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Multi-objective Combinatorial Optimization Problems: A Survey

E. L. ULUNGU and J. TEGHEM
Department of Mathematics and Operational Research,
Faculté Polytechnique de Mons, 9 Rue de Houdain, B-7000 Mons,
Belgium

ABSTRACT

In the last 20 years many multi-objective linear programming (MOLP) methods with continuous variables have been developed. However, in many real-world applications discrete variables must be introduced. It is well known that MOLP problems with discrete variables can have special difficulties and so cannot be solved by simply combining discrete programming methods and multi-objective programming methods.

The present paper is intended to review the existing literature on *multi-objective combinatorial optimization* (MOCO) problems. Various classical combinatorial problems are examined in a multi-criteria framework. Some conclusions are drawn and directions for future research are suggested.

KEY WORDS Assignment and allocation problems
Knapsack problems Location problems
Multi-objective combinatorial optimization (MOCO)
Network flow problems Transportation problems
Transhipment problems Travelling salesman problem

1. MOCO: A CHALLENGE FOR THE 1990S

In 1986 Teghem and Kunsch [19, 20] published two surveys on the general structure of multi-objective integer linear programming; the characterization of the efficient solution set was analysed in the first and the main interactive techniques for these problems were described in the second. The motivation of these papers was to introduce discrete variables in some applications related to planning in a multi-objective framework [18]. During the same period another review focused on 0–1 programming was published by Rasmussen [15].

Combinatorial optimization provides a powerful tool to model many real-world applications. Nevertheless, the *multi-objective spirit* is not yet prevalent and much progress remains to be made in this direction. This is clearly apparent from the bibliographical study on the applications of multi-objective mathematical programming, based on 504 papers, published by White [24]: only a few are modelled with discrete variables owing to their inherent difficulties and the lack of sufficient theoretical foundations.

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Multi-objective combinatorial optimization (MOCO) thus provides a real challenge for the future, namely to introduce multi-objective approaches into the large class of combinatorial optimization applications. We are presently doing research in this field (see Section 3) and are aware of the lack of any useful synthesis of the field. We should, none the less, mention the annotated bibliography and taxonomy of papers related to the multi-objective design of transportation networks by Current and Min [3], in which some information is given on the methods.

The aim of the present paper is to address this need by a descriptive and comparative survey of many varied papers published in the field of MOCO. Various classical combinatorial optimization problems with multiple objectives will be examined in two sections. Section 2 contains problems which have received much attention: the family of problems which—with a single objective—can be solved in polynomial time, e.g. assignment and allocation; transportation and network flow problems. Section 3 is devoted to other classical combinatorial optimization problems: location; travelling salesman, set-covering and knapsack problems. In Section 4 we offer some conclusions derived from this survey of the literature.

(1) Some connections are made between several studies.
(2) Some elements of a classification are given—other than that given by the nature of the problem.
(3) The weaknesses of some papers are discussed, explaining in large part the relatively poor methodology used in existing MOCO applications.
(4) Some directions for further research are suggested.

Note that multi-objective shortest-path and scheduling problems are not discussed here for the following reasons.

(1) A specific study on the shortest-path problem has recently been published by Ulungu and Teghem [21].
(2) Combinatorial optimization can be an efficient tool for scheduling problems, but they form a real specific class of problems—one with its own models (single machine, several parallel machines, flow permutation shop, flow shop, job shop, etc.) and its own criteria (makespan, mean sojourn time, lateness, tardiness, etc.). Thus there is need for a separate study. Moreover, several multi-objective models are already described in specialized books and theses [e.g. 4, 9].

The general framework of MOCO is

$$[\text{'min'}_{X\in D}\ z_k(X)=c_kX, \quad k=1,\ldots,p] \tag{P}$$

where

$$D=\{X{:}X\in LD, X\in B^n\}, \qquad LD=\{X{:}AX\leqslant b\}$$

with $A\,(m\times n)$, $c_k(1\times n)$, $X\,(n\times 1)$, $b\,(m\times 1)$ and $B=\{0,1\}$.

In the following $C(p\times n)$ denotes the matrix formed by the p row vectors c_k and (LP) denotes the linear relaxation of problem (P).

A solution X^* of D (or LD) is *efficient* if there does not exist any other solution $X\in D$ (or LD) such that $z_k(X)\leqslant z_k(X^*)$, $k=1,\ldots,p$, with at least one strict inequality.

The vector $z(X^*)$ of values $z_k(X^*)$ for $k=1,\ldots,p$ is said to be *non-dominated* in the space of objective functions. Let $E(\cdot)$ denote the set of all efficient solutions of problem $(\cdot)$.

It is well known [e.g. 17] that $E(LP)$ may be characterized by the optimal solutions of the single-objective and parametrized problem

$$\left[\min_{X \in LD} \sum_{k=1}^{p} \alpha_k z_k(X)\right] \qquad (LP_\alpha)$$

with $\alpha_k > 0\ \forall k$ and $\sum_{k=1}^{p} \alpha_k = 1$.

This fundamental principle, called *Geoffrion's theorem*, is no longer valid for problem (P_α)—defined as problem (LP_α) in which LD is replaced by D—because the set D is not convex. Effectively, the set SE(P) of optimal solutions of problem (Pα) is only a subset of E(P); these solutions are called *the supported efficient solutions*, while the solutions belonging to E(P) − SE(P) are the *non-supported efficient solutions*.

Nevertheless, Bowman [1] proved that the set E(P) is characterized by the optimal solutions of the problem

$$\left[\min_{X \in D} \|CX - \bar{Z}\|_\lambda\right] \qquad (P_\lambda)$$

where Z is a goal point in the objective space such that $\bar{Z} \leqslant CX\ \forall X \in D$ and $\|CX - \bar{Z}\|_\lambda = \max_k \lambda_k |c_k - \bar{Z}_k|$ is the generalized Tchebytcheff norm, the vector λ being normalized.

Note that another characterization of E(P) is given by Burkard *et al.* [2] specifically for the case of binary variables.

These two characterizations are mainly of theoretical interest; they do not provide computational procedures to generate E(P) explicitly.

As in the case of MOLP with continuous variables, two different challenges must be considered: (1) the explicit determination of E(P) and (2) the design of interactive procedures which generate a good compromise, i.e. a satisfying efficient solution.

Most of the papers examined describe specific methods for a particular MOCO. Our study is limited to published papers to be found in the open literature up to 1991, but, of course, without any claim of exhaustivity. Technical reports have been disregarded. Only the basic ideas of each approach are given. Many relationships are made with methods developed for single-criterion combinatorial optimization problems and a knowledge of these methods is sometimes a prerequisite to the understanding of the present survey. Good references are the books [16, 5, 14, 13]. Some books focus on a particular problem, e.g. Lawler *et al.* [10] for the travelling salesman problem, Martello and Toth [11] for the knapsack problem and Merchandani and Francis [12] for the location problem.

2. PROBLEMS OF CLASS P

The majority of papers published in operational research journals and proceedings concerning MOCO are related to the assignment problem (AP), the transportation problem (TP) and the network flow problem (NFP).

For $p = 1$ it is well known that the assignment, transportation and network flow problems belong to the class P of problems [e.g. 14]. The main reason is that the optimal solution is also the optimal solution of the linear relaxation (LP) because of the *unimodularity property* (the constraint matrix A is totally unimodular and the right-hand-side vector b is integer [e.g. 5, 16]), so that the primal-dual approach is a very efficient one to solve this type of problem [e.g. 16].

The consequence in a multi-objective framework is that problem (P_α) can be replaced by its linear relaxation (LP_α) to provide the supported efficient solution set SE(P). Nevertheless—and this is fundamental for the understanding of this research field—in spite of the unimodularity property, it remains true that

$$E(\mathrm{P}) - SE(\mathrm{P}) \neq \varnothing$$

Even for this easy class of problems there generally exist non-supported efficient solutions. As an illustration, consider the following simple instance of an MOAP (see Section 2.1) given by White [34] with $n=3$ and $p=2$:

$$\min\ 3x_{11} + 2x_{12} + x_{13} + 2x_{21} + 3x_{22} + 100x_{23} + x_{31} + 100x_{32} + 3x_{33}$$
$$\min\ x_{11} + 4x_{12} + 3x_{13} + x_{21} + x_{22} + 100x_{23} + 3x_{31} + 100x_{32} + x_{33}$$

subject to

$$\sum_{j=1}^{3} x_{ij} = 1, \quad i=1,2,3, \qquad \sum_{i=1}^{3} x_{ij} = 1, \quad j=1,2,3$$
$$x_{ij} = (0,1), \quad (i,j) = 1,2,3$$

The two supported efficient solutions are

$$X^{(1)} \equiv \{x_{13}^{(1)} = x_{22}^{(1)} = x_{31}^{(1)} = 1\}, \qquad z(X^{(1)}) = (5,7)$$
$$X^{(2)} \equiv \{x_{11}^{(2)} = x_{22}^{(2)} = x_{33}^{(2)} = 1\}, \qquad z(X^{(2)}) = (9,3)$$

while the solution

$$X^{(3)} \equiv \{x_{12}^{(3)} = x_{21}^{(3)} = x_{33}^{(3)} = 1\}, \qquad z(X^{(3)}) = (7,6)$$

is also an efficient one but non-supported.

This shows that MOCO problems have their own specific difficulties.

Note that the location problem does not belong to the same class of problems but is nevertheless largely present in the existing literature, probably owing to the natural multi-objective context of this problem (see Section 2.4).

2.1 Multi-objective assignment and allocation problems

The paper of Charnes *et al.* [25] is the first to have drawn attention to the multi-objective extension of the assignment problem

$$\left[\begin{array}{lll} \text{'min'} z_k(X) = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^k x_{ij}, & k=1,\ldots,p & \\ \sum_{j=1}^{n} x_{ij} = 1, & i=1,\ldots,n & (1) \\ \sum_{i=1}^{n} x_{ij} = 1, & j=1,\ldots,n & (2) \\ x_{ij} = (0,1), & i,j=1,\ldots,n & (3) \end{array}\right] \qquad \text{(MOAP)}$$

We note four contributions to the analysis of this problem.

2.1.1. Malhotra et al. [30] (p=2)
The authors consider the bicriteria case and extensively use the two dual problems

$$\begin{cases} \max \sum_{i=1}^{n} u_i^{(k)} + \sum_{j=1}^{n} v_j^{(k)} \\ u_i^{(k)} + v_i^{(k)} \leqslant c_{ij}^{(k)}, \qquad k=1,2 \end{cases}$$

to generate the set of non-dominated pairs (z_1, z_2) in the objective space. An enumerative procedure is described to generate this set in the order of increasing values z_1. At each step the feasibility and infeasibility of the dual constraints are used to determine the admissible edge incident at the current basis; among the set of possible new bases, the one with the best value z_1 is selected; this basis corresponds to a new non-dominated pair $(z_1 z_2)$. Note that all the E(MOAP) set is obtained.

2.1.2. Gilbert et al.'s interactive approach [26]
If $X^{(m)}$ represents the best current compromise at the mth step, the calculation step consists of solving $p-1$ single-objective problems

$$\begin{cases} \min z_k(X) \\ z_{\bar{k}}(X) \leqslant z_{\bar{k}}(X^{(m)}) - \epsilon_k^{(m)} \\ z_l(X) \leqslant z_l(X^{(m)}), \qquad l=1,\ldots,p, \quad l \neq k, \bar{k} \end{cases}$$

for $k=1,\ldots,p$, $k \neq \bar{k}$, where $\bar{k}$ is the objective selected by the decision maker for improvement over its current level $z_{\bar{k}}(X^{(m)})$. The optimal solution of each of these problems is efficient (if they are unique; otherwise a selection must be made among the optimal solutions to obtain an efficient solution); the decision maker is then requested to choose the new best compromise $X^{(m+1)}$ among them.

2.1.3. Multi-objective assignment problem variations [34, 31, 32]
A theoretical paper of White [34] analyses a particular MOAP

$$\text{'min'}\ z_j(X) = \sum_{i=1}^{n} c_{ij} x_{ij}$$

subject to equations (1)–(3). It corresponds to the previous formalism by setting $c_{ij}^k = c_{ij}\delta_{jk}$, where

$$\delta_{jk} = \begin{cases} 1 & \text{if } j=k \\ 0 & \text{if } j \neq k \end{cases}$$

For this problem he proves that $E(\text{MOAP}) \equiv SE(\text{MOAP})$.

In [31] Murphy and Ignizio deal with a complex multi-objective quadratic assignment problem with mixed variables. A two-stage heuristic method is proposed: firstly, a penalty heuristic that builds a feasible solution, related to the more important objective; secondly, an exchange heuristic to incorporate the lower-priority design goals.

A hierarchical MOAP is described by Phillips [32]. Using some specific weights w_k, this particular problem can be formulated with the unique composite function

$$\sum_{k=1}^{p} \sum_{i=1}^{n} \sum_{j=1}^{n} w_k c_{ij}^k x_{ij}$$

Exploiting the particular structure of AP, the author yields acceptable weights which are generally smaller than those usually required in this approach for a general hierarchical multi-objective problem.

2.1.4. Applications [27, 28, 29, 35]
Goal-programming techniques are used in all these case studies (see also [24]). The general idea is thus, by excess and by default, to introduce the deviation variables d_k^+ and d_k^- with respect to a certain *a priori* goal g_k, so that goal constraints are obtained (see (*) below). If some priorities expressed by some weights p_k are given, this results in a single-objective problem defined by the global weighted deviation function:

$$\begin{array}{ll} \min & \sum_{k=1}^{p} p_k d_k^+ \\ \text{subject to} & \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}^k x_{ij} + d_k^- - d_k^+ = g_k \quad \forall k \qquad (*) \\ \text{and} & (1)\text{–}(3) \end{array}$$

2.2. The multi-objective transportation problem
This problem is a direct extension of the preceding one and is defined by

$$\left[\begin{array}{lll} \text{'min'} z_k(X) = \sum_{i=1}^{r}\sum_{j=1}^{s} c_{ij}^k x_{ij}, & k=1,\ldots,p & \\ \sum_{j=1}^{s} x_{ij} = a_i, & i=1,\ldots,r & (1) \\ \sum_{i=1}^{r} x_{ij} = b_j, & j=1,\ldots,s & (2) \\ x_{ij} \geqslant 0 \text{ and integer}, & \forall i, \forall j & (3) \end{array}\right] \qquad \text{(MOTP)}$$

(It is not restrictive to suppose equality constraints with $\sum_{i=1}^{r} a_i = \sum_{j=1}^{s} b_j$.)

2.2.1. Aneja and Nair's bicriteria method [36]
At each step of their iterative procedure they solve a single-objective transportation problem defined by the criterion

$$\min \sum_{i=1}^{r}\sum_{j=1}^{s} (\alpha_1 c_{ij}^1 + \alpha_2 c_{ij}^2) x_{ij}$$

where $\alpha_1 = z_2(X_r) - z_2(X_s)$ and $\alpha_2 = z_1(X_s) - z_1(X_r)$, X_r and X_s being two extreme (basic) supported efficient solutions. The algorithm is initiated with optimal efficient solutions for each objective separately.

If no new efficient solution is found, X_r and X_s are two adjacent extreme supported efficient solutions; otherwise, if a new efficient solution X_t is obtained, the pairs (X_r, X_t) and (X_t, X_s) are considered for further iterations. Only the supported efficient solutions can be found. Note that this method is very similar to the one of the same authors for a bicriteria shortest-path problem [21].

2.2.2. Enumerative method of Diaz [37]

The procedure is initiated with an efficient optimal solution for a single objective. At each step a non-basic variable of the current efficient solution, say x_{hl}, is selected and the efficiency of the adjacent extreme solution with this variable basic is checked. The following problem is considered:

$$\begin{cases} \max \sum_{k=1}^{p} y_k \\ \sum_{i=1}^{r}\sum_{j=1}^{s} d_{ij}^k x_{ij} - d_{hl}^k x_{hl} + y_k = 0, & k=1,\ldots,p \\ x_{ij} \geqslant 0 & \forall (i,j) \\ y_k \geqslant 0, & k=1,\ldots,p \end{cases}$$

where $d_{ij}^k = c_{ij}^k - u_i^k - v_j^k$ is the reduced cost. The adjacent extreme solution is efficient if this problem has a finite optimal solution. When all non-basic variables of each generated efficient solution have been checked, the procedure stops. Observe that this is an adaptation of the Steuer–Evans method for general MOLP [17] to the particular structure of MOTP. Only supported efficient solutions are obtained.

Remark. In [38] Diaz proposed a method to find a unique satisfying efficient solution by using two particular utility functions.

2.2.3. Enumerative method of Isermann [39]

The author applies his previous study concerning duality to MOLP.* If problem (MOTP) is written as a vector optimization problem, the following dual problem is introduced:

$$\begin{bmatrix} \text{'max'} \ \sum_{i=1}^{r} a_i U_i + \sum_{j=1}^{s} b_j V_j \\ (U_i, V_j) \in \Delta \end{bmatrix} \qquad \text{(D)}$$

where

$$\Delta = \{(U_i, V_j) \mid \nexists y_{ij} \neq 0 \ \forall (ij): \sum_{i=1}^{r}\sum_{j=1}^{s} (U_i + V_j - C_{ij}) y_{ij} \geqslant 0\}$$

with C_{ij} a $p \times 1$ vector of components c_{ij}^k $(k=1,\ldots,p)$ and U_i and V_j vectors of components u_i^k and v_j^k respectively.

The method is based on the following two results.

Theorem 1

A feasible solution $\bar{X}$ of (MOTP) is efficient iff $\exists(\bar{U}_i, \bar{V}_j) \in \Delta$ such that

$$\sum_{i=1}^{r}\sum_{j=1}^{s} C_{ij}\bar{x}_{ij} = \sum_{i=1}^{r} a_i \bar{U}_i + \sum_{j=1}^{s} b_j \bar{V}_j$$

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*Isermann, H., 'On some relation between dual pair of multiple objective linear programs', *Z. Oper. Res.*, **22**, 33–41 (1978).

Let $\mathscr{B}$ be the set of $r+s$ basic pairs (i,j) of a feasible basic solution $\bar{X}$. A solution $(\bar{U}_i, \bar{V}_j)$ of an $(r+s-1)$-vector-equation system is determined:

$$C_{ij} = \bar{U}_i + \bar{V}_j \quad \forall (i,j) \in \mathscr{B}$$

(setting e.g. $\bar{U}_1 = 0$).

Theorem 2
Let $D_{ij} = C_{ij} - \bar{U}_i - \bar{V}_j$; if the system

$$\sum_{i=1}^{r} \sum_{j=1}^{s} D_{ij} y_{ij} \leqslant 0$$

has a non-trivial solution, then $\bar{X}$ is efficient.

The procedure only generates supported efficient solutions.

Remark. Note that Isermann and Diaz both complete their analysis by the description of the efficient faces and edges of the non-dominated set in the continuous objective space.

2.2.4. Interactive methods of Ringuest and Rinks [*43*]
The two proposed algorithms have the same interactive phase: the decision maker (DM) must identify his most preferred solution in $S \subset E$(MOTP) (the initial set S is formed with efficient optimal solutions of each objective). Using this information, a new efficient set S is determined.

In the first algorithm S contains all extreme efficient solutions adjacent to the one identified by the DM (this is accomplished by Klingmann and Mote's method [49]).

In the second algorithm the unique new efficient solution optimizes a single weighted linear combination of the objectives (the weights are determined with the help of preference information given by the DM, as in Gonzalez *et al.*'s method [8]).

2.2.5. Miscellaneous papers [*40, 42, 44, 45, 46*]
These treat particular cases or describe applications, often by a goal-programming approach.

2.3. The multi-objective network flow or transhipment problem
Let $G(\mathscr{N}, \mathscr{A})$ be a network flow with a node set $\mathscr{N}$ and an arc set $\mathscr{A}$; the model can be stated as

$$\left[\begin{array}{ll} \text{'min'} z_k(X) = \sum_{(ij) \in \mathscr{A}} c_{ij}^k x_{ij}, & k = 1, \ldots, p \\ \sum_{j \in \mathscr{N} \setminus i} x_{ij} - \sum_{j \in \mathscr{N} \setminus i} x_{ji} = 0 & \forall i \in \mathscr{N} \\ l_{ij} \leqslant x_{ij} \leqslant u_{ij} & \forall (i,j) \in \mathscr{A} \\ x_{ij} \geqslant 0 \text{ and integer} & \end{array}\right] \qquad \text{(MNFP)}$$

where x_{ij} is the flow through arc (i,j), c_{ij}^k is the linear transhipment 'cost' for arc (i,j) in objective k and l_{ij} and u_{ij} are lower and upper bounds on x_{ij} respectively.

2.3.1. 'Out-of-kilter' method of Malhotra and Puri [51]
They consider the particular case $l_{ij}=0$ and $u_{ij}=U$ for all arcs (i,j), and then generalize the classic *out-of-kilter* method [e.g. 16] to the bicriteria framework. The *out-of-kilter* method for the single objective $z_l(X)$ with $l=1,2$ is based on the following three optimality conditions, one of which must be satisfied by every arc $(i,j)\in \mathscr{A}$ at the optimum (*in-kilter* status):

$$\begin{cases} \pi_j-\pi_i<c_{ij}^l, & x_{ij}=0 \qquad (\alpha_l)\\ \pi_j-\pi_i=c_{ij}^l, & 0\leqslant x_{ij}\leqslant U \qquad (\beta_l)\\ \pi_j-\pi_i>c_{ij}^l, & x_{ij}=U \qquad (\gamma_l)\end{cases}$$

where π_i is the dual variable associated with the ith primal constraint.

An arc $(i,j)\in \mathscr{A}$ is said to be *out-of-kilter* if it satisfies one of the two conditions below:

$$\begin{cases} \pi_j-\pi_i<c_{ij}^l, & x_{ij}>0 \qquad (\mathrm{a}_l)\\ \pi_j-\pi_i>c_{ij}^l, & x_{ij}<U \qquad (\mathrm{b}_l)\end{cases}$$

The first efficient solution $X^{(1)}$ of the iterative procedure is the optimal solution for criterion $z_1(X)$, giving the best value for criterion $z_2(X)$.

Initially all the arcs of $\mathscr{A}$ are thus in condition α_1,β_1 or γ_1. Consider the mth efficient solution $X^{(m)}$.

The set E_m of eligible arcs is composed of arcs with status (α_1,b_2), (β_1,a_2) or (β_1,b_2), (γ_1,a_2). Each arc in E_m is selected successively.

As in the single-criterion case, a flow-augmenting path is built including the selected eligible arc; a labelling procedure is applied to construct the path.

Each time a flow-augmenting path is obtained, the new solution is introduced in a list and this list is constructed to keep only the efficient solutions. If no flow-augmenting path is obtained, then the dual variables are changed as usual [e.g. 16]. The procedure stops when all arcs are in *in-kilter* status for objective 2.

Both non-supported and supported efficient solutions are generated.

2.3.2. The two-stage method of Pulat et al. [53] (p=2)
In a first step they generate all supported efficient solutions using the *unimodularity* property. The following parametric problem is solved by the *network simplex* method:

$$\min \sum_{(i,j)\in \mathscr{A}} (\alpha c_{ij}^{(1)}+(1-\alpha)c_{ij}^{(2)})x_{ij}$$

The various optimal solutions thus correspond to the integer points located on the efficient frontier of the convex hull of the non-dominated set in the objective space (see points X_1, X_2, X_3, X_4 and X_5 in Figure 1).

The second step consists of determining the non-supported efficient solutions, i.e. the possible integer points in the shaded region of Figure 1.

The authors prove that *'a non-supported efficient solution is a supported one for a modified problem, in which some bounds l_{ij} or u_{ij} are changed'*.

Thus if, for instance, $x_{ij}=l_{ij}$ in two adjacent supported efficient solutions, then $l_{ij}\leftarrow l_{ij}+1$.

Moreover, the authors propose a technique to generate the possible non-supported efficient solutions without solving a new network flow problem, but only using the information given by the reduced costs:

$$d_{ij}^k=c_{ij}^k-\pi_i-\pi_j, \quad k=1,2$$

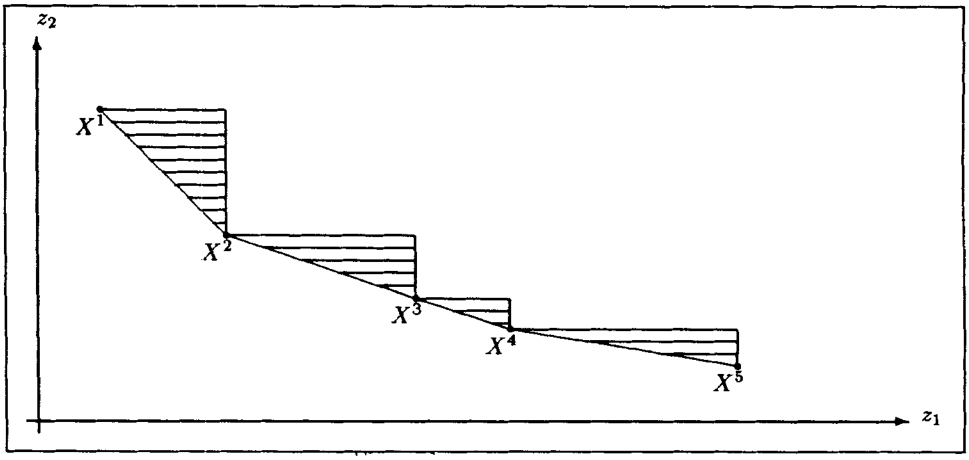

Figure 1.

2.3.3. *Complexity results and approximation of SE(P)*

Ruhe [54] obtains complexity results for a bicriteria (MNFP) by analysis of the pathological graph—introduced by Zadeh in 1973—with $2n+2$ vertices as shown in Figure 2 with $l_{ij}=0$, $\forall(i,j)$.

For this particular network he shows that there are 2^n supported efficient solutions, i.e. in a worst-case analysis the number of supported efficient solutions may be exponential in the number of nodes.

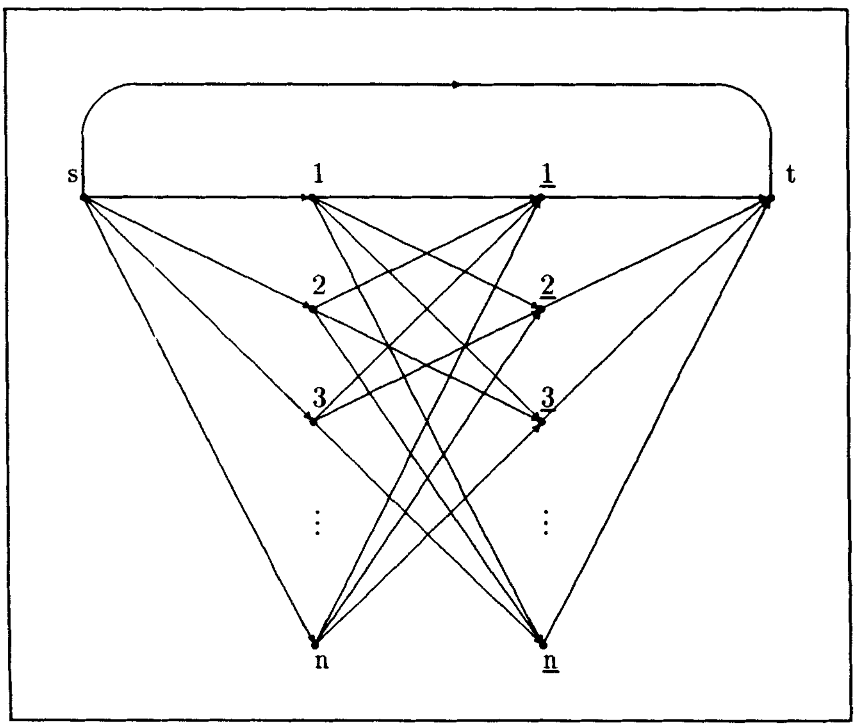

Figure 2.

For a bicriteria problem Fruhwirth *et al.* [48] proposed approximating the $SE(\mathrm{P})$ set using a so-called *sandwich algorithm*: this algorithm approximates a convex function f—in our case the piecewise linear function defined by the $SE(\mathrm{P})$ points—by two piecewise linear functions l and u which bound f from below and from above respectively.

2.3.4. Applications [47, 52, 50]
The first two problems are solved by a goal-programming approach, while the last is solved by an aggregation of criteria.

3. $\mathcal{NP}$-HARD PROBLEMS

The so-called classical combinatorial optimization problems have only received a very small amount of attention in a multi-objective framework. These problems (with a single objective) are $\mathcal{N}\mathcal{P}$-hard [10, 11, 12, 14, 13] and to our knowledge only a few papers exist on the *location problem* (LP), the *travelling salesman problem* (TSP), the *set-covering problem* (CP) and the *knapsack problem* (KP) in a multi-objective context. Moreover, it is often particular versions of the problem which are analysed.

3.1. The multi-objective location problem

We consider here the location problem modelled as a combinatorial optimization problem. We ignore multi-criteria decision-making problems in which it is necessary to choose between several potential sites characterized by their evaluations on different criteria. This problem is often called *multi-criteria warehouse location*. There exist a large number of applications of this type made by the so-called *French school*; some of them are mentioned in the book by Roy.* The approach used is there to construct an outranking relation on the set of potential sites. Green *et al.* [58], Eilon [57] and Lee and Luebbe [61] discuss the use of a goal-programming approach to solve this problem.

Neither do we consider papers which analyse, in a multi-objective framework, the so-called Weber problem [12], i.e. finding a centre in the continuous plane minimizing the sum of the weighted distances with n points. See also the interesting papers [56, 62, 64] which develop a geometrical approach to this problem.

One of the first papers to analyse the necessity of a multiple-objective approach for a location problem in the classical meaning used in combinatorial optimization—i.e. either the *min sum*, the so-called *median* problem, or the *min max*, the so-called *centre* problem—is the paper of Revelle *et al.* [65].

The multi-objective *min sum* location problem is generally formulated as

$$\left[\begin{array}{ll} \min \sum_{i=1}^{m} f_i^k z_i + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij}, & k=1,\ldots,p \\ \sum_{i=1}^{m} x_{ij} = 1, & j=1,\ldots,n \\ a_i z_i \leqslant \sum_{j=1}^{n} d_{ij} x_{ij} \leqslant b_i z_i, & i=1,\ldots,m \\ x_{ij} = (0,1) & \forall (i,j) \\ z_i = (0,1) & \forall i \end{array}\right] \qquad \text{(MLOP)}$$

*Roy, B., *Méthodologie Multicritere d'Aide à la Décision*, Paris: Economica, 1985.

where $z_i = 1$ if a facility is established at site i; $x_{ij} = 1$ if customer j is assigned to the facility at site i; d_{ij} is a certain usage of the facility at site i by customer j if he is assigned to that facility; a_i and b_i are possible limitations on the total customer usage permitted at the facility i; c_{ij}^k is, for objective k, a variable cost—or distance, etc.—if customer j is assigned to facility i; f_i^k is, for objective k, a fixed cost associated with the facility at site i.

Possibly the number of facilities may be fixed in advance, $\sum_{i=1}^{m} z_i = p$.

3.1.1. *The review of Revelle et al.* [66]

This review is focused on structuring objectives for these problems rather than on developing solution algorithms. The authors analyse a large number of possible criteria: population travel burden, population coverage, number of facilities, transport costs, profits, etc.

3.1.2. *Method of Ross and Soland* [67]; *computer system of Hultz et al.* [59]

For these, all the previous objectives can be formulated in the same manner. Thus, using Soland's characterization of the efficient solutions [e.g. 15], they have to solve the single-criterion problem

$$\left[\begin{array}{ll} \min \sum_{k=1}^{p} \alpha_k \left(\sum_{i=1}^{m} f_i^k z_i + \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^k x_{ij} \right) & \\ \sum_{i=1}^{m} x_{ij} = 1, & j = 1, \ldots, n \\ a_i z_i \leqslant \sum_{j=1}^{n} d_{ij} x_{ij} \leqslant b_i z_i, & i = 1, \ldots, m \\ \sum_{i=1}^{m} f_i^k z_i + \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^k x_{ij} \leqslant \lambda_k, & k = 1, \ldots, p \quad (*) \\ x_{ij} = (0,1), & \forall (i,j) \\ z_i = (0,1) & \forall i \end{array}\right] \qquad (\mathrm{P}_{\alpha,\lambda})$$

It is well known [12] that a location problem can be modelled, by addition of variables and modification of the writing of the constraints, like a *generalized assignment problem* (GAP). This is still true with the additional constraint (*). Thus an efficient solution of the multi-objective location problem can be generated by solving a GAP.

To help the DM, the authors propose an interactive approach. The procedure is a *satisfying* one in that it uses λ as a vector of satisfaction levels which define upper limits on acceptable values of the p objective functions. After choosing a level of satisfaction λ_k and specifying the relative importance α_k of each criterion function k, the DM obtains an efficient solution.

Thus a sequence of efficient solutions is generated through an iterative process alternating between the DM changing weights α and/or satisfying level λ and an optimization routine solving the resultant problem $(\mathrm{P}_{\alpha,\lambda})$ modelled as a GAP.

In [59] a *Lagrangian* relaxation technique is used to solve the previous GAP. The results of an instance with 455 allocation variables x_{ij} and 22 location variables z_i are described.

3.1.3. *Applications*

Again the most common technique to solve multi-objective location applications is the goal-programming approach [55, 60, 68].

Note two interesting exceptions. In [69] a bi-objective problem is treated by a single-objective location model in which the second objective is parametrized in the constraint set; a parametric analysis is performed to propose a set of efficient solutions to the DM. In [63] a very similar approach results in mixed integer mathematical programming.

3.2. The multi-objective travelling salesman problem

The problem can be stated as

$$\begin{bmatrix} \text{`min'} \sum_{\rho} c^{(k)}_{i\rho(i)}, & k=1,\ldots,p \\ \rho \text{ is a tour of } \{1,\ldots,n\} & \end{bmatrix} \qquad \text{(MOTSP)}$$

3.2.1. Particular bicriteria problem of Fisher and Richter [70]

A set of n products is given and a cost optimal sequence of production is wanted (objective 1). The reliability of a given sequence consists of the probability of manufacturing without failure and losses on a machine. The reliability of a sequence can be computed by multiplying the reliability of all changes from one product to another (objective 2). The mathematical problem formulation is

$$\begin{cases} \min \sum_{\rho} c^{(1)}_{i\rho(i)} \\ \max \prod_{\rho} c^{(2)}_{i\rho(i)} \quad \text{with } 0 \leqslant c^{(2)}_{i\rho(i)} \leqslant 1 \\ \rho \text{ is a tour of } \{1,\ldots,n\} \end{cases}$$

The authors apply a dynamic programming approach to determine the set of efficient tours.

3.2.2. An aggregation approach of Gupta and Warburton [71]

To solve the MOTSP, they first use a *Tchebytcheff* aggregation procedure $\Rightarrow$ *min* $\max_k \sum_{\rho} c^k_{(i\rho(i))}$; then they apply the Held and Karp procedure [10] to determine the optimal solution of this problem and finally they propose the well-known Lin and Kernighan heuristic [10] to construct a satisfying tour.

3.2.3. The multi-objective vending problem of Keller and Goodchild [72]

The classical TSP is modified in the following sense: each node has an associated reward R_i; it is not necessary to visit all the nodes, but the total reward collected by visiting as many nodes as possible must be maximized. Thus the problem is

$$\begin{cases} \min \sum_{\rho} c_{i\rho(i)} \\ \max \sum_{\rho} R_{\rho(i)} \\ \rho \text{ is a subtour of } K \cup \{O\} \end{cases}$$

where K is any subset of $\{1,\ldots,n\}$ and O is a particular node corresponding to a depot from which the subtour is initiated.

The method consists of introducing one objective in the constraint in the following way:

(1) Solve a classical single-criterion TSP $\Rightarrow \tilde{z}$ the optimal value.
(2) Define an integer value L.
(3) Examine the following constraint by increasing value l up to L:

$$i \in K \quad \text{if } c_{iO} + c_{Oi} < (1/l)\tilde{z}$$

Thus a subset of nodes is examined at each iteration and an exchange heuristic is defined to find a good solution inside this subset of nodes.

3.3. The multi-objective set-covering problem

The mathematical formulation of the problem is

$$\begin{bmatrix} \text{'min' } z_k = \sum_{j=1}^{n} c_j^{(k)} x_j, & k=1,\ldots,p & \\ \sum_{j=1}^{n} t_{ij} x_j \geqslant 1, & i=1,\ldots,m & (*) \\ x_j = (0,1), & j=1,\ldots,n & \end{bmatrix} \qquad \text{(MOCP)}$$

where

$$t_{ij} = \begin{cases} 1 & \text{if sink } i \text{ is covered by source } j \\ 0 & \text{otherwise} \end{cases}$$

$$c_j^k > 0 \quad \forall (k,j)$$

3.3.1 Particular problem of Harnett and Ignizio [75]

Here all the sinks cannot always be covered because of the limited number L of sources in the solution: thus constraints (*) are replaced by

$$\sum_j x_j \leqslant L$$

The two criteria considered are to minimize the associated cost to the sources j and to maximize the number of covered sinks, i.e.

$$\begin{cases} \min z_1(X) = \sum_{j=1}^{n} c_j x_j \\ \max z_2(X) = \sum_{i=1}^{m} \min\left(\sum_{j=1}^{n} t_{ij} x_j,\ 1\right) \end{cases}$$

The authors propose a heuristic method maximizing the ratio $z_2(X)/z_1(X)$.

3.3.2. Daskin and Stern [72] *model for emergency medical service vehicle deployment*

Let i and j represent n zones ($i, j = 1, \ldots, n = m$):

(1) $t_{ij} = 1$ if the expected response time for an ambulance in j to respond to a demand in i is inferior to a prespecified upper bound T;
(2) $x_j = 1$ if an ambulance is located in zone j;
(3) $c_j = 1$, $\forall j$.

The single-objective set-covering problem means that all zones must be covered by at least one vehicle. Nevertheless, it appears necessary to also maximize the number of ambulances that can respond to calls in each zone i within time T, i.e.

$$\max z_2(X) = \sum_{i=1}^{n} s_i$$

where $s_i = \sum_j (t_{ij} x_j - 1)$ represents the additional vehicles capable of responding to a call in zone i in a time inferior to T.

A simple weighted aggregation function is introduced:

$$\min \lambda z_1(X) - z_2(X)$$

and the optimal solution of the corresponding single-objective problem is analysed as a function of the parameter λ.

3.3.3. The maximum-covering/shortest-path problem of Current et al. [*73*]

This problem is more related to the multi-objective shortest-path problem [21] and is solved by Cohon's approach.*

3.4. The multi-objective knapsack problem

This problem is characterized by its unique constraint and can be formulated as

$$\begin{bmatrix} \text{'max'} \sum_{j=1}^{n} c_j^{(k)} x_j, & k = 1, \ldots, p \\ \sum_{j=1}^{n} w_j x_j \leqslant W & (1) \\ x_j = (0,1), & (2) \end{bmatrix} \qquad \text{(MOKP)}$$

Several contexts are possible for application of this useful model [11].

3.4.1. Media selection problem of Dyer et al. [*76*]

In this interpretation the most effective magazines j must be selected subject to the constraint W on the advertising budget; the coefficients c_j^k are defined by the percentage p_j^k of the number N_j of readers of magazine j possessing a desired target characteristic k (income level, education level, age, etc.), i.e.

$$c_j^k = p_j^k N_j$$

The authors propose in a first step to apply the well-known AHP method of Saaty† to determine a global effectiveness coefficient e_j for each magazine j from the p_j^k. In the second step they obtain a unique efficient solution by solving the single-objective knapsack problem

$$\begin{cases} \max \sum_{j=1}^{n} e_j N_j x_j \\ \text{subject to (1) and (2)} \end{cases}$$

*Cohon, J. L., *Multiobjective Programming and Planning*, New York: Academic, 1978.

†Saaty, T. L., *The Analytic Hierarchy Process*, New York: McGraw Hill, 1980.

They obtained a unique efficient solution.

3.4.2. Bicriteria capital-budgeting problem of Rosenblatt and Sinuany-Stern [77]
Let p_j and v_j be respectively the expected present value and the variance of project j; let w_j be the expected cash outflow of project j and W be the available budget. A bicriteria problem can be formulated as

$$\begin{cases} \min z_1(X) = \sum_{j=1}^{n} p_j x_j \\ \max z_2(X) = \sum_{j=1}^{n} v_j x_j \\ \text{subject to (1) and (2)} \end{cases}$$

The authors introduce a weighted aggregating function

$$\min [\alpha z_1(X) - (1-\alpha) z_2(X)]$$

and propose an implicit enumeration procedure to determine the supported efficient solutions.

4. COMPARATIVE ANALYSIS AND DIRECTIONS OF FURTHER RESEARCH

4.1 Comparative analysis

4.1.1
Compared with the vast literature on single-objective combinatorial optimization, MOCO appears to have been substantially ignored. Two reasons probably explain this.

(a) The 'multi-objective paradigm'* is not yet really implemented among the circle of research workers in combinatorial optimization, who are still almost totally influenced by the classical single-objective optimization approach. The large majority of combinatorial optimization problems are '$\mathcal{NP}$-hard', so the development of more efficient techniques is certainly an important, lively and much needed research area. Yet we believe that this is no reason to ignore the issue of multi-objective combinatorial optimization problems.

(b) MOCO presents new inherent difficulties which are not easy to tackle.

(1) It is difficult—often seemingly impossible—to obtain theoretical properties characterizing the efficient solutions of MOCO problems.
(2) Moreover, the set of these solutions is so large—and, of course, not continuous—that it is hard to generate completely, except perhaps for bicriteria cases of moderate size.
(3) Interactive approaches are not easy to develop. Effectively, to take into account the preferences of the DM—expressing, for instance, the relaxation or the improvement of an objective—it is necessary to introduce a supplementary constraint, which often induces the loss of the particular structure of the initial combinatorial optimization problem.

Nevertheless, independently of these reasons, we have been surprised by the poor literature on this subject.

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*Roy, B., *Méthodologie Multicritere d'Aide à la Décision*, Paris: Economica, 1985.

Table I. Summary

	Type of model				Type of method						Type of solution		
MOCO	Gen.	Bicr.	Part.	Appl.	Aggr.	Hier.	GP	Inter.	Heur.	Spec.	*SE*(P)	*E*(P)	Un.Ss.
MOAP													
[26]			×	×				×		×			×
[27]			×	×			×						×
[28]			×	×			×						×
[29]			×	×			×						×
[30]		×								×		×	
[31]			×						×				×
[32]		×			×	×							×
[34]			×							×		×	
[35]			×	×			×						×
MOTP													
[36]		×			×						×		
[37]	×									×	×		
[38]	×				×								×
[39]	×									×	×		
[40]			×	×			×						×
[41]			×	×			×						×
[42]	×			×	×	×							×
[43]	×							×					×
[44]		×	×		×								×
[45]		×		×			×		×				×
[46]			×	×			×						×
MONFP													
[47]			×	×			×						×
[48]		×							×	×	×		
[49]	×									×	×	×	
[50]			×	×		×	×						×
[51]		×								×		×	
[52]			×	×			×						×
[53]		×			×					×		×	
[54]		×	×										
MOLOP													
[55]				×		×	×						×
[59, 67]	×				×			×		×			×
[60]				×			×						×
[63]		×	×	×						×			
[68]				×			×						×
[69]		×		×						×		×	
MOTSP													
[70]		×	×							×		×	
[71]	×				×				×				×
[72]		×	×						×	×			×
MOSCP													
[73]		×	×							×			×
[74]		×	×	×	×	×			×				×
[75]		×	×		×				×				×
MOKP													
[76]	×			×	×					×			×
[77]		×			×				×		×		

Key: Aggr., aggregation; Appl., application; Bicr., bicriteria; Gen., general; GP, goal programming; Heur., heuristics; Hier., hierarchical; Inter., interactive; Part., particular; Spec., specific; Un.Ss., unique solution or subset of solutions.

4.1.2
The comparative analysis contained in Table I specifies some characteristics of the papers found in the literature (see Table I: $E(\mathrm{P})$ versus $SE(\mathrm{P})$; type of method; type of model) and emphasizes either several connections between them or the broad outline in the field.

In particular we note the following.

(1) Among the studies devoted to the generation of the efficient set, only bicriteria problems are examined and these procedures are in our opinion only applicable for small problems; moreover, often the non-supported efficient solutions are avoided. (Some authors seem to ignore that even if the total unimodularity property is verified, the set $E(\mathrm{P}) - SE(\mathrm{P})$ is generally not empty).
(2) Among the more interesting papers, many are focused entirely on a particular problem [e.g. 30, 51, 55, 67, 69]. We should emphasize some exceptions [e.g. 36, 53, 63] in which some principles of resolution can possibly be applied to other problems.
(3) It is surprising that very few papers—even for applications—discuss interactive approaches, which have been found to be very successful in the case of continuous MOLP problems.
(4) Most applications described in the MOCO literature are treated by goal-programming techniques or sometimes by hierarchical treatment of the objectives, which can certainly be useful in particular cases—especially if used in an interactive procedure—but which also appear to be limited in the multicriteria context.
(5) Development of heuristics is an important subfield of single-objective combinatorial optimization problems. Clearly this is not the case for MOCO: heuristics are rare and only developed for very particular models.

4.2. Directions of further research
Nevertheless, MOCO is a promising field for applications: in fact, there is no rational reason why MOCO would not be a useful tool in many cases where combinatorial optimization and multiple-objective optimization separately are so widely used in practice.

Clearly the results of the present survey suggest that much important research remains if we are to accept the challenge of MOCO. In our opinion further research must be oriented principally in three directions.

4.2.1. A 'theoretical' direction
To investigate whether some specific characterizations of efficient solutions exist for some MOCO models: examples of this type of research are the papers [34, 56].

Nevertheless, on the basis of our experience such theoretical properties are unlikely to be found for a large class of problems: only very specific models can be analysed in this way.

4.2.2. A 'methodological' direction
To adapt some existing methods of combinatorial optimization to the multicriteria case: good examples are the papers [30, 51, 53, 69] and, for supported efficient solutions [36, 39, 67].

Clearly some important progress can still be made in this direction, but in any case it appears that the possible methods would be efficient only for the bicriteria case and for small-dimensional problems.

4.2.3. A 'practical' direction
To design general heuristic procedures to generate a representative subset of efficient solutions or to determine interactively a satisfying compromise for the decision maker.

These heuristics can, of course, be based on the particular structure of the considered combinatorial optimization problem. Yet a major drawback will be that such procedures will be much too specific and often inapplicable if some particular elements perturb the classical model, as is common in combinatorial optimization.

We think that heuristics would be more successful if they were flexible enough to be adapted to a large set of applications.

Metaheuristics such as *simulated annealing* [23], *tabu search* [6] and the *genetic algorithm* [7] promise a new efficient way to solve combinatorial optimization models independently of their mathematical structure and without requiring too much effort in implementation; they generate excellent solutions in a very short time. Thus in our opinion the adaptation of these metaheuristics to a multi-objective environment is certainly one of the more promising research directions.

4.3. Conclusions

Consequently, we have oriented our own research in the two directions 4.2.2 and 4.2.3.

(a) We have recently developed [33] three new methods to enumerate E(MOAP) in the bicriteria assignment problem:

(1) the first can be seen as an improvement of the principle described in [30];
(2) the second is a particularization of the Gonzales *et al.* [8] method to the MOAP case;
(3) the third is an enumerative branch-and-bound technique.

These methods can be easily adapted to obtain interactive approaches and extended without too much effort to the MOTP problem.

(b) We are intensively working [22] on the use of metaheuristics to generate an approximation of E(P), but also to develop an interactive method integrating regularly the preferences of the DM.

First experiments implemented for the MOKP appear very promising: large instances can be treated and this approach seems easy to adapt to any MOCO problem.

In conclusion, we hope that this survey will be—as it was for us—a basis for and an incitement to further successful research work in this exciting field.

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