COMPUTE THE BILATERAL CVA OF AN EQUITY FORWARD

Final Report

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VERSION HISTORY

Version	Revision Date	Reason	Sections Updated
1.0	*21 Mar*	*Build and test Stock Price Simulator, NDV of Equity Forward, Hazard Rate Calibration and Default Event Simulation*	*1* *2.1 – 2.4 ,* *3.1-3.5*
2.0	*4 Apr*	*Incorporated all the classes built last time (equity forward, MC generator, HazardCalibrate and DefaultTimeGenerator) to calculate a preliminary BVA(CVA + DVA) for an equity forward.* ● *Simulate the investor and counterparty default event simultaneously* ● *Calculate the expected exposure at default for both investor and Counterparty.* ● *Compute the Bilateral Valuation Adjustment: BVA = DVA - CVA.*	*2.5,* *3.6*
3.0	*19 Apr*	*Build Class AT1P* ● *Provide alternate way for hazard rate calibration* ● *Compare the calibrated volatilities with that in Brigo's textbook.* *Undergo the study of Wrong Way Risk modelling on CVA* ● *In progress of debugging / test case preparation / validating the mathematical framework*	*1.* *2.6 – 2.8* *3.7*
4.0	*03 May*	*AT1P implementation* *WWR calibration and implementation*	*3.7* *3.8*
Final Report	*17 May*	*Create user input interface for selection of different BVA methodology* *Perform overall programming test , directional test, sensitivity test*	*5*

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1. Backgrounds and motivations

Counterparty Credit Risk (CCR) has become a major concern for banks and its importance has grown significantly since 2008, the financial crisis. There are 4 reasons that CCR has to be properly taken into account in banking business.

First, the volume of OTC derivatives transactions s has increased significantly during last decade. According to the Bank of International Settlements (BIS), the volume of transactions has been increased drastically from USD 90 trillion in year 2000 to USD 670 trillion in year 2008. The potential loss of OTC derivatives can be disastrous when there is a market bubbles burst.

Second, the unseen confident crisis with the extreme market volatility in 2008 result a huge P&L loss when the OTC derivatives are marked-to-market and valued on a fair value basis. During the crisis, Banks are unwilling to lend money to each other and rather prefer to invest in government bonds during crisis. The credit spread and liquidity spread were widened significantly. Impaired level of confidence by market participants posed an impact on instrument value. According to Basel Committee, it is estimated that 75% of the losses that occurred on OTC derivatives portfolios during 2007-2009 were not due to counterparty defaults but by rating downgrades and credit spread volatility which have an impact on MTM instruments.

Third, there is in fact an existence of potential systemic risk. Most of large financial institutions have huge portfolios of derivatives with other large financial institutions as counterparties. If one of these financial institution fail, other financial will face huge losses. The domino effect intensify the default probability of every market participants. The Concept of “Too big to fail” is no longer valid. Every market participant can evade from the market shock and treat itself as standalone non-defaultable.

Fourth, 2007-2009 crisis revealed the regulators and banks did not deal with CCR efficiently. Basel Committee identified the loopholes in Basel II framework concerning CCR measurement. Consequently, there is social and political pressure for more regulation and turning to revamp of CCR measurement implemented in Basel III.

Basel III Introduced a new capital charge for Credit Value Adjustment (CVA) losses with the purpose of capturing risk and loss related to credit spread volatility. It aims to allocate more capital resource to CCR. After complete launch of Basel III 2018, financial market will open a new chapter of risk measurement which rely on more sophisticated modelling technique by system to cater for the complexity and plurality of OTC derivatives.

2. Objectives

The Objective of our project is to calculate the price of a single equity forward which taking the counterparty credit risk of both investor and counterparty into account. To quantify the counterparty credit risk, we measure both credit Valuation adjustment (CVA) and debt valuation adjustment (DVA). They are defined as the difference between the portfolio value before and after adjustment for the default risk of the counterparty and default risk of the investors respectively.

In general, the fair value of equity forward will be found by subtracting the bilateral Credit Value Adjustment (BVA) on the counterparty from the Non Default Value (NDV) of an equity forward contract. By BVA, it is not just the difference of DVA and CVA computed each as if in a world where only one name can default. BVA has accounted for both default of investor and counterparty in both term (i.e. consider first to default).

3. Mathematical framework

3.1 Underlying Stock Price

Stock Price is assumed to follow Geometric Brownian Motion with drift rate μ

$$ln(S_t) = ln(S_0) + \left(\mu - \frac{\sigma^2}{2}\right) t + \sigma\, \mathrm{N}(0,1)$$

The stock price at particular time is found by Monte Carlo Scheme, taking average of all the simulated stock prices at that particular time. Explicit Euler Scheme is used for the discretization of geometric Brownian motion.

$$S_{t+\Delta t} = S_t + S_t(\mu\Delta t + \sigma\sqrt{\Delta T}\, Z_i)\ where\ Z_i\ be\ the\ iid\ N(0,1)$$

3.2 Non-Defaulting Forward Price

Non-default forward value of equity forward at time = t is given by:

$$\text{forward\_value}_{time=t} = \left(S_t e^{(r-q)(T-t)} - K\right)e^{-r(T-t)}$$

As we are calculating CVA as of now, we need to discount the forward value from $time = t \to 0$:

$$\text{PV}(\text{forward\_value}_{time=t}) = \left(S_t e^{(r-q)(T-t)} - K\right)e^{-rT}$$

3.3 Using CDS to calibrate the intensity function *(Reference [1])*

Reduced models are the models that are commonly used in the market to find out the implied default probability from the market CDS quotes. Assume piecewise constant hazard rate.

$$\gamma(\mathrm{t}) = \gamma_i\ for\ t \in [T_{i-1}, T_i), \quad (\gamma_1, \gamma_2, \ldots \ldots \gamma_i \ldots \ldots)$$

The cumulative hazard function is:

$$\mathit{\Gamma}_j = \sum_{i=1}^{j} (T_i - T_{i-1})\gamma_i$$

The CDS pricing will be by formula:

$$CDS_{a,b}(0, R, LGD) =$$
$$= R \sum_{i=a+1}^{b} \gamma_i \int_{T_{i-1}}^{T_i} \exp\left(-\mathit{\Gamma}_{i-1} - \gamma_i(u - T_{i-1})\right) P(0,u)(u - T_{i-1})du + R \sum_{i=a+1}^{b} P(0,T_i)\alpha_i e^{-\Gamma(T_i)}$$
$$-LGD \sum_{i=a+1}^{b} \gamma_i \int_{T_{i-1}}^{T_i} \exp\left(-\mathit{\Gamma}_{i-1} - \gamma_i(u - T_{i-1})\right) P(0,u)du$$

Where R is the CDS spread. Now in the market $T_a = 0$ and we have fair R quotes for $T_b = 1y, 2y, 3y, \ldots .10y$ with T_i 's resetting quarterly. With the above formula, setting the CDS pricing to 0 with the market quote, the piecewise constant hazard rate can be back out one by one through bootstrapping technique. γ_i can be found out one by one .

$$CDS_{0,1y}\left(0, R_{0,1y}^{MKT}, LGD\,;\ \gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 =: \gamma^1\right) = 0\,;$$
$$CDS_{0,2y}\left(0, R_{0,2y}^{MKT}, LGD\,;\ \gamma^1\,;\ \gamma_5 = \gamma_6 = \gamma_7 = \gamma_8 =: \gamma^2\right) = 0\ \ldots\ldots$$

3.4 Simulation of default event

For each particular stock path, there is a likelihood of default in the middle of the contract period. To get the exact default date, we simulate the default date by using a uniform Random Variable U [0, 1] as a proxy of the "target survival probability level".

$$\inf(\, j \geq 0: \exp(-\Gamma_j) = \exp(-\sum_{i=1}^{j}(T_i - T_{i-1})\gamma_i) > U(0,1))$$

After finding the first j such that the implied survival probability is greater than the target survival probability level, default happen in the subsequent time bucket. The length of the time buckets is pre-specified from the term structure of the CDS tenor. Finally

$$e^{\sum_{i=1}^{j-1} -\Gamma_i - \gamma_i(\tau - T_{i-1})} = U(0,1)$$

Where τ is the default time point within T_{i-1} to T_i

We assume no default when $T_j > T$: *the contract period*

3.5 Bilateral Counterparty Risk *(Reference [2])*

Two names involved in the transactions and subject to default risk as:

Investor is indicated with symbol I and Counterparty is indicated with symbol C,

τ_I & τ_c be the default time of the investor and counterparty respectively, T be the final maturity of the payoff .

When first to default is considered, the BVA calculated is as followed. If the investor defaults first, the DVA term will be activated and CVA term vanishes. If the counterparty default first, the DVA will vanish and CVA term will be activated.

Let $\tau = \min(\tau_I, \tau_C)$ be the stopping time due to trigger of default event.

Case 1: $\tau > T$ (No default)

In case the default time is out of the period of contract, both parties can fulfil the agreement of the contract.

$CVA_i = DVA_i = 0$

Case 2: $\tau = \tau_C$ (Counterparty Default)

NPV(τ) : Upon Default by either party, Net Present Value of the residual payoff until maturity is computed.

If NPV is negative, Investor has to completely pay back NPV (τ_C) to the counterparty

If NPV is positive, Investor can only claim a recovery fraction of the NPV (τ_C) from the counterparty

$$CVA_i = LGD\ of\ Counterparty_{\ i} \cdot max(NPV(\tau_{c,i}), 0)$$

$$DVA_i = 0$$

Where i indicates the i-th simulated stock path and i = 1, 2… n

Case 3: $\tau = \tau_I$ (Investor Default)

If NPV is positive, Investor can claim full amount of the NPV (τ_I) from the counterparty
If NPV is negative, Investor only pay a recovery fraction of the NPV (τ_I) to the counterparty

$$DVA_i = LGD\ of\ Investor\ _i \cdot max(-NPV(\tau_{I,i}), 0)$$

$$CVA_i = 0$$

Where i indicates the i-th simulated stock path and i = 1, 2… n

$\text{CVA} = \frac{\sum_{i=1}^{n} CVA_i}{n}$ Where n is the total number of simulated default events

$\text{DVA} = \frac{\sum_{i=1}^{n} DVA_i}{n}$ Where n is the total number of simulated default events

3.6 AT1P Model *(Reference [2])*

We consider a structure model called Analytical-Tractable First Passage (AT1P) Model to model the survival probability, $\mathbb{Q}\ (\tau > \mathrm{T})$

a) It is assumed that the value of firm V is governed by below dynamics:

$$\mathrm{d}V_t = V_t(r_t - k_t)dt + V_t\sigma_t dW_t$$

r_t is the risk free rate , k_t is the payoff ratio and σ_t is the instantaneous volatility.

b) It is assumed the default barrier H(t)

$$\mathrm{H(t)} = Hexp(\int_0^t (r_u - k_u - B\sigma_u^2)du$$

c) τ is the default time and defined at the first time where V(t) hits H(t) from above starting from $\mathrm{V_0} > H$. In maths, $\tau = \inf(\mathrm{t} \geq 0: V_t < H(t))$

d) The survival probability is given analytically by

$$\mathbb{Q}\ (\tau > \mathrm{T}) = \left[\Phi\left(\frac{\log\frac{V_0}{H} + \frac{2B-1}{2}\int_0^T \sigma_u^2 du}{\sqrt{\int_0^T \sigma_u^2 du}}\right) - \left(\frac{H}{V_0}\right)^{2B-1}\Phi\left(\frac{\log\frac{H}{V_0} + \frac{2B-1}{2}\int_0^T \sigma_u^2 du}{\sqrt{\int_0^T \sigma_u^2 du}}\right)\right]$$

$\mathrm{V_0} =$ inital value of the firm. In our setting, we set $\mathrm{V_0} = 1$

$\mathrm{H} =$ level of liabilities, depending on safety convenant and capital structure of the company.

B = control parameter for the volatility of company asset. In our setting, we set B = 0

$\Phi =$ cumulative distribution function (CDF)of the standard Normal distribution.

e) In a simplified and discrete version:

$$\mathbb{Q}\ (\tau > \mathrm{T}) = \left[\Phi\left(\frac{\log\frac{1}{H} - \frac{1}{2}\sum_0^T \sigma_u^2}{\sqrt{\sum_0^T \sigma_u^2}}\right) - \frac{1}{H}\Phi\left(\frac{\log\frac{H}{1} - \frac{1}{2}\sum_0^T \sigma_u^2}{\sqrt{\sum_0^T \sigma_u^2}}\right)\right]$$

Given an AT1P model with calibrated sigma and the above assumption, we can model the default behaviour of investor and counterparty and hence another way to calculate BVA.

3.7 CVA and Wrong Way Risk *(Reference [3, 4])*

When the default probability is positive correlated with the exposure, this is referred to "Wrong-Way "Risk. There is a tendency for a counterparty to default when the investor's exposure is relatively high. By the method suggested by Hull and White, the default probability q (t) will depend on the path followed the value of the portfolio up until time t. We define hazard rate h (t) is an affine function of the dealer's exposure to the counterparty with below two simple models

1) $\mathrm{h(t)} = \exp[\,a(t) + bw(t)]$ *(h(t) increases exponentially when w(t) take more extreme value)*
2) $\mathrm{h(t)} = \ln(1 + \exp(a(t) + bw(t))$ *(h(t) increases linearly when w(t) take more extreme value)*

w (t) = value to the investor of its portfolio with the counterparty at time t.

b = parameter which measures the sensitivity of h (t) to w (t)

(I.e. b>0, h (t) increases with w (t) which corresponding to wrong way risk, vice versa b<0 corresponding to right way risk).

If b is given, the remaining function $a(t)$ can be determined by setting the average survival probability , across all simulation, up to time t_i equals the survival probability calculated from the term structure of credit spreads. This means that

$$\frac{1}{n}\sum_{j=1}^{n}\exp\left(\sum_{i=1}^{k}-h_{ij}(T_i - T_{i-1})\right) = \exp\left(\frac{-R_k T_k}{LGD}\right)$$

1) $\mathrm{h}_{ij} = \exp[\,a(t) + bw(t)]$
2) $\mathrm{h}_{ij} = \ln(1 + \exp(a(t) + bw(t))$

$where\ T_k = 1y, 2y, 3y, \ldots .10y$ with T_i 's resetting quarterly, n = number of total simulations.

To model WWR consistently in the bilateral Case, we extend the model by John Hull to DVA calculation and we assume hazard rate of investor is also sensitivity to the value to the counterparty of his/her portfolio with us at time t .With exact framework, all the parameters are denoted with an apostrophe " ' " on the original sign.

$\mathrm{h}'(\mathrm{t}) = \exp[\,a'(t) + b'w'(t)]$

h'(t) = hazard rate of the investor at time t
w'(t) = value to the counterparty of its portfolio with investor at time t

However, using the John Hull's model, the hazard rates are set to correlate with product payoff at future dates. A limitation was that only UCVA/UDVA can be calculated (first-to-default is not embedded)

In our project, b and b’ are adjustment to the hazard rate of counterparty and investor respectively. They are used to model the effect on wrong way rate and right way risk on the exposure of equity forward. Below table summarizes the how changing b and b’ affect the UCVA and UDVA from perspective of investor.

			Case when b =	0	+ ve (WWR)	- ve (RWR)	+ ve (WWR)	- ve (RWR)
			Case when b' =	0	+ ve (WWR)	- ve (RWR)	- ve (RWR)	+ ve (WWR)
	Type	Payoff	Payoff at Maturity	UCVA (I) and UDVA(I) are found as an reference	UCVA' (I) > UCVA (I) UDVA' (I) > UDVA (I)	UCVA' (I) < UCVA (I) UDVA' (I) < UDVA (I)	UCVA' (I) > UCVA (I) UDVA' (I) < UDVA (I)	UCVA' (I) < UCVA (I) UDVA' (I) > UDVA (I)
Investor 's point of view	Long	S-K	S>K +ve Exposure	Normal UCVA (I)	UCVA(I) is larger	UCVA(I) is smaller	UCVA(I) is larger	UCVA(I) is smaller
			S<K -ve Exposure	Normal UDVA (I)	UDVA(I) is larger	UDVA(I) is smaller	UDVA(I) is smaller	UDVA(I) is larger
	Short	K-S	S>K -ve Exposure	Normal UDVA (I)	UDVA(I) is larger	UDVA(I) is smaller	UDVA(I) is smaller	UDVA(I) is larger
			S<K +ve Exposure	Normal UCVA (I)	UCVA(I) is larger	UCVA(I) is smaller	UCVA(I) is larger	UCVA(I) is smaller

b = sensitivity of hazard rate of counterparty to investor's portfolio value
b' = sensitivity of hazard rate of investor to counterparty's portfolio value
S = Stock Price at maturity
K = Strike Price of and Equity Forward
WWR = Wrong Way Risk
RWR = Right Way Risk
UCVA (I) = unilateral CVA computed by Investor
UDVA (I) = unilateral DVA computed by Investor
UCVA' (I) = default probability adjusted UCVA (I)
UDVA' (I) = default probability adjusted UDVA (I)

3.8 Collateral

We provide a model independent formulas that gives an arbitrage-free valuation for bilateral counterparty risk adjustments with inclusive of collateralization.

Assuming that we are from investor's view, let C_t as the amount of collateral at time t where if $C_t > 0$, it is favourable to the investor that he can use it to net off his exposures at default if his counter-party defaulted and vice versa.

For the initial setting, the amount of the collateral should be zero before the deal is started or after the deal is matured

The collateral-inclusive bilateral credit and debit valuation adjustment is given by:

$$\mathrm{E}_t[\overline{\Pi}\,(t,T;C)] = \mathrm{E}_t[\Pi(t,T)] + C_{BVA}(t,T;C)$$

$\Pi(t,T)$ is the sum of net cash flow of the claim under consideration (not including the collateral account) without investor or counterparty default risk between time t and time T, discounted back to time t as seen from the point of view by investor.

Similarly, $\overline{\Pi}\,(t,T;C)$ will be the net cash flows of the claim under counterparty and investor default risk and inclusive of collateral netting.

We start by showing the formula in the case where all exposures are evaluated at mid-market, $\varepsilon_{I,t} \doteq \varepsilon_{C,t} \doteq V_t$, where $\varepsilon_{I,t}$ denotes the on-default exposure of the investor to the counterparty at time t and $\varepsilon_{C,t}$ denotes the on-default exposure of the counterparty to the investor at time t.

The collateralized bilateral valuation adjustment (i.e. CCVA and CDVA) is:

$$\begin{aligned}\text{CBVA}(t,T;C) = &-\mathbb{E}_t\left[\mathbf{1}_{\{\tau=\tau_C<T\}}D(t,\tau)\text{LGD}_C(\varepsilon_\tau^+ - C_\tau^+)^+\right]\\ &-\mathbb{E}_t\left[\mathbf{1}_{\{\tau=\tau_C<T\}}D(t,\tau)\text{LGD}'_C(\varepsilon_\tau^- - C_\tau^-)^+\right]\\ &-\mathbb{E}_t\left[\mathbf{1}_{\{\tau=\tau_I<T\}}D(t,\tau)\text{LGD}_I(\varepsilon_\tau^- - C_\tau^-)^-\right]\\ &-\mathbb{E}_t\left[\mathbf{1}_{\{\tau=\tau_I<T\}}D(t,\tau)\text{LGD}'_I(\varepsilon_\tau^+ - C_\tau^+)^-\right]\end{aligned}$$

If collateral re-hypothecation is not allowed (LGD posted of counterparty = LGD posted by investor = 0), then the above formula simplifies to:

$$\begin{aligned}\text{CBVA}(t,T;C) = &-\mathbb{E}_t\left[\mathbf{1}_{\{\tau=\tau_C<T\}}D(t,\tau)\text{LGD}_C(\varepsilon_\tau^+ - C_\tau^+)^+\right]\\ &-\mathbb{E}_t\left[\mathbf{1}_{\{\tau=\tau_I<T\}}D(t,\tau)\text{LGD}_I(\varepsilon_\tau^- - C_\tau^-)^-\right]\end{aligned}$$

Where $X^+ = \max(X,0), \quad X^- = \min(X,0).$

3.9 Summary of table

Section	**Content**	**Section**	**Content**	**Remarks**
# 3.1	Underlying Stock	# 4.2	Stock Simulator	
# 3.2	Non-Defaulting Forward Price	# 4.3	Forward Value	
# 3.3	CDS calibrating intensity model	# 4.4	Hazard Rate Calibrate	
# 3.4	Simulation of default event	# 4.5	Default Time Generator	Default event simulation is based on simple intensity model. For AT1P model (#2.6) and WWR (#2.7). The default event simulation will be an extension version.
# 3.5	Bilateral Counterparty Risk	# 4.6	BVA Calculator	
# 3.6	AT1P	# 4.7	AT1P	
# 3.7	Incorporation of Wrong Way Risk	# 4.8	HazardRateWWR	
# 3.8	Collateral	# 4.9	Stock Simulator & BVA calculator with collateral inclusive	

In section 4, we will go into the programming detail on how the classes are constructed to incorporate the mathematics model quoted above. The table above summarizes the linkage between section 3 and section 4 for easy reference.

3.10 Assumptions used in the project

Mathematical part:

1. Stock Price is assumed to follow Geometric Brownian Motion
2. Hazard rates are assumed to be piecewise constant
3. No default after the maturity of the forward contract
4. Default barrier H(t) for the AT1P model is assumed to be constant and be 0.4
5. Control parameter for the volatility of company asset is set to be 0
6. Default time between investor and counterparty are assumed to be independent
7. Forward value is valuated in mid-market value (i.e. No bid-offer adjustment)
8. Collateral is assumed to be cash only
9. The sensitivity of hazard rate h(t) to exposure v(t) is assumed to be constant across the tenor in WWR approach.

Programming part:

1. CVA and DVA values are only on a same single equity forward contract at each time of program run
2. Number of days between value date and settlement date is set to be 2 days
3. Drift rate used in the stock simulation is used as the discount yield in the calculation of forward present value
4. Valuation date is always set to be today
5. Number of simulation for the default date and the stock price are set to be 5000 and 10 respectively
6. Time step of Monte Carlo simulation is set to be 1 day
7. Day count conversion used is Actual365Fixed throughout the whole program
8. Recovery rate for the investor and counterparty are both set to be 0.4
9. Maturity of the forward is a whole number of years according to the test driver setting
10. Maximum tenor for the CDS is set to be 10 years according to the test driver setting
11. Yield for evaluating the CDS in the hazard rate model for generating the survival probability is set to be 0
12. Collateral yields for both investor and counterparty are set to be 0
13. CDS spread was assumed to be non-zero for investor & counterparty.

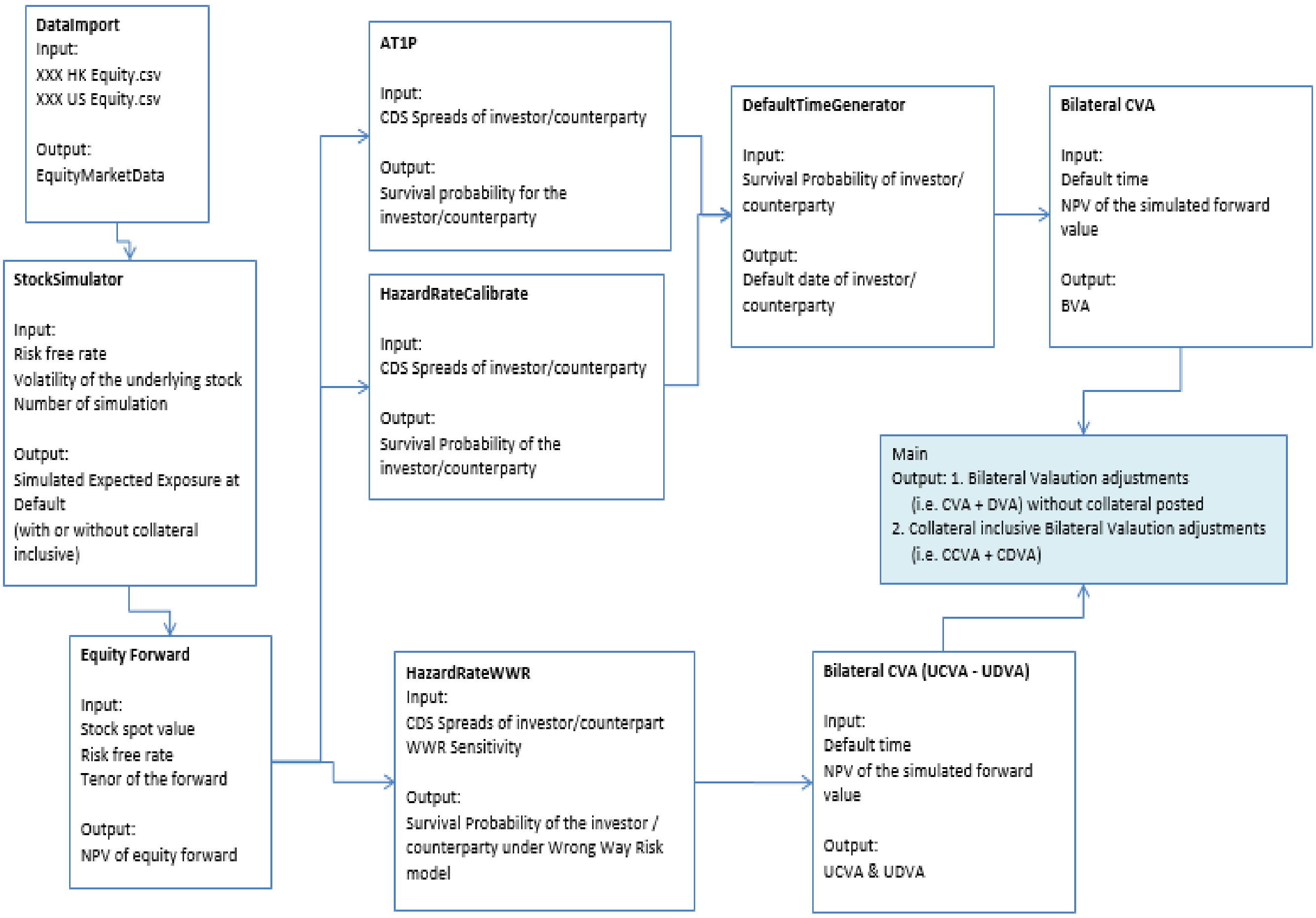
4. Programming detail

4.1 Overview of the pricing engine

Below diagram outlines the simplified building blocks of whole pricing engine. At the first stage, some basic information has to be selected by user via infilling a designated csv data file. Those basic information includes name of the underlying, spot price, dividend yield, CDS spread etc.

The program allows flexibility in calibrating the hazard rate for the investor and counterparty by selecting different method. The programing detail for each method will be explained in detail in coming section

The stock price and the value of equity forward have to be calculated in each time step. Once the hazard rate is calibrated for both investor and counterparty, the first Default time will be simulated accordingly and finding which party will default first. In each simulation, the possible DVA and CVA are calculated based on the simulated result (who default and when default). The final BVA computed will be taking the average for every simulated result.



4.2 Class: Stock Simulator

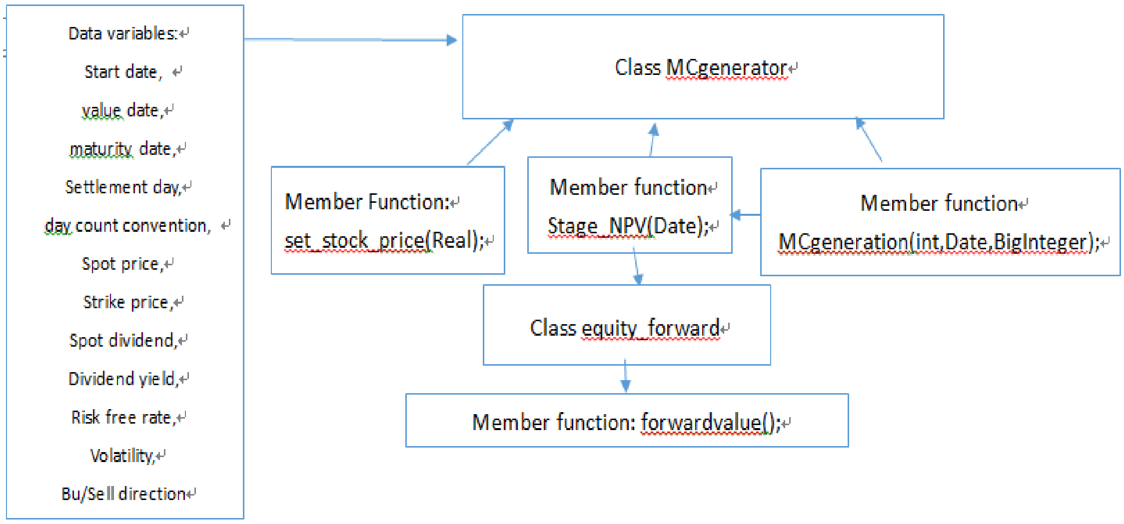
i. File Name:

MCgenerator.h, MCgenerator.cpp

ii. Descriptions:

Class "MCgenerator" is used for performing the Monte Carlo simulation on the stock price and invoking the class “equity_forward” to calculate the corresponding forward non defaultable value on the terminal stock prices of each price path

iii. Building Block Diagram:



iv. Test Cases and result cross checking:

```
C:\Users\user\Desktop\MAFS5220\project\forward pricing...
Spot: 100
Strike: 100
risk free rate: 0.01
Dividend: 0
Dividend yield: 0
Long/ Short: Long
Tenor (year): 1

Path: 1, Terminal stock price of the path: 98.73058517
Path: 2, Terminal stock price of the path: 128.2223689
Path: 3, Terminal stock price of the path: 179.7736991
Path: 4, Terminal stock price of the path: 197.4847165
Path: 5, Terminal stock price of the path: 263.5282549
Path: 6, Terminal stock price of the path: 195.0607149
Path: 7, Terminal stock price of the path: 153.2939027
Path: 8, Terminal stock price of the path: 126.4895962
Path: 9, Terminal stock price of the path: 122.1485678
Path: 10, Terminal stock price of the path: 145.550382
Path: 1, Forward path NDV: -0.2716857734
Path: 2, Forward path NDV: 29.220098
Path: 3, Forward path NDV: 80.77142811
Path: 4, Forward path NDV: 98.48244558
Path: 5, Forward path NDV: 164.5259839
Path: 6, Forward path NDV: 96.05844391
Path: 7, Forward path NDV: 54.2916318
Path: 8, Forward path NDV: 27.48732525
Path: 9, Forward path NDV: 23.14629686
Path: 10, Forward path NDV: 46.54811103

The expected forwad price is (using vector): 62.02600787
請按任意鍵繼續 . . .

微軟速成 半 :
```

```
C:\Users\user\Desktop\MAFS5220\project\forward pricing...
Spot: 100
Strike: 100
risk free rate: 0.01
Dividend: 0
Dividend yield: 0
Long/ Short: Long
Tenor (year): 1

Path: 1, Terminal stock price of the path: 77.67920108
Path: 2, Terminal stock price of the path: 102.3732581
Path: 3, Terminal stock price of the path: 50.21362174
Path: 4, Terminal stock price of the path: 63.99214121
Path: 5, Terminal stock price of the path: 100.2188329
Path: 6, Terminal stock price of the path: 115.3101395
Path: 7, Terminal stock price of the path: 90.5887214
Path: 8, Terminal stock price of the path: 147.8594706
Path: 9, Terminal stock price of the path: 111.93144
Path: 10, Terminal stock price of the path: 66.73954799
Path: 1, Forward path NDV: -21.32306987
Path: 2, Forward path NDV: 3.370987112
Path: 3, Forward path NDV: -48.78864921
Path: 4, Forward path NDV: -35.01012973
Path: 5, Forward path NDV: 1.216561982
Path: 6, Forward path NDV: 16.30786857
Path: 7, Forward path NDV: -8.413549546
Path: 8, Forward path NDV: 48.85719967
Path: 9, Forward path NDV: 12.92916907
Path: 10, Forward path NDV: -32.26272296

The expected forwad price is (using vector): -6.311633491
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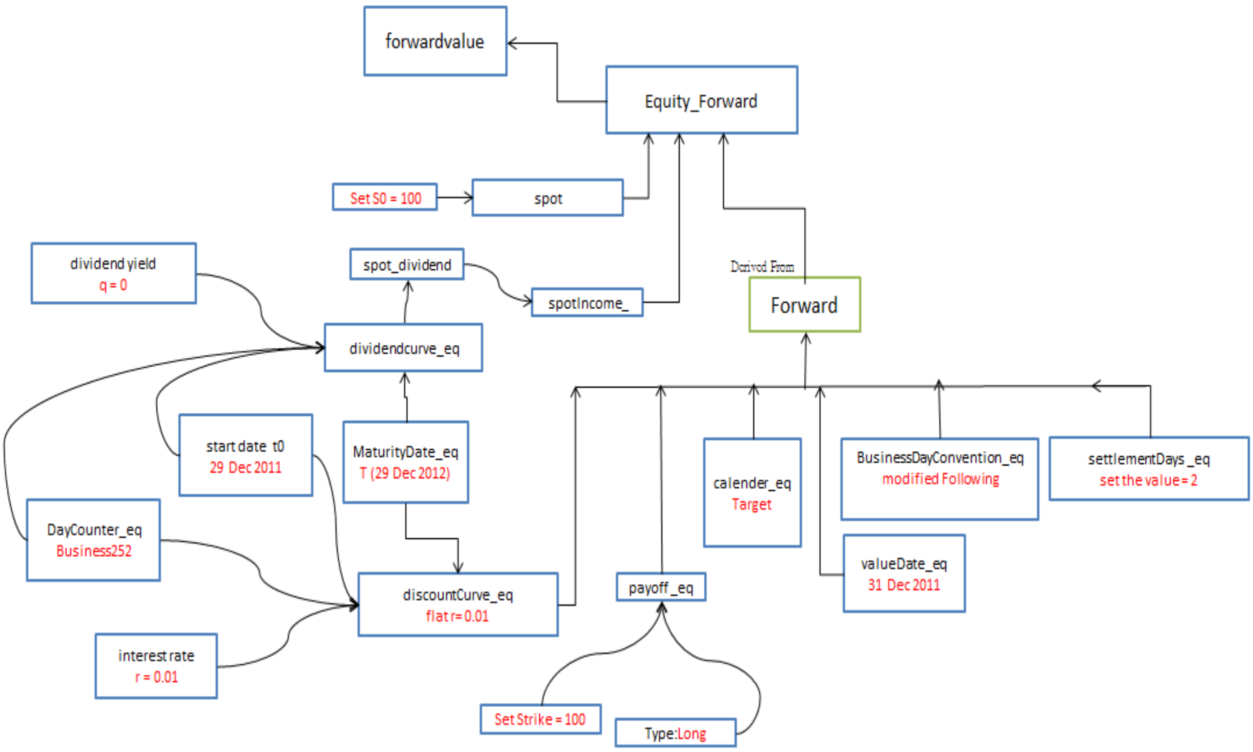
```

4.3 Class: Forward Value

i. File Name:
equity_forward.h, equity_forward cpp

ii. Descriptions:
It is used for calculating the non-defaultable present value of the equity forward. The variable setup is making reference from Felix's tutorial setting.

iii. Building Block Diagram:



iv. Test Cases and result cross checking:

- We build 10 test cases with different sets of stock values, strike, risk free rate, dividend, dividend yield, tenors and direction of the position as inputs.
- We verify whether the non defaultable value of the forwards match with the theoretical method performed by excel (*refer to testcase.xls for more detail calculation*)

Test Case	**#1**	**#2**	**#3**	**#4**	**#5**	**#6**	**#7**	**#8**	**#9**	**#10**
S	100	200	1000	100	100	100	200	1000	100	100
K	100	100	200	200	200	100	100	200	200	200
risk free rate	0.01	0.01	0.01	0.05	0.01	0.01	0.01	0.01	0.01	0.01
Dividend	0	0	0	1	1	0	0	0	1	1
dividend yield	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
tenor (year)	1.00274	2.00274	2.00274	3.00274	3.00274	1.00274	2.00274	2.00274	3.00274	3.00274
discounted dividend	0	0	0	1.161993	1.310287	0	0	0	1.310287	1.310287
Net Spot present value	100	200	1000	98.83801	98.68971	100	200	1000	98.68971	98.68971
Type	Long	Long	Long	Long	Long	Short	Short	Short	Short	Short
Discounted strike	99.00227	98.01718	196.0344	172.118	194.0838	99.00227	98.01718	196.0344	194.0838	194.0838
Equity forward value	0.997729	101.9828	803.9656	-73.28	-95.3941	-0.99773	-101.983	-803.966	95.39408	95.39408

10 Cases Result calculated from program

C:\Users\user\Desktop\MAFS5220\project\forward pricing...

```
The following test is to test the results of forward NDV calculated by class equ
ity_forward

Spot: 100
Strike: 100
risk free rate: 0.01
Dividend: 0
Dividend yield: 0.1
Long/ Short: Long
Tenor (year): 1
The NDV of equity forward is: 0.9977290532

Spot: 200
Strike: 100
risk free rate: 0.01
Dividend: 0
Dividend yield: 0.1
Long/ Short: Long
Tenor (year): 2
The NDV of equity forward is: 101.9828181

Spot: 1000
Strike: 200
risk free rate: 0.01
Dividend: 0
Dividend yield: 0.1
Long/ Short: Long
Tenor (year): 2
The NDV of equity forward is: 803.9656362

Spot: 100
Strike: 200
risk free rate: 0.05
Dividend: 1
Dividend yield: 0.1
Long/ Short: Long
Tenor (year): 3
The NDV of equity forward is: -73.28000927

Spot: 100
Strike: 200
risk free rate: 0.01
Dividend: 1
Dividend yield: 0.1
Long/ Short: Long
Tenor (year): 3
The NDV of equity forward is: -95.39407677

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```
Spot: 100
Strike: 100
risk free rate: 0.01
Dividend: 0
Dividend yield: 0.1
Long/ Short: Short
Tenor (year): 1
The NDV of equity forward is: -0.9977290532

Spot: 200
Strike: 100
risk free rate: 0.01
Dividend: 0
Dividend yield: 0.1
Long/ Short: Short
Tenor (year): 2
The NDV of equity forward is: -101.9828181

Spot: 1000
Strike: 200
risk free rate: 0.01
Dividend: 0
Dividend yield: 0.1
Long/ Short: Short
Tenor (year): 2
The NDV of equity forward is: -803.9656362

Spot: 100
Strike: 200
risk free rate: 0.01
Dividend: 1
Dividend yield: 0.1
Long/ Short: Short
Tenor (year): 3
The NDV of equity forward is: 95.39407677

Spot: 100
Strike: 200
risk free rate: 0.01
Dividend: 1
Dividend yield: 0.1
Long/ Short: Short
Tenor (year): 3
The NDV of equity forward is: 95.39407677

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4.4 Class: HazardRateCalibrate

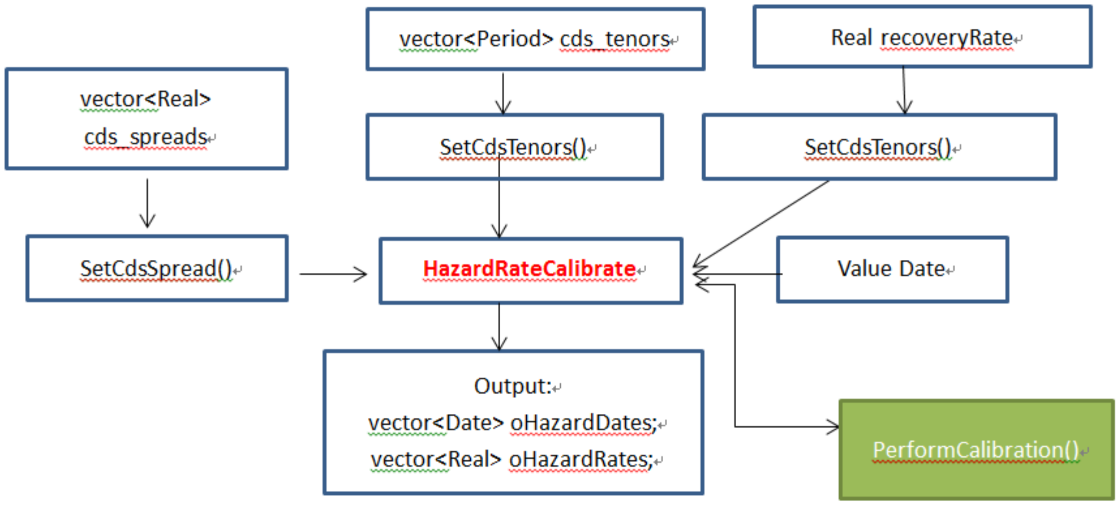
i. File Name:

HazardRateCalibrate.h, HazardRateCalibrate.cpp

ii. Descriptions:

It is used for calibrating stepwise hazard rate term structure given CDS spread inputted by users. In the class, we utilize the quantlib functions <DefaultProbabilityHelper> and <PiecewiseDefaultCurve> for computing the piecewise constant hazard rate.

iii. Building Block Diagram:



iv. Test cases and result cross checking:

- We build 1 test case with 1 set of CDS spread as inputs.

Tenor	3 mths	6 mths	1 yrs	2 yrs
CDS spread (bps)	15bps	30 bps	45 bps	60 ps

- Assuming the interest rate is zero, we verify the computed hazard rate by comparing to the manually bootstrapped hazard rate.

```
##### Test case of class: HazardRateCalibrate


Hazard rate at maturity June 22nd, 2015 was = 0.0247275 versus theo hazard rate = 0.0250785
Hazard rate at maturity September 22nd, 2015 was = 0.0747274 versus theo hazard rate = 0.0761928
Hazard rate at maturity March 22nd, 2016 was = 0.100193 versus theo hazard rate = 0.105287
Hazard rate at maturity March 22nd, 2017 was = 0.125588 versus theo hazard rate = 0.145182
```

4.5 Class: DefaultTimeGenerator

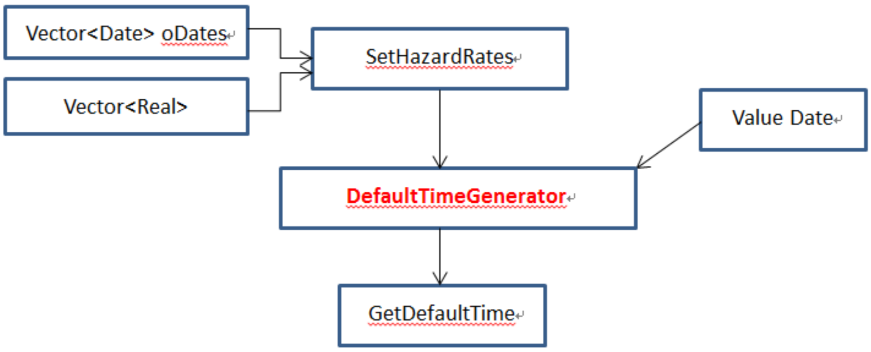
i. File Name:

DefaultTimeGenerator.h, DefaultTimeGenerator.cpp

ii. Descriptions:

It is used for generating default time of single individuals given hazard rate term structure inputted by users.

iii. Building Block Diagram:



iv. Test cases and result cross check:

- We build 2 test cases with different sets of hazard rates term structure as inputs.
- We verify whether the number of default cases roughly matches the expected number of default cases predicted by intensity model

Test case 1

Tenor	6 mths	1 yr	2 yrs	3 yrs	4 yrs	5 yrs	7 yrs	10 yrs
Hazard Rate	0.00147	0.00205	0.0029	0.00376	0.00468	0.00523	0.00676	0.00776

Test case 2

Tenor	6 mths	1 yr	2 yrs	3 yrs	4 yrs
Hazard Rate	0.03	0.02	0.01	0.1	0.15

```
##### Test case of class: DefaultTimeGenerator

Test 1: sample 1000, 10 tenors of hazard rates
270-th trial = September 22nd, 2017
420-th trial = October 2nd, 2015
615-th trial = July 10th, 2017
854-th trial = March 2nd, 2016

Theo default case # within 2 years is 5 while actual default case # = 4

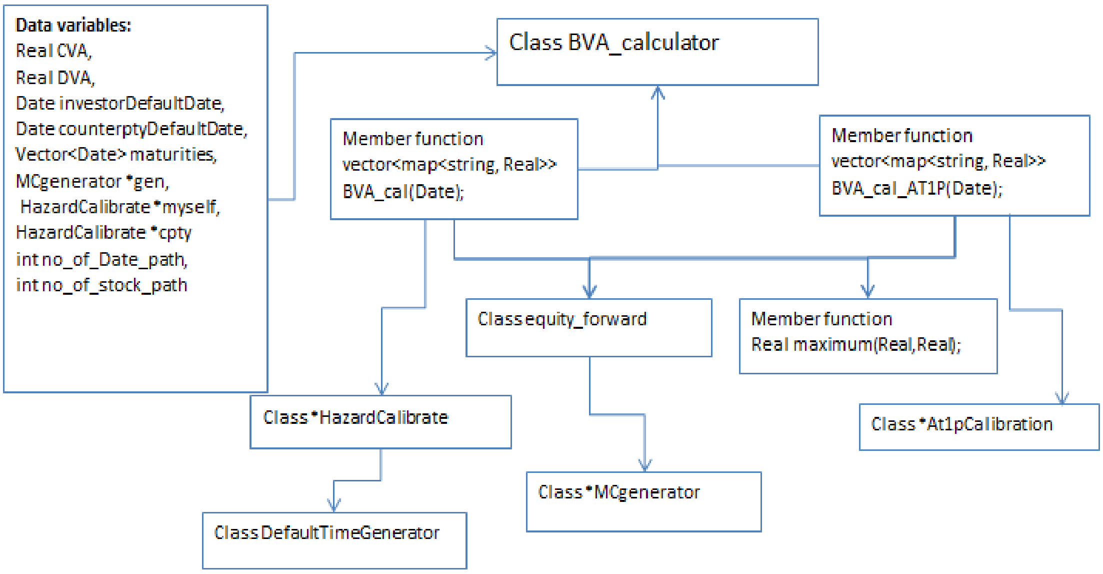
Test 2: sample 1000, 5 tenors of hazard rates
241-th trial = May 19th, 2017
311-th trial = February 17th, 2016
344-th trial = November 1st, 2017
361-th trial = July 21st, 2016
369-th trial = November 27th, 2017
389-th trial = January 25th, 2016
394-th trial = June 7th, 2017
425-th trial = October 12th, 2015
436-th trial = April 4th, 2016
489-th trial = July 3rd, 2017
565-th trial = December 11th, 2017
569-th trial = August 5th, 2016
580-th trial = July 12th, 2017
602-th trial = September 8th, 2016
679-th trial = February 10th, 2016
702-th trial = February 15th, 2016
749-th trial = September 5th, 2016
811-th trial = November 4th, 2015
818-th trial = July 20th, 2017
879-th trial = February 11th, 2016
902-th trial = September 27th, 2017
962-th trial = April 29th, 2016
983-th trial = September 18th, 2017
Theo default case # within 2 years is 26 while actual default case # = 23
```

4.6 Class: BVA calculator

i. File Name:
BVA_calculator.h, BVA_calculator.cpp

ii. Descriptions:
It is used to calculate the BVA (i.e. CVA+DVA) of the equity forward with counterparty's CDS spread, recovery rate and parameters of the forward

iii. Building Block Diagram:



iv. Test cases and result cross check:

- We build 3 test cases each with different CDS spread of investor & Counterparty as inputs.
- We verify whether the number of CVA & DVA components in BVA moved as expected.

Test Case 1 (Investor and Counterparty has same CDS Spreads)

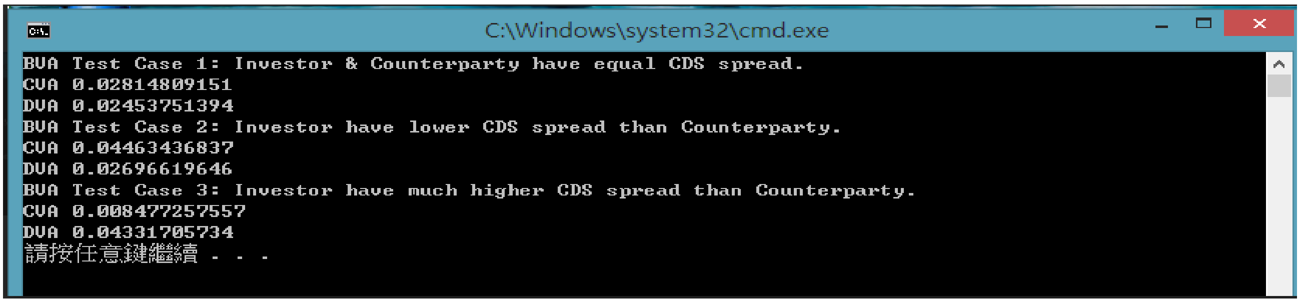
Tenor	6 mths	1 yr	2 yrs	3 yrs	4 yrs	5 yrs	7 yrs	10 yrs
Investor	0.0147	0.0205	0.029	0.0376	0.0468	0.0523	0.0676	0.0776
Counterparty	0.0147	0.0205	0.029	0.0376	0.0468	0.0523	0.0676	0.0776

Test Case 2 (Counterparty has a higher CDS Spreads)

Tenor	6 mths	1 yr	2 yrs	3 yrs	4 yrs	5 yrs	7 yrs	10 yrs
Investor	0.0147	0.0205	0.029	0.0376	0.0468	0.0523	0.0676	0.0776
Counterparty	0.0247	0.0405	0.049	0.0476	0.0568	0.0623	0.0776	0.0876

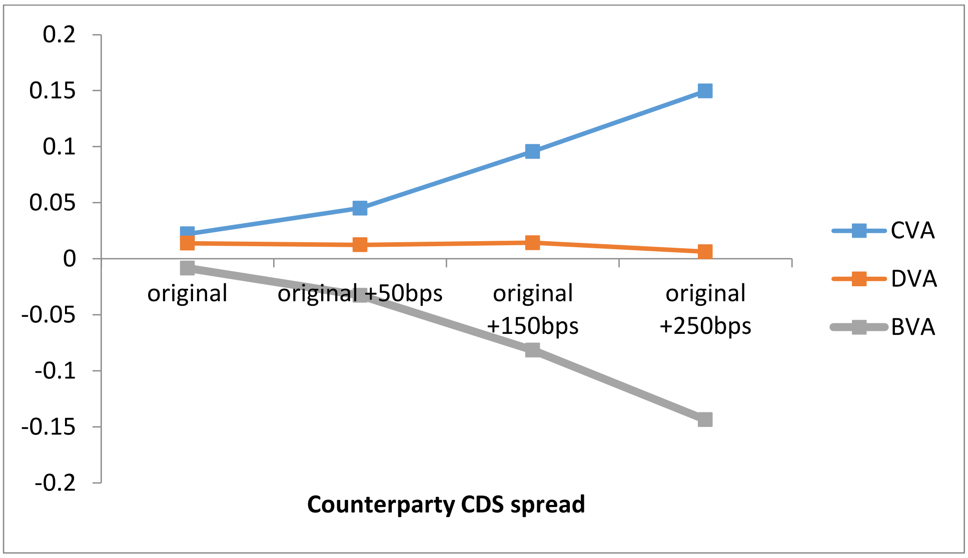
Test Case 3 (Investor has a higher CDS Spreads)

Tenor	6 mths	1 yr	2 yrs	3 yrs	4 yrs	5 yrs	7 yrs	10 yrs
Investor	0.0247	0.0405	0.049	0.0476	0.0568	0.0623	0.0776	0.0876
Counterparty	0.0047	0.0105	0.019	0.0276	0.0368	0.0423	0.0576	0.0676

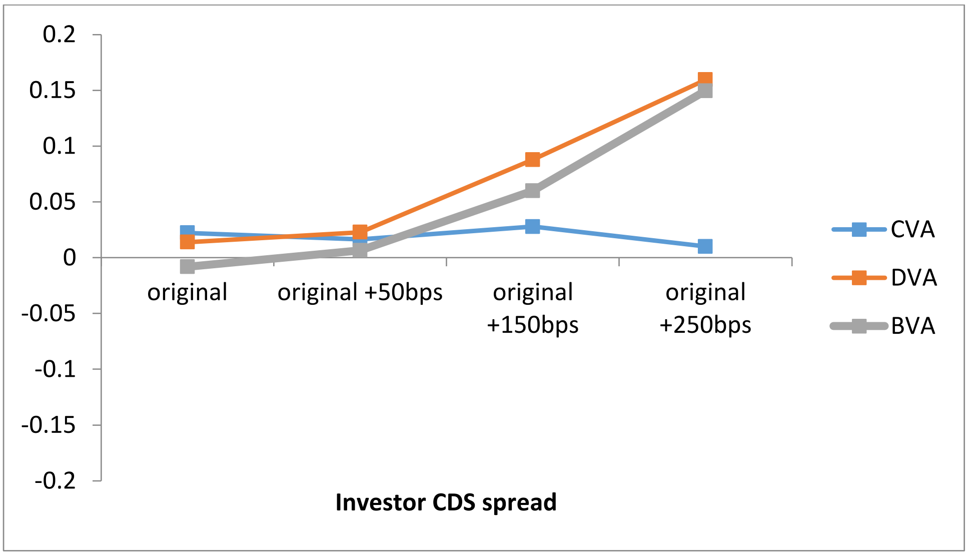


Sensitivity Test of increasing CDS spread of counterparty and investor.

Counterparty CDS spread	CVA	DVA	BVA
original	0.02219	0.013926	-0.00826
original +50bps	0.045119	0.012376	-0.03274
original +150bps	0.095742	0.014288	-0.08145
original +250bps	0.149631	0.00623	-0.1434



Investor CDS spread	CVA	DVA	BVA
original	0.02219	0.013926	-0.00826
original +50bps	0.016389	0.022803	0.006414
original +150bps	0.027888	0.087842	0.059954
original +250bps	0.009984	0.159412	0.149428



4.7 Class: AT1P

i. File Name:

AT1P.h, AT1P.cpp

ii. Descriptions:

- Class “VolatilityTS” is used for storing the volatility term structures generated by the calibration of CDS spreads of AT1P model and then to calculate the survival probability

- Class "AT1P" is used for calibrating the CDS spreads in order to generating the survival probability of the firm so that we can simulate the default date afterwards

- Class "CreditDefaultSwapV2" is used for calculating NPV of a credit default swap so that we can perform CDS calibration for finding the volatility term, structures implied by the corresponding CDS spreads

- Class “FunctionCdsNpv” is used for minimizing NPV for calibrating sigma(t)

Algorithm for using the classes together:

Part 1: Model calibration σ(t):

Step 1: Find an initial guess on the value of $\widehat{\sigma}_{t=6\mathrm{mths}}$ (assuming also $\widehat{\sigma}_t$ follows a stepwise constant pattern for each t)

Step 2: Substitute $\widehat{\sigma}_{t=6\mathrm{mths}}$ into AT1P’s $\mathbb{Q}\,(\tau > T)$ formula

Step 3: Substitute $\mathbb{Q}\,(\tau > T,\ \widehat{\sigma}_{t=6\mathrm{mths}})$ into CDS NPV formula

Step 4: Repeat Step 1-3 for n times with different value of $\widehat{\sigma}_{t=6\mathrm{mths}}$.

Step 5: Pick the $\widehat{\sigma}_{t=6\mathrm{mths}}$ that generated CDS NPV closest to zero.

Step 6: Repeat Step 1 to 5 to estimate the value of$\widehat{\sigma}_{t=1\mathrm{yr}}$, $\widehat{\sigma}_{t=3\mathrm{yrs}}$, $\widehat{\sigma}_{t=5\mathrm{yrs}}$,…, $\widehat{\sigma}_{t=10\mathrm{yrs}}$ step by step.

Part 2: Model implementation:

To generate default time, we can use a calibrated AT1P model with steps as below:

Step 1: Simulate an Uniform Random Variable $u \sim U(0,1)$

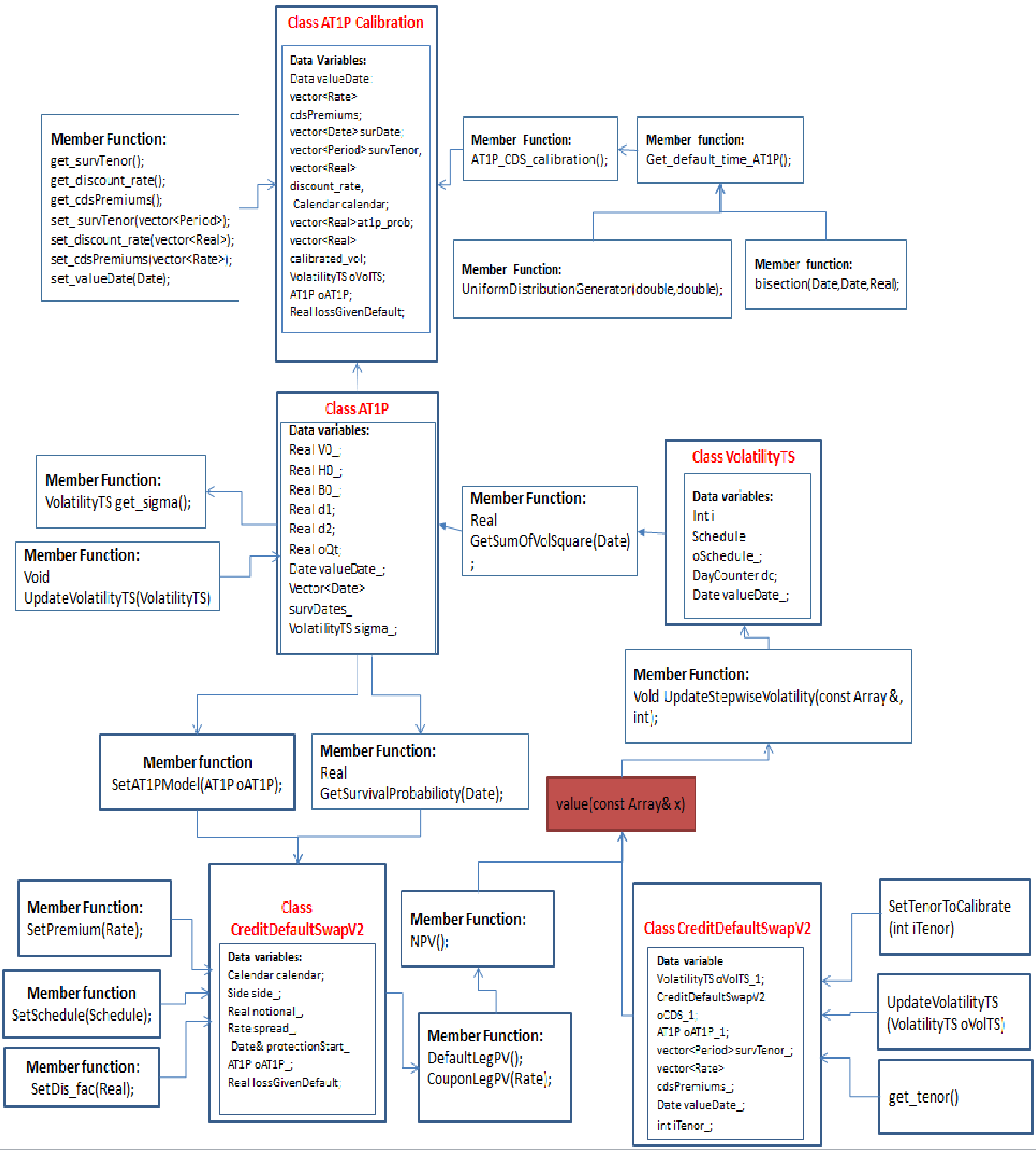
Step 2: Find an initial guess τ and calculate the AT1P’s $\mathbb{Q}\,(\tau > T,\ \widehat{\sigma}_t)$

Step 3: Repeat Step 2 for n times with different value of τ

Step 4: Pick the τ such that AT1P’s $\mathbb{Q}\,(\tau > T,\ \widehat{\sigma}_t)$ closest to value of u. This is the default time.

Step 5: Repeat Step 1 to 4 to generate 10000 default times.

iii. Building Block Diagram:



iv. Test cases and result cross check:

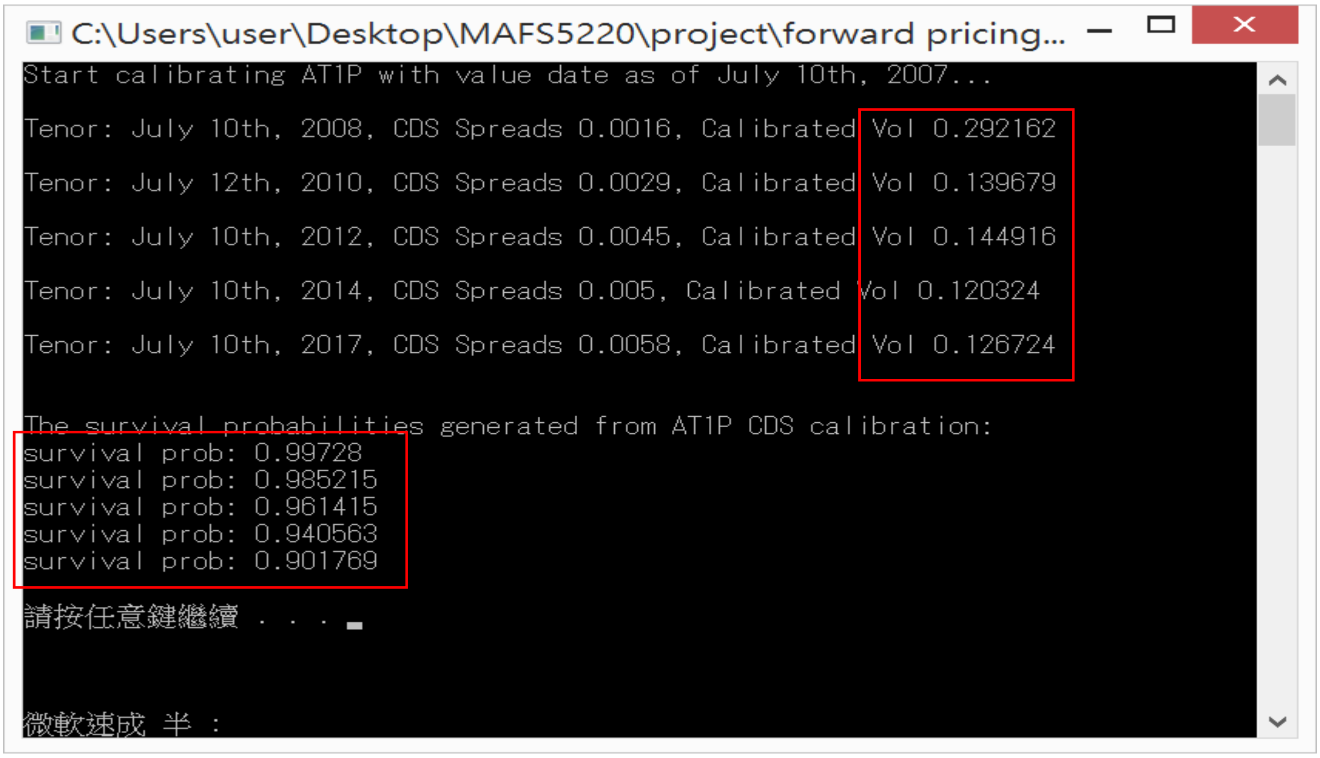
Results of $\widehat{\sigma}_t$ calibration by our group on $\widehat{\sigma}_t$versus that in Brigo's book:

1

Tenor	CDS Premium (bps)
1Yr	16
3Yr	29
5Yr	45
7Yr	50
10Yr	58

T_i	R_i (bps)	λ_i	Surv (Int)	σ_i	Surv (AT1P)
10 Jul 2007			100.0%		100.0%
1y	16	0.267%	99.7%	29.2%	99.7%
3y	29	0.601%	98.5%	14.0%	98.5%
5y	45	1.217%	96.2%	14.5%	96.1%
7y	50	1.096%	94.1%	12.0%	94.1%
10y	58	1.407%	90.2%	12.7%	90.2%

(Reference [2] Brigo's book p.59 Table 3.1)



```
C:\Users\user\Desktop\MAFS5220\project\forward pricing...
Start calibrating AT1P with value date as of July 10th, 2007...

Tenor: July 10th, 2008, CDS Spreads 0.0016, Calibrated Vol 0.292162

Tenor: July 12th, 2010, CDS Spreads 0.0029, Calibrated Vol 0.139679

Tenor: July 10th, 2012, CDS Spreads 0.0045, Calibrated Vol 0.144916

Tenor: July 10th, 2014, CDS Spreads 0.005, Calibrated Vol 0.120324

Tenor: July 10th, 2017, CDS Spreads 0.0058, Calibrated Vol 0.126724

The survival probabilities generated from AT1P CDS calibration:
survival prob: 0.99728
survival prob: 0.985215
survival prob: 0.961415
survival prob: 0.940563
survival prob: 0.901769

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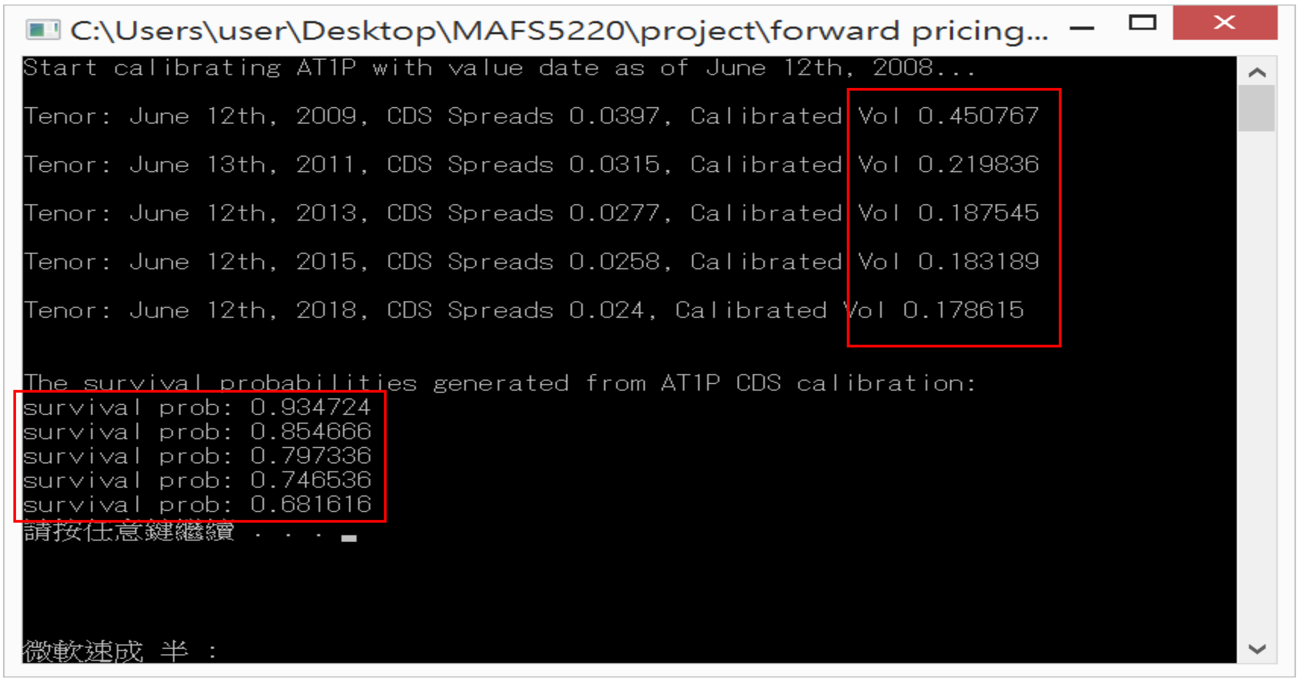
2

Tenor	CDS Premium (bps)
1Yr	397
3Yr	315
5Yr	277
7Yr	258
10Yr	240

T_i	R_i (bps)	λ_i	Surv (Int)	σ_i	Surv (AT1P)
12 Jun 2008			100.0%		100.0%
1y	397	6.563%	93.6%	45.0%	93.5%
3y	315	4.440%	85.7%	21.9%	85.6%
5y	277	3.411%	80.0%	18.6%	79.9%
7y	258	3.207%	75.1%	18.1%	75.0%
10y	240	2.907%	68.8%	17.5%	68.7%

(Reference [2] Brigo's book p.60 Table 3.2*)*



```
C:\Users\user\Desktop\MAFS5220\project\forward pricing...
Start calibrating AT1P with value date as of June 12th, 2008...

Tenor: June 12th, 2009, CDS Spreads 0.0397, Calibrated Vol 0.450767

Tenor: June 13th, 2011, CDS Spreads 0.0315, Calibrated Vol 0.219836

Tenor: June 12th, 2013, CDS Spreads 0.0277, Calibrated Vol 0.187545

Tenor: June 12th, 2015, CDS Spreads 0.0258, Calibrated Vol 0.183189

Tenor: June 12th, 2018, CDS Spreads 0.024, Calibrated Vol 0.178615

The survival probabilities generated from AT1P CDS calibration:
survival prob: 0.934724
survival prob: 0.854666
survival prob: 0.797336
survival prob: 0.746536
survival prob: 0.681616
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3

Tenor	CDS Premium (bps)
1Yr	1437
3Yr	902
5Yr	710
7Yr	636
10Yr	588

T_i	R_i (bps)	λ_i	Surv (Int)	σ_i	Surv (AT1P)
12 Sep 2008			100.0%		100.0%
1y	1437	23.260%	79.2%	62.2%	78.4%
3y	902	9.248%	65.9%	30.8%	65.5%
5y	710	5.245%	59.3%	24.3%	59.1%
7y	636	5.947%	52.7%	26.9%	52.5%
10y	588	6.422%	43.4%	29.5%	43.4%

(Reference [2] Brigo's book p.60 Table 3.3)

```
C:\Users\user\Desktop\MAFS5220\project\forward pricing...
Start calibrating AT1P with value date as of September 12th, 2008...
Tenor: September 14th, 2009, CDS Spreads 0.1437, Calibrated Vol 0.625359
Tenor: September 12th, 2011, CDS Spreads 0.0902, Calibrated Vol 0.307445
Tenor: September 12th, 2013, CDS Spreads 0.071, Calibrated Vol 0.245672
Tenor: September 14th, 2015, CDS Spreads 0.0636, Calibrated Vol 0.274649
Tenor: September 12th, 2018, CDS Spreads 0.0588, Calibrated Vol 0.302247

The survival probabilities generated from AT1P CDS calibration:
survival prob: 0.77953
survival prob: 0.652026
survival prob: 0.586762
survival prob: 0.519224
survival prob: 0.425795
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- We have tested the results of AT1P CDS calibration for the CDS spreads by using the same configurations and the CDS spreads suggested in Brigo's book. We found that our results are different from Brigo's results by around 1% on each tenor especially in the 3rd tests. It is believed that the difference is mainly due to the discount factor used in our test and the one used in Brigo's example. Brigo did not specify the discount factor used in his book.

- Making use of US treasury par yields from below web site, we back out the continuous discounting factor for 3 position dates. (http://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yield).

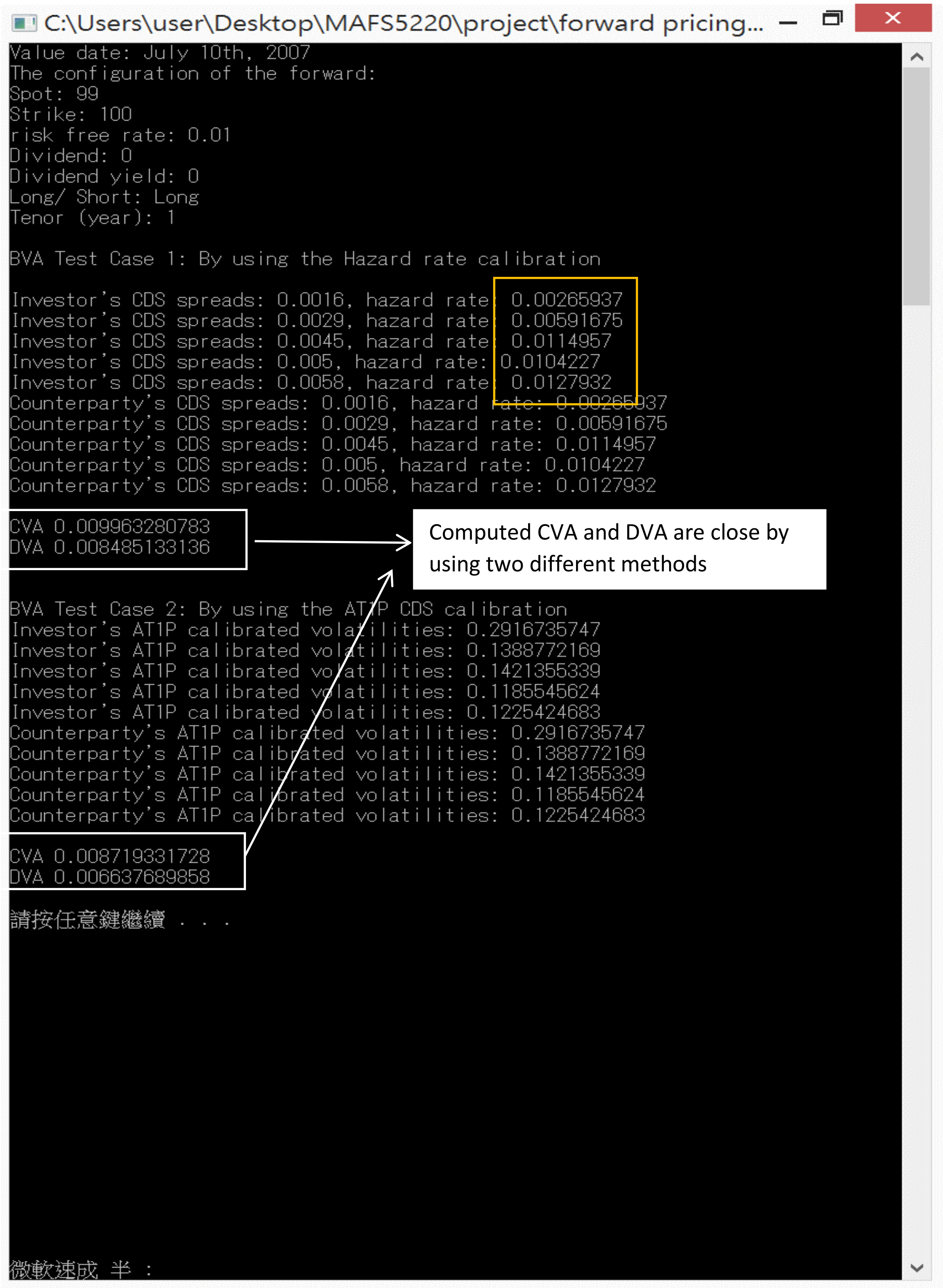
Date	1 mo	3 mo	6 mo	1 yr	2 yr	3 yr	5 yr	7 yr	10 yr	20 yr	30 yr
7/10/2007	4.74	4.95	5.03	4.97	4.85	4.87	4.93	4.97	5.03	5.21	5.14
6/12/2008	1.92	2	2.28	2.62	3.03	3.35	3.68	3.9	4.23	4.83	4.77
9/12/2008	1.37	1.49	1.84	2.02	2.23	2.45	2.97	3.32	3.74	4.36	4.3

Remarks: The yield current is quite flat in 2007 and become downwards steepening in 2008

- We tried another test to assume a flat yield curve, using the 3M Libor rate as a proxy, for Jul 2007 (5.3597%), Jun 2008 (2.7654%) and Sep 2008 (3.1217%). The result in 2007 is satisfactory as the calibrated volatilities all match Brigo's results.

- For Jun 2008 and Sep 2008, the results are still not match Brigo's results, but it having a closer result when compared to using the US treasury par yields.

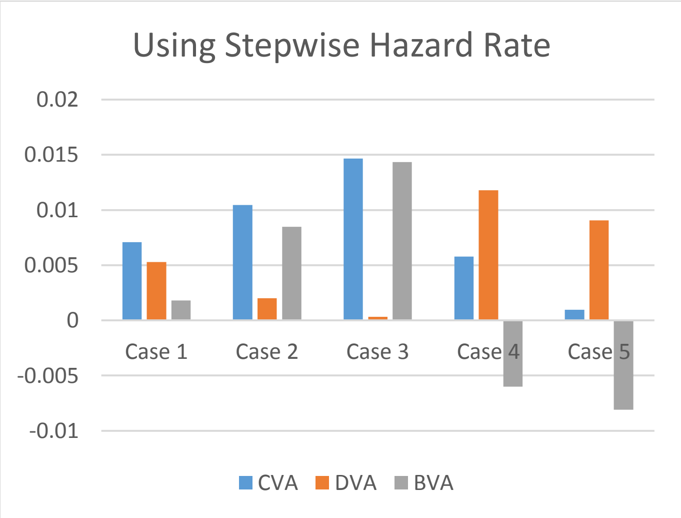
- We have calculated the BVA by using the Hazard rate calibration and the AT1P CDS calibration and we found that the figures generated by both methods are close by using same set of parameters.

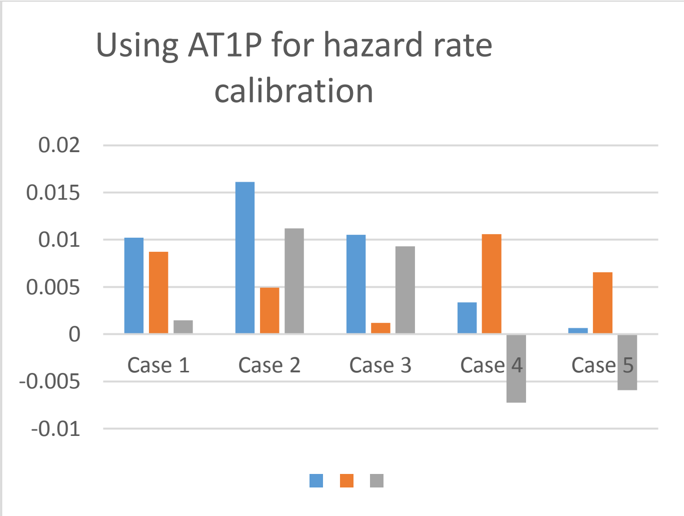


Perform test on BVA by stepwise hazard rate and AT1P (Spot = strike = 100, risk free rate = 1%, Tenor = 1 year)

Stepwise hazard rate	CVA	DVA	BVA
Case 1: (Investor has the same CDS spreads with the counterparty)	0.0070887	0.0052756	0.001813
Case 2: (CDS spreads of Investor is one-half of that the counterparty)	0.0104649	0.0019959	0.008469
Case 3: (CDS spreads of Investor is one-tenth of that the counterparty)	0.0146752	0.0003311	0.014344
Case 4: (CDS spreads of counterparty is one-half of that the investor)	0.0057904	0.0117984	-0.00601
Case 5: (CDS spreads of Investor is ten times of the counterparty)	0.0009661	0.0090692	-0.0081

AT1P	CVA	DVA	BVA
Case 1: (Investor has the same CDS spreads with the counterparty)	0.010214	0.008737	0.001477
Case 2: (CDS spreads of Investor is one-half of that the counterparty)	0.016118	0.004915	0.011203
Case 3: (CDS spreads of Investor is one-tenth of that the counterparty)	0.010509	0.001208	0.009301
Case 4: (CDS spreads of counterparty is one-half of that the investor)	0.003361	0.010589	-0.00723
Case 5: (CDS spreads of Investor is ten times of the counterparty)	0.000645	0.006566	-0.00592





The value CVA is slightly higher than that of DVA in cases that the CDS spreads of the investor and counterparty are the same because the risk free interest rate is positive so that in the long run, the forward value of the stock price will go up and the exposure for the investor will thus be increased.

Since the change of the CDS spreads for CDS spreads for either counterparty or investor, the results of CVA and DVA have changed and the results are reasonable.

4.8 Class: HazardRateWWR

i. File Name:

- HazardRateWWR.h,
- HazardRateWWR.cpp

ii. Descriptions:

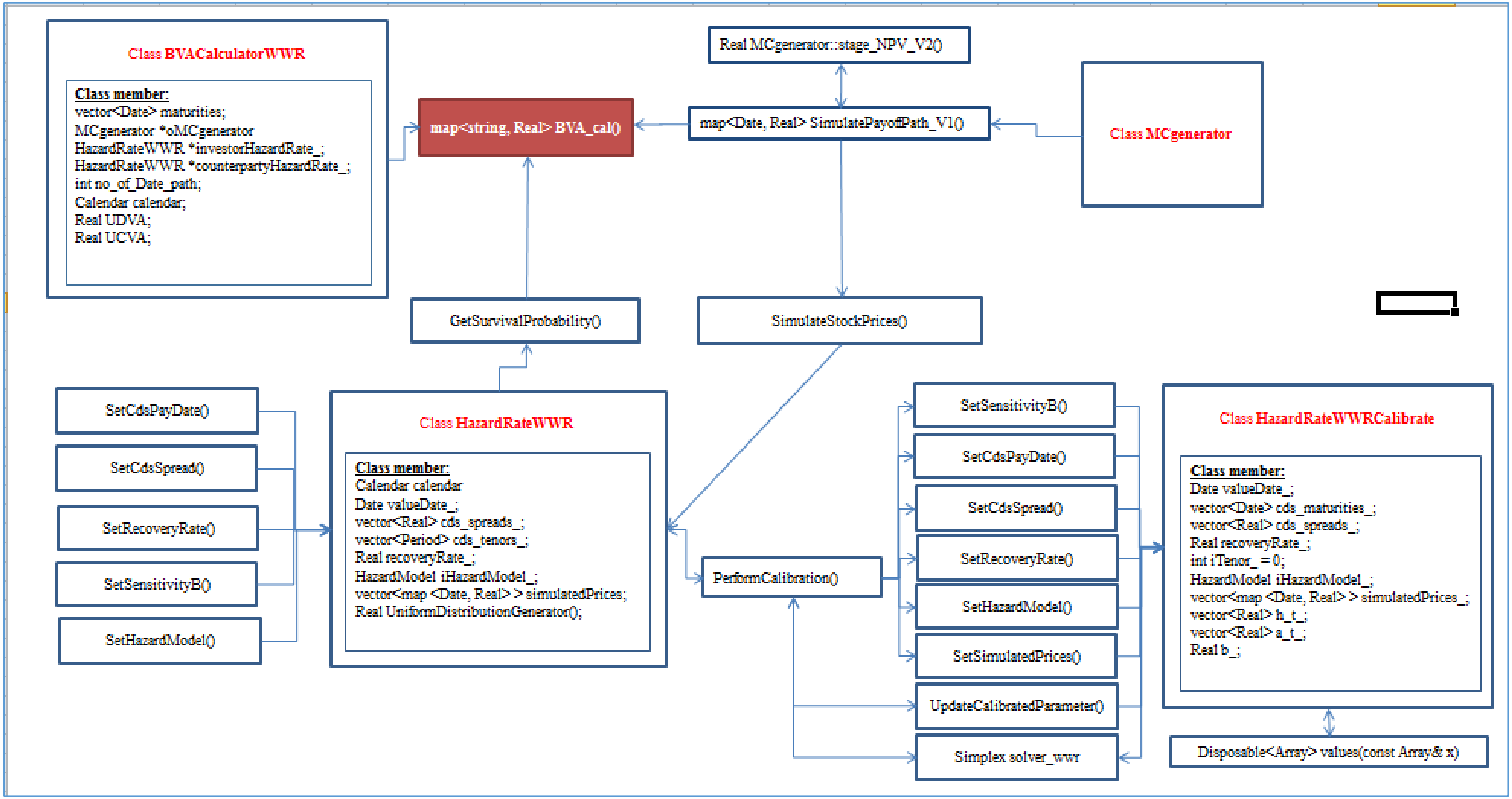
- class “HazardRateWWR” is used for calling class "HazardRateWWRCalibrate" and implementing the WWR model by generating survival probability for BVA calculation.
- class "HazardRateWWRCalibrate " is used for calibrating WWR model by equating survival probability of model and actual.

Remarks:

- As mentioned in weibo, BVA WWR was derived from UDVA & UCVA but not DVA & CVA (first-to-default was ignored). The reason was that only payoff at certain tenor could be used to calibrate hazard rate. Only postponed payoff can be used to calculate valuation adjustment, and hence UCVA/UDVA (First-to-default cannot be consider in this case). This was a limitation of this model.
- In the implementation of HazardRateWWR, we already assumed sensitivity b of the model was user input. The reason was that estimate of b could be quite subjective. To estimate b, one of the method could be by studying the correlation of CDS spread to underlying spot. To achieve this we collected short-term CDS data and stock price of 3 banks as example. (HSBC, SCB, MorganStanley) We can possibly use this correlation as estimate to b.

	2888 HK Equity	*2888 HK Equity CDS 1yr*	*2888 HK Equity CDS 2r*	*5 HK Equity*	*5 HK Equity CDS 1 yr*	*5 HK Equity CDS 2 yr*	*MS US Equity*	*MS US Equity CDS 1Yr*	*MS US Equity CDS 2Yr*
2888 HK Equity	1								
2888 HK Equity CDS 1yr	0.240689056	1							
2888 HK Equity CDS 2r	0.275125103	0.982194295	1						
5 HK Equity	0.575918942	0.140619363	0.164515482	1					
5 HK Equity CDS 1 yr	0.330093635	0.422196733	0.45405971	0.084585413	1				
5 HK Equity CDS 2 yr	0.018787629	0.051200545	0.145300923	-0.066684503	0.678564534	1			
MS US Equity	-0.390161925	0.121297091	0.062350819	-0.130864393	-0.173680051	-0.237913909	1		
MS US Equity CDS 1Yr	0.003463053	0.24710497	0.299825241	-0.239187253	0.179060824	0.330637839	-0.014904211	1	
MS US Equity CDS 2Yr	-0.03076008	0.243080338	0.321084075	-0.190366193	0.266002428	0.389224715	0.069419923	0.846253074	1

iii. Building Block Diagram:



iv. Test cases and result cross check:

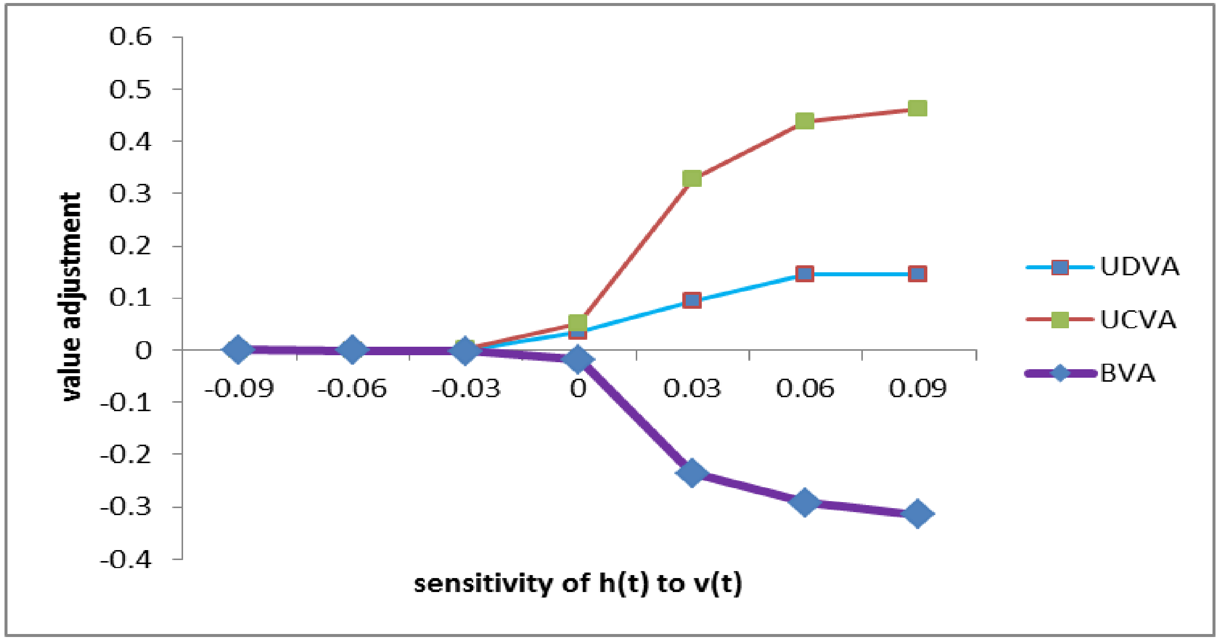
Parameter	CDS spread	Hazard Model	Sensitivity b
Value	Investor: flat at 125bps Counterparty: flat at 125bps	Exponential	Investor: +/-0.03 Counterparty: +/-0.03

Case #	**Forward Type**	**Results from John/Alan's paper when sensitivity b of the model was shifted by:**	**Results from our group when sensitivity b of the model was shifted by:**		
1	long forward contract	CVA ($ millions) for b = 0 0.048 Impact of b = 0.03 per $mm on: CVA 54.8%	b=0 b=0.03	UCVA to investor perspective = 0.04441219096 UCVA to investor perspective = 0.07605845453	CVA impact = **+71.2%**
2	short forward contract	Impact of b = 0.03 per $mm on: CVA 40.5%	b=0 b=-0.03	UCVA to investor perspective = 0.04358231484 UCVA to investor perspective = 0.06013862224	CVA impact = **+38.0%**
3	long forward contract	CVA ($ millions) for b = 0 0.048 Impact of b = –0.03 per $mm on: CVA –37.5%	b=0 b=-0.03	UCVA to investor perspective = 0.04441219096 UCVA to investor perspective = 0.02384691635	CVA impact = **-46.3%**
4	short forward contract	Impact of b = –0.03 per $mm on: CVA –33.9%	b=0 b=-0.03	UCVA to investor perspective = 0.04358231484 UCVA to investor perspective = 0.02590125036	CVA impact = **-40.6%**

The % CVA impact (from shift in b) roughly matches with the one by John/Alan's paper.

Below is the trend of UDVA/UCVA/BVA when we input different value of b = sensitivity of h(t) to v(t).

The result trend matches our expectation. When exposure was high, the hazard rate will be higher as b increase. This will increase the UCVA and UDVA.



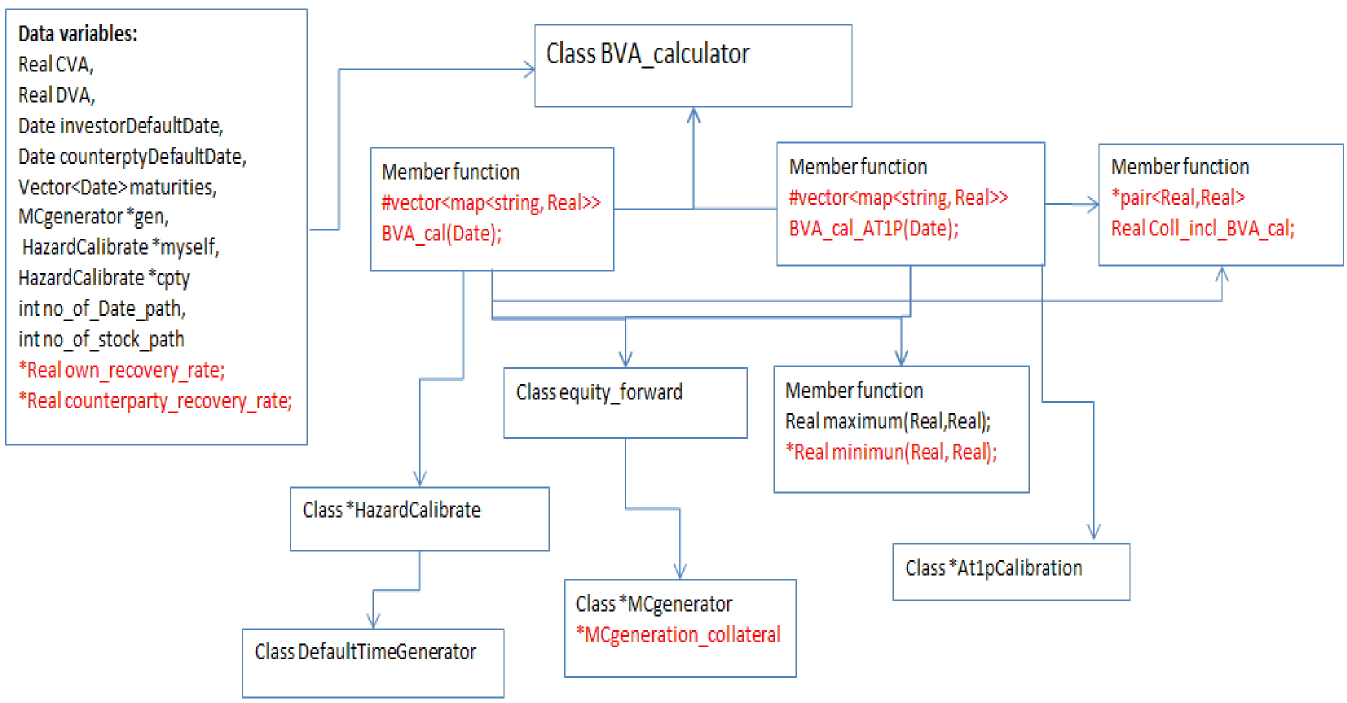
4.9 Class: Stock Simulator & BVA calculator with collateral inclusive
(This section is an extension to the classes described in section 4.2 and 4.6)

i. File Name:
MCgenerator.h, MCgenerator.cpp
BVA_calculator.h, BVA_calculator.cpp

ii. Descriptions:
The class MCgenerator is used for performing the Monte Carlo simulation on the stock price and in addition to the original classical simulation, a new function “MCgeneration_collateral” is added to the class MCgenerator. This function calculates the forward value and collateral amount with the interest accrual at every time step of the simulation and finally returns a pair of real values, forward expected exposure and expected present value of the collateral amount.

The class BVA_calculator is used to calculate the BVA (i.e. CVA+DVA) of the equity forward. The functions for calculating the BVA. BVA_cal and BVA_cal_AT1P have been amended for including the choice for choosing whether or not to calculate the collateral inclusive BVA with the present value of collateral simulated in the class MCgenerator. Also, for the calculation of the BVA, a new function Coll_incl_BVA_cal is added to calculate the BVA by using the formula from chapter 13 of Brigo’s book.

iii. Building Block Diagram:



* New functions/ variables in addition to the original class
\# Function has been amended to calaulate the collateral inclusive BVA

iv. Test cases

From the screen below two trial of BVA Calculation, assuming no re-hypothecation, the BVA figures with collateral are obviously less than that without collateral posted.

Trial #1

```
C:\Users\user\Desktop\MAFS5220\project\forward pricing...
Value date: May 16th, 2015
The configuration of the forward:
Spot: 100
Strike: 100
risk free rate: 0.01
Dividend: 0
Dividend yield: 0
Long/ Short: Long
Tenor (year): 1
Investor's collateral LGD: 0
Counterparty's collateral LGD: 0

Test Case 1:
BVA by the Hazard rate calibration with collateral posted (No re-hypothecation)

CVA 0.00161908569
DVA 0.002125282329

Test Case 2:
BVA by the Hazard rate calibration without collateral posted
CVA 0.007828558267
DVA 0.003988201207

Test Case 3:
BVA by the AT1P CDS calibration with collateral posted (No re-hypothecation)

CVA 0.01058848121
DVA 0.005950051099

Test Case 4:
BVA by the AT1P CDS calibration without collateral posted
CVA 0.006467509718
DVA 0.006749025486
```

The magnitude of CVA and DVA become smaller with collateral

Trial #2

```
C:\Users\user\Desktop\MAFS5220\project\forward pricing...
Value date: May 16th, 2015
The configuration of the forward:
Spot: 100
Strike: 100
risk free rate: 0.01
Dividend: 0
Dividend yield: 0
Long/ Short: Long
Tenor (year): 1
Investor's collateral LGD: 0
Counterparty's collateral LGD: 0

Test Case 1:
BVA by the Hazard rate calibration with collateral posted (No re-hypothecation)

CVA 0.001334318691
DVA 0.003577426321

Test Case 2:
BVA by the Hazard rate calibration without collateral posted
CVA 0.006021535412
DVA 0.004543845641

Test Case 3:
BVA by the AT1P CDS calibration with collateral posted (No re-hypothecation)

CVA 0.006714364049
DVA 0.003037720007

Test Case 4:
BVA by the AT1P CDS calibration without collateral posted
CVA 0.008624792562
DVA 0.007589833083
```

5. User defined interfaces

In order to apply the BVA calculation to real markets, we have developed a GUI for user to input details of the equity forward which they would like to calculate BVA. This GUI will then pass the user inputs, together with market data, to our BVA Calculator to generate the BVA results.

Remarks:

- User can select which hazard rate model they would like to use. (Stepwise / AT1P / Collateral inclusive / WWR)
- User need to input details of the equity forward like underlying stock / counterparty / strike / year to maturity / long short direction / LGD etc., by answering the questions asked in the GUI.

Example(s) of user inputs to the GUI

```
D:\Work\Project\UAT1\Debug\UAT1.exe

+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
+                                                                 +
+ MAFS5220 Group 4: Bilateral Value Adjustment of Equity Forward +
+                                                                 +
+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

Please select which model to use in calculating BVA: (No.1-4)
1) Stepwise Hazard Rate model without Collateralization
2) Stepwise Hazard Rate model with Collateralization (no re-hypothecation)
3) AT1P Hazard Rate model without Collateralization
4) AT1P Hazard Rate model with Collateralization (no re-hypothecation)
5) Hazard Rate with WWR
[Ans]: 2

Please input the BBG code of investor in the Equity Forward:
[Ans]: 5 hk equity

Please input the BBG code of underlying stock of the Equity Forward:
[Ans]: gs us equity

Please input the strike of the Equity Forward:
[Ans]: 202.62

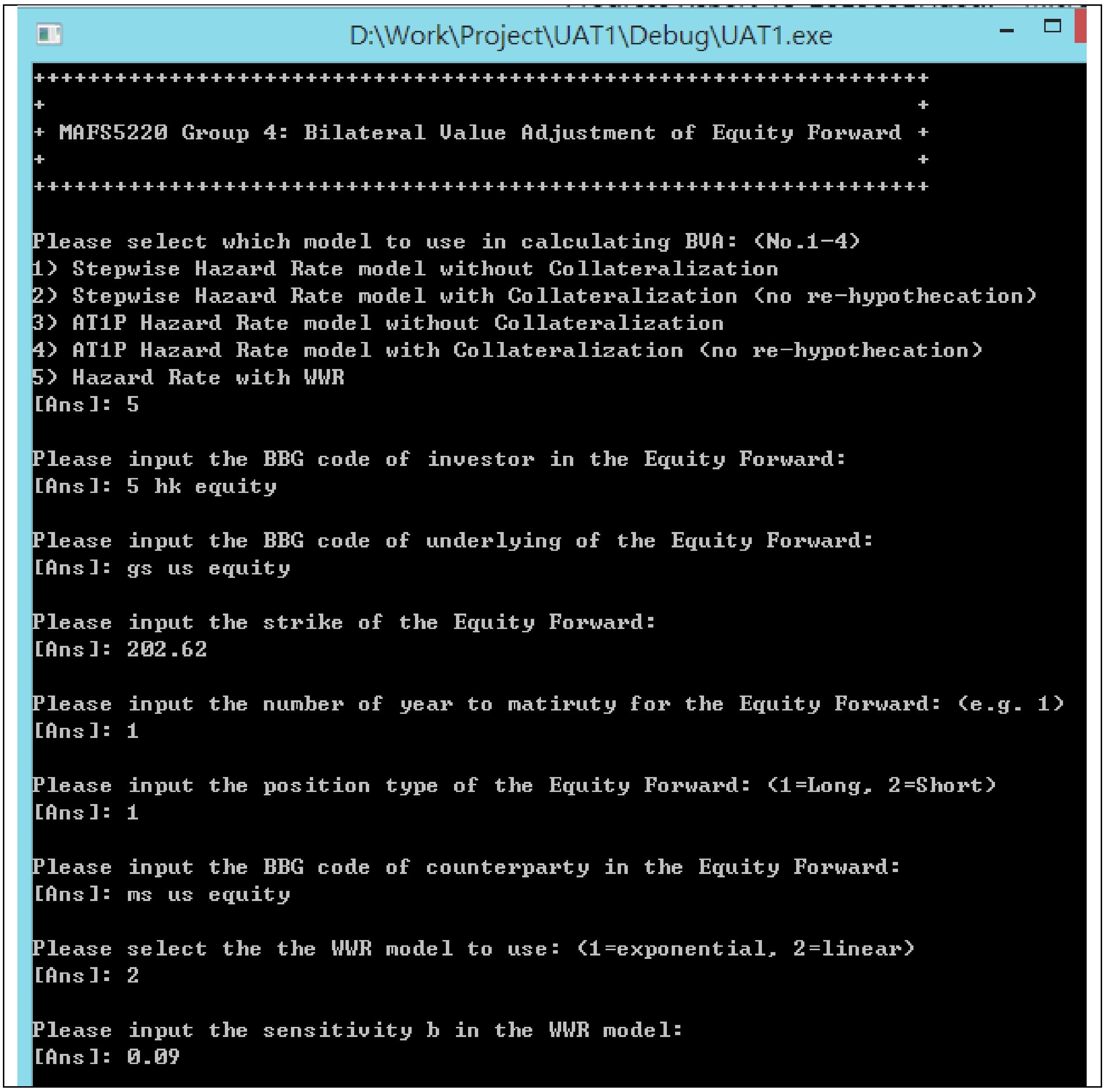
Please input the number of year to matiruty for the Equity Forward: (e.g. 1)
[Ans]: 1

Please input the position type of the Equity Forward: (1=Long, 2=Short)
[Ans]: 1

Please input the BBG code of counterparty of the Equity Forward:
[Ans]: ms us equity

Please input the collateral LGD of investor: (e.g. 0.4)
[Ans]: 0.4

Please input the collateral LGD of counterparty: (e.g. 0.4)
[Ans]: 0.4
```



```
D:\Work\Project\UAT1\Debug\UAT1.exe
++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
+                                                                  +
+ MAFS5220 Group 4: Bilateral Value Adjustment of Equity Forward +
+                                                                  +
++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

Please select which model to use in calculating BVA: (No.1-4)
1) Stepwise Hazard Rate model without Collateralization
2) Stepwise Hazard Rate model with Collateralization (no re-hypothecation)
3) AT1P Hazard Rate model without Collateralization
4) AT1P Hazard Rate model with Collateralization (no re-hypothecation)
5) Hazard Rate with WWR
[Ans]: 5

Please input the BBG code of investor in the Equity Forward:
[Ans]: 5 hk equity

Please input the BBG code of underlying of the Equity Forward:
[Ans]: gs us equity

Please input the strike of the Equity Forward:
[Ans]: 202.62

Please input the number of year to matiruty for the Equity Forward: (e.g. 1)
[Ans]: 1

Please input the position type of the Equity Forward: (1=Long, 2=Short)
[Ans]: 1

Please input the BBG code of counterparty in the Equity Forward:
[Ans]: ms us equity

Please select the the WWR model to use: (1=exponential, 2=linear)
[Ans]: 2

Please input the sensitivity b in the WWR model:
[Ans]: 0.09
```

6. References

[1]: Interest Rate – Models Theory and Practice: With Smile, Inflation and Credit. – Damiano Brigo, Fabio Mercurio

[2]:Counterparty Credit Risk , Collaterial Funding - Damiano Brigo, Massimo Morini , Andrea Pallavicini – chapter 3, 12

[3]: CVA and Wrong Way Risk – John Hull and Alan White – Section 5

[4]: Stochastic Intensity Models of Wrong Way Risk: Wrong Way : CVA Need Not Exceed Independent CVA Samim Ghamami and Lisa R. Goldberg