Testing Report

MAFS5220 Group 1

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1 AnalyticKouEuropeanEngineTests

Scope AnalyticKouEuropeanEngineTests is used to test whether the AnalyticKouEuropeanEngine returns a reasonable European option price, under the assumption that the underlying stock process is a Kou process. Kou process is a double exponential jump diffusion process. Its European option pricing formula is given by equation (20) in Kou's paper. [2]

Data Source One test case is from Kou's paper.[2] The other four test cases are created by ourselves, with zero jump intensity.

Pass Criterion The pass of the first test case requires the option price obtained from our engine to be one percent around the value claimed in Kou's paper. The others, with zero jump intensity, reduce to ordinary geometic Brownian motions, so the option prices should equal what can be obtained from Black-Scholes formula.

Testing Data and Results Two sets of test data and results are listed in the table.

	η_1	η_2	λ	p	σ	r	q	S_0	K	T	Expected	Actual
Kou's paper	10	5	1	0.4	0.16	0.05	0.00	100	98	0.5	9.15	9.07
Reduced case	$e^{0.5}$	$e^{1/1.5}$	0	0.5	0.1	0.03	0.05	100	80	1	17.54	17.54

Table 1: Kou Engine Test Data and Results

2 AT1PTest

Scope AT1P model class calibrates the piecewise constant volatility from the model-independent default probability term sturcture. The model-independent default probability termstructure is bootstrapped from the CDS spreads. Three test cases in AT1PTest are used to see whether the default probabilities calculated from the AT1P model actually fits the model-independent default probabilities.

Data Source Three test cases are from Brigo's book[1], page 59-60, table 3.1, 3.2, 3.3.

Pass Criterion The default probabilities calculated from the AT1P model should be two percent around the values claimed by Brigo.

Discussion Several things might slightly affect the outcome. Firstly, the interpolation method used in bootstrapping the default probability from maket quoted CDS spreads will affect the model-independent default probability curve, which in turn affects our AT1P calibration. It depends on whether you use a "Linear", "BackwardFlat" or other interpolation methods. We use "BackwardFlat" method.

Secondly, we theorectically should calibrate the AT1P volatilities by vanishing the CDS payments plus on default losses.(equation 3.6[1]) However, we calibrate by fitting the model default probabilities to the model-independent ones bootstrapped from CDS spreads. The advantage of the former approach is its more self-consistency, which requires no initial bootstrapping of default probability curve. The disadvantage is that the calibration is really time-consuming, because numerical integration is require every step of optimization. The latter approach actually divides the whole calibration process into two parts, it actually saves time and can reconstruct the default probability curve implied by the CDS spreads.

Testing Data and Results The testing data and result are listed in the table.

		1year	3 year	5year	7 year	10 year
Table 3.1	CDS spread (bps)	16	29	45	50	58
	expected survival prob	0.997	0.985	0.961	0.941	0.902
	actual survival prob	0.997	0.985	0.962	0.941	0.902
Table 3.2	CDS spread (bps)	397	315	277	258	240
	expected survival prob	0.936	0.857	0.800	0.751	0.688
	actual survival prob	0.936	0.855	0.798	0.748	0.685
Table 3.3	CDS spread (bps)	1437	902	710	636	588
	expected survival prob	0.784	0.655	0.591	0.525	0.434
	actual survival prob	0.786	0.654	0.591	0.526	0.434

Table 2: AT1P Test Data and Results

3 AT1PVarianceBugTest

Scope We once equate the "BlackVarianceCurve" with "BlackVolTermStructure". The "BlackVarianceCurve" describe the increase in variance of a process with respect to time, so usually the variance is monotonous. While the volatility term structure not necessarily have this property. The test case is used to reproduce the bug.

Data Source We construct the testing data by ourselves.

Pass Criterion We write a test case to reproduce the bug, and test case passes when the bug disappears.

4 KouProcessTests

Scope Test the Kou process constructor and the "evolve" member function of Kou process.

Data Source The data is constructed by ourselves.

Pass Criterion For the constructor test, we check whether all fields of a Kou process is correctly initialized. For the two sample path tests, we calculate the states of the process using excel spread sheet, then compare with the results obtained from the evolve function. The pass criterion is that the absolute errors are less than 0.001.

5 KouCalibratorTests

Scope The KouProcessCalibrator class calibrate the Kou process parameters from the market quoted option prices. This test case is to verify the calibration.

Data Source The data is from Yahoo Finance.

Pass Criterion After the Kou process is calibrated, we calculate the option prices using AnalyticKouEuropeanEngine. The pass criterion is that the calculation results to be around 10 percent of the market prices, which means the calibration can actually return the market prices.

Testing Data and Results The data is extracted from Yahoo Finance. We calibrate the Kou process parameters to the five option prices from the market.

Option Price	34.75	31.5	30	25.85	21
Strike	95.71	100	104.29	110	114.29

Table 3: Option Prices

Spot Price	Start Date	End Date	Interest Rate	Dividend
129.49	2/22/2015	1/22/2015	0.05	0.01

Table 4: Stock Information

6 CIRModelTest

Scope CIR model is a one factor short rate model, which can describe the entire discount curve. The CIRModelTest tests the calibration of CIR model to the input market bond prices.

Data Source The data is from Bloomberg. They are also used in the calculation of BVA of interest rate swap. You can refer to table 7 for the details of the discount curve.

Pass Criterion The CIR model parameters after calibration should reproduce the discount factors in accordance with the market bond prices. We calibrate the four parameters in CIR model to five discount factors, and allow one percentage of error.

7 AnalyticEquitySwapEngineTest

Scope AnalyticEquitySwapEngineTest is used to test the validity of AnalyticESEngine class. The AnalyticESEngine can calculate the value, or exposure, of equity swap at any time through the life of the contract.

Data Source We construct the testing data by ourselves.

Pass Criterion The theorectical value of equity swap at initiation should be zero. The pass criterion is that the initial value to be less than zero point one percent of the notional.

Testing Data and Results The information of the equity swap tested is listed in the table.

start price	spot price	N	T	K	σ	q	r	reference date	start date	expected	actual
35.12	35.12	100	5	0.04	0.20	0.03	0.04	2015/3/5	2015/3/5	0.0	0.04%

Table 5: AnalyticESEngine Test Data and Results

8 BVATest

Our BVA calculation is under Monte Carlo simulation framework. BVA is composed into two parts: CVA and DVA. They satisfy the relation: $BVA = CVA - DVA$. The MCBVAEngine class is inherited from template class BVAEngine. BVAEngine accepts pointer to model as parameter to calculate bilateral value adjustment of different models.

The BVA outcome obtained from Monte Carlo simulation can be tricky. The stability of CVA and DVA is mainly determined by the occurence of default events. We need to carry out a large number of simulations to get good CVA and DVA estimations. Even when CVA and DVA are promising, the BVA, which is the difference of the two numbers can fluctuate wildly when CVA and DVA are close to each other. So the accuracy of BVA is also largely affected by the relative creditworthiness of the counterparties.

8.1 InterestRateSwapBVATest

Scope InterestRateSwapBVATest test the Bilateral Value Adjustment of interest rate swap.

Data Source The data is from Bloomberg.

Pass Criterion Bloomberg has internal models to support the calculation of bilateral valuation adjustment of interest rate swap. To pass the test, our results should be around the results obtained from Bloomberg.

Testing Data and Results We carried out two tests. The first one is a five-year payer swap started on 2015/3/18. The second one is a five-year receiver swap started on 2015/5/5. For the second test case, we change the fixed rate to see its influence on the bilateral valuation adjustment of interest rate swap. The data used are listed in the tables below.
For a payer interest rate swap, the larger the fixed rate the less the positive exposure, which means less CVA and more DVA. So fixed rate level is negatively correlated with BVA. On the other hand, for a receiver vanilla swap, the fixed rate is positively correlated with BVA.

Type	Start Date	Maturity Date	fixed rate	Fixed Leg Frequency	Floating Leg Frequency
Payer	3/18/2015	3/18/2020	0.01551176	6 month	3 month

Table 6: Interest Rate Swap Information (Test 1)

Date	3/18/2015	6/18/2015	9/18/2015	12/18/2015	3/18/2016	6/18/2016	9/18/2016
Discount	1.0000	0.9993	0.9983	0.9968	0.9948	0.9922	0.9891
Date	12/18/2016	3/18/2017	6/18/2017	9/18/2017	12/18/2017	3/18/2018	6/18/2018
Discount	0.9855	0.9815	0.9771	0.9724	0.9675	0.9623	0.9570
Date	9/18/2018	12/18/2018	3/18/2019	6/18/2019	9/18/2019	12/18/2019	3/18/2020
Discount	0.9518	0.9464	0.9409	0.9355	0.9299	0.9242	0.9184

Table 7: Interest Rate Swap Discount Curve (Test1)

CDS Spreads (bps)	1 year	2 year	3 year	4 year	5 year
Issuer	316.21	507.27	906.06	1179.75	1269.78
Investor	340.21	474.3	597.06	656.24	779.95

Table 8: Interest Rate Swap Counterparty Information (Test 1)

Path Number	50,000	100,000	200,000	BloomBerg
Result (bps)	42.1388	41.8268	41.9586	41.37

Table 9: Interest Rate Swap BVA Results (Test 1)

Type	Start Date	Maturity Date	fixed rate	Fixed Leg Frequency	Floating Leg Frequency
Receiver	5/5/2015	5/5/2020	0.0167	6 month	3 month
Receiver	5/5/2015	5/5/2020	0.02	6 month	3 month

Table 10: Interest Rate Swap Information (Test 2)

Date	5/5/2015	8/5/2015	11/5/2015	2/5/2016	5/5/2016	8/5/2016	11/7/2016
Discount	1.0000	0.9993	0.9983	0.9969	0.9951	0.9927	0.9897
Date	2/6/2017	5/5/2017	8/7/2017	11/6/2017	2/5/2018	5/8/2018	8/6/2018
Discount	0.9863	0.9826	0.9782	0.9736	0.9688	0.9636	0.9584
Date	11/5/2018	2/5/2019	5/7/2019	8/5/2019	11/5/2019	2/5/2020	5/5/2020
Discount	0.9532	0.9475	0.9417	0.9361	0.9302	0.9241	0.9179

Table 11: Interest Rate Swap Discount Curve (Test 2)

8.2 EquitySwapBVATest

Scope EquitySwapBVATest test the Bilateral Value Adjustment of equity swap.

Data Source From Brigo's book [1] page 171

CDS Spreads (bps)	1 year	2 year	3 year	4 year	5 year
Issuer	80.01	254.03	503.31	620.66	699.91
Investor	33.03	44.24	57.2	68.5	82.79

Table 12: Interest Rate Swap Counterparty Information (Test 2)

Path Number	50,000	100,000	200,000	BloomBerg
Result	6.3772	6.4525	6.4266	6bps
Result	22.8686	23.3041	23.4845	23.14bps

Table 13: Interest Rate Swap BVA Results (Test 2)

Pass Criterion The two counterparties have similar credit conditions, so the CVA and DVA part of the BVA should be close to each other, which contributes a close-to-zero BVA. The relative error of the BVA can be quite big.

Testing Data and Results The information of counterparties and equity swap is in the below tables. We cannot find an instrument that perfectly fits the equity swap in Bloomberg, so we construct this BVA test data partly from Brigo[1].
For a payer equity swap, when the fixed rate decreases, the investor has larger positive exposure. So with all the other variables and counterparties conditions remain the same, the CVA will increase and DVA decrease, which leads to a bigger BVA. The BVA is negatively correlated with dividend yield.
When the volatility of the underlying stock increases, both the CVA and DVA increases, but the BVA basically remains unchange. (table 17) The volatility of the underlying stock has relatively small impact on BVA.

CDS Spreads (bps)	1 year	3 year	5 year	7 year	10 year
issuer	19	32	42	45	56
investor	19	32	42	45	56

Table 14: Equity Swap Counterparty Information

Start Price	Spot Price	T	σ	q	Fixed Rate	Reference Date	Start Date
20.0	20.0	5	0.20	0.008	0.04	2004/3/10	2004/3/10

Table 15: Equity Swap Information

Simulation	50,000	100,000	200,000
CVA	0.0782668	0.0814642	0.0800553
DVA	0.0402974	0.0401452	0.0387499
BVA	0.0379694	0.041319	0.0413054

Table 16: Equity Swap BVA Results

100,000 path	CVA	DVA	BVA
$\sigma = 0.3$	0.105279	0.067038	0.038241
$\sigma = 0.2$	0.081464	0.040145	0.041319
$\sigma = 0.1$	0.05127	0.01327	0.038

Table 17: Equity Swap BVA-σ Sensitivity

8.3 CrossCurrencySwapBVATest

Scope CrossCurrencySwapBVATest test the Bilateral Value Adjustment of cross currency swap.

Data Source The data is from Bloomberg.

100,000 path	CVA	DVA	BVA
Fixed Rate=0.02	0.087261	0.034418	0.052844
Fixed Rate=0.04	0.081464	0.040145	0.041319
Fixed Rate=0.06	0.065875	0.048869	0.017006

Table 18: Equity Swap BVA-Fixed Rate Sensitivity

Pass Criterion Bloomberg has internal models to support the calculation of bilateral valuation adjustment of cross currency swap. To pass the test, our results should be very close to the results obtained from Bloomberg. We allow big relative error, because the BVA is fairly close to zero and is likely to fluctuate a lot.

Testing Data and Results The information of counterparties and cross currency swap is in table 20 and table 21. The BVA calculated by Bloomberg is 7169.7, and our result is 50% bigger. There are several reasons for the disparity. First, we do not know the volatility of the FX rate, which is a necessary parameter to calculate the cross currency swap exposure. We estimate the volatility using historical data, and the result is around 12%. Secondly, the BVA calculation by Bloomberg does not take the first-to-default and wrong way risk into consideration. This also may account for some parts of the difference in the outcome.

Date	3/18/2015	9/18/2015	3/18/2016	9/18/2016	3/18/2017	9/18/2017	3/18/2018
Discount	1.0000	0.9983	0.9948	0.9891	0.9815	0.9724	0.9623
Date	9/18/2018	3/18/2019	9/18/2019	3/18/2020	3/18/2021	3/18/2022	3/18/2023
Discount	0.9518	0.9411	0.9301	0.9186	0.8955	0.8727	0.8501
Date	3/18/2024	3/18/2025	3/18/2026	3/18/2027	3/18/2030	3/18/2035	3/18/2040
Discount	0.8275	0.8055	0.7838	0.7625	0.7023	0.6131	0.5379

Table 19: Cross Currency Swap Discount Curve

CDS Spreads (bps)	1 year	2 year	3 year	4 year	5 year	7 year	10 year
Investor	30.235	41.876	54.027	65.151	78.457	102.7	122.061
Issuer	26.766	35.064	42.953	53.08	68.203	91.453	111.109

Table 20: Cross Currency Swap Counterparty Information

Reference Date	FX rate	FXVol	DomesticVol	ForeignVol
3/20/2015	0.161171	0.12	0.001	0.001
Type	ReceiveNominal	ReceiveRate	PayNominal	PayRate
Pay Foreign	1.00E+07	0.015676	62046000	0.03294

Table 21: Cross Currency Swap Information

CCS BVA	CVA	DVA	BVA
50000 path	32499.3	20758.2	11741.1
100000 path	33745.8	21378.3	12367.5
200000 path	32396.9	20346.4	12050.5
Bloomberg			7169.72

Table 22: Cross Currency Swap BVA Results

9 TVATest

Our TVA calculation is under Monte Carlo simulation framework. TVA is composed into three parts: CVA, DVA and FVA. And they satisfy the relation: $TVA = CVA - DVA + FVA$. The MCTVAEngine class is inherited from template class TVAEngine. TVAEngine accepts pointer

	CVA	DVA	BVA
150% Receiver Rate	29303.6	24892.1	4411.5
100% Receiver Rate	32499.3	20758.2	11741.1
50% Receiver Rate	36783.4	17470.1	19313.3

Table 23: Cross Currency Swap BVA-Receiver Rate Sensitivity

	CVA	DVA	BVA
150% Pay Rate	42807.3	15195	27612.3
100% Pay Rate	32499.3	20758.2	11741.1
50% Pay Rate	25229.5	29008.4	-3778.9

Table 24: Cross Currency Swap BVA-Payer Rate Sensitivity

to model as parameter to calculate total value adjustment of different models.
We calculate FVA by summing up the funding benefit and cost across small time periods. So we have to calculate the exposure of the instrument a lot of times for one single path. We can imagine the work load of calculating FVA compared with BVA. However, FVA exists even if no default occurs, and default events are actually rare, so the value of FVA is much more stable than CVA and DVA. So we need relatively smaller number of paths in Monte Carlo simulation, than BVA calculation, to get a convincing result.
It is quite hard to find good FVA testing data. We can estimate the FVA using liquidity spread of the investor and the average exposure.
The TVA results are represented as one dollar notional.

9.1 InterestRateSwapTVATest

Scope InterestRateSwapTVATest tests the Total Value Adjustment of interest rate swap.

Data Source The data is from Bloomberg.

Testing Data and Results The testing data used is the same as in the BVA calculation, please refer to BVA tests for detailed information. The simulation results are listed in the table 25.

Path Number	1,000	2,000	5,000
FVA (bps)	-40.20	-38.77	-39.96

Table 25: Interest Rate Swap TVA Results

9.2 EquitySwapTVATest

Scope EquitySwapTVATest tests the Total Value Adjustment of equity swap.

Data Source From Brigo's book [1] page 171

Testing Data and Results The testing data used is the same as in the BVA calculation, please refer to BVA tests for detailed information. The simulation results are listed in the table 26.

Path Number	1,000	2,000	5,000
FVA (bps)	-30.5	-30.2	-29.4

Table 26: Equity Swap TVA Results

9.3 CrossCurrencySwapTVATest

Scope CrossCurrencySwapTVATest tests the Total Value Adjustment of cross currency swap.

Data Source The data is from Bloomberg.

Testing Data and Results The testing data used is the same as in the BVA calculation, please refer to BVA tests for detailed information. The simulation results are listed in the table 27.

Path Number	1000	2000	5000
FVA (bps)	-20.916	-20.826	-20.800

Table 27: Cross Currency Swap TVA Results

10 CreditVaRTest

To calculate credit VaR using Monte Carlo simulation, we simulate a large number of path and obtain the loss distribution on default; if there is no default the loss is zero for this path. We then sort the losses and find the 99 percentile of the loss, which is the 99% credit VaR. In case of default events are rare, we should simulate much more paths than BVA calculation to obtain a smooth distribution of default loss over the 99 percentile. So the credit VaR calculation is more time-consuming than BVA.
The credit VaR are represented in one dollar notional.

10.1 InterestRateSwapCreditVaRTest

Scope InterestRateSwapCreditVaRTest tests the credit var calculation of interest rate swap.

Data Source The data is from Bloomberg.

Testing Data and Results The testing data used is the same as in the BVA calculation, please refer to BVA tests for detailed information.

10.2 EquitySwapCreditVaRTest

Scope EquitySwapCreditVaRTest tests the credit var calculation of equity swap.

Data Source From Brigo's book [1] page 171

Testing Data and Results The testing data used is the same as in the BVA calculation, please refer to BVA tests for detailed information.

Path	50,000	100,000	200,000
Credit VaR	0.1370435	0.1354615	0.135258

Table 28: Equity Swap Credit VaR Results

10.3 CrossCurrencySwapCreditVaRTest

Scope CrossCurrencySwapCreditVaRTest tests the credit var calculation of cross currency swap.

Data Source The data is from Bloomberg.

Testing Data and Results The testing data used is the same as in the BVA calculation, please refer to BVA tests for detailed information.

Path	50,000	100,000	200,000
Credit VaR	0.0866755	0.0870713	0.0864844

Table 29: Cross Currency Swap Credit VaR Results

References

[1] Damiano Brigo, Massimo Morini, and Andrea Pallavicini. *Counterparty Credit Risk, Collateral and Funding: With Pricing Cases For All Asset Classes*. John Wiley & Sons, 2013.

[2] Steven G Kou. A jump-diffusion model for option pricing. *Management science*, 48(8):1086–1101, 2002.