第四周第二次作业答案

4-2.1

解:

系统的冲激响应$h(t)=\mathcal{F}^{-1}[H(j\omega)]$，理想低通滤波器$h_I(t)=\mathcal{F}^{-1}[H_I(j\omega)]$

$$h_I(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}H_I(j\omega)e^{j\omega t}d\omega=\frac{1}{2\pi}\int_{-\omega_c}^{\omega_c}e^{j\omega(t-t_0)}d\omega=\frac{\omega_c}{\pi}S_a[\omega_c(t-t_0)]$$

解法一:

$$\begin{aligned}h(t)&=\frac{1}{2\pi}\int_{-\infty}^{+\infty}H(j\omega)e^{j\omega t}d\omega\\&=\frac{1}{2\pi}\int_{-\omega_c}^{\omega_c}\left[\frac{1}{2}+\frac{1}{2}\cos\left(\frac{\pi}{\omega_c}\omega\right)\right]e^{-j\omega t_0}e^{j\omega t}d\omega\\&=\frac{1}{4\pi}\cdot[\frac{e^{j\omega(t-t_0)}}{j(t-t_0)}|_{-\omega_c}^{\omega_c}]+\frac{1}{8\pi}\cdot[\frac{e^{j\omega(t-t_0+\frac{\pi}{\omega_c})}}{j(t-t_0+\frac{\pi}{\omega_c})}+\frac{e^{j\omega(t-t_0-\frac{\pi}{\omega_c})}}{j(t-t_0-\frac{\pi}{\omega_c})}|_{-\omega_c}^{\omega_c}]\\&=\frac{\omega_c}{2\pi}Sa[\omega_c(t-t_0)]+\frac{1}{4\pi}Sa[\omega_c(t-t_0+\frac{\pi}{\omega_c})]+\frac{1}{4\pi}Sa[\omega_c(t-t_0-\frac{\pi}{\omega_c})]\end{aligned}$$

解法二:

$\cos\left(\frac{\pi}{\omega_c}\omega\right)=\frac{e^{j\frac{\pi}{\omega_c}\omega}+e^{-j\frac{\pi}{\omega_c}\omega}}{2}$,

$$H(j\omega)=H_I(j\omega)\left[\frac{1}{2}+\frac{1}{2}\cos\left(\frac{\pi}{\omega_c}\omega\right)\right]=\frac{1}{2}H_I(j\omega)+\frac{e^{j\frac{\pi}{\omega_c}\omega}+e^{-j\frac{\pi}{\omega_c}\omega}}{4}H_I(j\omega)$$

由傅里叶变换的线性和时移特性得,

$$h(t)=\frac{\omega_c}{2\pi}Sa[\omega_c(t-t_0)]+\frac{1}{4\pi}Sa[\omega_c(t-t_0+\frac{\pi}{\omega_c})]+\frac{1}{4\pi}Sa[\omega_c(t-t_0-\frac{\pi}{\omega_c})]$$

理想低通滤波器和此系统的冲激响应波形图分别如图(a)和(b)所示，与理想低通滤波器的冲激响应相比，此系统的冲激响应也具有抽样函数的形状，但主瓣宽度变宽了，幅度也减小了，主峰为理想低通滤波器的一半。

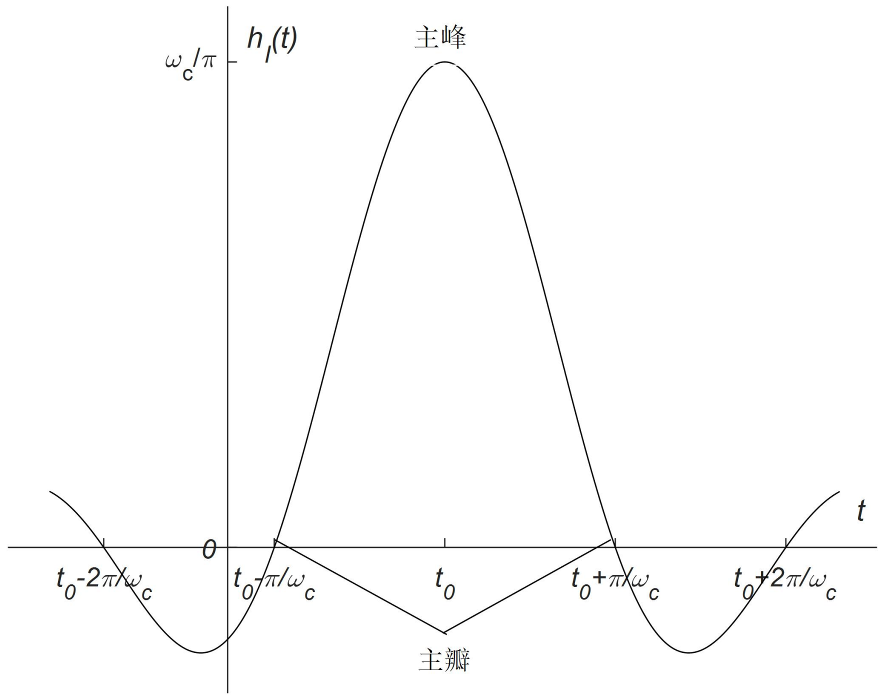
$h_l(t)$
主峰
ω_c/π
t
0
$t_0-2\pi/\omega_c$
$t_0-\pi/\omega_c$
t_0
$t_0+\pi/\omega_c$
$t_0+2\pi/\omega_c$
主瓣

图 a

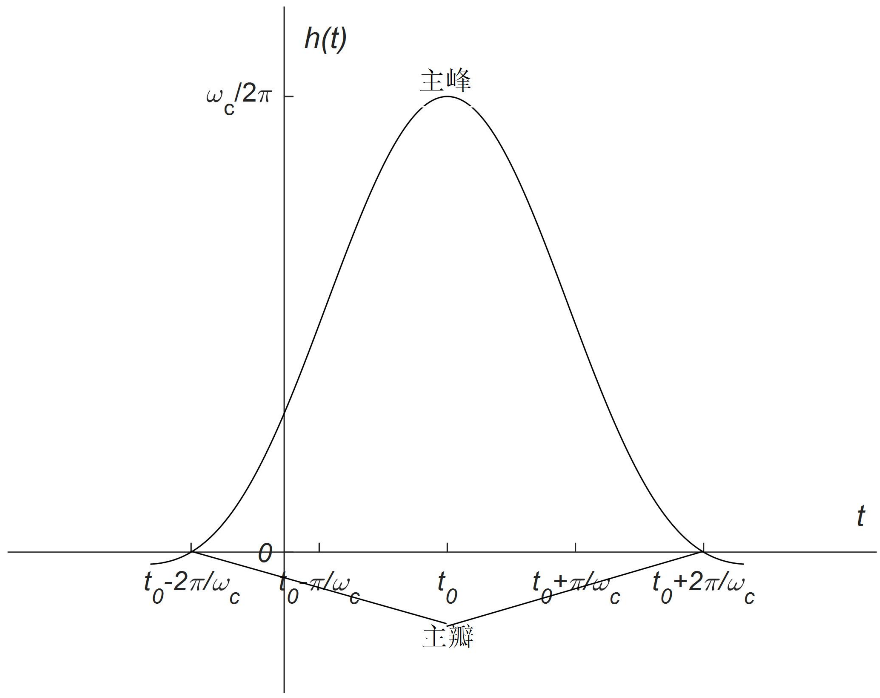
$h(t)$
主峰
$\omega_c/2\pi$
t
0
$t_0-2\pi/\omega_c$
$t_0-\pi/\omega_c$
t_0
$t_0+\pi/\omega_c$
$t_0+2\pi/\omega_c$
主瓣

图 b

4-2.2

解:

(1)$|H(j\omega)| = 1 - G_{2\omega_c}(\omega)$

故$h_1(t) = \delta(t) - \frac{\omega_c}{\pi} Sa(\omega_c t) = \delta(t) - 80Sa(80\pi t)$

故 $h(t) = \mathscr{F}^{-1}[H(j\omega)] = \mathscr{F}^{-1}[|H(j\omega)|e^{-i\omega t_0}] = h_1(t - t_0)$

$= \delta(t - t_0) - 80Sa(80\pi t - t_0)$

(2)由于高通系统的截止频率$\omega_c = 80\pi$，信号 f(t)中只有角频率大于 80π的频率分量方程能通过，故得 $y(t) = 0.2\cos 120\pi(t - t_0)$, $t \in R$.