Homework 9

November 15, 2019

Deadline

Due: November 22, 2019, 23:59. Good luck!

Problem 1

List all the elements of relation R

1. $A = \{1, 2, 3\}, B = \{0, 2, 4\}$, and $R = \{\langle x, y\rangle | x, y \in A \cup B\}$
2. $A = \{1, 2, 3, 4, 5\}, B = \{1, 2, 3\}$, and $R = \{\langle x, y\rangle | x \in A \wedge y \in B \wedge x = y^2\}$

Problem 2

Let R, S be relation on A, prove that:

$$R \circ (S \cup T) = (R \circ S) \cup (R \circ T)$$

Problem 3

Let R_1 and R_2 be relations on a set A represented by the matrices:

$$M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Find the matrices that represents:

1. $R_1 \cup R_2$
2. $R_1 \cap R_2$
3. $R_1 \oplus R_2$
4. $R_1 \circ R_2$

Problem 4

For set $A = \{1, 2, 3, 4\}$, the relation R on A:

$$R = \{\langle 1, 2\rangle, \langle 4, 3\rangle, \langle 2, 2\rangle, \langle 2, 1\rangle\}$$

Construct a relation R_1 on A for each following problem, such that $R \subseteq R_1$, $R_1 \neq E_A$ and:

1. R_1 is reflexive.
2. R_1 is symmetric.
3. R_1 is transitive.
4. R_1 is an equivalence relation, then write the quotient set of R_1 on A
5. R_1 is a compatible relation, then write the complete cover of R_1 on A

Problem 5

Let R_1 and R_2 are equivalence relations on a non-empty set A, determine whether the following relation is an equivalence relation on A. If so, prove it. If not, write a counterexample.

1. $(A \times A) - R_1$
2. R_1^2
3. $R_1 - R_2$

Problem 6

R is a relation on set A, prove that:

$$S = I_A \cup R \cup R^{-1} \text{ is a compatible relation on } A$$