第三周第一次作业答案

3-2.1

解：设$f(t)=a_0+\sum_{n=1}^{\infty}(a_n\cos(n\omega_1 t)+b_n\sin(n\omega_1 t))$，

$\omega_1=\frac{2\cdot\pi}{T}=2\cdot\pi\cdot f=2\cdot\pi\times 5\times 10^3=10^4\cdot\pi\ \ (rad\ /\ s)$，

$$\frac{1}{T}=f=5\times 10^3 Hz, E=10V, \tau=2\times 10^{-5}$$

由图可知，$f(t)$为偶函数，故$b_n=0,(n\geq 1)$。

$$a_0=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}f(t)\,dt=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}E\cdot[u\left(t+\frac{\tau}{2}\right)-u\left(t-\frac{\tau}{2}\right)]\,dt$$
$$=5\times 10^3\times E\cdot\tau$$
$$=5\times 10^3\times 10\times 2\times 10^{-5}$$
$$=1\text{V}$$

$$a_n=\frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}f(t)\cdot\cos(n\omega_1 t)\,dt$$
$$=\frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}E\cdot\left[u\left(t+\frac{\tau}{2}\right)-u\left(t-\frac{\tau}{2}\right)\right]\cdot\cos(n\omega_1 t)\,dt$$
$$=\frac{2\cdot E}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}\cos(n\omega_1 t)\,dt$$
$$=\frac{2\cdot E}{n\omega_1 T}\cdot\sin(n\omega_1 t)\quad\Big|_{-\frac{\tau}{2}}^{\frac{\tau}{2}}$$
$$=\frac{2\times 10\times 5\times 10^3}{n\cdot 10^4}\times 2\sin\left(\frac{n\omega_1\tau}{2}\right)$$
$$=\frac{20}{n\cdot\pi}\times\sin\left(\frac{n\cdot\pi}{10}\right)\qquad ,(n\geq 1)$$

直流分量$a_0=1$V。

基波的有效值：$\frac{\sqrt{a_1^2+b_1^2}}{\sqrt{2}}=\frac{\left|\frac{20}{\pi}\times\sin\left(\frac{\pi}{10}\right)\right|}{\sqrt{2}}\approx 1.39$V，

二次谐波的有效值：$\frac{\sqrt{a_2^2+b_2^2}}{\sqrt{2}}=\frac{\left|\frac{20}{2\cdot\pi}\times\sin\left(\frac{2\cdot\pi}{10}\right)\right|}{\sqrt{2}}\approx 1.32$V

三次谐波的有效值：$\frac{\sqrt{a_3^2+b_3^2}}{\sqrt{2}}=\frac{\left|\frac{20}{3\cdot\pi}\times\sin\left(\frac{3\cdot\pi}{10}\right)\right|}{\sqrt{2}}\approx 1.21$V