Types of control

There are many ways to manipulate the behavior of a dynamical system, and these control approaches are organized schematically in Fig. 8.1. Passive control does not require input energy, and when sufficient, it is desirable because of its simplicity, reliability, and low cost. For example, stop signs at a traffic intersection regulate the flow of traffic. Active control requires input energy, and these controllers are divided into two broad categories based on whether or not sensors are used to inform the controller. In the first category, open-loop control relies on a pre-programmed control sequence; in the traffic example, signals may be pre-programmed to regulate traffic dynamically at different times of day. In the second category, active control uses sensors to inform the control law. Disturbance feedforward control measures exogenous disturbances to the system and then feeds this into an open-loop control law; an example of feedforward control would be to preemptively change the direction of the flow of traffic near a stadium when a large crowd of people are expected to leave. Finally, the last category is closed-loop feedback control, which will be the main focus of this chapter. Closed-loop control uses sensors to measure the system directly and then shapes the control in response to whether the system is actually achieving the desired goal. Many modern traffic systems have smart traffic lights with a control logic informed by inductive sensors in the roadbed that measure traffic density.

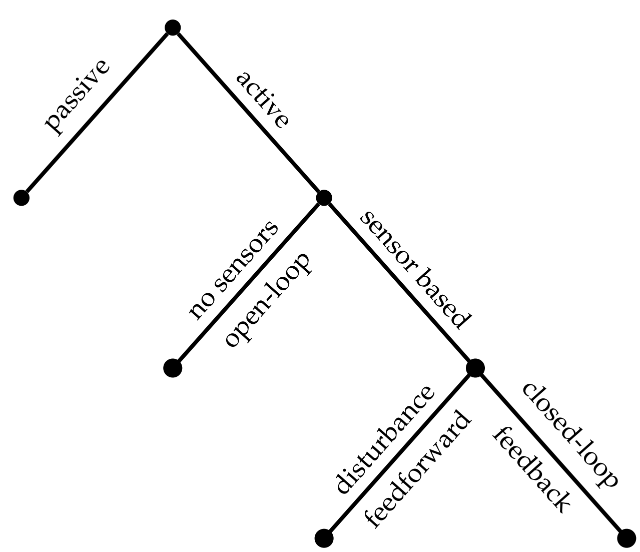

Figure 8.1: Schematic illustrating the various types of control. Most of this chapter will focus on closed-loop feedback control.

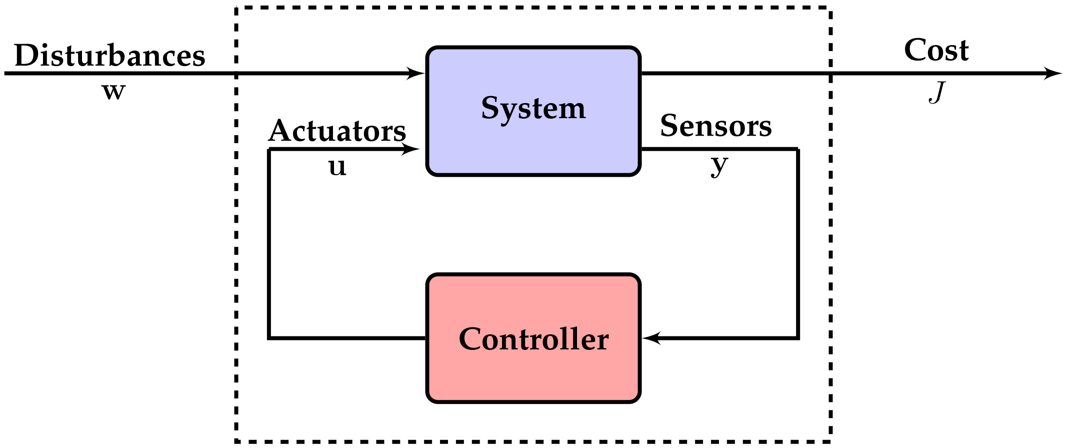

Figure 8.2: Standard framework for feedback control. Measurements of the system, $\mathbf{y}(t)$, are fed back into a controller, which then decides on the appropriate actuation signal $\mathbf{u}(t)$ to control the system. The control law is designed to modify the system dynamics and provide good performance, quantified by the cost J, despite exogenous disturbances and noise in $\mathbf{w}$. The exogenous input $\mathbf{w}$ may also include a reference trajectory $\mathbf{w}_r$ that should be tracked.

8.1 Closed-loop feedback control

The main focus of this chapter is closed-loop feedback control, which is the method of choice for systems with uncertainty, instability, and/or external disturbances. Figure 8.2 depicts the general feedback control framework, where sensor measurements, $\mathbf{y}$, of a system are fed back into a controller, which then decides on an actuation signal, $\mathbf{u}$, to manipulate the dynamics and provide robust performance despite model uncertainty and exogenous disturbances. In all of the examples discussed in this chapter, the vector of exogenous disturbances may be decomposed as $\mathbf{w} = \begin{bmatrix}\mathbf{w}_d^T & \mathbf{w}_n^T & \mathbf{w}_r^T\end{bmatrix}^T$, where $\mathbf{w}_d$ are disturbances to the state of the system, $\mathbf{w}_n$ is measurement noise, and $\mathbf{w}_r$ is a reference trajectory that should be tracked by the closed-loop system.

Mathematically, the system and measurements are typically described by a dynamical system:

$$\frac{d}{dt}\mathbf{x} = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{w}_d) \tag{8.1a}$$

$$\mathbf{y} = \mathbf{g}(\mathbf{x}, \mathbf{u}, \mathbf{w}_n). \tag{8.1b}$$

The goal is to construct a control law

$$\mathbf{u} = \mathbf{k}(\mathbf{y}, \mathbf{w}_r) \tag{8.2}$$

that minimizes a cost function

$$J \triangleq J(\mathbf{x}, \mathbf{u}, \mathbf{w}_r). \tag{8.3}$$

Thus, modern control relies heavily on techniques from optimization [74]. In general, the controller in (8.2) will be a dynamical system, rather than a static function of the inputs. For example, the Kalman filter in Sec. 8.5 dynamically estimates the full state $\mathbf{x}$ from measurements of $\mathbf{u}$ and $\mathbf{y}$. In this case, the control law will become $\mathbf{u} = \mathbf{k}(\mathbf{y}, \hat{\mathbf{x}}, \mathbf{w}_r)$, where $\hat{\mathbf{x}}$ is the full-state estimate.

To motivate the added cost and complexity of sensor-based feedback control, it is helpful to compare with open-loop control. For reference tracking problems, the controller is designed to steer the output of a system towards a desired reference output value $\mathbf{w}_r$, thus minimizing the error $\epsilon = \mathbf{y} - \mathbf{w}_r$. Open-loop control, shown in Fig. 8.3, uses a model of the system to design an actuation signal $\mathbf{u}$ that produces the desired reference output. However, this pre-planned strategy cannot correct for external disturbances to the system and is fundamentally incapable of changing the dynamics. Thus, it is impossible to stabilize an unstable system, such as an inverted pendulum, with open-loop control, since the system model would have to be known perfectly and the system would need to be perfectly isolated from disturbances. Moreover, any model uncertainty will directly contribute to open-loop tracking error.

In contrast, closed-loop feedback control, shown in Fig. 8.4 uses sensor measurements of the system to inform the controller about how the system is actually responding. These sensor measurements provide information about unmodeled dynamics and disturbances that would degrade the performance in open-loop control. Further, with feedback it is often possible to modify and stabilize the dynamics of the closed-loop system, something which is not possible with open-loop control. Thus, closed-loop feedback control is often able to maintain high-performance operation for systems with unstable dynamics, model uncertainty, and external disturbances.

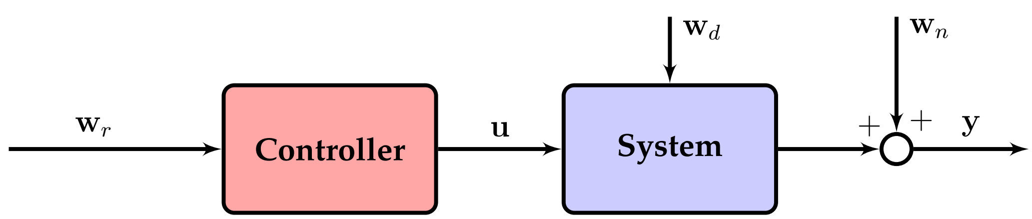

Figure 8.3: Open-loop control diagram. Given a desired reference signal $\mathbf{w}_r$, the open-loop control law constructs a control protocol $\mathbf{u}$ to drive the system based on a model. External disturbances ($\mathbf{w}_d$) and sensor noise ($\mathbf{w}_n$), as well as unmodeled system dynamics and uncertainty, are not accounted for and degrade performance.

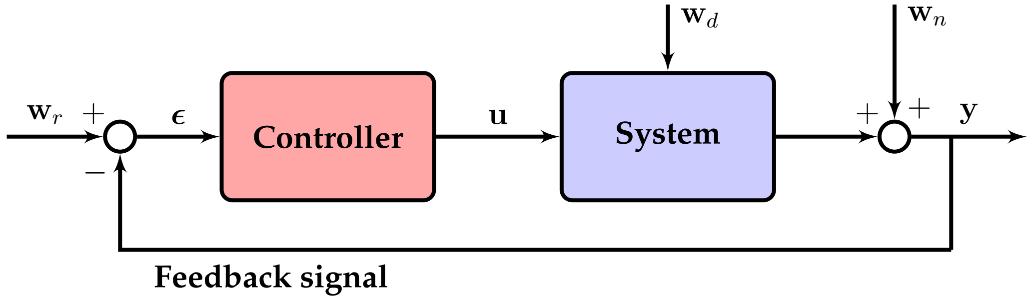

Figure 8.4: Closed-loop feedback control diagram. The sensor signal $\mathbf{y}$ is fed back and subtracted from the reference signal $\mathbf{w}_r$, providing information about how the system is responding to actuation and external disturbances. The controller uses the resulting error ε to determine the correct actuation signal $\mathbf{u}$ for the desired response. Feedback is often able to stabilize unstable dynamics while effectively rejecting disturbances $\mathbf{w}_d$ and attenuating noise $\mathbf{w}_n$.

Examples of the benefits of feedback control

To summarize, closed-loop feedback control has several benefits over open-loop control:

- It may be possible to stabilize an unstable system;
- It may be possible to compensate for external disturbances;
- It may be possible to correct for unmodeled dynamics and model uncertainty.

These issues are illustrated in the following two simple examples.

Inverted pendulum. Consider the unstable inverted pendulum equations, which will be derived later in Sec. 8.2. The linearized equations are:

$$\frac{d}{dt}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ g/L & d\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}u \tag{8.4}$$

where $x_1 = \theta$, $x_2 = \dot{\theta}$, u is a torque applied to the pendulum arm, g is gravitational acceleration, L is the length of the pendulum arm, and d is damping. We may write this system in standard form as

$$\frac{d}{dt}\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{B}u.$$

If we choose constants so that the natural frequency is $\omega_n = \sqrt{g/L} = 1$ and $d = 0$, then the system has eigenvalues $\lambda = \pm 1$, corresponding to an unstable saddle-type fixed point.

No open-loop control strategy can change the dynamics of the system, given by the eigenvalues of $\mathbf{A}$. However, with full-state feedback control, given by $u = -\mathbf{K}\mathbf{x}$, the closed-loop system becomes

$$\frac{d}{dt}\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{B}u = (\mathbf{A} - \mathbf{B}\mathbf{K})\,\mathbf{x}.$$

Choosing $\mathbf{K} = \begin{bmatrix}4 & 4\end{bmatrix}$, corresponding to a control law $u = -4x_1 - 4x_2 = -4\theta - 4\dot{\theta}$, the closed loop system $(\mathbf{A} - \mathbf{B}\mathbf{K})$ has stable eigenvalues $\lambda = -1$ and $\lambda = -3$.

Determining when it is possible to change the eigenvalues of the closed-loop system, and determining the appropriate control law $\mathbf{K}$ to achieve this, will be the subject of future sections.

Cruise control. To appreciate the ability of closed-loop control to compensate for unmodeled dynamics and disturbances, we will consider a simple model of cruise control in an automobile. Let u be the rate of gas fed into the engine, and let y be the car's speed. Neglecting transients, a crude model[1] is:

$$y = u. \tag{8.5}$$

Thus, if we double the gas input, we double the automobile's speed.

Based on this model, we may design an open-loop cruise controller to track a reference speed w_r by simply commanding an input of $u = w_r$. However, an incorrect automobile model (i.e., in actuality $y = 2u$), or external disturbances, such as rolling hills (i.e., if $y = u + \sin(t)$), are not accounted for in the simple open-loop design.

In contrast, a closed-loop control law, based on measurements of the speed, is able to compensate for unmodeled dynamics and disturbances. Consider the closed-loop control law $u = K(w_r - y)$, so that gas is increased when the measured velocity is too low, and decreased when it is too high. Then if the dynamics are actually $y = 2u$ instead of $y = u$, the open-loop system will have 50% steady-state tracking error, while the performance of the closed-loop system can be significantly improved for large K:

$$y = 2K(w_r - y) \quad \Longrightarrow \quad (1 + 2K)y = 2Kw_r \quad \Longrightarrow \quad y = \frac{2K}{1 + 2K}w_r. \tag{8.6}$$

For $K = 50$, the closed-loop system only has 1% steady-state tracking error. Similarly, an added disturbance w_d will be attenuated by a factor of $1/(2K + 1)$.

[1] A more realistic model would have acceleration dynamics, so that $\dot{x} = -x + u$ and $y = x$.

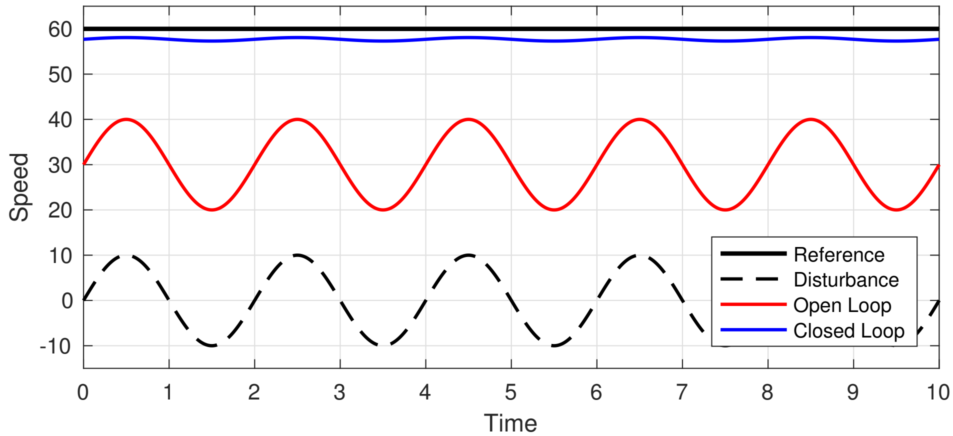

Figure 8.5: Open-loop vs. closed-loop cruise control.

As a concrete example, consider a reference tracking problem with a desired reference speed of 60 mph. The model is $y = u$, and the true system is $y = 0.5\,u$. In addition, there is a disturbance in the form of rolling hills that increase and decrease the speed by ± 10 mph at a frequency of 0.5 Hz. An open-loop controller is compared with a closed-loop proportional controller with $K = 50$ in Fig. 8.5 and Code 8.1. Although the closed-loop controller has significantly better performance, we will see later that a large proportional gain may come at the cost of robustness. Adding an integral term will improve performance.

Code 8.1: Compare open-loop and closed-loop cruise control.

```
clear all, close all, clc

t = 0:.01:10;             % time

wr = 60*ones(size(t)); % reference speed
d = 10*sin(pi*t);        % disturbance

aModel = 1;               % y = aModel*u
aTrue = .5;               % y = aTrue*u

uOL = wr/aModel;          % Open-loop u based on model
yOL = aTrue*uOL + d;      % Open-loop response

K = 50;                   % control gain, u=K(wr-y);
yCL = aTrue*K/(1+aTrue*K)*wr + d/(1+aTrue*K);
```