
0 1 1 1

Problem2

1. If R is reflexive, then s(R) and t(R) are both reflexive.

2. If R is transitive, then r(R) is transitive. And find a counterexample to show that s(R) is not transitive.

1. R is reflexive $\Rightarrow R_{i,i} = 1$
   令 $S=s(R)=R\cup R^{-1} \Rightarrow S_{i,i} = R_{i,i} \vee R^{-1}_{i,i} = 1 \Rightarrow$ s(R) is reflexive
   令 $T=t(R)=R\cup R^2 \ldots \cup R^n \Rightarrow T_{i,i} = R_{i,i} \vee R^2_{i,i} \vee \ldots \vee R^n_{i,i} = 1 \Rightarrow$ t(R) is reflexive
2. R is transive $\Rightarrow R_{ij} = 1 \wedge R_{jk} = 1 \longrightarrow R_{ik} = 1$
   令 $F=r(R)=R \cup R^0 \Rightarrow$ 对于任意互不相等的 ijk，若$F_{i,j} = F_{j,k} = 1$, 则$R_{i,j} = R_{j,k} = 1 \Rightarrow R_{i,k} = 1 \Rightarrow F_{i,k} = 1 \Rightarrow$ F is transive

Counterexample:

R=

1 1 1

0 1 0

0 0 0

Problem3

An equivalence closure e(R) for releation R is defined by:

1. e(R) is an equivalence relation.

2. For any equivalence R 0 , if R ⊆ R 0 , then e(R) ⊆ R 0

For a relation R on a non-empty set, prove that tsr(R) (defined in theorem 10.5.12) is the equivalence closure of R.

Obviously, tsr(R) is transitive.

r(R) is reflexive ⇒ s(r(R)) is reflexive ⇒ t(s(r(R))) is reflexive ⇒ tsr(R) is reflexive

ts(r(R)) ⊇ st(r(R)) ⇒ ts(r(R)) is symmetric

So tsr(R) is equivalence relation.

For any equivalence R_0, if $R \subseteq R_0$, then $r(R) \subseteq R_0$, then $s(r(R)) \subseteq R_0$, then $t(s(r(R))) \subseteq R_0$

So,tsy(R) is eauibalence closure of R.

Problem4

Determine wether f : Z × Z → Z is surjective if:

1. f(m, n) = m + n
2. f(m, n) = m − n
3. f(m, n) = |m| − |n|
4. f(m, n) = m 2 + n 2
5. f(m, n) = m 2 − n

1. T
2. T
3. T
4. F
5. T

Problem5

For every function below, answer the questions:

1. Whether the function is injective, surjective or bijective. If it is bijective, write down the expression of f −1

2. Write down the image of the function and the inverse image of a given set S.

3. The relation R = {<x, y>|x, y ∈ dom(f) ∧ f(x) = f(y)} is an equivalence relation on dom(f), find this relation for the function.

All the functions:

1. f : R → (0, ∞), f(x) = 2 x , S = [1, 2]

Injective,surjective,bijective f^{-1}: $(0, \infty) \rightarrow R, f(x) = 1/2$

R [0.5,1]

R={<x,y>|x,y ∈ dom(f) ∧ x=y}

2. f : N → N, f(n) = 2n + 1, S = {2, 3}

Injective, not surjective,not bijective

Image {x|x∈ N ∧ x ≡ 1mod(2)} {1}

R={<x,y>|x,y ∈ dom(f) ∧ x=y}

3. f : Z → N, f(x) = |x|, S = {0, 2}

Not injective,surjective,not bijective

Image N {0,2,-2}

{<x,y>||x|=|y|∧ x ∈ Z ∧ y ∈ Z}

4. f : N → N × N, f(n) = <n, n + 1>, S = <2, 2>

Injective,surjective,bijective $f^{-1}: N \times N \longrightarrow N, f(<n, n+1>) = n$

{<n,n+1>|n∈ N} ∅

{<x,y>|x ∈ N ∧ y ∈ N ∧ x = y}

5. $f : [0, 1] \rightarrow [0, 1], f(x) = (2x+1)/ 4$, $S = [0, 1/2]$
   Injective , not surjective, not bijective
   [1/4 , 3/4]   [0,1/2]
   $\{<x,y> | x,y \in [0,1] \wedge x = y\}$

Problem6

Let f, g $\in A_B$, and $f \cap g \neq \emptyset$, are $f \cap g$ and $f \cup g$ are functions? If so, prove it. If not, show the counterexample.

$f \cap g$ 是。$f \cap g = \{ <a,b> | <a,b> \in f \wedge <a,b> \in g\} \Rightarrow$ 若 $J = dom(f \cap g), K = ran(f \cap g)$则 $f \cap g \in J_K$

$f \cup g$ 不是, 若 $A = [0,1] = B, f(x) = x, g(x) = x^2$, 则 $<1/2,1/2> \in f \cup g \wedge <1/2,1/4> \in f \cup g$, 所以 $f \cup g$ 不是函数