

PyDDSBB: A Python Package for Simulation-Optimization using Data-Driven Branch-and-Bound Techniques

Jianyuan Zhai¹, Suryateja Ravutla¹, and Fani Boukouvala¹

¹ Department of Chemical and Biomolecular Engineering, Georgia Institute of Technology, Atlanta, GA
✉ Corresponding author

DOI: [10.xxxxxx/draft](https://doi.org/10.xxxxxx/draft)

Software

- [Review](#)
- [Repository](#)
- [Archive](#)

Editor: [Open Journals](#)

Reviewers:

- [@openjournals](#)

Submitted: 01 January 1970

Published: unpublished

License

Authors of papers retain copyright and release the work under a Creative Commons Attribution 4.0 International License ([CC BY 4.0](#)).

Summary

High-fidelity (HF) computer simulations are essential for quantitative analysis and decision-making (i.e., design or inverse optimization), however, often the objective functions and the constraints reliant on a simulation are only available as black-box functions (Bhosekar & Ierapetritou, 2018; Boukouvala et al., 2016; McBride & Sundmacher, 2019; Rios & Sahinidis, 2013). Sample generation using HF simulations is often expensive due to computational cost and time. Most of the equation/derivative-based optimization solvers cannot be directly applied to optimize problems with embedded HF simulations due to lack of closed-form equations and gradient information. As an alternative, machine learning (ML) surrogate models are often employed to approximate HF simulation data and expedite the optimization search (Bhosekar & Ierapetritou, 2018). However, training and building accurate models requires large amount of data model parameterizations are highly subjected to the available data. One may arrive at different solutions because the ML models may be nonconvex and the surrogate model parameterizations can be different due to re-initialization and slight changes in the data set. Finally, most derivative-free or black-box methods offer no information regarding the quality of the incumbent optimal solution (e.g., upper/lower bound on the optimum) (Amaran et al., 2016; Boukouvala et al., 2016; Larson et al., 2019; Zhai et al., 2021; Zhai & Boukouvala, 2022). To tackle these challenges, we recently proposed a data-driven equivalent of the spatial branch-and-bound (DDSBB) algorithm based on the concept of constructing convex underestimators of HF data from simulations and low-fidelity (LF) data from ML model approximations (Zhai et al., 2021; Zhai & Boukouvala, 2022).

Statement of need

Black-box optimization is a challenging problem due to lack of analytic equations and derivatives. Such problems arise in many fields of science and engineering, such as chemical process design, oilfield operations, protein folding, aircraft/vehicle design, and many more (McBride & Sundmacher, 2019). Developing novel and improved data-driven approaches for efficient optimization of such systems will help in quantitative analysis and improved decision making. Although many methods exist, one drawback of many sampling- or ML-based optimization methods is the high dependency on initial sampling and selection of the ML model type. PyDDSBB is a Python package for the proposed Data-Driven Spatial Branch-and-Bound Algorithm (Zhai et al., 2021; Zhai & Boukouvala, 2022). Instead of only using samples, or directly relying on a single ML model fit, pyDDSBB develops convex underestimators as relaxations of data generated from HF models and/or ML surrogates. These relaxations are embedded into the branch and bound algorithm, and the search space is progressively partitioned by branching and pruning heuristics. This approach allows for estimation of upper

42 and lower bounds on the optimum solution, which help in pruning spaces that have a very low
43 probability of containing a better optimum. Samples are added adaptively in the non-pruned
44 subspaces(Rios & Sahinidis, 2013; Zhai et al., 2021; Zhai & Boukouvala, 2022). Through
45 benchmarking of pyDDSB on a large pool of problems with dimensions 1-10, we have shown
46 that the solver can locate the global optimum for >90% of the problems with 2-3 dimensions
47 and >70% of the problems with 4-10 dimensions. Most importantly pyDDSB provides valid
48 bounds for overall >90% of the problems(Zhai & Boukouvala, 2022).

49 Overview and Description

50 The PyDDSB algorithm has been constructed using object-oriented programming, with
51 an intent of easier incorporation of further additions or extensions to the algorithm. The
52 algorithm has the option of employing Support Vector Regression models (trained by HF data)
53 to generate LF data that also inform the underestimators. PyDDSB utilizes the Scikit-learn
54 (Pedregosa et al., 2011) package for constructing and optimizing the surrogate models for
55 LF data generation and also offers the capability of user-based additions of new surrogate
56 models. The algorithm utilizes the package – PYOMO(Hart et al., 2011) to construct the
57 convex quadratic underestimators and also provides an option for integration of other type
58 of underestimators. Latin Hypercube Sampling (LHS) technique is used for generating the
59 samples and the samples are generated adaptively in the non-pruned subspaces by utilizing the
60 bounding information. Two possible paths for sampling and constructing the underestimators
61 in algorithm are shown in Figure 1.

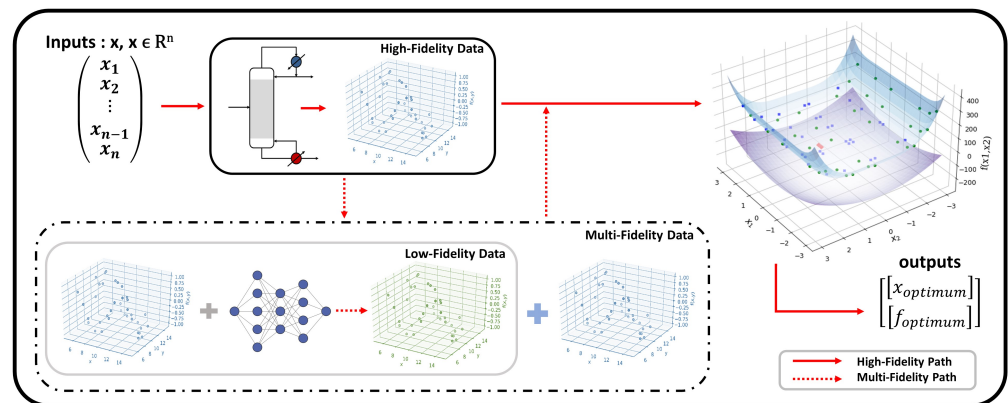
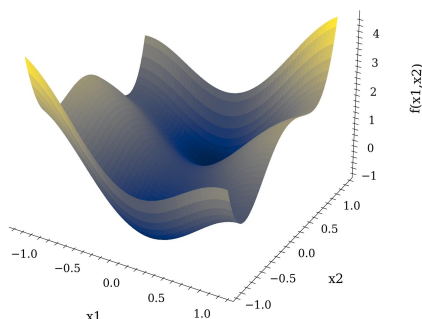


Figure 1: Overview of PyDDSB sampling and underestimator construction. Solid red line shows the HF path where only HF samples obtained from black-box simulation are used in constructing underestimators. Dotted red line shows the path for MF approach, in which HF samples are used to generate LF samples and, combined HF and LF samples are used to construct underestimators

62 While the objective function is assumed to be simulation-based (black-box), PyDDSB allows
63 the user to include simulation-based constraints (unknown or black-box) and equation-based
64 (known) constraints into the formulation. A variety of branching techniques (e.g., equal
65 bisection, longest side, ML-based prioritizing important variables, etc.)(Zhai et al., 2021) and
66 branch-and-bound heuristics are considered, and default options are recommended for the case
67 of box-constrained and general constrained problems. The HF simulation can be in the form of
68 a python function, or any external platform that could be connected via an API. The ultimate
69 goal of this software is to provide a user-friendly simulation-based optimization framework for
70 both expert and non-expert users, benefiting from the high-level features of Python. PyDDSB
71 follows the object-oriented programming paradigm and is designed to allow easy extension of
72 the core functionality by users and developers. The optimization of an example benchmark

73 black-box function and visualization of the outputs in shown in Figure 2

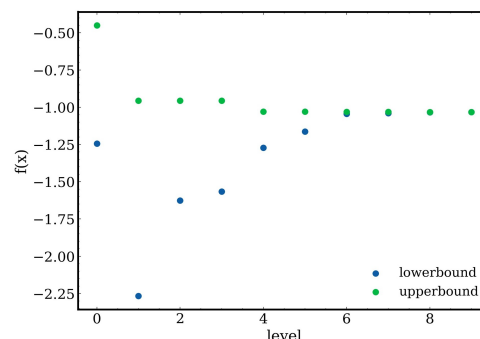
Benchmark function



Output

```
problem dimensionality : 2
absolute gap closed
Time elapsed: 2.69s
Current level: 9
Current node: 46
Number of samples used: 226
Current best upper bound : -1.0316242828345896
Current best lower bound : -1.032432292738122
Current absolute gap: 0.0008080099035323585
Current best optimizer: [[ 0.08881361 -0.71269176]]
yopt : -1.0316242828345896
xopt : [[ 0.08881361 -0.71269176]]
```

Lower and Upper bound evolution



Visualize branching

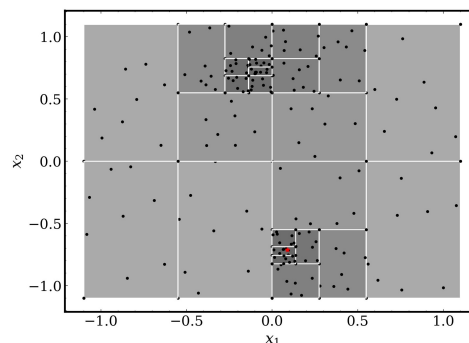


Figure 2: Optimization of sample benchmark function and visualizing the output. **Top left:** A non-convex benchmark function $f(x_1, x_2) = 4x_1^2 - 2.1x_1^4 + 0.3333x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$, available as black-box model. **Top right:** Evolution of upper and lower bounds found during the branching process of the sample space. **Bottom right:** Sampling and branching in the sample space visualized for the branch-and-bound process. Darker regions correspond to a higher level of branching and the red dot represents the optimum solution found. **Bottom left:** Algorithm output after termination with some information on sampling and optimum solution found

74 Acknowledgements

75 The authors acknowledge financial support from the National Science Foundation (NSF-1805724
76 & GR00003313) and DOE/AICHE RAPID Institute “Synopsis” project grant (GR10002225)

77 References

- 78 Amaran, S., Sahinidis, N. V., Sharda, B., & Bury, S. J. (2016). Simulation optimization: a
79 review of algorithms and applications. *Annals of Operations Research*, 240(1), 351–380.
- 80 Bhosekar, A., & Ierapetritou, M. (2018). Advances in surrogate based modeling, feasibility
81 analysis, and optimization: A review. *Computers & Chemical Engineering*, 108, 250–267.
- 82 Boukouvala, F., Misener, R., & Floudas, C. A. (2016). Global optimization advances in
83 mixed-integer nonlinear programming, MINLP, and constrained derivative-free optimization,
84 CDO. *European Journal of Operational Research*, 252(3), 701–727.
- 85 Hart, W. E., Watson, J.-P., & Woodruff, D. L. (2011). Pyomo: modeling and solving mathe-
86 matical programs in Python. *Mathematical Programming Computation*, 3(3), 219–260.

- 87 Larson, J., Menickelly, M., & Wild, S. M. (2019). Derivative-free optimization methods. *Acta*
88 *Numerica*, 28, 287–404.
- 89 McBride, K., & Sundmacher, K. (2019). Overview of surrogate modeling in chemical process
90 engineering. *Chemie Ingenieur Technik*, 91(3), 228–239.
- 91 Pedregosa, F., Varoquaux, G., Gramfort, A., Michel, V., Thirion, B., Grisel, O., Blondel, M.,
92 Prettenhofer, P., Weiss, R., & Dubourg, V. (2011). Scikit-learn: Machine learning in
93 Python. *The Journal of Machine Learning Research*, 12, 2825–2830.
- 94 Rios, L. M., & Sahinidis, N. V. (2013). Derivative-free optimization: a review of algorithms
95 and comparison of software implementations. *Journal of Global Optimization*, 56(3),
96 1247–1293.
- 97 Zhai, J., & Boukouvala, F. (2022). Data-driven spatial branch-and-bound algorithms for
98 box-constrained simulation-based optimization. *Journal of Global Optimization*, 82(1),
99 21–50.
- 100 Zhai, J., Shirpurkar, S., & Boukouvala, F. (2021). Data-driven Branch-and-bound Algorithms
101 for Constrained Simulation-based Optimization. In *Computer aided chemical engineering*
102 (Vol. 50, pp. 1067–1072). Elsevier. ISBN: 1570-7946

DRAFT