Homework 5

October 25, 2019

Deadline

Due: November 1, 2019, 23:59. Good luck!

Problem 1

Prove the following equations and tautological implication. (Hint:5.1, 5.2, 5.4.2 in the textbook)

1.

$$\begin{aligned}&\neg(\exists x)(\exists y)(P(x) \land P(y) \land Q(x) \land Q(y) \land R(x, y))\\ =&(\forall x)(\forall y)((P(x) \land P(y) \land Q(x) \land Q(y)) \to \neg R(x, y))\end{aligned}$$

2.

$$\begin{aligned}&(\exists x)(P(x) \to Q(x))\\ =&(\forall x)P(x) \to (\exists x)Q(x)\end{aligned}$$

3.

$$
\begin{aligned}
&(\forall y)(\exists x)((P(x) \to q) \vee S(y)) \\
=&((\forall x)P(x) \to q) \vee (\forall y)S(y)
\end{aligned}
$$

4.

$$
\begin{aligned}
&(\exists x)P(x) \to (\forall x)Q(x) \\
\Rightarrow&(\forall x)(P(x) \to Q(x))
\end{aligned}
$$

Solution.
Your solution here.

Problem 2

1. Prove $(\forall x)(P(x) \vee Q(x)) \wedge (\forall x)(Q(x) \to \neg R(x)) \Rightarrow (\exists x)(R(x) \to P(x))$ by deduction in 5.5.

2. Every student in the university is either an undergraduate or a postgraduate. Some students are male. John is not a postgraduate but he is male. Therefore, if John is a student in the university, he must be an undergraduate. Represent these statements in predicate logic and prove the conclusion ("if John is a student in the university, he must be an undergraduate") by resolution method in 5.6

Solution.
Your solution here.

Problem 3

Determine if the following deduction are right. Explain your reasons if the deduction is wrong.

1. If $(\forall x)P(x) \to Q(x)$, then $P(a) \to Q(a)$.

2. If $\neg(\exists x)(\neg P(x) \wedge \neg Q(x))$, then $\neg((\exists x)\neg P(x) \wedge (\exists x)\neg Q(x))$.

3. If $(\exists x)(\neg P(x) \wedge \neg Q(x))$, then $(\exists x)\neg P(x) \wedge (\exists x)\neg Q(x)$.

Solution.
Your solution here.

Problem 4

Determine if the following formulas are universally valid. If they are universally valid, give the proof. Otherwise, give the counterexample.

1. $((\exists x)P(x) \rightarrow (\exists x)Q(x)) \rightarrow (\exists x)(P(x) \rightarrow Q(x))$

2. $(\forall x)(\exists y)P(x,y) \rightarrow (\exists y)(\forall x)P(x,y)$

Solution.
Your solution here.

Problem 5

Encode $f(b) = b, f(b) = f(a), a = c, f(c) \neq b$ into a SAT problem and determine the satisfiability of it. You don't need to write the process of DPLL. Directly write its satisfiability.

Solution.
Your solution here.

Problem 6

Describe how to solve $f(b) = b, f(b) = f(a), a = c, b \neq c, f(c) \neq a$ by EUF solver. Draw the graph like that in slides.

Solution.
Your solution here.

Problem 7

Solve $f(i) - f(j) \neq 0 \wedge i - j = 0$ by the Nelson-Oppen Method.

Solution.
Your solution here.