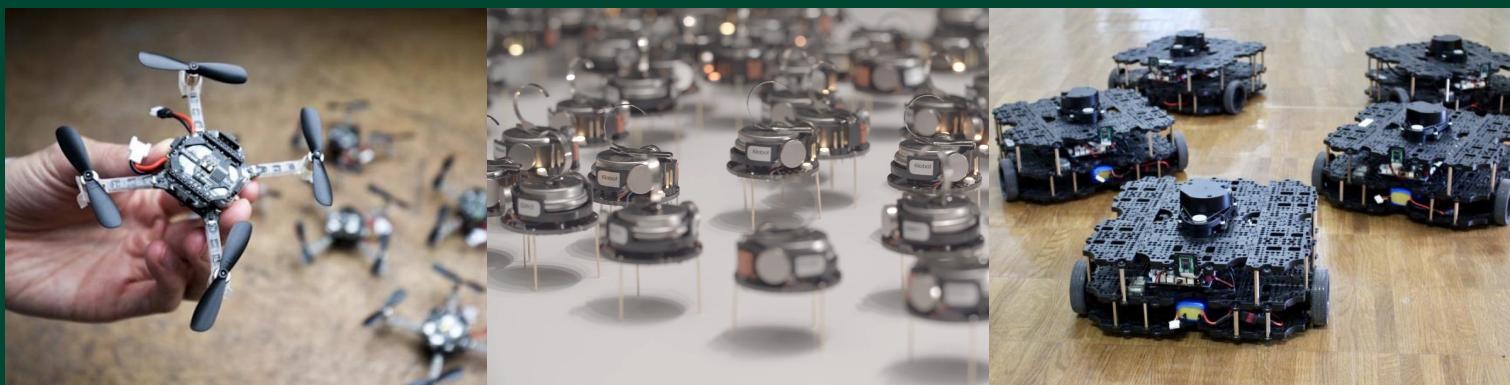


ECE693H, Spring 2025: Multi-robot System Design

“Multi-robot Systems 2”



Dr. Daniel Drew

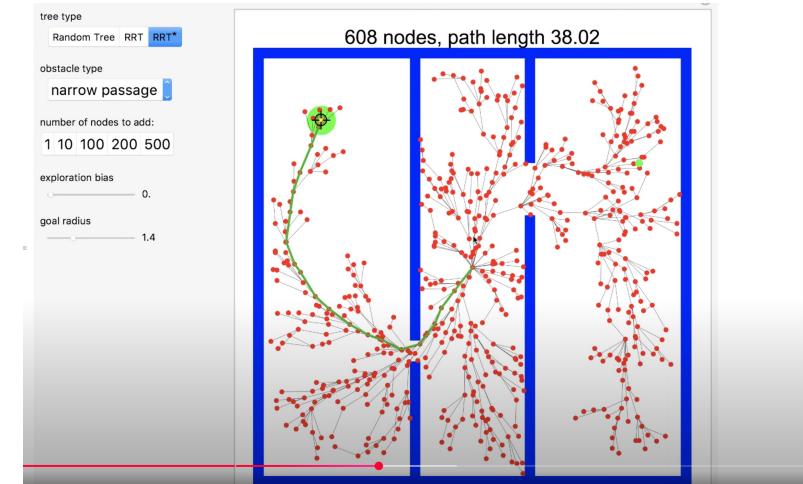
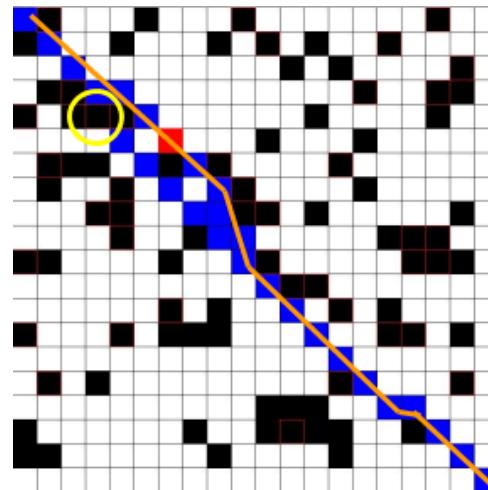
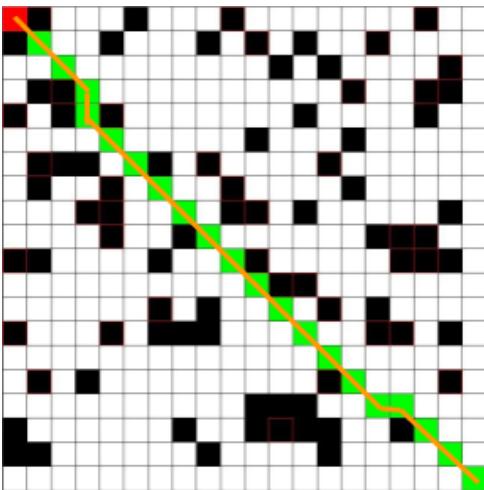


MRS Motion Planning Recap

For scenarios with an omniscient planner: A* is ubiquitous (e.g., video games)

For highly dynamic environments: D*-lite is state of the art

For large, high-dimensional state spaces: dRRT* is state of the art



Questions?

Class Overview

1. Lecture (~40 minutes)
2. Subsystem checkins (~20 minutes)

Multi-robot Systems: Lecture 3 of 4

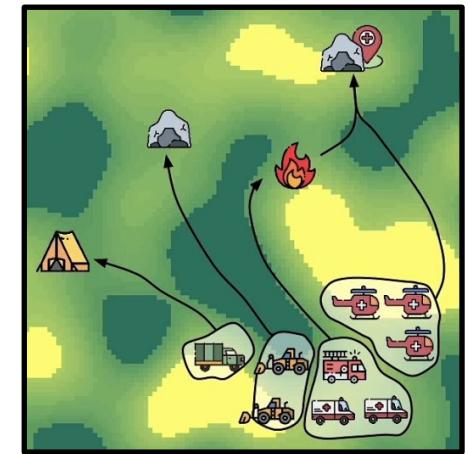
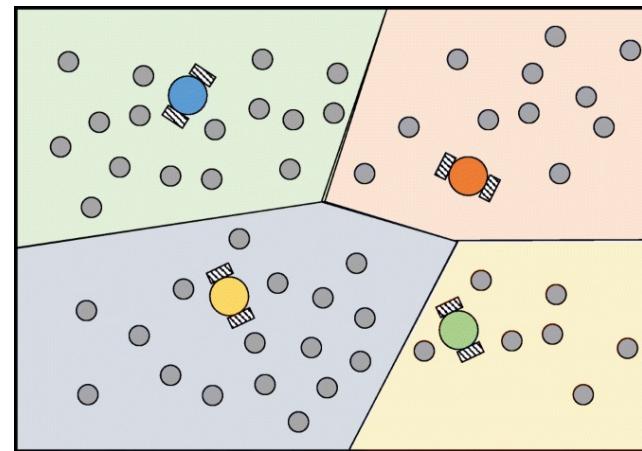
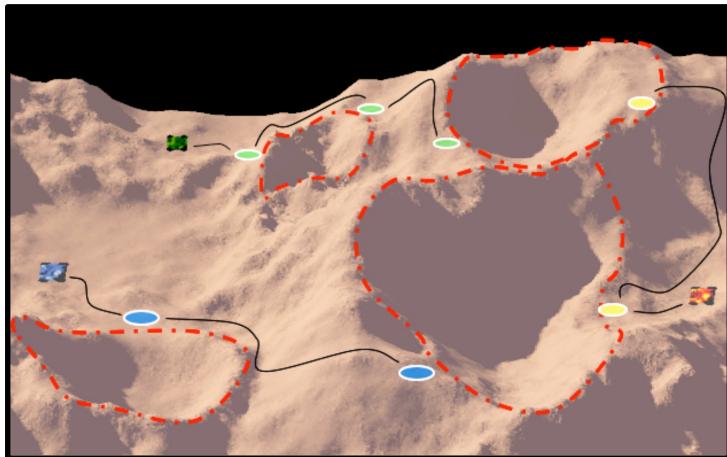
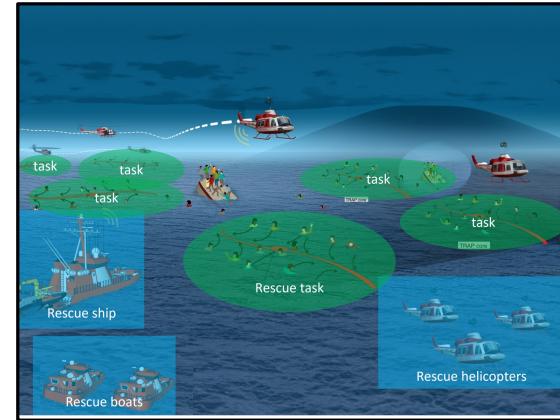
Lecture 1: MRS Survey

Lecture 2: Motion Planning

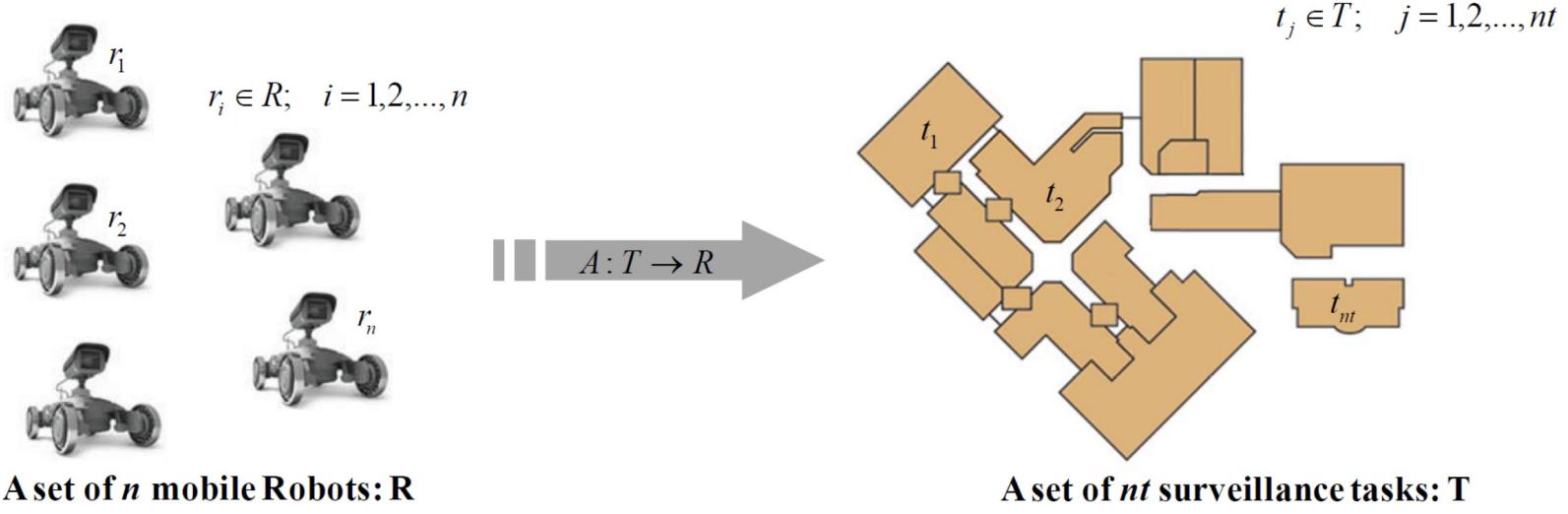
Lecture 3: Task Assignment (+Motion Planning if needed)

Lecture 4: Communication and Control Paradigms

Task Assignment Examples



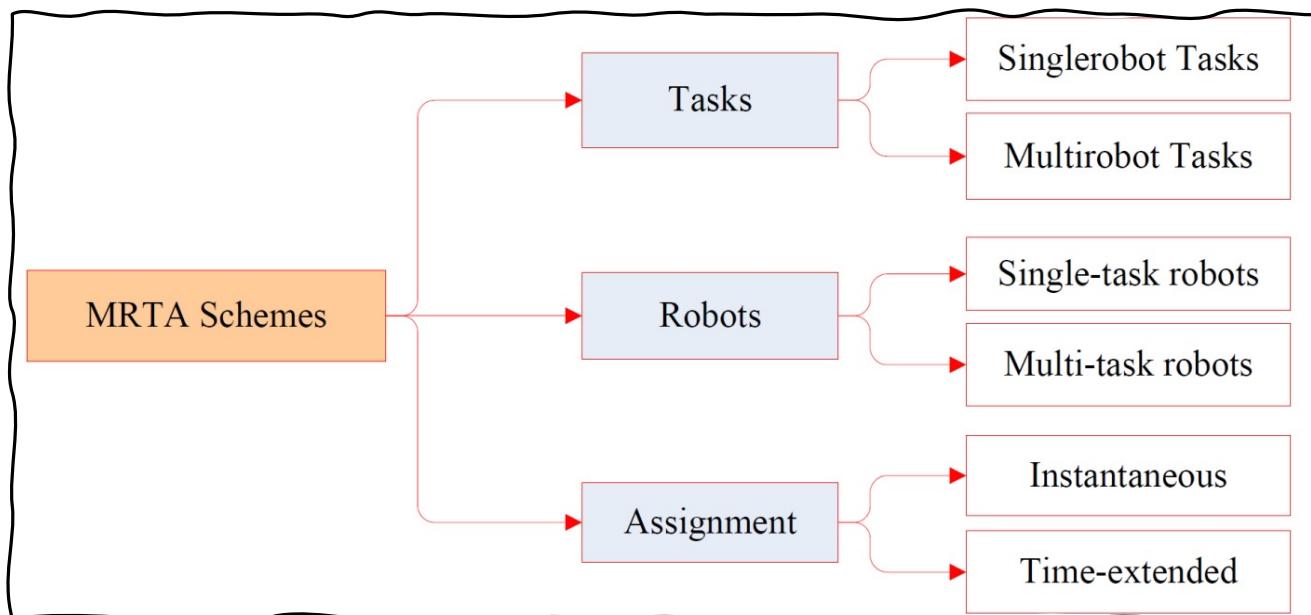
The Task Assignment Problem



U , the set of robot utilities, where u_{ij} is the utility of robot i to execute task j

The Multi-robot Task Assignment (MRTA) problem: assign set of robots to a set of tasks such that overall performance is optimized by some metric(s), subject to some set of constraints

Task Assignment: Taxonomies



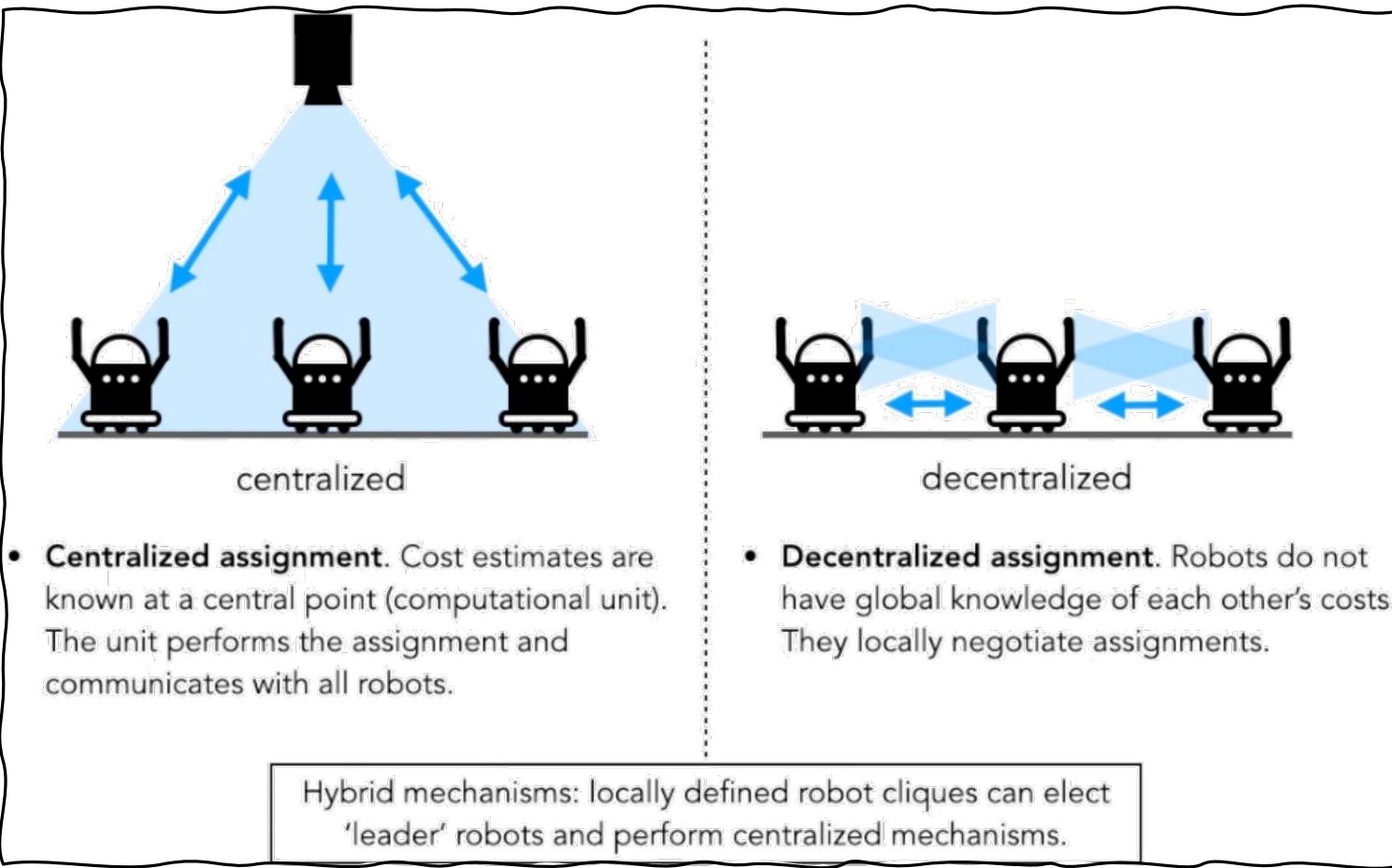
Instantaneous: Decisions made only based on current state

Time-extended: Planning over a finite time horizon

Offline: Assignments and planning done based on simulation / model

Online: Assignments and planning dynamically adjusted

Task Assignment: Taxonomies



Task Assignment: Two Major Paradigms

CENTRALIZED

“Hungarian Algorithm”

DECENTRALIZED

Market-based Allocation
(various approaches)

Approaches we will not talk about:

Centralized, math-intensive (e.g., mixed-integer linear programming and other multi-factor optimization methods)

Approaches we will talk about later in the context of swarms:

Genetic algorithms, ant colony optimization

The Hungarian Algorithm (HA)

Inputs:

Number of robots n and number of tasks m

Cost matrix \mathbf{C} , an $n \times m$ matrix (often padded to be square)

Cost metric definition for C_{ij}

Examples of cost metrics:

- Travel time or distance
- Energy consumption
- Execution time
- Task priority

Constraints on \mathbf{C} for typical HA:

- Finite, comparable values
- Non-negative costs
- Independence of costs (e.g., cost should not depend on other assignments)

The Munkres Assignment Algorithm

40	60	15
25	30	45
55	30	25



25	40	0
0	0	20
30	0	0

The Hungarian Algorithm (HA)

Step 1: Row Reduction } - $O(n^2)$

Step 2: Column Reduction } - $O(n^2)$

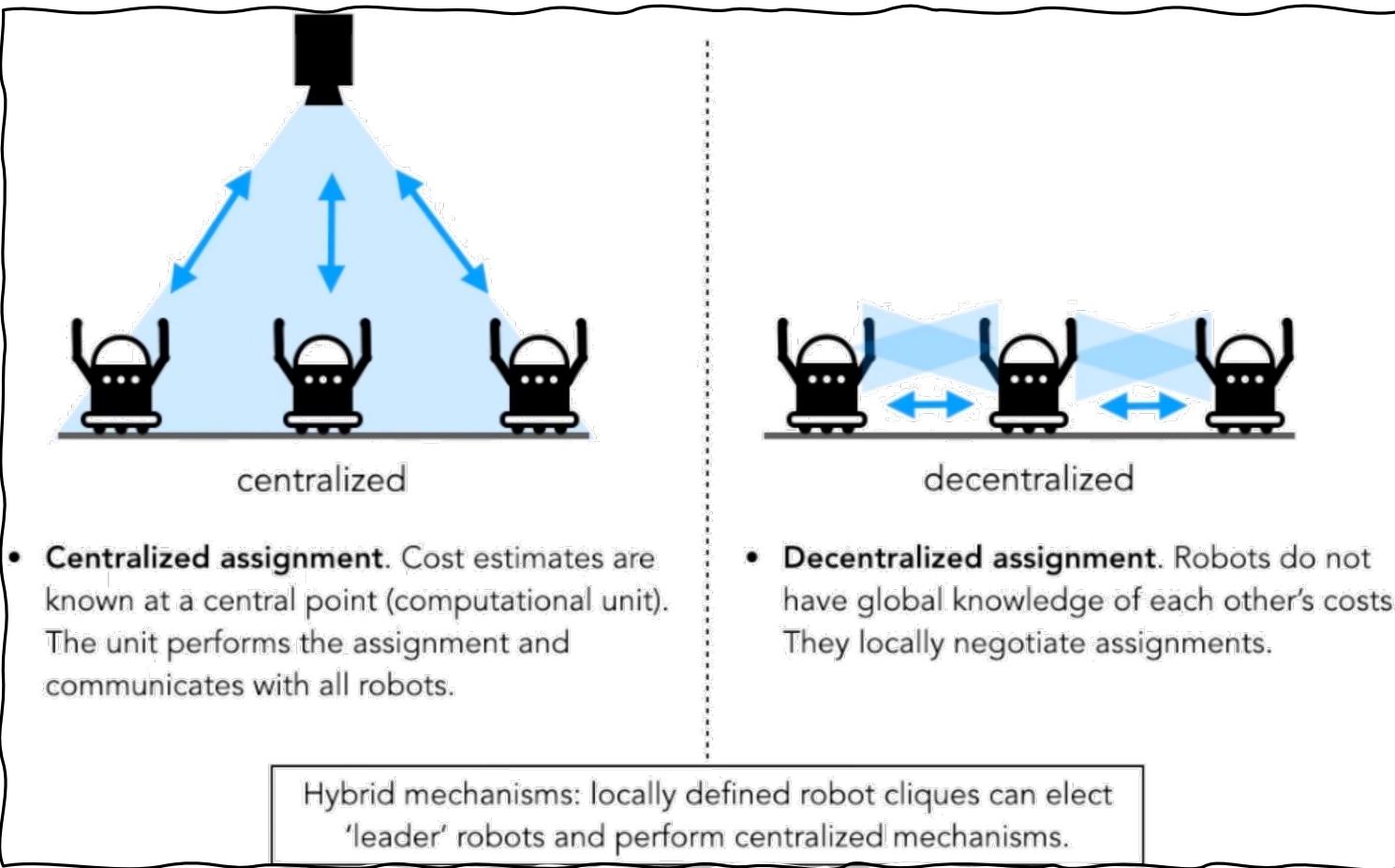
Step 3: Test for an optimal
assignment } - $n * O(n^2) = O(n^3)$

Step 4: Shift zeros } - $n * O(n^2) = O(n^3)$

Step 5: Making the final
assignment } - $O(n)$

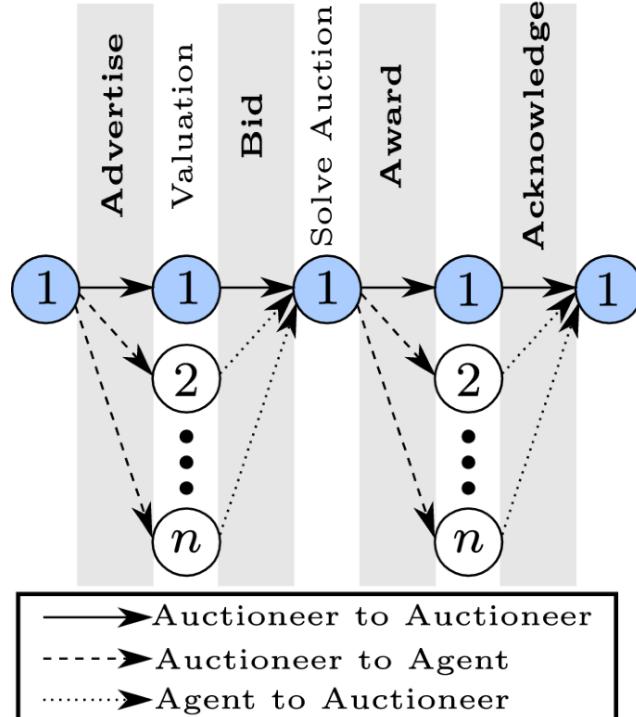
(reminder: big O notation describes how run time scales as input grows)

Task Assignment: Taxonomies



Market-based Assignment

One Auction Round



Key Assumptions:

- Robot revenue is a combination of personal and system profit
- Robots work to maximize revenue
- Robots are honest with each other

Based on the “**Efficient Market Hypothesis**” (an important basis for capitalism), this will result in near-optimal solutions to task prices.

Market-based Assignment: Single Task Trades

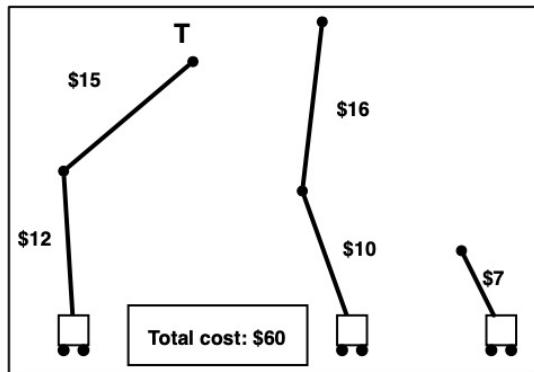
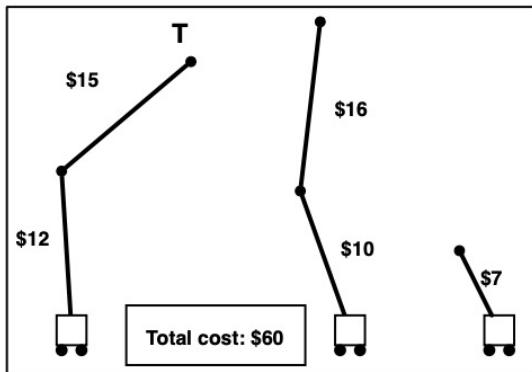
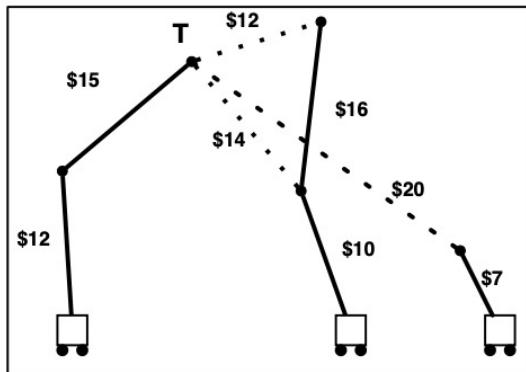


Figure 3.2: A single-task trade. (a) The initial allocation and schedules. Robot 1 can perform task T for a marginal cost of \$15. The global cost is \$60.

Market-based Assignment: Single Task Trades



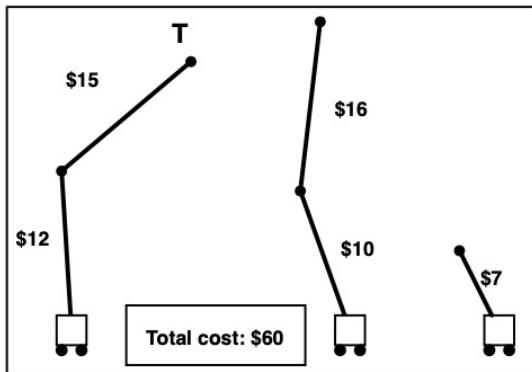
(a)



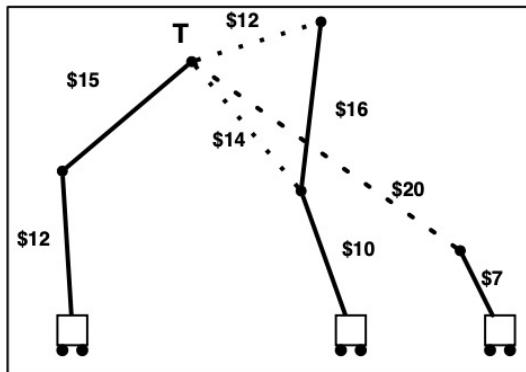
(b)

Figure 3.2: A single-task trade. (a) The initial allocation and schedules. Robot 1 can perform task T for a marginal cost of \$15. The global cost is \$60. (b) Robots 2 and 3 estimate their marginal costs for task T . It is determined that T can be inserted into robot 2's schedule for an additional cost of \$10 ($\$14 + \$12 - \16), or into robot 3's schedule for an additional cost of \$20.

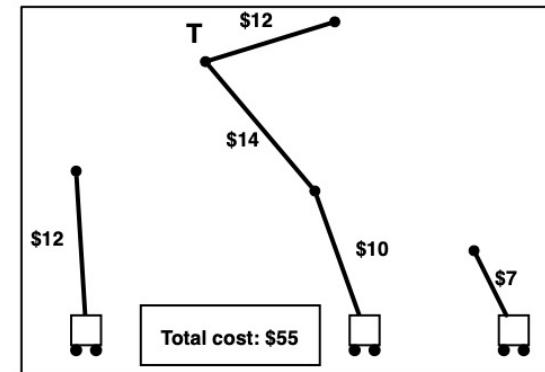
Market-based Assignment: Single Task Trades



(a)



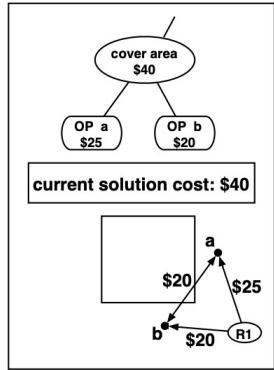
(b)



(c)

Figure 3.2: A single-task trade. (a) The initial allocation and schedules. Robot 1 can perform task T for a marginal cost of \$15. The global cost is \$60. (b) Robots 2 and 3 estimate their marginal costs for task T . It is determined that T can be inserted into robot 2's schedule for an additional cost of \$10 ($\$14 + \$12 - \16), or into robot 3's schedule for an additional cost of \$20. (c) Robot 2 is awarded task T . Global solution cost has dropped by \$5 to \$55.

Market-based Assignment: Complex Tasks → Task Trees



(a) Valuation of R_1

Figure 3.5: Simple example of a task tree auction for an area coverage task. (a) R_1 holds a task tree auction. The initial plan of R_1 is displayed along with R_1 's reserve price for the task tree.

Market-based Assignment: Complex Tasks → Task Trees

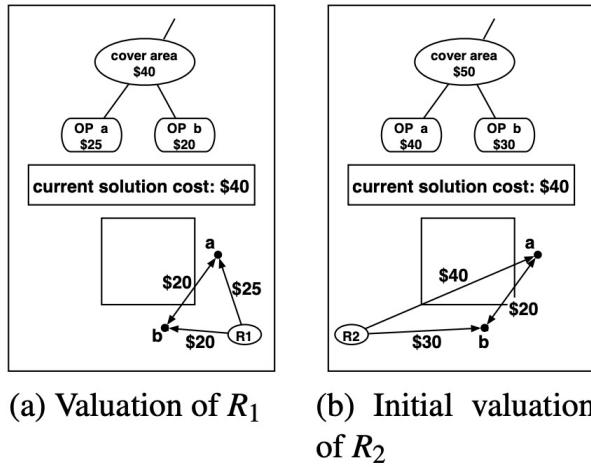


Figure 3.5: Simple example of a task tree auction for an area coverage task. (a) R_1 holds a task tree auction. The initial plan of R_1 is displayed along with R_1 's reserve price for the task tree. (b) R_2 's valuation of R_1 's tree, without replanning.

Market-based Assignment: Complex Tasks → Task Trees

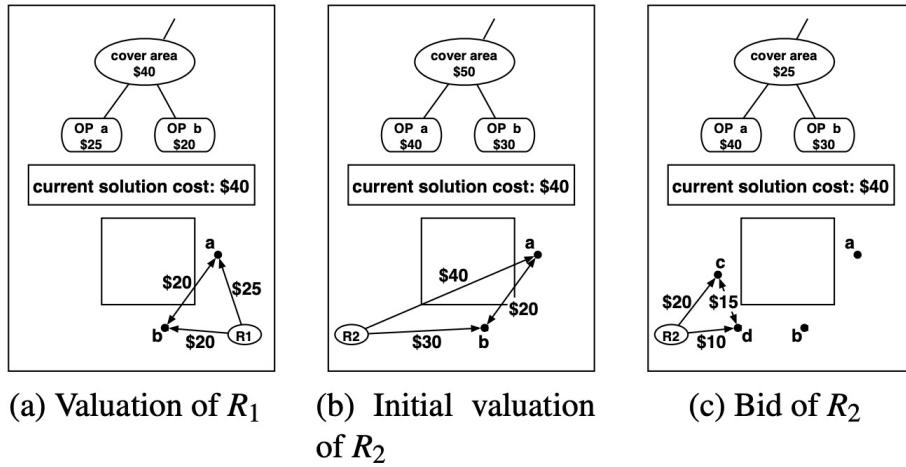


Figure 3.5: Simple example of a task tree auction for an area coverage task. (a) R_1 holds a task tree auction. The initial plan of R_1 is displayed along with R_1 's reserve price for the task tree. (b) R_2 's valuation of R_1 's tree, without replanning. (c) R_2 comes up with a different decomposition for the cover area task, and updates its bid accordingly.

Market-based Assignment: Complex Tasks → Task Trees

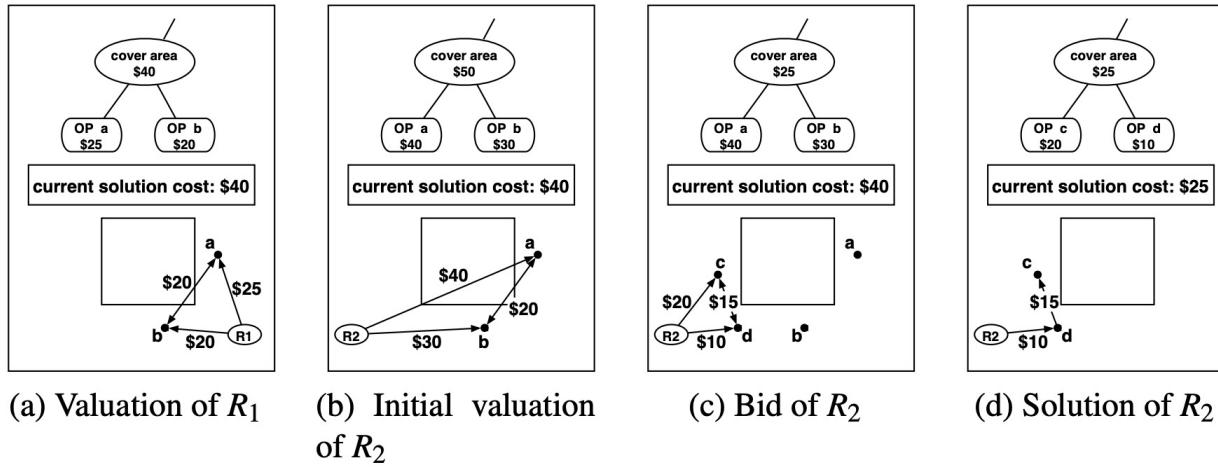


Figure 3.5: Simple example of a task tree auction for an area coverage task. (a) R_1 holds a task tree auction. The initial plan of R_1 is displayed along with R_1 's reserve price for the task tree. (b) R_2 's valuation of R_1 's tree, without replanning. (c) R_2 comes up with a different decomposition for the cover area task, and updates its bid accordingly. (d) The auction is cleared. R_2 is awarded the abstract area coverage task. R_2 's new plan is shown along with the associated task tree decomposition. The global solution cost has been reduced from \$40 to \$25.

Market-based Assignment: Complex Tasks → Task Trees

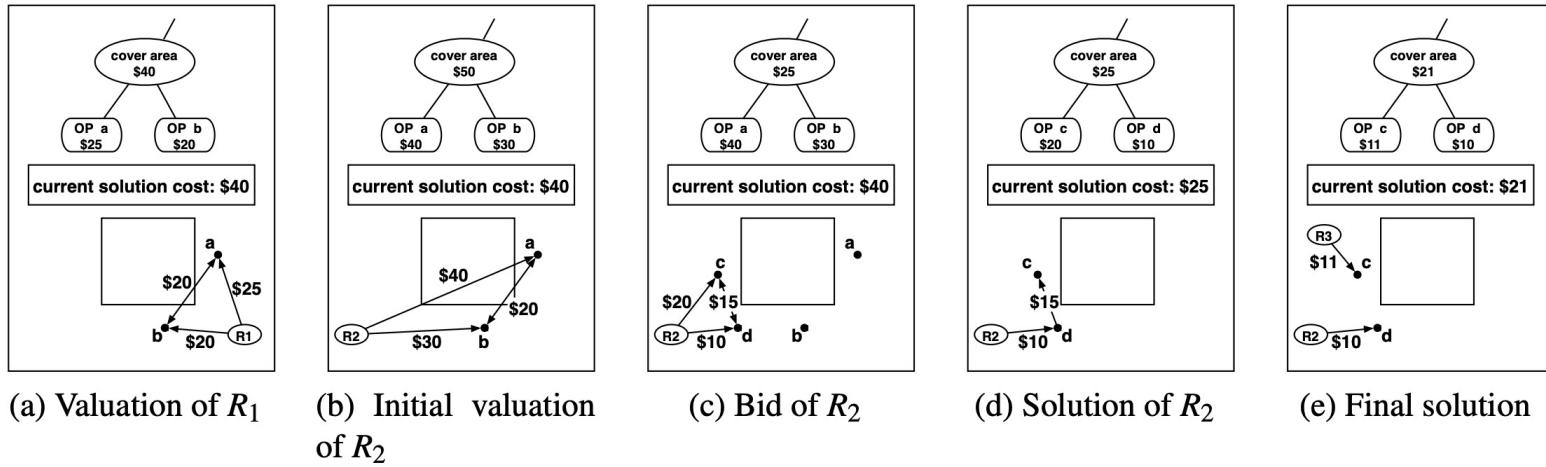


Figure 3.5: Simple example of a task tree auction for an area coverage task. (a) R_1 holds a task tree auction. The initial plan of R_1 is displayed along with R_1 's reserve price for the task tree. (b) R_2 's valuation of R_1 's tree, without replanning. (c) R_2 comes up with a different decomposition for the cover area task, and updates its bid accordingly. (d) The auction is cleared. R_2 is awarded the abstract area coverage task. R_2 's new plan is shown along with the associated task tree decomposition. The global solution cost has been reduced from \$40 to \$25. (e) R_2 holds another auction round, which results in task c being subcontracted out to R_3 . The global solution cost has been further reduced to \$21.

Market-based Assignment: *TraderBots*

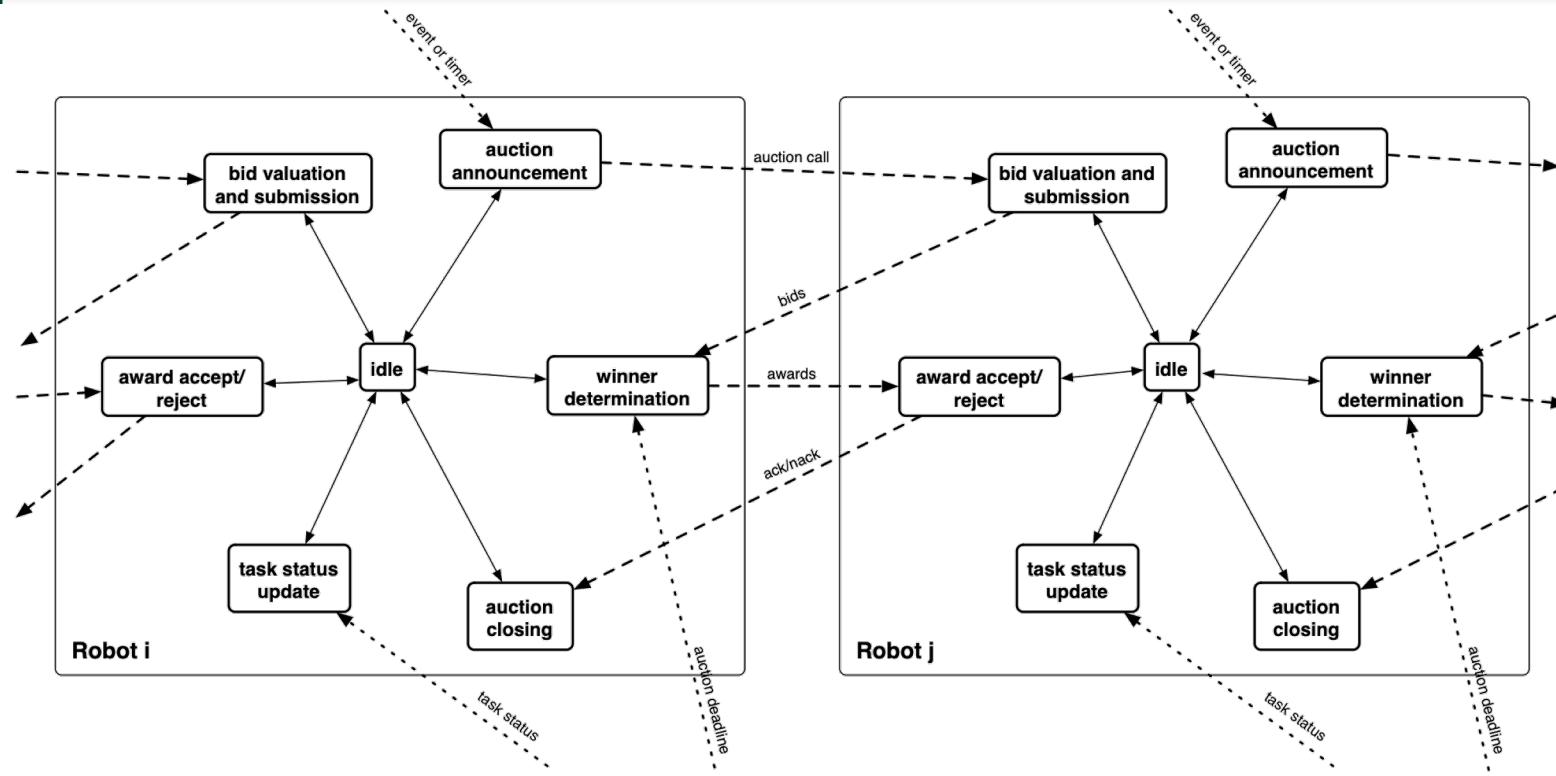
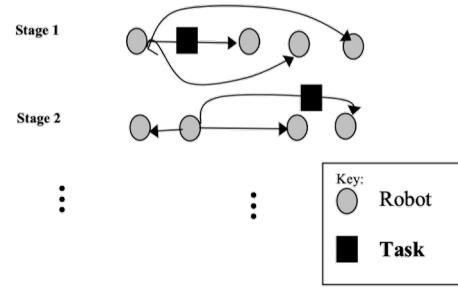


Figure 3.1: A state diagram of the *TraderBots* auction protocol. Dashed arrows represent messages and dotted arrows represent other events that trigger transitions into new states.

Market-based Assignment: *TraderBots*

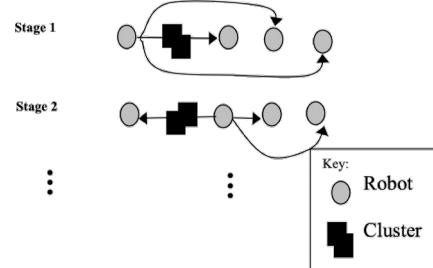
5.5.1 Two-Party, Single-Task (TPST) Negotiations

In this case, once the initial random task assignments are made, each of the robots, *in turn*, offers all its assigned tasks to all the other robots, *in turn*. Thus, interactions are limited to two parties at any given time as illustrated in Figure 21.



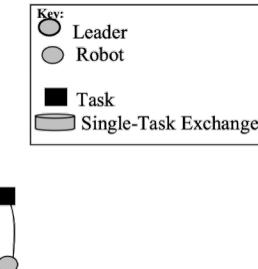
5.5.2 Two-Party, Multi-Task (TPMT) Negotiations

In this case, the previous case is repeated with clusters of tasks being the atomic unit of the negotiations as shown in Figure 22.

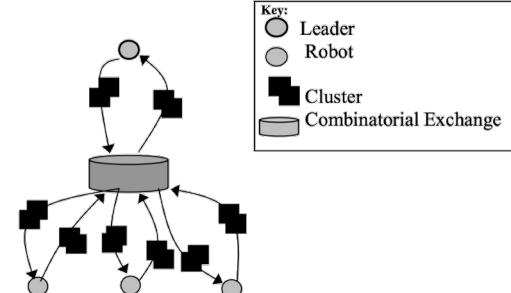


5.5.3 Leader Performing Multi-Party Single-Task (MPST) Optimizations

A leader, whose capability is restricted to dealing in single-task deals, is introduced in this case. The leader queries all the robots, and gathers all the tasks of all the robots along with each robot's state information. The leader then sets up an exchange by formulating single-task bids for the robots in the sub-group based on the gathered information.



5.5.4 Leader Performing Multi-Party, Multi-Task (MPMT) Optimizations



Contrasting These Approaches

When to use decentralized market-based approaches?

Agent count scalability (computation)

Communication challenges (network scaling, single point of failure)

Complex task interdependency (constraints on cost metric for HA)

How to choose between all the market-based approaches?

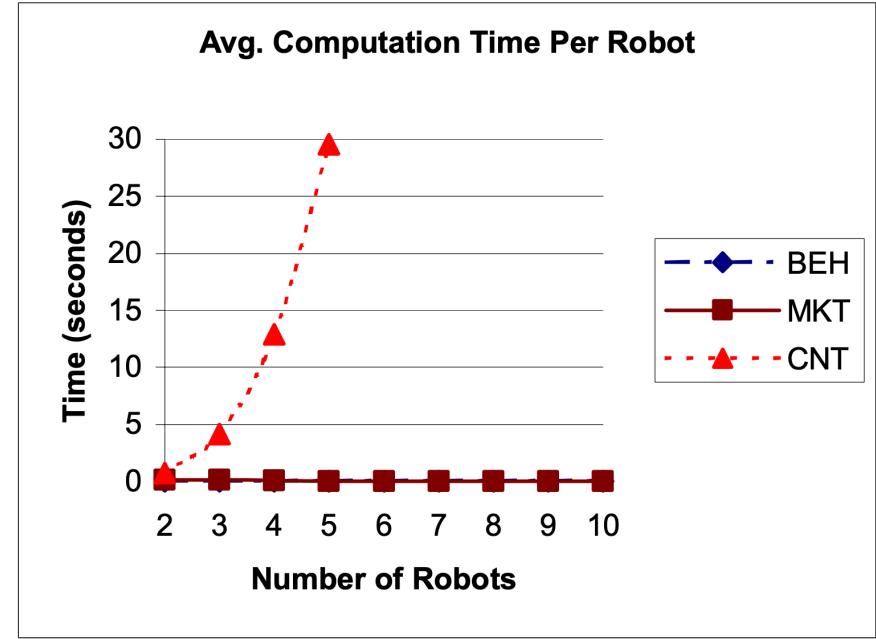
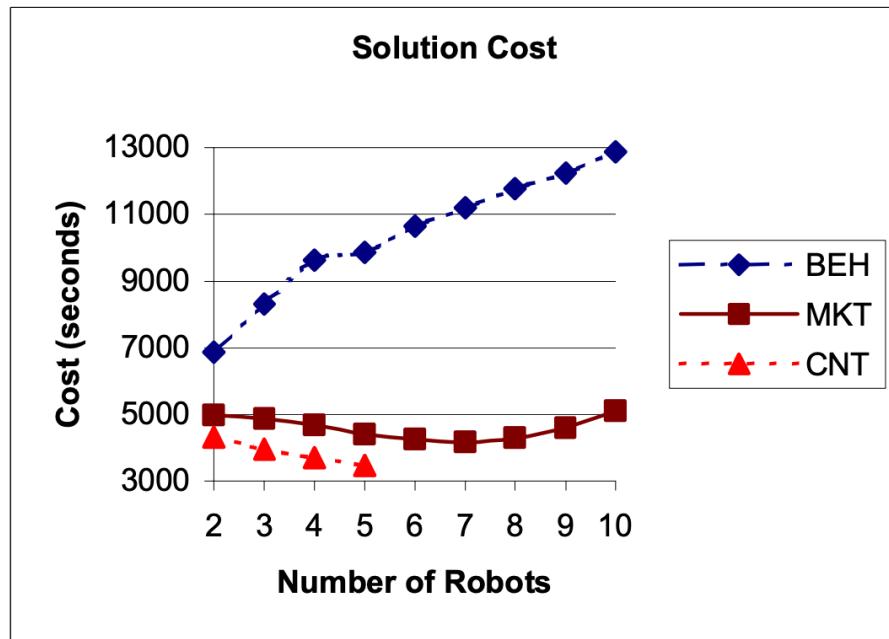
How do tasks arrive?

Number of robots and tasks?

MRS composition?

Optimality vs speed?

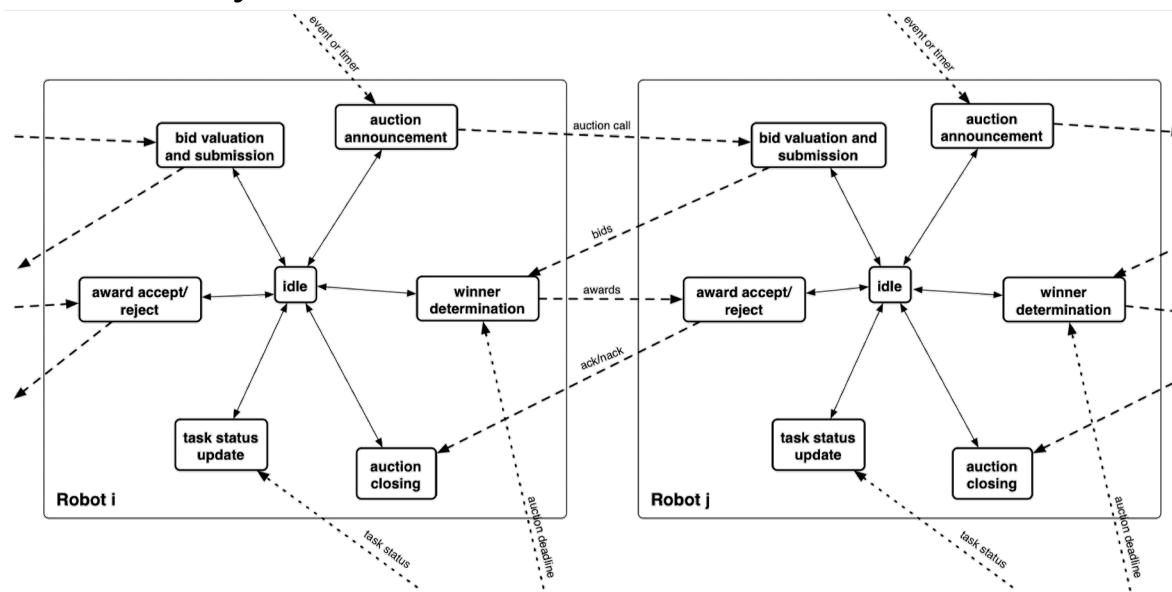
Contrasting These Approaches



Standard solutions?

Unlike for motion planning (e.g., with Nav2), there is no standard solution to MRTA in ROS2.

Exercise for you to think about: What would this look like?



Note: HA and fully centralized approaches are “easy” to implement.

1. Lecture (~40 minutes)
2. Subsystem check-in (~20 minutes)

Questions?

Lecture 1: MRS Survey

Lecture 2: Motion Planning

Lecture 3: Task Assignment (+Motion Planning if needed)

Lecture 4: Communication and Control Paradigms

693H Robotic Subsystems

- Second Design Review Presentations are **3/10, 3/12**
- You will be expected to have **functional prototypes** complete to show by then
- You will be expected to have **PCB designs and final BoMs** complete by then

