The SIFT (Scale Invariant Feature Transform) Detector and Descriptor

developed by David Lowe
University of British Columbia

Initial paper 1999
Newer journal paper 2004

Motivation

- The Harris operator is not invariant to scale and its descriptor was not invariant to rotation[1].

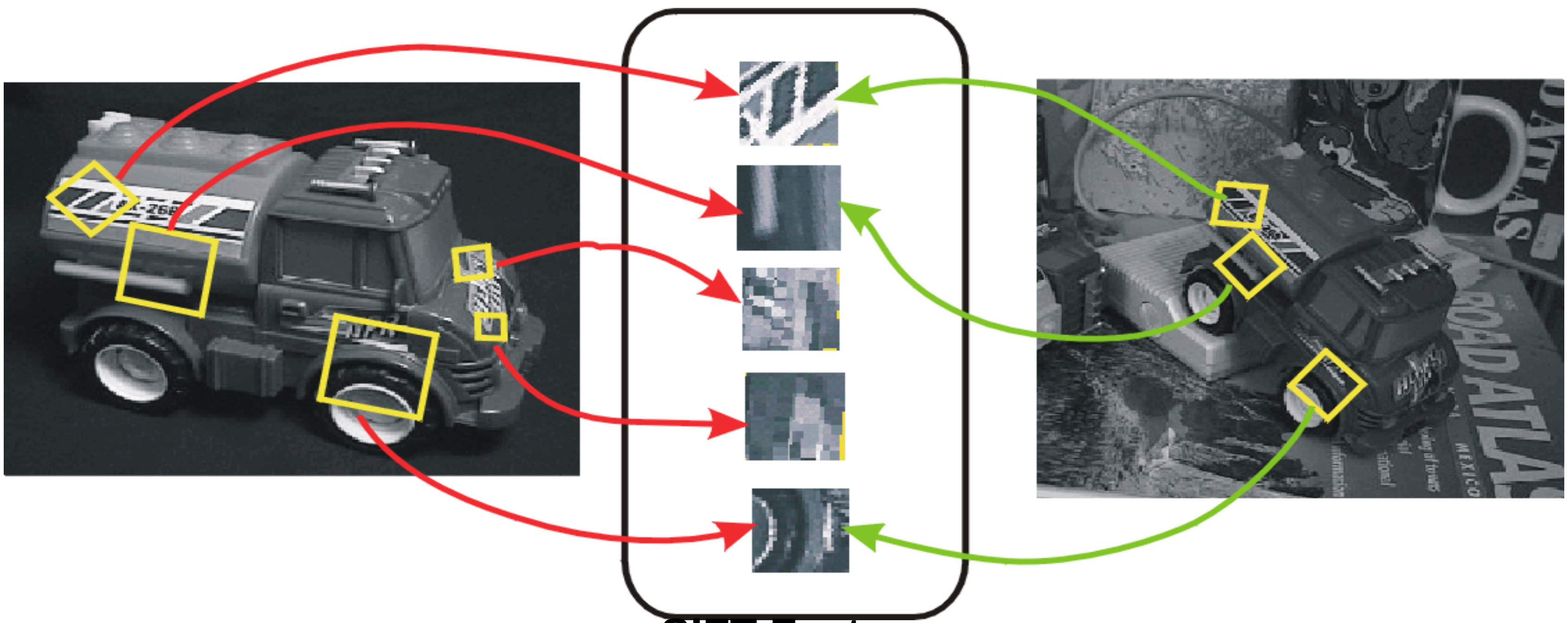
- For better image matching, Lowe's goal was to develop an operator that is invariant to scale and rotation.

- The operator he developed is both a detector and a descriptor and can be used for both image matching and object recognition.

[1]But Schmidt and Mohr developed a rotation invariant descriptor for it in 1997.

Idea of SIFT

- Image content is transformed into local feature coordinates that are invariant to translation, rotation, scale, and other imaging parameters



Claimed Advantages of SIFT

- **Locality:** features are local, so robust to occlusion and clutter (no prior segmentation)
- **Distinctiveness:** individual features can be matched to a large database of objects
- **Quantity:** many features can be generated for even small objects
- **Efficiency:** close to real-time performance
- **Extensibility:** can easily be extended to wide range of differing feature types, with each adding robustness

Overall Procedure at a High Level

1. Scale-space extrema detection

   Search over multiple scales and image locations.

2. Keypoint localization

   Fit a model to detrmine location and scale.
   Select keypoints based on a measure of stability.

3. Orientation assignment

   Compute best orientation(s) for each keypoint region.

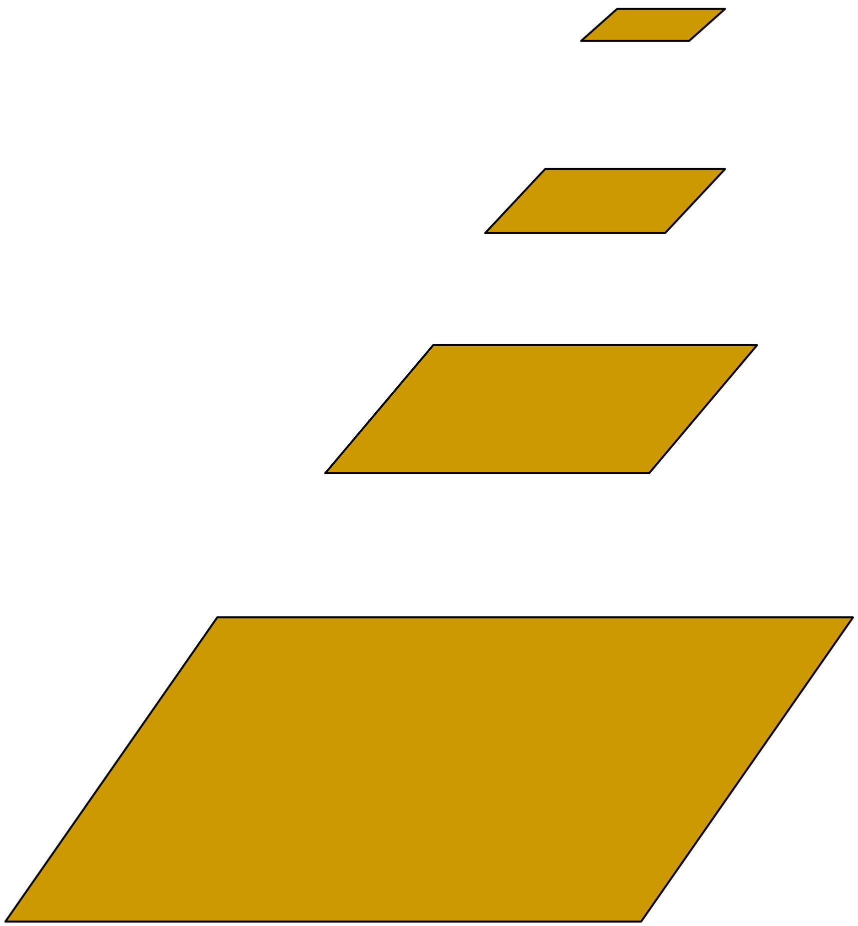
4. Keypoint description

   Use local image gradients at selected scale and rotation to describe each keypoint region.

1. Scale-space extrema detection

- Goal: Identify locations and scales that can be repeatably assigned under different views of the same scene or object.
- Method: search for stable features across multiple scales using a continuous function of scale.
- Prior work has shown that under a variety of assumptions, the best function is a Gaussian function.
- The scale space of an image is a function $L(x,y,\sigma)$ that is produced from the convolution of a Gaussian kernel (at different scales) with the input image.

Aside: Image Pyramids



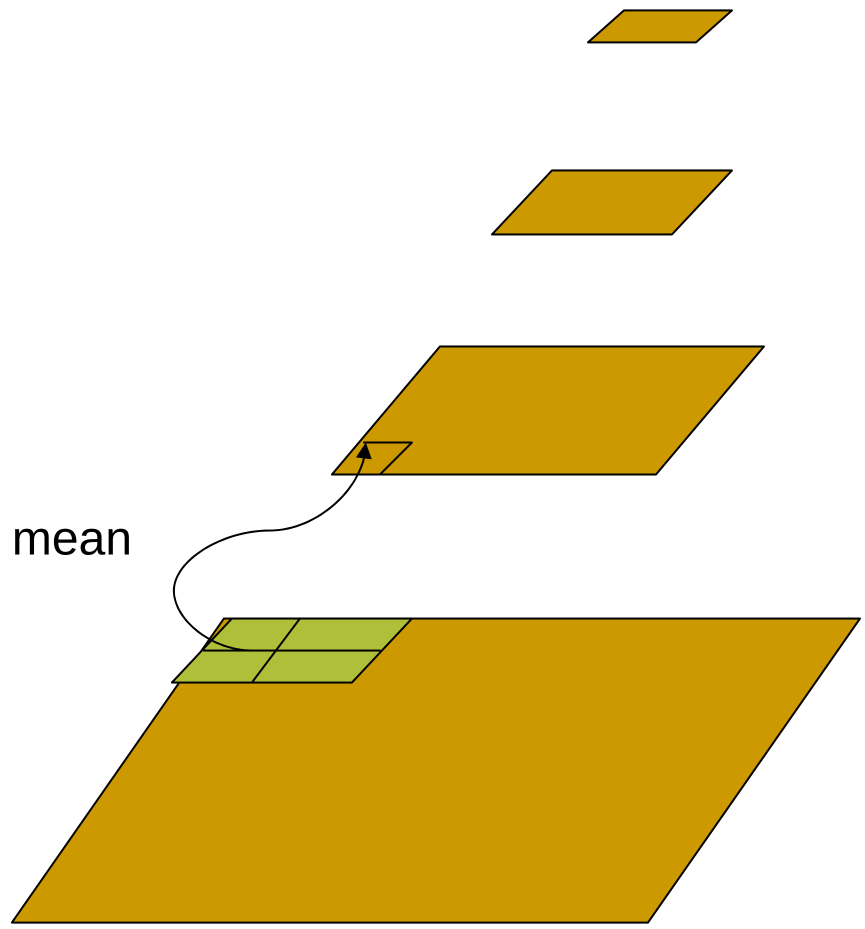
And so on.

3^{rd} level is derived from the 2^{nd} level according to the same funtion

2^{nd} level is derived from the original image according to some function

Bottom level is the original image.

Aside: Mean Pyramid



And so on.

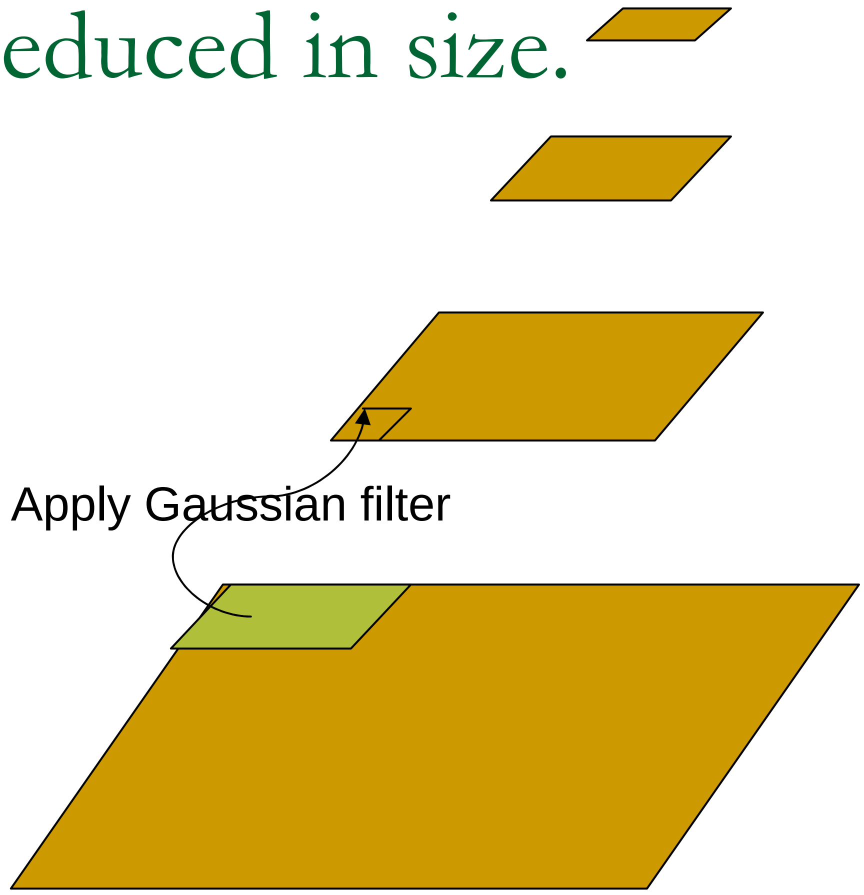
At 3rd level, each pixel is the mean of 4 pixels in the 2nd level.

At 2nd level, each pixel is the mean of 4 pixels in the original image.

Bottom level is the original image.

Aside: Gaussian Pyramid

At each level, image is smoothed and reduced in size.

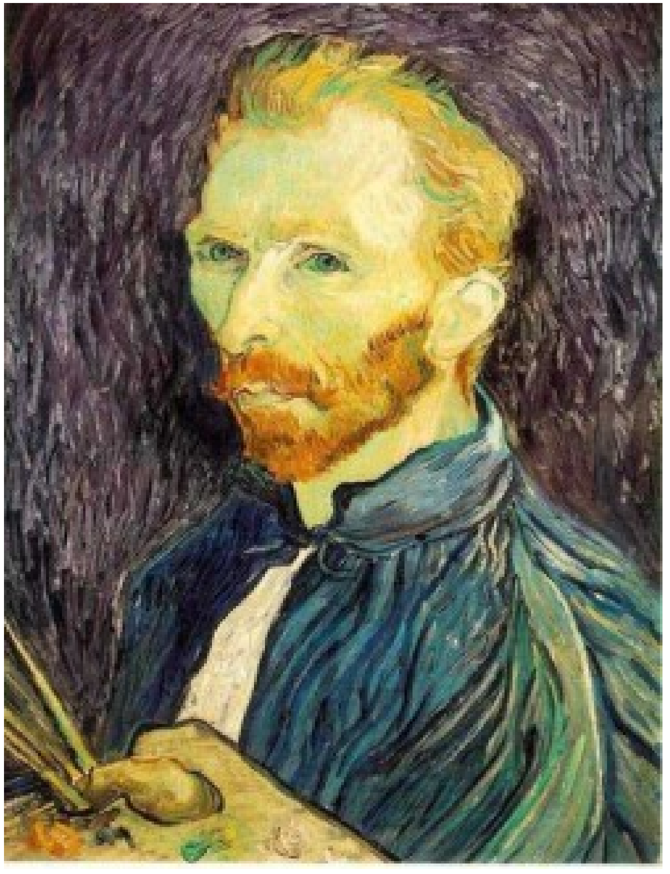


And so on.

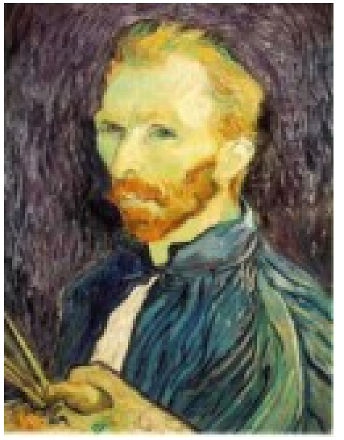
At 2nd level, each pixel is the result of applying a Gaussian mask to the first level and then subsampling to reduce the size.

Bottom level is the original image.

Example: Subsampling with Gaussian pre-filtering



Gaussian 1/2



G 1/4

G 1/8

Lowe's Scale-space extrema detection

- Scale-space function *L*
  - Gaussian convolution

$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2}$$

$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y),$$ where σ is the width of the Gaussian.

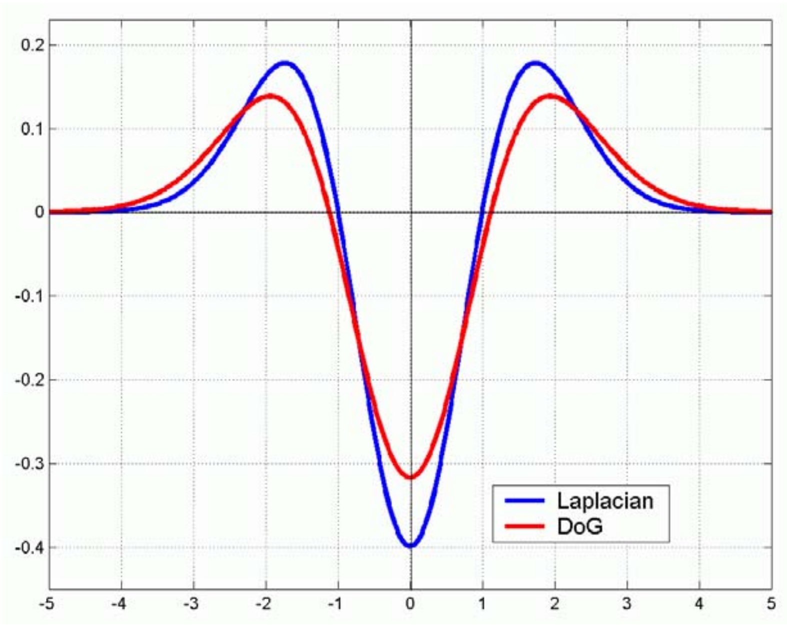
  - Laplacian of Gaussian kernel has been used in other work on scale invariance

  - Difference of Gaussian kernel is a close approximate to scale-normalized Laplacian of Gaussian

$$\begin{aligned} D(x, y, \sigma) &= (G(x, y, k\sigma) - G(x, y, \sigma)) * I(x, y) \\ &= L(x, y, k\sigma) - L(x, y, \sigma). \end{aligned}$$

2 scales:
σ and $k\sigma$

Scale-space extrema detection



- Gaussian is an ad hoc solution of heat diffusion equation

$$\frac{\partial G}{\partial \sigma} = \sigma \nabla^2 G.$$

- Hence

$$G(x, y, k\sigma) - G(x, y, \sigma) \approx (k-1)\sigma^2 \nabla^2 G.$$

- k is not necessarily very small in practice

Lowe's Pyramid Scheme

- Scale space is separated into octaves:
  - Octave 1 uses scale σ
  - Octave 2 uses scale 2σ
  - etc.

- In each octave, the initial image is repeatedly convolved with Gaussians to produce a set of scale space images.

- Adjacent Gaussians are subtracted to produce the DOG

- After each octave, the Gaussian image is down-sampled by a factor of 2 to produce an image ¼ the size to start the next level.

Lowe's Pyramid Scheme

s+2 filters

$\sigma_{s+1}=2^{(s+1)/s}\sigma_0$

.

.

$\sigma_i=2^{i/s}\sigma_0$

.

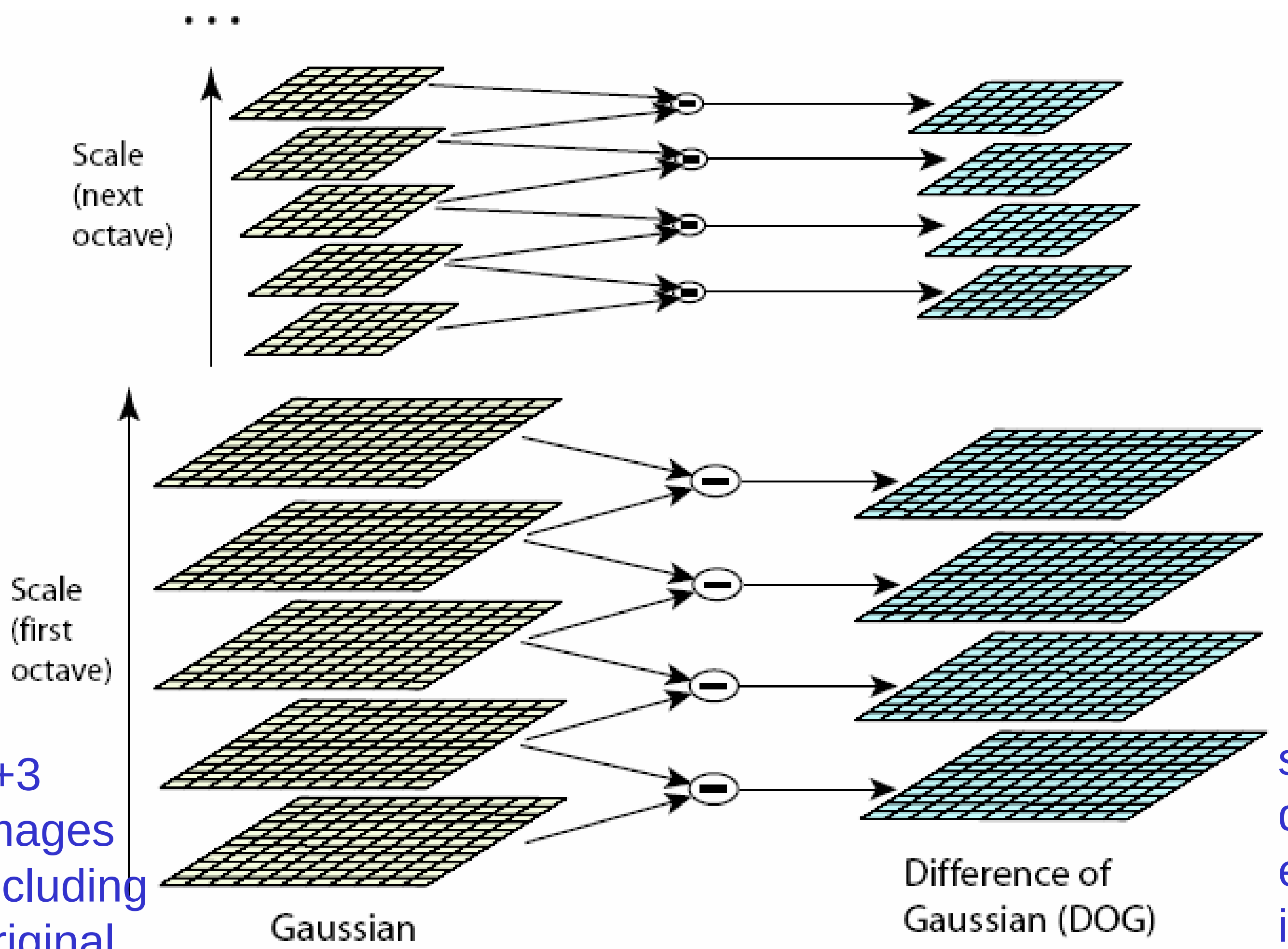
.

$\sigma_2=2^{2/s}\sigma_0$

$\sigma_1=2^{1/s}\sigma_0$

σ_0

s+3 images including original



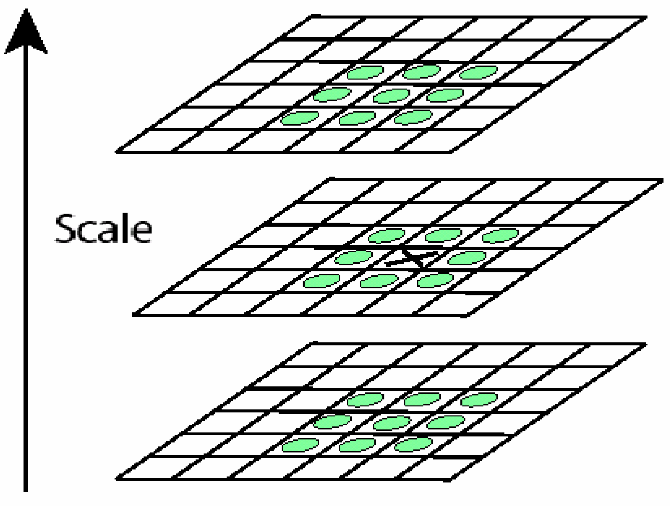
s+2 differ-ence images

The parameter **s** determines the number of images per octave.

Key point localization

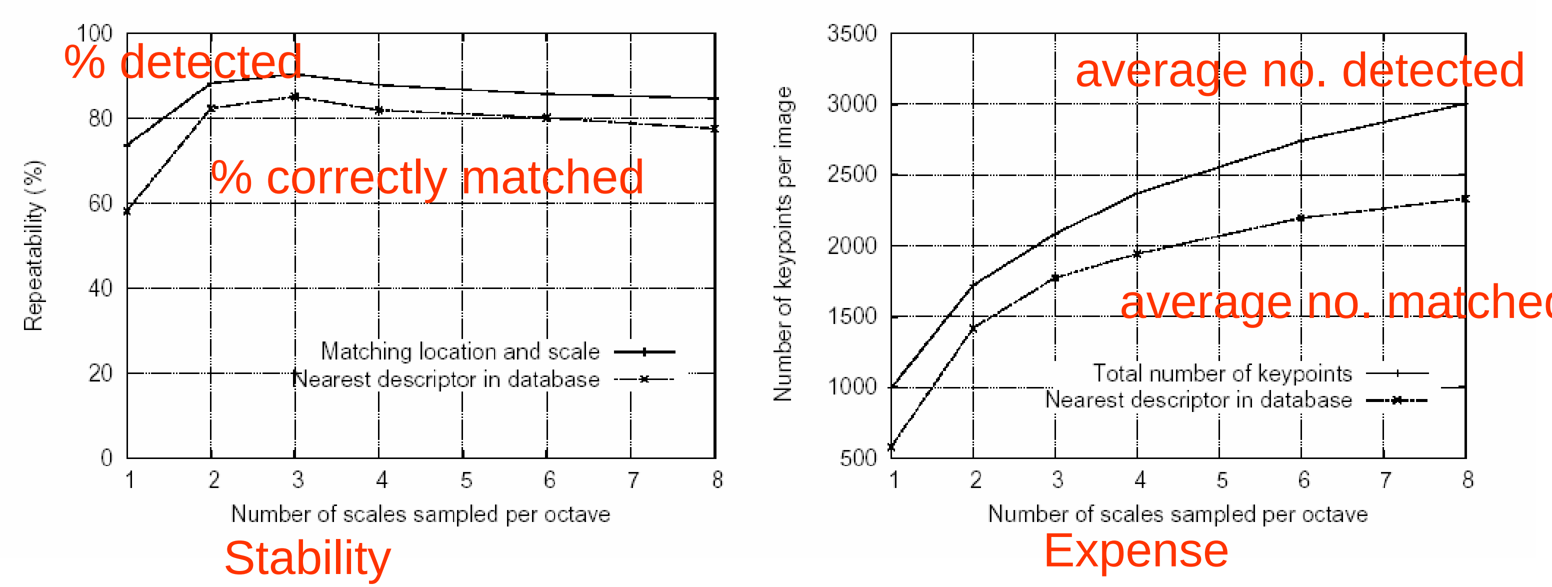
s+2 difference images.
top and bottom ignored.
s planes searched.

- Detect maxima and minima of difference-of-Gaussian in scale space
- Each point is compared to its 8 neighbors in the current image and 9 neighbors each in the scales above and below



For each max or min found, output is the **location** and the **scale**.

Scale-space extrema detection: experimental results over 32 images that were synthetically transformed and noise added.



- Sampling in scale for efficiency
  - How many scales should be used per octave? S=?
    - More scales evaluated, more keypoints found
    - S < 3, stable keypoints increased too
    - S > 3, stable keypoints decreased
    - S = 3, maximum stable keypoints found

2. Keypoint localization

- Detailed keypoint determination
  - Sub-pixel and sub-scale location scale determination
  - Ratio of principal curvature to reject edges and flats (like detecting corners)

Keypoint localization

- Once a keypoint candidate is found, perform a detailed fit to nearby data to determine
  - location, scale, and ratio of principal curvatures
- In initial work keypoints were found at location and scale of a central sample point.
- In newer work, they fit a 3D quadratic function to improve interpolation accuracy.
- The Hessian matrix was used to eliminate edge responses.

Eliminating the Edge Response

- Reject flats:
  - $|D(\hat{\mathbf{x}})|$ < 0.03
- Reject edges:

$$\mathbf{H} = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix}$$

Let α be the eigenvalue with larger magnitude and β the smaller.

$$\mathrm{Tr}(\mathbf{H}) = D_{xx} + D_{yy} = \alpha + \beta,$$

$$\mathrm{Det}(\mathbf{H}) = D_{xx}D_{yy} - (D_{xy})^2 = \alpha\beta.$$

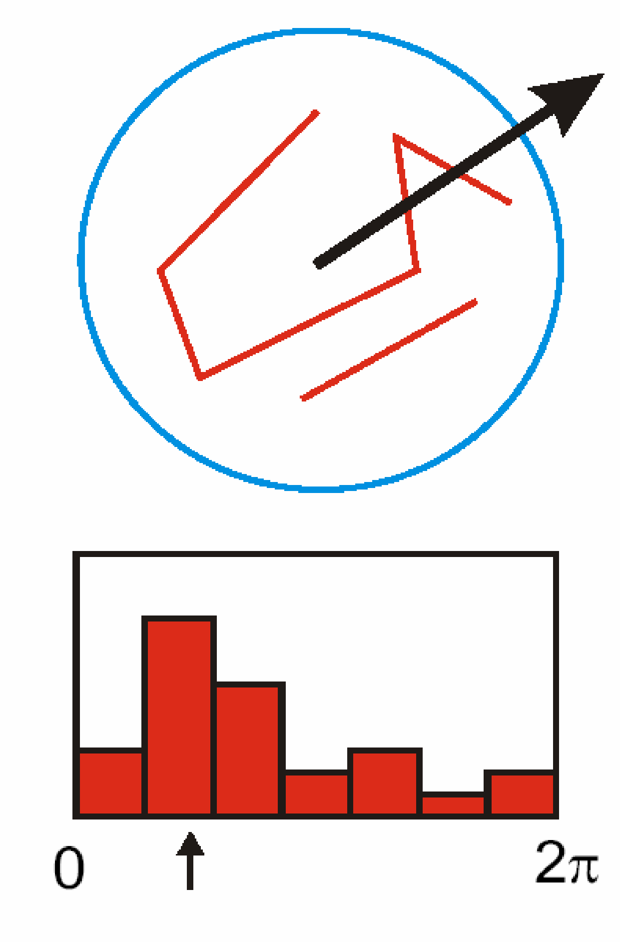
Let r = α/β.
So α = rβ

$$\frac{\mathrm{Tr}(\mathbf{H})^2}{\mathrm{Det}(\mathbf{H})} = \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(r\beta+\beta)^2}{r\beta^2} = \frac{(r+1)^2}{r},$$

$(r+1)^2/r$ is at a min when the 2 eigenvalues are equal.

  - r < 10
- What does this look like?

3. Orientation assignment

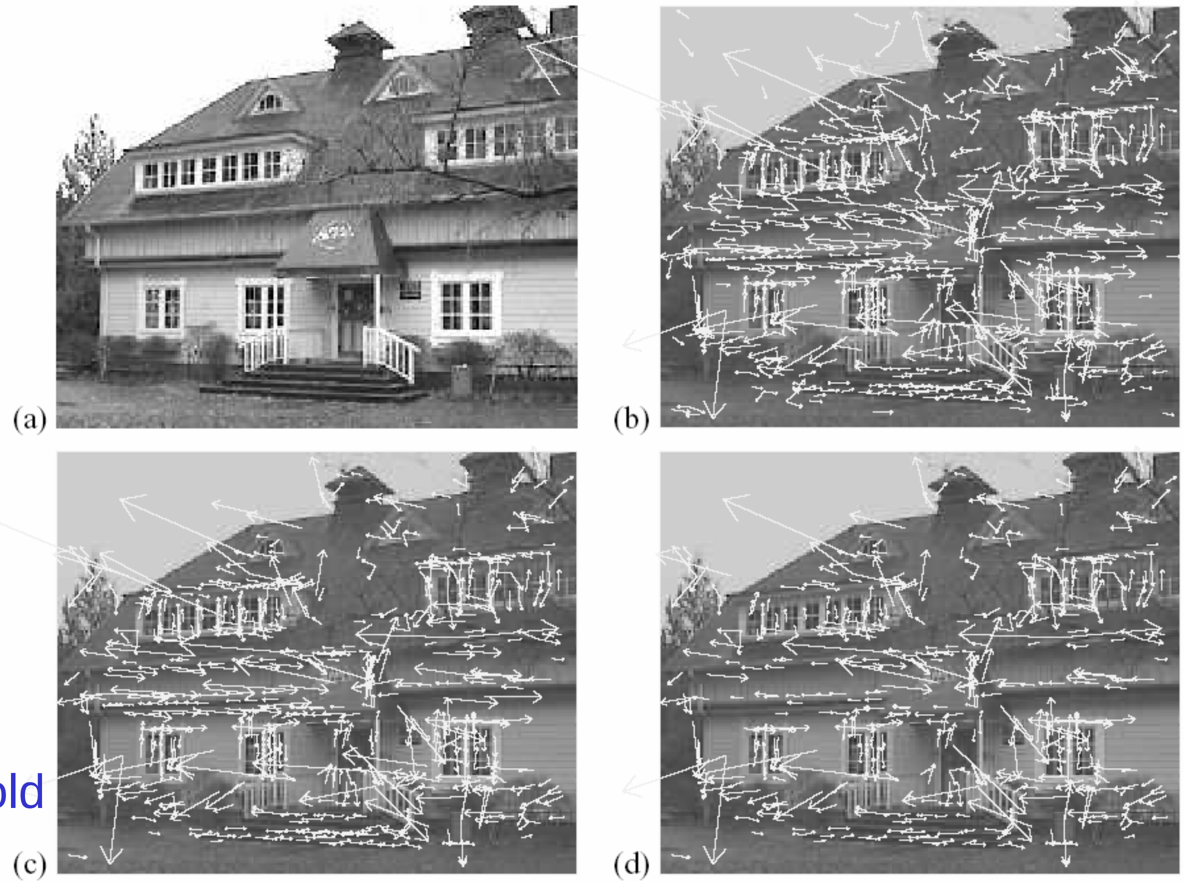


- Create histogram of local gradient directions at selected scale
- Assign canonical orientation at peak of smoothed histogram
- Each key specifies stable 2D coordinates (x, y, scale,orientation)

If 2 major orientations, use both.

Keypoint localization with orientation

233x189



832

initial keypoints

729

keypoints after gradient threshold

536

keypoints after ratio threshold

4. Keypoint Descriptors

- At this point, each keypoint has
  - location
  - scale
  - orientation
- Next is to compute a descriptor for the local image region about each keypoint that is
  - highly distinctive
  - invariant as possible to variations such as changes in viewpoint and illumination

Normalization

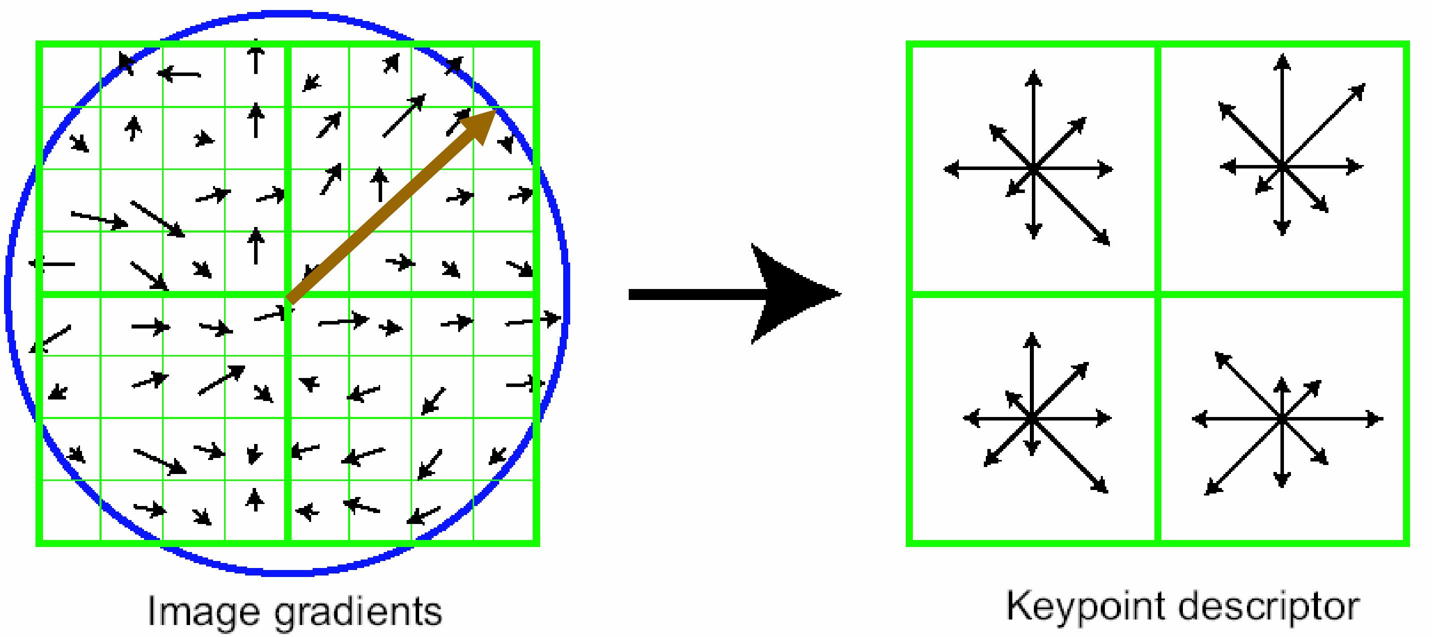
- Rotate the window to standard orientation
- Scale the window size based on the scale at which the point was found.

Lowe's Keypoint Descriptor

- use the normalized circular region about the keypoint
- compute gradient magnitude and orientation at each point in the region
- weight them by a Gaussian window overlaid on the circle
- create an orientation histogram over the 4 X 4 subregions of the window
- 4 X 4 descriptors over 16 X 16 sample array were used in practice. 4 X 4 times 8 directions gives a vector of 128 values.

Lowe's Keypoint Descriptor (shown with 2 X 2 descriptors over 8 X 8)

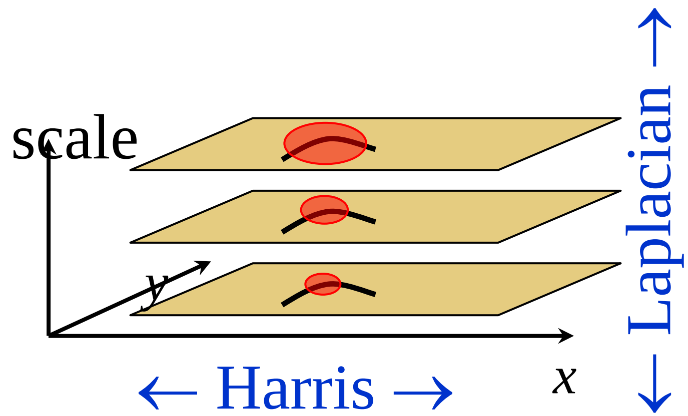
- Invariant to other changes (Complex Cell)



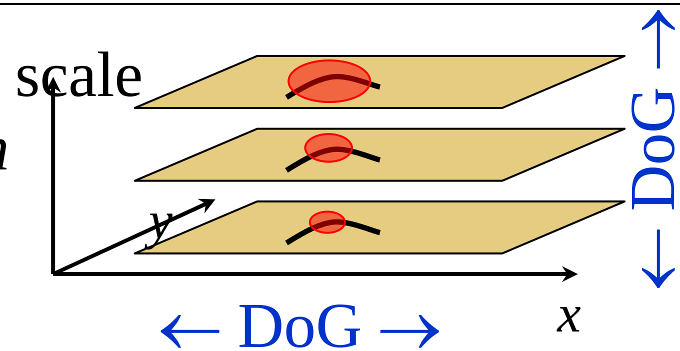
In experiments, 4x4 arrays of 8 bin histogram is used,
a total of 128 features for one keypoint

Scale Invariant Detectors

- Harris-Laplacian[1]
  Find local maximum of:
  - Harris corner detector in space (image coordinates)
  - Laplacian in scale



- SIFT (Lowe)[2]
  Find local maximum of:
  - Difference of Gaussians in space and scale



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[1] K.Mikolajczyk, C.Schmid. "Indexing Based on Scale Invariant Interest Points". ICCV 2001

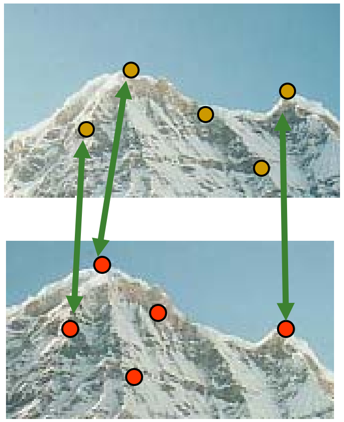
[2] D.Lowe. "Distinctive Image Features from Scale-Invariant Keypoints". IJCV 2004

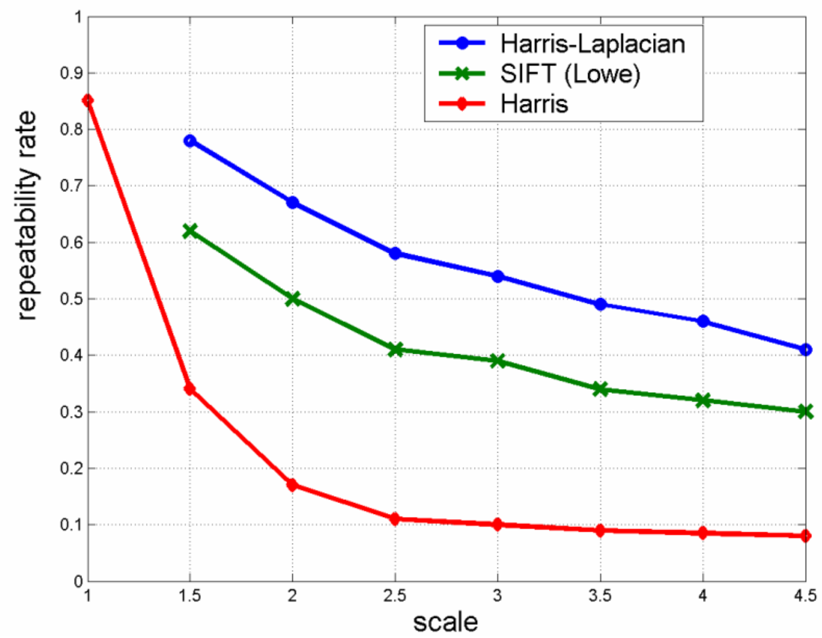
Scale Invariant Detectors

- Experimental evaluation of detectors w.r.t. scale change

Repeatability rate:

$$\frac{\#\ \text{correspondences}}{\#\ \text{possible correspondences}}$$





K.Mikolajczyk, C.Schmid. "Indexing Based on Scale Invariant Interest Points". ICCV 2001

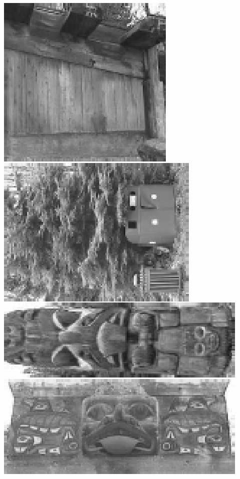
Schmid's Comparison with Harris-Laplacian

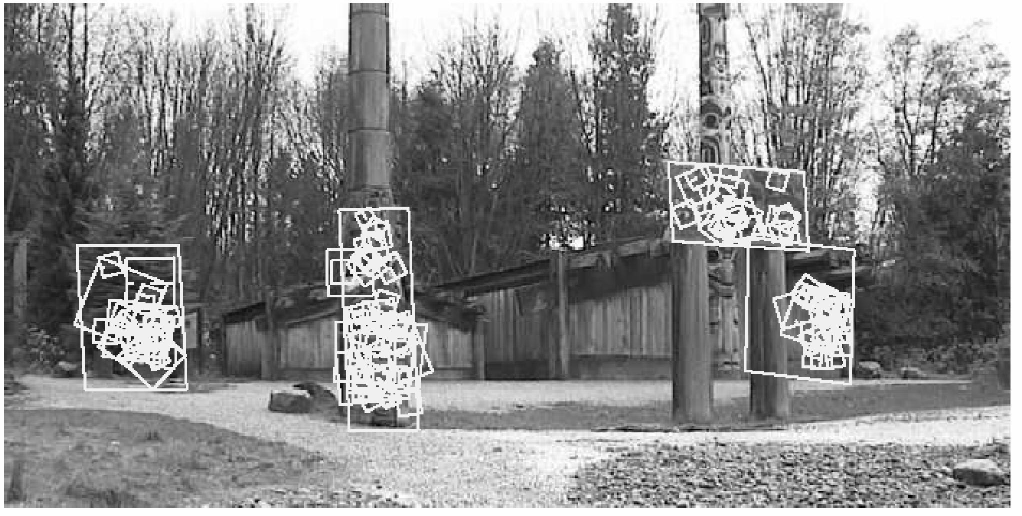
- Affine-invariant comparison
  - Translation-invariant – local features: both OK
  - Rotation-invariant
    - Harris-Laplacian
      - PCA
    - SIFT
      - Orientation
  - Shear-invariant
    - Harris-Laplacian
      - Eigenvalues
    - SIFT
      - No
- Within 50 degree of viewpoint, SIFT is better than HL, after 70 degree, HL is better.

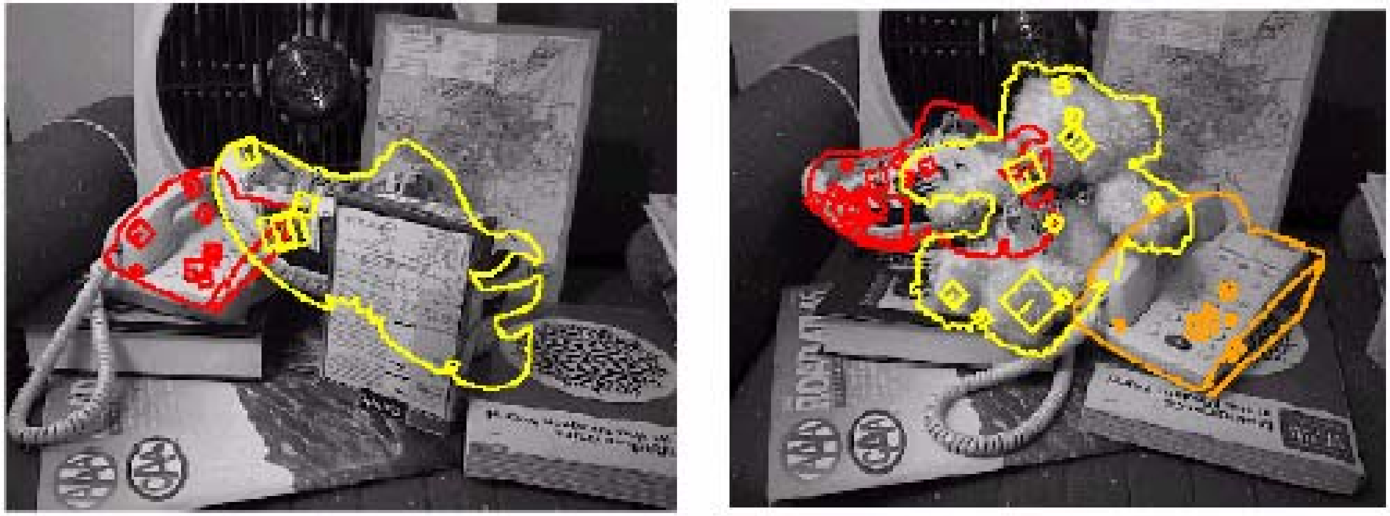
Comparison with Harris-Laplacian

- Computational time:
  - SIFT uses few floating point calculation
  - HL uses iterative calculation which costs much more

Using SIFT for Matching "Objects"







Uses for SIFT

- Feature points are used also for:
  - Image alignment (homography, fundamental matrix)
  - 3D reconstruction
  - Motion tracking
  - Object recognition
  - Indexing and database retrieval
  - Robot navigation
  - … other