CHAPTER 9

Filters

A *filter* is a circuit that is capable of passing a specific range of frequencies while blocking other frequencies. As you discovered in Chap. 2, the four major types of filters include *low-pass filters*, *high-pass filters*, *bandpass filters*, and *notch filters* (or *band-reject filters*). A low-pass filter passes low-frequency components of an input signal, while a high-pass filter passes high-frequency components. A bandpass filter passes a narrow range of frequencies centered around the filter's resonant frequency, while a notch filter passes all frequencies except those within a narrow band centered around the filter's resonant frequency.

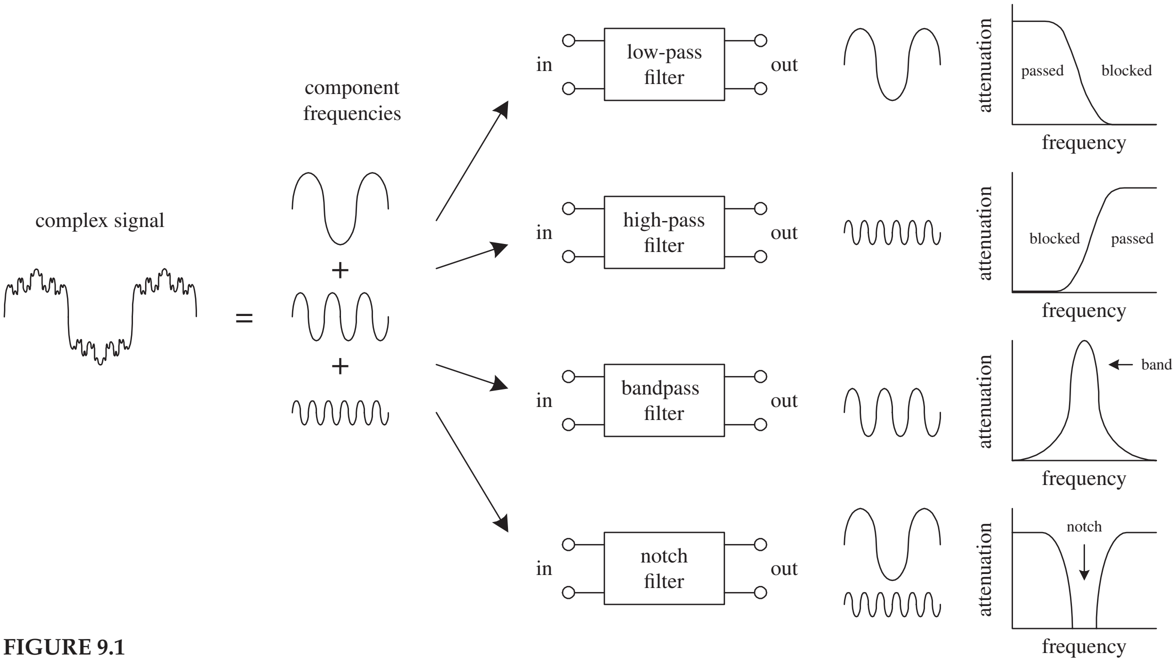

FIGURE 9.1

Filters have many practical applications in electronics. For example, within a dc power supply, filters can be used to eliminate unwanted high-frequency noise present within the ac line voltage, and they act to flatten out pulsing dc voltages generated by the supply's rectifier section. In radio communications, filters make it possible for a radio receiver to provide the listener with only the desired signal while rejecting all others. Likewise, filters allow a radio transmitter to generate only one signal while attenuating other signals that might interfere with different radio transmitters' signals. In audio electronics, filter networks called *crossover networks* are used to divert low audio signals to woofers, middle-range frequencies to midrange speakers, and high frequencies to tweeters. A high-pass filter is often used to eliminate 60 Hz mains hum from audio circuits. The list of filter applications is extensive.

There are two filter types covered in this chapter, namely, *passive filters* and *active filters.* Passive filters are designed using passive elements (e.g., resistors, capacitors, and inductors) and are most responsive to frequencies between around 100 Hz and 300 MHz. (The lower frequency limit results from the fact that at low frequencies the capacitance and inductance values become exceedingly large, meaning prohibitively large components are needed. The upper frequency limit results from the fact that at high frequencies parasitic capacitances and inductances wreak havoc.) When designing passive filters with very steep attenuation falloff responses, the number of inductor and capacitor sections increases. As more sections are added to get the desired response, greater is the chance for signal loss to occur. Also, source and load impedances must be taken into consideration when designing passive filters.

Active filters, unlike passive filters, are constructed from op amps, resistors, and capacitors—no inductors are needed. Active filters are capable of handling very low frequency signals (approaching 0 Hz), and they can provide voltage gain if needed (unlike passive filters). Active filters can be designed to offer comparable performance to *LC* filters, and they are typically easier to make, less finicky, and can be designed without the need for large-sized components. Also, with active filters, a desired input and output impedance can be provided that is independent of frequency. One major drawback with active filters is a relatively limited high-frequency range. Above around 100 kHz or so, active filters can become unreliable (a result of the op amp's bandwidth and slew-rate requirements). At radiofrequencies, it is best to use a passive filter.

9.1 Things to Know Before You Start Designing Filters

When describing how a filter behaves, a response curve is used, which is simply an attenuation (V_{out}/V_{in}) versus frequency graph (see Fig. 9.2). As you discovered in Chap. 2, attenuation is often expressed in decibels (dB), while frequency may be expressed in either angular form ω (expressed in rad/s) or conventional form f (expressed in Hz). The two forms are related by $\omega = 2\pi f$. Filter response curves may be plotted on linear-linear, log-linear, or log-log paper. In the case of log-linear graphs, the attenuation need not be specified in decibels.

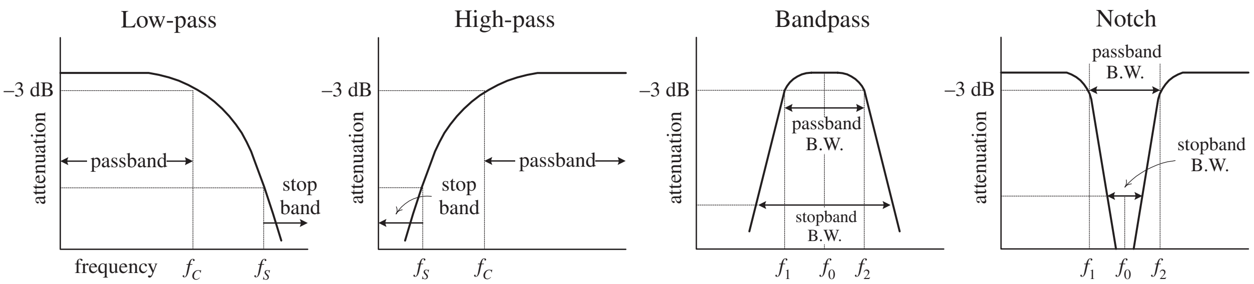

FIGURE 9.2

Here are some terms that are commonly used when describing filter response:

–3-dB Frequency (f_{3dB}). This represents the input frequency that causes the output signal to drop to –3 dB relative to the input signal. The –3-dB frequency is equivalent to the cutoff frequency—the point where the input-to-output power is reduced by one-half or the point where the input-to-output voltage is reduced by $1/\sqrt{2}$. For low-pass and high-pass filters, there is only one –3-dB frequency. However, for bandpass and notch filters, there are two –3-dB frequencies, typically referred to as f_1 and f_2.

Center frequency (f_0). On a linear-log graph, bandpass filters are geometrically symmetrical around the filter's resonant frequency or center frequency—provided the response is plotted on linear-log graph paper (the logarithmic axis representing the frequency). On linear-log paper, the central frequency is related to the –3-dB frequencies by the following expression:

$$f_0 = \sqrt{f_1 f_2}$$

For narrow-band bandpass filters, where the ratio of f_2 to f_1 is less than 1.1, the response shape approaches arithmetic symmetry. In this case, we can approximate f_0 by taking the average of –3-dB frequencies:

$$f_0 = \frac{f_1 + f_2}{2}$$

Passband. This represents those frequency signals that reach the output with no more than –3 dB worth of attenuation.

Stop-band frequency (f_s). This is a specific frequency where the attenuation reaches a specified value set by the designer. For low-pass and high-pass filters, the frequencies beyond the stop-band frequency are referred to as the *stop band.* For bandpass and notch filters, there are two stop-band frequencies, and the frequencies between the stop bands are also collectively called the *stop band.*

Quality factor (Q). This represents the ratio of the center frequency of a bandpass filter to the –3-dB bandwidth (distance between –3-dB points f_1 and f_2):

$$Q = \frac{f_0}{f_2 - f_1}$$

For a notch filter, use $Q = (f_2 - f_1)/f_0$, where f_0 is often referred to as the *null frequency.*

9.2 Basic Filters

In Chap. 2 you discovered that by using the reactive properties of capacitors and inductors, along with the resonant behavior of *LC* series and parallel networks, you could create simple low-pass, high-pass, bandpass, and notch filters. Here's a quick look at the basic filters covered in Chap. 2:

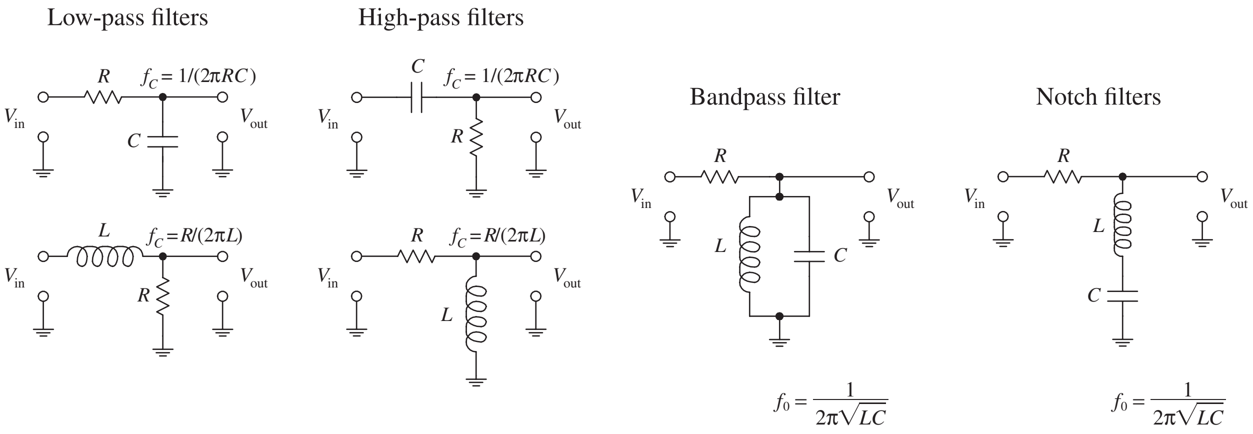

FIGURE 9.3

Now, all the filters shown in this figure have a common limiting characteristic, namely, a shallow 6-dB per octave falloff response beyond the –3-dB point(s). (You can prove this to yourself by going back to Chap. 2 and fiddling with the equations.) In certain noncritical applications, a 6-dB per octave falloff works fine, especially in cases where the signals you want to remove are set well beyond the –3-dB point. However, in situations where greater frequency selectivity is needed (e.g., steeper falloffs and flatter passbands), 6-dB per octave filters will not work. What is needed is a new way to design filters.

Making Filters with Sharper Falloff and Flatter Passband Responses

One approach used for getting a sharper falloff would be to combine a number of 6-dB per octave filters together. Each new section would act to filter the output of the preceding section. However, connecting one filter with another for the purpose of increasing the "dB per octave" slope is not as easy as it seems and in fact becomes impractical in certain instances (e.g., narrow-band bandpass filter design). For example, you have to contend with transient responses, phase-shift problems, signal degradation, winding capacitances, internal resistances, magnetic noise pickup, etc. Things can get nasty.

To keep things practical, what we will do is skip the hard-core filter theory (which can indeed get very nasty) and simply apply some design tricks that use basic response graphs and filter design tables. To truly understand the finer points of filter theory is by no means trivial. If you want in-depth coverage of filter theory, refer to a filter design handbook. (A comprehensive handbook written by Zverck covers almost everything you would want to know about filters.)

Let's begin by jumping straight into some practical filter design examples that require varying degrees of falloff response beyond 6 dB per octave. As you go through these examples, important new concepts will surface. First, we will discuss passive filters and then move on to active filters.

9.3 Passive Low-Pass Filter Design

Suppose that you want to design a low-pass filter that has a $f_{3dB} = 3000$ Hz (attenuation is –3 dB at 3000 Hz) and an attenuation of –25 dB at a frequency of 9000 Hz—which will be called the *stop frequency* f_s. Also, let's assume that both the signal-source impedance R_s and the load impedance R_L are equal to 50 Ω. How do you design the filter?

Step 1 (Normalization)

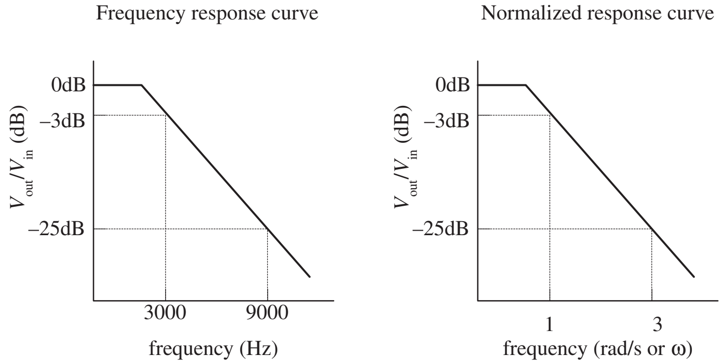

FIGURE 9.4

First, make a rough attenuation versus frequency graph to give yourself a general idea of what the response looks like (see far-left figure). Next, you must normalize the graph. This means that you set the –3-dB frequency f_{3dB} to 1 rad/s. The figure to the near left shows the normalized graph. (The reason for normalizing becomes important later on when you start applying design tricks that use normalized response curves and tables.) In order to determine the normalized stop frequency, simply use the following relation, which is also referred to as the *steepness factor:*

$$A_s = \frac{f_s}{f_{3dB}} = \frac{9000\text{ Hz}}{3000\text{ Hz}} = 3$$

This expression tells you that the normalized stop frequency is three times larger than the normalized –3-dB point of 1 rad/s. Therefore, the normalized stop frequency is 3 rad/s.

Step 2 (Pick Response Curve)

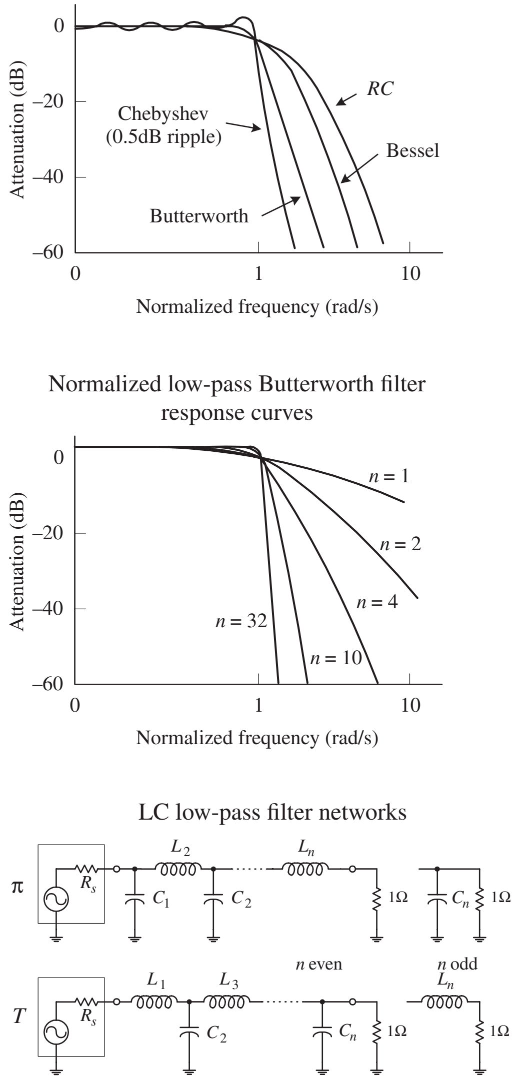

FIGURE 9.5

Next, you must pick a filter response type. Three of the major kinds to choose from include the Butterworth, Chebyshev, and Bessel. Without getting too technical here, what is going on is this: Butterworth, Chebyshev, and Bessel response curves are named after individuals who were able to model *LC* filter networks after a mathematical function called the *transfer function,* given here:

$$T(S) = \frac{V_{out}}{V_{in}} = \frac{N_m S^m + N_{m-1} S^{m-1} + \cdots + N_1 S + N_0}{D_n S^n + D_{n-1} S^{n-1} + \cdots + D_1 S + D_0}$$

The *N*'s in the equation are the numerator's coefficients, the *D*'s are the denominator's coefficients, and $S = j\omega$ ($j = \sqrt{-1}$, $\omega = 2\pi f$). The highest power *n* in the denominator is referred to as the order of the filter or the number of poles. The highest power *m* in the numerator is referred to as the number of zeros. Now, by manipulating this function, individuals (e.g., Butterworth, Chebyshev, and Bessel) were able to generate unique graphs of the transfer function that resembled the attenuation response curves of cascaded *LC* filter networks. What is important to know, for practical purposes, is that the number of poles within the transfer function correlates with the number of *LC* sections present within the cascaded filter network and determines the overall steepness of the response curve (the decibels per octave). As the number of poles increases (number of *LC* sections increases), the falloff response becomes steeper. The coefficients of the transfer function influence the overall shape of the response curve and correlate with the specific capacitor and inductor values found within the filter network. Butterworth, Chebyshev, and Bessel came up with their own transfer functions and figured out what values to place in the coefficients and how to influence the slope of the falloff by manipulating the order of the transfer function. Butterworth figured out a way to manipulate the function to give a maximally flat passband response at the expense of steepness in the transition region between the passband and the stop band. Chebyshev figured out a way to get a very steep transition between the passband and stop band at the expense of ripples present in the passband, while Bessel figured out a way to minimize phase shifts at the expense of both flat passbands and steep falloffs. Later we will discuss the pros and cons of Butterworth, Chebyshev, and Bessell filters. For now, however, let's concentrate on Butterworth filters.

Step 3 (Determine the Number of Poles Needed)

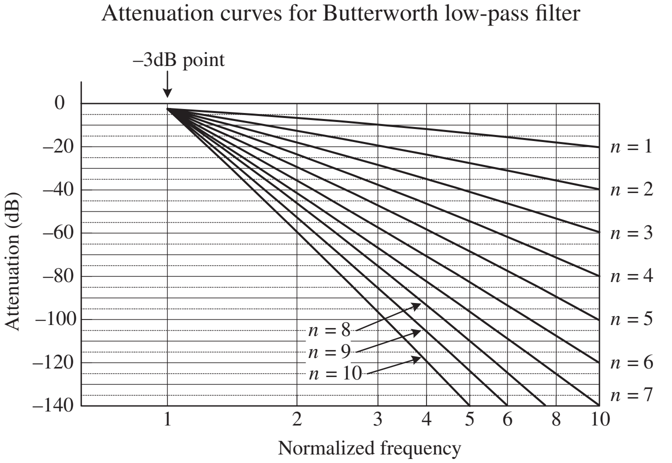

FIGURE 9.6

Continuing on with our low-pass filter problem, let's choose the Butterworth design approach, since it is one of the more popular designs used. The next step is to use a graph of attenuation versus normalized frequency curves for Butterworth low-pass filters, shown in the figure. (Response curves like this are provided in filter handbooks, along with response curves for Chebyshev and Bessel filters.) Next, pick out the single response curve from the graph that provides the desired –25 dB at 3 rad/s, as stated in the problem. If you move your finger along the curves, you will find that the $n = 3$ curve provides sufficient attenuation at 3 rad/s. Now, the filter that is needed will be a third-order low-pass filter, since there are three poles. This means that the actual filter that you will construct will have three *LC* sections.

Step 4 (Create a Normalized Filter)

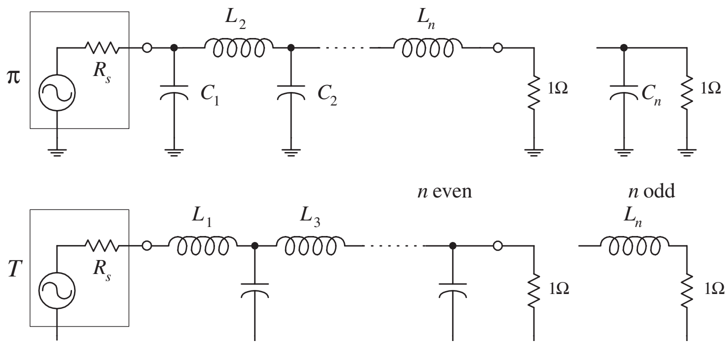

FIGURE 9.7

Now that you have determined the order of the filter, move on to the next step—creating a normalized *LC* filter circuit. (This circuit will not be the final filter circuit you will use—it will need to be altered.) The circuit networks that are used in this step take on either a π or the *T* configuration, as shown in the figure. If the source and load impedances match, either configuration can be used—though a π network is more attractive because fewer inductors are needed. However, if the load impedance is greater than the source impedance, it is better to use *T* configuration. If the load impedance is smaller than the source impedance, it is better to use the π configuration. Since the initial problem stated that the source and load impedances were both 50 Ω, choose the π configuration. The values of the inductors and capacitors are given in Table 9.1. (Filter handbooks will provide such tables, along with tables for Chebyshev and Bessel filters.) Since you need a third-order filter, use the values listed in the $n = 3$ row. The normalized filter circuit you get in this case is shown in Fig. 9.8.

TABLE 9.1 Butterworth Active Filter Low-Pass Values

π {T} n	R_S {1/R_S}	C_1 {L_1}	L_2 {C_2}	C_3 {L_3}	L_4 {C_4}	C_5 {L_5}	L_6 {C_6}	C_7 {L_7}
2	1.000	1.4142	1.4142					
3	1.000	1.0000	2.0000	1.0000				
4	1.000	0.7654	1.8478	1.8478	0.7654			
5	1.000	0.6180	1.6180	2.0000	1.6180	0.6180		
6	1.000	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176	
7	1.000	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450

Note: Values of L_n and C_n are for a 1-Ω load and –3-dB frequency of 1 rad/s and have units of H and F. These values must be scaled down. See text.

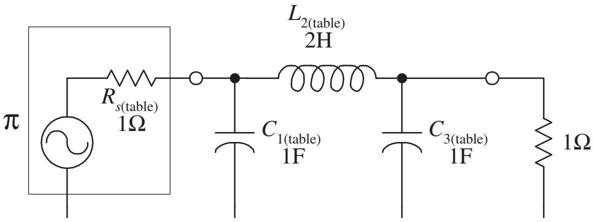

FIGURE 9.8

As mentioned a moment ago, this circuit is not the final circuit that we'll use. That is, the component values listed here will not work! This is so because the graphs and tables you used to get to this point used the normalized frequency within them. Also, you haven't considered the effects of the source and load impedances. In order to construct the final working circuit, you must frequency and impedance scale the component values listed in the circuit in Fig. 9.8. This leads us to the next step.

Step 5 (Frequency and Impedance Scaling)

$$L_{2(\text{actual})} = \frac{R_L L_{2(\text{table})}}{2\pi f_{3\text{dB}}} = \frac{(50\Omega)(2\text{ H})}{2\pi(3000\text{ Hz})} = 5.3\text{ mH}$$

$$C_{1(\text{actual})} = \frac{C_{1(\text{table})}}{2\pi f_{3\text{dB}} R_L} = \frac{1\text{ F}}{2\pi(3000\text{ Hz})(50\ \Omega)} = 1.06\ \mu\text{F}$$

$$C_{3(\text{actual})} = \frac{C_{3(\text{table})}}{2\pi f_{3\text{dB}} R_L} = \frac{1\text{ F}}{2\pi(3000\text{ Hz})(50\ \Omega)} = 1.06\ \mu\text{F}$$

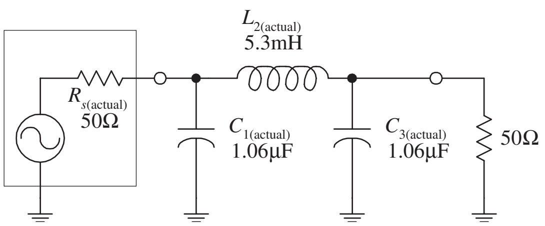

FIGURE 9.9

To account for impedance matching of the source and load, as well as getting rid of the normalized frequency, apply the following frequency and impedance scaling rules. To frequency scale, divide the capacitor and inductor values that you got from the table by $\omega = 2\pi f_c$. To impedance scale, multiply resistor and inductor values by the load impedance and divide the capacitor values by the load impedance. In other words, use the following two equations to get the actual component values needed:

$$L_{n(\text{actual})} = \frac{R_L L_{n(\text{table})}}{2\pi f_{3\text{dB}}}$$

$$C_{n(\text{actual})} = \frac{C_{n(\text{table})}}{2\pi f_{3dB} R_L}$$

The calculations and the final low-pass circuit are shown in the figure.

9.4 A Note on Filter Types

It was briefly mentioned earlier that Chebyshev and Bessel filters could be used instead of Butterworth filters. To design Chebyshev and Bessel filters, you take the same approach you used to design Butterworth filters. However, you need to use different low-pass attenuation graphs and tables to come up with the component values placed in the π and T LC networks. If you are interested in designing Chebyshev and Bessel filters, consult a filter design handbook. Now, to give you a better understanding of the differences between the various filter types, the following few paragraphs should help.

Butterworth filters are perhaps the most popular filters used. They have very flat frequency response in the middle passband region, although they have somewhat rounded bends in the region near the –3-dB point. Beyond the –3-dB point, the rate of attenuation increases and eventually reaches $n \times 6$ dB per octave (e.g., $n = 3$, attenuation = 18 dB/octave). Butterworth filters are relatively easy to construct, and the components needed tend not to require as strict tolerances as those of the other filters.

Chebyshev filters (e.g., 0.5-dB ripple, 0.1-dB ripple Chebyshev filter) provide a sharper rate of descent in attenuation beyond the –3-dB point than Butterworth and Bessel filters. However, there is a price to pay for the steep descent—the cost is a ripple voltage within the passband, referred to as the *passband ripple.* The size of the passband ripple increases with order of the filter. Also, Chebyshev filters are more sensitive to component tolerances than Butterworth filters.

Now, there is a problem with Butterworth and Chebyshev filters—they both introduce varying amounts of delay time on signals of different frequencies. In other words, if an input signal consists of a multiple-frequency waveform (e.g., a modulated signal), the output signal will become distorted because different frequencies will be displaced by different delay times. The delay-time variation over the passband is called *delay distortion,* and it increases as the order of the Butterworth and Chebyshev filters increases. To avoid this effect, a Bessel filter can be used. Bessel filters, unlike Butterworth and Chebyshev filters, provide a constant delay over the passband. However, unlike the other two filters, Bessel filters do not have as sharp an attenuation falloff. Having a sharp falloff, however, is not always as important as good signal reproduction at the output. In situations where actual signal reproduction is needed, Bessel filters are more reliable.

9.5 Passive High-Pass Filter Design

Suppose that you want to design a high-pass filter that has an $f_{3dB} = 1000$ Hz and an attenuation of at least –45 dB at 300 Hz—which we call the *stop frequency* f_s. Assume that the filter is hooked up to a source and load that both have impedances of 50 Ω and that a Butterworth response is desired. How do you design the filter? The trick, as you will see in a second, involves treating the high-pass response as an inverted low-pass response, then designing a normalized low-pass filter, applying some conversion tricks on the low-pass filter's components to get a normalized high-pass filter, and then frequency and impedance scaling the normalized high-pass filter.