Homework 15

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Problem 1

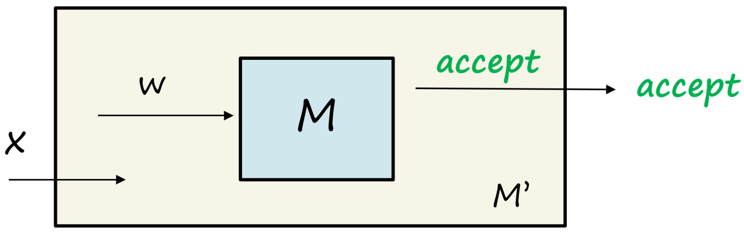
Show that the set of Turing-machine codes for TM's that accept all inputs that are palindromes (possibly along with some other inputs) is undecidable.

Solution.

We donate $L_p = \{M|$ M accepts all palindormes$\}$

We prove L_p is undecidable by reducing L_u to L_p

For TM string pair $\langle M, w \rangle$, we can construct a TM M' as follows:



M' firstly simulates M on w. If M accepts w, then M' accept its input; otherwise, M' does not accept its input.

If $\langle M, w \rangle \in L_u$, or M accepts w, then M' accepts all strings, so M' accepts all palindorems for sure, or $M' \in L_p$

If $\langle M, w \rangle \notin L_u$, or M doesn't accept w, then M' accepts no string, so M' doesn't accepts all palindorems, or $M' \notin L_p$

We've reduced L_u to L_p, then we have L_p is undecidable since L_u is undecidable.

Problem 2

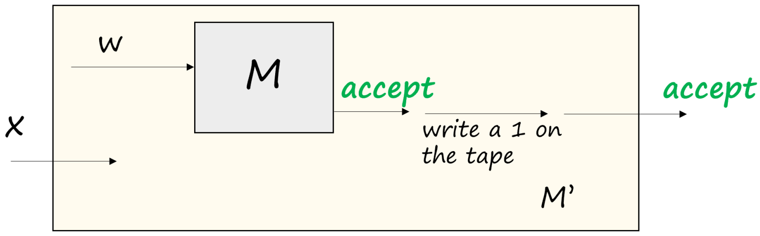
Show that the language of codes for TM's M that, when started with blank tape, eventually write a 1 somewhere on the tape is undecidable.

Solution.

We donate $L_1 = \{M|$M write a 1 somewhere when started with blank tape$\}$

We prove L_1 is undecidable by reducing L_u to L_1.

For TM string pair $\langle M, w \rangle$, we can construct a TM M' as follows:



M' first simulates M on w, we can assume that there's no 1 writed on the tape during this process. If there's any, we can replace 1 with another symbol that not in the alphabet. Then if M accepts w, M' write a 1 on the tape; otherwise, there's no 1 writed on the tape.

If $\langle M, w \rangle \in L_u$, or M accepts w, then M' finally write a 1 somewhere, so $M' \in L_1$

If $\langle M, w \rangle \notin L_u$, or M doesn't accept w, then M' does not write a 1 anywhere, so $M' \notin L_p$

We've reduced L_u to L_1, then we have L_1 is undecidable since L_u is undecidable.