Homework 10

November 18, 2019

Deadline

Due: November 25, 2019, 23:59. Good luck!

Problem 1

Determine whether the following relations on $\{0, 1, 2, 3\}$ are partial ordering. If not, show the reason.

1. $\{\langle 0,0\rangle, \langle 1,1\rangle, \langle 2,0\rangle, \langle 2,2\rangle, \langle 2,3\rangle, \langle 3,2\rangle, \langle 3,3\rangle\}$
2. $\{\langle 0,0\rangle, \langle 1,1\rangle, \langle 1,2\rangle, \langle 2,2\rangle, \langle 3,3\rangle\}$
3. $\{\langle 0,0\rangle, \langle 1,1\rangle, \langle 1,2\rangle, \langle 1,3\rangle, \langle 2,2\rangle, \langle 2,3\rangle, \langle 3,3\rangle\}$

Problem 2

Draw the hasse diagram of poset $\langle A, R\rangle$, and write the maximal, minimal, maximum and minimum elements of A

1. $A = \{a, b, c, d, e\}$

   $R = \{\langle a,d\rangle, \langle a,c\rangle, \langle a,b\rangle, \langle a,e\rangle, \langle b,e\rangle, \langle c,e\rangle, \langle d,e\rangle\} \cup I_A$
2. $A = \{a, b, c, d\}$

   $R = \{\langle c,d\rangle\} \cup I_A$

Problem 3

Let R be a partial order on set A, $B \subseteq A$, prove that $R \cap B \times B$ is a partial order on B.

Problem 4

Find $r(R), s(R), t(R)$ for the relation given blow.

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Problem 5

Theorem 10.5.5: Let R_1, R_2 be relations on a non-empty set A, if $R_1 \subseteq R_2$ prove that (only using definition):

1. $r(R_1) \subseteq r(R_2)$
2. $s(R_1) \subseteq s(R_2)$
3. $t(R_1) \subseteq t(R_2)$

Problem 6

Theorem 10.5.6: Let R_1, R_2 be relations on a non-empty set A, prove that (only using definition):

1. $s(R_1) \cup s(R_2) = s(R_1 \cup R_2)$
2. $t(R_1) \cup t(R_2) \subseteq t(R_1 \cup R_2)$