1.A

$A = \{0; 2; 4\}$

$W = \{1; 3; 4\}$

De Morgan's Laws:

B. $(A \cap B)' = A' \cup B'$

A. $(A \cup B)' = A' \cap B'$

A

1) $A \cup W = \begin{matrix} A = \{0; \_; 2; \_; 4; \_\} \\ W = \{\_; 1; \_; 3; 4\ \_\} \end{matrix} = \{0; 1; 2; 3; 4\}$

2) $(A \cup W)' = \{5\}$

3) $A' = \{1; 3; 5\}$

$W' = \{0; 2; 5\}$ $\Rightarrow$

4) $A' \cap W' = \{5\}$

$\Rightarrow$ 2) = 4) $(A \cup W)' = \{5\}$

$\Updownarrow$

$(A' \cap W') = \{5\}$

$\Rightarrow (A \cup W)' = A' \cap W'$

B

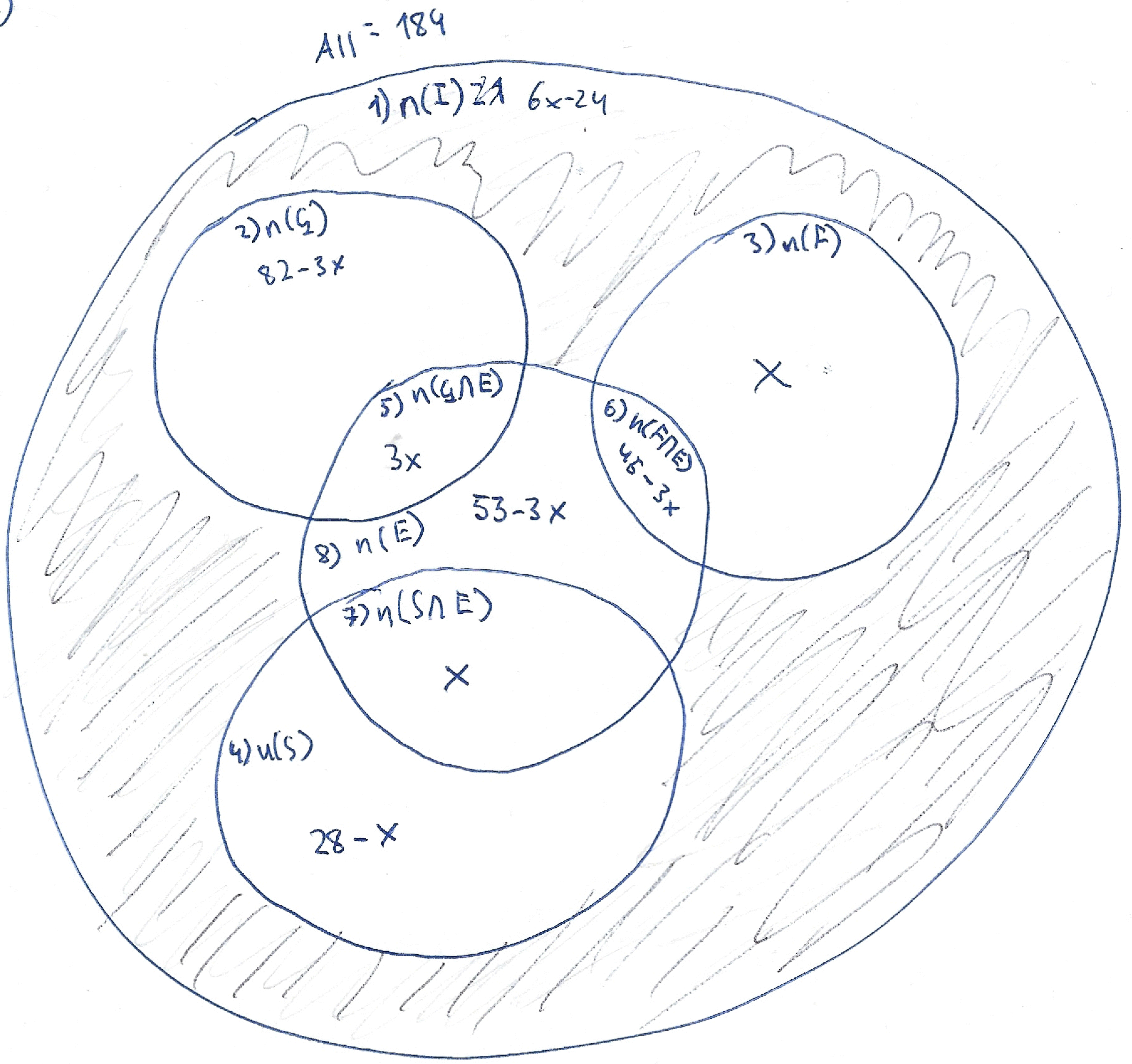
1) $A \cap W = \{4\}$

2) $(A \cap W)' = \{0; 1; 2; 3; 5\}$ $\Rightarrow (A \cap W)' = A' \cup W'$

3) $A' \cup W' = \{0; 1; 3; 5\}$

1. B

(I)



1) $n(I)$ - Italian only $6x-24$
2) $n(G) = 82-3x$ - German and Italian
3) $n(F) = x$ - French and Italian
4) $n(S) = 28-x$ - Spain and Italian
5) $n(G \cap E) = 3x$ -> German; English and Italian
6) $n(F \cap E) = 45-3x$ -> French; English and Italian
7) $n(S \cap E) = x$ -> Spain; English and Italian
8) $n(E) = 53-3x$ -> English and Italian

(II)
$$\begin{cases} 82-3x \geq 0 \\ x > 0 \\ 28-x \geq 0 \\ 45-3x \geq 0 \\ 53-3x \geq 0 \end{cases} \Rightarrow \begin{cases} x \leq 28 \\ x \leq 27.3 \\ x \leq 15 \\ x > 0 \\ x \geq 4 \end{cases} \Rightarrow x \in (\overset{4}{\cancel{0}}; 15]$$

(III)

Only italian: $184-[(82-3x)+(x)+(28-x)+(3x)+(45-3x)+(x)+(53-3x)]=$

$=184-(-6x+208)=-24+6x \geq 0 \Rightarrow x \geq 4$

$6x-24:0$

Only italian: $\{6\times 15-24;\ 0\}$: Only italian: $\{0;\ 66\}$

only German or French:

Only French: $\{4;\ 15\}$

Only German: $\{82-12;\ 82-45\}=\{70;\ 37\}=\{37;\ 70\}$

(IV)

Number of three-language students:

$$\left.\begin{aligned} &n(G\cap E)=3x \\ +\,&n(F\cap E)=45-3x \\ +\,&n(S\cap E)=x \end{aligned}\right\} 45+x,\ \text{if } x=15 \Rightarrow 60 \text{ students}$$

Answer: 60 students will earn extra money after the graduation

2.1 Let's $\frac{\partial s}{\partial t} = D$, then:

$$D^2 \cdot S + 8 \cdot D \cdot S + 20 \cdot S = 0$$
$$S \cdot (D^2 + 8D + 20) = 0$$

So, $S = 0$ or $D^2 + 8 \cdot D + 20 = 0$

Descreminant $= 8^2 - 4 \cdot 20 = -64$

$$D_1 = \frac{-8-8i}{2} = -4-2i$$

$$D_2 = \frac{-8+8i}{2} = -4+2i$$

General solution:

$$S(t) = A \cdot e^{(-4+2i) \cdot t} + B \cdot e^{(-4-2i) \cdot t}$$

$\begin{cases} S = 10 \\ t = 0 \end{cases}$ $S(t) \Rightarrow 10 = A \cdot e^{(-4+2i) \cdot 0} + B \cdot e^{(-4-2i) \cdot 0}$

$$10 = A + B$$

$\begin{cases} \frac{\partial S}{\partial t} = -30 \\ t = 0 \end{cases}$ $\frac{\partial S(t)}{\partial t} = \underbrace{(-4+2i) \cdot A \cdot e^{(-4+2i) \cdot t}}_{\Downarrow} + \underbrace{B \cdot e^{(-4-2i) \cdot t} \cdot (-4-2i)}_{\Downarrow}$

$$-30 = (-4+2i) \cdot A + B \cdot (-4-2i)$$

$\left[\begin{array}{l} 10 = A + B \\ -30 = (-4+2i) \cdot A + (-4-2i) \cdot B \end{array}\right. \Rightarrow A = 10 - B$

$$-30 = (-4+2i)(10 - B) + (-4-2i) \cdot B$$
$$-30 = -40 + 20i + \cancel{4B} - 2i \cdot B - \cancel{4B} - 2i \cdot B$$
$$-70 - 20i = -4i \, B$$

$\left[\begin{array}{l} B = 17.5 \cdot i^{(-1)} + 5 \\ A = 5 - 17.5 \cdot i^{(-1)} \end{array}\right.$

Compleet solution:

$$S(t) = (5 - 17.5 \cdot i^{(-1)}) \cdot e^{(-4+2i) \cdot t} + (17.5 \cdot i^{(-1)}) \cdot e^{(-4-2i) \cdot t}$$

2.B

I)

Let's assume:

$R_t = M^t$:

$t=0$

$6M^2 - 5M + 1 = 0 \Rightarrow (M - \frac{1}{3})(M - \frac{1}{2}) = 0$

$X_t = A \cdot (\frac{1}{3})^t + B \cdot (\frac{1}{2})^t$

if $X_t = K$

$6K - 5K + K = 40$

$K = 20$

so, Combine two solutions:

General solution is:

$X_t = A(\frac{1}{3})^t + B \cdot (\frac{1}{2})^t + 20$

for:

$X_0 = 46$

$46 = A + B + 20$

for:

$x_1 = 30$

$30 = \frac{1}{3} \cdot A + \frac{1}{2} \cdot B + 20$

$60 = 2A + 3B$

$\begin{cases} A + B = 26 \\ 2A + 3B = 60 \end{cases}$

$\Downarrow$

$B = 8$

$A = 18$

Complete solution:

$X_t = 18 (\frac{1}{3})^t + 8 \cdot (\frac{1}{2})^t + 20$

II) $P_t = R_t - C_t \Rightarrow P_t = [18 \cdot (\frac{1}{3})^t + 8 \cdot (\frac{1}{2})^t + 20] - [27 \cdot (\frac{1}{3})^t + 12 \cdot (\frac{1}{2})^t + 30]$

$P_t = -9 \cdot (\frac{1}{3})^t - 4 \cdot (\frac{1}{2})^t - 10$, for $t \geq 0$

$P_t(t=0) = -9 - 4 - 10 = -23$

$P_t(t=1) = -9 \cdot \frac{1}{3} - 4 \cdot \frac{1}{2} - 10 = -15$

$P_t(t=2) = -9 \cdot \frac{1}{9} - 4 \cdot \frac{1}{4} - 10 = -12$

$P_t(t=3) = -9 \cdot \frac{1}{27} - 4 \cdot \frac{1}{8} - 10 = -\frac{1}{3} - \frac{1}{2} - 10 = -10\frac{5}{6}$

so, $\lim_{t \to \infty} (-9 \cdot \underbrace{(\frac{1}{3})^t}_{\to 0} - 4 \underbrace{(\frac{1}{2})^t}_{\to 0} - 10) =$

$= -10$

Answer: product will never be profitable; Limit of profit Function is negative $\lim_{t \to \infty} (P_t) = -10$

3.

Year	$\sum$ Number of customers	$\sum$ Total purchases ($000)	AVG purchases per customer ($000)
2019	580 + 600 + 50 + 40 = 1270	600 + 540 + 90 + 8 = 1238	~~1270~~ 1238/1270 = 0.97
2020	500 + 640 + 40 + 45 = 1225	730 + 750 + 80 + 12 = 1572	1572/1225 = 1.28
2021	510 + 640 + 38 + 35 = 1223	750 + 900 + 70 + 10 = 1730	1730/1223 = 1.41

I

Year	Wine total purchases per year (000$)	Index of wine Total purchases
2019	1238	$100 = \boxed{100}$
2020	1572	$100 \times \frac{1572}{1238} = \boxed{127}$
2021	1730	$100 \times \frac{1730}{1238} = \boxed{139.7}$

II

year	Total number of customers per year	Index of customor Total number
2019	1270	100 100
2020	1225	$100 \times \frac{1225}{1270} = 96.5$
2021	1223	$100 \times \frac{1223}{1270} = 96.3$

III

year	AVG purchases of wine per customer per year ($000)	Index of AVG purchases of wine per customer per year
2019	0.97	$100 = 100$
2020	1.28	$100 \times \frac{1.28}{0.97} = 131.6$
2021	1.41	$100 \times \frac{1.41}{0.97} = 145.1$

IV

year	GMV	N	AVG
2019	100	100	100
2020	127	96.5	131.6
2021	139.7	96.3	145.1

GMV ↗
N ↘
AVG ↗

by looking into data it's noticable, that increasment in purchase amount from 100 to 139 goese Due to increasment of AVG revenue per customor {from 100 to 145 by 2 years}. Also, it's notisable a slight fall in customors {from 100 to 96.3 by 2 years}.

5. What production $b = \begin{pmatrix}20\\10\\20\end{pmatrix}\begin{matrix}P\\Q\\R\end{matrix}$

$X = (I-A)^{-1}\cdot b$, where x - is ~~needed~~ necessary export

$$(I-A) = \begin{array}{ccc} P & Q & R \end{array}$$
$$(I-A) = \begin{pmatrix} 1-0.8 & 0-0.2 & 0-0.12 \\ 0-0 & 1-0 & 0-0.4 \\ 0-0.2 & 0-0.5 & 1-0.5 \end{pmatrix} = \begin{pmatrix} 0.2 & -0.2 & -0.12 \\ 0 & 1 & -0.4 \\ -0.2 & -0.5 & 0.5 \end{pmatrix}$$

$R_1 = R_1/0.2$

$$(I-A)^{-1} = \left(\begin{array}{ccc|ccc} 0.2 & -0.2 & -0.12 & 1 & 0 & 0 \\ 0 & 1 & -0.4 & 0 & 1 & 0 \\ -0.2 & -0.5 & 0.5 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & -1 & -0.6 & 5 & 0 & 0 \\ 0 & 1 & -0.4 & 0 & 1 & 0 \\ -0.2 & -0.5 & 0.5 & 0 & 0 & 1 \end{array}\right) =$$

$R_3 = R_1\cdot 0.2 + R_3$

$R_1 = R_1 + R_2$

$R_3 = R_3 + R_2\cdot 0.7$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & -1 & -0.6 & 5 & 0 & 0 \\ 0 & 1 & -0.4 & 0 & 1 & 0 \\ 0 & -0.7 & 0.38 & 1 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 5 & 1 & 0 \\ 0 & 1 & -0.4 & 0 & 1 & 0 \\ 0 & 0 & 0.1 & 1 & 0.7 & 1 \end{array}\right) \to$$

$R_3 = R_3/0.1$

$$\to \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 5 & 1 & 0 \\ 0 & 1 & -0.4 & 0 & 1 & 0 \\ 0 & 0 & 1 & 10 & 7 & 10 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 15 & 8 & 10 \\ 0 & 1 & 0 & 4 & 3.8 & 4 \\ 0 & 0 & 1 & 10 & 7 & 10 \end{array}\right)$$

$R_1 = R_1 + R_3$

$R_2 = R_2 + R_3\cdot 0.4$

$$(I-A)^{-1} = \begin{pmatrix} 15 & 8 & 10 \\ 4 & 3.8 & 4 \\ 10 & 7 & 10 \end{pmatrix}$$

to produce necessary export we must produce:

$$\begin{pmatrix} 15 & 8 & 10 \\ 4 & 3.8 & 4 \\ 10 & 7 & 10 \end{pmatrix}\begin{pmatrix} 20 \\ 10 \\ 20 \end{pmatrix} = \begin{pmatrix} 580 \\ 198 \\ 470 \end{pmatrix}$$

Production of 580 units P will consume:

$0.8 \times 580 = 464$ of P
$0 \times 580 = 0 \cdot Q$
$0.2 \times 580 = 116$ of R

Production of 198 units Q will consume:

$0.2 \times 198 = 39.6$ of P
$0 \times 198 = 0$ of Q
$0.5 \times 198 = 99$ of R

Production of 470 units R will consume:

$0.12 \times 470 = 56.4$ of P
$0.4 \times 470 = 188$ of Q
$0.5 \times 470 = 235$ of R

So, P: total production: 580
total consumtion: $(464 + 39.6 + 56.4) = 560$
export: 20

Q: total production: 198
total consumtion: 188
export: 10

R: total production: 470
total consumtion: $(116 + 99 + 235) = 450$
export $= 20$

Answer: $\left\{ \begin{array}{l} 580 \text{ of } P \\ 198 \text{ of } Q \\ 470 \text{ of } R \end{array} \right\}$ Production is necessary to achive export goals.

6.A
(I)

$EWMA = \hat{T}_t = \alpha \cdot X_t + (1-\alpha) \cdot \hat{T}_{t-1}$

Day N t / Day Number	% Humidity $X(t)$	$\hat{T}_t(\alpha=0.3)$	$\lvert E_r \rvert (\alpha=0.3)$	$\hat{T}_t(\alpha=0.5)$	$\lvert E_r \rvert (\alpha=0.5)$
1	23				
2	22.5	23	0.5	23	0.5
3	22	22.85	0.85	22.75	0.75
4	23	22.6	0.41	22.43	0.58
5	24	22.72	1.28	22.8	1.2
6	26.5	23.1	3.40	23.36	3.14
7	25	24.12	0.88	24.8	0.2
8	24	24.38	0.38	24.56	0.56
9		24.27		24.19	
			$\Sigma = 7.7$ MAE = 1.1		$\Sigma = 6.93$ MAE = 0.99

$\alpha = 0.3$

$\hat{T}_2 = 23 \cdot 0.3 + 0.7 \times 23 = 23$

$\hat{T}_3 = 22.5 \times 0.3 + 0.7 \times 23 = 22.85$

$\hat{T}_4 = 22 \times 0.3 + 0.7 \times 22.85 = 22.6$

$\hat{T}_5 = 23 \times 0.3 + 0.7 \times 22.6 = 22.72$

$\hat{T}_6 = 24 \times 0.3 + 0.7 \times 22.72 = 23.1$

$\hat{T}_7 = 26.5 \times 0.3 + 0.7 \times 23.1 = 24.12$

$\hat{T}_8 = 25 \times 0.3 \times 0.7 \times 24.12 = 24.38$

$\hat{T}_9 = 24 \times 0.3 + 0.7 \times 24.38 = 24.27$

$\alpha = 0.3$
$\hat{T}_9 = 24.27$
$MAE = \frac{7.7}{7} = 1.1$

$\alpha = 0.5$

$\hat{T}_2 = 23 \cdot 0.5 + 0.5 \times 23 = 23$

$\hat{T}_3 = 22.5 \times 0.5 + 0.5 \times 23 = 22.75$

$\hat{T}_4 = 22 \times 0.5 + 0.5 \times 22.75 = 22.43$

$\hat{T}_5 = 23 \times 0.5 + 0.5 \times 22.43 = 22.8$

$\hat{T}_6 = 24 \times 0.5 + 0.5 \times 22.8 = 23.36$

$\hat{T}_7 = 26.5 \times 0.5 + 0.5 \times 23.36 = 24.8$

$\hat{T}_8 = 25 \times 0.5 + 0.5 \times 24.8 = 24.56$

$\hat{T}_9 = 24 \times 0.5 + 0.5 \times 24.56 = 24.19$

$\alpha = 0.5$
$\hat{T}_9 = 24.19$
$MAE = 0.99$

Answer!

$\left\{ \begin{array}{l} \alpha = 0.3 \\ \hat{T}_9 = 24.27 \\ MAE = 1.1 \end{array} \right\}$, $\left\{ \begin{array}{l} \alpha = 0.5 \\ \hat{T}_9 = 24.19 \\ MAE = 0.99 \end{array} \right\}$

6.A.

(II)

α	MAE
0.4	1.087
0.6	1.091
0.7	1.128
0.3	1.1
0.5	0.989

Optimum value of Alpha is 0.5, becaus it's produce the lowest $MAE_{\alpha=0.5} = 0.989$

(III)

For 8th observation data-set in is not relevant to tune-up model for systemic trend prediction. But, if consider, that EWMA will be fedded more data. it is posible to add systematic trend term.

Them, basic aquattion: $\hat{T}_t = \alpha \cdot X_t + (1-\alpha) \cdot \hat{T}_{t-1}$

will adapted:

$$\begin{cases} \hat{T}_t = \hat{T}_{t-1} + \hat{C}_{t-1} + \alpha(X_t - (\hat{T}_{t-1} + \hat{C}_{t-1})) \\ \hat{C}_t = \hat{C}_{t-1} + \beta((\hat{T}_t - \hat{T}_{t-1}) - \hat{C}_{t-1}) \\ \begin{Bmatrix} \beta > 0 \\ \beta < 1 \end{Bmatrix} \end{cases}$$

where: $\hat{C}$ - systomatic trend

6.B

	Tom	Dick	Harriet	Fied Fact
AGE	51	58	54	55
Weight	67	67	71	70
Height	1.61	1.69	1.85	1.64
Height (SM)	161	169	185	164

I

$$MAE_{TOM} = \frac{1}{3}\cdot(|55-51|+|70-67|+|1.64-1.61|) = 2.34$$

$$MAE_{Dick} = \frac{1}{3}\cdot(|55-58|+|70-67|+|1.64-1.69|) = 2.01$$

$$MAE_{Harriet} = \frac{1}{3}\cdot(|55-54|+|70-71|+|1.64-1.85|) = 0.73$$

So, Harriet has minimum $\{MaE = 0.73\}$, that's mean, that Harriet has better estimations

II

$$MAE_{TOM\_2} = \frac{1}{3}\cdot(|55-51|+|70-67|+|164-161|) = 3.34 \quad \text{Min}$$

$$MAE_{Dick\_2} = \frac{1}{3}\cdot(|55-58|+|70-67|+|164-169|) = 3.67$$

$$MAE_{Harriet\_2} = \frac{1}{3}\cdot(|55-54|+|70-71|+|164-185|) = 7.67 \quad \text{Max}$$

By appling height in sm, Harriet become the worst estimator with higher MAE (7.67)

Tom become bettor estimator with MAE = 3.34

III

$$MARE_{TOM} = \frac{1}{3}\cdot\left(\frac{|55-51|}{55} + \frac{|70-67|}{70} + \frac{|1.64-1.61|}{1.64}\right) = 0.045$$

$$MARE_{Dick} = \frac{1}{3}\cdot\left(\frac{|55-58|}{55} + \frac{|70-67|}{70} + \frac{|1.64-1.69|}{1.64}\right) = 0.043$$

$$MARE_{Harriet} = \frac{1}{3}\left(\frac{|55-54|}{55} + \frac{|70-71|}{70} + \frac{|1.64-1.85|}{1.64}\right) = 0.854$$

The best estimate was produced by Dick, with smallest MARE 0.43.

MARE is more accurate measure of error in high dispersed values prediction

MARE allow to ignore outliers

7. A.I

1)

	A	B	C	DE	F	G
A	-					
B	40	-				
C	69	33	-			
DE	120	100	74	-		
F	55	78	90	95		
G	35	72	96	110	31	-

picking up D and E sities, hence they have the smallest destination(18)

- $\max(\overset{120}{D-A}; \overset{100}{A-E}) = 120 = (DE; A)$
- $\max(\overset{100}{D-B}; \overset{80}{E-B}) = 100 = (DE; B)$
- $\max(\overset{70}{D-C}; \overset{74}{E-C}) = 74 = (DE; C)$
- $\max(\overset{95}{D-F}; \overset{19}{E-F}) = 95 = (DE; F)$
- $\max(\overset{110}{D-G}; \overset{50}{E-G}) = 110 = (DE; G)$

2)

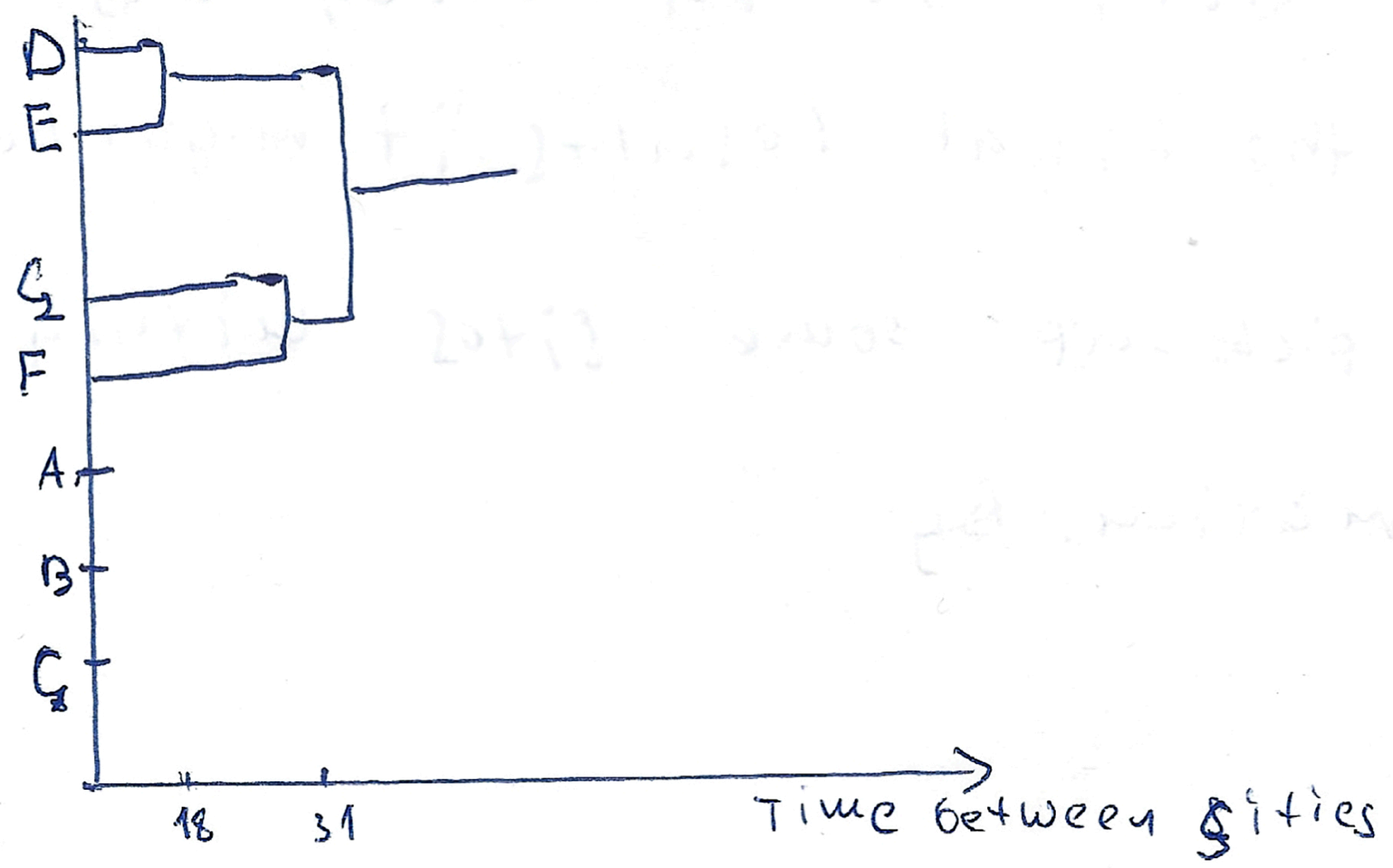
	A	B	C	DE	GF
A	-				
B	40	-			
C	69	33	-		
DE	120	100	74	-	
GF	55	78	96	110	-

Next one: pick G and F with has minimum Value of 31

55. $\max(G-A; F-A) = \max(55; 35) \Rightarrow (GF; A)$
78. $\max(G-B; F-B) = \max(72; 78) \Rightarrow (GF; B)$
96. $\max(G-C; F-C) = \max(90; 96) \Rightarrow (GF; C)$
110. $\max(G-DE; F-DE) = \max(95; 110) \Rightarrow (GF)$

Most convinient cites are: DE and GF

Dendogram of complete linkage of choice 3 cities from 7



So, thouse pairs has the ~~les~~ smallest time between them: DE : 18
GF : 31

(II) For purpose of picking Distribution center time between sities not the best measure ~~of~~.

pros:

- Easy to understand
- Quantative

cons:

- time between sities ignore road capacity
- time between sities could change by wheather

condition

- ignore population and sales potential in each sites

better solution is to use mixed metirix:

- Time
- KM
- AVG trafic jam

(III) So, before the clustering it could be applicablo to exclude sites with low population and sales potentials. At the final results it might be an option to pick-up some sites within population information. ~~By~~

8

$$Y_1 = 0.4\cdot X_1 \cancel{+} - 1.2\cdot X_2 + 0.3\, X_3 + 0.2\, X_4 \quad \text{for } DF_1$$

$$Y_2 = 0.5\cdot X_1 - 2.2\cdot X_2 + 0.6\cdot X_3 + 0.1\cdot X_4 + 0.5\cdot X_5 \quad \text{for } DF_2$$

I $\underline{DF_1}$

if DF_1 > Cut-off => group B

for Df_1 · cutt off = 3

predict

		A	B	total
Actual	A	180	20	200
	B	10	40	50
				250

$$APER = \frac{10+20}{200+50} = 0.12 \text{ for } DF_1$$

$\underline{DF_2}$

		A	B	total
Actual	A	190	10	200
	B	5	45	50
				250

$$APER = \frac{15}{250} = 0.06 \text{ for } DF_2$$

II

DF1

$$Y_1(U\,187) = 0.4\times 20 \cancel{+} - 1.2\cdot 12 + 0.3\times 20 + 0.2\cdot 10 = 1.6$$

critical point for DF_1 is 3, so $Y_1(U\,187) < 3$ =>

By using DF_1 unit 187 goes to <u>group A</u>

DF_2

$$\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array}$$

$$Y_2(U\,187) = 0.5\times 20 - 2.2\times 12 + 0.6\times 20 + 0.1\times 10 + 0.5\times 5 = -0.9$$

critical point for DF_2 is 1, so for $Y_2(U\,187) < 1$ =>

By using DF_2 unit 187 goes to <u>group A</u>

III

DF1

		Train A	B	Total
Act	A	180	20	200
	B	10	40	50

APER = 0.12

DF2

		Train A	B	total
Act	A	190	10	200
	B	5	45	50

APER = 0.06

Total cost

$DF_1 = 25 \times 20 + 10 \times 50 = 1000$

$DF_2 = 10 \times 25 + 5 \times 50 + 250 \times 3 = 1250$

In situation of nigligble cost for all Values $(x_1 : x_5)$, better clasification with less cost will be DF_2 (AER 0.06 Versus $AER_{DF_2} = 0.12$) and Cost : ~~100~~ 500 Versus 1000.

But, by adding additional cost for X_5 clasification, the cheaper solution will be DF_1 (1000).

Other words: it's Always balanse between cost and AER

IV

DF1

		Predict A	B	
Actual	A	850	50	900
	B	40	60	100
				1000

$AER = \frac{40+50}{1000} = 0.09$

IV AER = 0.09
APER = 0.12

For DF1 we obseove AER lower than APER, it might be in case when descrominational function was optimse for 250 sample. Hence as usual we expect AER be more than APER. It's posiole when circumstance have changed and model couldn't work propper with out of sample result.