Homework4

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Problem 1

1. **Not wffs**
2. **Not wffs**
3. **Not wffs**
4. **Wffs**
5. **Wffs**

Problem 2

X is bound and Z is free

The scope of the first ∀x is P and Q

The scope of the second ∀x is R

Problem3

1. **f(A,B) means a line connects point A and point B , E(X,Y)means line X and line Y are the same line.**

   (∀A)(∀B) ((∀X)(∀Y) (E(X,f(A,B)) ∧ E(Y,f(A,B)) →E(X,Y)))

2. **P(X)means X works in Shanghai**
   Q(X)means X lives in Shanghai

   ∃X (P(X) ∧ ¬Q(X))

3. **P(Y) means date Y is tomorrow and date Y is fine**
   Q(X) means student X will go swimming

   (∀Y) P(Y)→(∃X) Q(X)

Problem 4

任意一个正整数都是有理数而并非所有有理数都是正整数。

Problem 5

1. **P(x) means X is not smaller than 2**

   (∃X)P(X)

2. ((P(a)→Q(a)) ∨ (P(a)→Q(b)) ∨ (P(a)→Q(c))) ∧ ((P(b)→Q(a)) ∨ (P(b)→Q(b)) ∨ (P(b)→Q(c))) ∧ ((P(c)→Q(a)) ∨ (P(c)→Q(b)) ∨ (P(c)→Q(c)))

= (¬P(a) ∨ Q(a) ∨ Q(b) ∨ Q(c)) ∧ (¬P(b) ∨ Q(a) ∨ Q(b) ∨ Q(c)) ∧ (¬P(c) ∨ Q(a) ∨ Q(b) ∨ Q(c))

3. True
   False
   False
   True
   True
   False

Problem 6

1. $(\forall x)(P(x) \rightarrow (\exists y)Q(x, y))$

   $=(\forall x)(\neg P(x) \vee (\exists y)Q(x, y))$

   $=(\forall x)(\exists y)((\neg P(x) \vee Q(x, y))$

   $=(\forall x)(\exists y)S(x,y)$

   $(\exists x)P(x, y) \leftrightarrow (\forall z)Q(z)$

   $=(\neg(\exists x)P(x, y) \vee (\forall z)Q(z)) \wedge ((\exists x)P(x, y) \vee \neg(\forall z)Q(z))$

   $=((\forall x)\neg P(x, y) \vee (\forall z)Q(z)) \wedge ((\exists x)P(x, y) \vee (\exists z)\neg Q(z))$

   $=(\forall x)(\forall z)(\neg P(x, y) \vee Q(z)) \wedge (\exists x)(P(x, y) \vee \neg Q(x))$

   $=(\forall x)(\forall z)(\neg P(x, y) \vee Q(z)) \wedge (\exists m)(P(m, y) \vee \neg Q(m))$

   $=(\forall x)(\forall z)(\exists m)((\neg P(x, y) \vee Q(z)) \wedge (P(m, y) \vee \neg Q(m)))$

   $=(\forall x)(\forall z)(\exists m)S(x,y,z,m)$

2. $(\forall x)(P(x) \rightarrow (\exists y)Q(x, y)) \vee (\forall z)R(z)$

   $=(\forall x)(\neg(P(x) \vee ((\exists y)Q(x, y)) \vee (\forall z)R(z)))$

   $=(\forall x)(\neg P(x) \vee (\exists y)(\forall z)(Q(x, y) \vee R(z)) \quad)$

   $=(\forall x)(\exists y)(\forall z) (\neg P(x) \vee Q(x, y) \vee R(z))$

   $=(\forall x)(\exists y)(\forall z)M(x,y,z)$

   $=(\forall x)(\forall z)M(x,f(x),z)$

(∃y)(∀x)(∀z)(∃u)(∀v)P(x, y, z, u, v)

=(∀x)(∀z)(∀v)P(x, a, z, f(x,z), v)

Problem 7

P = true Q =true

It will not return unsat

当改变一个变元的真值后使得 **profits** 为 **5** 时返回 **sat**

Problem 8

∅	**|| 1 ∨ 2, 1 ∨ 2, 2 ∨ 3, 3 ∨ 4, 1 ∨ 4**	
1	**||1 ∨ 2, 1 ∨ 2, 2 ∨ 3, 3 ∨ 4, 1 ∨ 4**	
1 2 4	**||1 ∨ 2, 1 ∨ 2, 2 ∨ 3, 3 ∨ 4, 1 ∨ 4**	
1 2 4 3	**||1 ∨ 2, 1 ∨ 2, 2 ∨ 3, 3 ∨ 4, 1 ∨ 4**	**3 ∨ 4 not profit**
1	**||1 ∨ 2, 1 ∨ 2, 2 ∨ 3, 3 ∨ 4, 1 ∨ 4**	
1 2	**||1 ∨ 2, 1 ∨ 2, 2 ∨ 3, 3 ∨ 4, 1 ∨ 4**	
12 3	**||1 ∨ 2, 1 ∨ 2, 2 ∨ 3, 3 ∨ 4, 1 ∨ 4**	
1 2 3 4	**||1 ∨ 2, 1 ∨ 2, 2 ∨ 3, 3 ∨ 4, 1 ∨ 4**	**profit**