LOW-NOISE TECHNIQUES

CHAPTER 8

In many applications you're dealing with small signals, for which it is essential to minimize the degrading effects of amplifier "noise." Low-noise design is thus an important part of the art of electronics. The extensive detail (decorated with more than our usually paltry quota of equations!) in this chapter reflects the richness of the low-noise design. So does its length – it's the longest chapter in the book. Recognizing that many readers will have a less-than-passionate interest in the various subjects treated here, we offer the following guide.

A quick guide to this chapter The basics of noise are explained in §8.1 ("Noise"), which should be read first. Readers interested primarily in low-noise design with *op-amps* can then skip ahead to the discussion, tables, and graphs in §8.9 ("Noise in operational amplifier circuits"). Those interested in low-noise design with discrete transistors (or interested in gaining a fuller understanding of what's going on "under the hood" of op-amps) should read §8.5 ("Low-noise design with bipolar transistors") and §8.6 ("Low-noise design with JFETs"). Readers working with photodiode circuits and the like will want to read §8.11 ("Noise in transimpedance amplifiers").[1] For a discussion of noise *measurement*, go to §8.12 ("Noise measurements and noise sources") and §8.13 ("Bandwidth Limiting and RMS Voltage Measurement").

An even quicker guide to noise This chapter is lengthy, and it's filled with mathematical details and information about hundreds of transistors and op-amps. But noise doesn't have to be complicated. In a spirit of encapsulating the essence of noise, we offer a one-minute breathless end-of-class "takeaway":

> The random noise you care about is characterized by its *density* (rms noise amplitude in a 1 Hz band of frequency); voltage noise density is called e_n, and has units like $\mathrm{nV}/\sqrt{\mathrm{Hz}}$. Likewise, the symbol for noise *current* is i_n; a noise current at an amplifier's input flows through the signal's source resistance, creating its own noise voltage $e_n = i_n R_s$. If a noise source is uniform over frequency, it's called "white noise," and the rms noise voltage contained within a bandwidth B is just $v_n = e_n\sqrt{B}$. Knowing that, you can go to Tables 8.3a–8.3c on page 522, which lists e_n and i_n for a wide selection of op-amps, to figure out how much noise is added in an amplifier stage. Multiply by the amplifier's gain and, voilà, you've got the output noise.
>
> Amplifiers aren't the only source of noise. A resistor generates "Johnson noise", eq'n 8.4, and the discrete charges in a flow of current generate "shot noise", eq'n 8.6. Both of these are white noise.[2] Finally (with ten seconds remaining) – to figure the total noise in a circuit with multiple independent noise sources, you take the sum of the squares of each noise density, multiply by the bandwidth, then take the square root. Time's up. End of class.

8.1 "Noise"

In almost every area of measurement the ultimate limit of detectability of weak signals is set by noise – unwanted signals that obscure the desired signal. Even if the quantity being measured is not weak, the presence of noise degrades the accuracy of the measurement. Some forms of noise are unavoidable (e.g., real fluctuations in the quantity being measured), and they can be overcome only with the techniques of *signal averaging* and *bandwidth narrowing*.[3] Other forms of noise (e.g., radiofrequency interference and "ground loops") can be reduced or eliminated by a variety of tricks, including filtering and careful attention to wiring configuration and parts location. Finally, there is noise that arises in the amplification process itself, and it can be reduced through the techniques of low-noise amplifier design. Although the techniques of signal averaging

[1] And also the related discussion of stability and bandwidth in §*4x.3*.

[2] Things get more interesting when noise density varies with frequency, for example the notorious pink "flicker noise" that rises as $e_n \propto 1/\sqrt{f}$ at low frequencies. This fascinating (and infuriating) annoyance does not go unnoticed in this chapter!

[3] See §8.14, and also Chapter 15 of this book's second edition (1989).

can often be used to rescue a signal buried in noise, it always pays to begin with a system that is free of preventable interference and that possesses the lowest amplifier noise practicable.

We begin by talking about the origins and characteristics of the different kinds of noise that afflict electronic circuits. Then we launch into a discussion of bipolar-transistor (BJT) and field-effect transistor (FET) noise, including methods for low-noise design with a given signal source, and we present some design examples. After a short discussion of noise in differential and feedback amplifiers, we continue with low-noise design with op-amps, including transimpedance (current-to-voltage) amplifiers. Sections on noise measurements, bandwidth limiting, and lock-in detection follow, then a short discussion of power-supply noise. We conclude with a section on proper grounding and shielding and the elimination of interference and pickup.

Because the term *noise* can be applied to anything that obscures a desired signal,[4] noise can take the form of another signal (“interference”); most often, however, we use the term to describe “random” noise of a physical (often thermal) origin. Noise can be characterized by its frequency spectrum, its amplitude distribution, and the physical mechanism responsible for its generation. Let’s next look at the chief offenders:

Johnson noise: Random-noise voltage created by thermal fluctuations in a resistor.

Shot noise: Random statistical fluctuations in a flowing current caused by the discrete nature of electrical charge.

Flicker noise: Additional random noise, rising typically as $1/f$ in power at low frequencies, with a multitude of causes.

Burst noise: low-frequency noise typically seen as random jumps between a pair of levels, caused by material device defects.

8.1.1 Johnson (Nyquist) noise

Any old resistor just sitting on the table generates a noise voltage across its terminals known as Johnson noise (or Nyquist noise).[5] It has a flat frequency spectrum, meaning that there is the same noise power in each hertz of frequency (up to some limit, of course). Noise with a flat spectrum is also called “white noise.” The actual open-circuit noise voltage generated by a resistance R at temperature T is given by

$$v_{\text{noise}}(\text{rms}) = v_{\text{n}} = (4kTRB)^{\frac{1}{2}} \quad \text{V(rms)}, \tag{8.1}$$

where k is Boltzmann’s constant, T is the absolute temperature in Kelvins (K=°C + 273.16), and B is the bandwidth in hertz. Thus v_{noise}(rms) is what you would measure at the output if you drove a perfect noiseless bandpass filter (of bandwidth B) with the voltage generated by a resistor at temperature T. At room temperature (68°F = 20°C = 293K),

$$\begin{aligned} 4kT &= 1.62 \times 10^{-20} && \text{V}^2/\text{Hz} - \Omega, \\ (4kTR)^{\frac{1}{2}} &= 1.27 \times 10^{-10} R^{\frac{1}{2}} && \text{V/Hz}^{\frac{1}{2}} \\ &= 1.27 \times 10^{-4} R^{\frac{1}{2}} && \mu\text{V/Hz}^{\frac{1}{2}}. \end{aligned} \tag{8.2}$$

For example, a 10k resistor at room temperature has an open-circuit rms voltage of 1.3μV, measured with a bandwidth of 10 kHz (e.g., by placing it across the input of a good audio amplifier and measuring the output with a voltmeter). The source resistance of this noise voltage is just R. If you connect the terminals of the resistor together, you get a (short-circuit) current of

$$i_{\text{noise}}(\text{rms}) = v_{\text{noise}}(\text{rms})/R = v_{\text{nR}}/R = (4kTB/R)^{\frac{1}{2}}. \tag{8.3}$$

As we’ll see in §8.2.1, it’s convenient to express noise voltage (or current) as a *density* e_{n} (rms voltage per square root bandwidth). Johnson noise, with its flat (white) spectrum, has constant noise voltage-density

$$e_{\text{n}} = \sqrt{4kTR} \quad \text{V/Hz}^{\frac{1}{2}}, \tag{8.4}$$

from which the rms noise voltage in some limited bandwidth B is then simply $v_{\text{n}} = e_{\text{n}}\sqrt{B}$. Likewise, the short-circuit noise-current density is

$$i_{\text{n}} = \sqrt{4kT/R} \quad \text{A/Hz}^{\frac{1}{2}}. \tag{8.5}$$

Figure 8.1 plots the simple relationship between Johnson-noise voltage density and source resistance; also shown is the short-circuit noise current density. An easy number to remember, when choosing resistor values for low-noise amplifier designs, is that a 1kΩ resistor at room temperature generates an open-circuit noise voltage density of 4 nV/$\sqrt{\text{Hz}}$; scale by the square root of resistance for other values.[6]

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[4] As Lew Branscomb famously noted, “Nature does not ‘know’ what experiment a scientist is trying to do. God loves the noise as much as the signal.” See L. Branscomb, “Integrity in Science,” *Am. Sci.* **73**, 421–23 (1985).

[5] Experiment and formulas by J. B. Johnson, Letter to *Nature*, **119**, 50 (1927), *Phys. Rev.* **32**, “Thermal agitation of electricity in conductors,” 97–109, (1928), subsequent theory by H. Nyquist, *Phys. Rev.*, “Thermal agitation of electric charge in conductors,” **32**, 110–113, (1928).

[6] We find it handy to remember the values of q and of $4kT$ (which keeps

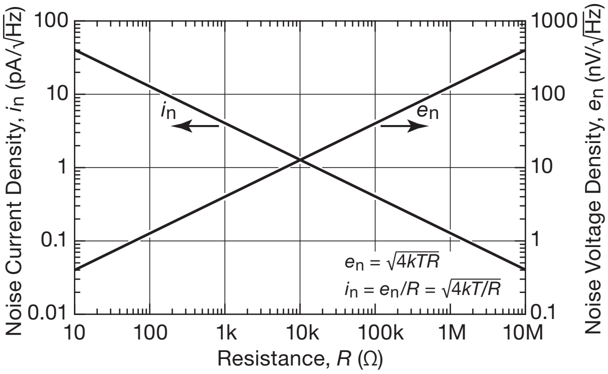

Figure 8.1. Open-circuit thermal noise-voltage and short-circuit thermal noise-current densities versus resistance at 25°C.

Here's a handy Johnson-noise mini-table, listing both voltage and current noise *densities* (units of $\mathrm{V}/\sqrt{\mathrm{Hz}}$ and $\mathrm{A}/\sqrt{\mathrm{Hz}}$), and noise within a 10 kHz band, for seven decade-related values of resistance:

	Johnson noise, at T=25°C			
	open circuit		short circuit	
R	e_n (nV/$\sqrt{\mathrm{Hz}}$)	$e_n\sqrt{B}$ B=10 kHz (μV)	i_n (pA/$\sqrt{\mathrm{Hz}}$)	$i_n\sqrt{B}$ B=10 kHz (pA)
100 Ω	1.28	0.128	12.8	1280
1k	4.06	0.406	4.06	406
10k	12.8	1.28	1.28	128
100k	40.6	4.06	0.406	40.6
1M	128	12.8	0.128	12.8
10M	406	40.6	0.041	4.06
100M	1280	128	0.0128	1.28

The amplitude of the Johnson-noise voltage at any instant is, in general, unpredictable, but it obeys a Gaussian amplitude distribution (Figure 8.2), where $p(V)dV$ is the probability that the instantaneous voltage lies between V and $V+dV$, and $v_n(\mathrm{rms})$ is the rms noise voltage, given earlier.[7]

The significance of Johnson noise is that it sets a lower limit on the noise voltage in any detector, signal source, or amplifier having resistance. The resistive part of a source impedance generates Johnson noise, as do the bias and load resistors of an amplifier. You will see how it all works out presently.

It is interesting to note that the physical analog of resistance (any mechanism of energy loss in a physical system, e.g., viscous friction acting on small particles in a liquid) has associated with it fluctuations in the associated physical quantity (in this case, the particles' velocity, manifest as the chaotic Brownian motion). Johnson noise is just a special case of this fluctuation–dissipation phenomenon.

Johnson noise should not be confused with the additional noise voltage created by the effect of resistance fluctuations when an externally applied current flows through a resistor. This "excess noise" has a $1/f$ spectrum (approximately) and is heavily dependent on the actual construction of the resistor. We will talk about it later.

8.1.2 Shot noise

An electric current is the flow of discrete electric charges, not a smooth fluidlike flow. The finiteness of the charge quantum results in statistical fluctuations of the current. If the charges act independently of each other, the fluctuating current's noise density is given by

$$i_n = \sqrt{2qI_{dc}} \quad \mathrm{A/Hz}^{\frac{1}{2}}, \tag{8.6}$$

where q is the electron charge (1.60×10^{-19} coulomb). This noise, like resistor Johnson noise, is white and Gaussian. So its amplitude, taken over a measurement bandwidth B, is just

$$i_{noise}(\mathrm{rms}) = i_{nR}(\mathrm{rms}) = i_n\sqrt{B} = (2qI_{dc}B)^{\frac{1}{2}} \quad \mathrm{A(rms)}. \tag{8.7}$$

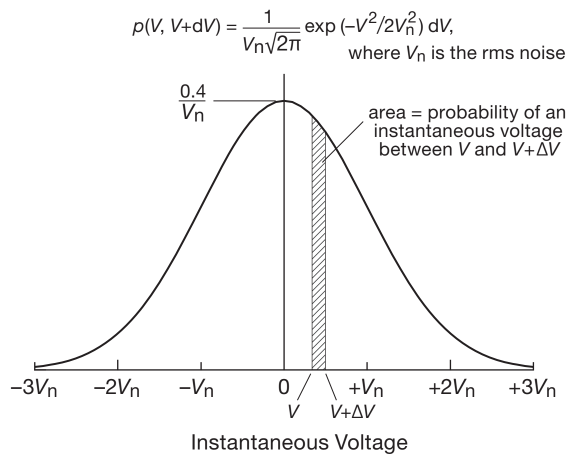

Figure 8.2. Johnson noise obeys a Gaussian distribution of amplitudes. The normalizing factor $0.4/V_n$ ensures dimensionless unit area under the bell curve (the "0.4" is actually $1/\sqrt{2\pi}$, about 0.3989).

popping up) together, because in SI units they are 1.6×10^{-19} and 1.6×10^{-20}, respectively.

[7] See also Figure 8.115, which plots the odds (over 9 decades) that the instantaneous amplitude exceeds some multiple of the rms amplitude.

For example, a "steady" current of 1 A actually has an rms fluctuation of 57 nA, measured in a 10 kHz bandwidth; i.e., it fluctuates by about 0.000006%. The relative fluctuations are larger for smaller currents: a "steady" current of 1 μA actually has an rms current-noise fluctuation, measured over a 10 kHz bandwidth, of 0.006%, i.e., −85 dB. At 1 pA dc, the rms current fluctuation (same bandwidth) is 57 fA, i.e., a 5.7% variation! Shot noise is "rain on a tin roof."

Here's a handy minitable listing shot-noise current density and shot noise current in a 10 kHz band for decadal currents spanning 12 orders of magnitude:

		Shot noise current $B = 10\,\text{kHz}$	
I_{dc}	i_n	$i_n\sqrt{B}$ (10 kHz)	$\frac{i_n\sqrt{B}}{I_{dc}}$
1 fA	18 aA/$\sqrt{\text{Hz}}$	1.8 fA	+5 dB
1 pA	0.57 fA/$\sqrt{\text{Hz}}$	57 fA	−25 dB
1 nA	18 fA/$\sqrt{\text{Hz}}$	1.8 pA	−55 dB
1 μA	0.57 pA/$\sqrt{\text{Hz}}$	57 pA	−85 dB
1 mA	18 pA/$\sqrt{\text{Hz}}$	1.8 nA	−115 dB

An important point: the shot-noise formula given earlier assumes that the charge carriers making up the current act independently. That is indeed the case for charges crossing a barrier, for example the current in a junction diode, where the charges move by diffusion; but it is not true for the important case of metallic conductors, where there are long-range correlations between charge carriers. Thus the current in a simple resistive circuit has far less noise than is predicted by the shot-noise formula. Another important exception to the shot-noise formula is provided by our standard transistor current-source circuit (Figure 2.32); we discuss this further in §8.3.5.

Exercise 8.1. A resistor is used as the collector load in a low-noise amplifier; the collector current I_C is accompanied by shot noise. Show that the output noise voltage is dominated by shot noise (rather than Johnson noise in the resistor) as long as the quiescent voltage drop across the load resistor is greater than $2kT/q$ (50mV, at room temperature).

8.1.3 1/*f* noise (flicker noise)

Shot noise and Johnson noise are irreducible forms of noise generated according to physical principles. The most expensive and most carefully made resistor has exactly the same Johnson noise as the cheapest carbon resistor of the same resistance. Real devices have, in addition, various sources of "excess noise." Real resistors suffer from fluctuations in resistance, generating an additional noise voltage (which adds to the ever-present Johnson noise) proportional to the dc current flowing through them. This noise depends on many factors having to do with the construction of the particular resistor, including the resistive material and especially the end-cap connections. Here is a listing of typical excess noise for various resistor types, given as rms microvolts per volt applied across the resistor, measured over one decade of frequency: (eq'n 8.13)

Carbon composition	0.10μV to 3.0μV
Carbon film	0.05μV to 0.3μV
Metal film	0.02μV to 0.2μV
Wire wound	0.01μV to 0.2μV

This noise has approximately a $1/f$ power spectrum (equal power per decade of frequency) and is sometimes called "pink noise." When plotted against voltage or current (rather than power) its *amplitude* falls as $1/\sqrt{f}$, as shown in Figure 8.3. Figure 8.4 shows how it looks in comparison with a sample of white noise and of what's sometimes called "red noise" ($1/f^2$ power spectrum); if you want to make your own, look ahead to Figure 8.93 to see how.

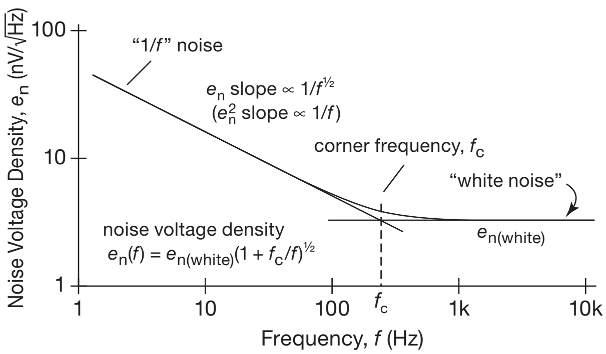

Figure 8.3. When plotted on log axes as noise voltage versus frequency, $1/f$ noise slopes downward with a slope of 1/2, i.e., as $1/f^{\frac{1}{2}}$ (it is noise *power* that goes as $1/f$).

You often see the notation f_c for the corner frequency at which the $1/f$ noise is the same as an underlying white-noise component.[8] The combined noise voltage density is

$$e_n(f) = e_{n(\text{white})}\sqrt{1 + f_c/f}, \tag{8.8}$$

from which the rms integrated noise voltage in a band extending from f_1 to f_2 can be calculated; see eq'n 8.59 on page 565.

[8] You can estimate f_c with the help of eq'n 8.27 on page 491.

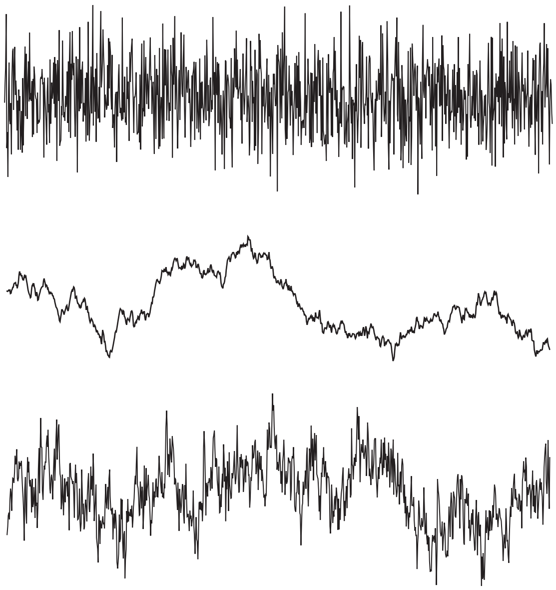

Figure 8.4. Three noises: top, "white noise" (uniform power per Hz); middle, "red noise" (power per Hz proportional to $1/f^2$); bottom, "pink noise" (or $1/f$ noise, power per Hz proportional to $1/f$).

Other noise-generating mechanisms often produce $1/f$ noise, examples being base-current noise in transistors and cathode current noise in vacuum tubes. Curiously enough, $1/f$ noise is present in nature in unexpected places, e.g., the speed of ocean currents, the flow of sand in an hourglass, the flow of traffic on Japanese expressways, and the yearly flow of the Nile measured over the last 2,000 years.[9] If you plot the loudness of a piece of classical music versus time, you get a $1/f$ spectrum! No unifying principle has been found for all the $1/f$ noise that seems to be swirling around us, although particular sources can often be identified in each instance.

8.1.4 Burst noise

Not all noise sources are characterized by a Gaussian (or even *smooth*) distribution of amplitudes. Most notorious among the exceptions is *burst noise* (also variously called *popcorn* noise, *bistable* noise, or *random telegraph signal* noise), seen occasionally in semiconductor devices (particularly in parts dating back to the 1970s and earlier). It consists of random jumps between two (usually) voltage levels, taking place on time scales of tens of milliseconds; when played out on a loudspeaker it sounds like the birth throes of popcorn. Figure 8.5 shows a typical waveform, the output of a vintage[10] 741 op-amp wired as a noninverting amplifier with G=100.

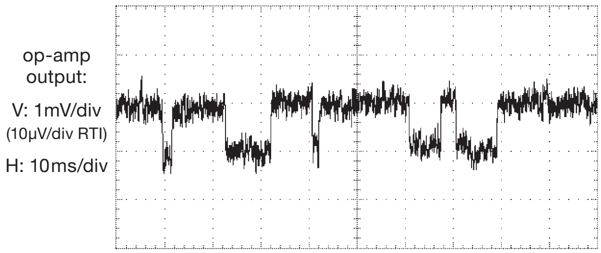

Figure 8.5. Burst noise from a 1973-vintage 741 op-amp, configured as an ×100 noninverting amplifier with grounded input. The output was bandpass filtered to 0.1 Hz–3 kHz, with 6 dB/octave rolloffs.

Viewed in the frequency domain, the effect of burst noise is a raised low frequency portion, without any obvious spectral peaks. You can see this in Figure 8.6, where the voltage noise spectrum of the noisy and quiet op-amp specimens are plotted.[11]

8.1.5 Band-limited noise

All circuits operate within some limited frequency band. So, although it's nice to talk about (and calculate with) noise-*density* quantities, what you usually care about is the rms noise voltage contained within some signal band of interest (called B in eq'n 8.1). In many cases you are dealing with a white noise source (e.g., Johnson noise or shot noise). If that is passed through a perfectly sharp bandpass filter (a "brick-wall" filter), the band-limited rms amplitude is simply $v_{n(rms)}=e_n\sqrt{B}$. But analog brick-wall filters aren't practical – so what you want to know is the equivalent bandwidth of a real filter, say a simple RC lowpass. It turns out that the equivalent brick-wall bandwidth is given by

$$B = \frac{\pi}{2} f_{3dB} = 1.57 f_{3dB} = \frac{1}{4RC} \quad \text{Hz,} \tag{8.9}$$

[9] A delightful reference is W. H. Press, "Flicker noises in astronomy and elsewhere," *Comm. on Astrophys.* **7**, 103–119 (1978). Available at `http://www.nr.com/whp/Flicker_Noise_1978.pdf`.

[10] Semiconductor manufacturers have worked hard to alleviate this problem (believed to be caused by intermittent trapping of charge carriers at defects and interfaces), and popcorn noise is largely a thing of the past. We tested ten samples of 741s, from six different manufacturers, before we found this one. A second sample from the same manufacturer and with the same date code showed no evidence of popcorn noise, as seen in the pair of spectra of Figure 8.6.

[11] It is possible that a related mechanism is responsible for similarly shaped noise plateaus seen in some JFETs; see for example the LSK389 in Figure 8.47.

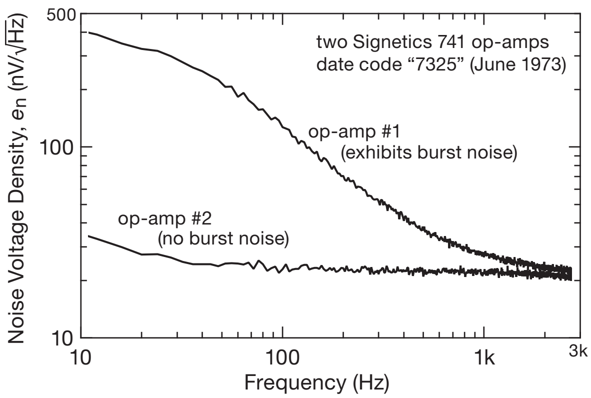

Figure 8.6. Spectrum of the burst noise produced by the same op-amp as used for Figure 8.5, along with that of a second op-amp from the same batch that exhibited no burst noise. The vertical scale shows RTI (referred to the input) rms noise densities.

where $f_{3\text{dB}}=1/2\pi RC$. You can use higher-order filters, of course, for example a 2-pole Butterworth lowpass filter; its equivalent brick-wall bandwidth is $B=1.11f_{3\text{dB}}$. For still higher-order filters (including bandpass) see the expressions in Table 8.4 on page 564. For slowly varying (or dc) signals you can instead do a simple averaging (as, for example, with an integrating ADC, see §13.8.3); in that case the equivalent noise bandwidth is $B=1/2T$, where T is the duration of the (uniform) averaging of the input signal. We'll have a bit more to say about this in §8.13.1.

Of course, the noise spectrum may be other than white (e.g., it may be $1/f$ noise or a combination of white noise with a $1/f$ rising tail at low frequencies). In such a case you cannot simply multiply the noise density by the square root of the bandwidth. Instead you have to integrate the (changing) noise density over the bandpass. For an ideal brick-wall bandpass that is just $v_n^2=\int e_n^2(f)df$ from the lower to upper frequency cutoff of the filter. For a realizable filter you have to integrate the noise density, multiplied by the filter's spectral response $H(f)$, over the bandpass: $v_n^2=\int|e_n(f)H(f)|^2df$. For an arbitrary noise spectrum this is what you would need to do. But life is simpler if you're dealing with classic noise spectra like $1/f$ flicker noise, in which case the noise integrals can be expressed analytically. We've gathered these together in Table 8.4 on page 564, which includes results for white-, pink-, and red-noise spectra, when band limited by brick-wall, single-pole, 2-pole Butterworth, and m-pole Butterworth bandpass filters (response from f_1 to f_2). Those tabulated formulas let you get the results for lowpass ($f_1=0$) or highpass filters ($f_2=\infty$), which are just special cases of the more general bandpass filter.

We discuss this in plenty of detail later in the chapter, at §8.13.

8.1.6 Interference

As we mentioned earlier, an interfering signal or stray pickup constitutes a form of noise. Here the spectrum and amplitude characteristics depend on the interfering signal. For example, 60 Hz powerline pickup has a sharp spectrum and relatively constant amplitude, whereas car ignition noise, lightning, and other impulsive interferences are broad in spectrum and spiky in amplitude. Other sources of interference are radio and television stations (a particularly serious problem near large cities), nearby electrical equipment, motors and elevators, subways, switching regulators, and television sets. Cellphones often dwarf all other sources of RF interference. Even when not in use, the cellphone transmits periodically to tell the cell tower its location, generating interference with a distinctive galloping rhythm.[12] The same goes for mobile computers that use the cellular network for Internet access.

In a slightly different guise you have the same sort of problem generated by anything that puts a signal into the parameter you are measuring. For example, an optical interferometer is susceptible to vibration, and a sensitive RF measurement (e.g., nuclear magnetic resonance, NMR or MRI) can be affected by ambient RF. Many circuits, as well as detectors and even cables, are sensitive to vibration and sound; they are *microphonic*, in the terminology of the trade.

Many of these noise sources can be controlled by careful shielding and filtering, as we discuss later in the chapter. At other times we are forced to take Draconian measures, involving massive stone tables (for vibration isolation), constant-temperature rooms, anechoic chambers, and electrically shielded ("Faraday cage") rooms.

8.2 Signal-to-noise ratio and noise figure

Before getting into the details of amplifier noise and low-noise design, we need to define a few terms that are often used to describe amplifier performance. These involve ratios of noise voltages, measured at the same place in the circuit. It is conventional to refer noise voltages to the input of an amplifier (although the measurements are usually made at the output), i.e., to describe source noise and amplifier noise in terms of microvolts *at the input* that would

[12] Which you can hear at `www.covingtoninnovations.com/michael/blog/0506/050622-cellnoise.mp3`.

generate the observed output noise. This makes sense when you want to think of the relative noise added by the amplifier to a given signal, independent of amplifier gain; it's also realistic, because most of the amplifier noise is usually contributed by the input stage. Unless we state otherwise, noise voltages are referred to the input (RTI).

8.2.1 Noise power density and bandwidth

In the preceding examples of Johnson noise and shot noise, the noise voltage you measure depends both on the measurement bandwidth B (i.e., how much noise you see depends on how fast you look) and on the variables (R and I) of the noise source itself. So it's convenient to talk about an rms noise-voltage "density" e_n:

$$v_{n(rms)} = e_n B^{\frac{1}{2}} = (4kTR)^{\frac{1}{2}} B^{\frac{1}{2}} \quad \text{Vrms}, \qquad (8.10)$$

where v_n is the rms noise voltage you would measure in a bandwidth B. White-noise sources have an e_n that doesn't depend on frequency, whereas pink noise, for instance, has a e_n that drops off at 3 dB/octave. You'll often see e_n^2, too, the mean squared noise density. Since e_n always refers to rms and e_n^2 always refers to mean square, you can just square e_n to get e_n^2! Sounds simple (and it is), but we want to make sure you don't get confused.

Note that B and the square root of B keep popping up. Thus, for example, for Johnson noise from a resistor R,

$$\begin{aligned}
e_{nR}(\text{rms}) &= (4kTR)^{\frac{1}{2}} && && \text{V/Hz}^{\frac{1}{2}},\\
e_{nR}^2 &= 4kTR && && \text{V}^2/\text{Hz},\\
v_{n(rms)} &= v_{nR} B^{\frac{1}{2}} && = (4kTRB)^{\frac{1}{2}} && \text{V},\\
v_n^2 &= v_{nR}^2 B && = 4kTRB && \text{V}^2.
\end{aligned}$$

On datasheets you may see graphs of e_n or e_n^2, with units like "nanovolts per root Hz" or "volts squared per Hz." The quantities e_n and i_n that will soon appear work just the same way.

When you add two signals that are uncorrelated (two noise signals, or noise plus a real signal), you add their noise *power*; that is, their *squared* amplitudes add:

$$v = (v_s^2 + v_n^2)^{\frac{1}{2}},$$

where v is the rms signal obtained by adding together a signal of rms amplitude v_s and a noise signal of rms amplitude v_n. The rms amplitudes[13] *do not* add.

[13] Which, we emphasize, are the convenient and familiar quantites found on datasheets, etc. For example, we're used to thinking of a 3 nV/$\sqrt{\text{Hz}}$ amplifier as quiet; it's hard to recognize a 0.9×10^{-17} V^2/Hz amplifier as the same thing.

8.2.2 Signal-to-noise ratio

Signal-to-noise ratio (SNR) is simply defined as

$$\text{SNR} = 10\log_{10}(v_s^2/v_n^2) = 20\log_{10}(v_s/v_n) \quad \text{dB} \qquad (8.11)$$

where the voltages are rms values, and some bandwidth and center frequency are specified; i.e., it is the ratio, in decibels, of the rms voltage of the desired signal to the rms voltage of the noise that is also present.[14] The "signal" itself may be sinusoidal, or a modulated information-carrying waveform, or even a noiselike signal itself. It is particularly important to specify the bandwidth if the signal has some sort of narrowband spectrum, because the SNR will decrease as the bandwidth is increased beyond that of the signal: the amplifier keeps adding noise power, while the signal power remains constant.

8.2.3 Noise figure

Any real signal source or measuring device generates noise because of Johnson noise in its source resistance (the real part of its complex source impedance). There may be additional noise, of course, from other causes. The *noise figure* (NF) of an amplifier is simply the ratio, in decibels, of the output of the real amplifier to the output of a "perfect" (noiseless) amplifier of the same gain, with a resistor of value R_s connected across the amplifier's input terminals in each case. That is, the Johnson noise of R_s is the "input signal":

$$\text{NF} = 10\log_{10}\left(\frac{4kTR_s + v_n^2}{4kTR_s}\right) \qquad (8.12)$$

$$= 10\log_{10}\left(1 + \frac{v_n^2}{4kTR_s}\right) \quad \text{dB}, \qquad (8.13)$$

where v_n^2 is the mean squared noise voltage per hertz contributed by the amplifier, with a noiseless (cold) resistor of value R_s connected across its input. This latter restriction is important, as you will see shortly, because the noise voltage contributed by an amplifier depends very much on the source impedance (Figure 8.7).

Noise figure is handy as a figure of merit for an amplifier when you have a signal source of a given source impedance and want to compare amplifiers (or transistors, for which the NF is often specified). The NF varies with frequency

[14] The expression in terms of squared amplitudes suggests a ratio in terms of *power*, which is the origin of the decibel ratio definition. But the "$20\log_{10}$" form is widely used, even when there is no actual power, for example with an open-circuit load (or, more confusingly, when the result is at odds with the actual power ratio, for example when expressing the ratio of amplitudes created by a signal transformer).