

Codifying Logical Fragments in ASP - A General Knowledge Base Approach

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Motivation

The fact that all Mathematics is Symbolic Logic is one of the greatest discoveries of our age; and when this fact has been established, the remainder of the principles of mathematics consists in the analysis of Symbolic Logic itself.

Bertrand Russell, “The Principles of Mathematics”

Overview

Our Approach

Modelling Subjects

Logic Fragments



Complexity Hierarchy

Results

Conclusion

Previous Works

- ▶ DL database by Evgeny Zolin (2005 - 2013)
- ▶ DL database by Mohamed Ibrahim (2019)

<div></div> <div>Complexity of reasoning in Description Logics</div> <div>Note: the information here is (subject to) incomplete and outdated often</div> <div>Base description logic: <i>Attributive Language with Complements</i></div> <div>$\mathcal{ALC} ::= A \mid \top \mid \bot \mid A \mid \neg C \mid C \sqcap D \mid C \sqcup D \mid \exists R.C \mid \forall R.C$</div>		<div></div> <div>Slide 10/11</div>
<div>Concept constructors</div> <div><input type="checkbox"/> \exists - functional($\exists R$) ($\exists R.C$)</div> <div><input type="checkbox"/> \exists - (un)qualified number restrictions: ($\geq n R$, $\leq n R$)</div> <div><input type="checkbox"/> Q - qualified number restrictions: ($\geq n(A, C)$, $\leq n(A, C)$)</div> <div><input type="checkbox"/> \exists - normal: ($\exists A$) or ($\exists A_1, \dots, \exists A_n$) ("one-of")</div> <div><input type="checkbox"/> μ - least fixpoint operator: $\mu X.C$</div> <div>Form: ... complex roles in number restrictions</div>	<div>Role constructors</div> <div><input type="checkbox"/> \neg - role inverse: R^{-1}</div> <div><input type="checkbox"/> \cap - role intersection: $R \sqcap S$</div> <div><input type="checkbox"/> \cup - role union: $R \sqcup S$</div> <div><input type="checkbox"/> \sim - role complement: $\sim R$, \bar{R} ($\bar{R} = \neg R$)</div> <div><input type="checkbox"/> \circ - role chain composition: $R \circ S$</div> <div><input type="checkbox"/> \vdash - reflexive-transitive closure: R^+</div> <div><input type="checkbox"/> id - concept identity: $\text{id}(C)$</div>	
<div>TBox (concept axioms)</div> <div><input checked="" type="checkbox"/> empty TBox</div> <div><input type="checkbox"/> asydc TBox ($G \sqsubseteq C$, A is a concept name; no cycles)</div> <div><input type="checkbox"/> general TBox ($C \sqsubseteq D$, for arbitrary concepts C and D)</div>	<div>RBox (role axioms)</div> <div><input type="checkbox"/> \exists - role transitivity: $\forall R$</div> <div><input type="checkbox"/> \leq - role hierarchy: $R \sqsubseteq S$</div> <div><input type="checkbox"/> \neq - complete role inclusions: $R \sqsubseteq \perp$, $R \sqsubseteq \perp$</div> <div><input type="checkbox"/> \neq - some additional features (click to see them)</div>	
Form: ... You have selected a Description Logic: \mathcal{ALC}		One role Form: $\exists R.C$ Form: $\exists R.C$
Complexity of reasoning problems		
Concept satisfiability	PSPACE-complete	• Hardness for \mathcal{ALC} see [80]. • Upper bound for \mathcal{ALC} see [22, Theorem 4.4].
Role satisfiability	PSPACE-complete	• Hardness follows from that for concept satisfiability. • Upper bound for \mathcal{ALC} see [22, Appendix A].
Important properties of the Description Logic:		
Finite model property	Yes	\mathcal{ALC} is a notational variant of the multi-modal logic \mathcal{K}_{MF} [6, [22]], for which the finite model property can be found in [4, Sect. 2.3].
Tree model property	Yes	\mathcal{ALC} is a notational variant of the multi-modal logic \mathcal{K}_{TM} [6, [22]], for which the tree model property can be found in [4, Proposition 2.14].

Why ASP?

- ▶ **A**nswer **S**et **P**rogramming
- ▶ semantics based on Gelfond-Lifschitz reduct
→ calculates all answer sets of a given program
- ▶ Advantages of ASP:
 - ▶ allows for arbitrary arity of predicates
 - ▶ analytical tools
 - ▶ feasibility of calculations
 - ▶ tests for ambiguous rules

Aims of Our Approach

- ▶ model different logic fragments and DLs in a *unified manner*
- ▶ model connections between logic fragments of different “families”
- ▶ *infer* all possible complexity results
- ▶ *query* the knowledge base
- ▶ find *explanations* for inferred results

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Families of Fragments

- ▶ *families* of logic fragments utilize the same basic semantics (e.g. DLs, FO, modal logic, ...)
- ▶ fragments are described by their traits (e.g. functionality, qualified number restriction)
⇒ traits may subsume other traits
- ▶ traits are classified in restrictions and features of the fragment (relative to chosen base fragment)

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Ex. Traits

	Features	Restrictions
DL	number restriction nominals inverse roles ...	horn logic no negation empty TBox ...
FOL	equality symbol ...	2 – variable fragment quantifier prefixes ...

Expressivity of Logic Fragments

- ▶ some fragments are more expressive than others
i.e. all satisfiability problems in one fragment can be expressed in the other
- ▶ for fragments within the same family, this can be easy to determine:
the fragment with more *features* and less *restrictions* is the more expressive
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Ex. Fragments

family(dl; fol).

fragment(fo2, fol).

has_trait(fo2, two_variable).

trait(fo2, restriction, fol).

fragment(alc_EC, dl).

has_trait(alc_EC, (empty_TBox; concept_SAT)).

trait(concept_SAT, restriction, dl).

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translation(alc_EC, fo2).

translation(X, Z) :- translation(X, Y), translation(Y, Z).

has_extra_features(L₁, L₂):- has_trait(L₁,A), -has_trait(L₂, A), trait(A, feature, F), fragment(L₁, F), fragment(L₂, F).

higher_logic(X, Y) :- has_extra_features(X, Y), not has_extra_features(Y, X), not has_extra_restrictions(X, Y).

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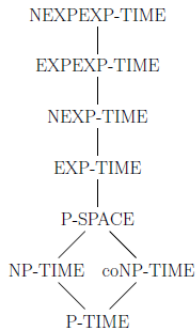
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 - ▶ expressivity can propagate model properties
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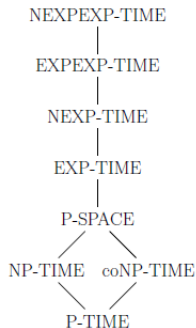
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- ▶ Complexity Classes: sets of problems with the same asymptotic complexity
- ▶ inclusion ordering hierarchy between complexity classes
- ▶ *smaller_complexity(X,X):- complexity_class(X).*
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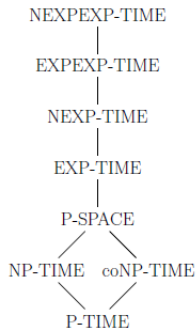
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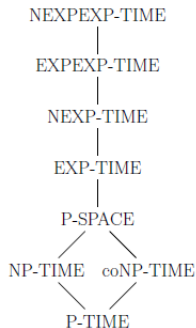
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Propagating Complexity Results

- ▶ some complexity results are known for logic fragments (depending on fragment and semantic)
- ▶ furthermore, expressivity propagates complexity results:
 - ▶ Hardness: more expressive fragments contain the hard problems of less expressive fragments
is_hard(L_1, C, S) :- is_hard(L_2, C, S), higher_logic(L_1, L_2).
 - ▶ Inclusion: less expressive fragments can be solved with the same resources as more expressive fragments
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- ▶ Querying for information:
facts can be retrieved from the knowledge base by filtering the inferred results
(using the `target_logic` in `query.lp`)
- ▶ additions to the knowledge base:
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Finding Justifications

- ▶ Justifications: minimal sets of facts that satisfy a given consequence
- ▶ useful metric for debugging and analysis of a query result
- ▶ Calculation:
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Conclusion

We

- ▶ have modelled different logic families in the same framework
- ▶ can impose an ordering of the expressivity on the different fragments
- ▶ can infer “new” results using this hierarchy
- ▶ can extend the knowledge base with new results
- ▶ can find a justification to provide some explanation for a result

What can still be done?

- ▶ from proof of concept to well-filled knowledge base
- ▶ graphical interface
- ▶ more general approach for justifications