

ML dseb62 w4 - Nguyen Tuan Duy

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1 Problem 2c

Ridge regression

Loss function:

$$\begin{aligned} L_{ridge} &= \frac{1}{2N} \sum_{i=1}^N (w_0 + w_1 x_i - y_i)^2 + \lambda w_1^2 \\ &= \frac{1}{2N} (\|Y - XW\|_2^2 + \lambda \|W\|^2) \end{aligned}$$

minimizing L_{ridge} is the same as minimizing $L = \|Y - XW\|_2^2 + \lambda \|W\|^2$

$$\begin{aligned} \frac{\delta L}{\delta W} &= 2X^T(Y - XW) + 2\lambda W = 0 \\ W &= (X^T X + I\lambda)^{-1} X^T Y \end{aligned}$$

Lasso regression:

$$L_{lasso} = \frac{1}{2N} \sum_{i=1}^N (w_0 + w_1 x_i - y_i)^2 + \lambda |w_1|$$

because $\lambda |w_1|$ cannot be derived we have to optimize the function using gradient descent

* algorithm:

we update $w = w - \alpha * dw$ and $b = b - \alpha * db$ where α is the learning rate and d

if $w_i > 0$:

$$dw = \frac{-2}{m} \sum_{i=1}^N (w_0 + w_1 x_i - y_i)^2 + \lambda$$

if $w_i \leq 0$:

$$dw = \frac{-2}{m} \sum_{i=1}^N (w_0 + w_1 x_i - y_i)^2 - \lambda$$

$$\text{and } db = \frac{-2}{m} (w_0 + w_1 x_i - y_i)$$