Homework11

518021911160窦嘉伟

Problem1

Find t(R) using warshall algorithm for the relation given blow.

MR=

0 1 0 0

0 0 1 0

0 0 0 1

0 1 0 0

Warshall:

i=1,j=1~4

0 1 0 0

0 0 1 0

0 0 0 1

0 1 0 0

i=2,j=1 i=2,j=2 i=2,j=3 i=2,j=4  
0 1 1 0 0 1 1 0

0 0 1 0 0 0 1 0

0 0 0 1 0 0 0 1

0 1 0 0 0 1 1 0

i=3,j=1 i=3,j=2 i=3,j=3 i=3,j=4

0 1 1 1 0 1 1 1 0 1 1 1 0 1 1 1

0 0 1 0 0 0 1 1 0 0 1 1 0 0 1 1

0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1

0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 1

i=4,j=1 i=4,j=2 i=4,j=3 i=4,j=4

0 1 1 1 0 1 1 1 0 1 1 1 0 1 1 1

0 0 1 1 0 1 1 1 0 1 1 1 0 1 1 1

0 0 0 1 0 0 0 1 0 1 1 1 0 1 1 1

0 1 1 1 0 1 1 1 0 1 1 1 0 1 1 1

T(R)

0 1 1 1

0 1 1 1

0 1 1 1

0 1 1 1

Problem2

1. If R is reﬂexive, then s(R) and t(R) are both reﬂexive.

1. If R is transitive, then r(R) is transitive. And ﬁnd a counterexample to show that s(R) is not transitive.

Counterexample:

R=

1 1 1

0 1 0

0 0 0

Problem3

An equivalence closure e(R) for releation R is deﬁned by:

1. e(R) is an equivalence relation.

2. For any equivalence R 0 , if R ⊆ R 0 , then e(R) ⊆ R 0

For a relation R on a non-empty set, prove that tsr(R) (deﬁned in theorem 10.5.12) is the equivalence closure of R.

Obviously，tsr(R) is transitive.

So tsr(R) is equivalence relation.

So,tsy(R) is eauibalence closure of R.

Problem4

Determine wether f : Z × Z → Z is surjective if:

1. f(m, n) = m + n

2. f(m, n) = m − n

3. f(m, n) = |m| − |n|

4. f(m, n) = m 2 + n 2

5. f(m, n) = m 2 − n

1. T

2. T

3. T

4. F

5. F

Problem5

For every function below, answer the questions:

1. Whether the function is injective, surjective or bijective. If it is bijective, write down the expression of f −1

2. Write down the image of the function and the inverse image of a given set S.

3. The relation R = {<x, y>|x, y ∈ dom(f) ∧ f(x) = f(y)} is an equivalence relation on dom(f), ﬁnd this relation for the function.

All the functions:

1. f : R → (0, ∞), f(x) = 2 x , S = [1, 2]

Injective,surjective,bijective

R [0.5,1]

R={<x,y>|x,y x=y}

1. f : N → N, f(n) = 2n + 1, S = {2, 3}

Injective, not surjective,not bijective

Image {x|x} {1}

R={<x,y>|x,y x=y}

1. f : Z → N, f(x) = |x|, S = {0, 2}

Not injective,surjective,not bijective

Image N {0,2,-2}

{<x,y>||x|=|y|}

4. f : N → N × N, f(n) = <n, n + 1>, S = <2, 2>

Injective,surjective,bijective

{<n,n+1>|n}

{<x,y>|

1. f : [0, 1] → [0, 1], f(x) = (2x+1)/ 4 , S = [0, 1/2 ]

Injective , not surjective, not bijective

[1/4 , 3/4] [0,1/2]

{<x,y>|x}

Problem6

Let f, g ∈ , and f ∩ g != ∅, are f ∩ g and f ∪ g are functions? If so, prove it. If not, show the counterexample.