Homework 5
518021911160 窦嘉伟

Problem 1

1. $\neg(\exists x)(\exists y)(P(x) \wedge P(y) \wedge Q(x) \wedge Q(y) \wedge R(x, y)) = (\forall x)(\forall y)((P(x) \wedge P(y) \wedge Q(x) \wedge Q(y)) \to \neg R(x, y))$

证明:

$\neg(\exists x)(\exists y)(P(x) \wedge P(y) \wedge Q(x) \wedge Q(y) \wedge R(x, y))$

$= (\forall x)(\forall y)\neg(P(x) \wedge P(y) \wedge Q(x) \wedge Q(y) \wedge R(x, y))$

$= (\forall x)(\forall y)(\neg(P(x) \wedge P(y) \wedge Q(x) \wedge Q(y)) \vee \neg(R(x, y)))$

$=(\forall x)(\forall y)((P(x) \wedge P(y) \wedge Q(x) \wedge Q(y)) \to \neg R(x, y))$

2. $(\exists x)(P(x) \to Q(x)) = (\forall x)P(x) \to (\exists x)Q(x)$

证明:

$(\exists x)(P(x) \to Q(x))$

$=(\exists x)(\neg P(x) \vee Q(x))$

$=(\exists x)\neg P(x) \vee (\exists x) Q(x)$

$=\neg(\forall x)P(x) \vee (\exists x) Q(x)$

$=(\forall x)P(x) \to (\exists x)Q(x)$

3. $(\forall y)(\exists x)((P(x) \to q) \vee S(y)) = ((\forall x)P(x) \to q) \vee (\forall y)S(y)$

$(\forall y)(\exists x)((P(x) \to q) \vee S(y))$

$=(\forall y)((\exists x)(P(x) \to q) \vee S(y))$

$=(\forall y)(((\forall x)P(x) \to q) \vee S(y))$

$=((\forall x)P(x) \to q) \vee (\forall y)S(y)$

4. $(\exists x)P(x) \to (\forall x)Q(x) \Rightarrow (\forall x)(P(x) \to Q(x))$

$(\exists x)P(x) \rightarrow (\forall x)Q(x)$

$=\neg(\exists x)P(x) \vee (\forall x)Q(x)$

$=(\forall x)\neg P(x) \vee (\forall y)Q(y)$

$\Rightarrow (\forall x)(P(x) \rightarrow Q(x))$

Problem 2

$(\forall x)(P(x) \vee Q(x)) \wedge (\forall x)(Q(x) \rightarrow \neg R(x)) \Rightarrow (\exists x)(R(x) \rightarrow P(x))$

(1)	$(\forall x)(P(x) \vee Q(x))$	前提
(2)	$\neg P(x)\rightarrow Q(x)$	全称量词消去
(3)	$(\forall x)(Q(x) \rightarrow \neg R(x))$	前提
(4)	$Q(x) \rightarrow \neg R(x)$	全称量词消去
(5)	$\neg P(x)\rightarrow\neg R(x)$	24 分离
(6)	$R(x)\rightarrow P(x)$	5 置换
(7)	$(\exists x)(R(x) \rightarrow P(x))$	存在量词引入

Every student in the university is either an undergraduate or a postgraduate. Some students are male. John is not a postgraduate but he is male. Therefore, if John is a student in the university, he must be an undergraduate. Represent these statements in predicate logic and prove the conclusion (”if John is a student in the university, he must be an undergraduate”) by resolution method in 5.6

P(x) means x is an undergraduate,Q(x) means x is a postgraduate ,M(x)means x is male and U(x) means x is in the university.

$(\forall x)(U(x)\rightarrow(P(x) \vee Q(x)))$

$(\exists x) M(x)$

$\neg Q(John) \wedge M(John)$
$U(John) \rightarrow P(John)$

证明

$U(John) \rightarrow P(John)$可通过证明 $(\forall x)(U(x) \rightarrow (P(x) \vee Q(x))) \wedge \neg Q(John) \rightarrow (U(John) \rightarrow P(John))$

令 $G = (\forall x)(U(x) \rightarrow (P(x) \vee Q(x))) \wedge \neg Q(John) \wedge \neg(U(John) \rightarrow P(John))$

$= (\forall x)(\neg U(x) \vee P(x) \vee Q(x)) \wedge \neg Q(John) \wedge (U(John) \wedge \neg P(John))$

子句集 $S = \{\neg U(x) \vee P(x) \vee Q(x), U(John), \neg P(John), \neg Q(John)\}$

(1)$\neg U(x) \vee P(x) \vee Q(x)$
(2)$U(John)$
(3) $\neg P(John)$
(4)$\neg Q(John)$
(5) $P(John) \vee Q(John)$ 12 归结
(6)$Q(John)$ 34 归结
(7)$\square$ 46 归结

Problem 3
1 wrong

如果能，有$((\forall x)P(x) \rightarrow Q(x)) \Rightarrow (P(a) \rightarrow Q(a))$

令 $G = ((\forall x)P(x) \rightarrow Q(x)) \wedge (P(a) \wedge \neg Q(a))$

$= (\neg P(b) \vee Q(x)) \wedge (P(a) \wedge \neg Q(a))$
$S = \{(\neg P(b) \vee Q(x)), P(a), \neg Q(a)\}$
无法归结

2 wrong

$\exists x$ 无法作用于$\wedge$

3 同上

Problem 4
1 universally valid

$((\exists x)P(x) \rightarrow (\exists x)Q(x)) \rightarrow (\exists x)(P(x) \rightarrow Q(x))$

令 $G = ((\forall x)\neg P(x) \vee (\exists x)Q(x)) \wedge (\forall x)(P(x) \wedge \neg Q(x))$

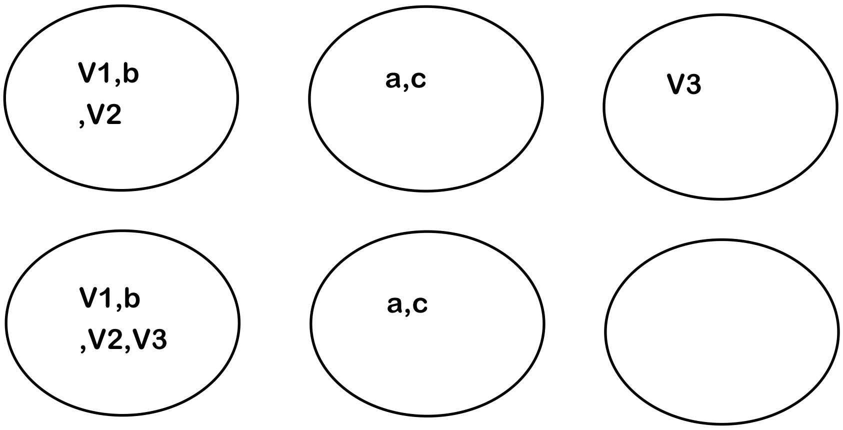
$S = \{\neg P(x) \vee Q(a), P(x), Q(x)\}$
归结得$\square$

2 not ex: $P(x,y)$表示 $x+y>0$

Problem 5
Unsatisfiable

Problem 6

f(b) = b, f(b) = f(a), a = c, b != c, f(c) != a
V1 = f(b) V2=f(a) V3=f(c)



Satisfiable

Problem 7
f(i) − f(j) != 0 ∧ i − j = 0
m-n!=0 ∧ i − j = 0 ∧ m=f(i) ∧ n=f(j)

m-n!=0 ∧ i − j = 0 **m=f(i) ∧ n=f(j)**
Arithmetic Solver **EUF**
i=j 与 **i!=j** 时，左边都 **unsat,**最终返回 **unsat**