Codifying Logical Fragments in ASP - A General Knowledge Base Approach

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Motivation

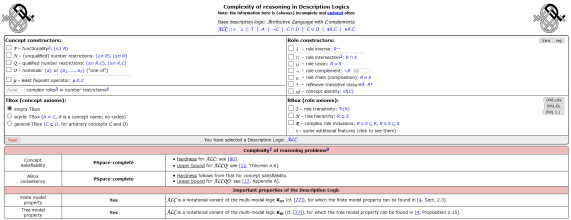
The fact that all Mathematics is Symbolic Logic is one of the greatest discoveries of our age; and when this fact has been established, the remainder of the principles of mathematics consists in the analysis of Symbolic Logic itself.

Bertrand Russell, "The Principles of Mathematics"

Overview

Previous Works

- DL database by Evgeny Zolin (2005 - 2013)
- DL database by Mohamed Ibrahim (2019)



Why ASP?

- **A**nswer **S**et **P**rogramming
- semantics based on Gelfond-Lifschitz reduct
  $\rightarrow$ calculates all answer sets of a given program
- Advantages of ASP:
  - allows for arbitrary arity of predicates
  - analytical tools
  - feasibility of calculations
  - tests for ambiguous rules

Aims of Our Approach

- model different logic fragments and DLs in a *unified manner*
- model connections between logic fragments of different "families"
- *infer* all possible complexity results
- *query* the knowledge base
- find *explanations* for inferred results

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Families of Fragments

- *families* of logic fragments utilize the same basic semantics (e.g. DLs, FO, modal logic, ...)
- fragments are described by their traits (e.g. functionality, qualified number restriction)
  $\Rightarrow$ traits may subsume other traits
- traits are classified in restrictions and features of the fragment (relative to chosen base fragment)

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Ex. Traits

	Features	Restrictions
DL	number restriction nominals inverse roles ...	horn logic no negation empty TBox ...
FOL	equality symbol	2 – variable fragment quantifier prefixes ...

Expressivity of Logic Fragments

- some fragments are more expressive than others
  i.e. all satisfiability problems in one fragment can be expressed in the other
- for fragments within the same family, this can be easy to determine:
  the fragment with more *features* and less *restrictions* is the more expressive
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Ex. Fragments

family(dl; fol).

fragment(fo2, fol).
has_trait(fo2, two_variable).
trait(fo2, restriction, fol).

fragment(alc_EC, dl).
has_trait(alc_EC, (empty_TBox; concept_SAT)).
trait(concept_SAT, restriction, dl).

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translation(alc_EC, fo2).

translation(X, Z) :- translation(X, Y), translation(Y, Z).

has_extra_features(L_1, L_2):- has_trait(L_1,A), -has_trait(L_2, A), trait(A, feature, F), fragment(L_1, F), fragment(L_2, F).

higher_logic(X, Y) :- has_extra_features(X, Y), not has_extra_features(Y, X), not has_extra_restrictions(X, Y).

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Model Properties

- fragments can also have certain model properties (e.g. finite model property)
- expressivity can propagate model properties
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Complexity Classes

- Complexity Classes: sets of problems with the same asymptotic complexity
- inclusion ordering hierarchy between complexity classes
- *smaller_complexity(X,X):- complexity_class(X).*
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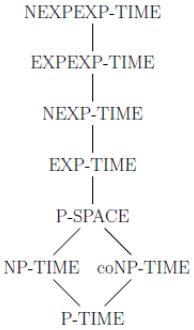
- equality results can be incorporated



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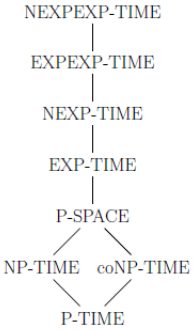

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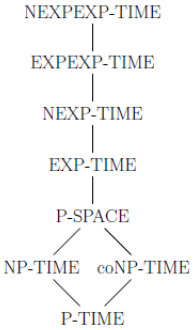

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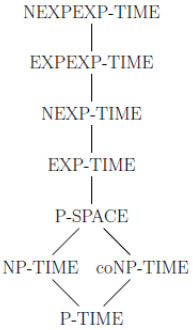

Propagating Complexity Results

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- furthermore, expressivity propagates complexity results:
  - Hardness: more expressive fragments contain the hard problems of less expressive fragments
    is_hard$(L_1, C, S) \coloneq$ *is_hard* (L_2, C, S), *higher_logic*(L_1, L_2).
  - Inclusion: less expressive fragments can be solved with the same resources as more expressive fragments
    has_complexity $(L_2, C, S) \coloneq$ *has_complexity* (L_1, C, S), *higher_logic*(L_1, L_2), C *!= undecidable*.
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- Querying for information:
  facts can be retrieved from the knowledge base by filtering the inferred results (using the `target_logic` in query.lp)
- additions to the knowledge base:
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- Justifications: minimal sets of facts that satisfy a given consequence
- useful metric for debugging and analysis of a query result
- Calculation:
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Conclusion

We

- have modelled different logic families in the same framework
- can impose an ordering of the expressivity on the different fragments
- can infer “new” results using this hierarchy
- can extend the knowledge base with new results
- can find a justification to provide some explanation for a result

What can still be done?

- from proof of concept to well-filled knowledge base
- graphical interface
- more general approach for justifications