SRM Institute of Science and Technology

DEPARTMENT OF MATHEMATICS

Transforms and Boundary Value Problems

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1. The order and degree of the PDE $\frac{\partial^2 z}{\partial x^2} + 2xy(\frac{\partial z}{\partial x})^2 + \frac{\partial z}{\partial y} = 5,$ respectively

   (A) $2, 1$ (B) $1, 2$ (C) $1, 1$ (D) $2, 2$

   ANSWER: A

2. The Partial Differential Equation corresponding to $Z = (x + a)(y + b)$ is

   (A) $p^2 + q^2 = z$ (B) $pq = z$ (C) $p^2 - q^2 = z$ (D) $z = p^2q^2$

   ANSWER: B

3. The complete solution of $\sqrt{p} + \sqrt{q} = 1$ is

   (A) $z = ax + (1 + \sqrt{a})^2 y + c$ (B) $z = ax + (1 - \sqrt{a})^2 y$

   (C) $z = ax + (1 - \sqrt{a})^2 y + c$ (D) $z = ax - (1 - \sqrt{a})^2 y + c$

   ANSWER: C

4. The general solution of $px + qy = z$ is

   (A) $f(x, y) = 0$ (B) $f(\frac{x}{y}, \frac{y}{z}) = 0$ (C) $f(xy, yz) = 0$ (D) $f(x^2 + y^2) = 0$

   ANSWER: B

5. The general solution of $(y - z)p + (z - x)q = x - y$ is

   (A) $f(x + y + z) = x^2 + y^2 + z^2$ (B) $f(xyz) = x^2 + y^2 + z^2$

   (C) $f(x + y + z) = xyz$ (D) $f(x^2 + y^2 + z^2) = x^2y^2z^2$

   ANSWER: A

6. The complete solution of $z = px + qy + p^2 + q^2$ is

   (A) $z = (x + a)(y + b)$ (B) $z = ax + by + c$

   (C) $z = ax + by + c^2 + d^2$ (D) $z = ax + by + a^2 + b^2$

   ANSWER: D

7. The solution of the linear PDE $(D^2 + 4DD' - 5D'^2)z = 0$ is

   (A) $z = f_1(y + x) + f_2(y + 5x)$ (B) $z = f_1(y - x) + f_2(y - 5x)$

   (C) $z = f_1(y + x) + f_2(y - 5x)$ (D) $z = f_1(y - x) + f_2(y + 5x)$

   ANSWER: C

8. The solution of $\frac{\partial^3 z}{\partial x^3} = 0$ is

   (A) $z = (1 + x + x^2)f(y)$ (B) $z = (1 + y + y^2)f(x)$

   (C) $z = f_1(y) + xf_2(y) + x^2f_3(y)$ (D) $z = f_1(x) + yf_2(x) + y^2f_3(x)$

   ANSWER: C

9. The solution of $p+q=z$ is

(A) $f(x+y, y+logz)$ (B) $f(xy, ylogz)$

(C) $f(x-y, y-logz)$ (D) $f(xy, y-logz)$

ANSWER: C

10. The particular solution of $(D^2-2DD'+D'^2)z = sinx$

(A) $-sinx$ (B) $sinx$ (C) $cosx$ (D) $-cosx$

ANSWER: A

11. The period of $sin5x$ is

(A) $\frac{8\pi}{5}$ (B) $\frac{6\pi}{5}$ (C) $\frac{4\pi}{5}$ (D) $\frac{2\pi}{5}$

ANSWER: D

12. If $f(x) = xsinx$ in $(-\pi, \pi)$ then the value of b_n in Fourier series expansion is

(A) 0 (B) 1 (C) 2 (D) 3

ANSWER: A

13. Fourier coefficient a_0 in the Fourier series expansion of a function represents the

(A) maximum value of the function (B) 2 mean value of the function

(C) minimum value of the function (D) mean value of the function

ANSWER: B

14. If the Fourier series of the function $f(x)$ in $(-\ell, \ell)$ has only cosine terms then $f(x)$ must be

(A) odd function (B) even function

(C) neither even nor odd function (D) multi-valued function

ANSWER: B

15. If $f(x) = x^2+x$ in $(0, \ell)$ then the even extension in $(-\ell, 0)$ is

(A) $-x^2-x$ (B) $-x^2+x$ (C) x^2+x (D) x^2-x

ANSWER: D

16. Compute the constant term $\frac{a_0}{2}$ of the Fourier series of $f(x)$ given by the following data:

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(A) 8.7 (B) 9.7 (C) 2.9 (D) 1.45

ANSWER: A

x	0	1	2	3	4	5
f(x)	9	18	24	28	26	20

17. Compute a_1 of the Fourier series of $f(x)$ given by the table:

    (A) -8.33 (B) -25 (C) -1.155 (D) 0.519

    ANSWER: A

18. The root mean square value of the function $f(x)$ over the interval (a, b) then $\bar{y} =$

    (A) $\sqrt{\int_a^b \mid f(x) \mid^2 dx}$ (B) $\sqrt{\frac{1}{b-a} \int_a^b \mid f(x) \mid^2 dx}$

    (C) $\sqrt{\frac{1}{a-b} \int_a^b \mid f(x) \mid^2 dx}$ (D) $\sqrt{\frac{1}{b-a} \int_a^b \mid f(x) \mid dx}$

    ANSWER: B

19. The period of $tan 2x$ is

    (A) $\frac{2\pi}{n}$ (B) $\frac{\pi}{n}$ (C) $\frac{\pi}{2}$ (D) $\frac{2}{\pi}$

    ANSWER: C

20. The Fourier cosine series of the function $f(t) = sin(\frac{\pi t}{\ell})$, $0 < t < \ell$ then the value of a_0 is

    (A) $\frac{1}{\pi}$ (B) $\frac{2}{\pi}$ (C) $\frac{3}{\pi}$ (D) $\frac{4}{\pi}$

    ANSWER: D

21. In one dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$, a^2 stands for

    (A) $\frac{T}{m}$ (B) $\frac{k}{c}$ (C) $\frac{m}{T}$ (D) $\frac{k}{m}$

    ANSWER: A

22. One dimensional wave equation is used to find the

    (A) time (B) displacement (C) heat flow (D) mass

    ANSWER: B

23. Heat flows from

    (A) higher to lower temperature (B) lower to higher temperature

    (C) constant temperature (D) uniform temperature

    ANSWER: A

24. The steady state temperature of the rod of length $20cm$ whose ends are kept at $30C$ and $80C$ is

    (A) $30 - \frac{5}{2}x$ (B) $30 + \frac{2}{5}x$ (C) $10 + \frac{5}{2}x$ (D) $30 + \frac{5}{2}x$

    ANSWER: D

25. The tension T caused by stretching the string before fixing it at the end points is

(A) decreasing (B) increasing (C) zero (D) constant

ANSWER: D

26. The amount of heat required to produce a given temperature change in a body is proportional to the

(A) mass of the body (B) weight of the body

(C) density of the body (D) Tension of the body

ANSWER: A

27. The one dimensional heat equation is of the form

(A) $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$ (B) $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$

(C) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ (D) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$

ANSWER: B

28. The proper solution of one dimensional heat flow equation is $u(x, t)$

(A) $Ax + B$ (B) $(Ae^{\lambda x} + Be^{-\lambda x})e^{\alpha^2 \lambda^2 t}$

(C) $(A \cos px + B \sin px)e^{-\alpha^2 \lambda^2 t}$ (D) $(Ae^{\lambda x} + Be^{-\lambda x})(Ce^{\lambda at} + De^{-\lambda at})$

ANSWER: C

29. The slope of the deflection curve in vibrating string is assumed to be

(A) small at all points and at all times

(B) large at all points and at all times

(C) small at all points but not at all times

(D) large at all points but not at all times

ANSWER: A

30. How many initial and boundary conditions are required to solve the equation $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$

(A) 1 (B) 2 (C) 3 (D) 4

ANSWER: D

31. If $F[f(x)] = F(s)$, then $F[f(x - a)] =$

(A) $e^{isa}F(s)$ (B) $e^{-isa}F(s)$ (C) $e^{isx}F(s)$ (D) $e^{is(x-a)}F(s)$

ANSWER: A

32. If $F[f(x)] = F(s)$, then $F[f(ax)] =$

(A) $F(\frac{s}{a})$ (B) $F(\frac{a}{s})$ (C) $\frac{1}{|a|}F(\frac{a}{s})$ (D) $\frac{1}{|a|}F(\frac{s}{a})$

ANSWER: D

33. If $F[f(x)] = F(s)$, then $F[e^{iax}f(x)] =$

(A) $F(s-a)$ (B) F(s+a) (C) $e^{isa}F(s)$ (D) $e^{-isa}F(s)$

ANSWER: B

34. If $f(x) = e^{-ax}$, then Fourier sine transform of $f(x)$ is

(A) $\sqrt{\frac{2}{\pi}}\frac{a}{s^2+a^2}$ (B) $\sqrt{\frac{2}{\pi}}\frac{s}{s^2+a^2}$ (C) $\sqrt{\frac{\pi}{2}}\frac{a}{s^2+a^2}$ (D) $\sqrt{\frac{\pi}{2}}\frac{s}{s^2+a^2}$

ANSWER: B

35. If $f(x) = e^{-ax}$, then Fourier cosine transform of $f(x)$ is

(A) $\sqrt{\frac{2}{\pi}}\frac{a}{s^2+a^2}$ (B) $\sqrt{\frac{2}{\pi}}\frac{s}{s^2+a^2}$ (C) $\sqrt{\frac{\pi}{2}}\frac{a}{s^2+a^2}$ (D) $\sqrt{\frac{\pi}{2}}\frac{s}{s^2+a^2}$

ANSWER: A

36. If $f(x) = \frac{1}{x}$, then Fourier sine transform of $f(x)$ is

(A) $\sqrt{\frac{\pi}{2}}$ (B) $\sqrt{\frac{2}{\pi}}$ (C) $\frac{\pi}{2}$ (D) $\frac{2}{\pi}$

ANSWER: A

37. Under Fourier cosine transform $f(x) = \frac{1}{\sqrt{x}}$ is

(A) cosine function (B) sine function

(C) self reciprocal function (D) complex function

ANSWER: C

38. If $F[f(x)] = F(s)$, then $\int_{-\infty}^{\infty} \mid f(x) \mid^2 dx =$

(A) $\int_{-\infty}^{\infty} \mid F_s(s) \mid^2 ds$ (B) $\int_{-\infty}^{\infty} \mid F_c(s) \mid^2 ds$

(C) $\int_{0}^{\infty} \mid F(s) \mid^2 ds$ (D) $\int_{-\infty}^{\infty} \mid F(s) \mid^2 ds$

ANSWER: D

39. The Fourier transform of a function $f(x)$ is

(A) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$ (B) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(s)e^{-isx}ds$

(C) $\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$ (D) $\frac{1}{\sqrt{2\pi}}\int_{0}^{\infty} f(x)e^{isx}dx$

ANSWER: A

40. The Fourier cosine transform of $5e^{-2x}$ is

(A) $\sqrt{\frac{2}{\pi}}\frac{10}{s^2+4}$ (B) $\sqrt{\frac{2}{\pi}}\frac{2}{s^2+4}$ (C) $\sqrt{\frac{2}{\pi}}\frac{s}{s^2+4}$ (D) $\sqrt{\frac{2}{\pi}}\frac{5s}{s^2+4}$

ANSWER: A

41. If $Z[f(n)] = F(z)$, then $Z(\frac{1}{3^n}) =$

(A) $\frac{z}{z-1}$ (B) $\frac{3z}{3z-1}$ (C) $\frac{z}{z-3}$ (D) $\frac{z}{z-3^n}$

ANSWER: B

42. If $Z[f(n)] = F(z)$, then $Z(3^n \sin \frac{n\pi}{2}) =$

(A) $\frac{z^2}{z^2+9}$ (B) $\frac{z}{z^2+9}$ (C) $\frac{\frac{z}{3}}{(\frac{z}{3})^2+1}$ (D) $\frac{z}{z-3^n}$

ANSWER: C

43. If $Z[f(n)] = F(z)$, then $Z[a^n n] =$

(A) $\frac{z}{(z-a)^2}$ (B) $\frac{az^2+a^2z}{(z-a)^3}$ (C) $\frac{az}{(z-a)^2}$ (D) $\frac{z^2+z}{(z-1)^3}$

ANSWER: C

44. If $Z[f(n)] = F(z)$, then $Z^{-1}[\frac{z}{(z-1)(z-2)}] =$

(A) $1 - 2^n$ (B) $2^n + 1$ (C) $-2^n - 1$ (D) $2^n - 1$

ANSWER: D

45. If $Z[f(n)] = F(z)$, then $Z^{-1}[\frac{z}{z-a}] =$

(A) a^n (B) na^n (C) n^2a^n (D) $(-a)^n$

ANSWER: A

46. If $Z[f(n)] = F(z)$, then the poles of $F(z) = \frac{z}{(z-1)(z-2)}$ are

(A) $z = -1,\ z = 2$ (B) $z = 1,\ z = 2$

(C) $z = 1,\ z = -2$ (D) $z = -1,\ z = -2$

ANSWER: B

47. If $F(z)z^{n-1} = \frac{z^n}{(z-1)(z-2)}$, then the residue of $F(z)z^{n-1}$ at each pole, respectively

(A) $1,\ 2^n$ (B) $-1,\ 2^n$ (C) $1,\ (-2)^n$ (D) $-1,\ -2$

ANSWER: B

48. If $Z[f(n)] = F(z)$, then $Z[(-3)^n] =$

(A) $\frac{z}{(z-3)^2}$ (B) $\frac{z}{z+3}$ (C) $\frac{z}{(z+3)^2}$ (D) $\frac{z}{z-3}$

ANSWER: B

49. If $Z[f(n)] = F(z)$, then $Z[K] =$

(A) $\frac{Kz}{z-1}$ (B) $\frac{Kz}{z+1}$ (C) $\frac{z}{z+1}$ (D) $\frac{z}{z-1}$

ANSWER:A

50. If $Z[f(n)] = F(z)$, then $Z[e^{-5n}] =$

(A) $\frac{z}{z+e^{-5}}$ (B) $\frac{z}{z-e^5}$ (C) $\frac{z}{z-e^{-5}}$ (D) $\frac{z}{z+e^5}$

ANSWER:C

51. If $Z[f(n)] = F(z)$, then $Z[\frac{1}{n!}] =$

(A) $e^{-\frac{1}{z}}$ (B) e^z (C) $e^{\frac{1}{z}}$ (D) e^{-z}

ANSWER: C

52. If $Z[f(n)] = F(z)$, then $Z^{-1}[\frac{z^2}{(z-a)^2}] =$

(A) $(n+1)(-a)^n$ (B) $(n-1)(-a)^n$ (C) $(n+1)(a)^n$ (D) $(n-1)(a)^n$

ANSWER: C

53. If $Z[f(n)] = F(z)$, then $Z[a^n \cos\frac{n\pi}{2}] =$

(A) $\frac{az^2}{z^2+a^2}$ (B) $\frac{z^2}{z^2+a^2}$ (C) $\frac{az^2}{z^2-a^2}$ (D) $\frac{az}{z^2+a^2}$

ANSWER: B

SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS

18MAB201T - Transforms and Boundary value problems

UNIT V – Z-Transform

TUTORIAL SHEET -15

PART-B		
Sl.No	**Questions**	**Answer**
1	Find $Z^{-1}\left[\dfrac{2z^2+3z}{(z+2)(z-4)}\right]$ by partial fraction method.	$\dfrac{(-2)^n}{6}+\dfrac{11(4)^n}{6}$
2	If $F(z)=\dfrac{3z}{(z-1)\ (z-2)}$, find the residue of $F(z)z^{n-1}$ at $z=2$.	$3.(2)^n$
3	If $F(z)=\dfrac{z+3}{(z+1)\ (z-2)}$, find the residue of $F(z)z^{n-1}$ at $z=-1$.	$\dfrac{2}{3}(-1)^n$
4	Using convolution Theorem evaluate $Z^{-1}\left[\dfrac{z^2}{(z-1)(z-3)}\right]$.	$\dfrac{1}{2}(3^{n+1}-1)$
5	Solve: $y_{n+1}-2y_n=0$ given $y_0=3$ using Z transforms.	$3.(2)^n$
PART-C		
6	Find the inverse Z transform of $\dfrac{8z^2}{(2z-1)(4z-1)}$ by using convolution theorem.	$2\left[\left(\dfrac{1}{2}\right)^n-\left(\dfrac{1}{2}\right)^{2n+1}\right]$
7	Find by Residue method if $Z^{-1}\left[\dfrac{2z^2+4z}{(z-2)^3}\right]$.	$n^2.(2)^n$
8	Find $Z^{-1}\left[\dfrac{z^3}{(z-2)(z-1)^2}\right]$ by using method of partial fraction.	$-3.1^n-n+4.2^n$
9	Solve: $y_{n+2}+6y_{n+1}+9y_n=2^n$, given $y_0=y_1=0$, using Z transforms.	$\dfrac{1}{25}.2^n-\dfrac{1}{25}(-3)^n+\dfrac{1}{15}n(-3)^n$
10	Solve $y_{n+2}-7y_{n+1}+12y_n=2^n$, given $y_0=0$, $y_1=0$, using Z transform method.	$\dfrac{1}{2}.2^n-3^n+\dfrac{1}{2}.\ 4^n$

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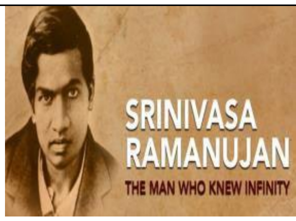
18MAB201T Transforms and Boundary Value Problems

UNIT – V : Z Transforms
Tutorial Sheet - 14



Sl.No.	Questions	Answer
	Part – B	
1	Find the z-transforms of $\sin\frac{n\pi}{2}$.	$z\left\{\sin\frac{n\pi}{2}\right\}=\frac{z}{z^2+1}$
2	Find the z-transforms of $\sin^3\left(\frac{n\pi}{6}\right)$.	$z\left\{\sin^3\left(\frac{n\pi}{6}\right)\right\}=\frac{3z}{4(z^2-z\sqrt{3}+1)}-\frac{z}{4(z^2+1)}$
3	Find the z-transforms of $\sin^2\left(\frac{n\pi}{4}\right)$.	$z\left\{\sin^2\left(\frac{n\pi}{4}\right)\right\}=\frac{z}{2(z-1)}-\frac{z^2}{2(z^2+1)}$
4	Use initial value theorem to find $f(0)$ when $\overline{f}(z)=\frac{ze^{aT}(ze^{aT}\cos bT)}{z^2e^{2aT}-2ze^{aT}\cos bT+1}$.	$f(0)=1$
5	Use final value theorem to find $f(\infty)$ when $\overline{f}(z)=\frac{Tze^{aT}}{(ze^{aT}-1)^2}$.	$f(\infty)=0$
	Part – C	
6	Find the inverse z-transforms of (i) $\frac{z^2+z}{(z-1)^2}$ (ii) $\frac{2z^2+4z}{(z-2)^3}$ by long division division method.	(i) $f(n)=2n+1$ (ii) $f(n)=n^2 2^n$
7	Find the inverse z-transform of $\frac{1+2z^{-1}}{1-z^{-1}}$ by long division division method.	$f(n)=1+2u(n-1)$
8	Find the inverse z-transform of $\frac{5z}{(2z-1)(z-3)}$ by partial fraction method.	$f(n)=3^n-\frac{1}{2^n}$
9	Find the inverse z-transform of $\frac{z^2+2z}{(z-1)(z-2)(z-3)}$ by partial fraction method.	$f(n)=\frac{3}{2}-4.2^n+\frac{5}{2}.3^n$
10	Find the inverse z-transform of $\frac{4z^2-12z}{z^3-3z+2}$ by partial fraction method.	$f(n)=\frac{20}{9}-\frac{8}{3}n-\frac{20}{9}(-2)^n$

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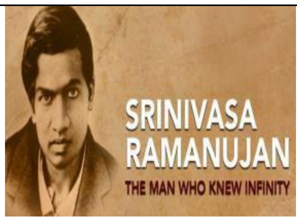
18MAB201T Transforms and Boundary Value Problems

UNIT - V : Z Transforms
Tutorial Sheet - 13



Sl.No.	Questions	Answer
	Part – B	
1	Find the z-transforms of $\cos\frac{n\pi}{2}$.	$z\left\{\cos\frac{n\pi}{2}\right\}=\frac{z^2}{z^2+1}$
2	Find the z-transforms of $\cos^2 t$.	$z\left\{\cos^2 t\right\}=\frac{z}{2(z-1)}+\frac{z(z-\cos 2T)}{2(z^2-2z\cos 2T+1)}$
3	If $z[f(n)]=F(z)$, then prove that $z[a^{-n}f(n)]=F(az)$.	
4	If $z[f(n)]=F(z)$, then prove that $z[a^n f(n)]=F\left(\frac{z}{a}\right)$.	
5	Find $z[n^2+a^{n+3}]$.	$\frac{z(z+1)}{(z-1)^3}+\frac{a^3z}{z-a}$
	Part – C	
6	f $z[f(n)]=F(z)$, then prove that $z[f(n-k)]=z^{-k}F(z)$ for $k>0$.	
7	Find $z[(n+1)(n+2)]$.	$\frac{z(z+1)}{(z-1)^3}+\frac{3z}{(z-1)^2}+\frac{2z}{z-1}$
8	Find $z\left[\frac{2n+3}{(n+1)(n+2)}\right]$.	$(z^2+z)\log\left(\frac{z}{z-1}\right)-z$
9	Find $z\left[\frac{1}{n(n-1)}\right]$.	$\left(\frac{z-1}{z}\right)\log\left(\frac{z-1}{z}\right)$
10	Find the z-transforms of $\cos\left(\frac{n\pi}{2}+\frac{\pi}{4}\right)$.	$\frac{1}{\sqrt{2}}\frac{z(z-1)}{(z^2+1)}$

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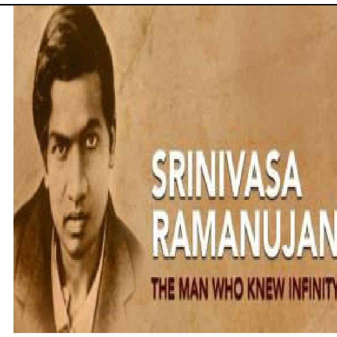
DEPARTMENT OF MEATHEMATICS

18MAB201T- TRANSFORMS AND BOUNDARY VALUE PROBLEMS

UNIT - I Partial Differential Equations
Tutorial Sheet - 3



Sl. No.	Questions	Answer
Part - A		
1	**Solve** $(D^2-3DD'+2D'^2)z=0.$	$z=\phi_1(y+x)+\phi_2(y+2x)$
2	**Solve** $(D^2-4DD'+4D'^2)z=e^{2x+y}.$	$z=\phi_1(y+2x)+x\phi_2(y+2x)+\frac{x^2}{2}e^{2x+y}$
3	**Solve** $(D^3-2D^2D')z=4\sin(x+y).$	$z=\phi_1(y)+x\phi_2(y)+\phi_3(y+2x)-4\cos(x+y)$
4	**Solve** $(D^2-6DD'+5D'^2)z=xy.$	$z=\phi_1(y+x)+\phi_2(y+5x)+\frac{x^3y}{6}+\frac{x^4}{4}$
5	**Solve** $(D^2-DD')z=\sin x\sin 2y.$	$z=\phi_1(y)+\phi_2(y+x)$ $-\frac{1}{3}(2\cos x\cos 2y-\sin x\sin 2y)$
Part - B		
6	**Solve** $(D^2+2DD'+D'^2)z=2\cos y-x\sin y.$	$z=\phi_1(y-x)+x\phi_2(y-x)$ $+x\sin y+2\cos y$
7	**Solve** $(D^3+D^2D'-DD'^2-D'^3)z=e^x\cos(2y).$	$z=\phi_1(y-x)+x\phi_2(y-x)+\phi_3(y+x)$ $+\frac{e^x}{25}(\cos 2y+2\sin 2y)$
8	**Solve** $(D^3-2D^2D')z=\sin(x+2y)+3x^2y.$	$z=\phi_1(y)+x\phi_2(y)+\phi_3(y+2x)$ $-\frac{1}{3}\cos(x+2y)+\frac{x^5y}{20}+\frac{x^6}{60}$
9	**Solve** $(D^2+DD'-6D'^2)z=y\cos x.$	$z=\phi_1(y+2x)+\phi_2(y-3x)$ $+\sin x-y\cos x$
10	**Solve** $(D^2-3DD'+2D'^2)z=(2+4x)e^{x+2y}.$	$z=\phi_1(y+x)+\phi_2(y+2x)$ $+\frac{2}{9}e^{x+2y}(11+6x)$

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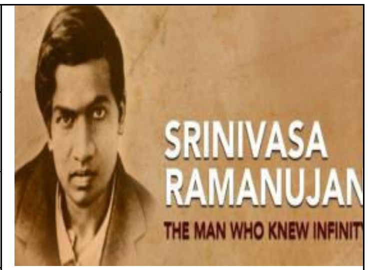
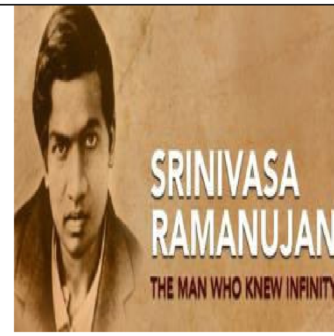
18MAB201T- TRANSFORMS AND BOUNDARY VALUE PROBLEMS

UNIT - I Partial Differential Equations
Tutorial Sheet - 1



Sl. No.	Questions	Answer
Part - A		
1	**Form the PDE by eliminating arbitrary constants 'a' and 'b' from** $(x-a)^2+(x-b)^2+z^2=c^2$	$(p^2+q^2+1)z^2=c^2$
2	**Form the PDE by eliminating arbitrary constants 'a' and 'b' from** $\log(az-1)=x+ay+b$	$p=q(z-p)$
3	**Eliminate the arbitrary function 'f' from** $z=f(x^2+y^2)$	$py=qx$
4	**Solve** $\sqrt{p}+\sqrt{q}=1$	$z=ax+(1-\sqrt{a})^2 y+c$
5	**Solve the equation** $pq+p+q=0$	$z=ax-\dfrac{a}{a+1}y+c$
Part - B		
6	**Form the PDE by eliminating 'f' from** $f(x^2+y^2+z^2, xyz)=0$	$x(y^2-z^2)p+y(z^2-x^2)q$ $=z(x^2-y^2)$
7	**Form the PDE by eliminating 'f' from** $z=xy+f(x^2+y^2+z^2)$	$p(y+xz)-q(x+yz)$ $=y^2-x^2$
8	**Form the PDE by eliminating 'f' from** $xyz=f(x+y+z)$	$x(y-z)p+y(z-x)q$ $=z(x-y)$
9	**Form the PDE by eliminating 'f' and 'g' from** $z=f(x+ct)+g(x-ct)$	$q^2=c^2p^2$
10	**Obtain the PDE by eliminating 'a' , 'b' and 'c' from** $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1$	$zs+pq=0$

	SRM Institute of Science and Technology Kattankulathur **DEPARTMENT OF MEATHEMATICS** **18MAB201T- TRANSFORMS AND BOUNDARY VALUE PROBLEMS** **UNIT - I Partial Differential Equations** **Tutorial Sheet - 2**	

Sl. No.	Questions	Answer
	Part - A	
1	**Find the singular integral of the PDE** $z = px + qy + p^2 - q^2$	$4z = y^2 - x^2$
2	**Find the complete integral of the PDE** $\sqrt{p} + \sqrt{q} = \sqrt{x}$	$z = \frac{x^2}{2} + ax - \frac{4\sqrt{a}}{3} x^{\frac{3}{2}} + ay + c$
3	**Solve** $yp = 2xy + \log q$	$z = x^2 + ax + \frac{e^{ay}}{a} + c$
4	**Find the complete integral of** $p + q = \sin x + \sin y$	$z = ax - \cos x - \cos y - ay + c$
5	**Solve** $p \tan x + q \tan y = \tan z$	$\phi\left(\frac{\sin x}{\sin y}, \frac{\sin y}{\sin z}\right) = 0$
	Part - B	
6	**Solve** $(3z - 4y)p + (4x - 2z)q = 2y - 3x$	$\phi\left(x^2 + y^2 + z^2, 2x + 3y + 4z\right) = 0$
7	**Solve** $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$	$\phi\left(\frac{x-y}{y-z}, xy + yz + zx\right) = 0$
8	**Solve** $(2z - y)p + (x + z)q + 2x + y = 0$	$\phi\left(x^2 + y^2 + z^2, z + 2y - x\right) = 0$
9	**Solve** $(y + z)p + (z + x)q = x + y$	$\phi\left(\frac{x-y}{y-z}, (x+y+z)(x-y)^2\right) = 0$
10	**Solve** $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$	$\phi\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}, xyz\right) = 0$

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DEPARTMENT OF MATHEMATICS

Transforms and Boundary Value Problems

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1. The order and degree of the PDE $\frac{\partial^2 z}{\partial x^2} + 2xy(\frac{\partial z}{\partial x})^2 + \frac{\partial z}{\partial y} = 5,$ respectively

   (A) $2, 1$ (B) $1, 2$ (C) $1, 1$ (D) $2, 2$

   ANSWER: A

2. The Partial Differential Equation corresponding to $Z = (x + a)(y + b)$ is

   (A) $p^2 + q^2 = z$ (B) $pq = z$ (C) $p^2 - q^2 = z$ (D) $z = p^2q^2$

   ANSWER: B

3. The complete solution of $\sqrt{p} + \sqrt{q} = 1$ is

   (A) $z = ax + (1 + \sqrt{a})^2y + c$ (B) $z = ax + (1 - \sqrt{a})^2y$

   (C) $z = ax + (1 - \sqrt{a})^2y + c$ (D) $z = ax - (1 - \sqrt{a})^2y + c$

   ANSWER: C

4. The general solution of $px + qy = z$ is

   (A) $f(x, y) = 0$ (B) $f(\frac{x}{y}, \frac{y}{z}) = 0$ (C) $f(xy, yz) = 0$ (D) $f(x^2 + y^2) = 0$

   ANSWER: B

5. The general solution of $(y - z)p + (z - x)q = x - y$ is

   (A) $f(x + y + z) = x^2 + y^2 + z^2$ (B) $f(xyz) = x^2 + y^2 + z^2$

   (C) $f(x + y + z) = xyz$ (D) $f(x^2 + y^2 + z^2) = x^2y^2z^2$

   ANSWER: A

6. The complete solution of $z = px + qy + p^2 + q^2$ is

   (A) $z = (x + a)(y + b)$ (B) $z = ax + by + c$

   (C) $z = ax + by + c^2 + d^2$ (D) $z = ax + by + a^2 + b^2$

   ANSWER: D

7. The solution of the linear PDE $(D^2 + 4DD' - 5D'^2)z = 0$ is

   (A) $z = f_1(y + x) + f_2(y + 5x)$ (B) $z = f_1(y - x) + f_2(y - 5x)$

   (C) $z = f_1(y + x) + f_2(y - 5x)$ (D) $z = f_1(y - x) + f_2(y + 5x)$

   ANSWER: C

8. The solution of $\frac{\partial^3 z}{\partial x^3} = 0$ is

   (A) $z = (1 + x + x^2)f(y)$ (B) $z = (1 + y + y^2)f(x)$

   (C) $z = f_1(y) + xf_2(y) + x^2f_3(y)$ (D) $z = f_1(x) + yf_2(x) + y^2f_3(x)$

   ANSWER: C

9. The solution of $p+q=z$ is

(A) $f(x+y, y+logz)$ (B) $f(xy, ylogz)$

(C) $f(x-y, y-logz)$ (D) $f(xy, y-logz)$

ANSWER: C

10. The particular solution of $(D^2-2DD'+D'^2)z=sinx$

(A) $-sinx$ (B) $sinx$ (C) $cosx$ (D) $-cosx$

ANSWER: A

11. The period of $sin5x$ is

(A) $\frac{8\pi}{5}$ (B) $\frac{6\pi}{5}$ (C) $\frac{4\pi}{5}$ (D) $\frac{2\pi}{5}$

ANSWER: D

12. If $f(x)=xsinx$ in $(-\pi,\pi)$ then the value of b_n in Fourier series expansion is

(A) 0 (B) 1 (C) 2 (D) 3

ANSWER: A

13. Fourier coefficient a_0 in the Fourier series expansion of a function represents the

(A) maximum value of the function (B) 2 mean value of the function

(C) minimum value of the function (D) mean value of the function

ANSWER: B

14. If the Fourier series of the function $f(x)$ in $(-\ell,\ell)$ has only cosine terms then $f(x)$ must be

(A) odd function (B) even function

(C) neither even nor odd function (D) multi-valued function

ANSWER: B

15. If $f(x)=x^2+x$ in $(0,\ell)$ then the even extension in $(-\ell,0)$ is

(A) $-x^2-x$ (B) $-x^2+x$ (C) x^2+x (D) x^2-x

ANSWER: D

16. Compute the constant term $\frac{a_0}{2}$ of the Fourier series of $f(x)$ given by the following data:

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(A) 8.7 (B) 9.7 (C) 2.9 (D) 1.45

ANSWER: A

x	0	1	2	3	4	5
f(x)	9	18	24	28	26	20

17. Compute a_1 of the Fourier series of $f(x)$ given by the table:

(A) -8.33 (B) -25 (C) -1.155 (D) 0.519

ANSWER: A

18. The root mean square value of the function $f(x)$ over the interval (a, b) then $\bar{y} =$

(A) $\sqrt{\int_a^b \mid f(x) \mid^2 dx}$ (B) $\sqrt{\frac{1}{b-a}\int_a^b \mid f(x) \mid^2 dx}$

(C) $\sqrt{\frac{1}{a-b}\int_a^b \mid f(x) \mid^2 dx}$ (D) $\sqrt{\frac{1}{b-a}\int_a^b \mid f(x) \mid dx}$

ANSWER: B

19. The period of $tan2x$ is

(A) $\frac{2\pi}{n}$ (B) $\frac{\pi}{n}$ (C) $\frac{\pi}{2}$ (D) $\frac{2}{\pi}$

ANSWER: C

20. The Fourier cosine series of the function $f(t) = sin(\frac{\pi t}{\ell})$, $0 < t < \ell$ then the value of a_0 is

(A) $\frac{1}{\pi}$ (B) $\frac{2}{\pi}$ (C) $\frac{3}{\pi}$ (D) $\frac{4}{\pi}$

ANSWER: D

21. In one dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$, a^2 stands for

(A) $\frac{T}{m}$ (B) $\frac{k}{c}$ (C) $\frac{m}{T}$ (D) $\frac{k}{m}$

ANSWER: A

22. One dimensional wave equation is used to find the

(A) time (B) displacement (C) heat flow (D) mass

ANSWER: B

23. Heat flows from

(A) higher to lower temperature (B) lower to higher temperature

(C) constant temperature (D) uniform temperature

ANSWER: A

24. The steady state temperature of the rod of length $20cm$ whose ends are kept at $30C$ and $80C$ is

(A) $30 - \frac{5}{2}x$ (B) $30 + \frac{2}{5}x$ (C) $10 + \frac{5}{2}x$ (D) $30 + \frac{5}{2}x$

ANSWER: D

25. The tension T caused by stretching the string before fixing it at the end points is

(A) decreasing (B) increasing (C) zero (D) constant

ANSWER: D

26. The amount of heat required to produce a given temperature change in a body is proportional to the

(A) mass of the body (B) weight of the body

(C) density of the body (D) Tension of the body

ANSWER: A

27. The one dimensional heat equation is of the form

(A) $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$ (B) $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$

(C) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ (D) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$

ANSWER: B

28. The proper solution of one dimensional heat flow equation is $u(x, t)$

(A) $Ax + B$ (B) $(Ae^{\lambda x} + Be^{-\lambda x})e^{\alpha^2 \lambda^2 t}$

(C) $(A \cos px + B \sin px)e^{-\alpha^2 \lambda^2 t}$ (D) $(Ae^{\lambda x} + Be^{-\lambda x})(Ce^{\lambda at} + De^{-\lambda at})$

ANSWER: C

29. The slope of the deflection curve in vibrating string is assumed to be

(A) small at all points and at all times

(B) large at all points and at all times

(C) small at all points but not at all times

(D) large at all points but not at all times

ANSWER: A

30. How many initial and boundary conditions are required to solve the equation $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$

(A) 1 (B) 2 (C) 3 (D) 4

ANSWER: D

31. If $F[f(x)] = F(s)$, then $F[f(x - a)] =$

(A) $e^{isa}F(s)$ (B) $e^{-isa}F(s)$ (C) $e^{isx}F(s)$ (D) $e^{is(x-a)}F(s)$

ANSWER: A

32. If $F[f(x)] = F(s)$, then $F[f(ax)] =$

(A) $F(\frac{s}{a})$ (B) $F(\frac{a}{s})$ (C) $\frac{1}{|a|}F(\frac{a}{s})$ (D) $\frac{1}{|a|}F(\frac{s}{a})$

ANSWER: D

33. If $F[f(x)] = F(s)$, then $F[e^{iax}f(x)] =$

(A) $F(s-a)$ (B) F(s+a) (C) $e^{isa}F(s)$ (D) $e^{-isa}F(s)$

ANSWER: B

34. If $f(x) = e^{-ax}$, then Fourier sine transform of $f(x)$ is

(A) $\sqrt{\frac{2}{\pi}}\frac{a}{s^2+a^2}$ (B) $\sqrt{\frac{2}{\pi}}\frac{s}{s^2+a^2}$ (C) $\sqrt{\frac{\pi}{2}}\frac{a}{s^2+a^2}$ (D) $\sqrt{\frac{\pi}{2}}\frac{s}{s^2+a^2}$

ANSWER: B

35. If $f(x) = e^{-ax}$, then Fourier cosine transform of $f(x)$ is

(A) $\sqrt{\frac{2}{\pi}}\frac{a}{s^2+a^2}$ (B) $\sqrt{\frac{2}{\pi}}\frac{s}{s^2+a^2}$ (C) $\sqrt{\frac{\pi}{2}}\frac{a}{s^2+a^2}$ (D) $\sqrt{\frac{\pi}{2}}\frac{s}{s^2+a^2}$

ANSWER: A

36. If $f(x) = \frac{1}{x}$, then Fourier sine transform of $f(x)$ is

(A) $\sqrt{\frac{\pi}{2}}$ (B) $\sqrt{\frac{2}{\pi}}$ (C) $\frac{\pi}{2}$ (D) $\frac{2}{\pi}$

ANSWER: A

37. Under Fourier cosine transform $f(x) = \frac{1}{\sqrt{x}}$ is

(A) cosine function (B) sine function

(C) self reciprocal function (D) complex function

ANSWER: C

38. If $F[f(x)] = F(s)$, then $\int_{-\infty}^{\infty} \mid f(x) \mid^2 dx =$

(A) $\int_{-\infty}^{\infty} \mid F_s(s) \mid^2 ds$ (B) $\int_{-\infty}^{\infty} \mid F_c(s) \mid^2 ds$

(C) $\int_{0}^{\infty} \mid F(s) \mid^2 ds$ (D) $\int_{-\infty}^{\infty} \mid F(s) \mid^2 ds$

ANSWER: D

39. The Fourier transform of a function $f(x)$ is

(A) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$ (B) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(s)e^{-isx}ds$

(C) $\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$ (D) $\frac{1}{\sqrt{2\pi}}\int_{0}^{\infty} f(x)e^{isx}dx$

ANSWER: A

40. The Fourier cosine transform of $5e^{-2x}$ is

(A) $\sqrt{\frac{2}{\pi}}\frac{10}{s^2+4}$ (B) $\sqrt{\frac{2}{\pi}}\frac{2}{s^2+4}$ (C) $\sqrt{\frac{2}{\pi}}\frac{s}{s^2+4}$ (D) $\sqrt{\frac{2}{\pi}}\frac{5s}{s^2+4}$

ANSWER: A

41. If $Z[f(n)] = F(z)$, then $Z(\frac{1}{3^n}) =$

(A) $\frac{z}{z-1}$ (B) $\frac{3z}{3z-1}$ (C) $\frac{z}{z-3}$ (D) $\frac{z}{z-3^n}$

ANSWER: B

42. If $Z[f(n)] = F(z)$, then $Z(3^n \sin \frac{n\pi}{2}) =$

(A) $\frac{z^2}{z^2+9}$ (B) $\frac{z}{z^2+9}$ (C) $\frac{\frac{z}{3}}{(\frac{z}{3})^2+1}$ (D) $\frac{z}{z-3^n}$

ANSWER: C

43. If $Z[f(n)] = F(z)$, then $Z[a^n n] =$

(A) $\frac{z}{(z-a)^2}$ (B) $\frac{az^2+a^2z}{(z-a)^3}$ (C) $\frac{az}{(z-a)^2}$ (D) $\frac{z^2+z}{(z-1)^3}$

ANSWER: C

44. If $Z[f(n)] = F(z)$, then $Z^{-1}[\frac{z}{(z-1)(z-2)}] =$

(A) $1 - 2^n$ (B) $2^n + 1$ (C) $-2^n - 1$ (D) $2^n - 1$

ANSWER: D

45. If $Z[f(n)] = F(z)$, then $Z^{-1}[\frac{z}{z-a}] =$

(A) a^n (B) na^n (C) n^2a^n (D) $(-a)^n$

ANSWER: A

46. If $Z[f(n)] = F(z)$, then the poles of $F(z) = \frac{z}{(z-1)(z-2)}$ are

(A) $z = -1,\ z = 2$ (B) $z = 1,\ z = 2$

(C) $z = 1,\ z = -2$ (D) $z = -1,\ z = -2$

ANSWER: B

47. If $F(z)z^{n-1} = \frac{z^n}{(z-1)(z-2)}$, then the residue of $F(z)z^{n-1}$ at each pole, respectively

(A) $1,\ 2^n$ (B) $-1,\ 2^n$ (C) $1,\ (-2)^n$ (D) $-1,\ -2$

ANSWER: B

48. If $Z[f(n)] = F(z)$, then $Z[(-3)^n] =$

(A) $\frac{z}{(z-3)^2}$ (B) $\frac{z}{z+3}$ (C) $\frac{z}{(z+3)^2}$ (D) $\frac{z}{z-3}$

ANSWER: B

49. If $Z[f(n)] = F(z)$, then $Z[K] =$

(A) $\frac{Kz}{z-1}$ (B) $\frac{Kz}{z+1}$ (C) $\frac{z}{z+1}$ (D) $\frac{z}{z-1}$

ANSWER:A

50. If $Z[f(n)] = F(z)$, then $Z[e^{-5n}] =$

(A) $\frac{z}{z+e^{-5}}$ (B) $\frac{z}{z-e^5}$ (C) $\frac{z}{z-e^{-5}}$ (D) $\frac{z}{z+e^5}$

ANSWER:C

51. If $Z[f(n)] = F(z)$, then $Z[\frac{1}{n!}] =$

(A) $e^{-\frac{1}{z}}$ (B) e^z (C) $e^{\frac{1}{z}}$ (D) e^{-z}

ANSWER: C

52. If $Z[f(n)] = F(z)$, then $Z^{-1}[\frac{z^2}{(z-a)^2}] =$

(A) $(n+1)(-a)^n$ (B) $(n-1)(-a)^n$ (C) $(n+1)(a)^n$ (D) $(n-1)(a)^n$

ANSWER: C

53. If $Z[f(n)] = F(z)$, then $Z[a^n \cos\frac{n\pi}{2}] =$

(A) $\frac{az^2}{z^2+a^2}$ (B) $\frac{z^2}{z^2+a^2}$ (C) $\frac{az^2}{z^2-a^2}$ (D) $\frac{az}{z^2+a^2}$

ANSWER: B

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Ramapuram Campus

Department of Mathematics

Continuous Assessment I

18MAB201T/ Transforms and Boundary Value Problems

Part-A

1.	The order and degree of a PDE $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=0$ is		(CLO-1 Remember)
	a) 2,1 b) 1,2 c) 2,2 d) 1,1	**Ans: (a)**	
2.	The order and degree of a PDE $\left(\frac{\partial z}{\partial x}\right)^3+\frac{\partial^2 z}{\partial y^2}=\cos(x+y)$ is		(CLO-1 Remember)
	a)1,2 b) 2,1 c) 1,3 d) 3,1	**Ans: (b)**	
3.	While forming the PDE, if the number of arbitrary constants to be eliminated is equal to the number of independent variables,then the resulting PDE will be of __________ order.		(CLO-1 Remember)
	a)1^{st} b) 2^{nd} c) 3^{rd} d) >1	**Ans: (a)**	
4.	The complete integral of $z=px+qy+p^2q^2$ is		(CLO-1 Apply)
	a)$z=ax+by+a^2b^2$ b) $z=px+qy$ c) $z=ax+by$ d) $z=ax+by$+c	**Ans: (a)**	
5.	The solution of $(D^2-3DD'+2D'^2)Z=0$ is		(CLO-1 Apply)
	a) $z=\varphi_1(y+x)+\varphi_2(y+2x)$ b) $z=Ae^x+Be^{2x}$ c) $z=\varphi_1(y+2x)+\varphi_2(y-x)$ d) a$z=Ae^x+Be^{2x}$	**Ans: (a)**	
6.	The P.I of $(D^2+4DD'^2)z=e^x$ is		(CLO-1 Apply)
	a) e^x b) e^{-x} c) e^{2x} d) 0	**Ans: (a)**	
7	The P.I of $(D^3-2D^2D')z=4\sin(x+y)$ is		(CLO-1 Apply)
	a) $4\sin(x+y)$ b) $-4\cos(x+y)$ c) $4\cos(x+y)$ d) 0	**Ans: (b)**	

8.	The complementary function of $(D^2+2DD'+D'^2)Z = xy$ is		
	a) $\varphi_1(y-x)+x\varphi_2(y-x)$ b) $(A+Bx)e^{-x}$ c) $\varphi_1(y-2x)+x\varphi_2(y-x)$ d)2x $(A+Bx)e^{-x}$	**Ans: (a)**	(CLO-1 Apply)
9.	The P.I of $\frac{\partial^3 z}{\partial z^3}-2\frac{\partial^3 z}{\partial x^2 \partial y}=e^{x+2y}$ is		
	a) $\frac{1}{3}e^{x+2y}$ b) $-\frac{1}{3}e^{x+2y}$ c) $-e^{x+2y}$ d) $x\frac{1}{3}e^{x+2y}$	**Ans: (b)**	(CLO-1 Apply)
10.	The P.I of $(D^2-2DD')z=e^{2x}$ *is*		
	a) e^{2x} b) $\frac{1}{4}e^{-2x}$ c) $\frac{1}{4}e^{2x}$ d) 0	**Ans: (a)**	(CLO-1 Apply)
11.	The P.I of $(D^2-2DD'+D'^2)z=\cos(x-3y)$ is		
	a) $-\frac{1}{16}\cos(x-3y)$ b) $\frac{1}{16}\cos(x-3y)$ c) $\cos(x-3y)$ d) 0	**Ans: (a)**	(CLO-1 Apply)
12	The complete integral of $z=px+qy+p^2q^2$ is		(CLO-1 Apply)
	(a)$z=ax+by+a^2b^2$ b) $z=px+qy$ c) $z=ax+by$ d) $z=px+qy$+2	**Ans: (a)**	
13	The complete integral of F(p,q) =0 is		(CLO-1 Apply)
	a) 0 b) px + qy + c c) Z = ax + f(a)y + c d)1	**Ans: (c)**	
14.	While forming the PDE, if the number of arbitrary constants to be eliminated is more than the number of independent variables, then the resulting PDE will be of __________ order.		(CLO-1 Remember)
	a) 1^{st} b) 2^{nd} and higher c) only 3^{rd} d) only 2^{nd}	**Ans: (b)**	
15.	The complete integral of z= px+qy+p+q is		
	a)z=ax+by+c b) z= ax+by+a+b c)z=ax+by+b d) z= ax+by+a	**Ans: (b)**	(CLO-1 Apply)
16.	The P.I of $(D^2-2DD'+D'^2)Z=8e^{x+2y}$ is		(CLO-1 Apply)

	a) e^{x+2y} b) $8e^{x+2y}$ c) 8 d) 0	**Ans: (b)**	
17.	The P.I of $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x\partial y}-6\frac{\partial^2 z}{\partial y^2}=\cos(3x+2y)$ is		(CLO-1 Apply)
	a) $\frac{1}{9}\cos(3x+2y)$ b) $-\frac{1}{9}\cos(3x+2y)$ c) 0 d) 1	**Ans: (a)**	

18.	The solution of $(D^3-3D^2D'+2DD'^2)z=0$ is		(CLO-1 Apply)
	a) $z=f_1(y)+f_2(y+x)+f_3(y+2x)$ b) $z=f_1(y)+f_2(y-x)+f_3(y+2x)$ c) $z=f_1(y)+f_2(y+x)+f_3(y-2x)$ d) xy $z=f_1(y)+f_2(y+x)+f_3(y-2x)$	Ans: (a)	
19.	The P.I of $(D^2-2DD')z=4\sin(x+y)$ is		(CLO-1 Apply)
	a) $4\sin(x+y)$ b) $-4\cos(x+y)$ c) $4\cos(x+y)$ d) 0	Ans: (b)	
20.	The solution of $r-4s+4t=0$ is		(CLO-1 Apply)
	a) $z=(A+Bx)e^{2x}$ b) $z=\varphi_1(y+2x)+x\varphi_2(y+2x)$ c) $z=\varphi_1(y+x)+\varphi_2(y+2x)$ d) z=1	Ans: (b)	
21.	The complete integral of $\sqrt{p}+\sqrt{q}=1$ is		(CLO-1 Apply)
	a) $z=ax+\left(1-\sqrt{a}\right)^2y+c$ b) $z=px+\left(1-\sqrt{a}\right)^2y+c$ c)$z=x+\left(1+\sqrt{a}\right)^2y+c$ d) $z=px+\left(1+\sqrt{a}\right)^2y+c$	**Ans: (a)**	
22.	The complete integral of $z=px+qy+p^2+q^2$ is		(CLO-1 Apply)
	a) $z=ax+by+a^2+b^2$ b) $z=px+qy$ c) $z=ax+by$ d) z=ay+bx	**Ans: (a)**	
23.	The solution of $(D^2-DD'-6D'^2)Z=0$ is		(CLO-1 Apply)

	a) $z=\varphi_1(y+3x)+\varphi_2(y-2x)$ b) $z=ax$ c) $z=Ae^x+Be^{2x}$ d) $z=\varphi_1(y+2x)+\varphi_2(y-x)$	**Ans: (a)**	
24.	Equation of the form Pp+Qq=R is called ----------		(CLO-1 Remember)
	a) Clairaut's type b) Lagrange's Linear equations c) Euler form d) Laurent's Form	**Ans: (b)**	
25.	The P.I of $(D^2-2DD'+D'^2)z=\sin(x-3y)$ is		(CLO-1 Apply)
	a) $-\frac{1}{16}\sin(x-3y)$ b) $\frac{1}{16}\sin(x-3y)$ c) $\sin(x-3y)$ d) 0	**Ans: (a)**	
26	The solution of $r-4s+4t=0$ is		(CLO-1 Apply)
	a) $z=(A+Bx)e^{2x}$ b) $z=\varphi_1(y+2x)+x\varphi_2(y+2x)$ c) $z=\varphi_1(y+x)+\varphi_2(y+2x)$ d) z=6	**Ans: (b)**	
27	While forming the PDE, if the number of arbitrary constants to be eliminated is more than the number of independent variables, then the resulting PDE will be of __________ order.		(CLO-1 Remember)
	a) 1st b) 2nd and higher c) 3rd and higher d) only 2nd	**Ans: (b)**	
28	The order and degree of a PDE $\frac{\partial z}{\partial x}+\frac{\partial^2 z}{\partial y^2}=\cos(x+y)$ is		(CLO-1 Apply)
	a) 1,2 b) 2,1 c) 1,3 d) 3,1	**Ans: (b)**	
29	The solution of $yp=2yx+\log q$ is a) $\boldsymbol{z=x^2+ax+\frac{1}{a}e^{ay}+b}$ b) $z=y^2+bx+\frac{1}{a}e^{ay}-b$ c) $z=a^2+ax+\frac{1}{a}e^{ay}+b$ d) $z=x^2-ax+\frac{1}{a}e^{ay}-b$	**Ans: (a)**	(CLO-1 Apply)
30	The complete solution of $p+q=sinx+siny$ is		(CLO-1 Apply)

	a)$z = a(x+y) - cosx - cosy + b$ b)$z = a(x-y) + cosx - cosy + b$ **c)$z = a(x-y) - cosx - cosy + b$** d)$z = a(x-y) - cosx + cosy + b$	**Ans: (c)**	
31	The solution of $z^2 = 1 + p^2 + q^2$ is a)$sinh^{-1}z = \frac{1}{\sqrt{1+a^2}}(x - ay) + b$ b)$cosh^{-1}z = \frac{1}{\sqrt{1+a^2}}(x - ay) - b$ c)$sinh^{-1}z = \frac{1}{\sqrt{1+a^2}}(x + ay) + b$ **d)$cosh^{-1}z = \frac{1}{\sqrt{1+a^2}}(x + ay) + b$**	**Ans: (d)**	(CLO-1 Apply)
32	The complete Integral of $pq = 4$ is **a) $z = ax + \frac{4}{a}y + c$** b) $z = cx - \frac{4}{a}y + c$ c) $z = ax - \frac{6}{a}y + c$ d) $z = x + \frac{4}{a}y + c$	**Ans: (a)**	(CLO-1 Apply)
33	The Particular Integral of $(D + D')^2 z = e^{x-y}$ is a)$\frac{x}{2}e^{x-y}$ b)$\frac{1}{2}e^{x-y}$ **c)$\frac{x^2}{2}e^{x-y}$** d) $\frac{x^2}{6}e^{x-y}$	**Ans: (c)**	(CLO-1 Apply)
34	The Particular Integral of $[D^2 - 6DD' + 5D'^2]z = e^x sinhy$ is **a) $\frac{-x}{8}e^{x+y} - \frac{1}{24}e^{x-y}$** b) $\frac{-x}{16}e^{x+y} - \frac{1}{8}e^{x-y}$	**Ans: (a)**	(CLO-1 Apply)

	c) $\frac{1}{8}e^{x+y} + \frac{1}{8}e^{x-y}$ d) $\frac{1}{16}e^{x+y} - \frac{x}{8}e^{x-y}$		
35.	The Particular Integral of $[D^2 - 6DD' + 5D'^2]z = xy$ is **a)** $\frac{x^3y}{6} + \frac{x^4}{4}$ b) $\frac{x^3y}{6} - \frac{x^4}{4}$ c) $\frac{x^3y}{26} + \frac{x^4}{24}$ d) $\frac{x^3y}{26} + \frac{x^4y}{4}$	**Ans: (a)**	(CLO-1 Apply)
36	The complete integral of z=px+qy+p-q is		(CLO-1 Apply)
	(a)z= ax+by+a-b b) z=ax+by c) $z = ax + by$+c d) $z = px + qy$+2	**Ans: (a)**	

SRM INSTITUTE OF SCIENCE AND TECHNOLOGY
RAMAPURAM CAMPUS
DEPARTMENT OF MATHEMATICS

Year/Sem : II/III
Branch: Common to All branches

Unit I – Partial Differential Equations

1.Form PDE of $z = ax + a^2y^2 + b$ by eliminating arbitrary constants

(a) $\boldsymbol{q = 2p^2y}$ (b) $q = 2py$ (c) $p = 2q^2y$ (d) $p = 2qy$

Solution:

Given $z = ax + a^2y^2 + b$ ------(1)

Differentiate (1) partially with respect to $x\ and\ y$,

$$p = a \quad ---\rightarrow (2)$$

$$q = 2a^2y \quad --\rightarrow (3)$$

Substituting (2) in (3) we get $q = 2p^2y$ which is the required PDE.

2. Form PDE of $2z = (ax + y)^2 + b$, by eliminating arbitrary constants

(a) $px + qy = p^2$ (b) $py + qx = q^2$ **(c)** $\boldsymbol{px + qy = q^2}$ (d) $p^2x + q^2y = q^2$

Solution:

Given $2z = (ax + y)^2 + b$ ------(1)

Differentiate (1) partially with respect to $x\ and\ y$,

$$2p = 2a(ax + y) \quad ---\rightarrow (2)$$

$$2q = 2(ax + y) \quad --\rightarrow (3)$$

Dividing (2) by (3) we get $\frac{p}{q} = a\ \ substituting\ in\ (3)\ we\ get$

$px + qy = q^2$ which is the required PDE.

3. Form PDE of $z = axe^y + \frac{a^2e^{2y}}{2} + b$, by eliminating arbitrary constants

(a) $p = xq + p^2$ **(b)** $\boldsymbol{q = xp + p^2}$ (c) $p = xq + q^2$ (d) $q = x + p^2$

Solution

Given $z = axe^y + \frac{a^2e^{2y}}{2} + b$ ------(1)

Differentiate (1) partially with respect to $x\ and\ y$,

$$p = ae^y \quad ---\rightarrow\ (2)$$

$$q = axe^y + a^2e^{2y} \quad --\rightarrow (3)$$

Substituting (2) in (3) we get $q = xp + p^2$ which is the required PDE.

4. Form PDE of $z = f(x^2 - y^2)$ by eliminating arbitrary function

(a) $\boldsymbol{yp + xq = 0}$ (b) $yp - xq = 0$ (c) $yq + xp = 0$ (d) $yq - xp = 0$

Solution:

Given $z = f(x^2 - y^2) --\rightarrow (1)$

Differentiate (1) partially with respect to $x\ and\ y$,

$$p = 2xf^{`}(x^2 - y^2) --\rightarrow (2)$$

$$q = -2yf^{`}(x^2 - y^2) --\rightarrow (3)$$

From (2) & (3) $f^{`}(x^2 - y^2) = \frac{p}{2x}$ & $f^{`}(x^2 - y^2) = \frac{q}{-2y}$

So, $\frac{p}{2x} = \frac{q}{-2y} \Rightarrow yp + xq = 0\ which\ is\ the\ required\ PDE.$

5. Form PDE of $\varphi(x^2 + y^2 + z^2, lx + my + nz) = 0$

(a) $\boldsymbol{\frac{2x+2zp}{2y+2zq} = \frac{l+np}{m+nq}}$ (b) $\frac{2y+2zq}{2x+2zp} = \frac{l+np}{m+nq}$

(c) $\frac{2x+2zp}{2y+2zq} = \frac{m+nq}{l+np}$ (d) $\frac{2x-2zp}{2y-2zq} = \frac{l+np}{m+nq}$

Solution:

$$x^2 + y^2 + z^2 = \varphi(\ lx + my + nz) --\rightarrow (1)$$

Differentiate Partially (1) with respect to $x\ and\ y$

$$2x + 2zp = (l + np)\varphi^{`}(lx + my + nz) --\rightarrow (2)$$

$$2y + 2zq = (m + nq)\varphi^{`}(lx + my + nz) --\rightarrow (3)$$

$\frac{(2)}{(3)} \Rightarrow \frac{2x+2zp}{2y+2zq} = \frac{l+np}{m+nq}$ which is the required PDE

6.Form a PDE of $z = f(x^2 + y^2)$ $by\ eliminating\ the\ arbitrary\ function$

(a) $px = yq$ (b) $p + y = xq$ (c) $py = x + q$ **(d)** $\boldsymbol{py = xq}$

Solution:

Given $z = f(x^2 + y^2) -\!\!-\!\!\to (1)$

Differentiate partially (1) with respect to $x\ and\ y\ we\ get$

$$p = 2x\, f^{`}(x^2 + y^2)$$
$$q = 2y\, f^{`}(x^2 + y^2)$$

Therefore, $\frac{p}{q} = \frac{x}{y}$ $\Rightarrow py = xq\ which\ is\ the\ required\ PDE$

7. Solve $p^2 + q^2 = 4$

(a) $\boldsymbol{z = ax \pm \sqrt{4 - a^2} + c}$ (b) $z = ax \pm \sqrt{4 + a^2} + c$

(c) $z = ax \pm \sqrt{4 - a} + c$ (d) $z = ax \pm \sqrt{4 + a} + c$

Solution:

Given $p^2 + q^2 = 4 - \,-\!\to (1)$

Let us assume that $z = ax + by + c -\!\!-\!\!\to (2)$ be a solution of (1)

Partially differentiating (2) with respect to $x\ and\ y$ we get

$$p = a, q = b$$

Substituting above in (1) we get

$$a^2 + b^2 = 4 -\!\!-\!\!\to (3)$$

From (3) we get $b = \pm\sqrt{4 - a^2}$

Substituting in (2) we get $z = ax \pm \sqrt{4 - a^2} + c$ which is the complete integral of (1). There is no singular integral of this type $f(p, q) = 0$

8. Find the complete integral of $p = q$

(a) $z = a(x - y) + c$ (b) $z = 2ax + c$

(c) $z = ax + y + c$ **(d)** $\boldsymbol{z = a(x + y) + c}$

Solution:

Given $p = q -\!\!-\!\!\to (1)$

Let us assume that $z = ax + by + c -\!\!-\!\!\to (2)$ be a solution of (1)

Partially differentiating (2) with respect to $x\ and\ y$ we get

$$p = a, q = b$$

Substituting above in (1) we get

$$a = b -\!\!-\!\!\to (3)$$

Substituting in (2) we get $z = a(x+y)+c$ which is the complete integral of (1).

9.*Find the complete solution of* $\sqrt{p}+\sqrt{q} \;=1$

(a) $z = ax + \left(1+\sqrt{a}\right)^{\frac{1}{2}}y + c$ **(b)** $\boldsymbol{z = ax + \left(1-\sqrt{a}\right)^{\frac{1}{2}}y + c}$

(c) $z = ax - \left(1-\sqrt{a}\right)^{\frac{1}{2}}y + c$ (d) $z = ax - \left(1+\sqrt{a}\right)^{\frac{1}{2}}y + c$

Solution:

Given $\sqrt{p}+\sqrt{q}-1=0 \quad --\rightarrow (1)$

The Complete Solution is given by $z = ax+by+c \quad --\rightarrow (2)$

Replace $p\ by\ a$ and $q\ by\ b\ \ in\ (1), we\ get\ \sqrt{a}+\sqrt{b}-1=0$

$\sqrt{b} = 1-\sqrt{a} \Rightarrow b = \left(1-\sqrt{a}\right)^{\frac{1}{2}}$

Substituting in (2), we get $z = ax + \left(1-\sqrt{a}\right)^{\frac{1}{2}}y + c$ which is required complete solution.

10. Find the Singular integral of $z = px+qy+pq$

(a) $z = xy$ (b) $z = -\frac{x}{y}$

(c) $\boldsymbol{z = -xy}$ (d) $z = \frac{x}{y}$

Solution:

The Complete Integral is $z = ax+by+ab --\rightarrow (1)$

Partially differentiating (1) with respect to $`a`\ and\ `b`\ \ and\ equating\ to\ 0$

$$\frac{\partial z}{\partial a} = x+b = 0 --\rightarrow (2)$$
$$\frac{\partial z}{\partial b} = y+a = 0 --\rightarrow (3)$$

From (2) and (3) $a=-y, b=-x\ substituting\ in\ (1)$

$$z = -xy - xy + xy$$

So, $z = -xy\ \ is\ required\ singular\ integral.$

11.Solve $p + q = 1$

(a) $f(x + y, y - z) = 0$ (b) $f(x - y, y + z) = 0$

(c) $\boldsymbol{f(x - y, y - z) = 0}$ (d) $f(x + y, y + z) = 0$

Solution:

$$p + q = 1 \; in \; the \; form \; of \; pP + qQ = R$$

$$\frac{dx}{1} = \frac{dy}{1} = \frac{dz}{1}$$

Comparing 1st and 2nd & 2nd and 3rd term and integrating

$$\int dx = \int dy \qquad \& \quad \int dy = \int dz$$

$$x - y = c_1, \quad y - z = c_2$$

Solution is $f(x - y, y - z) = 0$

12.Find the complete integral of $p^2 = qz$

(a) $z = ke^{a(x-ay)}$ (b) $z = ke^{x+ay}$

(c) $z = ke^{x-ay}$ **(d)** $\boldsymbol{z = ke^{a(x+ay)}}$

Solution:

Given $p^2 = qz \longrightarrow (1)$ which of the form $f(p, q, z) = 0$

Let $u = x + ay$,*where `a` is arbitary constant.*

Replace $p \; by \frac{dz}{du} \; and \; q \; by \; a\left(\frac{dz}{du}\right) \; in \; (1)$, we get

$$\frac{dz}{du} = az$$

$$\frac{dz}{z} = adu$$

Integrating both sides , $logz = au + c \quad (or) \; z = ke^{au}$

The complete integral is given by $z = ke^{a(x+ay)}$

13. Find the complete integral of $z = pq$

(a) $\boldsymbol{4az = (x + ay + b)^2}$ (b) $4az = (x + ay + b)^3$

(c) $4az = (x - ay + b)^2$ (d) $4az = (x - ay + b)^3$

Solution:

Given $z = pq \longrightarrow (1)$ which of the form $f(p, q, z) = 0$

Let $u = x + ay$,*where `a` is arbitary constant.*

Replace $p \; by \frac{dz}{du} \; and \; q \; by \; a\left(\frac{dz}{du}\right) \; in \; (1)$, we get

$$z = a\left(\frac{dz}{du}\right)^2$$
$$\frac{dz}{du} = \pm\sqrt{\frac{z}{a}}$$
$$\frac{dz}{\sqrt{z}} = \pm\frac{du}{\sqrt{a}}$$

Integrating on both sides, $\pm 2\sqrt{az} = u + k$

Squaring on both sides and substituting $u = x + ay$ $we\ get$

$4az = (x + ay + b)^2$ which is the required complete integral.

14. Solve $p^2 + q^2 = x + y$

(a) $z = \frac{2}{3}\,(x+k)^{\frac{3}{2}} + \frac{3}{2}\,(y-k)^{\frac{3}{2}}$ (b) $z = \frac{3}{2}\,(x+k)^{\frac{3}{2}} + \frac{2}{3}\,(y-k)^{\frac{3}{2}}$

(c) $\boldsymbol{z = \frac{2}{3}\,(x+k)^{\frac{3}{2}} + \frac{2}{3}\,(y-k)^{\frac{3}{2}}}$ (d) $z = \frac{3}{2}\,(x+k)^{\frac{3}{2}} + \frac{3}{2}\,(y-k)^{\frac{3}{2}}$

Solution:

The given problem can be written as

$$p^2 - x = y - q^2 \dashrightarrow (1)$$

This is of the form $f_1(x,p) = f_2(y,q)$ and there is no singular integral for this type. We will find the complete integral.

Let $p^2 - x = y - q^2 = k\ (say)$

Then $p = \sqrt{x+k}$, $q = \sqrt{y-k}$

We know that $dz = pdx + qdy$

Integrating both sides

$$z = \int \sqrt{x+k}\ dx + \int \sqrt{y-k}\ dy$$
$$z = \frac{2}{3}\,(x+k)^{\frac{3}{2}} + \frac{2}{3}\,(y-k)^{\frac{3}{2}}$$

which is the required solution

15.Solve $yp = 2yx + \log q$

(a) $\boldsymbol{z = (x^2 + kx) + \frac{e^{ky}}{k} + C}$ (b) $z = (x^2 - kx) - \frac{e^{ky}}{k} + C$

(c) $z = (x^2 + kx) - \frac{e^{ky}}{k} + C$ (d) $z = (x^2 - kx) + \frac{e^{ky}}{k} + C$

Solution:

The given problem can be written as

$$p - 2x = \frac{logq}{y} \dashrightarrow (1)$$

This is of the form $f_1(x,p) = f_2(y,q)$ and there is no singular integral for this type. We will find the complete integral.

$$p - 2x = \frac{logq}{y} = k\ (say)$$

i.e., $p - 2x = k$, $\frac{logq}{y} = k$

$$p = 2x + k, \quad logq - ky = 0$$

$$p = 2x + k, \quad q = e^{ky}$$

$$z = \int pdx + \int qdy$$

$$z = \int (2x + k)dx + \int e^{ky}\, dy$$

$$z = (x^2 + kx) + \frac{e^{ky}}{k} + C$$

$$which\ is\ the\ required\ solution$$

16. Solve $xp + yq = x$

(a) $\varphi\left(\frac{y}{x}, x - z\right) = 0$ (b) $\varphi(xy, x - z) = 0$

(c) $\boldsymbol{\varphi\left(\frac{x}{y}, x - z\right) = 0}$ (d) $\varphi\left(\frac{x}{y}, \frac{z}{x}\right) = 0$

Solution:

This is of Lagrange`s type of PDE where $P = x, Q = y, R = x$

The subsidiary equations are $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{x}$

Taking first two $\frac{dx}{x} = \frac{dy}{y}$ $and\ integrating\ we\ get\ logx = logy + logc_1$

$$ie., \frac{x}{y} = c_1 \quad So, u = \frac{x}{y}$$

Taking first and last, $\frac{dx}{x} = \frac{dz}{x}$ $\Rightarrow dx = dz\ and\ integrating\ we\ get\ x = z + c_2$

So $v = x - z$

The Solution is given by $\varphi(u, v) = 0\ ie., \varphi\left(\frac{x}{y}, x - z\right) = 0$

17.*Solve* $\left(D^2 - 4DD^` - 5D^{`2}\right)z = 0$

(a)$z = f_1(y-x) + f_2(y-5x)$ **(b)**$\boldsymbol{z = f_1(y-x) + f_2(y+5x)}$

(c)$z = f_1(y+x) + f_2(y+5x)$ (d) $z = f_1(y-x) + f_2(y-5x)$

Solution:

Replace $D\ by\ m\ and\ D^`by\ 1$

The auxiliary equation is given by $m^2 - 4m - 5 = 0$

$$\Rightarrow m = 5, -1$$

$The\ general\ solution\ is$ given by

$$z = f_1(y-x) + f_2(y+5x)$$

18. Solve $25r - 40s + 16t = 0$

(a)$\boldsymbol{z = f_1(5y+4x) + f_2(5y+4x)}$ (b) $z = f_1(5y+4x) + f_2(4y+5x)$

(c)$z = f_1(4y+5x) + f_2(5y+4x)$ (d) $z = f_1(y+4x) + f_2(5y+x)$

Solution

Since $r = \frac{\partial^2 z}{\partial x^2} = D^2 z$, $t = \frac{\partial^2 z}{\partial y^2} = D^{`2} z$ $s = \frac{\partial^2 z}{\partial x \partial y} = DD^` z$

The auxiliary equation is given by $25m^2 - 40m + 16 = 0 \Rightarrow (5m-4)^2 = 0$

So, $m = \frac{4}{5}, \frac{4}{5}$

$The\ general\ solution\ is$ given by

$$z = f_1(5y+4x) + f_2(5y+4x)$$

19. *Solve* $\frac{\partial^3 z}{\partial z^3} = 0$

(a) $z = f_1(y) + x^2 f_2(y) + x f_3(y)$ (b) $z = f_1(y) - x f_2(y) + x^2 f_3(y)$

(c)$\boldsymbol{z = f_1(y) + x f_2(y) + x^2 f_3(y)}$ (d)$z = f_1(y) + x f_2(y) - x^2 f_3(y)$

Solution:

The auxiliary equation is given by $m^3 = 0$

$$m = 0,0,0$$

Hence the general solution is given by

$$z = f_1(y) + x f_2(y) + x^2 f_3(y)$$

20. Find the particular integral of $\frac{\partial z}{\partial x} + \frac{\partial z}{dy} = sinx$

(a) $\boldsymbol{-cosx}$ (b) $cosx$ (c) $-sinx$ (d) $sinx$

Solution:

$P.I = \frac{1}{D+D^{`}} sinx = \int sinx\, dx = -cosx$

$P.I = -cosx$

21. Find the Particular Integral of $\left(D^2 - 2DD^{`} + D^{`2}\right)z = \cos(x - 3y)$

(a) $\boldsymbol{\frac{-1}{16} \cos(x - 3y)}$ (b) $\frac{1}{16} \cos(x - 3y)$

(c) $\frac{-1}{32} \cos(x - 3y)$ (d) $\frac{x}{16} \cos(x - 3y)$

Solution:

$$P.I = \frac{1}{D^2 - 2DD^{`} + D^{`2}} \cos(x - 3y)$$

Replace $D^2\ by - 1, D^{`2}\ by - 9, DD^{`}\ by\ 3$

$$P.I = \frac{1}{-1-6-9} \cos(x - 3y) \quad = \frac{-1}{16} \cos(x - 3y)$$

22. Find the Particular Integral of $\frac{\partial^2 z}{\partial x^2} - 5\frac{\partial^2 z}{\partial x \partial y} + 6\frac{\partial^2 z}{\partial y^2} = e^{x+y}$

(a) $\boldsymbol{\frac{1}{2}\, e^{x+y}}$ (b) $\frac{1}{2}\, e^{x-y}$ (c) $\frac{1}{2}\, e^{2x+y}$ (d) $\frac{1}{2}\, e^{2x-y}$

Solution:

The given equation can be written as

$$\left(D^2 - 5DD^` + 6D^{`2}\right)z = e^{x+y}$$

$$P.I = \frac{1}{D^2 - 5DD^` + 6D^{`2}}\ e^{x+y}$$

$$= \frac{1}{1-5+6} e^{x+y} = \frac{1}{2}\ e^{x+y}$$

23. Solve $\left(D^3 - 6D^2D^` + 11DD^{`2} - 6D^{`3}\right)z = 0$

(a) $z = f_1(y-x) + f_2(y+2x) + f_3(y+3x)$

(b) $\boldsymbol{z = f_1(y+x) + f_2(y+2x) + f_3(y+3x)}$

(c) $z = f_1(y+x) + f_2(y-2x) + f_3(y+3x)$

(d) $z = f_1(y-x) + f_2(y+2x) + f_3(y-3x)$

Solution:

The auxiliary Equation is given by $m^3 - 6m^2 + 11m - 6 = 0$

Solving $m = 1,2,3$

C.F is $f_1(y+x) + f_2(y+2x) + f_3(y+3x)$

$$where\ f_1, f_2, f_3\ are\ arbitrary\ functions$$

Solution is $z = f_1(y+x) + f_2(y+2x) + f_3(y+3x)$

24. Find the Particular Integral of $(D^2 + 3DD^` - 4D^{`2})z = siny$

(a) $-\frac{1}{4}siny$ (b) $\frac{1}{4}cosy$ (c)-$\frac{1}{4}cosy$ **(d)** $\boldsymbol{\frac{1}{4}siny}$

Solution:

$P.I = \frac{1}{D^2+3DD^`-4D^{`2}}siny$

Replace D^2 by $0, D^{`2}$ $by -1, DD^`$ by 0

$$P.I = \frac{1}{0+0-4(-1)}siny = \frac{1}{4}siny$$

25. Solve $(D^3 - 3D^2D^` + 4D^{`3})z = 0$

(a) $z = f_1(y+x) + f_2(y+2x) + xf_3(y-2x)$

(b) $z = f_1(y+x) + f_2(y+2x) + xf_3(y+2x)$

(c) $z = f_1(y+x) + f_2(y-2x) + xf_3(y-2x)$

(d) $\boldsymbol{z = f_1(y-x) + f_2(y+2x) + xf_3(y+2x)}$

Solution:

The auxiliary equation is $m^3 - 3m^2 + 4 = 0$

The roots are $m = -1,2,2$

C.F = $f_1(y-x) + f_2(y+2x) + xf_3(y+2x)$

Solution is $z = f_1(y-x) + f_2(y+2x) + xf_3(y+2x)$

SRM Institute of Science and Technology

Ramapuram Campus

Department of Mathematics

Question Bank of Module-2(Fourier Series)

(2020–2021-ODD)

Subject.Code: 18MAB201T

Subject.Name: Transforms and Boundary Value Problems

Year/Sem: II/III **Part-A(1*20=20)** **Branch: Common to All branches**

Module-2(Fourier Series) **1*20=20**

1.	$\sin x$ is a periodic function with period (a) π (b) $\frac{\pi}{2}$ (c) 2π (d) 4π	ANS - **c**	(CLO-2, Remember)
2.	Which one of the following function is an even function (a) (a) $\sin x$ (b) x (c) e^x (d) x^2	ANS - **d**	(CLO-2, Remember)
3.	$\int_{-a}^{a} f(x)dx = 0$ if f(x) is (a)odd (b) even (c) periodic (iv) zero	ANS- **a**	(CLO-2, Remember)
4.	$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$ if f(x) is (a) even (b) odd (c) neither even nor odd (iv) periodic	ANS - **a**	(CLO-2, Remember)
5.	$\int_{-\pi}^{\pi} \lvert x \rvert dx$ is equal to (a) $2\int_{0}^{\pi} xdx$ (b) 0 (c) $2\int_{0}^{\pi} (-x)dx$ (iv) $4\int_{0}^{\pi/2} xdx$	ANS - **a**	(CLO-2, Remember)

6.	$\tan x$ is a periodic function with period (a) π (b) 2π (c) 3π (d) $\pi/2$	ANS - **a**	(CLO-2, Remember)
7.	The constant a_0 of the Fourier series for the function f(x) = x is $0 \le x \le 2\pi$ (a) 2π (b) π (c) 3π (d) 0	ANS - **b**	(CLO-2, Apply)
8.	The constant a_0 of the Fourier series for the function f(x) = k, $0 \le x \le 2\pi$ (a) k (b) 2k (c) 0 (d) $\frac{k}{2}$	ANS - **b**	(CLO-2, Apply)
9.	If f(x) is an odd function in $(-l,l)$ then value of a_n in the Fourier series expansion of f(x) is (a) $\frac{2}{l}\int_0^l f(x)\cos nx\,dx$ (b) 0 (c) $\frac{2}{l}\int_0^l f(x)\sin nx\,dx$ (d) $\frac{1}{l}\int_{-l}^l x\,dx$	ANS – b	(CLO-2, Remember)
10.	If f(x) is an even function in $(-\pi,\pi)$ then the value of b_n in the Fourier series expansion of $f(x)$ is (a) $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$ (b) $\frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$ (c) 0 (d) $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$	ANS – **c**	(CLO-2, Remember)
11.	The RMS value of f(x) in $a \le x \le b$ is (a) 0 (b) $\sqrt{\frac{\int_a^b [f(x)]^2 dx}{b-a}}$ (c) $\sqrt{\frac{\int_a^b [f(x)]^2 dx}{b+a}}$ (d) $\sqrt{\frac{\sqrt{\int_a^b f(x)dx}}{b-a}}$	ANS – b	(CLO-2, Remember)
12.	The RMS value of f(x) = x in $-1 \le x \le 1$ is (a) 1 (b) 0 (c) $\frac{1}{\sqrt{3}}$ (d) -1	ANS – **c**	(CLO-2, Apply)
13.	If $\bar{y}$ is the RMS value of f(x) in $(0,2l)$ then $\frac{a_0^2}{4}+\frac{1}{2}\sum_{n=1}^{\infty}(a_n^2+b_n^2)$ is	ANS –	(CLO-2, Remember)

	(a) $\frac{\bar{y}^2}{2}$ (b) $\bar{y}$ (c) $\frac{\bar{y}}{2}$ (d) $\bar{y}^{-2}$	**d**	
14.	Half range cosine series for f(x) in $(0,\pi)$ is (a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx$ (b) $\frac{a_0^2}{4}+\frac{1}{2}\sum_{n=1}^{\infty}(a_n^2+b_n^2)$ (c) $\sum_{n=1}^{\infty} b_n \sin nx$ (d) $\sum_{n=1}^{\infty} a_n \cos nx$	ANS – **a**	(CLO-2, Remember)
15.	Half range sine series for f(x) in $(0,\pi)$ is (a) $\sum_{n=1}^{\infty} a_n \cos nx$ (b) $\frac{a_0}{2}+\sum a_n \cos nx$ (c) $\sum_{n=1}^{\infty} b_n \sin nx$ (d) $\frac{a_0}{2}-\sum a_n \cos nx$	ANS - **c**	(CLO-2, Remember)
16.	The function defined by $f(x)=\begin{cases} x, & -\pi \le x \le 0 \\ -x, & 0 \le x \le \pi \end{cases}$ is (a) odd (b) neither odd nor even (c) periodic (d) even	ANS- **d**	(CLO-2, Remember)
17.	The function $f(x)=\begin{cases} g(x), & 0 \le x \le \pi \\ -g(-x), & -\pi \le x \le 0 \end{cases}$ is (a) even function (b) odd function (c) increasing function (d) periodic function	ANS - **b**	(CLO-2, Remember)
18.	The value of Fourier series of f(x) in $0 < x < 2\pi$ at x = 0 is (a) f(0) (b) $f(2\pi)$ (c) $\frac{f(0)+f(2\pi)}{2}$ (d) 0	ANS- **c**	(CLO-2, Remember)
19.	A function f(x) with period T if (a) $f(x+T)=f(T)$ (b) $f(x+T)=f(x)$ (c) $f(x+T)=-f(x)$ (d) $f(x+T).f(x)=0$	ANS- **b**	(CLO-2, Remember)
20.	An example for a function which neither even nor odd (a) $x\sin x$ (b) e^{ax} (c) $x^2 \sin x$ (d) $x\cos x$	ANS- **b**	(CLO-2, Apply)
21.	Write the formula for finding Euler's constant of a_0 Fourier series in $(0,2\pi)$		

	$a)\ a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)dx$ $\quad b)\ a_0 = \frac{2}{\pi}\int_0^{2\pi} f(x)dx$ $c)\ a_0 = \frac{l}{\pi}\int_0^{2\pi} f(x)dx$ $\quad d)\ a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)dx$	Ans (a)	(CLO-2 Remember)
22.	Sum the Fourier series for $f(x) = \begin{cases} x & 0<x<1 \\ 2 & 1<x<2 \end{cases}$ $at\ \ x=0$ (a) 2 (b) 1 (c) 3 (d)0	Ans (b)	(CLO-2 Remember)
23.	Sum the Fourier series for $f(x) = \begin{cases} x & 0<x<1 \\ 2 & 1<x<2 \end{cases}$ $at\ \ x=1$ (a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{3}{2}$ (d) $\frac{1}{4}$	Ans (c)	(CLO-2 Remember)
24.	What is the constant term a_0 and the coefficient of cosnx, a_n in the Fourier series expansion of *f(x)* = *x* - *x*^{3} in (-π, π)? (a) $\frac{\pi}{3},0$ (b) $0,\pi$ (c) $0,\frac{\pi}{2}$ (d) 0,0	Ans (d)	(CLO-2 Remember)
25	Write the formula for finding Euler's constant of a_n Fourier series in (0,2π) $a)\ a_n = \frac{1}{2\pi}\int_0^{2\pi} f(x)\cos nx dx$ $\quad b)\ a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx dx$ $c)\ a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx dx$ $\quad d)\ a_n = \frac{1}{\pi}\int_0^{\pi} f(x)\cos nx dx$	Ans (b)	(CLO-2 Remember)
26.	State Parseval's Identity for full range expression of *f(x)* as Fourier series in (0,2*l*) $a)\ \frac{1}{2l}\int_0^{2l}[f(x)]^2\,dx = \frac{a_0^2}{4} + \frac{1}{2}\sum_1^{\infty}(a_n^2 + b_n^2)$ $b)\ \frac{1}{l}\int_0^{l}[f(x)]^2\,dx = \frac{a_0^2}{4} + \frac{1}{2}\sum_1^{\infty}(a_n^2 + b_n^2)$ $c)\ \frac{1}{l}\int_0^{2l}[f(x)]^2\,dx = \frac{a_0^2}{2} + \frac{1}{2}\sum_1^{\infty}(a_n^2 + b_n^2)$ $d)\ \frac{1}{2l}\int_0^{2l}[f(x)]^2\,dx = \frac{a_0^2}{4} + \frac{1}{4}\sum_0^{\infty}(a_n^2 + b_n^2)$		
27	What is the constant term a_0 and the coefficient of cosnx, a_n in the Fourier series expansion of *f(x)* = *x*^{3} in (-π, π)? (a)0,0 (b) $\pi,1$ (c) $\frac{\pi}{2},0$ (d) $\frac{\pi}{3},0$	Ans (a)	(CLO-2 Remember)

28	Write the formula for finding Euler's constant of b_n Fourier series in (0,2π) $a) b_n = \frac{1}{2\pi}\int_0^{\pi} f(x)\sin nx dx$ $b)\ b_n = \frac{1}{\pi}\int_0^{\pi} f(x)\sin nx dx$ $c) b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx dx$ $d)\ b_n = \frac{1}{2\pi}\int_0^{\pi} f(x)\sin nx dx$	Ans (c)	(CLO-2 Remember)
29.	Find a Fourier sine series for the function *f(x)*=1; *0<x< π.* $a)\ \frac{4}{\pi}\sum_{n=1,3,5}^{\infty}\frac{\sin nx}{n}$ $b)\ \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\sin nx}{n}$ $c)\ \frac{4}{3\pi}\sum_{n=1,3,5}^{\infty}\frac{\sin nx}{2n}$ $d)\ \frac{2}{\pi}\sum_{n=1,3,5}^{\infty}\frac{\sin nx}{n}$	Ans (a)	(CLO-2 Remember)
30	Find a_n in expanding e^{-x} as Fourier series in (-π, π) $a)\ \frac{(-1)^{n+1}}{(1+n^2)}2\cosh\pi$ $b)\ \frac{(-1)^n}{(1+n^2)}\cosh\pi$ $c)\ \frac{(-1)^n}{\pi(1+n^2)}2\cosh\pi$ $d)\ \frac{(-1)^{n+1}}{\pi(1+n^2)}\cosh\pi$	Ans (c)	(CLO-2 Remember)
31	Find the value of a_n for f(x)= c in (0,10) in cosine series expansion (a) 10 (b) c (c) c/10 (d)0	Ans (d)	(CLO-2 Remember)
32	If f(x) is an odd function defined in (-*l* , *l*) what are the values of a_0 and a_n.? (a) 0,0 (b) 0,2l (c) 2l,0 (d)1,1	Ans (a)	(CLO-2 Remember)
33	Find b_n in the expansion of cosx as a Fourier series in (-π, π) (a) $\frac{\pi}{3}$ (b) π (c) $\frac{\pi}{2}$ (d) 0	Ans (d)	(CLO-2 Remember)
34	Find a_0 in the expansion of $f(x) = \begin{cases} -\pi, -\pi < x < 0 \\ x, 0 < x < \pi \end{cases}$ (a) $\frac{\pi}{2}$ (b) π (c) $-\frac{\pi}{2}$ (d) $-\pi$	Ans (c)	(CLO-2 Remember)
35	finding Euler's constant of a_0 for $f(x) = \frac{1}{2}(\pi - x)$ in -π < x < π (a) $\frac{\pi}{3}$ (b) π (c) $\frac{\pi}{2}$ (d) 0	Ans (b)	(CLO-2 Remember)
36	Find the fourier constant a_n of periodicity 3 for f(x) = 2x-x^2 in 0<x<3	Ans	(CLO-2

	$a)\left(\frac{-3}{n^3\pi^2}\right)$ $b)\left(\frac{9}{n^3\pi^2}\right)$ $c)\left(\frac{-9}{n^2\pi^2}\right)$ $d)\left(\frac{9}{n^2\pi^2}\right)$	(c)	Remember)

SRM INSTITUTE OF SCIENCE AND TECHNOLOGY
RAMAPURAM CAMPUS
DEPARTMENT OF MATHEMATICS

Year/Sem : II/III

Branch: Common to All branches

Unit 2 – Fourier Series

1. Write the formula for finding Euler's constants of a Fourier series in $0 \le x \le 2\pi$.

Solution:

Euler's constants of a Fourier series in $0 \le x \le 2\pi$ is given by

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\,\sin nx\,dx$$

2. Write the formula for finding Euler's constants of a Fourier series in $0 \le x \le 2l$.

Solution:

Euler's constants of a Fourier series in $0 \le x \le l$ is given by

$$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx$$

$$a_n = \frac{1}{l}\int_0^{2l} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{l}\int_0^{2l} f(x)\,\sin nx\,dx$$

3. Write the formula for Fourier constants for f(x) in the interval $-\pi \le x \le \pi$.

Solution:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,\sin nx\,dx$$

4. Write the formula for Fourier constants for f(x) in the interval $-l \le x \le l$.

Solution:

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\,\sin nx\,dx$$

5. Find the constant value a_0 of the Fourier series for the function f(x) = k, $0 \le x \le 2\pi$.

Solution:

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx \qquad = \frac{1}{\pi}\int_0^{2\pi} k\,dx \qquad = \frac{k}{\pi}(2\pi) \qquad = 2k$$

a) K b) **2k** c) 0 d) k/2

6. Find the constant value a_0 of the Fourier series for the function f(x) = x, $0 \le x \le \pi$.

Solution:

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx \qquad = \frac{2}{\pi}\int_0^{\pi} x\,dx \qquad = \frac{2}{\pi}\frac{(\pi)^2}{2} \qquad = \pi$$

a) π b)2 π c)0 d) π /2

7. $If\ f(x) = e^x\ in\ -\pi \le x \le \pi$, find a_n

Solution:

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx \qquad = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x \cos nx\,dx \quad = \frac{1}{\pi}\left\{\frac{e^x}{(1+n^2)}(\cos nx + n\sin nx)\right\}_{-\pi}^{\pi}$$

$$= \frac{(-1)^n}{\pi(1+n^2)}\left(e^{\pi} - e^{-\pi}\right)$$

a) $\frac{(-1)^n}{\pi(1+n^2)}\left(e^{\pi} - e^{-\pi}\right)$ b) $\frac{(-1)^n}{\pi(1-n^2)}\left(e^{\pi} - e^{-\pi}\right)$

c) $\frac{(-1)^n}{\pi(1-n^2)}\left(e^{\pi} + e^{-\pi}\right)$ d) $\frac{(-1)^n}{\pi}\left(e^{\pi} - e^{-\pi}\right)$

8. a) $If\ f(x) = e^x\ in\ -\pi \le x \le \pi$, find b_n

Solution:

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx \qquad = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x \sin nx\,dx \quad = \frac{1}{\pi}\left\{\frac{e^x}{(1+n^2)}(\sin nx - n\cos nx)\right\}_{-\pi}^{\pi}$$

$$= \frac{n(-1)^{n+1}}{\pi(1+n^2)}\left(e^{\pi} - e^{-\pi}\right)$$

a) $\frac{n(-1)^{n+1}}{\pi(1+n^2)}\left(e^{\pi} - e^{-\pi}\right)$ b) $\frac{1}{\pi(1-n^2)}\left(e^{\pi} - e^{-\pi}\right)$

c) $\frac{2}{\pi(1-n^2)}\left(e^{\pi} + e^{-\pi}\right)$ d) $\frac{(-1)^n}{\pi}\left(e^{\pi} - e^{-\pi}\right)$

b)

9. Check whether the function is even or odd, where $f(x)=\begin{cases} 1+\dfrac{2x}{l}, & -l \le x \le 0 \\ 1-\dfrac{2x}{l}, & 0 \le x \le l \end{cases}$

Solution:

$$for\ -l \le x \le 0,\ f(-x) = 1+\frac{2(-x)}{l} = 1-\frac{2x}{l} = f(x), where\ 0 \le x \le l$$

$\Rightarrow$ the given function is even function

a) Even function
b) odd function
c) constant function
d) neither even nor odd

10. Find the constant value a_n of the Fourier series for the function f(x) = x, $0 \le x \le \pi$.

Solution:

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx = \frac{2}{\pi}\int_0^{\pi} x\cos nx\,dx$$

$$= \frac{2}{\pi}\left[x\frac{\sin nx}{n} + \frac{\cos nx}{n^2}\right]_0^{\pi} = \frac{2}{\pi}\left(\frac{(-1)^n - 1}{n^2}\right)$$

a) $\frac{2}{\pi}\left(\frac{(-1)^n - 1}{n^2}\right)$ b) 2π c) $\frac{2(-1)^n}{\pi}$ d) $\pi/2$

11. Find the constant value b_n of the Fourier series for the function f(x) = x, $-\pi \le x \le \pi$.

Solution:

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx = \frac{2}{\pi}\int_0^{\pi} x\sin nx\,dx$$

$$= \frac{2}{\pi}\left[x\frac{-\cos nx}{n} + \frac{\sin nx}{n^2}\right]_0^{\pi} = \frac{-2(-1)^n}{n}$$

a) $\frac{2}{\pi}\left(\frac{(-1)^n - 1}{n^2}\right)$ b) $\frac{(-1)^n}{\pi}$ c) $\frac{-2(-1)^n}{n}$ d) $\pi/2$

12. Find the constant term of the Fourier series for the function f(x) = |x|, $-\pi \le x \le \pi$.

Solution:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{2}{\pi}\int_0^{\pi} x\,dx = \pi$$

a) π b) 2π c) 0 d) $\pi/2$

13. Find the Fourier coefficient b_n of the Fourier series for the function f(x) = x, $0 \le x \le 2\pi$.

Solution:

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_0^{2\pi} x\sin nx\,dx = \frac{1}{\pi}\left[x\frac{-\cos nx}{n} + \frac{\sin nx}{n^2}\right]_0^{2\pi}$$

$$= \left(-2(-1)^n\right)/n$$

a) π b) $\left(-2(-1)^n\right)/n$ c)0 d)3 π

14. Half-range cosine series for f(x) in $(0,\pi)$is

a) $\frac{(a_0)}{2} + \sum_{n=1}^{\infty} a_n \cos nx$ b) $\sum_{n=1}^{\infty} b_n \cos nx$ c) $\sum_{n=1}^{\infty} a_n \cos nx$

d) $\frac{(a_0)^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty}(a_n)^2 + (b_n)^2$

15. If f(x) = x^2 in $(-\pi,\pi)$ then the value of b_n is?

Solution:

Since the given function is even in the given interval, the Fourier coefficient b_n value is zero in this case.

a) 1 b)0 c)-1 d)2

16. In the Fourier series expansion of f(x) = sin x in $(-\pi,\pi)$. What is the value of a_n?

Solution:

The function $f(-x) = \sin(-x) = -sinx = -f(-x)\ so\ f(x) is\ odd\ function$

So a_n = 0

a) 1 b)0 c) π d) $-\pi$

17. Find the constant value a_0 from the following table:

X	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
F(x) = y	1	1.4	1.9	1.7	1.5	1.2	1

Solution:

$$a_0 = \frac{2\sum y}{n} = \frac{2(1+1.4+1.9+1.7+1.5+1.2)}{6} = 2.9$$

a) 1.9 b)2.9 c)4.9 d)6.9

18. What is the sum of the Fourier series at a point x = 1 where the function has a finite discontinuity.

Solution:

$$f(x) = \frac{f(x+x_0) + f(x-x_0)}{2}$$

Here $x_0 = 1$, $f(x) = \frac{f(x+1)+f(x-1)}{2}$

a) $\mathrm{f(x)} = \frac{\mathrm{f(x+1)+f(x-1)}}{2}$ b)$f(x) = \frac{f(x+1)}{2}$ c)$f(x) = \frac{f(x-1)}{2}$

d) $f(x) = f(x+1) + f(x-1)$

19. In the expansion of $f(x) = sinhx\ in\ (-\pi,\pi)$ as a Fourier Series, find the coefficient of a_n.

Solution:

$$f(x) = \sin hx = \frac{e^x - e^{-x}}{2}$$

$$f(-x) = \frac{e^{-x}-e^{x}}{2} = \frac{-(e^{x}-e^{-x})}{2} = -\sin hx = -f(x)$$

So, $f(x) is\ an\ odd\ function, the\ fourier\ coefficient\ \ a_n\ is\ 0$

a) 0 b)1 c)2 d)3

20. To what value, the Fourier series corresponding to f(x) = x^2 in (0, 2 π) converges at x = 0?

Solution:

The Fourier series converges to $\dfrac{f(0)+f(2\pi)}{2} = 2\pi^2$

a)π b)2π c)π^2 d)$\mathbf{2\pi^2}$

21. Examine whether the function $f(x) = \frac{1}{1-x}$, can be expanded in Fourier series in any interval including $x = 1$

Solution:

At $x = 1, the\ function\ f(x) = \frac{1}{1-x}\ \ is\ not\ continuous$

By Dirichlet`s condition, we cannot expand $f(x)$ as a Fourier series.

a) Can be expanded since f(x) is not continuous
b) Cannot be expanded since f(x) does not satisfies Dirichlet`s condition
c) Can be expanded since f(x) satisfies Dirichlet`s Condition
d) Cannot be expanded since f(x) is continuous

22. Find the constant term in the Fourier series corresponding to $f(x) = x - x^3\ in\ (-\pi, \pi)$

Solution:

$f(x) = x - x^3$
$f(-x) = -x + x^3 \quad = -(x - x^3) = -f(x)$
$f(x) is\ an\ odd\ function\ (-\pi, \pi)$

Hence Constant term $a_0 = 0$

a) 0 b)1 c)2 d)3

23. Find the R.M.S Value of the function $f(x) = x\ in\ (0, l)$.

Solution:

$$\mathrm{R.M.S} = \frac{\sqrt{\int_0^l \mathrm{x}^2\mathrm{dx}}}{l} = \frac{\sqrt{\left(\frac{\mathrm{x}^3}{3}\right)_0^l}}{l} = \sqrt{\frac{l^3}{3l}} = \frac{l}{\sqrt{3}}$$

a) $\frac{l}{\sqrt{3}}$ b)$\frac{\mathrm{l}}{\sqrt{2}}$ c)$\frac{\mathrm{l}^2}{\sqrt{3}}$ d) $\frac{\mathrm{l}^2}{\sqrt{2}}$

24. Find the value of $\mathrm{a_n}$in the cosine series expansion of $\mathrm{f(x) = k\ in\ (0,10)}$.

Solution:

$$a_n = \frac{1}{5}\int_0^{10} k\cos\frac{n\pi x}{10}\,dx = \frac{k}{5}\left[\frac{sin\frac{n\pi x}{10}}{\frac{n\pi}{10}}\right]_0^{10} = 0$$

a) 0 b)10 c)20 d)30

25. Find the R.M.S Value of the function $f(x) = k\ in\ (-l, l)$.

Solution:

$$\text{R.M.S} = \frac{\sqrt{\int_{-l}^{l} \text{k}^2 \text{dx}}}{2l} == \frac{\sqrt{\text{k}^2(\text{x})_{-l}{}^{l}}}{2l} = k$$

a) $\frac{\mathbf{l}}{\sqrt{3}}$ b) $\mathbf{k}$ c) $\frac{l^2}{\sqrt{3}}$ d) $\frac{l^2}{\sqrt{2}}$

26. In the Fourier series expansion of f(x) = |sin x| in $(-\pi,\pi)$. What is the value of b_n?

Solution:

$f(-x) = |\sin(-x)| = |-sinx| = |sinx| = f(x)$

$since\ f(x)\ is\ an\ odd\ function\ b_n = 0$

a) 1 b)0 c) π d) $-\pi$

27. Find b_1, if f(x) = k in 0 < x < π .

Solution:

$$b_1 = \frac{2}{\pi}\int_0^{\pi} f(x)sinx\ dx = \frac{2}{\pi}\int_0^{\pi} ksinx\ dx$$

$$= \frac{2}{\pi}k(-cosx)_0{}^{\pi} = \frac{2}{\pi}(-(-1)+1) = \frac{4}{\pi}$$

a) $\frac{2}{\pi}$ b) $\frac{\mathbf{4}}{\boldsymbol{\pi}}$ c) $\frac{1}{2\pi}$ d) $\frac{1}{4\pi}$

28. Find the R.M.S Value of the function $f(x) = 2x\ in\ (0,3)$.

Solution:

$$R.M.S = \frac{\sqrt{\int_0^3 4x^2 dx}}{3} = \frac{\sqrt{4\left(\frac{x^3}{3}\right)_0^3}}{3} = \frac{6}{3} = 2$$

a) 2 b)6 c)9 d)0

29. F(x) = x + x^2 in $(-\pi,\pi)$is ____________

Solution:

$F(-x) = -x + (-x)^2 = -x + x^2 \neq -f(x)$so it is neither ever nor odd function

a) Odd function b)Even function

c) Constant function d)Neither odd nor even

30. Find the series value $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} + \ldots$, if f(x) = x^2 in the interval (-π , π), $a_0 = 2\pi^2/3$, $a_n = 4(-1)^{n+1}/n^2$

Solution:

The Fourier cosine series is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^{n+1}}{n^2}\cos nx$$

When x = 0,

$$f(0) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^{n+1}}{n^2}$$

$$\frac{f(-\pi)+f(\pi)}{2} = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^{n+1}}{n^2}$$

$$\pi^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^{n+1}}{n^2} \quad \Rightarrow \quad \frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$$

a) $\frac{\pi^2}{12}$ b) $\frac{\pi^2}{8}$ c) $\frac{\pi^2}{2}$ d)0

Answers

5	b	6	a	7	a	8	a	9	a	10	a	11	c	12	a	13	b
14	a	15	b	16	b	17	b	18	a	19	a	20	d	21	b	22	a
23	a	24	a	25	b	26	b	27	b	28	a	29	d	30	a		

SRM Institute of Science and Technology

Ramapuram Campus

Department of Mathematics

Question Bank of Module-3(Application of PDE)

(2020–2021-ODD)

Subject.Code: 18MAB201T

Subject.Name: Transforms and Boundary Value Problems

Year/Sem: II/III **Part-A (1*20=20)** **Branch: Common to All branches**

1.	The proper solution of the problems on vibration of string is	1 mark	
	(a) $y(x,t)=(Ae^{\lambda x}+Be^{-\lambda x})(Ce^{\lambda at}+De^{-\lambda at})$ (b) $y(x,t)=(Ax+B)(Ct+D)$ (c) $y(x,t)=(A\cos\lambda x+B\sin\lambda x)(C\cos\lambda at+D\sin\lambda at)$ (d) $y(x,t)=(Ax+B)$	Ans (c)	(CLO-3 Remember)
2.	The one dimensional wave equation is	1 mark	
	(a) $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ (b) $\frac{\partial^2 y}{\partial t^2}=a^2\frac{\partial^2 y}{\partial x^2}$ (c) $\frac{\partial y}{\partial t}=a\frac{\partial^2 y}{\partial x^2}$ (d) $\frac{\partial^2 y}{\partial x^2}=a\frac{\partial^2 y}{\partial t^2}$	Ans (b)	(CLO-3 Remember)
3.	In wave equation $\frac{\partial^2 y}{\partial t^2}=a^2\frac{\partial^2 y}{\partial x^2}$, a^2 stands for	1 mark	
	(a) $\frac{T}{m}$ (b) $\frac{k}{c}$ (c) $\frac{m}{T}$ (d) $\frac{k}{m}$	Ans (a)	(CLO-3 Remember)
4.	In heat equation $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$, α^2 stands for	1 mark	
	(a) $\frac{k}{\rho}$ (b) $\frac{T}{m}$ (c) $\frac{k}{\rho c}$ (d) $\frac{k}{c}$	Ans (c)	(CLO-3 Remember)
5.	The one dimensional heat equation in steady state is	1 mark	
	(a) $\frac{\partial u}{\partial t}=0$ (b) $\frac{\partial^2 u}{\partial t^2}=0$ (c) $\frac{\partial^2 u}{\partial x^2}-\frac{\partial u}{\partial t}=0$ (d) $\frac{\partial^2 u}{\partial x^2}=0$	Ans (d)	(CLO-3 Remember)

6.	The proper solution of $u_t = \alpha^2 u_{xx}$ is	1 mark	
	(a) $u = (Ax+B)C$ (b) $u = (A\cos\lambda x + B\sin\lambda x)e^{-\alpha^2\lambda^2 t}$ (c) $u = (Ae^{\lambda x} + Be^{-\lambda x})e^{\alpha\lambda t}$ (d) $u = At + B$	Ans (b)	(CLO-3 Remember)
7.	The proper solution in steady state heat flow problems is	1 mark	
	(a) $u = (Ae^{\lambda x} + Be^{-\lambda x})e^{\alpha\lambda t}$ (b) $u = Ax + B$ (c) $u = (A\cos\lambda x + B\sin\lambda x)e^{-\alpha^2\lambda^2 t}$ (d) $u = (Ae^{\lambda x} + Be^{-\lambda x})(Ce^{\lambda at} + De^{-\lambda at})$	Ans (b)	(CLO-3 Remember)
8.	The one dimensional heat equation is	1 mark	
	(a) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ (b) $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ (c) $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$ (d) $\frac{\partial u}{\partial x} = \alpha^2 \frac{\partial^2 u}{\partial t^2}$	Ans (b)	(CLO-3 Remember)
9.	How many initial and boundary conditions are required to solve $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$	1 mark	
	(a) Four (b) Two (c) Three (d) Five	Ans (c)	(CLO-3 Remember)
10.	How many initial and boundary conditions are required to solve $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$	1 mark	
	(a) Two (b) Three (c) Five (d) Four	Ans (d)	(CLO-3 Remember)
11.	One dimensional wave equation is used to find	1 mark	
	(a) Temperature (b) Displacement (c) Time (d) Mass	Ans (b)	(CLO-3 Remember)
12.	One dimensional heat equation is used to find	1 mark	
	(a) Density (b) Temperature distribution (c) Time (d) Displacement	Ans (b)	(CLO-3 Remember)
13.	Heat flows from___________temperature	1 mark	
	(a) Higher to Lower (b) Uniform (c) Lower to higher (d) Stable	Ans (a)	(CLO-3 Remember)
14.	The tension T caused by stretching the string before fixing it at the end points is	1 mark	

	(a) Increasing (b) Decreasing (c) Constant (d) Zero	Ans (c)	(CLO-3 Remember)
15.	A string is stretched between two fixed points x = 0 and x = l. The initial conditions are	1 mark	
	(a) $y(0, t) = 0, y(x,t) = 0$ (b) $y(x, 0) = 0, \frac{\partial y}{\partial t}(x, 0) = 0$ (c) $y(0, t) = 0, y(l, t) = 0$ (d) $\left(\frac{\partial y}{\partial x}\right)_{(0,t)} = 0, \left(\frac{\partial y}{\partial x}\right)_{(l,t)} = 0$	Ans (c)	(CLO-3 Apply)
16.	The amount of heat required to produce a given temperature change in a body is proportional to	1 mark	
	(a) Weight of the body (b) Mass of the body (c) Density of the body (d) Tension of the body	Ans (b)	(CLO-3 Remember)
17.	The general solution for the displacement y(x,t) of the string of length l vibrating between fixed end points with initial velocity zero and initial displacement f(x) is	1 mark	
	(a) $\sum B_n \sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi at}{l}\right)$ (b) $\sum B_n \sin\left(\frac{n\pi x}{l}\right)\sin\left(\frac{n\pi at}{l}\right)$ (c) $\sum B_n \cos\left(\frac{n\pi x}{l}\right)\sin\left(\frac{n\pi at}{l}\right)$ (d) $\sum B_n \sin\left(\frac{n\pi x}{l}\right)$	Ans (a)	(CLO-3 Remember)
18.	The steady state temperature of a rod of length l whose ends are kept at 30° and 40° is	1 mark	
	(a) $u = \frac{10x}{l} + 30$ (b) $u = \frac{20x}{l} + 30$ (c) $u = \frac{10x}{l} + 20$ (d) $u = \frac{10x}{l}$	Ans (a)	(CLO-3 Apply)
19.	When the ends of a rod is non-zero for one dimensional heat flow equation, the temperature function u(x, t) is modified as the sum of steady state and transient state temperatures. The transient part of the solution which	1 mark	
	(a) Increases with increase of time (b) Decreases with increase of time (c) Increases with decrease of time (d) Decreases with decrease of time	Ans (b)	(CLO-3 Remember)
20.	A rod of length l has its ends A and B kept at 0° and 100° respectively, until steady state conditions prevail. Then the initial condition is given by	1 mark	
	(a) $u(x, 0) = ax + b + 100l$ (b) $u(x, 0) = \frac{100x}{l}$ (c) $u(x, 0) = 100xl$ (d) $u(x,0) = (x + l)100$	Ans (b)	(CLO-3 Apply)

21.	In wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$, a^2 stands for	1 mark	
	(a) $\frac{Tension}{Mass}$ (b) $\frac{Temperature}{Mass}$ (c) $\frac{Time}{Mass}$ (d) $\frac{Mass}{Time}$	Ans (a)	(CLO-3 Remember)
22.	In heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, α^2 stands for	1 mark	
	(a) diffusivity (b)time (c) tension (d) mass	Ans (a)	(CLO-3 Remember)
23.	In heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, $\alpha^2 = \frac{k}{\rho c}$, here k stands for	1 mark	
	(a)Thermal conductivity (b)time (c) zero (d) mass	Ans (a)	(CLO-3 Remember)
24.	In heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, $\alpha^2 = \frac{k}{\rho c}$, here ρ stands for	1 mark	
	(a) density (b) tension (c)mass (d) zero	Ans (a)	(CLO-3 Remember)
25.	In heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, $\alpha^2 = \frac{k}{\rho c}$, here c stands for	1 mark	
	(a) specific heat (b) tension (c)mass (d) zero	Ans (a)	(CLO-3 Remember)
26.	In heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, the steady state condition is	1 mark	
	(a) $\frac{\partial u}{\partial t} = 0$ (b) $\frac{\partial^2 u}{\partial t^2} = 0$ (c) $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ (d) $\frac{\partial u}{\partial x} = 0$	Ans (a)	(CLO-3 Remember)
27.	In heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, the unsteady state solution is	1 mark	
	(a) $u = (Ax + B)C$ (b) $u = (A \cos\lambda x + B \sin \lambda x)e^{-\alpha \lambda t}$ (c) $u = (Ae^{\lambda x} + Be^{-\lambda x})e^{\alpha\lambda t}$ (d) $u = At + B$	Ans (b)	(CLO-3 Remember)
28.	If $B^2 - 4AC = 0$, then the 2nd order partial differential equation is classified as	1 mark	

	(a) Elliptic (b) Hyperbolic (c)parabolic (d) Laplace equation	Ans (c)	(CLO-3 Remember)
29.	If $B^2 - 4AC < 0$, then the 2nd order partial differential equation is classified as	1 mark	
	(a) Elliptic (b) Hyperbolic (c)parabolic (d) Laplace equation	Ans (a)	(CLO-3 Remember)
30.	If $B^2 - 4AC > 0$, then the 2nd order partial differential equation is classified as	1 mark	
	(a) Elliptic (b) Hyperbolic (c)parabolic (d) Laplace equation	Ans (b)	(CLO-3 Remember)
31.	The one dimensional heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ is classified as	1 mark	
	(a) Elliptic (b) Hyperbolic (c)parabolic (d) Laplace equation	Ans (c)	(CLO-3 Remember)
32.	The one dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = \alpha^2 \frac{\partial^2 y}{\partial x^2}$ is classified as	1 mark	
	(a) Elliptic (b) Hyperbolic (c)parabolic (d) Laplace equation	Ans (b)	(CLO-3 Remember)
33.	The partial differential equation $u_{xx} + 2u_{xy} + u_{yy} = 0$ is classified as	1 mark	
	(a) Elliptic (b) Hyperbolic (c)parabolic (d) Laplace equation	Ans (c)	(CLO-3 Remember)
34.	The partial differential equation $xf_{xx} + yf_{yy} = 0$, $x > 0, y > 0$ is classified as	1 mark	
	(a) Elliptic (b) Hyperbolic (c)parabolic (d) Laplace equation	Ans (a)	(CLO-3 Remember)
35.	The partial differential equation $xf_{xx} + yf_{yy} = 0$, $x < 0, y > 0$ is classified as	1 mark	
	(a) Elliptic (b) Hyperbolic (c)parabolic (d) Laplace equation	Ans (b)	(CLO-3 Remember)
36.	The partial differential equation $u_{xx} + 4u_{xy} + 4u_{yy} = 0$ is classified as	1 mark	

	(a) Elliptic (b) Hyperbolic (c)parabolic (d) Laplace equation	Ans (c)	(CLO-3 Remember)
37.	The partial differential equation $2u_{xx} + 3u_{xy} + 4u_{yy} = 0$ is classified as	1 mark	
	(a) Elliptic (b) Hyperbolic (c)parabolic (d) Laplace equation	Ans (a)	(CLO-3 Remember)
38.	The partial differential equation $u_{xx} - 3u_{xy} + 2u_{yy} = 0$ is classified as	1 mark	
	(a) Elliptic (b) Hyperbolic (c)parabolic (d) Laplace equation	Ans (b)	(CLO-3 Remember)
39.	The partial differential equation $f_{xx} + f_{xy} + f_{yy} + f_{y} = 0$ is classified as	1 mark	
	(a) Elliptic (b) Hyperbolic (c)parabolic (d) Laplace equation	Ans (a)	(CLO-3 Remember)
40.	The partial differential equation $2f_{xx} - f_{xy} - f_{yy} + 2f_{y} = 0$ is classified as	1 mark	
	(a) Elliptic (b) Hyperbolic (c)parabolic (d) Laplace equation	Ans (b)	(CLO-3 Remember)

SRM INSTITUTE OF SCIENCE AND TECHNOLOGY
RAMAPURAM CAMPUS
DEPARTMENT OF MATHEMATICS

Year/Sem : II/III
Branch: Common to All branches

Unit III – Applications of Partial Differential Equations

1.Write the possible solutions of one dimensional wave equation

(a) $\boldsymbol{y(x,t) = (c_1 e^{px} + c_2 e^{-px})(c_3 e^{pat} + c_4 e^{-pat})}$

$\boldsymbol{y(x,t) = (c_5 cospx + c_6 sinpx)(c_7 cospat + c_8 sinpat)}$

$\boldsymbol{y(x,t) = (c_9 x + c_{10})(c_{11} t + c_{12})}$

(b) $y(x,t) = (c_1 e^{px} + c_2 e^{-px})(c_3 e^{pat} + c_4 e^{-pat})$

$y(x,t) = (c_5 cospx - c_6 sinpx)(c_7 cospat + c_8 sinpat)$

$y(x,t) = (c_9 x + c_{10})(c_{11} t + c_{12} t^2)$

(c) $y(x,t) = (c_1 e^{px} + c_2 e^{-px})(c_3 e^{at} + c_4 e^{-at})$

$y(x,t) = (c_5 cospx + c_6 sinpx)(c_7 cospat + c_8 sinpat)$

(d) $y(x,t) = (c_1 e^{px} + c_2 e^{-px})(c_3 e^{pat} + c_4 e^{-pat})$

$y(x,t) = (c_9 x + c_{10} x^2)(c_{11} t + c_{12})$

Solution:

The possible solution of one dimensional wave equation $\boldsymbol{\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}}$ ***are***

- $y(x,t) = (c_1 e^{px} + c_2 e^{-px})(c_3 e^{pat} + c_4 e^{-pat})$
- $y(x,t) = (c_5 cospx + c_6 sinpx)(c_7 cospat + c_8 sinpat)$
- $y(x,t) = (c_9 x + c_{10})(c_{11} t + c_{12})$

2. Write the all the possible solutions of one dimensional heat flow equation

(a) $u(x,t) = c_1(c_2x^2 + c_3)$,

$u(x,t) = c_3e^{\alpha^2p^2t}(c_4e^{px} + c_5e^{-px})$,

$u(x,t) = c_6e^{-\alpha^2p^2t}(c_7cospx + c_8sinpx)$

(b) $\boldsymbol{u(x,t) = c_1(c_2x + c_3)}$ **,**

$\boldsymbol{u(x,t) = c_3e^{\alpha^2p^2t}(c_4e^{px} + c_5e^{-px})}$ **,**

$\boldsymbol{u(x,t) = c_6e^{-\alpha^2p^2t}(c_7cospx + c_8sinpx)}$

(c) $u(x,t) = c_1(c_2x + c_3)$,

$u(x,t) = c_3e^{-\alpha^2p^2t}(c_4e^{px} + c_5e^{-px})$,

$u(x,t) = c_6e^{\alpha^2p^2t}(c_7cospx + c_8sinpx)$

(d) $u(x,t) = c_1(c_2x + c_3)$,

$u(x,t) = c_3e^{p^2t}(c_4e^{px} + c_5e^{-px})$,

$u(x,t) = c_6e^{-p^2t}(c_7cospx + c_8sinpx)$

Solution:

The possible solutions of one dimensional heat flow equation $\frac{\partial u}{\partial t} = \boldsymbol{\alpha^2 \frac{\partial^2 u}{\partial x^2}}$ are

- $u(x,t) = c_1(c_2x + c_3)$
- $u(x,t) = c_3e^{\alpha^2p^2t}(c_4e^{px} + c_5e^{-px})$
- $u(x,t) = c_6e^{-\alpha^2p^2t}(c_7cospx + c_8sinpx)$

3. Write all boundary conditions for one dimensional wave equation with zero initial velocity

(a) $y(0,t) = 0\ , for\ all\ t > 0$

$y(l,t) = l\ , for\ all\ t > 0$

$\frac{\partial y(x,0)}{\partial t} = 0$

$y(x,0) = f(x)$

(b) $y(0,t) = l\ , for\ all\ t > 0$

$y(l,t) = 0\ , for\ all\ t > 0$

$y(x,0) = 0$

$\frac{\partial y(x,0)}{\partial t} = f(x)$

(c) $y(0,t) = 0\ , for\ all\ t > 0$

$y(l,t) = 0\ , for\ all\ t > 0$

$y(x,0) = 0$

$\frac{\partial y(x,0)}{\partial t} = f(x)$

(d) $\boldsymbol{y(0,t) = 0\ , for\ all\ t > 0}$

$\boldsymbol{y(l,t) = 0\ , for\ all\ t > 0}$

$\boldsymbol{\frac{\partial y(x,0)}{\partial t} = 0}$

$\boldsymbol{y(x,0) = f(x)}$

Solution

One dimensional wave equation is $\boldsymbol{\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}}$**, boundary conditions are**

- $y(0,t) = 0\ , for\ all\ t > 0$
- $y(l,t) = 0\ , for\ all\ t > 0$
- $\frac{\partial y(x,0)}{\partial t} = 0\ (Initial\ Velocity = 0)$
- $y(x,0) = f(x)$

4. Write all boundary conditions for one dimensional heat flow equation

(a) $u(0,t) = l \; for \; all \; t > 0$
$u(l,t) = 0 \; for \; all \; t > 0$
$u(x,0) = f(x) \; for \; all \; x \; in \; (0,l)$

(b) $u(0,t) = 0 \; for \; all \; t > 0$
$u(l,t) = 0 \; for \; all \; t > 0$
$u(x,0) = 0 \; for \; all \; x \; in \; (0,l)$

(c) $\boldsymbol{u(0,t) = 0 \; for \; all \; t > 0}$
$\boldsymbol{u(l,t) = 0 \; for \; all \; t > 0}$
$\boldsymbol{u(x,0) = f(x) \; for \; all \; x \; in \; (0,l)}$

(d) $u(0,t) = 0 \; for \; all \; t > 0$
$u(l,t) = l \; for \; all \; t > 0$
$u(x,0) = f(x) \; for \; all \; x \; in \; (0,l)$

Solution:

One dimensional heat flow equation $\frac{\partial u}{\partial t} = \boldsymbol{\alpha^2 \frac{\partial^2 u}{\partial x^2} \; are}$

- $u(0,t) = 0 \; for \; all \; t > 0$
- $u(l,t) = 0 \; for \; all \; t > 0$
- $u(x,0) = f(x) \; for \; all \; x \; in \; (0,l)$

5. A string is stretched between two fixed points at a distance $2l$ apart and the points of the string are given velocity $f(x), x$ being the distance from end point. Formulate the problem to find the displacement of the string at any time

(a) $\boldsymbol{y(0,t) = 0 \, , for\ all\ t > 0}$

$\boldsymbol{y(2l,t) = 0 \, , for\ all\ t > 0}$

$\boldsymbol{y(x,0) = 0\ for\ all\ x\ in\ (0,2l)}$

$\boldsymbol{\frac{\partial y(x,0)}{\partial t} = f(x) forall\ x\ in\ (0,2l)}$

(a) $y(0,t) = 0 \, , for\ all\ t > 0$

$y(2l,t) = 0 \, , for\ all\ t > 0$

$\frac{\partial y(x,0)}{\partial t} = 0 forall\ x\ in\ (0,2l)$

$y(x,0) = f(x)\ for\ all\ x\ in\ (0,2l)$

(a) $y(0,t) = 2l \, , for\ all\ t > 0$

$y(2l,t) = 0 \, , for\ all\ t > 0$

$y(x,0) = 0\ for\ all\ x\ in\ (0,2l)$

$\frac{\partial y(x,0)}{\partial t} = f(x) forall\ x\ in\ (0,2l)$

(a) $y(0,t) = 0 \, , for\ all\ t > 0$

$y(2l,t) = 2l \, , for\ all\ t > 0$

$y(x,0) = 0\ for\ all\ x\ in\ (0,2l)$

$\frac{\partial y(x,0)}{\partial t} = f(x) forall\ x\ in\ (0,2l)$

Solution:

One dimensional wave equation is $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$**, boundary conditions are**

- $y(0,t) = 0\,, for\ all\ t > 0$
- $y(2l,t) = 0\,, for\ all\ t > 0$
- $y(x,0) = 0\ for\ all\ x\ in\ (0,2l)$
- $\frac{\partial y(x,0)}{\partial t} = f(x) forall\ x\ in\ (0,2l)$

6.A string of length $2l$ is stretched to a constant tension T, is fastened at both the ends and hence fixed. The mid points of the string is taken to a height $'b'$ and then released from rest in that position. Find the equation of the string in its initial position

(a) $y(x,0) = \begin{cases} \frac{bx}{l} & when\ 0 < x < l \\ \frac{b}{l}(2l + x) & when\ l < x < 2l \end{cases}$

(b) $\boldsymbol{y(x,0) = \begin{cases} \frac{bx}{l} & when\ 0 < x < l \\ \frac{b}{l}(2l - x) & when\ l < x < 2l \end{cases}}$

(c) $y(x,0) = \begin{cases} -\frac{bx}{l} & when\ 0 < x < l \\ \frac{b}{l}(2l - x) & when\ l < x < 2l \end{cases}$

(d) $y(x,0) = \begin{cases} -\frac{bx}{l} & when\ 0 < x < l \\ \frac{b}{l}(2l + x) & when\ l < x < 2l \end{cases}$

Solution:

The initial displacement of the string is in the form

$$y(x,0) = \begin{cases} \frac{bx}{l} & when\ 0 < x < l \\ \frac{b}{l}(2l - x) & when\ l < x < 2l \end{cases}$$

7.Classify the equation $U_{xx} - y^4 U_{yy} = 2y^3 U_y$

(a) Elliptic **(b) Hyperbolic** (c)Parabolic (d) Concentric

Solution:

Here $A = 1,\ B = 0, C = -y^4$

$$B^2 - 4AC = 0 - 4(1)(-y^4) = 4y^4 > 0$$

The equation is hyperbolic

8. Classify the equation $x^2 f_{xx} + (1 - y^2) f_{yy} = 0\ when\ x = 0$

(a) Elliptic (b) Hyperbolic **(c)Parabolic** (d) Concentric

Solution:

Here $A = x^2,\ B = 0, C = 1 - y^2$

$$B^2 - 4AC = 0 - 4(x^2)(1 - y^2) = 0 \quad (since\ x = 0)$$

The equation is parabolic

9. Classify the equation $4U_{xx} + 4U_{xy} + U_{yy} + 2U_x - U_y = 0$

(a) Elliptic (b) Hyperbolic **(c)Parabolic** (d) Concentric

Solution:

Here $A = 4,\ B = 4, C = 1$

$$B^2 - 4AC = 16 - \ 4(4)(1) = 0$$

The equation is parabolic

10. Classify $U_{xx} + U_{yy} = 0$

(a) Elliptic (b) Hyperbolic (c)Parabolic (d) Concentric

Solution:

Here $A = 1,\ B = 0, C = 1$

$$B^2 - 4AC = -1 < 0$$

The equation is elliptic

11.Classify $U_{xx} + 5U_{xy} + 4U_{yy} + U_x + U_y = 0$
(a) Elliptic **(b) Hyperbolic** (c)Parabolic (d) Concentric

Solution:
Here $A = 1,\ B = 5, C = 4$
$$B^2 - 4AC = 25 - 16 > 0$$
The equation is hyperbolic

12.Classify $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$
(a) Elliptic **(b) Hyperbolic** (c)Parabolic (d) Concentric

Solution:

Here $A = 1,\ B = 0, C = -c^2$
$$B^2 - 4AC = 0 - 4(1)(-c^2) = 4c^2 > 0$$
The equation is hyperbolic

13. Classify $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$
(a) Elliptic (b) Hyperbolic **(c)Parabolic** (d) Concentric

Solution:
Here $A = c^2,\ B = 0, C = 0$
$$B^2 - 4AC = 0 - 4(0)(c^2) = 0$$
The equation is parabolic

14. Write the conditions for classification of PDE to be hyperbolic, parabolic and hyperbolic

(a) $\boldsymbol{B^2 - 4AC < 0}$ $[\boldsymbol{Elliptic\ Equation}]$
$\boldsymbol{B^2 - 4AC = 0}$ $[\boldsymbol{Parabolic\ Equation}]$
$\boldsymbol{B^2 - 4AC > 0}$ $[\boldsymbol{Hyperbolic\ Equation}]$

(a) $B^2 - 4AC > 0$ $[Elliptic\ Equation]$
$B^2 - 4AC < 0$ $[Parabolic\ Equation]$
$B^2 - 4AC = 0$ $[Hyperbolic\ Equation]$

(a) $B^2 - 4AC = 0$ $[Elliptic\ Equation]$
$B^2 - 4AC > 0$ $[Parabolic\ Equation]$

$B^2 - 4AC < 0$ $[Hyperbolic\ Equation]$

(a) $B^2 - 4AC = 0$ $[Elliptic\ Equation]$
$B^2 - 4AC > 0$ $[Parabolic\ Equation]$
$B^2 - 4AC < 0$ $[Hyperbolic\ Equation]$

Solution:
Let a second order PDE in the function u of the two independent variables x, y be of the form

$$A(x,y)\frac{\partial^2 u}{\partial x^2} + B(x,y)\frac{\partial^2 u}{\partial x \partial y} + C(x,y)\frac{\partial^2 u}{\partial y^2} + f\left(x, y, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right) = 0 \quad --\rightarrow (1)$$

Equation (1) is classified as elliptic, parabolic or hyperbolic at the points of a given region R depending on whether

$$B^2 - 4AC < 0\ [Elliptic\ Equation]$$
$$B^2 - 4AC = 0\ [Parabolic\ Equation]$$
$$B^2 - 4AC > 0\ [Hyperbolic\ Equation]$$

15.Classify $x^2U_{xx} + 2xyU_{xy} + (1+y^2)U_{yy} = 0$
(a) Elliptic (b) Hyperbolic (c)Parabolic (d) Concentric

Solution:
Here $A = x^2,\ B = 2xy, C = (1+y^2)$

$$B^2 - 4AC = 2xy - 4(x^2)(1+y^2) = -4x^2 < 0$$

The equation is elliptic

16. Classify $\frac{\partial^2 u}{\partial x \partial y} = \left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial u}{\partial y}\right) + xy$
(a) Elliptic **(b) Hyperbolic** (c)Parabolic (d) Concentric

Solution:
Here $A = 0,\ B = 1, C = 0$

$$B^2 - 4AC = 1 - 4(0)(0) = 1 > 0$$

The equation is hyperbolic

17. *A rod of length l cm whose one side is kept at $20°C$ and the other end is kept at $50°C$ is maintained until steady state prevails. Find the steady state temperature.*

(a) $u(x) = \frac{20}{l}x - 20$

(b) $u(x) = \frac{20}{l}x + 30$

(c) $\boldsymbol{u(x) = \frac{30}{l}x + 20}$

(d) $u(x) = \frac{30}{l}x - 20$

Solution:

$$u(x) = ax + b$$

When $x = 0, u(0) = b \Rightarrow b = 20C$

When $x = l, u(l) = al + 20 \Rightarrow 50C = al + 20$

$$al = 30 \Rightarrow a = \frac{30}{l}$$

$$so, u(x) = \frac{30}{l}x + 20$$

18. *A rod of length l cm whose one side is kept at $0°C$ and the other end is kept at $100°C$ is maintained until steady state prevails. Find the steady state temperature.*

(a) $\boldsymbol{u(x) = \frac{100}{l}x}$

(b) $u(x) = \frac{10}{l}x$

(c) $u(x) = \frac{-100}{l}x$

(d) $u(x) = \frac{100}{l}x^2$

Solution:

$$u(x) = ax + b$$

When $x = 0, u(0) = b \Rightarrow b = 0$

When $x = l, u(l) = al + 0 \;\Rightarrow\; 100 = al$

$$al = 100 \Rightarrow a = \frac{100}{l}$$

$$so, u(x) = \frac{100}{l}x$$

19. *A rod of length $l\ cm$ whose one side is kept at $0°C$ and the other end is kept at $120°C$ is maintained until steady state prevails. Find the steady state temperature.*

(a) $u(x) = -\frac{120}{l}x$

(b) $\boldsymbol{u(x) = \frac{120}{l}x}$

(c) $u(x) = \frac{120}{l}x^2$

(d) $u(x) = \frac{120}{l}$

Solution:

$$u(x) = ax + b$$

When $x = 0, u(0) = b \;\Rightarrow b = \; 0$

When $x = l, u(l) = al + 0 \;\Rightarrow\; 120 = al$

$$al = 120 \Rightarrow a = \frac{120}{l}$$

$$so, u(x) = \frac{120}{l}x$$

20. *A rod of length $20\ cm$ whose one side is kept at $30°C$ and the other end is kept at $70°C$ is maintained until steady state prevails. Find the steady state temperature.*

(a) $u(x) = 3x - 20$

(b) $u(x) = 2x - 30$

(c) $\boldsymbol{u(x) = 2x + 30}$

(d) $u(x) = 3x + 30$

Solution:

$$u(x) = ax + b$$

When $x = 0, u(0) = b \Rightarrow b = 30$

When $x = 20, u(20) = 70 = 20a + 30$

$$\Rightarrow 40 = 20a$$

$$\Rightarrow a = 2$$

$$so, u(x) = 2x + 30$$

21. A Uniform string of length `l` is struck in such a way that an initial velocity of V_0 is imparted to the portion of the string between $\frac{l}{4}$ $and\, \frac{3l}{4}$ while the string is in its equilibrium position. Write the wave equation and its boundary conditions

(a) $\boldsymbol{\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}}$,

$$\boldsymbol{y(0,t) = 0, t > 0}$$
$$\boldsymbol{y(l,t) = 0, t > 0}$$
$$\boldsymbol{y(x,0) = 0}$$
$$\boldsymbol{\frac{\partial y(x,0)}{\partial t} = \begin{cases} 0 & 0 < x < \frac{l}{4} \\ v_0 & \frac{l}{4} \le x \le \frac{3l}{4} \\ 0 & \frac{3l}{4} < x \le l \end{cases}}$$

(b) $\frac{\partial^2 y}{\partial x} = a^2 \frac{\partial^2 y}{\partial t^2}$,

$$y(0,t) = 0, t > 0$$
$$y(l,t) = 0, t > 0$$
$$y(x,0) = x$$
$$\frac{\partial y(x,0)}{\partial t} = \begin{cases} v_0 & 0 < x < \frac{l}{4} \\ 1 & \frac{l}{4} \le x \le \frac{3l}{4} \\ 0 & \frac{3l}{4} < x \le l \end{cases}$$

(c) $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$,

$y(0,t) = 0, t > 0$

$y(l,t) = 0, t > 0$

$\frac{\partial y(x,0)}{\partial t} = 0$

$$y(x,0) = \begin{cases} 0 & 0 < x < \frac{l}{4} \\ v_0 & \frac{l}{4} \leq x \leq \frac{3l}{4} \\ 0 & \frac{3l}{4} < x \leq l \end{cases}$$

(d) $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$,

$y(0,t) = 0, t > 0$

$y(l,t) = 0, t > 0$

$\frac{\partial y(x,0)}{\partial t} = 0$

$$y(x,0) = \begin{cases} 0 & 0 < x < \frac{l}{4} \\ -v_0 & \frac{l}{4} \leq x \leq \frac{3l}{4} \\ 1 & \frac{3l}{4} < x \leq l \end{cases}$$

Solution:

The wave equation is $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$

The boundary conditions are

$y(0,t) = 0, t > 0$

$y(l,t) = 0, t > 0$

$y(x,0) = 0$

$$\frac{\partial y(x,0)}{\partial t} = \begin{cases} 0 & 0 < x < \frac{l}{4} \\ v_0 & \frac{l}{4} \leq x \leq \frac{3l}{4} \\ 0 & \frac{3l}{4} < x \leq l \end{cases}$$

22. Write the one dimensional wave equation and also general solution for the displacement $y(x,t)$ of the string l vibrating between fixed end points with initial zero and initial displacement $f(x)$.

(a) $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$, $\sum B_n \cos\left(\frac{n\pi x}{l}\right) \sin\left(\frac{n\pi a t}{l}\right)$

(a) $\boldsymbol{\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}}$ **,** $\boldsymbol{\sum B_n \sin\left(\frac{n\pi x}{l}\right) \cos\left(\frac{n\pi a t}{l}\right)}$

(a) $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$, $\sum \sin\left(\frac{n\pi x}{l}\right) \cos\left(\frac{n\pi a t}{l}\right)$

(a) $\frac{\partial^2 y}{\partial x^2} = a^2 \frac{\partial^2 y}{\partial t^2}$, $\sum B_n \sin\left(\frac{n\pi x}{l}\right) \cos\left(\frac{n\pi a t}{l}\right)$

Solution:

The one dimensional wave equation is given by $\boldsymbol{\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}}$

The general solution for the displacement $y(x,t) is\ given\ by$

$$\sum B_n \sin\left(\frac{n\pi x}{l}\right) \cos\left(\frac{n\pi a t}{l}\right)$$

23. Classify $x^2 f_{xx} + (1-y^2)f_{yy}) = 0\ for -1 < y < 1, -\infty < x < \infty$

$$and\ x \neq 0$$

(a) Elliptic (b) Hyperbolic (c)Parabolic (d) Concentric

Solution:

Here $A = x^2,\ B = 0, C = 1 - y^2$

$$B^2 - 4AC = 0 - 4(x^2)(1-y^2) = 4x^2(y^2-1) < 0$$

(Since $x^2 is\ always\ positive\ in\ -\infty < x < \infty\ \&$

$$in\ -1 < y < 1, y^2 - 1\ is\ negative$$)

The equation is elliptic

24. Classify the one dimensional wave equation

(a) Elliptic **(b) Hyperbolic** (c)Parabolic (d) Concentric

Solution:

The one dimensional wave equation is given by $\boldsymbol{\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}}$

Here $A = a^2,\ B = -1, C = 0$

$$B^2 - 4AC = (-1)^2 - 4(a^2)(0) = 1 > 0$$

The equation is hyperbolic.

25. Write the one dimensional heat flow equation and also general solution for $u(x,t)$ where $u(x,0) = f(x)$.

(a) $\boldsymbol{\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2},\ \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}}$

(b) $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2},\ \sum_{n=1}^{\infty} B_n \cos\frac{n\pi x}{l}$

(c) $\frac{\partial u}{\partial x} = \alpha^2 \frac{\partial^2 u}{\partial t^2},\ \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}$

(d) $\frac{\partial u}{\partial x} = \alpha^2 \frac{\partial^2 u}{\partial t^2},\ \sum_{n=1}^{\infty} B_n \cos\frac{n\pi x}{l}$

Solution:

One dimensional heat flow equation $\frac{\partial u}{\partial t} = \boldsymbol{\alpha^2 \frac{\partial^2 u}{\partial x^2}}$

Solution is $u(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}$

where $B_n = b_n = \frac{2}{l}\int f(x)\ \sin\frac{n\pi x}{l}\ dx$

SRM Institute of Science and Technology

Ramapuram Campus

Department of Mathematics

Question Bank of Module-IV(Fourier Transforms)

(2020–2021-ODD)

Subject.Code: 18MAB201T

Subject.Name: Transforms and Boundary Value Problems

Year/Sem: II/III **Part-A (1*20=20)** **Branch: Common to All branches**

1.	The Fourier transform of a function f(x) is	1 mark	
	a) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\, e^{ist} dt$ b) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\, e^{isx} dx$ c) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\, e^{isx} dx$ d) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(s)\, e^{isx} dx$	Ans (b)	(CLO-4 Remember)
2.	The Fourier transform of f(x)= $e^{-\frac{x^2}{2}}$ is	1 mark	
	a) e^{-s^2} b) $\frac{1}{e^{\frac{s^2}{2}}}$ c) $\frac{1}{e^{-\frac{s^2}{2}}}$ d) $\frac{1}{e^{x^2}}$	Ans (a)	(CLO-4 Remember)
3.	The Fourier cosine transform of e^{-ax} is	1 mark	
	a) $\sqrt{\frac{2}{\pi}}\frac{a}{a^2+x^2}$ b) $\sqrt{\frac{1}{\pi}}\frac{s}{s^2+a^2}$ c) $\sqrt{\frac{1}{\pi}}\frac{a}{s^2+a^2}$ d) $\sqrt{\frac{2}{\pi}}\frac{a}{s^2+a^2}$	Ans (d)	(CLO-4 Remember)
4.	Under Fourier cosine transform $f(x) = e^{-a^2 x^2}$ is----	1 mark	

	--function		
	a) self-reciprocal b) cosine c) inverse function d) sine	Ans (a)	(CLO-4 Remember)
5.	The Fourier sine transform of x $e^{-\frac{x^2}{2}}$ is	1 mark	
	a) 0 b) $se^{-\frac{s^2}{2}}$ c) $\frac{1}{e^{x^2}}$ d) 1	Ans (b)	(CLO-4 Remember)
6.	$F[f(ax)] = \frac{1}{a}F\left(\frac{s}{a}\right)$	1 mark	
	a) $\frac{1}{s}F\left(\frac{s}{a}\right)$ b) $\frac{1}{a}F\left(\frac{a}{s}\right)$ c) $\frac{1}{a}F\left(\frac{s}{a}\right)$ d) $\frac{1}{s}F\left(\frac{as}{a}\right)$	Ans (c)	(CLO-4 Remember)
7.	The $F[f(x-a)] =$	1 mark	
	a) $e^{ias}F(a)$ b) $e^{ias}F(x)$ c) $e^{iax}F(a)$ d) $e^{ias}F(s)$	Ans (d)	(CLO-4 Remember)
8.	$F\left[e^{iax}f(x)\right] =$	1 mark	
	a) F(s+a) b) F(s-a) c) F(sa) d) F(s/a)	Ans (a)	(CLO-4 Remember)
9.	F[f(x) cosax] =	1 mark	
	a) [f(a)+f(s-a)]/ 2 b) [f(sa)+f(s+a)]/ 2 c) [f(s+a)+f(s-a)]/ 2 d) [f(s+a)-f(s-a)]/ 2	Ans (c)	(CLO-4 Remember)
10.	F[f(x) *g(x)] =	1 mark	
	a) F(s) + G(s) b) F(s) - G(s) c) F(s)G(s) d) F(s)|G(s)	Ans (c)	(CLO-4 Remember)

11.	If F(s) = F [f(x)] then $\int_{-\infty}^{\infty}\lvert f(x)\rvert^2dx =$	1 mark	
	a) $\int_{-\infty}^{\infty}\lvert f(x)\rvert^2dx$ b) $\int_{-\infty}^{\infty}\lvert f(s)\rvert^2ds$ c) $\int_{0}^{\infty}\lvert f(x)\rvert^2dx$ d) $\int_{-0}^{\infty}\lvert f(s)\rvert^2ds$	Ans (b)	(CLO-4 Remember)
12.	$F[xf(x)] =$	1 mark	
	a) $\frac{dF(s)}{ds}$ b) $i\frac{dF(s)}{ds}$ c) $-i\frac{dF(s)}{ds}$ d) $-\frac{dF(s)}{ds}$	Ans (c)	(CLO-4 Remember)
13.	$F_C[xf(x)] =$	1 mark	
	a) $\frac{dF_S(s)}{ds}$ b) $i\frac{dF_S(s)}{ds}$ c) $-i\frac{dF_S(s)}{ds}$ d) $-\frac{dF_S(s)}{ds}$	Ans (a)	(CLO-4 Remember)
14.	$F_s[xf(x)] =$	1 mark	
	a) $\frac{dF_c(s)}{ds}$ b) $i\frac{dF_c(s)}{ds}$ c) $-i\frac{dF_c(s)}{ds}$ d) $-\frac{dF_c(s)}{ds}$	Ans (d)	(CLO-4 Remember)

15.	The relation between Fourier transform and Laplace transform is	1 mark	
	a) $F[f(x)] = \frac{1}{\sqrt{2\pi}} L[g(x)]$ b) $F[f(x)] = \frac{1}{\sqrt{\pi}} L[g(x)]$ c) $F[f(x)] = \frac{1}{\sqrt{2}} L[g(x)]$ d) $F[f(x)] = \frac{-1}{\sqrt{\pi}} L[g(x)]$	Ans (a)	(CLO-4 Remember)
16.	The Fourier cosine transform of $F_C[e^{-4x}]$	1 mark	
	a) $\sqrt{\frac{2}{\pi}} \frac{4}{16+s^2}$ b) $\sqrt{\frac{2}{\pi}} \frac{4}{4+s^2}$ c) $\sqrt{\frac{\pi}{2}} \frac{4}{16+s^2}$ d) $\sqrt{\frac{\pi}{2}} \frac{4}{4+s^2}$	Ans (a)	(CLO-4 Remember)
17.	The Fourier transform of an odd function of x is	1 mark	
	a) an odd function of s b) even function of s c) an odd function of x d) even function of x	Ans (a)	(CLO-4 Remember)
18.	The Fourier transform of an even function of x is	1 mark	
	a) an odd function of s b) even function of s c) an odd function of x d) even function of x	Ans (b)	(CLO-4 Remember)
19.	The Fourier sine transform of $F_S[\frac{1}{x}]$	1 mark	
	a) $\sqrt{\frac{2}{\pi}}$ b) $\sqrt{\frac{1}{\pi}}$ c) $\sqrt{\frac{\pi}{2}}$ d) $\sqrt{\frac{\pi}{4}}$	Ans (c)	(CLO-4 Remember)

20.	$F[e^{ibx}f(x)] =$	1 mark	
	a) F(s/b) b) F(s+b) c) F(bs) d) F(s-b)	Ans (b)	(CLO-4 Remember)
21	If f(x) is a function in(-l,l) and satisfies dirichlets conditions then	1 mark	
	a) $f(x) = \frac{1}{\pi}\int_0^\infty \int_{-\infty}^\infty f(t)\, cos\boldsymbol{\lambda}(\mathbf{t}-\mathbf{x})dtd\boldsymbol{\lambda}$ b) $f(x) = \frac{1}{\pi}\int_0^\infty \int_{-\infty}^\infty f(t)\, cos\mathbf{x}(\mathbf{t}-\mathbf{x})dxd\boldsymbol{\lambda}$ c) $f(x) = \frac{1}{2\pi}\int_0^\infty \int_{-\infty}^\infty f(t)\, cos\boldsymbol{\lambda}(\mathbf{t}-\mathbf{x})dtd\boldsymbol{\lambda}$ d) $f(x) = \frac{2}{\pi}\int_0^\infty \int_{-\infty}^\infty f(t)\, cos\boldsymbol{\lambda}(\mathbf{t}-\mathbf{x})dtd\boldsymbol{\lambda}$	Ans (c)	(CLO-4 Remember)
22	Under Fourier cosine transform $f(x) = \frac{1}{\sqrt{x}}$	1 mark	
	a) Self-reciprocal function b) Cosine function c) Inverse function d) Complex function	Ans (a)	(CLO-4 Remember)
23	If F(f(x)) = F(s) and f(x)→0 as x→±∞ then F(f'(x)) is	1 mark	
	a) –isF(s) b) is F(s) c) sF(s) d) –F(s)	Ans (a)	(CLO-4 Remember)
24	Find the Fourier sine transform of e^{-ax}, a>0	1 mark	
	a) $F_s[e^{-ax}] = \sqrt{\frac{2}{\pi}}[\frac{s}{s^2+a^2}]$ b) $F_c[e^{ax}] = \sqrt{\frac{2}{\pi}}[\frac{a}{s^2+a^2}]$ c) $F_s[e^{-ax}] = \sqrt{\frac{1}{\pi}}[\frac{s}{s^2+a^2}]$ d) $F_s[e^{ax}] = \sqrt{\frac{2}{\pi}}[\frac{s}{s^2+a^2}]$	Ans(a)	(CLO-4 Remember)

25	Find the Fourier Cosine transform of e^{-ax}, a>0	1 mark	
	a) $F_c[e^{-ax}] = \sqrt{\frac{2}{\pi}}[\frac{a}{s^2+a^2}]$ b) $F_c[e^{ax}] = \sqrt{\frac{2}{\pi}}[\frac{a}{s^2+a^2}]$ c) $F_s[e^{-ax}] = \sqrt{\frac{2}{\pi}}[\frac{s}{s^2+a^2}]$ d) $F_s[e^{ax}] = \sqrt{\frac{2}{\pi}}[\frac{s}{s^2+a^2}]$	An(a)	(CLO-4 Remember)
26	Modulation theorem F[f(x)cosax]	1 mark	
	a) $\frac{1}{2}[F(s+a)+f(s-a)]$ b) $\frac{1}{2}[F(s+a)-f(s-a)]$ c) $\frac{1}{4}[F(s+a)+f(s-a)]$ d) $\frac{1}{4}[F(s+a)+f(s-a)]$	Ans (a)	(CLO-4 Remember)
27	Find the fourier transform of $f(x) = \begin{cases} x, & \lvert x\rvert < a \\ 0, & \lvert x\rvert \geq a \end{cases}$	1 mark	
	a) $i\sqrt{\frac{2}{\pi}}\left[\frac{sinsa-ascossa}{s^2}\right]$ b) $\sqrt{\frac{1}{\pi}}\left[\frac{sinsa-ascossa}{s^2}\right]$ c) $i\sqrt{\frac{2}{\pi}}\left[\frac{sinsa-ascossa}{s}\right]$ d) $i\sqrt{\frac{2}{\pi}}\left[\frac{sinsa-ascossa}{s^3}\right]$	Ans (a)	(CLO-4 Remember)
28	Find the fourier transform of $f(x) = \begin{cases} 1, & \lvert x\rvert < a \\ 0, & \lvert x\rvert \geq a \end{cases}$	1 mark	
	a) $\sqrt{\frac{2}{\pi}}\left[\frac{sinsa}{s}\right]$ b) $\sqrt{\frac{1}{\pi}}\left[\frac{sinsa}{s^2}\right]$ c) $i\sqrt{\frac{2}{\pi}}\left[\frac{ascossa}{s}\right]$	Ans (a)	(CLO-4 Remember)

	d) $i\sqrt{\frac{2}{\pi}}\left[\frac{sinsa}{s^3}\right]$		
29	Find the fourier cosine transform of $e^{-\lvert x\rvert}$	1 mark	
	a) $\frac{\pi}{2}\ e^{-\lvert x\rvert}$ b) $\frac{\pi}{4}\ e^{-\lvert x\rvert}$ c) $\frac{\pi}{2}\ e^{\lvert x\rvert}$ d) $\frac{1}{2}\ e^{-\lvert x\rvert}$	Ans (a)	(CLO-4 Remember)
30	Find the Fourier Cosine transform of $3e^{-5x}+5e^{-2x}$	1 mark	
	a) $F_c[e^{-ax}] = \sqrt{\frac{2}{\pi}}[\frac{15}{s^2+25}+\frac{10}{s^2+4}]$ b) $F_s[e^{-ax}] = \sqrt{\frac{2}{\pi}}[\frac{15}{s^2+25}+\frac{10}{s^2+4}]$ c) $F_c[e^{-ax}] = \sqrt{\frac{1}{\pi}}[\frac{15}{s^2+25}+\frac{10}{s^2+4}]$ d) $F_s[e^{-ax}] = \sqrt{\frac{1}{\pi}}[\frac{15}{s^2+25}+\frac{10}{s^2+4}]$	Ans (a)	(CLO-4 Remember)
31	Find the Fourier sine transform of e^{-3x}	1 mark	
	a) $F_s[e^{-ax}] = \sqrt{\frac{2}{\pi}}[\frac{s}{s^2+3^2}]$ b) $F_c[e^{ax}] = \sqrt{\frac{2}{\pi}}[\frac{a}{s^2+3^2}]$ c) $F_s[e^{-ax}] = \sqrt{\frac{1}{\pi}}[\frac{s}{s^2+3^2}]$ d) $F_s[e^{ax}] = \sqrt{\frac{2}{\pi}}[\frac{s}{s^2+3^2}]$	Ans (a)	(CLO-4 Remember)
32	Find the Fourier sine transform of $\frac{1}{x}$	1 mark	

	a) $\sqrt{\frac{\pi}{2}}$ b) $\sqrt{\frac{\pi}{4}}$ c) $\sqrt{\frac{1}{2}}$ d) $\sqrt{\frac{1}{\pi}}$	Ans(b)	(CLO-4 Remember)
33	Find the Fourier transform of $e^{-a^2x^2}$.	1 mark	
	(a) $F(S) = \frac{1}{\sqrt{2}} e^{\frac{-s^2}{4a^2}}$ (b) $F(S) = \frac{1}{\sqrt{3}} e^{\frac{-s^2}{4a^2}}$ (c) $F(S) = \frac{1}{a\sqrt{4}} e^{\frac{-s^2}{4a^2}}$ d) $F(S) = \frac{1}{a\sqrt{2}} e^{\frac{-s^2}{4a^2}}$	Ans (d)	(CLO-4 Remember)
34	Find the Fourier cosine transform of e^{-x^2}	1 mark	
	(a) $\frac{\sqrt{\pi}}{2a} e^{-\frac{s^2}{4a^2}}$ (b) $\frac{1}{\sqrt{3}} e^{\frac{-s^2}{4a^2}}$ (c) $\frac{1}{a\sqrt{4}} e^{\frac{-s^2}{4a^2}}$ (d) $\frac{1}{a\sqrt{2}} e^{\frac{-s^2}{4a^2}}$	Ans (a)	(CLO-4 Remember)
35	evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)^2}$	1 mark	
	a) $\frac{\pi}{3a^2}$ b) $\frac{3\pi}{4a^3}$ c) $\frac{\pi}{4a^3}$ d) $\frac{\pi}{a^3}$	Ans(c)	(CLO-4 Remember)
36	Find the Fourier cosine transform of e^{-3x}	1 mark	

	a) $\sqrt{\frac{2}{\pi}}\left[\frac{3}{3^2+s^2}\right]$ (b) $\sqrt{\frac{2}{\pi}}\left[\frac{2}{4+s^3}\right]$ (c) $\sqrt{\frac{2}{\pi}}\left[\frac{b}{b+s^3}\right]$ (d) $\sqrt{\frac{2}{\pi}}\left[\frac{1}{1+s^2}\right]$	Ans (a)	(CLO-4 Remember)

SRM INSTITUTE OF SCIENCE AND TECHNOLOGY
RAMAPURAM CAMPUS
DEPARTMENT OF MATHEMATICS

Year/Sem : II/III

Branch: Common to All branches

Unit 4 – Fourier Transforms

1. The Fourier Transforms of $f(x) = 1 \; in \; a < x < b \; is$
   (a) $\boldsymbol{F[f(x)] = \frac{1}{is\sqrt{2\pi}}[e^{ibs} - e^{ias}]}$
   (b) $F[f(x)] = \frac{1}{\sqrt{2\pi}}[e^{ibs} - e^{ias}]$
   (c) $F[f(x)] = \frac{1}{i\sqrt{\pi}}[e^{ibs} - e^{ias}]$
   (d) $F[f(x)] = \frac{1}{is\sqrt{2\pi}}[e^{ias} - e^{ibs}]$

   Solution: $F[f(x)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$

$$= \frac{1}{\sqrt{2\pi}}\int_a^b 1.\,e^{isx}dx \;\; = \frac{1}{\sqrt{2\pi}}\left[\frac{e^{isx}}{is}\right]_a^b = \frac{1}{is\sqrt{2\pi}}[e^{ibs} - e^{ias}]$$

   Ans: a

2. The Fourier Transforms of $f(x) = e^{-a|x|}, a > 0 \; is$
   (a) $\sqrt{\frac{1}{2\pi}}\left[\frac{a}{a^2+s^2}\right]$ **(b)** $\boldsymbol{\sqrt{\frac{2}{\pi}}\left[\frac{a}{a^2+s^2}\right]}$ (c) $\sqrt{\frac{1}{\pi}}\left[\frac{1}{a^2+s^2}\right]$ (d) $\sqrt{\frac{2a}{\pi}}\left[\frac{s}{a^2+s^2}\right]$

   Solution: $F\left[e^{-a|x|}\right] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-a|x|}e^{isx}dx$

$$= \frac{1}{\sqrt{2\pi}}\int_0^a (cos\; sx + isinsx)e^{-a|x|}dx$$

$$= \frac{2}{\sqrt{2\pi}}\int_0^a cos\; sx\; e^{-ax}dx \;\; = \;\; \sqrt{\frac{\boldsymbol{2}}{\boldsymbol{\pi}}}\left[\frac{\boldsymbol{a}}{\boldsymbol{a^2+s^2}}\right]$$

   Ans: b

3. $If \;\; F[f(x)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx = F(s), \; then \; F[\int_a^x f(x)dx] =$

   (a) $\frac{1}{is}F(s)$ (b) $-\frac{1}{2is}F(s)$ **(c)** $\boldsymbol{-\frac{1}{is}F(s)}$ (d) $-\frac{2}{is}F(s)$

   Solution:
   Let $\int_a^x f(x)dx = g(x) \quad i.e., \;\; f(x)dx = g'(x)$
   If $F[g(x)] = G(s), \;\; F[g'(x)] = \; -isG(s)$

$F[f(x)] = -isF[g(x)] = -\frac{\mathbf{1}}{\boldsymbol{is}}\boldsymbol{F}(\boldsymbol{s})$

Ans: c

4. The Fourier Transforms of $f(x) = e^{ikx}, a < x < b\ is$

(a) $\frac{1}{i(ks)\sqrt{2\pi}}[e^{i(ks)b} - e^{i(ks)a}]$

(b) $\frac{1}{i(k-s)\sqrt{2\pi}}[e^{i(k-s)b} - e^{i(k-s)a}]$

(c) $\frac{2}{i(k+s)\sqrt{\pi}}[e^{i(k+s)b} - e^{i(k+s)a}]$

(d) $\boldsymbol{\frac{1}{i(k+s)\sqrt{2\pi}}[e^{i(k+s)b} - e^{i(k+s)a}]}$

Solution: $F[f(x)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$

$$F\left[e^{ikx}\right] = \frac{1}{\sqrt{2\pi}}\int_a^b e^{ikx}e^{isx}dx$$

$$= \frac{1}{\sqrt{2\pi}}\int_a^b e^{i(k+s)x}dx \quad = \frac{1}{\sqrt{2\pi}}\left[\frac{e^{i(k+s)x}}{i(k+s)}\right]_a^b = \frac{1}{i(k+s)\sqrt{2\pi}}[e^{i(k+s)b} - e^{i(k+s)a}]$$

Ans:d

5. The Fourier Transforms of $f(x) = 1, |x| \le a\ is$

(a) $\boldsymbol{\sqrt{\frac{2}{\pi}}\left(\frac{\sin as}{s}\right)}$ (b) $\sqrt{\frac{1}{\pi}}\left(\frac{\sin as}{s}\right)$ (c) $\sqrt{\frac{1}{2\pi}}\left(\frac{\cos as}{s}\right)$ (d) $\sqrt{\frac{2}{\pi}}\left(\frac{\cos as}{s}\right)$

Solution: $F[f(x)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$

$$= \frac{1}{\sqrt{2\pi}}\int_{-a}^{a} 1.e^{isx}dx \quad = \frac{1}{\sqrt{2\pi}}\left[\frac{e^{isx}}{is}\right]_{-a}^{a} = \frac{1}{is\sqrt{2\pi}}[e^{isa} - e^{-ias}]$$

$$= \frac{1}{is\sqrt{2\pi}}[2i\sin as] = \sqrt{\frac{2}{\pi}}\left(\frac{\sin as}{s}\right)$$

Ans:a

6. If $f(x) = 1\ and \quad F[f(x)] = \sqrt{\frac{2}{\pi}}\left(\frac{\sin as}{s}\right)\ then\ by\ Parseval's\ Identity, \int_{-\infty}^{\infty}\left(\frac{\sin as}{s}\right)^2 ds\ is$

(a) $2\pi a$ **(b)** $\boldsymbol{\pi a}$ (c) π/a (d) 2π

Solution: $\int_{-a}^{a} 1dx = \int_{-\infty}^{\infty}\sqrt{\frac{2}{\pi}}\left(\frac{\sin as}{s}\right)^2 ds$

$2a = \frac{2}{\pi}\int_{-\infty}^{\infty}\left(\frac{\sin as}{s}\right)^2 ds \;=\; \int_{-\infty}^{\infty}\left(\frac{\sin as}{s}\right)^2 ds \;=\; \pi a$

Ans: b

7. $If\ F_s[x\,f(x)] = -\frac{d}{ds}F_c[\,f(x)],\ then\ F_s[x\,e^{-ax}]\ is$

(a) $\sqrt{\frac{2}{\pi}}\left[\frac{as}{(a^2+s^2)^2}\right]$ (b) $\sqrt{\frac{1}{\pi}}\left[\frac{2as}{(a^2+s^2)^2}\right]$ **(c)** $\sqrt{\frac{2}{\pi}}\left[\frac{2as}{(a^2+s^2)^2}\right]$ (d) $\sqrt{\frac{2}{\pi}}\left[\frac{2\pi}{(a^2+s^2)^2}\right]$

Solution: $F_s[x\,e^{-ax}] = -\frac{d}{ds}F_c[e^{-ax}]$

$$= -\frac{d}{ds}\sqrt{\frac{2}{\pi}}\left[\frac{a}{(a^2+s^2)}\right] = \sqrt{\frac{2}{\pi}}\left[\frac{2as}{(a^2+s^2)^2}\right]$$

Ans:c

8. $If\ F_c[x\,f(x)] = \frac{d}{ds}F_s[\,f(x)],\ then\ F_c[x\,e^{-ax}]\ is$

(a) $\sqrt{\frac{2}{\pi}}\left[\frac{a^2}{(a^2+s^2)^2}\right]$ (b) $\sqrt{\frac{2}{\pi}}\left[\frac{s^2}{(a^2+s^2)^2}\right]$

(c) $\sqrt{\frac{2}{\pi}}\left[\frac{a^2-s^2}{(a^2-s^2)^2}\right]$ **(d)** $\sqrt{\frac{2}{\pi}}\left[\frac{a^2-s^2}{(a^2+s^2)^2}\right]$

Solution: $F_c[x\,e^{-ax}] = \frac{d}{ds}F_s[e^{-ax}]$

$$= \frac{d}{ds}\sqrt{\frac{2}{\pi}}\left[\frac{s}{(a^2+s^2)}\right] = \sqrt{\frac{2}{\pi}}\left[\frac{a^2-s^2}{(a^2+s^2)^2}\right].$$

Ans: d

9. $If\ f(x) = e^{-ax}\ \ and\ F_c[x\,e^{-ax}] = \sqrt{\frac{2}{\pi}}\left[\frac{a}{(a^2+s^2)}\right]$
$then\ by\ Parseval's\ Identity\ \int_0^{\infty}\frac{1}{(a^2+s^2)^2}ds\ \ is$

(a) $\frac{\pi}{4a^3}$ (b) $\frac{\pi}{a^3}$ (c) $\frac{\pi}{2a^2}$ (d) $\frac{3\pi}{4a^3}$

Solution:

By Parseval's Identity, $\int_0^{\infty}|F_c[\,s]|^2 ds \;=\; \int_0^{\infty}|f(x)|^2 dx$

$$\frac{2a^2}{\pi}\int_0^{\infty}\frac{1}{(a^2+s^2)^2}ds \;=\; \int_0^{\infty}e^{-2ax}dx$$

$$\int_0^{\infty}\frac{1}{(a^2+s^2)^2}ds \;=\; \frac{\pi}{4a^3}$$

Ans: a

10. $If\ f(x) = e^{-ax}\ and\ F_s[\,e^{-ax}] = \sqrt{\frac{2}{\pi}}\left[\frac{s}{(a^2+s^2)}\right]$

$then\ by\ Parseval's\ Identity\ \int_0^\infty \frac{\boldsymbol{s^2}}{\boldsymbol{(a^2+s^2)^2}} ds\ \ is$

(a) $\frac{\pi}{4a^3}$ **(b)** $\boldsymbol{\frac{\pi}{4a}}$ (c) $\frac{\boldsymbol{\pi}}{\boldsymbol{2a^2}}$ (d) $\frac{\boldsymbol{3\pi}}{\boldsymbol{4a^3}}$

Solution: By Parseval's Identity,

$$\int_0^\infty |F_c[\,s]|^2 ds \ = \int_0^\infty |f(x)|^2 dx$$

$$\boldsymbol{\frac{2}{\pi}} \int_0^\infty \frac{\boldsymbol{s^2}}{\boldsymbol{(a^2+s^2)^2}} ds \ = \int_0^\infty e^{-2ax} dx$$

$$\int_0^\infty \frac{s^2}{(a^2+s^2)^2} ds \ = \ \frac{\pi}{4a}$$

Ans: b

11. If $\frac{\boldsymbol{2}}{\boldsymbol{\pi}} \int_0^\infty \frac{\boldsymbol{10}}{\boldsymbol{(s^2+4)(s^2+25)}} ds = \int_0^\infty e^{-7x} dx$,

then $\int_0^\infty \frac{\boldsymbol{dx}}{\boldsymbol{(x^2+4)(x^2+25)}} \ =$

(a) $\frac{\pi}{120}$ **(b)** $\boldsymbol{\frac{\pi}{160}}$ **(c)** $\boldsymbol{\frac{\pi}{140}}$ (d) $\frac{\boldsymbol{3\pi}}{\boldsymbol{180}}$

Solution:

$$\int_0^\infty \frac{dx}{(s^2+4)(s^2+25)} \ = \ \frac{\pi}{20} \int_0^\infty e^{-7x} dx \ = \ \frac{\pi}{140}$$

Ans: c

12. If $\int_0^\infty e^{-ax} \cos bx\, dx \ = \frac{\boldsymbol{a}}{\boldsymbol{(a^2+b^2)}}$, then FCT of $f(x) = 3e^{-5x} + 5e^{-2x}$ is

(a) $\sqrt{\frac{2}{\pi}}\left[\frac{25}{(25+s^2)} + \frac{10}{(4+s^2)}\right]$ (b) $\sqrt{\frac{2}{\pi}}\left[\frac{5}{(25+s^2)} + \frac{20}{(4+s^2)}\right]$

(c) $\sqrt{\frac{2}{\pi}}\left[\frac{10}{(25+s^2)} + \frac{15}{(4+s^2)}\right]$ **(d)** $\boldsymbol{\sqrt{\frac{2}{\pi}}\left[\frac{15}{(25+s^2)} + \frac{10}{(4+s^2)}\right]}$

Solution:

$$F_c[f(x)] = 3F_c[e^{-5x}] + 5F_c[e^{-2x}]$$

$$= \sqrt{\frac{2}{\boldsymbol{\pi}}}\left\{\boldsymbol{3}\left[\frac{\boldsymbol{5}}{\boldsymbol{(5^2+s^2)}}\right] + \boldsymbol{5}\left[\frac{\boldsymbol{2}}{\boldsymbol{(2^2+s^2)}}\right]\right\}$$

$$= \sqrt{\frac{2}{\boldsymbol{\pi}}}\left[\frac{\boldsymbol{15}}{\boldsymbol{(25+s^2)}} + \frac{\boldsymbol{10}}{\boldsymbol{(4+s^2)}}\right]$$

Ans: d

13. By Fourier Transforms identity, if

$$\frac{2ab}{\pi} \int_0^\infty \frac{1}{(s^2+a^2)(s^2+b^2)} ds = \int_0^\infty e^{-(a+b)x} dx \ ,$$

Then $\int_0^\infty \frac{1}{(x^2+a^2)(x^2+b^2)}dx$ is

(a) $\frac{\pi}{2ab(a+b)}$ (b) $\frac{\pi}{2b(a+b)}$ (c) $\frac{\pi}{2a(a+b)}$ (d) $\frac{\pi}{2ab(a-b)}$

Solution:

$$\int_0^\infty \frac{1}{(s^2+a^2)(s^2+b^2)}ds = \frac{\pi}{2ab}\int_0^\infty e^{-(a+b)x}dx \ ,$$

$$= \frac{\pi}{2ab(a+b)}$$

Ans: a

14. If $F_c\left[\frac{e^{-ax}}{x}\right] = -\frac{1}{\sqrt{2\pi}}\log(a^2+s^2)$ then $F_c\left[\frac{e^{-ax}-e^{-bx}}{x}\right]$ is

(a) $\frac{1}{\sqrt{2\pi}}\log\left(\frac{b^2}{a^2+s^2}\right)$ **(b)** $\frac{1}{\sqrt{2\pi}}\log\left(\frac{b^2+s^2}{a^2+s^2}\right)$

(c) $\frac{1}{\sqrt{2\pi}}\log\left(\frac{a^2}{a^2+s^2}\right)$ (d) $\frac{1}{\sqrt{2\pi}}\log\left(\frac{b^2-s^2}{a^2+s^2}\right)$

Solution:

$F_c\left[\frac{e^{-ax}-e^{-bx}}{x}\right] = -\frac{1}{\sqrt{2\pi}}\log(a^2+s^2) + \frac{1}{\sqrt{2\pi}}\log(b^2+s^2)$

$$= \frac{1}{\sqrt{2\pi}}\log\left(\frac{b^2+s^2}{a^2+s^2}\right)$$

Ans:b

15. If $F_s\left[\frac{e^{-ax}}{x}\right] = -\sqrt{\frac{2}{\pi}}tan^{-1}\left(\frac{a}{s}\right)$ then $\int_0^\infty\left(\frac{e^{-ax}-e^{-bx}}{x}\right)\sin sx\,dx$ is

(a) $\left(tan^{-1}\left(\frac{a}{s}\right) - tan^{-1}\left(\frac{b}{s}\right)\right)$ (b) $\left(tan^{-1}\left(\frac{b}{s}\right) + tan^{-1}\left(\frac{a}{s}\right)\right)$

(c) $\left(tan^{-1}\left(\frac{b}{s}\right) - tan^{-1}\left(\frac{a}{s}\right)\right)$ (d) $\left(tan^{-1}\left(\frac{a}{s}\right) + tan^{-1}\left(\frac{b}{s}\right)\right)$

Solution:

$F_c\left[\frac{e^{-ax}-e^{-bx}}{x}\right] = \sqrt{\frac{2}{\pi}}\left(tan^{-1}\left(\frac{b}{s}\right) - tan^{-1}\left(\frac{a}{s}\right)\right)$

$\sqrt{\frac{2}{\pi}}\int_0^\infty\left(\frac{e^{-ax}-e^{-bx}}{x}\right)\sin sx\,dx = \sqrt{\frac{2}{\pi}}\left(tan^{-1}\left(\frac{b}{s}\right) - tan^{-1}\left(\frac{a}{s}\right)\right)$

$\int_0^\infty\left(\frac{e^{-ax}-e^{-bx}}{x}\right)\sin sx\,dx = \left(tan^{-1}\left(\frac{b}{s}\right) - tan^{-1}\left(\frac{a}{s}\right)\right)$

Ans: c

16. If $f(x) = \frac{2a}{\pi}\int_0^\infty \frac{cos\,sx}{s^2+a^2}ds$ (where $f(x) = e^{-ax}, a > 0$)

then by Inverse Fourier Transform $\int_0^\infty \frac{\cos mx}{x^2+a^2}dx$ is

(a) $\frac{\pi}{2a}e^{-ms}$ (b) $\frac{\pi}{2m}e^{-ma}$ (c) $\frac{\pi}{4a}e^{-ma}$ **(d)** $\frac{\pi}{2a}e^{-ma}$

Solution:

$$\int_0^\infty \frac{\cos sx}{x^2+a^2}ds = \frac{\pi}{2a}e^{-ax}$$

By changing the dummy variables of integration s=x and x=m then

$$\int_0^\infty \frac{\cos mx}{x^2+a^2}dx = \frac{\pi}{2a}e^{-ma}$$

Ans:d

17. If $f(x) = \frac{2}{\pi}\int_0^\infty \frac{s\sin sx}{s^2+a^2}ds$ (where $f(x) = e^{-ax}, a > 0$)

then by Inverse Fourier Transform $\int_0^\infty \frac{x\sin mx}{x^2+a^2}dx$ is

(a) $\frac{\pi}{2}e^{-am}$ (b) $\frac{\pi}{2m}e^{-ma}$ (c) $\frac{\pi}{4a}e^{-ma}$ (d) $\frac{\pi}{2a}e^{-ma}$

Solution:

$$\int_0^\infty \frac{s\sin sx}{x^2+a^2}ds = \frac{\pi}{2}e^{-ax}$$

By changing the dummy variables of integration s=x and x=m then

$$\int_0^\infty \frac{x\sin mx}{x^2+a^2}dx = \frac{\pi}{2}e^{-am}$$

Ans: a

18. If $\int_0^\infty \frac{\sin t}{t}dt = \frac{\pi}{2}$, then $F_s\left[\frac{1}{x}\right]$ is

(a) $\sqrt{\frac{2}{\pi}}$ **(b)** $\sqrt{\frac{\pi}{2}}$ (c) $\sqrt{\frac{1}{\pi}}$ (d) $\sqrt{\frac{1}{2\pi}}$

Solution: $F_s\left[\frac{1}{x}\right] = \sqrt{\frac{2}{\pi}}\int_0^\infty \frac{1}{x}\sin sx\,dx$

$$Put\ sx = \theta\ then\ \frac{1}{x} = \frac{s}{\theta}$$

$dx = \frac{d\theta}{s}$, then $F_s\left[\frac{1}{x}\right] = \sqrt{\frac{\pi}{2}}$

Ans: b

19. If $\int_0^\infty e^{-ax}\cos bx\,dx = \frac{a}{(a^2+b^2)}$, then FCT of $f(x) = e^{-2x} + 3e^{-x}$ is

(a) $\sqrt{\frac{2}{\pi}}\left[\frac{2}{(24+s^2)} + \frac{3}{(9+s^2)}\right]$ (b) $\sqrt{\frac{2}{\pi}}\left[\frac{1}{(25+s^2)} + \frac{2}{(4+s^2)}\right]$

(c) $\sqrt{\frac{2}{\pi}}\left[\frac{2}{(4+s^2)}+\frac{3}{(1+s^2)}\right]$ (d) $\sqrt{\frac{2}{\pi}}\left[\frac{5}{(25+s^2)}+\frac{1}{(4+s^2)}\right]$

Solution: $F_c[f(x)] = F_c[e^{-2x}] + 3F_c[e^{-x}]$

$$\sqrt{\frac{2}{\pi}}\left\{\left[\frac{2}{(4+s^2)}\right]+3\left[\frac{1}{(1+s^2)}\right]\right\}$$

$$=\sqrt{\frac{2}{\pi}}\left[\frac{2}{(4+s^2)}+\frac{3}{(1+s^2)}\right]$$

Ans:c

20. The Fourier sine Transforms of $f(x) = 1, 0 < x < 1\ is$

(a) $\sqrt{\frac{2}{\pi}}\left[\frac{\cos s}{s}\right]$ (b) $\sqrt{\frac{2}{\pi}}\left[\frac{1+\cos s}{s}\right]$ (c) $\sqrt{\frac{1}{\pi}}\left[\frac{2-\cos s}{s}\right]$ **(d)** $\sqrt{\frac{2}{\pi}}\left[\frac{1-\cos s}{s}\right]$

Solution: $F_s[f(x)] = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\sin sx\ dx = \sqrt{\frac{2}{\pi}}\int_0^1 \sin sx\ dx =$

$$\sqrt{\frac{2}{\pi}}\left[-\frac{\cos sx}{s}\right]_0^1 = \sqrt{\frac{2}{\pi}}\left[\frac{1-\cos s}{s}\right]$$

$$=\frac{1}{is\sqrt{2\pi}}[2i\sin as] = \sqrt{\frac{2}{\pi}}\left(\frac{\sin as}{s}\right)$$

Ans:d

21. If $\int_0^\infty e^{-ax}\sin bx\,dx \ = \frac{b}{(a^2+b^2)}$, then FST of $f(x) = e^{-2x} + 3e^{-x}$ is

(a) $\sqrt{\frac{2}{\pi}}\left[\frac{s}{(4+s^2)}+\frac{3s}{(1+s^2)}\right]$ (b) $\sqrt{\frac{2}{\pi}}\left[\frac{s}{(25+s^2)}+\frac{2s}{(4+s^2)}\right]$

(c) $\sqrt{\frac{2}{\pi}}\left[\frac{2}{(4+s^2)}-\frac{3}{(1+s^2)}\right]$ (d) $\sqrt{\frac{2}{\pi}}\left[\frac{5s}{(25+s^2)}+\frac{s}{(4+s^2)}\right]$

Solution: $F_s[f(x)] = F_c[e^{-2x}] + 3F_c[e^{-x}]$

$$\sqrt{\frac{2}{\pi}}\left\{\left[\frac{s}{(4+s^2)}\right]+3\left[\frac{s}{(1+s^2)}\right]\right\}$$

$$=\sqrt{\frac{2}{\pi}}\left[\frac{s}{(4+s^2)}+\frac{3s}{(1+s^2)}\right]$$

Ans:a

22. The Fourier cosine Transforms of $f(x) = 1, 0 < x < 1\ is$

(a) $\sqrt{\frac{2}{\pi}}\left[\frac{\cos s}{s-a}\right]$ **(b)** $\sqrt{\frac{2}{\pi}}\left[\frac{\sin s}{s}\right]$ (c) $\sqrt{\frac{1}{\pi}}\left[\frac{\cos s}{s}\right]$ **(d)** $\sqrt{\frac{2}{\pi}}\left[\frac{1-\sin s}{s}\right]$

Solution: $F_c[f(x)] = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos sx\, dx = \sqrt{\frac{2}{\pi}}\int_0^1 \cos sx\ dx$

$$= \sqrt{\frac{2}{\pi}}\left[\frac{\sin sx}{s}\right]_0^1 = \sqrt{\frac{2}{\pi}}\left[\frac{\sin s}{s}\right]$$

Ans:b

23. If $\frac{2}{\pi}\int_0^\infty \frac{s}{(s^2+4)(s^2+25)}ds = \int_0^\infty e^{-4x}dx$,

then $\int_0^\infty \frac{x\,dx}{(x^2+4)(x^2+25)} =$

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{16}$ **(c)** $\frac{\pi}{14}$ **(d)** $\frac{\pi}{8}$

Solution:

$$\int_0^\infty \frac{sds}{(s^2+4)(s^2+25)} = \frac{\pi}{2}\int_0^\infty e^{-4x}dx = \frac{\pi}{8}$$

Put s=x.

Ans: d

24. By Fourier Transforms identity, if $\frac{2ab}{\pi}\int_0^\infty \frac{1}{(s^2+a^2)(s^2+b^2)}ds = \int_0^\infty e^{-(a+b)x}dx$,

Then $\int_0^\infty \frac{1}{(x^2+9)(x^2+16}dx$ is

(a) $\frac{\pi}{162}$ (b) $\frac{\pi}{186}$ **(c)** $\frac{\pi}{168}$ (d) $\frac{\pi}{816}$

Solution:

$$\int_0^\infty \frac{1}{(s^2+3^2)(s^2+4^2)}ds = \frac{\pi}{2(3)(4)}\int_0^\infty e^{-7x}dx = \frac{\pi}{168}$$

Ans: c$e^{-7x}dx$

25. The Fourier Transforms of $f(x) = 1\ in\ 1 < x < 2\ is$

(a) $F[f(x)] = \frac{1}{is\sqrt{2\pi}}[e^{i2s} - e^{is}]$

(b) $F[f(x)] = \frac{1}{\sqrt{2\pi}}[e^{is} + e^{i2s}]$

(c) $F[f(x)] = \frac{1}{i\sqrt{\pi}}[e^{ib} - e^{ia}]$

(d) $F[f(x)] = \frac{1}{is\sqrt{2\pi}}[e^{is} - e^{i2s}]$

Solution: $F[f(x)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$

$$= \frac{1}{\sqrt{2\pi}}\int_{1}^{2} 1.e^{isx}dx \quad = \frac{1}{\sqrt{2\pi}}\left[\frac{e^{isx}}{is}\right]_1^2 = \frac{1}{is\sqrt{2\pi}}[e^{i2s} - e^{is}]$$

Ans: a

SRM Institute of Science and Technology

Ramapuram Campus

Department of Mathematics

Question Bank of Module-V(Z-Transform)

(2020–2021-ODD)

Subject.Code: 18MAB201T

Subject.Name: Transforms and Boundary Value Problems

Year/Sem: II/III **Part-A (1*20=20)** **Branch: Common to All branches**

1.	What is z(5)?	1 mark	
	$a)\ \frac{z}{z-1}$ $b)\ \frac{5z}{z-1}$ $c)\ \frac{z}{5(z-1)}$ $d)\ \frac{z-1}{z}$	Ans (b)	(CLO-5 Remember)
2.	$z[(-1)^n] =$ ________	1 mark	
	$a)\ \frac{z+1}{z}$ $b)\ \frac{z}{2z-1}$ $c)\ \frac{z}{(z+1)}$ $d)\ \frac{-z}{z+1}$	Ans (c)	(CLO-5 Remember)
3.	$z[(a)^n u(n)] =$ __________	1 mark	
	$a)\ \frac{z}{z-a}\ if\ \lvert z\rvert < a$ $b)\ \frac{z}{z+a}\ if\ \lvert z\rvert > a$ $c)\ \frac{z}{z-a}\ if\ \lvert z\rvert > a$ $d)\ \frac{z}{z+a}\ if\ \lvert z\rvert > a$	Ans (c)	(CLO-5 Remember)
4.	What is $z[(-2)^n]$?	**1** mark	
	$a)\ \frac{z+2}{z}$ $b)\ \frac{z}{z-2}$ $c)\ \frac{-z}{(z+2)}$ $d)\ \frac{z}{z+2}$	Ans (d)	(CLO-5 Remember)
5.	What is $z[n]$?	**1** mark	
	$a)\ \frac{z}{(z-1)^2}$ $b)\ \frac{-z}{(z+1)^2}$ $c)\ \frac{2z}{(z-1)^2}$ $d)\ \frac{z}{(z+1)^2}$	Ans (a)	(CLO-5 Remember)

6.	$z[e^{-5n}] =$ ________	1 mark	
	$a)\ \frac{z}{z+e^{-5}}$ $b)\ \frac{z}{z-e^{-5}}$ $c)\ \frac{z}{z+e^{5}}$ $d)\ \frac{2z}{z+e^{5}}$	Ans (b)	(CLO-5 Remember)
7.	What is Z-Transform of na^n ?	1 mark	
	$a)\ \frac{az}{(z+a)^2}$ $b)\ \frac{az}{(z-a)^2}$ $c)\ \frac{z}{(z-a)^2}$ $d)\ \frac{z}{(z+a)^2}$	Ans (b)	(CLO-5 Remember)
8.	What is $z[n^2]$?	1 mark	
	$a)\ \frac{az}{(z+a)^2}$ $b)\ \frac{az}{(z-a)^2}$ $c)\ \frac{z}{(z-a)^2}$ $d)\ \frac{z}{(z+a)^2}$	Ans (a)	(CLO-5 Remember)
9.	If z[f(t)]=F(z) then $\lim_{n\to\infty} F(z) = ?$	1 mark	
	$a)\ f(0)$ $b)\ f(1)$ $c)\ f(\infty)$ d) $\lim_{t\to\infty} f(t)$	Ans (a)	(CLO-5 Remember)
10.	What is Z-Transform of $z\left[\frac{1}{n!}\right]$?	1 mark	
	$a)\ e^{1/z}$ $b)\ e^{z}$ $c)\ e^{-1/z}$ $d)\ e^{-z}$	Ans (a)	(CLO-5 Remember)
11.	$z\left[\sin\frac{n\pi}{2}\right] =$ ____________	1 mark	
	$a)\ \frac{z}{(z^2+1)}$ $b)\ \frac{z}{(z^2-4)}$ $c)\ \frac{z}{(z^2-1)}$ $d)\ \frac{2z}{(z^2+1)}$	Ans (a)	(CLO-5 Remember)
12.	$z\left[\cos\frac{n\pi}{2}\right] =$ ____________	1 mark	
	$a)\ \frac{z^2}{(z^2+1)}$ $b)\ \frac{z}{(z-1)}$ $c)\ \frac{z}{(z^2+1)}$ $d)\ \frac{z^2}{(z^2-1)}$	Ans (a)	(CLO-5 Remember)

13.	$z^{-1}\left[\dfrac{z}{z-a}\right]=$ ________	1 mark	
	a) a^{n+1} b) a c) a^{n} d) a^{n-1}	Ans (c)	(CLO-5 Remember)
14.	$z^{-1}\left[\dfrac{z}{(z-a)^2}\right]=$ ________	1 mark	
	a) na^{n+1} b) na c) na^{n} d) na^{n-1}	Ans (d)	(CLO-5 Remember)
15.	$z^{-1}\left[\dfrac{1}{(z-a)}\right]=$ ________	1 mark	
	a) a^{n+1} b) a c) a^{n} d) a^{n-1}	Ans (d)	(CLO-5 Remember)
16.	$z^{-1}\left[e^{1/z}\right]=$ ________	1 mark	
	a) $\dfrac{1}{n+1}$ b) $\dfrac{1}{(n+1)!}$ c) $\dfrac{1}{(n-1)!}$ d) $\dfrac{1}{n!}$	Ans (d)	(CLO-5 Remember)
17.	$z^{-1}\left[\dfrac{z}{(z-1)^2}\right]=$ ________	1 mark	
	a) n b) $n+1$ c) $n-1$ d) $\dfrac{1}{n}$	Ans (a)	(CLO-5 Remember)
18.	$z^{-1}\left[\dfrac{az}{(z-1)^2}\right]=$ ________	1 mark	
	a) na^{n+1} b) na c) na^{n} d) na^{n-1}	Ans (c)	(CLO-5 Remember)
19.	$z^{-1}\left[\dfrac{z}{(z+1)}\right]=$ ________	1 mark	

	$a)\ (-1)^n$ $b)\ (-1)^{n+1}$ $c)\ (-1)^{n-1}$ $a)\ n(-1)^n$	Ans (a)	(CLO-5 Remember)
20.	What is $z[f(n)*g(n)]$	1 mark	
	$a)\ F(z).G^{-1}(z)$ $b)\ F^{-1}(z).G^{-1}(z)$ $c)\ F^{-1}(z).G(z)$ $d)\ F(z).G(z)$	Ans (d)	(CLO-5 Remember)
21.	$Z(a^n.n)$		
	(a) $\frac{az}{(z+a)^2}$ (b) $\frac{z}{(z-a)^2}$ (c) $\frac{az}{(z-a)^2}$ (d) $\frac{z}{a(z-a)^2}$	Ans (c)	(CLO-5 **Remember)**
22.	$Z[5.3^n - 2(-1)^n]$		
	(a) $5\left(\frac{z}{z-3}\right)-2\left(\frac{z}{z+1}\right)$ (b) $4\left(\frac{z}{z-3}\right)+2\left(\frac{z}{z-1}\right)$ (c) $5\left(\frac{z}{z-4}\right)+2\left(\frac{z}{z+1}\right)$ (d) $5\left(\frac{z}{z-3}\right)+\left(\frac{z}{z-2}\right)$	Ans (a)	(CLO-5 Remember)
23.	$Z\left(\frac{1}{n}\right)$		
	(a) $z\log\left(\frac{z}{z-1}\right)$ (b) $z\log\left(\frac{z-1}{z}\right)$ (c) $\log\left(\frac{z-1}{z}\right)$ (d) $\log\left(\frac{z}{z-1}\right)$	Ans (d)	(CLO-5 Remember)
24.	$Z^{-1}\left(\frac{1}{(z-\frac{1}{2})(z-\frac{1}{3})}\right)$ residue at $z=\frac{1}{2}$		

	(a) $\frac{6}{2^{n+1}}$ (b) $\frac{6}{2^{n-1}}$ (c) $\frac{6}{2^{n+2}}$ (d) $-\frac{6}{2^{n+1}}$	Ans (b)	(CLO-5 Remember)
25.	$Z^{-1}\left(\frac{8z^2}{(2z-1)(4z-1)}\right)$		
	(a) $\left(\frac{1}{2}\right)^n * \left(\frac{1}{4}\right)^n$ (b) $(1)^n * \left(\frac{1}{4}\right)^n$ (c) $\left(\frac{1}{8}\right)^n * \left(\frac{1}{4}\right)^n$ (d) $\left(\frac{1}{2}\right)^{n-1} * \left(\frac{1}{4}\right)^n$	Ans (a)	(CLO-5 Remember)
26.	$y(n+2)-4y(n+1)+4y(n)=0 \ where \ \ y(0)=1, y(1)=0$		
	(a) $Y(z)(z^2-4z+4)=z^2+4z$ (b) $Y(z)(z^2+4z+4)=z^2-4z$ (c) $Y(z)(z^2-4z+4)=z^2-4z$ (d) $Y(z)(z^2-4z-4)=z^2+4z$	Ans (c)	(CLO-5 Remember)
27.	Z [t] is		
	(a) $Z[t]=T\sum_{n=1}^{n=\infty} n z$ (b) $Z[t]=T\sum_{n=1}^{n=\infty} n z^{-n}$ (c) $Z[t]=T\sum_{n=1}^{n=\infty} \frac{1}{n} z^{-n}$ (d) $Z[t]=T\sum_{n=1}^{n=\infty} z^{-n}$	Ans (b)	(CLO-5 Remember)
28.	Z $\left[-n^2\right]$ is		
	(a) $-\frac{z(z+1)}{(z-1)^3}$ (b) $\frac{z}{(z-1)^2}$ (c) $\frac{z(z+1)}{(z-1)^3}$ (d) $\frac{Tz}{(z-1)^2}$	Ans (a)	(CLO-5 Remember)

29.	Z $\left(e^{3t-7}\right)$ is		
	(a) $\frac{e^{7}z}{z+e^{-3t}}$ (b) $\frac{e^{-7}z}{z-e^{-3t}}$ (c) $\frac{e^{-7}}{z-e^{-t}}$ (d) $\frac{e^{7}}{z+e^{-t}}$	Ans (b)	(CLO-5 Remember)
30.	Z- Transform formula		
	(a) $Z[f(n)]=\sum_{n=1}^{n=\infty} f(n)z^{-n}$ (b) $Z[f(n)]=\sum_{n=1}^{n=\infty}\left(f(n)z\right)^{-n}$ (c) $Z[f(n)]=\sum_{n=1}^{n=\infty} z^{n}$ (d) $Z[f(n)]=\sum_{n=1}^{n=\infty} f(n)z^{-n}$	Ans (a)	(CLO-5 Remember)
31.	Inverse Z transform of $\frac{z^2}{\left(z-\frac{1}{2}\right)(z-4)}$ is		
	(a) $\left[2\left(\frac{1}{2}\right)^n-\left(\frac{1}{4}\right)^n\right]$ (b) $\left[2\left(\frac{1}{2}\right)^n+\left(\frac{1}{4}\right)^n\right]$ (c) $\left[\left(\frac{1}{2}\right)^n-\left(\frac{1}{4}\right)^n\right]$ (d) $\left[\left(\frac{1}{2}\right)^n+\left(\frac{1}{4}\right)^n\right]$	Ans (a)	(CLO-5 Remember)
32.	Z $\left(e^{t+7}\right)$ is		
	(a) $\frac{e^{10}z}{z+e^{-t}}$ (b) $\frac{e^{7}z}{z-e^{t}}$ (c) $\frac{e^{10}}{z-e^{-t}}$ (d) $\frac{e^{10}}{z+e^{-t}}$	Ans (b)	(CLO-5 Remember)
33.	Z$[n(2n-1)]$ is		
	(a) $\frac{2z}{(z+1)^2}$ (b) $\frac{z(z+3)}{(z-1)^3}$ (c) $\frac{2z}{(z-1)^3}$ (d) $\frac{z}{(z-1)}$	Ans (b)	(CLO-5 Remember)
34.	Z $[n-1]$ is		

	$a)\ \frac{z}{(z-1)^2}$ $b)\ \frac{-2z}{(z+1)^2}$ $c)\ \frac{2z(1-z)}{(z-1)^2}$ $d)\ \frac{z}{(z+1)^2}$	Ans (c)	(CLO-5 Remember)
35.	inverse Z- transform of $\frac{z}{z^2-7z+10}$		
	(a) $-\frac{1}{3(z-5)}+\frac{1}{3(z+2)}$ (b) $-\frac{1}{3(z-5)}-\frac{1}{3(z-2)}$ (c) $\frac{1}{3(z-5)}-\frac{1}{3(z-2)}$ (d) $-\frac{1}{(z-5)}+\frac{1}{(z-2)}$	Ans (c)	(CLO-5 Remember)
36.	$Z(\sin \omega t)$ is		
	(a) $\frac{z \sin \omega T}{z^2-2z\cos \omega T-1}$ (b) $\frac{z \sin \omega T}{z^2-2z\cos \omega T+1}$ (c) $\frac{z^2 \sin \omega T}{z^2-2z\cos \omega T+1}$ (d) $\frac{z \sin \omega T}{z^2+2z\cos \omega T+1}$	Ans (b)	(CLO-5 Remember)

SRM INSTITUTE OF SCIENCE AND TECHNOLOGY
RAMAPURAM CAMPUS
DEPARTMENT OF MATHEMATICS

Year/Sem : II/III
Branch: Common to All branches

Unit V – Z Transforms

1.Find $Z(n)$

(a) $\frac{\mathbf{z}}{(\mathbf{z-1})^2}$ (b) $\frac{z}{(z+1)^2}$ (c) $\frac{z}{z-1}$ (d) $\frac{z}{z+1}$

Solution:

$$Z(n) = \sum_{0}^{\infty} nz^{-n}$$
$$= \frac{1}{z} + \frac{2}{z^2} + \frac{3}{z^3} + \cdots$$
$$= \frac{1}{z}\left(1 - \frac{1}{z}\right)^{-2} = \frac{z}{(z-1)^2}$$

2. Find Z-Transform of na^n

(a) $\frac{z}{(z+a)^2}$ (b) $\frac{z}{(z-a)^2}$ **(c)** $\frac{\boldsymbol{az}}{(\boldsymbol{z-a})^2}$ (d) $\frac{az}{(z+a)^2}$

Solution:

$$Z(nf(n)) = -z\frac{d}{dz}Z(a^n)$$
$$= -z\frac{d}{dz}\left(\frac{z}{z-a}\right)$$
$$= -z\left(-\frac{a}{(z-a)^2}\right)$$
$$= \frac{az}{(z-a)^2}$$

3. Find Z-Transform of $a^n \frac{1}{n!}$

(a) $e^{-\frac{a}{z}}$ **(b)** $e^{\frac{a}{z}}$ (c) $e^{\frac{1}{z}}$ (d) $e^{-\frac{1}{z}}$

Solution

$$Z\left(a^n \frac{1}{n!}\right) = Z\left(\frac{1}{n!}\right)_{z \to \frac{z}{a}}$$

$$= \left(e^{\frac{1}{z}}\right)_{z \to \frac{z}{a}}$$

$$= e^{\frac{a}{z}}$$

4. Find Z-Transform of $r^n e^{in\theta}$ by eliminating arbitrary function

(a) $\frac{z}{z-re^{i\theta}}$ (b) $\frac{z}{z+re^{i\theta}}$ (c) $\frac{1}{z-re^{i\theta}}$ (d) $\frac{-1}{z-re^{i\theta}}$

Solution:

We know that $Z(a^n) = \frac{z}{z-a}$

Put $a = re^{i\theta}$

$$Z\left(r^n e^{in\theta}\right) = \frac{z}{z - re^{i\theta}}$$

5. Find $Z(n^2)$

(a) $-z\left[\frac{1+z}{(z-1)^3}\right]$ (b) $z\left[\frac{1+z}{(z-1)^3}\right]$

(c) $-z\left[\frac{1+z}{(z+1)^3}\right]$ (d$z\left[\frac{1+z}{(z+1)^3}\right]$

Solution:

$$Z(n^2) = -z\frac{d}{dz}Z(n)$$

$$= -z\frac{d}{dz}\left(\frac{z}{(z-1)^2}\right)$$

$$= -z\left[\frac{1+z}{(z-1)^3}\right]$$

6.Find $Z(n(n-1))$

(a) $\frac{z}{(z+1)^3}$ (b) $\frac{z}{(z-1)^3}$ (c) $\frac{2z}{(z+1)^3}$ **(d)** $\boldsymbol{\frac{2z}{(z-1)^3}}$

Solution:

$$\begin{aligned} Z(n(n-1)) &= Z(n^2-n) \\ &= Z(n^2) - Z(n) \\ &= \frac{z(z+1)}{(z-1)^3} - \frac{z}{(z-1)^2} \\ &= \frac{2z}{(z-1)^3} \end{aligned}$$

7. Find $Z(n(1+5^n))$

(a) $\boldsymbol{\frac{z}{(z-1)^2} + \frac{5z}{(z-5)^2}}$ (b) $\frac{z}{(z+1)^2} + \frac{5z}{(z+5)^2}$

(c) $\frac{z}{(z-1)^2} + \frac{5z}{(z+5)^2}$ (d) $\frac{z}{(z+1)^2} + \frac{5z}{(z-5)^2}$

Solutionz9:

$$\begin{aligned} Z\left(n(1+5^n)\right) &= Z(n) + Z(n.5^n) \\ &= \frac{z}{(z-1)^2} + \frac{5z}{(z-5)^2} \end{aligned}$$

8. Find the inverse Z-Transform of $\frac{z}{z+1} + \frac{7z}{z-3}, for\ n > 0$

(a) $(-1)^n + 7(-3)^n$ (b) $(-1)^n - 7(3)^n$

(c) $(-1)^n - 7(-3)^n$ **(d)** $\boldsymbol{(-1)^n + 7(3)^n}$

Solution:

$$Z^{-1}\left(\frac{z}{z+1} + \frac{7z}{z-3}\right) = (-1)^n + 7(3)^n$$

***9.**Find* $Z^{-1}\left(\frac{2z}{2z-1}\right)$

(a) $\left(\frac{1}{2}\right)^n$ **(b)** $\boldsymbol{\left(\frac{1}{2}\right)^n}$

(c) $\left(\frac{1}{2}\right)^n$ (d) $\left(\frac{1}{2}\right)^n$

Solution:

$$Z^{-1}\left(\frac{2z}{2z-1}\right) = \frac{2}{2}Z^{-1}\left(\frac{z}{z-\frac{1}{2}}\right) = \left(\frac{1}{2}\right)^n$$

10. Find $Z^{-1}\left(\frac{z}{4z+1}\right)$

(a) $(-1)^n\left(\frac{1}{3}\right)^{n+1}$ (b) $(-2)^n\left(\frac{1}{3}\right)^{n+1}$

(c) $\boldsymbol{(-1)^n\left(\frac{1}{4}\right)^{n+1}}$ (d) $(-2)^n\left(\frac{1}{4}\right)^{n+1}$

Solution:

$$Z^{-1}\left(\frac{z}{4z+1}\right) = \frac{1}{4}Z^{-1}\left(\frac{z}{z+\frac{1}{4}}\right)$$

$$= \frac{1}{4}\left(-\frac{1}{4}\right)^n = (-1)^n\left(\frac{1}{4}\right)^{n+1}$$

11.Find $\boldsymbol{Z}\big(3^n(1+n)\big)$

(a) $\frac{z}{z+3}+\frac{3z}{(z+3)^2}$ (b) $\frac{z}{z+3}+\frac{3z}{(z-3)^2}$

(c) $\boldsymbol{\frac{z}{z-3}+\frac{3z}{(z-3)^2}}$ (d) $\frac{z}{z-3}+\frac{3z}{(z+3)^2}$

Solution:

$$\boldsymbol{Z}\big(3^n(1+n)\big) = Z(3^n)+Z(n.3^n)$$

$$= \frac{z}{z-3}+\frac{3z}{(z-3)^2}$$

12.If $F(z)z^{n-1} = \frac{z^n}{(z-1)(z-2)}$ $then\ find\ residue\ at\ largest\ pole$

(a) 2^{-n} (b) 3^{-n}

(c) 3^n **(d)**$\boldsymbol{2^n}$

Solution:

$$Res_{z=2}F(z)z^{n-1} = \lim_{z\to 2}(z-2)\frac{z^n}{(z-1)(z-2)}$$

$$= \frac{2^n}{2-1} = 2^n$$

13. If $F(z)z^{n-1} = \frac{z^n}{(z-1)(z-2)}$ $then\ find\ residue\ at\ smallest\ pole$

(a) $\boldsymbol{-1}$ (b) $(-1)^n$

(c) 1 (d) $(-2)^n$

Solution:

$$Res_{z=1}F(z)z^{n-1} = \lim_{z\to 2}(z-1)\frac{z^n}{(z-1)(z-2)}$$
$$= \frac{1}{-1} = -1$$

14. Find $y(z) for\ the\ difference\ Equation\ y_{n+1} - y_n = 0, y_0 = 1$

(a) $y(z) = \frac{z}{z+1}$ (b) $y(z) = \frac{z}{1-z}$

(c) $\boldsymbol{y(z) = \frac{z}{z-1}}$ (d) $y(z) = \frac{-z}{z+1}$

Solution:

$$Z(y_n) - zy_0$$
$$zy(z) - zy_0 - y(z) = 0$$
$$(z-1)y(z) = z$$
$$y(z) = \frac{z}{z-1}$$

15.Find $Z[(e^n)^{100} + (e^n)^{200})]$

(a) $\boldsymbol{\frac{z}{z-e^{100}} + \frac{z}{z-e^{200}}}$ (b) $\frac{z}{z+e^{100}} + \frac{z}{z-e^{200}}$

(c) $\frac{z}{z-e^{100}} + \frac{z}{z+e^{200}}$ (d) $\frac{z}{z+e^{100}} + \frac{z}{z+e^{200}}$

Solution:

$$Z[(e^n)^{100} + (e^n)^{200})] = Z\ ((e^{100})^n) + Z(((e^{100})^n)$$

$$= \frac{z}{z-e^{100}} + \frac{z}{z-e^{200}}$$

16. Using Final value theorem evaluate $f(z) = \frac{1+z^{-1}}{1-0.25z^{-1}}$

(a) 0 (b) 1

(c) 2 (d) 3

Solution:

Let $F(z) = \frac{z+1}{z-0.25}$

$$\lim_{z\to 1}(z-1)F(z) = \lim_{z\to 1}(z-1)\frac{z+1}{z-0.25} = 0$$

17. Find the initial value of $F(z) = \frac{z}{2z^2-3z+1}$

(a) 0 (b) 1 (c)-1 (d)2

Solution:

$f(0) = \lim_{z\to\infty} \boldsymbol{F(z)}$

$$0 = \frac{1}{z\left(2-\frac{3}{z}+z^2\right)} = \frac{z}{z^2\left(2-\frac{3}{z}+\frac{1}{z^2}\right)}$$

18. Evaluate $Z[(k-1)a^{k-1}]$

(a)$\boldsymbol{\frac{a}{(z-a)^2}}$

(b) $\frac{a}{(z+a)^2}$

(c)$\frac{1}{(z-a)^2}$

(d) $\frac{1}{(z+a)^2}$

Solution

$Z[(k-1)a^{k-1}] = z^{-1}Z[ka^k]$

$$= z^{-1}\left(\frac{az}{(z-a)^2}\right) = \frac{a}{(z-a)^2}$$

19. *Find* $Y(z)$ $for\ y_{n+2} + 4y_n = 0\ , y_0 = 0, y_1 = -2$

(a) $Y(z) = \frac{z}{z^2+4}$ (b)$Y(z) = \frac{2z}{z^2-4}$

(c)$\boldsymbol{Y(z) = \frac{2z}{z^2+4}}$ (d)$Y(z) = \frac{-2z}{z^2+4}$

Solution:

$$z^2Y(z) - z^2y(0) - zy(1) - 4Y(z) = 0$$

$$z^2Y(z) - z(2) + 4Y(z) = 0$$

$$Y(z) = \frac{2z}{z^2 + 4}$$

20.Find $Z\{e^{3n}\}$

(a) $-\boldsymbol{cosx}$ (b) $cosx$ (c) $-sinx$ (d) $sinx$

Solution:

$$Z\{e^{3n}\} = Z\{(e^3)^n\} = \frac{z}{z - e^3}$$

21. Find $Z\{(-3)^n\}$

(a) $\boldsymbol{\frac{z}{z+3}}$ (b) $\frac{-z}{z+3}$ (c) $\frac{z}{z-3}$ (d) $\frac{2z}{z+3}$

Solution:

$$Z\{a^n\} = \frac{z}{z - a}$$

$$Z\{(-3)^n\} = \frac{z}{z - (-3)} = \frac{z}{z + 3}$$

22. Find $Z\{2^n + (-5)^n\}$

(a) $\frac{z}{z+2} + \frac{z}{z-5}$ (b) $\frac{z}{z-2} + \frac{z}{z-5}$ **(c)** $\boldsymbol{\frac{z}{z-2} + \frac{z}{z+5}}$ (d) $\frac{z}{z+2} + \frac{z}{z-5}$

Solution:

$$Z\{2^n + (-5)^n\} = Z\{2^n\} + Z\{(-5)^n\}$$

$$= \frac{z}{z - 2} + \frac{z}{z + 5}$$

23. Find $Z\{4.8^n + (-4)(-6)^n\}$

(a) $\frac{4z}{z-8} + \frac{4z}{z+6}$

(b) $\boldsymbol{\frac{4z}{z-8} - \frac{4z}{z+6}}$

(c) $\frac{4z}{z+8} - \frac{4z}{z-6}$

(d) $\frac{4z}{z+8} + \frac{4z}{z-6}$

Solution:1

$$1Z\{4.8^n + (-4)(-6)^n\} = 4Z\{8^n\} + (-4)Z\{(-6)^n\}$$
$$= \frac{4z}{z-8} - \frac{4z}{z+6}$$

24.Find $Z^{-1}\left\{\frac{z}{z+7} + 2.\frac{z}{z-3}\right\}$

(a) $(7)^{\mathrm{n}} + 2(-3)^n$ (b) $(-7)^{\mathrm{n}} - 2(3)^n$

(c)$(7)^{\mathrm{n}} + 2(3)^n$ **(d)** $\boldsymbol{(-7)^{\mathrm{n}} + 2(3)^n}$

Solution:

$$Z^{-1}\left\{\frac{z}{z+7} + 2.\frac{z}{z-3}\right\} = Z^{-1}\left\{\frac{z}{z+7}\right\} + 2\,Z^{-1}\left\{\frac{2}{z-3}\right\}$$
$$= (-7)^{\mathrm{n}} + 2(3)^n$$

25.Find $Z^{-1}\left\{\frac{z}{z-8} + 3.\frac{z}{z+3}\right\}$

(a) $(8)^{\mathrm{n}} - 3(-3)^n$ (b) $(-8)^{\mathrm{n}} + 3(3)^n$

(c)$(-8)^{\mathrm{n}} + 3(-3)^n$ **(d)**$\boldsymbol{(8)^{\mathrm{n}} + 3(-3)^n}$

Solution:

$$Z^{-1}\left\{\frac{z}{z-8} + 3.\frac{z}{z+3}\right\} = Z^{-1}\left\{\frac{z}{z-8}\right\} + 3\,Z^{-1}\left\{\frac{2}{z+3}\right\}$$
$$= (8)^{\mathrm{n}} + 3(-3)^n$$

UNIT-I

PARTIAL DIFFERENTIAL EQUATIONS

1. The complete integral of $p = q$ is ____

(i) $z = ax + by$ (ii) $z = a(x + y) + b$ (iii) $z = ax + by + c$

(iv) $z = ax - by + a$

2. The complete integral of $q = 2py$ is ____

(i) $z = ax + ay^2 + b$ (ii) $z = ax^2 - ay^2 + b$ (iii) $z = ax + by$

(iv) $z = 2xy$.

3. The complete integral of $pq = 1$ is

(i) $az = a^2x + y + ac$ (ii) $z = ax + ay + c$ (iii) $az = x + y + c$

(iv) $z = x + y + c$.

4. The solution to $pq = x$ is

(i) $z = \frac{x^2}{2a} + ay + c$ (ii) $z = \frac{y^2}{2a} + ax + c$ (iii) $z = x + y + 1$

(iv) $z = x - ay$.

5. The partial differential equation formed by eliminating arbitrary constant is

$z = (x + a)(y + b)$ is

(i) $z = p + q$ (ii) $z = p - q$ (iii) $z = \frac{p}{q}$ (iv) $z = pq$.

6. The partial differential equation formed by eliminating arbitrary constant in

$z = ax + by + ab$

(i) $z = px + qy + ab$ (ii) $z = ax + by + pq$ (iii) $z = px + qy + pq$

(iv) $c = px + qy + pq$

7. The partial differential equation formed by eliminating arbitrary function in $z = f(x^2 + y^2)$ is

(i) $xp = yq$ (ii) $xy = pq$ (iii) $xq = yp$ (iv) $x + p = y + q$

8. The general integral of $z = xp + yq$ is

(i) $\Phi\left(\frac{x}{y}, \frac{y}{z}\right) = 0$ (ii) $\Phi(x + y, y + z) = 0$ (iii) $\Phi\left(x - y, \frac{x}{z}\right) = 0$

(iv) $\Phi\left(\frac{x}{y}, y + z\right) = 0$.

9. The general integral of $p + q = 1$ is

(i) $x - y = f(y - z)$ (ii) $\Phi(x + y, y - z) = 0$ (iii) $f(x - y, y - z) = 0$

(iv) $x = y + f(y + z)$

10. The solution to $z^2 = pq$ is

(i) $x + ay + c = \sqrt{a}\, logz$ (ii) $x - ay + c = logz$ (iii) $ax + y = \log az$

(iv) $ax + y = \text{a} \log z$

11. The solution which has number of arbitrary constants equal to number of independent variables is

(i) general integral (ii) complete integral (iii) particular integral

(iv) singular integral

12. The complete integral of $p^2 + q^2 = x + y$ is

(i) $z = \frac{2}{3}(x - a)^{\frac{3}{2}} + \frac{2}{3}(y - a)^{\frac{3}{2}} + b$ (ii) $z = \frac{2}{3}(x + a)^{\frac{3}{2}} + \frac{2}{3}(y + a)^{\frac{3}{2}} + b$

(iii) $z = \frac{2}{3}(x + a)^{\frac{3}{2}} + \frac{2}{3}(y - a)^{\frac{3}{2}} + b$ (iv) $z = \frac{2}{3}(x + a)^{\frac{3}{2}} + \frac{2}{3}(a - y)^{\frac{3}{2}} + b$

13. Solve $\left(D^3 - 7DD'^2 - 6D'^3\right)z = 0$.

(i) $z = f_1(y - x) + f_2(y - 2x) + f_3(y + 3x)$

(ii) $z = f_1(y - x) + f_2(y + 2x) + f_3(y - 3x)$

(iii) $z = f_1(y + x) + f_2(y - 2x) + f_3(y + 3x)$

(iv) $z = f_1(y + x) + f_2(y - 2x) + f_3(y + 3x)$

14. Solve $(D^3 - 3D^2D')z = 0$.

(i) $z = f_1(y - x) + f_2(y - 2x) + f_3(y + 2x)$

(ii) $z = f_1(y) + f_2(y) + f_3(y + 3x)$

(iii) $z = f_1(y) + xf_2(y) + f_3(y + 3x)$

(iv) $z = f_1(y) + f_2(y) + f_3(y - 3x)$

15. The complementary function of $\left(D^3 - 3D^2D' + 4D'^3\right)z = e^{2x+y}$.

(i) $f_1(y - x) + f_2(y - 2x) + f_3(y + 2x)$

(ii) $f_1(y + x) + f_2(y + 2x) + xf_3(y - 2x)$

(iii) $f_1(y + x) + xf_2(y + 2x) + f_3(y + 2x)$

(iv) $f_1(y - x) + f_2(y + 2x) + xf_3(y + 2x)$

16. The particular integral of $(D^3 - 2D^2D')z = e^{x+2y}$ is

(i) $\frac{e^{x+2y}}{3}$ (ii) $\frac{e^x}{3}$ (iii) e^{x+2y} (iv) $\frac{-e^{x+2y}}{3}$

17. The particular integral of $(D^2)z = x^3y$ is

(i) $\frac{x^5y}{20}$ (ii) x^3y (iii) x^4y^2 (iv) x^2y^2

18. The partial differential equation $u_{xx} = u_{yy}$ is of the form

(i) parabolic (ii) elliptic (iii) hyperbolic (iv) none of these

19. The particular integral of $(D^2 + 2DD' + D'^2)z = sinhy$ is

(i) $tanhy$ (ii) $coshy + sinhy$ (iii) $coshy - sinhy$ (iv) $sinhy$

20. The complete integral of $z = px + qy + p^2 + q^2$ is

(i) $z = ax + by + a^2 + b^2$ (ii) $z = ax + by + a^2 - b^2$

(iii) $z = ax + by + c^2 + d^2$ (iv) $z = ax - by + c^2 - d^2$

21. If complete integral is $z = ax + by - 3ab$, the singular integral is

(i) $z = x + y$ (ii) $z = \frac{x}{y}$ (iii) $z = xy$ (iv) $xy = 3z$.

22. The partial differential equation is elliptic if $B^2 - 4AC$

(i) >0 (ii) ≥ 0 (iii) ≤ 0 (iv) < 0

23. The degree and order of $\frac{\partial^3 z}{\partial x^3} + \left(\frac{\partial^3 z}{\partial x \partial y^2}\right)^2 + \frac{\partial z}{\partial y} = \sin(x + 2y)$ is

(i) 2, 3 (ii) 1, 3 (iii) 3, 2 (iv) 1, 3

24. $x^2 f_{xx} + (1 - y^2) f_{yy} = 0$ is elliptic in _______ region

(i) $x \neq 0, |y| < 1$ (ii) $x \neq 0, |y| > 1$ (iii) $x = 0$ (iv) $|y| < 1$

25. The general solution of $p\, tan\, x + q\, tan\, y = \tan z$

(i) $f\left(\frac{\sin x}{\sin y}, \frac{\sin y}{\sin z}\right) = 0$ (ii) $f\left(sin\left(\frac{x}{y}\right), sin\left(\frac{y}{z}\right)\right) = 0$

(iii) $f(\sin x, siny) = 0$ (iv) $f(\sin y, \sin z) = 0$

MA1003 - Transforms and Boundary Value Problems

Unit-IV : Fourier Transforms

1. If f(x) is a function defined in (-l,l) and satisfies Dirichlet's conditions then

   a. $f(x) = \frac{1}{\pi}\int_0^{\infty}\int_{-\infty}^{\infty} f(t)\cos\lambda(t-x)\,dx\,d\lambda$

   $f(x) = \frac{1}{\pi}\int_0^{\infty}\int_{-\infty}^{\infty} f(t)\cos x(t-x)\,dx\,d\lambda$. b.

   b. $f(x) = \frac{1}{2\pi}\int_0^{\infty}\int_{-\infty}^{\infty} f(t)\cos\lambda(t-x)\,dx\,d\lambda$

   $f(x) = \frac{2}{\pi}\int_0^{\infty}\int_{-\infty}^{\infty} f(t)\cos\lambda(t-x)\,dx\,d\lambda$ d.

2. The Fourier transform of a function f(x) is

   a. $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\,e^{ist}\,dt$

   b. $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\,e^{isx}\,dx$

   c. $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\,e^{isx}\,dx$

   d. $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(s)\,e^{isx}\,dx$

3. The Fourier transform of f(x)=$e^{-\frac{x^2}{2}}$ is

   a. $e^{-\frac{s^2}{2}}$

   b. $\frac{1}{e^{\frac{s^2}{2}}}$

   c. $\frac{1}{e^{-\frac{s^2}{2}}}$

   d. $\frac{1}{e^{x^2}}$

4. The Fourier cosine transform of e^{-ax} is

   a. $\sqrt{\frac{2}{\pi}}\frac{a}{a^2+x^2}$

   b. $\sqrt{\frac{1}{\pi}}\frac{s}{s^2+a^2}$

   c. $\sqrt{\frac{1}{\pi}}\frac{a}{s^2+a^2}$

   d. $\sqrt{\frac{2}{\pi}}\frac{a}{s^2+a^2}$

5. Under Fourier cosine transform $f(x) = \frac{1}{\sqrt{x}}$ is

   a. self-reciprocal function

   b. cosine function

   c. inverse function

   d. complex function

6. The Fourier sine transform of x $e^{-\frac{x^2}{2}}$ is

   a. 0

   b. $s\,e^{-\frac{s^2}{2}}$

   c. $\frac{1}{e^{x^2}}$

   d. none of the above

7. $F[f(ax)] = \frac{1}{a}F\left(\frac{s}{a}\right)$

   a. $\frac{1}{s}F\left(\frac{s}{a}\right)$

   b. $\frac{1}{a}F\left(\frac{a}{s}\right)$

c. $\frac{1}{a}F\left(\frac{s}{a}\right)$

d. $\frac{1}{s}F\left(\frac{as}{a}\right)$

8. $F[f(x-a)] =$

a. $e^{ias}F(a)$

b. $e^{ias}F(x)$

c. $e^{iax}F(a)$

d. $e^{ias}F(s)$

9. $F[e^{iax}f(x)] =$

a. F(s+a)

b. F(s-a)

c. F(sa)

d. F(s/a)

10. F[f(x) cosax] =

a. [f(a)+f(s-a)]/ 2

b. [f(sa)+f(s+a)]/ 2

c. [f(s+a)+f(s-a)]/ 2

d. none of the above

11. F[f(x) *g(x)] =

a. F(s) + G(s)

b. F(s) - G(s)

c. F(s)G(s)

d. F(s)|G(s)

12. If F(s) = F [f(x)] then $\int_{-\infty}^{\infty}|f(x)|^2dx =$

a. $\int_{-\infty}^{\infty}|f(x)|^2dx$

b. $\int_{-\infty}^{\infty}|f(s)|^2ds$

c. $\int_{0}^{\infty}|f(x)|^2dx$

d. $\int_{-0}^{\infty}|f(s)|^2ds$

13. $F[xf(x)] =$

a. $\frac{dF(s)}{ds}$

b. $i\frac{dF(s)}{ds}$

c. $-i\frac{dF(s)}{ds}$

d. $-\frac{dF(s)}{ds}$

14. $F_C[xf(x)] =$

a. $\frac{dF_S(s)}{ds}$

b. $i\frac{dF_S(s)}{ds}$

c. $-i\frac{dF_S(s)}{ds}$

d. $-\frac{dF_S(s)}{ds}$

15. $F_s[xf(x)] =$

a. $\frac{dF_C(s)}{ds}$

b. $i\frac{dF_C(s)}{ds}$

c. $-i\frac{dF_C(s)}{ds}$

d. $-\frac{dF_C(s)}{ds}$

16. The relation between Fourier transform and Laplace transform is

a. $F[f(x)] = \frac{1}{\sqrt{2\pi}} L[g(x)]$

b. $F[f(x)] = \frac{1}{\sqrt{\pi}} L[g(x)]$

c. $F[f(x)] = \frac{1}{\sqrt{2}} L[g(x)]$

c. $F[f(x)] = \frac{-1}{\sqrt{\pi}} L[g(x)]$

17. The Fourier cosine transform of $F_C[e^{-4x}]$

a. $\sqrt{\frac{2}{\pi}} \frac{4}{16+s^2}$ b $\sqrt{\frac{2}{\pi}} \frac{4}{4+s^2}$

c. $\sqrt{\frac{\pi}{2}} \frac{4}{16+s^2}$ d $\sqrt{\frac{\pi}{2}} \frac{4}{4+s^2}$

18. The Fourier transform of an odd function of x is

a. an odd function of s b. even function of s c. an odd function of x d. even function of x

19. The Fourier transform of an even function of x is

a. an odd function of s b. even function of s c. an odd function of x d. even function of x

20. . The Fourier sine transform of $F_S[\frac{1}{x}]$

a. $\sqrt{\frac{2}{\pi}}$

b $\sqrt{\frac{1}{\pi}}$

c. $\sqrt{\frac{\pi}{2}}$

d $\sqrt{\frac{\pi}{4}}$

MA1003 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS

UNIT-II : FOURIER SERIES

1. $\sin x$ is a periodic function with period

   (a) π (b) $\frac{\pi}{2}$ (c) 2π (d) 4π

2. Which one of the following function is an even function

   (a) $\sin x$ (b) x (c) e^x (d) x^2

3. $\int_{-a}^{a} f(x)dx = 0$ if f(x) is

   (a) odd (b) even (c) periodic (iv) zero

4. $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$ if f(x) is

   (a) even (b) odd (c) neither even nor odd (iv) periodic

5. $\int_{-\pi}^{\pi} |x|\,dx$ is equal to

   (a) $2\int_0^{\pi} x dx$ (b) 0 (c) $2\int_0^{\pi}(-x)dx$ (iv) $4\int_0^{\pi/2} x dx$

6. $\tan x$ is a periodic function with period

   (a) π (b) 2π (c) 3π (d) $\pi/2$

7. The constant a_0 of the Fourier series for the function f(x) = x is $0 \le x \le 2\pi$

   (a) π (b) 2π (c) 3π (d) 0

8. The constant a_0 of the Fourier series for the function f(x) = k, $0 \le x \le 2\pi$

   (a) k (b) 2k (c) 0 (d) $\frac{k}{2}$

9. If f(x) is an odd function in $(-l, l)$ then value of a_n in the Fourier series expansion of f(x) is

   (a) $\frac{2}{l}\int_0^{l} f(x)\cos nx\,dx$ (b) 0 (c) $\frac{2}{l}\int_0^{l} f(x)\sin nx\,dx$ (d) $\frac{1}{l}\int_{-l}^{l} x\,dx$

10. If f(x) is an even function in $(-\pi, \pi)$ then the value of b_n in the Fourier series expansion of f(x) is

    (a) $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$ (b) $\frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$ (c) 0 (d) $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$

11. The RMS value of f(x) in $a \le x \le b$ is

(a) 0 (b) $\sqrt{\dfrac{\int_a^b [f(x)]^2 dx}{b-a}}$ (c) $\sqrt{\dfrac{\int_a^b [f(x)]^2 dx}{b+a}}$ (d) $\sqrt{\dfrac{\sqrt{\int_a^b f(x)dx}}{b-a}}$

12. The RMS value of f(x) = x in $-1 \le x \le 1$ is

(a) 1 (b) 0 (c) $\dfrac{1}{\sqrt{3}}$ (d) -1

13. If $\bar{y}$ is the RMS value of f(x) in $(0, 2l)$ then $\dfrac{a_0^2}{4} + \dfrac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)$ is

(a) $\dfrac{\bar{y}^2}{2}$ (b) $\bar{y}$ (c) $\dfrac{\bar{y}}{2}$ (d) $\bar{y}^2$

14. Half range cosine series for f(x) in $(0, \pi)$ is

(a) $\dfrac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$ (b) $\dfrac{a_0^2}{4} + \dfrac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)$ (c) $\sum_{n=1}^{\infty} b_n \sin nx$ (d) $\sum_{n=1}^{\infty} a_n \cos nx$

15. Half range sine series for f(x) in $(0, \pi)$ is

(a) $\sum_{n=1}^{\infty} a_n \cos nx$ (b) $\dfrac{a_0}{2} + \sum a_n \cos nx$ (c) $\sum_{n=1}^{\infty} b_n \sin nx$ (d) $\dfrac{a_0}{2} - \sum a_n \cos nx$

16. The function defined by $f(x) = \begin{cases} x, & -\pi \le x \le 0 \\ -x, & 0 \le x \le \pi \end{cases}$ is

(a) odd (b) neither odd nor even (c) periodic (d) even

17. The function $f(x) = \begin{cases} g(x), & 0 \le x \le \pi \\ -g(-x), & -\pi \le x \le 0 \end{cases}$ is

(a) even function (b) odd function (c) increasing function (d) periodic function

18. The value of Fourier series of f(x) in $0 < x < 2\pi$ at x = 0 is

(a) $f(0)$ (b) $f(2\pi)$ (c) $\dfrac{f(0) + f(2\pi)}{2}$ (iv) 0

19. The value of Fourier series f(x) in $0 < x < 2l$ at x = l is

(a) f(l) (b) f(-l), (c) f(0) (d) f(2l)

20. The value of Fourier series of $f(x) = x^2$ in $0 < x < 2$ at x = 1 is

(a) 0 (b) 4 (c) 1 (d) -1

21. Which of the following is an even function of x?

(a) x^2 (b) $x^2 - 4x$ (c) $\sin(2x) + 3x$ (d) $x^3 + 6$

22. A function f(x) with period T if

(a) $f(x+T)=f(T)$ (b) $f(x+T)=f(x)$ (c) $f(x+T)=-f(x)$

(d) $f(x+T).f(x)=0$

23. For half range cosine series of $f(x)=\cos x$ in $(0,\pi)$ the value of a_0 is

(a) 4 (b) $\frac{2}{\pi}$ (c) $\frac{4}{\pi}$ (d) 0

24. An example for a function which neither even nor odd

(a) $x\sin x$ (b) e^{ax} (c) $x^2 \sin x$ (d) $x\cos x$

25. If f(x) is discontinuous at x = a, then the Fourier series at x = a is

(a) $\frac{f(a^-)-f(a^+)}{2}$ (b) $f(a^-)-f(a^+)$ (c) $\frac{f(a^-)-f(a^+)}{3}$ (d) $\frac{f(a^-)+f(a^+)}{2}$

MA1003 TRANSFORMS AND BOUNDARY VALUE PROBLEMS
Unit 3 One dimensional wave and heat equation
Objective type questions

1. The proper solution of the problems on vibration of string is

   (a) $y(x,t)=(Ae^{\lambda x}+Be^{-\lambda x})(Ce^{\lambda at}+De^{-\lambda at})$ (b) $y(x,t)=(Ax+B)(Ct+D)$

   (c) $y(x,t)=(A\cos\lambda x+B\sin\lambda x)(C\cos\lambda at+D\sin\lambda at)$ (d) $y(x,t)=(Ax+B)$

2. The one dimensional wave equation is

   (a) $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ (b) $\frac{\partial^2 y}{\partial t^2}=a^2\frac{\partial^2 y}{\partial x^2}$ (c) $\frac{\partial y}{\partial t}=a\frac{\partial^2 y}{\partial x^2}$ (d) $\frac{\partial^2 y}{\partial x^2}=a\frac{\partial^2 y}{\partial t^2}$

3. In wave equation $\frac{\partial^2 y}{\partial t^2}=a^2\frac{\partial^2 y}{\partial x^2}$, a^2 stands for

   (a) $\frac{T}{m}$ (b) $\frac{k}{c}$ (c) $\frac{m}{T}$ (d) $\frac{k}{m}$

4. In heat equation $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$, α^2 stands for

   (a) $\frac{k}{\rho}$ (b) $\frac{T}{m}$ (c) $\frac{k}{\rho c}$ (d) $\frac{k}{c}$

5. The one dimensional heat equation in steady state is

   (a) $\frac{\partial u}{\partial t}=0$ (b) $\frac{\partial^2 u}{\partial t^2}=0$ (c) $\frac{\partial^2 u}{\partial x^2}-\frac{\partial u}{\partial t}=0$ (d) $\frac{\partial^2 u}{\partial x^2}=0$

6. The proper solution of $u_t=\alpha^2 u_{xx}$ is

   (a) $u=(Ax+B)C$ (b) $u=(A\cos\lambda x+B\sin\lambda x)e^{-\alpha^2\lambda^2 t}$

   (c) $u=(Ae^{\lambda x}+Be^{-\lambda x})e^{\alpha^2\lambda^2 t}$ (d) $u=At+B$

7. The proper solution in steady state heat flow problems is

   (a) $u=(Ae^{\lambda x}+Be^{-\lambda x})e^{\alpha^2\lambda^2 t}$ (b) $u=Ax+B$

   (c) $u=(A\cos\lambda x+B\sin\lambda x)e^{-\alpha^2\lambda^2 t}$ (d) $u=(Ae^{\lambda x}+Be^{-\lambda x})(Ce^{\lambda at}+De^{-\lambda at})$

8. The one dimensional heat equation is

   (a) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ (b) $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ (c) $\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2}$ (d) $\frac{\partial u}{\partial x}=\alpha^2\frac{\partial^2 u}{\partial t^2}$

9. How many initial and boundary conditions are required to solve $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$

(a) Four (b) Two (c) Three (d) Five

10. How many initial and boundary conditions are required to solve $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$

(a) Two (b) Three (c) Five (d) Four

11. One dimensional wave equation is used to find

(a) Temperature (b) Displacement (c) Time (d) Mass

12. One dimensional heat equation is used to find

(a) Density (b) Temperature distribution (c) Time (d) Displacement

13. Heat flows from _________ temperature

(a) Higher to Lower (b) Uniform (c) Lower to higher (d) Stable

14. The tension T caused by stretching the string before fixing it at the end points is

(a) Increasing (b) Decreasing (c) Constant (d) Zero

15. A string is stretched between two fixed points x = 0 and x = l. The initial conditions are

(a) $y(0,t)=0,\ y(x,t)=0$ (b) $y(x,0)=0,\ \frac{\partial y}{\partial t}(x,0)=0$

(c) $y(0,t)=0,\ y(l,t)=0$ (d) $\left(\frac{\partial y}{\partial x}\right)_{(0,t)}=0,\ \left(\frac{\partial y}{\partial x}\right)_{(l,t)}=0$

16. The amount of heat required to produce a given temperature change in a body is proportional to

(a) Weight of the body (b) Mass of the body

(c) Density of the body (d) Tension of the body

17. The general solution for the displacement y(x,t) of the string of length l vibrating between fixed end points with initial velocity zero and initial displacement f(x) is

(a) $\sum B_n \sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi at}{l}\right)$ (b) $\sum B_n \sin\left(\frac{n\pi x}{l}\right)\sin\left(\frac{n\pi at}{l}\right)$

(c) $\sum B_n \cos\left(\frac{n\pi x}{l}\right)\sin\left(\frac{n\pi at}{l}\right)$ (d) $\sum B_n \sin\left(\frac{n\pi x}{l}\right)$

18. The steady state temperature of a rod of length l whose ends are kept at 30° and 40° is

(a) $u = \frac{10x}{l} + 30$ (b) $u = \frac{20x}{l} + 30$ (c) $u = \frac{10x}{l} + 20$ (d) $u = \frac{10x}{l}$

19. When the ends of a rod is non-zero for one dimensional heat flow equation, the temperature function u(x, t) is modified as the sum of steady state and transient state temperatures. The transient part of the solution which

(a) Increases with increase of time (b) Decreases with increase of time

(c) Increases with decrease of time (d) Decreases with decrease of time

20. A rod of length l has its ends A and B kept at 0° and 100° respectively, until steady state conditions prevail. Then the initial condition is given by

(a) $u(x,0) = ax + b + 100l$ (b) $u(x,0) = \frac{100x}{l}$ (c) $u(x,0) = 100xl$ (d) $u(x,0) = (x+l)100$

ANSWERS

1. (c) $y(x,t) = (A\cos\lambda x + B\sin\lambda x)(C\cos\lambda at + D\sin\lambda at)$

2. (b) $\frac{\partial^2 y}{\partial t^2} = a^2\frac{\partial^2 y}{\partial x^2}$

3. (a) $\frac{T}{m}$

Answers

1. (c) 2π
2. (d) x^2
3. (a) odd
4. (a) even
5. (a) $2\int_0^\pi x\,dx$
6. (a) π
7. (b) 2π
8. (b) 2k
9. (b) 0
10. (c) 0
11. (b) $\sqrt{\dfrac{\int_a^b [f(x)]^2 dx}{b-a}}$
12. (c) $\dfrac{1}{\sqrt{3}}$
13. (d) y^{-2}
14. (a) $\dfrac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$
15. (c) $\sum_{n=1}^{\infty} b_n \sin nx$
16. (d) even
17. (b) odd function
18. (c) $\dfrac{f(0)+f(2\pi)}{2}$
19. (a) f(l)
20. (c) 1
21. (a) x^2
22. (b) $f(x+T) = f(x)$
23. (d) 0
24. (b) e^{ax}
25. (d) $\dfrac{f(a^-)+f(a^+)}{2}$

15MA201 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS

Unit V – Z transforms and Difference Equations

Objective type questions

1. What is $z(5)$?

   (a) $\frac{z}{z-1}$ (b) $5.\frac{z}{z-1}$ (c) $\frac{1}{5}.\frac{z}{z-1}$ (d) $\frac{z-1}{z}$

2. $z[(-1)^n] = ?$

   (a) $\frac{z+1}{z}$ (b) $\frac{z}{z-1}$ (c) $\frac{z}{1+z}$ (d) $\frac{-z}{z+1}$

3. Radius of curvature of $z[a^n]$ is

   (a) $|z|<a$ (b) $|z|>a$ (c) $|z|>|a|$ (d) $|z|<|a|$

4. What is $z[(-2)^n]$?

   (a) $\frac{z}{z+2}$ (b) $\frac{-z}{z+2}$ (c) $\frac{-z}{z-2}$ (d) $\frac{z}{z-2}$

5. Find $z\left[\frac{1}{7^n}\right]$

   (a) $\frac{7z}{z-1}$ (b) $\frac{7z}{7z-1}$ (c) $\frac{z}{7z-1}$ (d) $\frac{z}{z-1}$

6. $z[e^{-5n}] = ?$

   (a) $\frac{z}{z+e^{-5}}$ (b) $\frac{z}{z-e^{-5}}$ (c) $\frac{z}{z-e^{-1}}$ (d) $\frac{z}{z+e^{-1}}$

7. What is z-transform of na^n?

   (a) $\frac{az}{(z-a)^2}$ (b) $\frac{z}{(z-a)^2}$ (c) $\frac{a}{(z-a)^2}$ (d) $\frac{z}{(z-a)^3}$

8. What is $z(n^2)$?

   (a) $\frac{z}{(z-1)^3}$ (b) $\frac{z(z+1)}{z^3}$ (c) $\frac{z(z+1)}{(z-1)^3}$ (d) $\frac{z+1}{(z-1)^3}$

9. If $z[f(t)] = F(z)$ then $\lim_{z\to\infty} F(z) = ?$

   (a) $f(0)$ (b) $f(1)$ (c) $\lim_{t\to\infty} f(t)$ (d) $f(\infty)$

10. What is z-transform of $\frac{1}{n!}$?

(a) $e^{1/z}$ (b) e^{z} (c) $e^{-1/z}$ (d) e^{-z}

11. Radius of curvature of $f(n) = u(n - n_0)$ is

(a) $|z| > 1$ (b) $|z| < \infty$ (c) $1 < |z| < \infty$ (d) $|z| < 1$

12. Radius of curvature of $f(n) = a^{n+1}u(n+1)$

(a) $|z| > |a|$ (b) $|z| > 0$ (c) $|z| < |a|$ (d) $|z| > \frac{1}{|a|}$

13. If $z[f(k)] = F(z)$ then $z[f(-k)] = ?$

(a) F(z) (b) $F\left(\frac{1}{z}\right)$ (c) F(k) (d) $F\left(\frac{1}{k}\right)$

14. $z\left[\sin\frac{n\pi}{2}\right] = ?$

(a) $\frac{z^2}{z^2-1}$ (b) $\frac{z}{z^2+4}$ (c) $\frac{z}{z^2+1}$ (d) $\frac{z^2}{z^2+1}$

15. $z\left[\cos\frac{n\pi}{2}\right] = ?$

(a) $\frac{z}{z^2+1}$ (b) $\frac{z^2}{z^2+1}$ (c) $\frac{z}{z^2-1}$ (d) $\frac{z^2}{z^2-4}$

16. Find $z^{-1}\left(\frac{z}{z-a}\right)$

(a) a^{n+1} (b) a (c) a^{n} (d) a^{n-1}

17. What is $z^{-1}\left(\frac{z}{(z-a)^2}\right)$

(a) a^{n-1} (b) na^{n+1} (c) na^{n-1} (d) a^{n+1}

18. What is $z^{-1}\left(\frac{1}{z-a}\right)$?

(a) na^{n-1} (b) a^{n+1}

(c) a^{n-1}

(d) na^n

19. What is $z[f(n) * g(n)]$?

(a) $F(z).G^{-1}(z)$ (b) $F^{-1}(z).G^{-1}(z)$ (c) $F(z).G(z)$ (d) $F^{-1}(z).G(z)$

20. $z^{-1}(e^{1/z})$=?

(a) $\frac{1}{n+1}$ (b) $\frac{1}{(n+1)!}$ (c) $\frac{1}{n!}$ (d) $\frac{1}{n}$

21. Find $z^{-1}\left(\frac{az}{(z-a)^2}\right)$

(a) a^{n+1} (b) a^n (c) na^n (d) a^{n-1}

22. Find $z^{-1}\left(\frac{z}{(z-1)^2}\right)$

(a) n+1 (b) n (c) n-1 (d) $\frac{1}{n}$

23. Poles of $\varphi(z) = \frac{z^n}{(z-1)(z-2)}$ are

(a) z=1, z=0 (b) z=1, z=2 (c) z=0, z=2 (d) z=0

24. $\varphi(z) = \frac{z^n(2z+4)}{(z-2)^3}$ has a pole 2 of order?

(a) 2 (b) 1 (c) 3 (d) 4

25. Poles of $\varphi(z) = \frac{z^n(z+1)}{(z-1)^3}$ are

(a) z=1 (b) z=-1 (c) z=0 (d) z=3

26. The difference equation formed by eliminating 'a' in $u_n = a2^{n+1}$ is

(a) $u_{n+1} - 2u_n = 0$ (b) $u_{n+1} = 0$ (c) $u_{n+1} - u_n = 0$ (d) $u_n = 0$

27. Solution of $u_n = 5u_{n-1}, n \geq 1, u_0 = 2$ is

(a) $u_n = 5^n$ (b) $u_n = 2.5^n$ (c) $u_n = 2^n$ (d) $u_n = 5.2^n$

4. (c) $\frac{k}{\rho c}$

5. (d) $\frac{\partial^2 u}{\partial x^2} = 0$

6. (b) $u = (A\cos\lambda x + B\sin\lambda x)e^{-\alpha^2\lambda^2 t}$

7. (b) $u = Ax + B$

8. (b) $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$

9. (c) Three

10. (d) Four

11. (b) Displacement

12. (b) Temperature distribution

13. (a) Higher to lower

14. (c) Constant

15. (c) $y(0,t) = 0,\ y(l,t) = 0$

16. (b) Mass of the body

17. (a) $\sum B_n \sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi at}{l}\right)$

18. (a) $u = \frac{10x}{l} + 30$

19. (b) Decreases with increase of time

20. (b) $u(x,0) = \frac{100x}{l}$

ANSWERS

1. (b)
2. (c)
3. (c)
4. (a)
5. (b)
6. (b)
7. (a)
8. (c)
9. (a)
10. (a)
11. (c)
12. (a)
13. (b)
14. (c)
15. (b)
16. (c)
17. (c)
18. (c)
19. (c)
20. (c)
21. (c)
22. (b)
23. (b)
24. (c)
25. (a)
26. (a)
27. (b)

SRM UNIVERSITY

DEPARTMENT OF MATHEMATICS

MULTIPLE CHOICE QUESTIONS

Subject Code: MA1003
Subject Name: TRANSFORMS AND BOUNDARY VALUE PROBLEMS

UNIT – I PARTIAL DIFFERENTIAL EQUATIONS

1. The order and degree of a PDE $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=0$ is
   a) 2,1 (b) 1,2 c) 2,2 d) 1,1 — Ans (a)

2. The order degree of a PDE $\left(\frac{\partial z}{\partial x}\right)^3+\frac{\partial^2 z}{\partial y^2}=\cos(x+y)$ is
   a) 1,2 b) 2,1 c) 1,3 d) 3,1 — Ans (b)

3. While forming the PDE, if the number of arbitrary constants to be eliminated is equal to the number of independent variables, the resulting PDE will be of ________ order.
   a) 1st b) 2nd c) 3rd d) >1 — Ans (a)

4. The complete integral of F(p,q) =0 is
   a) 0 b) px + qy + c c) Z = ax + by + c d) none — Ans (c)

5. The complete integral of $\sqrt{p}+\sqrt{q}=1$ is
   a) $z=ax+(1-\sqrt{a})^2y+c$ b) $z=px+(1-\sqrt{a})^2y+c$
   c) 0 d) None — Ans (a)

6. The complete integral of $z=px+qy+p^2q^2$ is
   a) $z=ax+by+a^2b^2$ b) $z=px+qy$ c) $z=ax+by$ d) none — Ans (a)

7. The solution of $(D^2-3DD'+2D'^2)Z=0$ is
   a) $z=\varphi_1(y+x)+\varphi_2(y+2x)$ b) $z=Ae^x+Be^{2x}$
   c) $z=\varphi_1(y+2x)+\varphi_2(y-x)$ d) None. — Ans (a)

8. The solution of $r-4s+t=0$ is
   a) $z=(A+Bx)e^{2x}$ b) $z=\varphi_1(y+2x)+x\varphi_2(y+2x)$
   c) $z=\varphi_1(y+x)+\varphi_2(y+2x)$ d) none. — Ans (b)

9. The P.I of $(D^2-2DD'+D'^2)Z=8e^{x+2y}$ is
   a) e^{x+2y} b) $8e^{x+2y}$ c) 8 d) 0 — Ans (b)

10. The P.I of $(D^2-3DD'+2D'^2)Z=2\cosh(3x+4y)$ is
    a) $\frac{2}{5}\cosh(3x+4y)$ b) $-\frac{2}{5}\cosh(3x+4y)$ c) $\frac{3}{5}\cosh(3x+4y)$ d) none — Ans (a)

11. The P.I of $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = \cos(3x+2y)$ is

a) $\frac{1}{9}\cos(3x+2y)$ b) $-\frac{1}{9}\cos(3x+2y)$ c) 0 d) none. Ans (a)

12. The P.I of $(D^3 - 2D^2D')z = 4\sin(x+y)$ is

a) $4\sin(x+y)$ b) $-4\cos(x+y)$ c) $4\cos(x+y)$ d) 0 Ans (b)

13. The P.I of $\left(D^2 + 4DD'^2\right)z = e^x$ is

a) e^x b) e^{-x} c) e^{2x} d) 0 Ans (a)

14. The complementary function of $\left(D^2 + 2DD' + D'^2\right)Z = xy$ is

a) $\varphi_1(y-x) + x\varphi_2(y-x)$ b) $(A+Bx)e^{-x}$

c) $\varphi_1(y-2x) + x\varphi_2(y-x)$ d) none Ans (a)

15. The P.I of $\left(D^2 - 3DD' + 2D'^2\right)Z = \sin(x-2y)$ is

a) $-\frac{1}{15}\sin(x-2y)$ b) $\frac{1}{15}\sin(x-2y)$ c) 0 d) none Ans (a)

16. The solution of $\left(D^3 - 3D^2D' + 2DD'^2\right)z = 0$ is

a) $z = f_1(y) + f_2(y+x) + f_3(y+2x)$ b) $z = f_1(y) + f_2(y-x) + f_3(y+2x)$

c) $z = f_1(y) + f_2(y+x) + f_3(y-2x)$ d) none Ans (a)

17. The solution of $\left(D^3 + DD'^2 - D^2D' - D'^3\right)z = 0$ is

a) $z = \varphi(y+x) + f(y+ix) + F(y-ix)$ b) $z = f_1(y+ix) + f_2(y-ix) + f_3(y)$

c) $z = \varphi(y-x) + f(y+ix) + F(y+ix)$ d) none Ans (a)

18. The P.I of $\frac{\partial^3 z}{\partial z^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = e^{x+2y}$ is

a) $\frac{1}{3}e^{x+2y}$ b) $-\frac{1}{3}e^{x+2y}$ c) $-e^{x+2y}$ d) none Ans (b)

19. The P.I of $(D^2 - 2DD')z = e^{2x}$ *is*

a) e^{2x} b) $\frac{1}{4}e^{-2x}$ c) $\frac{1}{4}e^{2x}$ d) 0 Ans (c)

20. The P.I of $\left(D^2 - 2DD' + D'^2\right)z = \cos(x-3y)$ is

a) $-\frac{1}{16}\cos(x-3y)$ b) $\frac{1}{16}\cos(x-3y)$ c) $\cos(x-3y)$ d) 0 Ans (a)

21. If $B^2 - 4AC < 0$, then linear PDE is called

a) Elliptic b) parabolic c) hyperbolic d) none Ans (a)

22. If $B^2 - 4AC = 0$, then linear PDE is called

a) Elliptic b) parabolic c) hyperbolic d) none Ans (b)

23. If $B^2 - 4AC > 0$, then linear PDE is called

a) Elliptic b) parabolic c) hyperbolic d) none Ans (c)

24. The PDE $xu_{xx} + u_{yy} = 0, x > 0$ is

a) Elliptic b) parabolic c) hyperbolic d) none Ans (a)

25. The PDE $xu_{xx} + u_{yy} = 0, x < 0$ is

a) Elliptic b) parabolic c) hyperbolic d) none Ans (c)

26. The Laplace equation in two dimensions $u_{xx} + u_{yy} = 0$ is classified as

a) Elliptic type b) parabolic type c) hyperbolic type d) none Ans (a)

27. The one dimensional heat equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{a^2}\frac{\partial u}{\partial t}$ is classified as

a) Elliptic type b) parabolic type c) hyperbolic type d) none Ans (b)

28. The one dimensional wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{a^2}\frac{\partial^2 u}{\partial t^2}$ is classified as

a) Elliptic type b) parabolic type c) hyperbolic type d) none Ans (c)

29. Which one of the following is classified as Elliptic?

a) Poisson's equation b) 1-D heat equation c) 1-D Wave equation d) none Ans (a)

30. The equation $u_{xx} + 2u_{xy} + u_{yy} = 0$ is parabolic

a) At all points b) only at x > 0 c) only at x < 0 d) none Ans (a)

31. The equation $x^2 f_{xx} + (1 - y^2) f_{yy} = 0, x \neq 0, -1 < y < 1$ is

a) Elliptic type b) parabolic type c) hyperbolic type d) none Ans (a)

UNIT – II FOURIER SERIES

1. Which of the following function is periodic in the interval $(0, \pi)$?

a) $\sin x$ b) $\cos x$ c) $\tan x$ d) $\sec x$ Ans (c)

2. Which of the following function is periodic with period 2π?

a) $\sin x$ b) $\tan x$ c) $\cot x$ d) None

Ans (a)

3. The function $f(x) = \cot x$ is periodic with period ......

a) 2π b) 4π c) π d) None Ans (c)

4. The smallest period of the following function is ......

a) $\sin x$ b) $\sin 2x$ c) $\cos 2x$ d) $\tan x$ Ans (a)

5. The Fourier series expansion of an odd function contains........only
   a) Sine terms b) cosine terms c) sine and cosine d) None Ans (b)

6. The Fourier series expansion of an even function contains........only
   a) Sine terms b) cosine terms c) sine and cosine d) None Ans (a)

7. If $f(x)$ is an odd function in $(-\pi,\pi)$, then the value of a_0 is ......
   a) 1 b) 0 c) -1 d) None Ans (b)

8. If $f(x)$ is an even function in $(-\pi,\pi)$, then the value of b_n is ......
   a) 1 b) 0 c) -1 d) None

Ans (b)

9. If $f(x)=x\sin x$ in $(-\pi,\pi)$ then the value of b_n is ....
   a) 1 b) 0 c) -1 d)None Ans (b)

10. If $f(x)=|x|$ in $(-\pi,\pi)$ then the value for a_0 is ....
   a) π b) 2π c) $\frac{\pi^2}{3}$ d) $\frac{2\pi^2}{3}$ Ans (c) C

11. If $f(x)=x^2$ in $(-\pi,\pi)$ then the value for b_n is ....
   a) 1 b) 0 c)-1 d)None Ans (b)

12. In the Fourier series expansion of $f(x)=|\sin x|$ in $(-\pi,\pi)$. What is the value of b_n?
   a) 1 b) 0 c) π d) None Ans (b)

13. The function $f(x)=x\cos x$ in $(-\pi,\pi)$ is .... Function
   a) Odd b) even c) neither even nor odd d) None Ans (b)

14. The function $f(x)=x^2\sin x$ in $(-\pi,\pi)$ is .... Function
   b) Odd b) even c) neither even nor odd d) None Ans (a)

15. The constant term of the function $f(x)=x-x^3$ in $(-\pi,\pi)$ is ....
   a) 0 b) π c) -1 d) 1 Ans (a) C

16. If the expansion of $f(x)=\sinh x$ in $(-\pi,\pi)$, then a_n is..........
   a) 0 b) π c) 1 d)-1 Ans (a)

17. The Root mean square value of a function $y=f(x)$ over a given interval (a,b) is ......
   a) $\bar{y}=\sqrt{\frac{\int_a^b y^2dx}{b-a}}$ b) $\bar{y}=\sqrt{\frac{\int_a^b y\,dx}{a-b}}$ c) $\bar{y}^2=\sqrt{\frac{\int_a^b y^2dy}{b-a}}$ d) None Ans (a)

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18. The right and left hand limit for the function $f(x)=(x-1)^2$ in the interval $(0,1)$ is .....
a) 0,0 b) 1,1 c) 1,0 d) 0,1 Ans (b)

19. The right and left hand limit for the function $f(x)=\frac{1}{1-x}$ in the interval $(0,1)$ is .....
a) 0,0 b) 1,1 c) 1,0 d) 0,1 Ans (c)

20. If $x=a$ is a point of discontinuity of $f(x)$, then the value of the Fourier series at $x=a$ is ....
a) $\frac{1}{2}[f(a+)+f(a-)]$ b) $\frac{1}{2}[f(a+)-f(a-)]$
c) $[f(a+)+f(a-)]$ d) $2[f(a+)+f(a-)]$ Ans (a)

21. If $f(x)$ has equally q spaced points then b_n is ....
a) 2(mean value of $f(x)\sin nx$) b) (mean value of $f(x)\sin nx$)
c) q (mean value of $f(x)\sin nx$) d) $\frac{2}{q}$(mean value of $f(x)\sin nx$) Ans (a)

22. The process of finding the Fourier series for the function given by the numerical values is known as.............
a) Complex Analysis b) Numerical Methods c) Harmonic Analysis d) None Ans (c)

UNIT – III ONE DIMENSIONAL WAVE AND HEAT EQUATION

1. A boundary value problem is a differential equation together with
a) Unknown variable b) known variable
c) boundary condition d) none Ans (c)

2. The boundary conditions are the set of additional restraints along with
a) Partial differential equation b) differential equation
c) Any equation d) none Ans (b)

3. The wave equation is a
a) Hyperbolic b) elliptic c) parabolic d) none Ans (a)

4. The heat equation is
a) Hyperbolic b) elliptic c) parabolic d) none Ans (c)

5. The Laplace's equation is
a) Hyperbolic b) elliptic c) parabolic d) none Ans (b)

6. The heat equation $u_t=k\nabla^2 u$ where u refers
a) Temperature b) wave c) time d) none Ans (a)

7. In the wave equation $u_{tt}-c^2\nabla^2 u=0$, u is the
a) Temperature b) displacement from rest c) Initial temperature d) none Ans (b)

8. In the Laplace's equation $\nabla^2 u=0$, u is the
a) Temperature b) displacement
c) steady state temperature d) none Ans (c)

9. The suitable solution of a finite string with fixed ends executing transverse vibration is
a) $y(x,t) = (A\cos\lambda x + B\sin\lambda x)(C\cos\lambda ct + D\sin\lambda ct)$ b) $y(x,t) = (A\cos\lambda x + B\sin\lambda x)$
c) $y(x,t) = (C\cos\lambda ct + D\sin\lambda ct)$ d) none Ans (a)

10. The solution of wave equation is if the string is at rest
a) $u(x,t) = \phi(x+ct) + \phi(x-ct)$ b) $u(x,t) = \frac{1}{2}[\phi(x+ct) + \phi(x-ct)]$
c) $u(x,t) = \phi(x+ct)$ d) none Ans (b)

11. One of the initial conditions on vibrating strings due to initial displacement is
a) $y(x,0) = f(x) \neq 0$ b) $y(x,t) = 0$ c) $y(x,0) = f(x) = 0$ d) none Ans (b)

12. One of the initial conditions on vibrating strings due to initial displacement is
a) $y(x,0) = 0$ b) $\frac{\partial y(x,0)}{\partial t} = 0$ c) $\frac{\partial y(x,0)}{\partial t} \neq 0$ d) none Ans (b)

13. One of the initial conditions on vibrating strings due to initial velocity is
a) $y(x,0) = 0, \frac{\partial y(x,0)}{\partial t} = f(x)$ b) $y(x,0) = f(x), \frac{\partial y(x,0)}{\partial t} = 0$
c) $y(x,t) = 0$ d) none Ans (b)

14. The one dimensional heat equation describes the flow of heat in a body of
a) String b) homogeneous material c) wave d) none Ans (b)

15. In the heat equation $u_t = c^2 u_{xx}$ where c^2 refers
a) Thermal constant b) thermal conductivity
c) thermal diffusivity d) none Ans (c)

16. In $C^2 = \frac{K}{\sigma\rho}$, ρ represents
a) Density of material b) specific heat capacity
b) constant d) none Ans (a)

17. In $C^2 = \frac{K}{\sigma\rho}$, σ represents
a) Density b) specific heat capacity c) constant d) none Ans (b)

18. The suitable solution of heat equation is
a) $u(x,t) = e^{-c^2\lambda^2 t}(A\cos\lambda x + B\sin\lambda x)$ b) $u(x,t) = e^{c^2\lambda^2 t}(A\cos\lambda x + B\sin\lambda x)$
c) $u(x,t) = (A\cos\lambda x + B\sin\lambda x)$ d) none Ans (a)

19. The initial condition on zero boundary condition is
a) $u(x,0) = 0$ b) $u(x,t) = t$ c) $u(x,0) = f(x)$ d) none Ans (c)

20. In steady state
a) $\frac{\partial u}{\partial x} = 0$ b) $\frac{\partial u}{\partial t} = 0$ c) $\frac{\partial^2 u}{\partial t^2} = 0$ d) none Ans (b)

UNIT – IV FOURIER TRANSFORMS

1. Fourier transform pair is represented by
(a) F(s) & $F^{-1}[F(s)]$ (b) F(s) & F[F(s)]
(c) f(x) & $F^{-1}[f(x)]$ (d) none Ans (a)

2. If F(s) is the Fourier transform of f(x) then F[f(x) cos ax] in terms of 'F' is
(a) F(s+a)+F(s-a) (b) $\frac{1}{2}$[F(s+a)+F(s-a)]
(b) (c) $\frac{1}{2}$[F(s+a)-F(s-a)] (d) F(s+a)-F(s-a) Ans (b)

3. If F(s) is the Fourier transform of f(x) then Fourier transform of f(x-a) is
(a)e^{iax} f(x) (b) e^{ias} f(s) (c) $e^{s^2/2}$ (d) $\frac{1}{a}f(s/a)$ Ans (b)

4. If F(s) = F[f(x)] then F[e^{iax} f(x)]=
(a) e^{iax} f(x) (b) e^{ias} f(s) (c) F(s+a) (d) $\frac{1}{a}f(s/a)$ Ans (c)

5. Parseval's identity for Fourier transform is
(a) $\int_{-\infty}^{\infty}|F(s)|^2 ds = \int_{-\infty}^{\infty}|f(x)|^2 dx$ (b) $\int_{-\infty}^{\infty}(F(s))^2 ds = \int_{-\infty}^{\infty}(f(x))^2 dx$
(c) $\int_{-\infty}^{\infty}|F(s)|^2 ds = \int_{-\infty}^{\infty}|f(x)|^2 dx$ (d) $\int_{-\infty}^{\infty}(F(s))^2 ds = \int_{-\infty}^{\infty}(f(x))^2 dx$ Ans (c)

6. The self reciprocal for $F[e^{-x^2/2}]$ is given by
(a) $e^{-s^2/2}$ (b) $e^{s^2/2}$ (c) $e^{s/2}$ (d) none Ans (a)

7. Find the Fourier cosine transform of e^{-x}
(a) $\sqrt{\frac{2}{\pi}}(\frac{s}{1+s^2})$ (b) $\sqrt{\frac{2}{\pi}}(\frac{1}{1+s^2})$
(c) $\sqrt{\frac{2}{\pi}}(\frac{a}{a+s^2})$ (d) $\sqrt{\frac{2}{\pi}}(\frac{a}{1+s^2})$ Ans (b)

7. If F[f(x)] = F(s) then F[f(x) cos ax] = $\frac{1}{2}$[F(s+a)+F(s-a)] is called ....
(a) Change of scale property (b) shifting property
(c) Modulation property (d) none Ans (c)

8. Fourier Transform is also known as ....
(a) Inverse Fourier transforms (b) Finite Fourier transforms
(c) Complex Fourier transforms (d) Infinite Fourier transforms Ans (c & d)

9. Find the Fourier sine transform of $\frac{1}{x} =$

(a) $\sqrt{\pi/2}$ (b) $\pi/2$ (c) $\sqrt{\pi/3}$ (d) none Ans (a)

11. If F(s) = F[f(x)] then F[x^nf(x)] is............

(a) $(-i)^n \frac{d^n}{ds^n} F(s)$ (b) $(i)^n \frac{d^n}{ds^n} F(s)$ (c) $(i)^n \frac{d}{ds} F(s)$ (d) $(-i) \frac{d^n}{ds^n} F(s)$ Ans (a)

12. The Convolution theorem for Fourier Transform is $f * g =$ ....

(a) $\frac{1}{\sqrt{2\pi}} \int_0^\infty f(t)g(x-t)dt$ (b) $\frac{1}{\sqrt{2\pi}} \int_0^\infty f(t)g(t)dt$

(c) $\frac{1}{2\pi} \int_{-\infty}^\infty f(t)g(x-t)dt$ (d) $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty f(t)g(x-t)dt$ Ans (d)

13. Fourier Transform must satisfy...............
(a) Dirichlet's condition (b) absolutely integrable
(c)Continuity (d) All three. Ans (d)

14. The Inverse Fourier sine transform f(x) = ...

(a) $\sqrt{\frac{2}{\pi}} \int_0^\infty F_s(s) \sin x \, dx$ (b) $\sqrt{\frac{2}{\pi}} \int_0^\infty F_s(s) \sin sx \, ds$

(c) $\sqrt{\frac{2}{\pi}} \int_0^\infty F(s) \sin sx \, ds$ (d) $\sqrt{\frac{2}{\pi}} \int_{-\infty}^\infty F_s(s) \sin x \, ds$ Ans (b)

15. The inverse Fourier cosine transforms f(x) =

(a) $\sqrt{\frac{2}{\pi}} \int_0^\infty F_c(s) \cos x \, dx$ (b) $\sqrt{\frac{2}{\pi}} \int_0^\infty F(s) \cos sx \, ds$

(c) $\sqrt{\frac{2}{\pi}} \int_0^\infty F_c(s) \cos sx \, ds$ (d) $\sqrt{\frac{2}{\pi}} \int_{-\infty}^\infty F_c(s) \cos sx \, ds$ Ans (c)

UNIT – V Z - TRANSFORMS

1. Z[a^nu(n)] exists only if

(a) $|z| < |a|$ (b) $|z| \le |a|$ (c) $|z| = |a|$ (d) $|z| > |a|$ Ans (d)

2. u(n) – u(n -1) is

(a) δ(n) (b) f(n) (c) k(n) (d) δ(k) Ans (a)

3. Z transform of e^{at} is

(a) $z/\ z-e^a$ (b) $z/\ z-e^{-a}$ (c) $z/\ z-e2^a$ (d) $z/\ z-e^{-a}$ Ans(a)

4. Z transform of a^n is

(a) $\frac{z}{z+a}$ (b) $\frac{z}{z-a}$ (c) $\frac{z}{z\pm a}$ (d) $\frac{2z}{z+a}$ Ans(b)

5. By shifting theorem if $Z[f(t)] = F(z)$, then $Z[e^{-at} f(t)]$ is

(a) $F[ze^{aT}]$ (b)) $F[ze^{at}]$ (c)) $F[ze^{bT}]$ (d) $F[ze^{-aT}]$ Ans (a)

6. Two sided Z transform is defined as

(a) $\sum_{n\to-\infty}^{\infty} x(n+1)z^{-n}$ (b) $\sum_{n\to\infty}^{\infty} x(n)z^{-n}$ (c) $\sum_{n\to-\infty}^{\infty} x(n)z^{n}$ (d) $\sum_{n\to-\infty}^{\infty} x(n)z^{-n}$ Ans (d)

7. One side Z transform is defined as

(a) $\sum_{n=0}^{\infty} x(n+1)z^{-n}$ (b) $\sum_{n=0}^{\infty} x(n)z^{-n}$ (c) $\sum_{n=0}^{\infty} x(n)z^{n}$ (d) $\sum_{n\to-\infty}^{\infty} x(n)z^{-n}$ Ans (b)

8. The series of one sided Z transform is
(a) divergent (b) convergent (c) obsolutely convergent (d) continous Ans (b)

9. Radius of convergence of $\sum_{n=0}^{\infty} x(n)z^{-n}$ is

(a) $\underset{n\to\infty}{Lt} \left| \frac{x(n+1)}{x(n)} \right|$ (b) x(n+1) (c) x(n) (d) limit $n \to 0$ Ans (a)

10. Z-transform plays an important role in analysis of
(a) Continous time signals (b) discrete time signals (c) invariant time signals

(d) random time signals Ans (b)

11. By intial value theorem $Z[f(t)] = F(z)$ then f(0) is
(a) $f(0) = \underset{z\to\infty}{Lt} F(z)$ (b) $f(0) = \underset{z\to\infty}{Lt} Z(z)$ (c) 1 (d) 0 Ans (a)

12. By final value theorem $Z[f(t)] = F(z)$ then $\underset{t\infty}{Lt}$ f(t) is

(a) F(z) (b)(z-1) (c) $\underset{z\to\infty}{Lt}$ (z -1)F(z) (d) f(0) Ans (c)

13. Convolution theorem states that if w(n) is the convolution of two sequences x(n) and y(n)

then Z[w(n)] is

(a) Z[x(n)] (b) Z[y(n)] (c) W(z) (d) Z[x(n)] Z[y(n)] Ans (d)

14. Find $Z\{(-1)^n\}$

(a) z/z -1 if z < 1 (b)) z if z > 1 (c)) z/z+1 if z > 1 (d)) z/z+1 if z < 1 Ans (c)

15. Inverse Z transform of $\frac{z}{z-a}$ is

(a) a^n (b) b^n (c) a^m (d) a^{-n} Ans (a)

16.Inverse Z transform of $\frac{az}{(z-a)^2}$ is

(a) m^{am} (b)n^{an} (c) ^{a2} (d) $^{a-n}$ Ans (b)

17.Inverse Z transform of z^{-k} is

(a) $\delta(k)$ (b) $\delta(n)$ (c)$\delta(n-k)$ (d) $\delta(n+k)$ Ans (c)

18. Inverse Z transform of $\frac{1}{z-a}$ is

(a) a^{n-1} (b) a^{n+1} (c) a^n (d) Ans (a)

19. Solve : $y_{n+1} - 2y_n = 1$ given $y_0 = 0$.

(a) 2n (b) 2^n (c) $2^n - 1$ (d) 2n +1 Ans (b)

20. Solve : : $y_{n+1} - 3y_n = 1$ given $y_0 = 1$.

(a) 3n (b) 3^{n-1} (c)3^n (d) 2n +1 Ans (c)