Sets

1. Of a group of 20 students, 10 are interested in music, 7 are interested in photography, and 4 like swimming. Furthermore 4 are interested in both music and photography, 3 are interested in both music and swimming, 2 are interested in both photography and swimming and 1 is interested in music, photography and swimming. How many students are interested in photography but not in music and swimming?
2. In a class of 50 students, there are 2 choices for optional subjects. It is found that 18 students have physics as an optional subject but not chemistry and 25 students have chemistry as an optional subjects but physics.
   (i) How many students have both optional subjects?
   (ii) How many students have chemistry as an optional subject?
   (iii) How many students have physics as an optional subject?
3. A survey was conducted among 1000 people. Of these 595 are democrats, 595 wear glasses and 550 like ice-cream; 395 of them are democrats who wear glasses, 350 of them are democrats who like ice-cream, and 400 of them wear glasses and like ice-cream; 250 of them are democrats who wear glasses and like ice-cream:
   (i) How many of them who are not democrats, do not wear glasses and do not like ice-cream?
   (ii) How many of them are democrats who do not wear glasses, but do like ice-cream?
4. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find
   (i) The number of people who read at least one of the three newspapers.
   (ii) The number of people who read exactly one newspaper.

Permutations and Combinations

1. A dance pair means a woman and man dancing together. How many such dance pairs can be formed from a group of 6 women and 10 men?
2. Eight chairs are numbered 1 to 8. Two women and three men are to occupy one chair each. First the women choose the chairs from amongst the chairs 1 to 4 and then men select from the remaining chairs. Find the number of possible arrangements.
3. Calculate the number of ways to paint 12 offices so that 3 of them will be green, 2 of them pink, 2 of them yellow and the remaining ones whites?
4. How many permutations can be made with the letters of the word CONSTITITION and:
   (i) In how many ways vowels occur together?
   (ii) In how many ways consonants and vowels occur alternatively?
   (iii) How many of these will have the letter N both at the beginning and at the end?
5. A palindrome is a word that reads the same forward and backward. How many seven-letter palindromes can be made out of English alphabet?
6. In how many ways can 20 boys and 7 girls stand in a circle so that no two girls are next to each other?

Principal of Mathematical Induction

1. A jigsaw puzzle consists of a number of pieces. Two or more pieces with matched boundaries can be put together to form a "big" piece. Finally, when all piece are put together as one single block, the jigsaw puzzle is set to be solved. Putting two blocks with matched boundaries together is counted as one move. Use principle of mathematical induction to prove that for a jigsaw puzzle with n pieces it will take n-*1* moves to solve the puzzle.
2. Use mathematical induction to show that:
$$1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$$
3. Show that $n^4 - 4n^2$ is divisible by 3 for all $n \geq 2$ by induction.
4. Use mathematical induction to show that:
$$1.2.3 + 2.3.4 + 3.4.5 + \cdots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

Relations

1. What is a Poset? Draw a Hasse Diagram for the given Poset: ({2, 4,5,10, 12, 20, 25} ; |).
2. Consider two sets A and B, A = {1, 2, 3, 4} and B = {3, 4, 5, 6}. Find the elements of each relation R stated below. Also, find the domain and range of R.
   (i) $a \in A$ is related to $b \in B$, i.e., aRb if and only if $a < b$.
   (ii) $a \in A$ is related to $b \in B$, i.e., aRb if and only if a and b are both odd numbers.
3. Let R be a binary relation on the set of all positive integers such that R = { (a, b) | a-b is an odd positive integer}
   Is R i) Reflexive? ii) Symmetric? iii) Antisymmetric? iv) Transitive?

Logic

1. Let L(x,y) be the statement "x loves y", where the universe of discourse for both x and y consists of all people in the world. Use quantifies to express each of the following statements:
   (i) Everybody loves somebody.
   (ii) There is somebody whom everybody loves.
   (iii) Nobody loves everybody.
   (iv) There is someone whom no one loves.
2. Consider the following advertisement of a game:
   (i) There are three statements in this advertisement.
   (ii) Two of them are not true.
   (iii) The average increase in IQ scores of people who learned this game is more than 20 points.

   Prove that the statement (iii) is true using truth table.
3. Translate "Everybody has somebody who is his or mother" into predicate calculus.
4. Show that the following argument is valid:
   "If Mohan is a lawyer then he is ambitious. If Mohan is an early riser, then he does not like rice. If Mohan is ambitious then is early riser. Then if Mohan is a lawyer, then he does not like rice."
5. Find the converse and contrapositive of the following statement:
   (i) If I go to market, then I buy a pen.
6. Let P, Q and R be the propositions as follows:
   P : You go to school.
   Q : You appear in the exam.
   R : You pass the exam.
   Write the following statements in symbolic form:
   (i) You do not go to school and you do not appear in the exam.
   (ii) If you do not go to school and you do not appear in the exam, then you do not pass the exam.

(iii) You go to school and you appear in the exam, but you do not pass the exam.

7. Consider the following statements:
   Riya is preparing food. If Riya is preparing food then Riya is not going to school. If Riya is not going to school then her father does not make her take the examination.
   Using the rules of inference prove "Riya's father does not make her take the examination."
8. Prove using contradiction method that the following premises are consistent:
   $$(r \rightarrow \neg q, r \vee s, s \rightarrow \neg q, p \rightarrow q) \rightarrow \neg p$$
9. Show that $\neg p$ is tautologically implied by $\neg(p \wedge \neg q), \neg q \vee r, \neg r$
10. Show that $(p \rightarrow q) \wedge (r \rightarrow q)$ and $(p \vee r) \rightarrow q$ are logically equivalent.

Functions

1. Given $f(x) = x^2 + 1$ and $g(x) = x + 2$. Find $f \, o \, g$ and $g \, o \, f$ where f and g are functions from R to R.
2. Let f be the function from the set X = {2, 3, 4, 5, 6, 7} into the set Y = {0, 1, 2, 3, 4} defined by $f(x) = 2x \ (mod\ 5)$. Write f as a set of ordered pairs. Is f one-to-one or onto Y?
3. Use the definition of big-theta to prove that $7x^2 - 9x + 1 = \theta(x^2)$
4. Determine whether the function $f(x) = x^2 + 2$ is a bijection from W to W, where W is set of whole numbers.
5. Give a big-O estimate for $f(n) = 3n \log(n!) + (n^2 + 3) \log n$, where n is positive integer.