

Homework12

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Problem1

Let $A = \{1, 2, 3, 4, 5\}$, $B = \{a, b, c, d\}$, and $f_1 : A \rightarrow B = \{\langle 1, c \rangle, \langle 2, c \rangle, \langle 3, b \rangle, \langle 4, a \rangle, \langle 5, d \rangle\}$, $f_2 : B \rightarrow A = \{\langle a, 2 \rangle, \langle b, 5 \rangle, \langle c, 1 \rangle, \langle d, 3 \rangle\}$. Determine whether f_1, f_2 have left or right inverse. If so, find the left or right inverse for each function.

F1:

No left inverse

No right inverse

F2:

No left inverse

No right inverse

Problem2

充分性:

假设 h 是单射, 则 $h \circ f = h \circ g \Rightarrow (\forall x)(x \in A \rightarrow (\exists y)(y \in A \wedge x h \circ f y \wedge x h \circ g y))$
 $\Rightarrow (\forall x)(x \in A \rightarrow (\exists y)(y \in A \wedge (\exists t_1)(\exists t_2)(x h t_1 \wedge t_1 f y \wedge x h t_2 \wedge t_2 g y)))$
 又 h 是单射 $\Rightarrow t_1 = t_2 \Rightarrow f = g$

必要性:

$h \circ f = h \circ g$

$\Rightarrow (\forall x)(x \in A \rightarrow (\exists y)(y \in A \wedge x h \circ f y \wedge x h \circ g y))$

$\Rightarrow (\forall x)(x \in A \rightarrow (\exists y)(y \in A \wedge (\exists t_1)(\exists t_2)(x h t_1 \wedge t_1 f y \wedge x h t_2 \wedge t_2 g y)))$

$f = g \Rightarrow t_1 = t_2 \Rightarrow h$ 是单射

Problem3

Design a DFA accepting the language $(a|b)^* c^+$ over the alphabet $\{a, b, c\}$. (Transition table, transition diagram or giving the transition functions are all acceptable). And show how it accepts the string "abaacc" by showing all the changes of states in whole process.

	$q_0 \rightarrow q_1$		
	a	b	c
q0	q0	q0	q1
*q1	q2	q2	q1
q2	q2	q2	q2

$$\begin{aligned} \delta^*(q_0, \epsilon) &= q_0; \\ \delta^*(q_0, a) &= \delta(\delta^*(q_0, \epsilon), a) = \delta(q_0, a) = q_0; \\ \delta^*(q_0, ab) &= \delta(\delta^*(q_0, a), b) = \delta(q_0, b) = q_0; \end{aligned}$$

$$\begin{aligned}
\delta^*(q_0, aba) &= \delta(\delta^*(q_0, ab), a) = \delta(q_0, a) = q_0; \\
\delta^*(q_0, abaa) &= \delta(\delta^*(q_0, aba), a) = \delta(q_0, a) = q_0; \\
\delta^*(q_0, abaac) &= \delta(\delta^*(q_0, abaa), c) = \delta(q_0, c) = q_1; \\
\delta^*(q_0, abaacc) &= \delta(\delta^*(q_0, abaac), c) = \delta(q_1, c) = q_1;
\end{aligned}$$

Problem4

Design a Turing Machine for the language $\{w \mid w \text{ has an equal number of } 0\text{'s and } 1\text{'s}\}$ over input alphabet $\Sigma = \{0, 1\}$. (Transition table, transition diagram or giving the transition functions are all acceptable) And show how it accepts the string 100011 by instantaneous descriptions.

	0	1	X	Y	B
q0	(q1,X,R)	(q5,Y,R)	(q0,X,R)	(q0,Y,R)	(q4,B,R)
q1	(q1,0,R)	(q2,Y,L)	---	(q1,Y,R)	---
q2	(q2,0,L)	---	(q0,X,R)	(q2,Y,L)	---
*q4	---	---	---	---	----
q5	(q6,0,L)	(q5,1,R)	---	---	---
q6	---	(q6,1,L)	(q6,X,L)	(q0,Y,R)	---

100011 \rightarrow X00011 \rightarrow XY0011 \rightarrow XYX011 \rightarrow XYX0Y1 \rightarrow XYXXY1 \rightarrow XYXXYX