Homework 12

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December 11, 2019

Problem 1

Let $A = \{1, 2, 3, 4, 5\}$,$B = \{a, b, c, d\}$, and $f_1 : A \to B = \{\langle 1, c\rangle, \langle 2, c\rangle, \langle 3, b\rangle, \langle 4, a\rangle, \langle 5, d\rangle\}$, $f_2 : B \to A = \{\langle a, 2\rangle, \langle b, 5\rangle, \langle c, 1\rangle, \langle d, 3\rangle\}$. Determine whether f_1,f_2 have left or right inverse. If so , find the left or right inverse for each function.

Solution.

1. Right Inverse. $\{\langle a, 4\rangle, \langle b, 3\rangle, \langle c, 1\rangle, \langle d, 5\rangle\}$
2. Left Inverse. $\{\langle 1, c\rangle, \langle 2, a\rangle, \langle 3, d\rangle, \langle 4, c\rangle, \langle 5, b\rangle\}$

Problem 2

Let $h \in A_A$, prove that the state "for any $f, g \in A_A$, if $h \circ f = h \circ g$ then we have $f = g$" is true if and only if h is injective, or:

$$(\forall f)(\forall g)((f \in A_A \land g \in A_A \land h \circ f = h \circ g) \to f = g) \Leftrightarrow h \text{ is injective}$$

Solution.

Method 1

1. $\Leftarrow$: If h is injective

   Then h has a left inverse m such that $m \circ h = I_A$. Then :

   $h \circ f = h \circ g$
   $\Rightarrow m \circ (h \circ f) = m \circ (h \circ g)$
   $\Leftrightarrow (m \circ h) \circ f = (m \circ h) \circ g$
   $\Rightarrow I_A \circ f = I_A \circ g$
   $\Rightarrow f = g$

2. $\Rightarrow$: If $(\forall f)(\forall g)((f \in A_A \land g \in A_A \land h \circ f = h \circ g) \to f = g)$

$(\forall f)(\forall g)((f \in A_A \wedge g \in A_A \wedge h \circ f = h \circ g) \rightarrow f = g)$
$\Rightarrow (\forall f)(\forall g)((f \in A_A \wedge g \in A_A \wedge f \neq g) \rightarrow (h \circ f \neq h \circ g))$

We can let $f(x) = x_1, g(x) = x_2, x_1, x_2 \in A$, then:

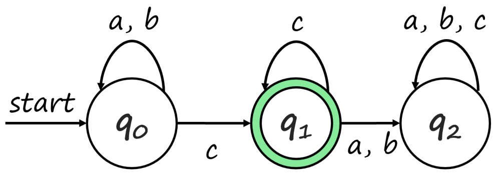
$(\forall f)(\forall g)((f \in A_A \wedge g \in A_A \wedge f \neq g) \rightarrow (h \circ f \neq h \circ g))$
$\Rightarrow (\forall x_1)(\forall x_2)((x_1 \in A \wedge x_2 \in A \wedge x_1 \neq x_2) \rightarrow h(x_1) \neq h(x_2))$
$\Rightarrow h$ is injective

Problem 3

Design a DFA accepting the language $(a|b)^*c^+$ over the alphabeta $\{a, b, c\}$. (Transition table, transition diagram or giving the transition functions are all acceptable). And show how it accepts the string "*abaacc*" by showing all the changes of states in whole process.

Solution.

	a	b	c
$\rightarrow q_0$	q_0	q_0	q_1
$*q_1$	q_2	q_2	q_1
q_2	q_2	q_2	q_2

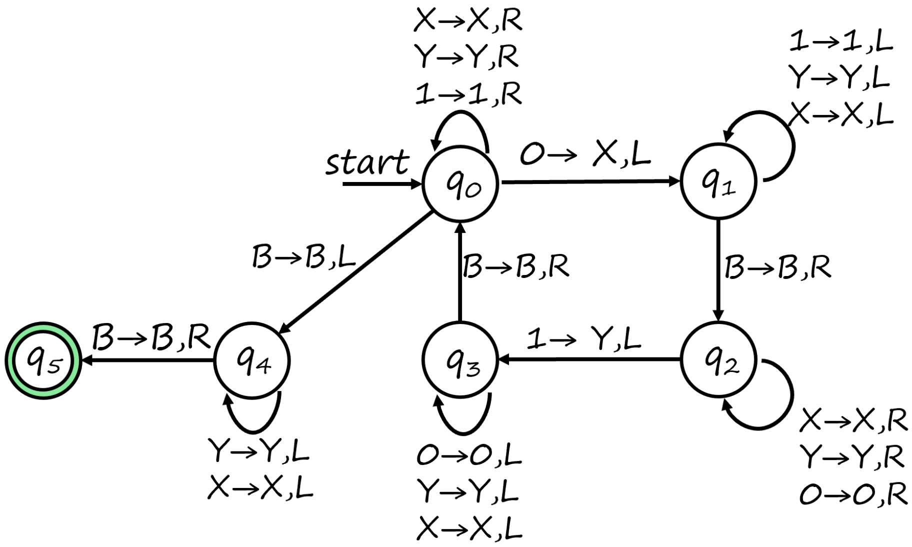


$\hat{\delta}(q_0, a) = q_0$
$\hat{\delta}(q_0, ab) = \delta(q_0, b) = q_0$
$\hat{\delta}(q_0, aba) = \delta(q_0, a) = q_0$
$\hat{\delta}(q_0, abaa) = \delta(q_0, a) = q_0$
$\hat{\delta}(q_0, abaac) = \delta(q_0, c) = q_1$
$\hat{\delta}(q_0, abaacc) = \delta(q_1, c) = q_1$

Problem 4

Design a Turing Machine for the language $\{w|w$ has an equal nurnber of 0's and 1's$\}$ over input alphabeta $\Sigma = \{0, 1\}$. (Transition table, transition diagram or giving the transition functions are all acceptable) And show how it accepts the string 100011 by instantaneous descriptions.

Solution.



$q_0 100011 \vdash_M 1q_0 00011 \vdash_M q_1 1X0011 \vdash_M q_1 B1X0011 \vdash_M q_2 1X0011 \vdash_M q_3 BYX0011$
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