ML dseb62 w1

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1 Problem 1

	x1	x2	x3	x4	x5	p(y)
y1	0.01	0.02	0.03	0.1	0.1	0.26
y2	0.05	0.1	0.05	0.07	0.2	0.47
y3	0.1	0.05	0.03	0.05	0.04	0.27
p(x)	0.16	0.17	0.11	0.22	0.34	1

a) Based on the table above:

$$\begin{aligned} p(x) &= p(x1) + p(x2) + p(x3) + p(x4) \\ &= 0.16 + 0.17 + 0.11 + 0.22 + 0.34 \\ &= 1 \end{aligned} \tag{1}$$

$$\begin{aligned} p(y) &= p(y1) + p(y2) + p(y3) \\ &= 0.26 + 0.47 + 0.27 \\ &= 1 \end{aligned} \tag{2}$$

b) we have the following formula for conditional probability:

$$p(x_i | y = y_1) = \frac{p(x_i, y_1)}{p(y_1)} \tag{3}$$

by applying this formula we can calculate

$$\begin{aligned} p(x_1 | Y = y_1) &= \frac{p(x_1, y_1)}{p(y_1)} \\ &= \frac{0.01}{0.26} \\ &= \frac{1}{26} \end{aligned} \tag{4}$$

by performing the same operation on all values of X, we have the following table:

i	1	2	3	4	5
$p(x_i\|Y = y_1)$	1/26	1/13	3/26	5/13	5/13

similarly, we can calculate $p(x|Y = y_3)$ and obtain the following result:

i	1	2	3	4	5
$p(x_i\|Y = 3)$	10/27	5/27	1/9	5/27	4/27

2 Problem 2

For discrete variables:

$$
\begin{aligned}
E_y[E_x[x|y]] &= E_y[\sum_x x * p(x|y)] \\
&= \sum_y [\sum_x x p(x|y)] p(y) \\
&= \sum_y \sum_x x p(x, y) \\
&= \sum_x x \sum_y p(x, y) \\
&= \sum_x x p(x) \\
&= E_x[x]
\end{aligned}
\tag{5}
$$

For continuous variables:

$$
\begin{aligned}
E_y[E_x[x|y]] &= \int E_x[x|y] f_y(y) dy \\
&= \int \int x f_{X|Y}(x|y) f_y(y) dx dy \\
&= \int \int x f(x, y) dx dy \\
&= \int x dx \int f(X, y) dy \\
&= \int x f(x) dx \\
&= E_x[x]
\end{aligned}
\tag{6}
$$

3 Problem 3

Let x be the event that people use product X
Let y be the event that people use product Y
According to the problem, we have:
$p(x) = 0.207$
$p(y) = 0.5$
$p(x|y) = 0.365$
a)

$$\begin{aligned}
p(x, y) &= p(x|y) * p(y) \\
&= 0.365 * 0.5 \\
&= 0.1825
\end{aligned} \tag{7}$$

b) $p(\bar{x}) = 1 - p(x) = 0.793$
$p(\bar{x}|y) = 1 - p(x|y) = 1 - 0.365 = 0.635$

$$\begin{aligned}
p(y|\bar{x}) &= \frac{p(y, \bar{x})}{p(\bar{x}} \\
&= \frac{p(\bar{x}|y)p(y)}{p(\bar{x})} \\
&= \frac{0.635 * 0.5}{0.793} \\
&= 0.4
\end{aligned} \tag{8}$$

4 Problem 4

raw-score expression for the variance:

$$\begin{aligned}
\sigma^2 &= \frac{\sum_N (x - \mu)^2}{N} \\
&= \frac{\sum_N (x^2 - 2x\mu + \mu^2)}{N} \\
&= \frac{\sum_N x^2}{N} - \frac{\sum_N 2x\mu}{N} + \frac{\sum_N \mu^2}{N} \\
&= \frac{\sum_N x^2}{N} - 2\mu^2 + \frac{N * \mu^2}{N} \\
&= \frac{\sum_N x^2}{N} - 2\mu^2 + \mu^2 \\
&= \frac{\sum_N x^2}{N} - \mu^2 \\
&= E[x^2] - E[x]^2
\end{aligned} \tag{9}$$

5 Problem 5

Let x_i be the event that the player choose door i (i $\in$ {1, 2, 3})
Let c_i be the event that the car is in door i (i $\in$ {1, 2, 3})

Let m_i be the event that Monty chooses door (i $\in$ {1, 2, 3}) Suppose that Monty chooses door 3 and player chooses door 1, we have $p(x1|c1) = \frac{1}{3}$

$p(c1) = 1/3$

$p(x1) = 1$

$p(m3|c2, x1) = 1$

$p(m3|c1, x1) = \frac{1}{2}$

$p(m3|x1) = \frac{1}{2}$

The event that we win a car by staying is that the car is in door 1 given that we chooses door 1 and monty chooses door 3:

$$
\begin{aligned}
p(c1|x1, m3) &= \frac{p(c1, x1, m3)}{p(x1, m3)} \\
&= \frac{p(m3|x1, c1)}{p(m3|x1)p(x1)} \\
&= \frac{\frac{1}{2}\frac{1}{3}}{\frac{1}{2}} \\
&= \frac{1}{3}
\end{aligned}
\tag{10}
$$

The event that we lose a car by switching is that the car is in door 2 given that we chooses door 2 and monty chooses door 3:

$$
\begin{aligned}
p(c2|x1, m3) &= \frac{p(c2, x1, m3)}{p(x1, m3)} \\
&= \frac{p(m3|x1, c2)}{p(m3|x1)p(x1)} \\
&= \frac{1\frac{1}{3}}{\frac{1}{2}} \\
&= \frac{2}{3}
\end{aligned}
\tag{11}
$$

In conclusion, the probability that we win by staying is only $\frac{1}{3}$ whereas if we switch, the probability that we win is $\frac{2}{3}$, thus switching is better in this case