

1.B

$$n(I) = 184$$

$$n(E) = 100$$

$$n(G) = 82$$

$$n(F) = 45$$

$$n(S) = 28$$

$$n(G \cap F) = 0$$

$$n(G \cap S) = 0$$

$$n(F \cap S) = 0$$

$$n(E \cap$$

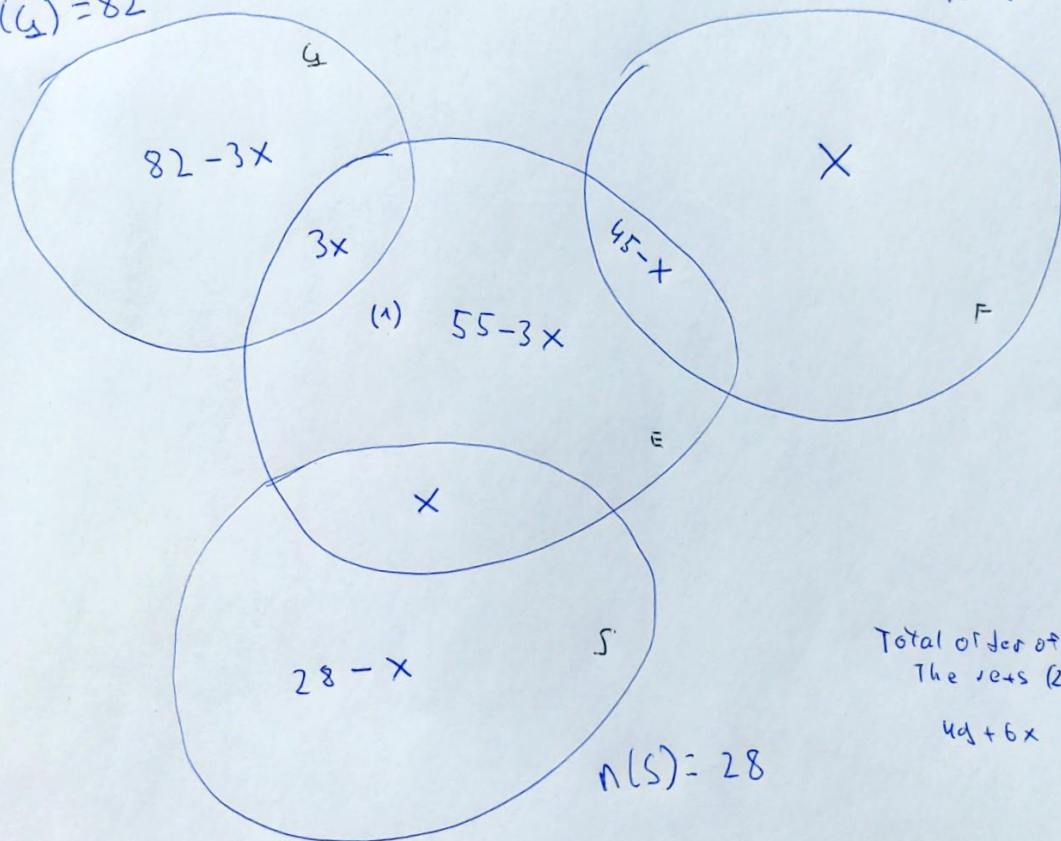
$$n(F) = x$$

$$n(S \cap E) = x$$

$$n(G \cap E) = 3x$$

$$n(G) = 82$$

$$n(F) = 45$$



Total order of
The sets (2)

$$49 + 6x$$

$$1) 100 - 3x - x - 45 + x = 55 - 3x$$

$$2) 184 - 82 + 3x - x - 28 + x - 55 + 3x - 24x + 6x$$

$$2) 184 - [(82 - 3x) + (3x) + (x) + (45 - 3x) + (55 - 3x) + (x) + (28 - x)] = -26 - 5x \Rightarrow x \leq$$

$$2) 184 - 82 + 3x - 3x - x - 47 + x$$

$$2) 184 - 82 + 3x - x - 28 + x - 55 + 3x$$

$$\text{II. We use: } 3x \leq 55 \Rightarrow x \leq 18$$

$$x > 0$$

$$184 \leq 49 + 6x$$

2.B.II

$$P_0 = R_0 - C_0 \Rightarrow P_0 = \left[18 \cdot \left(\frac{1}{3}\right)^0 + 20 + 8 \cdot \left(\frac{1}{2}\right)^0 \right] - \left[27 \cdot \left(\frac{1}{3}\right)^0 + 12 \cdot \left(\frac{1}{2}\right)^0 + 10 \right]$$
$$= (27 + 18) \cdot \left[\frac{1}{3}\right]^t + (8 - 12) \cdot \left[\frac{1}{2}\right]^t - 10 = -9 \cdot \left(\frac{1}{3}\right)^t - 4 \cdot \left(\frac{1}{2}\right)^t - 10$$

$$P_0 = -9 \cdot \left(\frac{1}{3}\right)^0 - 4 \cdot \left(\frac{1}{2}\right)^0 - 10 = -23$$

$$P_1 = -9 \cdot \left(\frac{1}{3}\right)^1 - 4 \cdot \left(\frac{1}{2}\right)^1 - 10 = -15$$

$$P_2 = -9 \cdot \left(\frac{1}{3}\right)^2 - 4 \cdot \left(\frac{1}{2}\right)^2 - 10 = -12$$

Answer: So, gas product will never be profitable at all because of negative revenue function in condition of positive productions

2.A

let's

$$S_t = M^t$$

2.B.I

let's assume

$$R_t = M^t :$$

$$6m^2 - 5m + 1 = 0 \Rightarrow (m - \frac{1}{3})(m - \frac{1}{2}) = 0$$

$$x_t = A\left(\frac{1}{3}\right)^t + B\left(\frac{1}{2}\right)^t$$

$$\text{if } x_t = k$$

$$6k - 5k + k = 40$$

$$k = 20$$

So, combine two solution parts:

General solution is

$$x_t = A\left(\frac{1}{3}\right)^t + B\left(\frac{1}{2}\right)^t + 20$$

$$x_0 = 46$$

$$46 = A + B + 20$$

$$\begin{cases} A + B = 26 \\ 2A + 3B = 60 \end{cases}$$

$$B = 8$$

$$A = 18$$

$$x_1 = 30$$

$$30 = \frac{1}{3} \cdot A + \frac{1}{2} \cdot B + 20$$

$$60 = 2A + 3B$$

Complete solution is:

$$x_t = 18\left(\frac{1}{3}\right)^t + 8\left(\frac{1}{2}\right)^t$$

4.4

$$P_n(t + \Delta t) = \lambda \Delta t P_{n-1}(t) + (1 - \lambda \Delta t) \cdot P_n(t) + o(\Delta t)$$

So, rearranging and letting $\Delta t \rightarrow 0$

$$\frac{dP_n}{dt} = -\lambda P_{n-1} \quad \text{for } n \geq 1$$

$$\frac{dP_0/dt}{P_0} = -\lambda$$

$$\frac{dP_1}{dt} = -\lambda P_1 + \lambda P_0 = -\lambda P_1 + \lambda e^{-\lambda t}$$

$$\frac{dP_1}{dt} = \lambda e^{-\lambda t}$$

$$e^{\int \lambda dt} = e^{\lambda t}$$

hence:

$$P_1 = e^{-\lambda t} \int \lambda e^{-\lambda t} e^{\lambda t} dt = \lambda t e^{-\lambda t}$$

Answer: Non often together and rate reasonably constant

Poisson should work well

$$u.BI \quad p_2 = p^2 (1-p) = 0.5^2 (1-0.5) = 0.125$$

$$u.BII \quad L = \frac{0.1}{0.2-0.1} = 1$$

5.

$$A = \begin{pmatrix} 0.8 & 0.2 & 0.12 \\ 0 & 0 & 0.4 \\ 0.2 & 0.5 & 0.5 \end{pmatrix} \begin{matrix} P \\ Q \\ R \end{matrix} \quad \begin{matrix} I \\ II \\ III \end{matrix}$$

$$(E - A) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} - A = \begin{pmatrix} 0.2 & -0.2 & -0.12 \\ 0 & 1 & -0.4 \\ -0.2 & -0.5 & 0.5 \end{pmatrix}$$

$$(E - A)^{-1} = \begin{pmatrix} 0.2 & -0.2 & -0.12 \\ 0 & 1 & -0.4 \\ -0.2 & -0.5 & 0.5 \end{pmatrix}$$

$$\Delta = 0.2 \cdot (1 \cdot 0.5 - (-0.5 \cdot (-0.4))) - 0 \cdot (-0.2 \cdot 0.5 - (-12)) + (-0.2) \cdot (-0.2 \cdot 0.4) - 1 \cdot (0.12) = 0.02$$

$$B^T = \begin{pmatrix} 0.2 & 0 & -0.2 \\ -0.2 & 1 & -0.5 \\ -0.12 & 0.4 & 0.5 \end{pmatrix}$$

$$B_{1,1}^T = (-1)^{1+1} \begin{pmatrix} 1 & -0.5 \\ -0.4 & 0.5 \end{pmatrix}$$

$$\Delta_{11} = (1 \cdot 0.5 - (-0.4 \cdot (-0.5))) = 0.5$$

$$B_{1,2}^T = (-1)^{1+2} \begin{pmatrix} -0.2 & -0.5 \\ -0.12 & 0.5 \end{pmatrix}$$

$$\Delta_{12} = (-0.2 \cdot 0.5 - (-0.12 \cdot (-0.5))) = -0.16$$

$$\Delta_{1,3} = (-0.2 \cdot 0.4 - (-0.12 \cdot 1)) = 0.2$$

$$\Delta_{2,1} = (0.4 \cdot 0.5 - (-0.4 \cdot 1)) = 0.08$$

$$\Delta_{2,2} = 0.076$$

$$\Delta_{2,3} = 0.08$$

$$\Delta_{3,1} = 0.2$$

$$\Delta_{3,2} = 0.14$$

$$\Delta_{3,3} = 0.2$$

$$B^{-1} = \frac{\Delta}{0.02} \begin{pmatrix} 0.3 & 0.16 & 0.2 \\ 0.08 & 0.076 & 0.08 \\ 0.2 & 0.14 & 0.2 \end{pmatrix}$$

$$B^{-1} = \begin{pmatrix} 15 & 8 & 10 \\ 4 & 3.8 & 4 \\ 10 & 7 & 10 \end{pmatrix}$$

System of basic equations

$$0.12x_1 - 0.2x_2 - 0.12x_3 = y_1$$

$$0x_1 + 1x_2 - 0.4x_3 = y_2$$

$$-0.2x_1 - 0.3x_2 + 0.5x_3 = y_3$$

$$X = (B^{-1} \cdot Y) = \begin{pmatrix} 15 & 8 & 10 \\ 4 & 3.8 & 4 \\ 10 & 7 & 10 \end{pmatrix} \begin{pmatrix} 20 \\ 10 \\ 20 \end{pmatrix} = \begin{pmatrix} 580 \\ 198 \\ 470 \end{pmatrix}$$

Production

$$580 - (464 + 0 + 116) = 0$$

$$198 - (39.6 + 0 + 99) = 59.4$$

$$470 - (56.4 + 188 + 335) = -9.4$$

inter-industry

industry	consuming industry	Final product	GROSS Domestic product	2	3
1	464	39.6	56.4	20	380
2	0	0	188	10	102
3	116	99	235	20	470
Net product	0	59.4	-9.4	50	
GROSS domestic	580	198	470		1248

Day	% Humidity	Forecast	Error	Error ²	Absolute error
1	23				
2	22,5	23	-0,5	0,25	-0,5
3	22	22,35	-0,35	0,1225	-0,3
4	23	22,3	-0,7	0,49	-0,7
5	24	23,3	-0,7	0,49	-0,7
6	26,5	24,75	1,75	3,0625	1,8
7	25	26,05	-0,7	1,1025	-0,7
8	24	24,7		0,49	-0,7
		16,8			

$$\lambda = 0.3$$

$$RMSE = 0.9263$$

$$MAE = 0.55/7 = 0.078$$

Day	Alpha = 0.5 Humidity	Forecast	Error	Error ²	Absolute error
1	23				
2	22,5	23	-0,5	0,25	-0,5
3	22	22,25	-0,25	0,0625	-0,3
4	23	22,5	0,5	0,25	0,5
5	24	23,5	0,5	0,25	0,5
6	26,5	25,25	1,25	1,5625	1,3
7	25	25,75	-0,75	0,5625	-0,8
8	24	24,5	-0,5	0,25	-0,5
9		(12)			
Total				3,1875	0,25

$$RMSE = 0,6248$$

$$MAE = 0,25/7 = 0,0357$$

Answer :

I Alpha = 0.3
 Forecast: 16,8
 $RMSE = 0,0263$
 $MAE = 0,078$

Alpha = 0.5
 Forecast = 12
 $RMSE = 0,6248$
 $MAE = 0,0357$

6.4.11

The lower the MAE \Rightarrow the better \Rightarrow Alpha = 0,5 is optimal value

MAE

$\alpha_1 = 0.3 \quad 0,078$
 $\alpha_2 = 0.5 \quad 0,036$
 $\alpha_3 = 0.4 \quad 1,087$
 $\alpha_4 = 0.6 \quad 1,091$
 $\alpha_5 = 0.7 \quad 1,128$

3.

2019

	<u>N_{cast}</u>	<u>GMV</u>	<u>AVG GMV</u>	<u>w x (P/P₀)</u>
I	580	600	1.03	597.4
II	600	540	0.9	486
III	50	90	1.8	
IV	40	8	0.20	
Total	1270	1238		

$$L_{aspejos}_{2019} = \frac{\cancel{1270}}{\cancel{123}} \frac{1238}{1270} = 0.974$$

Laspeyre

2020

		<u>GMV</u>
I	580	600
II	600	540
III	50	90
IV	40	8

$$s(t) = c_1 \cdot e^{-4t} \cdot \cos(2t) + c_2 \cdot e^{-4t} \cdot \sin(2t)$$

Particular solution:

$$\frac{d^2 s(t)}{dt^2} + 8 \frac{ds(t)}{dt} + 20 s(t) = 10e^{-3t}$$

$$s_p(t) = a_1 \cdot e^{-3t}$$

General solution:

$$s(t) = 2 \cdot e^{-3t} + c_1 \cdot e^{-4t} \cdot \cos(2t) + c_2 \cdot e^{-4t} \cdot \sin(2t)$$

$$t=0 \Rightarrow s(t) = 2 \cdot 1 + c_1 \cdot 1 + c_2 \cdot 0 = 10$$

$$\underline{c_1 = 8}$$

3.4

$$A = \{0, 2, 4\}$$

$$W = \{1, 3, 4\}$$

De Morgan's Law:

$$(A \cap B)^c = A^c \cup B^c$$

$$(A \cup B)^c = A^c \cap B^c$$

$$(A \cap W) = \{4\}$$

A