EIE 3333 Data and Computer Communications (2019/20)

Suggested Solutions to Tutorial 3

Unit 3: Data Link Layer: Error Detection and Correction

Review Questions

3. *Distinguish between forward error correction versus error correction by retransmission.*

[Answer]

In ***forward error correction***, the receiver tries to correct the corrupted codeword; in ***error detection by retransmission***, the corrupted message is discarded (the sender needs to retransmit the message).

4. *If we want to detect two-bit errors, what should be the minimum Hamming distance?*

[Answer]

The Hamming distance $d_{min} = s + 1$. Since $s = 2$, we have $d_{min} = 3$.

5. *In CRC, show the relationship between the following entities (size means the number of bits):*
   a. The size of the dataword and the size of the codeword
   b. The size of the divisor and the remainder

[Answer]

(a) The only relationship between the size of the codeword and dataword is the one based on the definition: $\mathbf{n = k + r}$., where ***n*** is the size of the codeword, ***k*** is the size of the dataword, and ***r*** is the size of the remainder.

(b) The ***remainder*** is always ***one bit smaller*** than the ***divisor***

8. *Can the value of a checksum be all 0s (in binary)? Defend your answer. Can the value be all 1s (in binary)? Defend your answer.*

[Answer]

The value of a checksum can be all 0s (in binary). This happens when the value of the sum (after wrapping) becomes all 1s (in binary). ***It is almost impossible for the value of a checksum to be all 1s***. For this to happen, the value of the sum (after wrapping) must be all 0s which means all data units must be 0s.

9. *Assume we are sending data items of 16-bit length. If two data items are swapped during transmission, can the traditional checksum detect this error? Explain.*

[Answer]

The error cannot be detected because the sum of items is not affected in this swapping.

<u>Problems</u>

1. *What is the maximum effect of a 2-ms burst of noise on data transmitted at the following rates:*
   a. 1500 kps
   b. 12 kbps
   c. 100 kbps
   d. 100 Mbps

[Solution]

We can say that **(vulnerable bits) = (data rate) × (burst duration)**

a.	vulnerable bits	$= (1{,}500) \times (2 \times 10^{-3})$	= **3 bits**
b.	vulnerable bits	$= (12 \times 10^{3}) \times (2 \times 10^{-3})$	= **24 bits**
c.	vulnerable bits	$= (100 \times 10^{3}) \times (2 \times 10^{-3})$	= **200 bits**
d.	vulnerable bits	$= (100 \times 10^{6}) \times (2 \times 10^{-3})$	= **200,000 bits**

Comment: The last example shows how a noise of small duration can affect so many bits if the data rate is high.

2. *Assuming even parity, find the parity bit for each of the following data units.*
   a. 1001011
   b. 0001100
   c. 1000000
   d. 1110111

[Solution]

	Dataword		Number of 1s		Parity	Codeword
a.	1001011	→	4 (even)	→	**0**	**0** 1001011
b.	0001100	→	2 (even)	→	**0**	**0** 0001100
c.	1000000	→	1 (odd)	→	**1**	**1** 1000000
d.	1110111	→	6 (even)	→	**0**	**0** 1110111

3. *Calculate the Hamming pairwise distances and determine the minimum Hamming distance among the following codewords:*
   a. *00000, 10101, 01010*
   b. *000000, 010101, 101010, 110110*

[Solution]

(a)

	00000	10101	01010
00000	0	2	2
10101	3	0	5
01010	2	5	0

Hence, the minimum Hamming distance is 2.

(b)

	000000	010101	101010	110110
000000	0	3	3	4
010101	3	0	6	6
101010	3	6	0	3
110110	4	6	3	0

Hence, the minimum Hamming distance is 3.

4. *Would you expect that the inclusion of a parity bit with each character would change the probability of receiving a correct message?*

[Solution]

The inclusion of a parity bit extends the message length. There are more bits that can be in error since the parity bit is now included. The parity bit may be in error when there are no errors in the corresponding data bits. Therefore, the inclusion of a parity bit with each character would change the probability of receiving a correct message.

5. *Consider a frame consisting of two characters of four bits each. Assume that the probability of bit error is 10^{-3} and that is independent for each bit.*
   (a) *What is the probability that the received frame contains at least one error?*
   (b) *Now add a parity bit to each character. What is the probability?*

[Solution]

a. We have:

Pr [single bit in error] = 10^{-3}

Pr [single bit not in error] = $1 - 10^{-3} = 0.999$

Pr [8 bits not in error] = $(1 - 10^{-3})^8 = (0.999)^8 = 0.992$

Pr [at least one error in frame] = $1 - (1 - 10^{-3})^8 = 0.008$

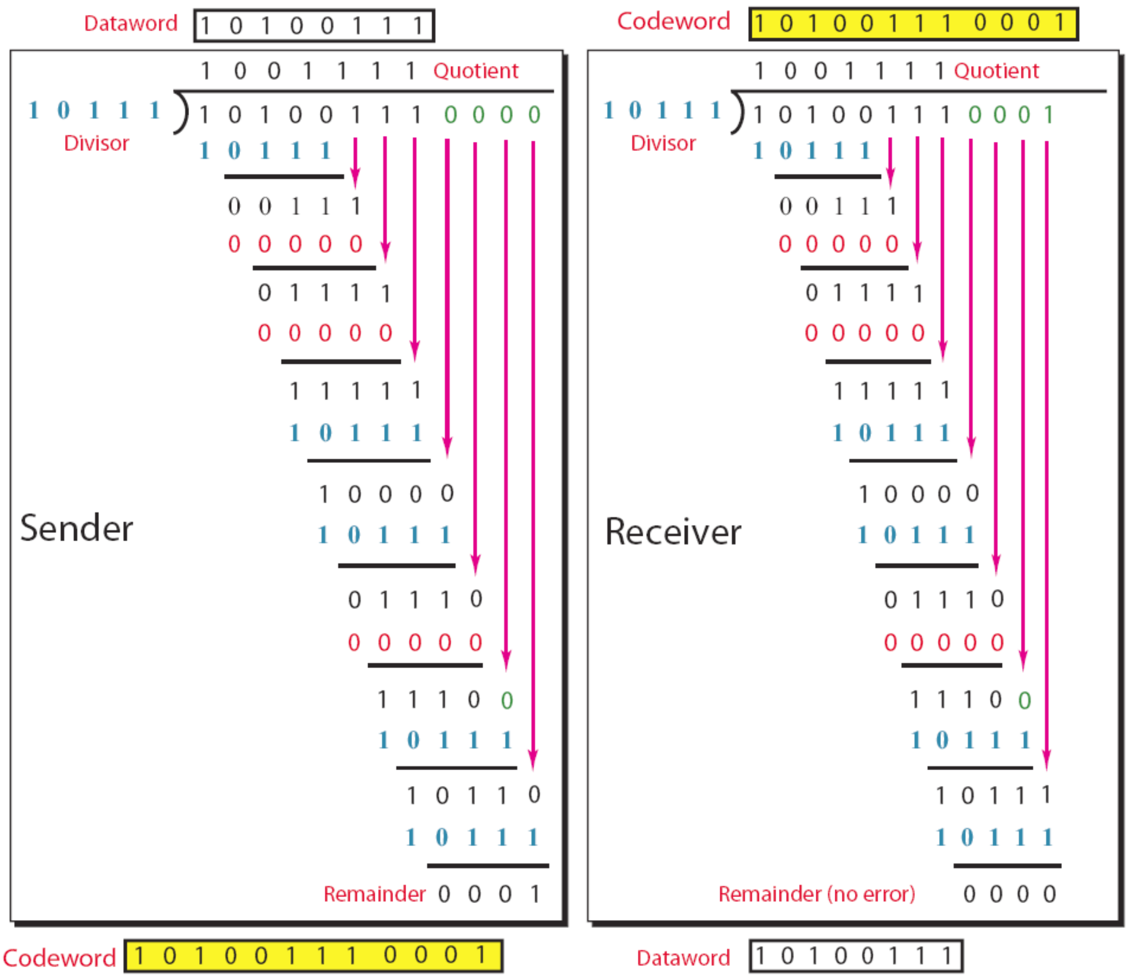
b. Pr [at least one error in frame] = $1 - (1 - 10^{-3})^{10} = 1 - (0.999)^{10} = 0.01$

6. *Given the dataword 10100111 and the divisor 10111*

   a. Show the generation of the codeword at the sender site.

   b. Show the checking of the codeword at the receiver site (assume no error).

[Solution]



7. A sender has two data items to send: 0x4567 and 0xBA98. What is the value of the checksum?

[Solution]

4	**5**	**6**	**7**	
B	**A**	**9**	**8**	
0	**0**	**0**	**0**	Checksum (initial)
F	F	F	F	Sum
0	**0**	**0**	**0**	Checksum (to send)

8. *Assume that the probability that a bit in a data unit is corrupted during transmission is p. Find the probability that x number of bits are corrupted in an n-bit data unit for each of the following cases.*
   - *a. n=8, x=1, p=0.2*
   - *b. n=16, x=3, p=0.3*
   - *c. n=32, x=10, p=0.4*

[Solution]

The probability that x number of bits are corrupted in an n-bit data unit is $C(n, x)\, p^x\, (1 - p)^{n-x}$

a. P [1-bit error in 8-bit unit] = $C(8, 1)\,(0.2)^1\,(0.8)^7 \approx 0.34$
b. P [3-bit error in 16-bit unit] = $C(16, 3)\,(0.3)^3\,(0.7)^{13} \approx 0.15$
c. P [10-bit error in 32-bit unit] = $C(32, 10)\,(0.4)^{10}\,(0.6)^{22} \approx 0.09$