Homework 8

November 11, 2019

Deadline

Due: November 20, 2019, 23:59. Good luck!

Problem 1

Let universal set $E = \{1, 2, 3, 4, 5\}$, set $A = \{1, 4\}$, $B = \{1, 2, 5\}$, $C = \{2, 4\}$, calculating the sets:

1. $A \cup -B$
2. $(A \cap B) \cup -C$
3. $(A \cup B) \oplus C$
4. $\mathcal{P}(A) - \mathcal{P}(B)$

Problem 2

Let A, B and C be sets. Prove that:

1. $A \oplus B = (A \cup B) \cap (\overline{B} \cup \overline{A})$
2. $(A - B) - C \subseteq A - C$
3. $(B - A) \cup (C - A) = (B \cup C) - A$
4. $\bigcup(\mathcal{P}(A)) = A$
5. $\bigcup(A \cup B) = (\bigcup A) \cup (\bigcup B)$

Problem 3

Definition 9.5.1 Transitive Set :A set of sets A is called transitive set if any element of A's element is an element of A, or :

$$A \text{ is a transitive set} \Leftrightarrow (\forall x)(\forall y)((x \in y \wedge y \in A) \to x \in A)$$

For example $A = \{\varnothing, \{\varnothing\}, \{\varnothing, \{\varnothing\}\}\}$ is a transitive set. Prove that:

$$A \text{ is a transitive set} \Leftrightarrow \bigcup A \subseteq A$$

Problem 4

Let A, B and C be sets. Prove that:

1. $A \subseteq C \wedge B \subseteq C \Leftrightarrow A \cup B \subseteq C$
2. $A = B \Leftrightarrow A \subseteq B \wedge B \subseteq A$
3. $\mathcal{P}(A) \subseteq \mathcal{P}(B) \Leftrightarrow A \subseteq B$
4. $\mathcal{P}(A) = \mathcal{P}(B) \Leftrightarrow A = B$
5. $A \subseteq B \Rightarrow \bigcup A \subseteq \bigcup B$

Problem 5

Definition 9.7.2 Singular Set: A set A is called singular set if there exists an infinite sequence of A's elements, $A_0 \in A, A_1 \in A, A_2 \in A, ..., A_n \in A, ...$(not necessarily distinct) such that:

$$... \in A_{n+1} \in A_n \in ... \in A_2 \in A_1 \in A_0$$

Prove that:

1. If $x \in x$, then $\{x\}$ is a singular set.
2. There don't exist set A and B such that $(A \in B \wedge B \in A)$. (Tips: Constructing a singular set based on A and B and using theorem 9.7.9)

Problem 6

Let A be a set, prove that $\{A\}$ is a set :

1. Using the axiom of paring.
2. Without using the axiom of paring.

Problem 7

Let A be a set of sets and $|A| = n(n > 0)$. Prove that:

$$|\bigcup A| > n \Rightarrow (\exists A_0)(A_0 \in A \wedge |A_0| > 1)$$

Problem 8

Find the number of integers such that $1 \leq x \leq 2019$ and x is relatively prime to 2020.

Problem 9

Prove that:

1. $[0, 1] \approx [a, b]$, where $a < b, a \in \mathbb{R}$ and $b \in \mathbb{R}$
2. $[0, 1] \approx \mathbb{R}$