Subject: **EIE413 / EIE4413 Digital Signal Processing**

Semester: **Semester 2, 2016/2017**

Quiz 6: **Overlap and add**

Name:______________________________ **Student ID:** ________________

A. Question

Given that x is a real data sequence with length equal to 29,185. x is fed to a 64-tap finite impulse response (FIR) filter with real coefficients h to obtain the output y. It is known that the total number of real multiplications required for using the direct approach to compute the linear convolution between x and h to obtain y is 1,867,840.

The filtering operation can also be implemented using the overlap-and-add method. If x can be divided into R non-overlapping segments, each with L data, such that,

$$x[n] = \sum_{r=0}^{R-1} x_r[n]$$
$$where \quad x_r[n] = \begin{cases} x[n] & rL \le n \le (r+1)L - 1 \\ 0 & otherwise \end{cases} \qquad \text{(Q1)}$$

y is equal to the sum of the linear convolutions between h and x_r, for all r.

To speed up the operations, we can use the decimation-in-time (DIT) fast Fourier transform (FFT) algorithm for the computation of linear convolutions.

(i) Assume that the parameter R in (Q1) is chosen as 449. Determine the total number of real multiplications required to obtain y if every linear convolution between h and x_r is computed using the DIT FFT approach. (Hint: you may assume that every complex multiplication requires 4 real multiplications for its implementation. Also, the FFT of h needs to be computed only once. It can then be used for all other segments.)

(ii) Repeat (i) by changing R to 65. Give the total number of real multiplications required.

(iii) From the result of (i) and (ii), deduce the strategy for selecting the value of R as far as the computational complexity is concerned. Explain also if there is any side effect to the system (such as delay) where implementing such strategy.

B. Write your answer using the space on the other side of this sheet:

(i)

For each segment, the FFT length is equal to $N = L+P-1$, where $L = 29{,}185/R = 65$; P = no. of filter taps = 64. Hence $N = 128$.

The number of multiplications = $4*N/2*\log_2 N + R*4*(2*N/2*\log_2 N + N) = 1{,}840{,}896$.

(ii)
When selecting $R = 65$, $L = 29{,}185/65 = 449$; $P = 64$. Hence $N = 512$.

The number of multiplications = $2*N*\log_2 N + R*4*(N*\log_2 N + N) = 1{,}340{,}416$.

(iii)
As can be seen in (i), the saving by selecting $R = 449$ is not significant. The reduction is less than 2%. But when $R = 65$, the saving is now much significant. It is because if R is smaller, the number of data in each segment will become bigger. And the saving of FFT is much higher when the sequence length is longer. So when selecting R, the strategy should be choosing a number as small as possible such that the data in each segment are as many as possible. However, when there are more data in each segment, the system delay will also be more serious, since the FFT can start to operate only when all the data in the segment are available.