23MT2014

THEORY OF COMPUTATION

Topic:

PROPERTIES OF REGULAR LANGUAGE AND PUMPING LEMMA

Session – 9-a

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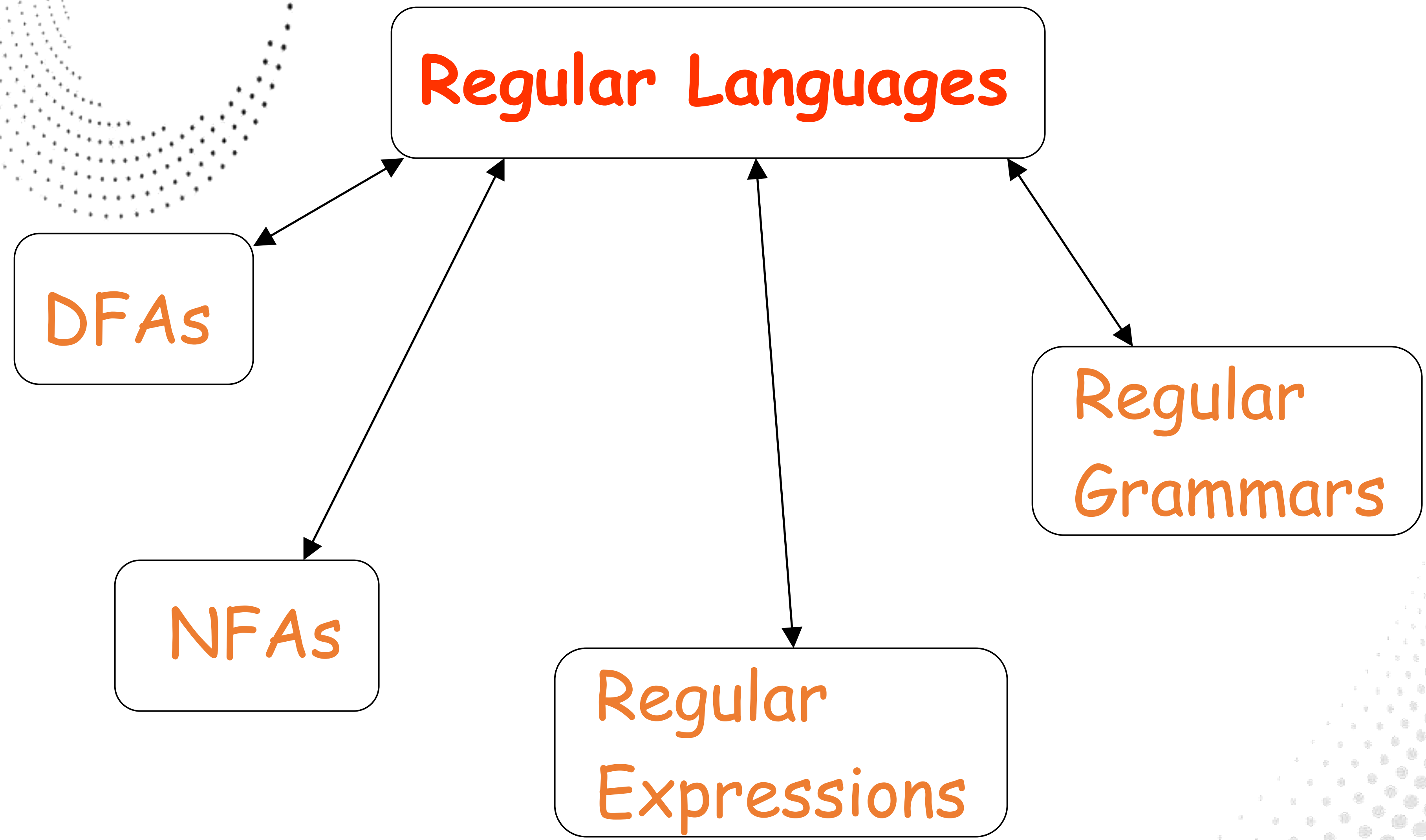
Standard Representations of Regular Languages



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When we say: We are given a Regular Language L

We mean: Language L is in a standard representation

Elementary Questions

about

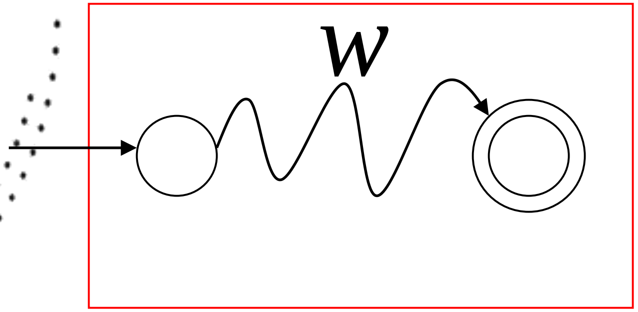
Regular Languages

Membership Question

Question: Given regular language L and string w how can we check if $w \in L$?

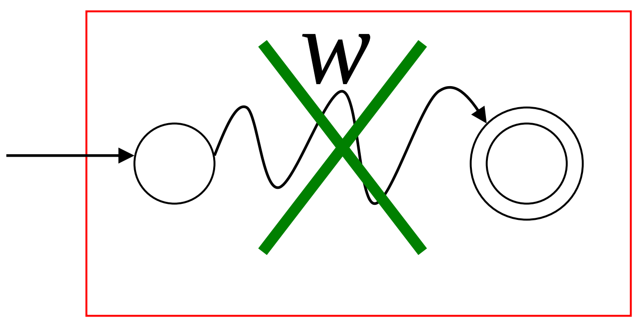
Answer: Take the DFA that accepts L and check if w is accepted

DFA



$$w \in L$$

DFA



$$w \notin L$$

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Question: Given regular language L how can we check if L is empty: $(L = \emptyset)$?

Answer: Take the DFA that accepts L

Check if there is any path from the initial state to a final state

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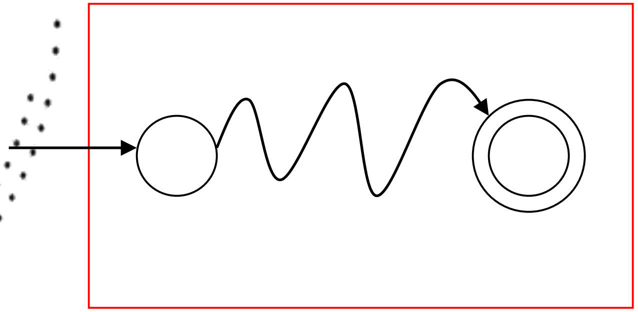
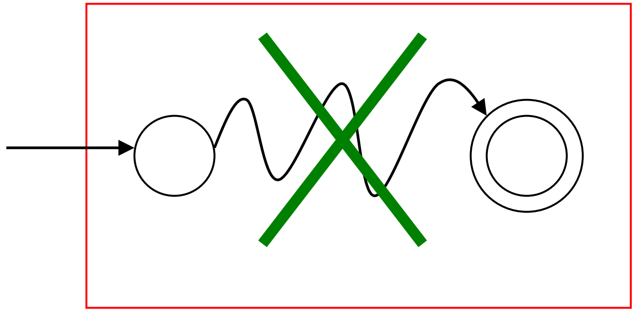
DFA



$$L \neq \varnothing$$

DFA



$$L = \varnothing$$

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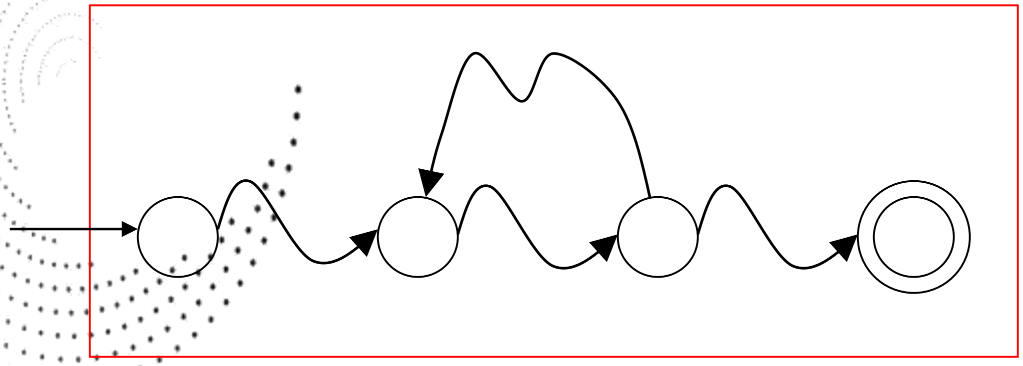
Question: Given regular language L how can we check if L is finite?

Answer: Take the DFA that accepts L

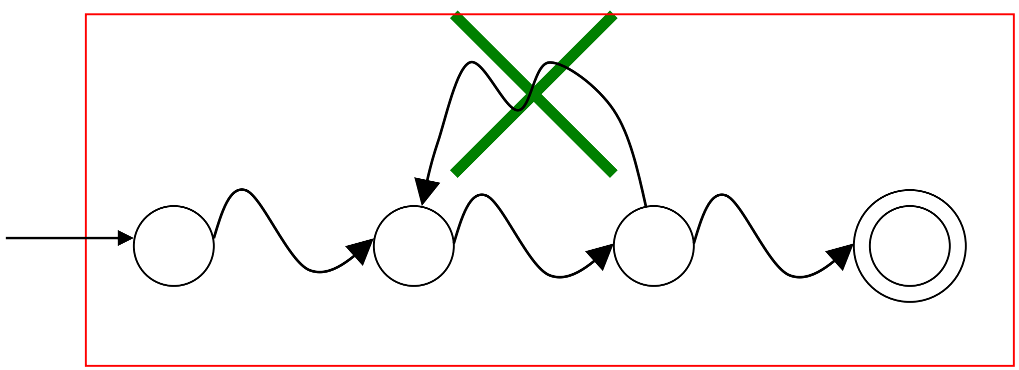
Check if there is a walk with cycle from the initial state to a final state

DFA



L is infinite

DFA



L is finite

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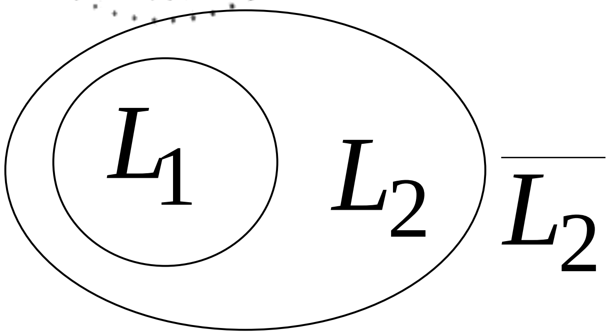
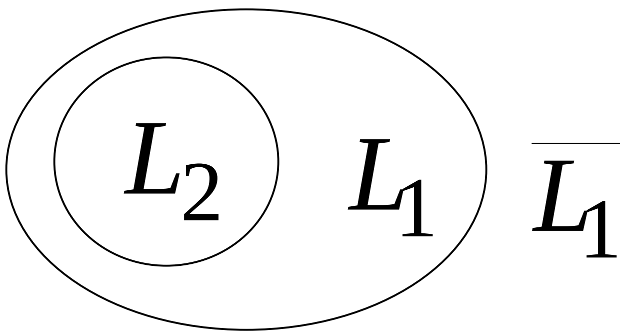
Question: Given regular languages L_1 and L_2 how can we check if $L_1 = L_2$?

Answer: Find if $(L_1 \cap \overline{L_2}) \cup (\overline{L_1} \cap L_2) = \varnothing$

$$(L_1 \cap \overline{L_2}) \cup (\overline{L_1} \cap L_2) = \emptyset$$

$$L_1 \cap \overline{L_2} = \emptyset \quad \text{and} \quad \overline{L_1} \cap L_2 = \emptyset$$





$$L_1 \subseteq L_2 \qquad L_2 \subseteq L_1$$

$$L_1 = L_2$$

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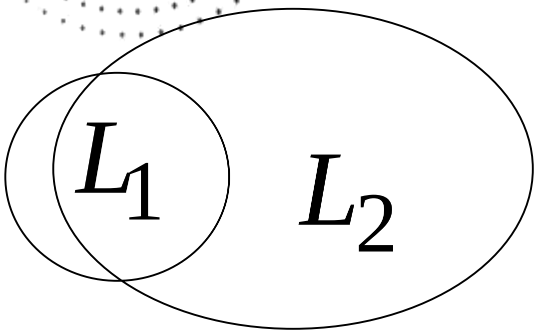
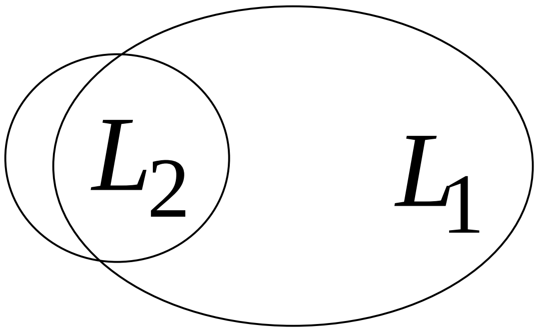
$$(L_1 \cap \overline{L_2}) \cup (\overline{L_1} \cap L_2) \neq \emptyset$$

$$L_1 \cap \overline{L_2} \neq \emptyset$$

or

$$\overline{L_1} \cap L_2 \neq \emptyset$$





$$L_1 \not\subset L_2 \qquad L_2 \not\subset L_1$$

$$L_1 \neq L_2$$

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Non-regular languages

$$\{a^n b^n : \ n \geq 0\}$$

Non-regular languages

$$\{vv^R : \ v \in \{a,b\}^*\}$$

Regular languages

$$a^*b$$

$$b^*c + a$$

$$b + c(a+b)^*$$

$$etc...$$

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How can we prove that a language L is not regular?

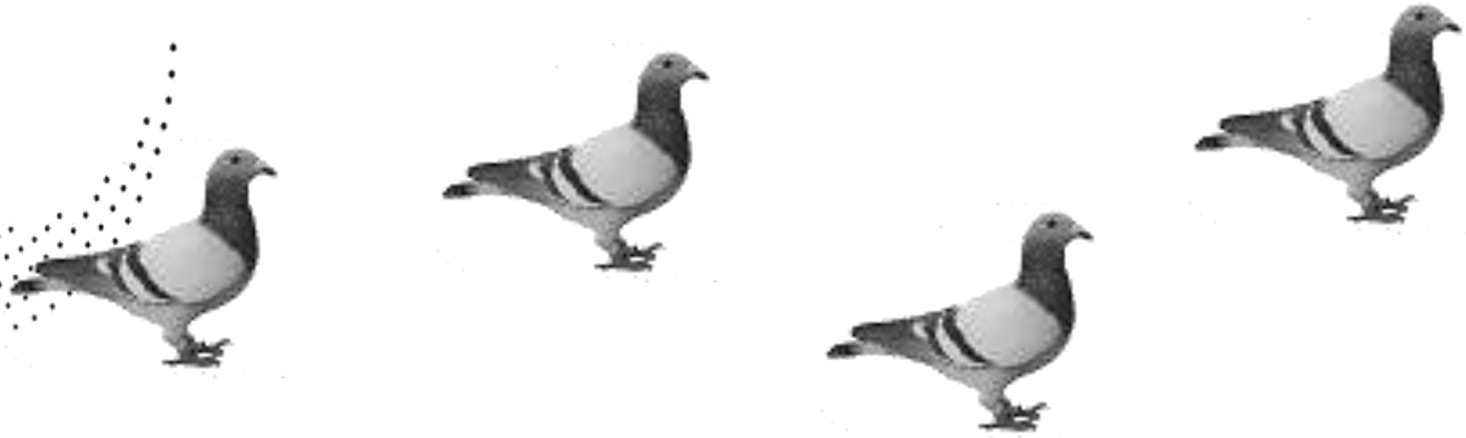
Prove that there is no DFA that accepts L

Problem: this is not easy to prove

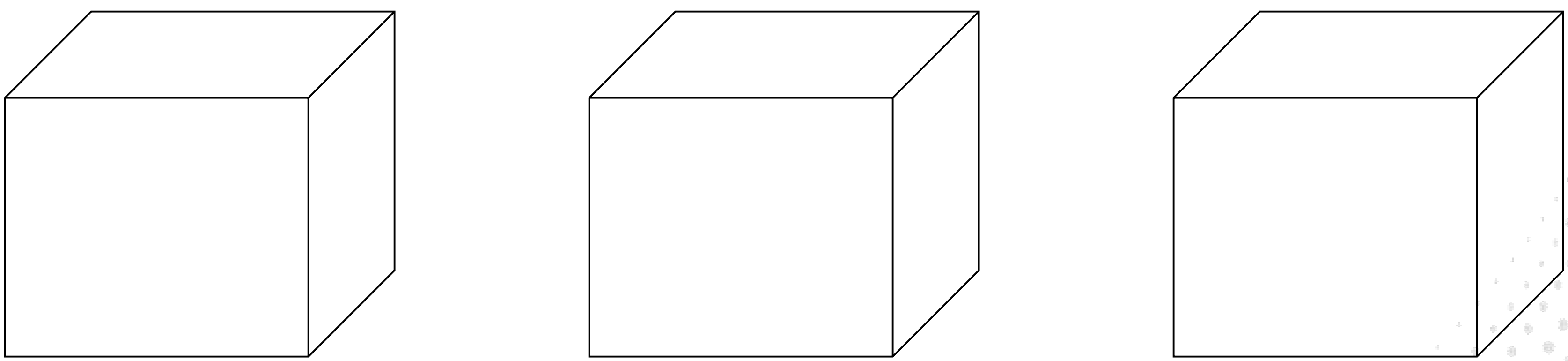
Solution: the Pumping Lemma !!!

The Pigeonhole Principle

4 pigeons



3 pigeonholes



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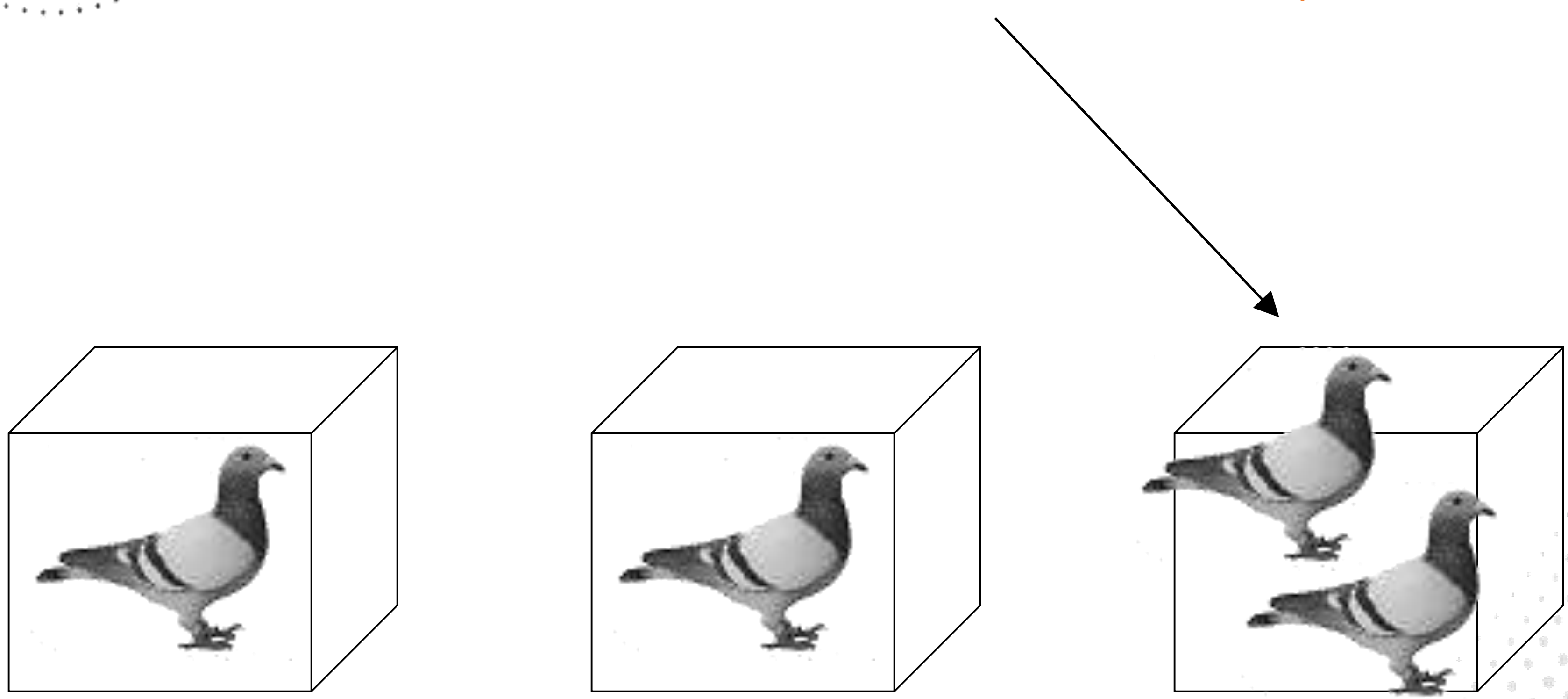
A pigeonhole must contain at least two pigeons



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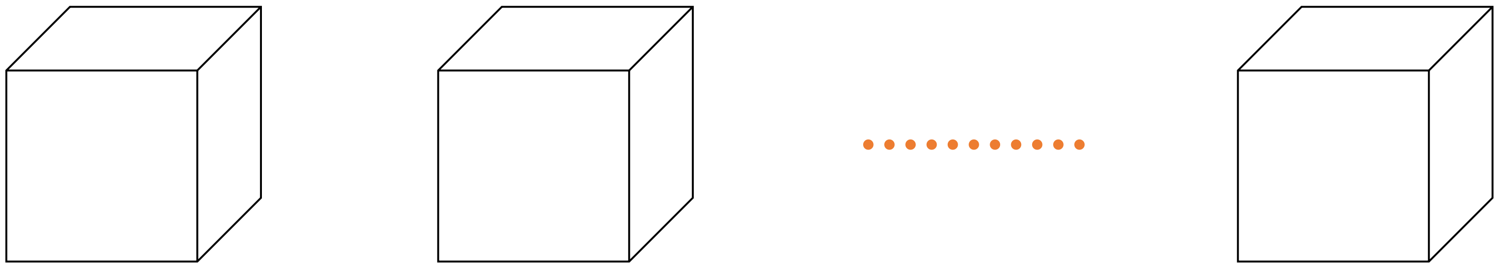
n pigeons

m pigeonholes $n > m$



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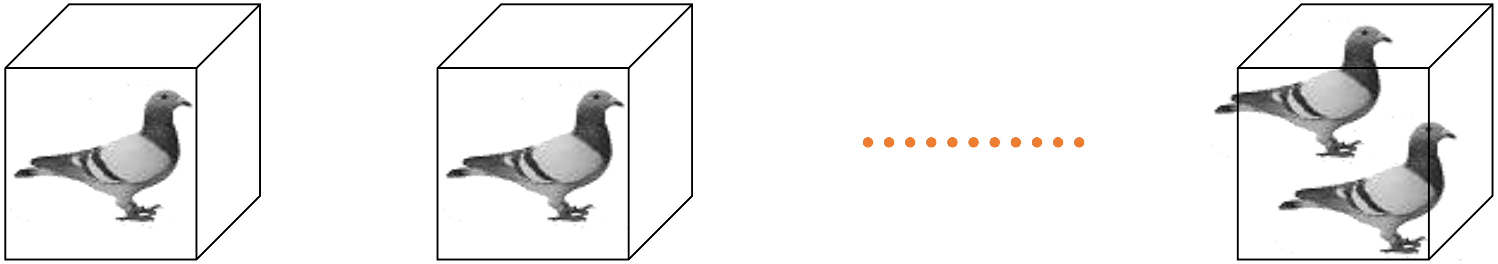
The Pigeonhole Principle

- n pigeons
- m pigeonholes

$$n > m$$

There is a pigeonhole with at least 2 pigeons



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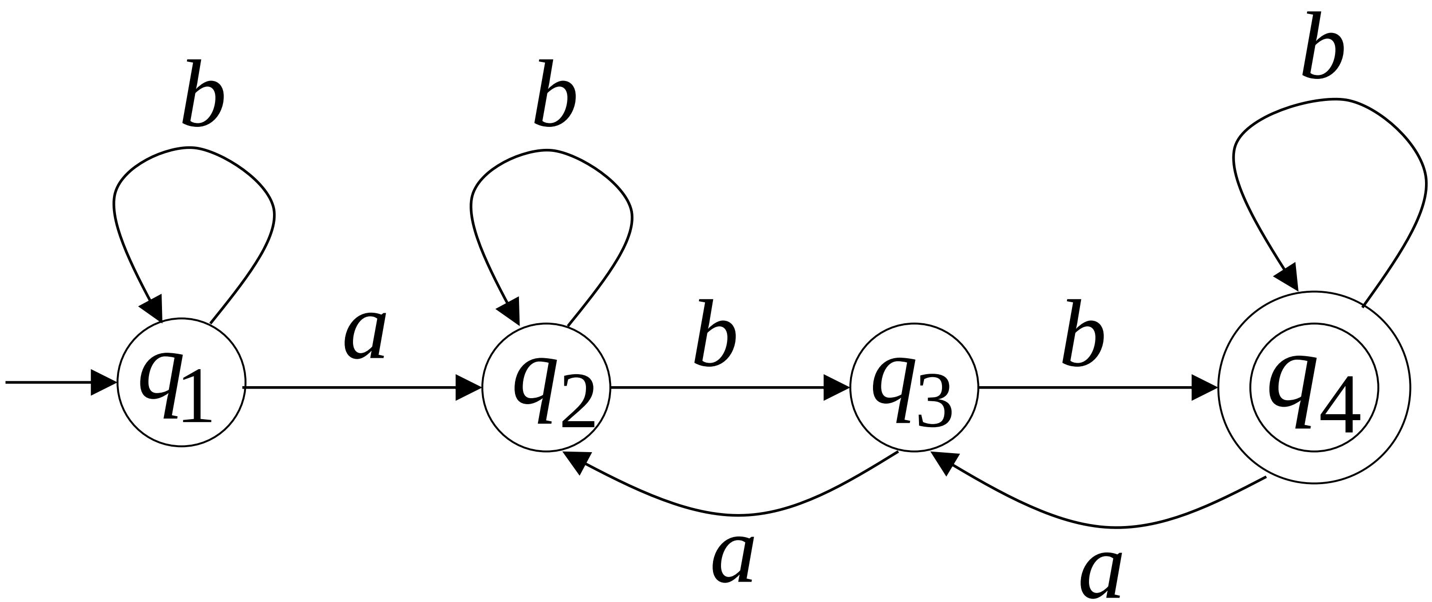
The Pigeonhole Principle

and

DFAs

DFA with 4 states

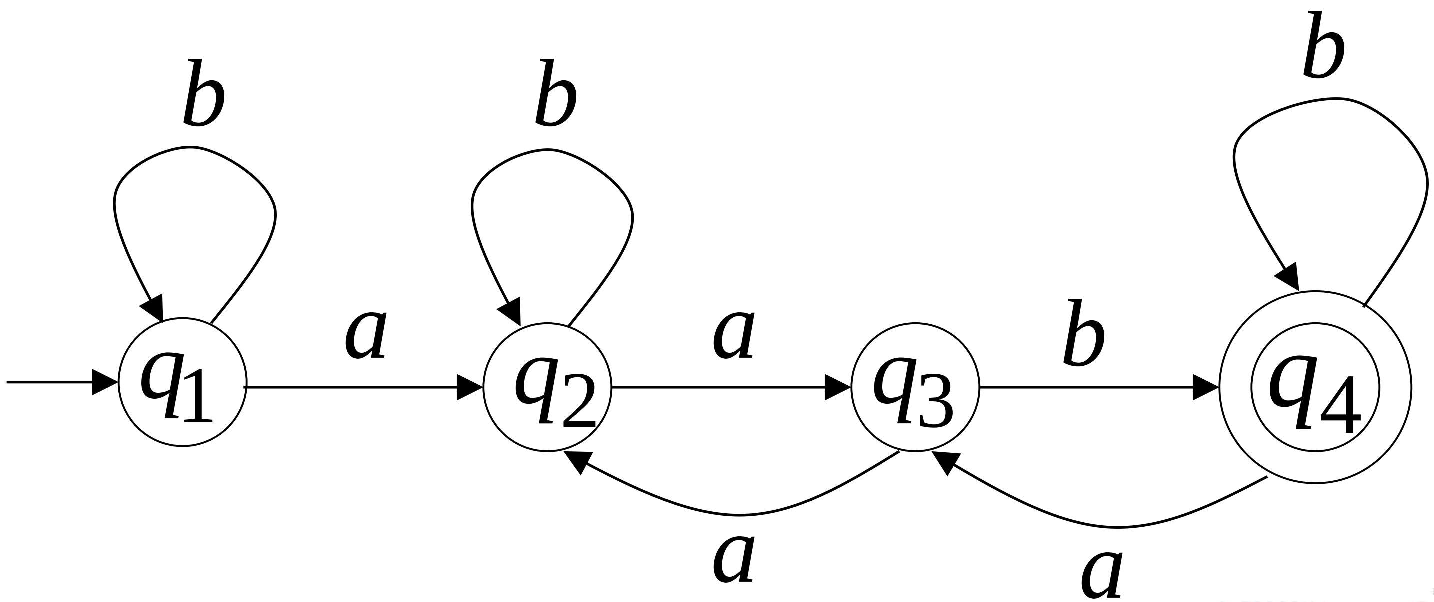


In walks of strings: a

aa

aab

no state is repeated



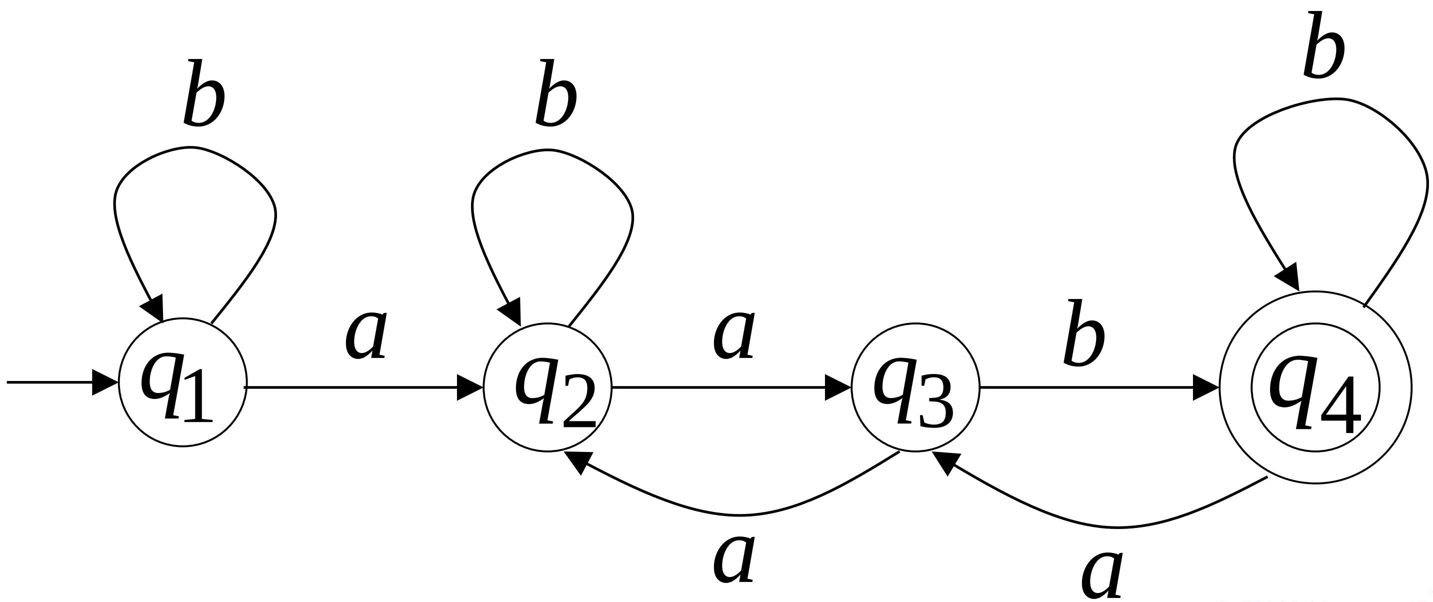
In walks of strings: *aabb*

bbaa

abbabb

abbbabbabb...

a state is repeated



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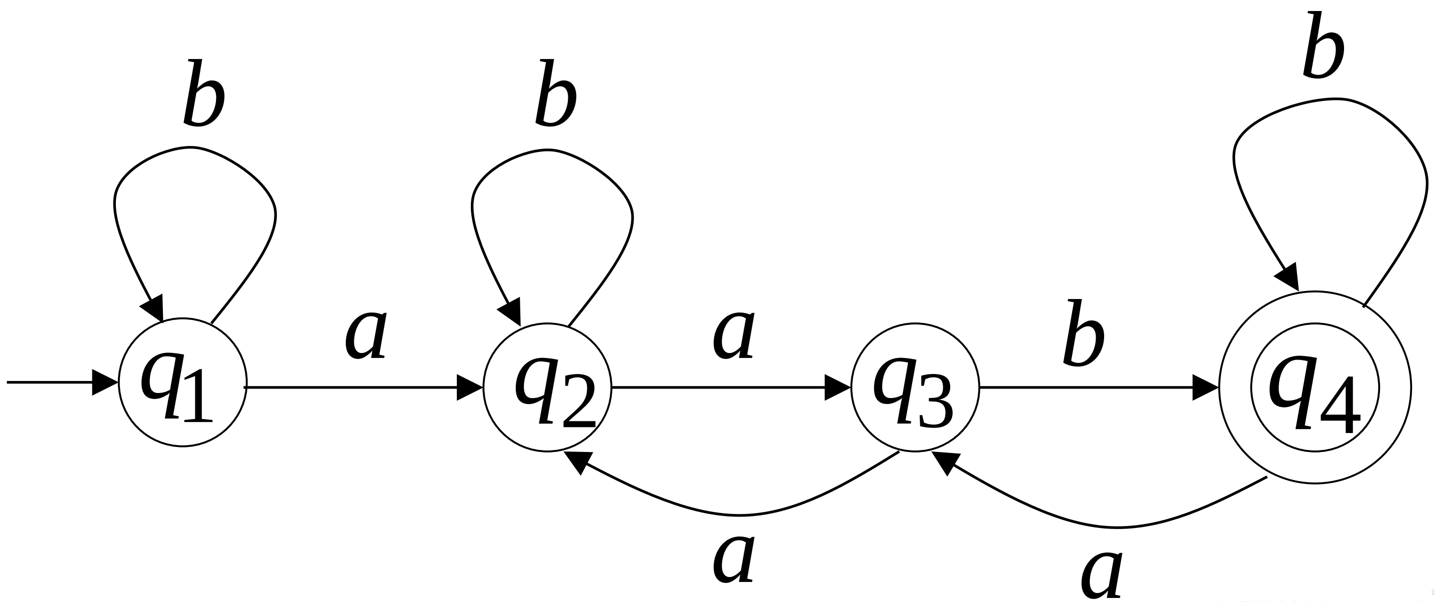
If string w has length $|w| \geq 4$:

Then the transitions of string w are more than the states of the DFA

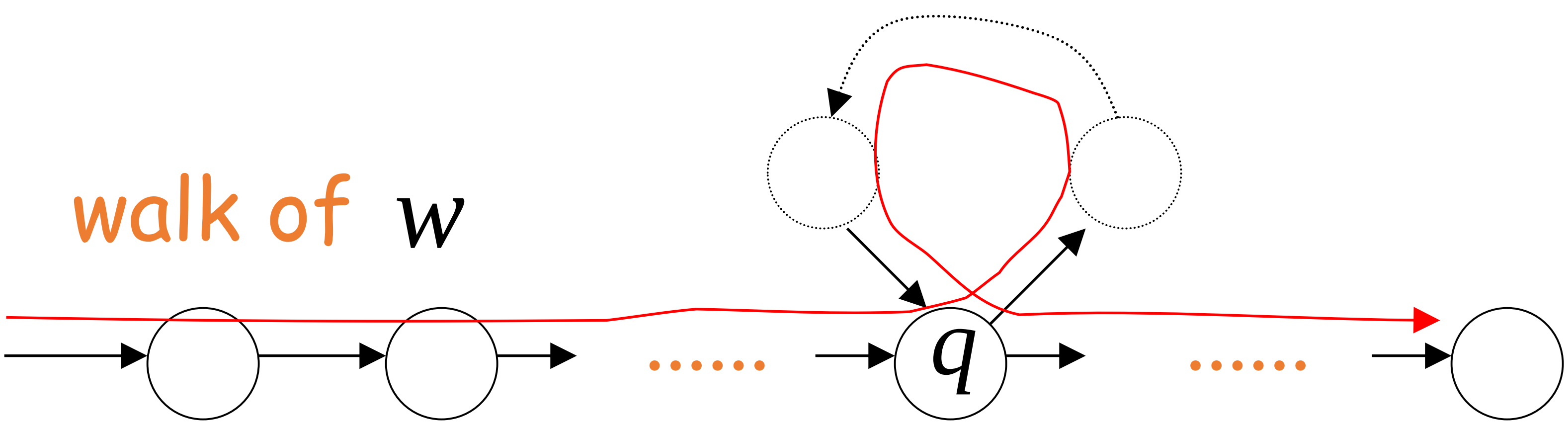
Thus, a state must be repeated



In general, for any DFA:

String w has length $\geq$ number of states

A state q must be repeated in the walk of w



Repeated state

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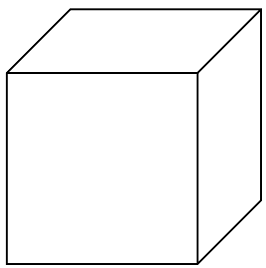
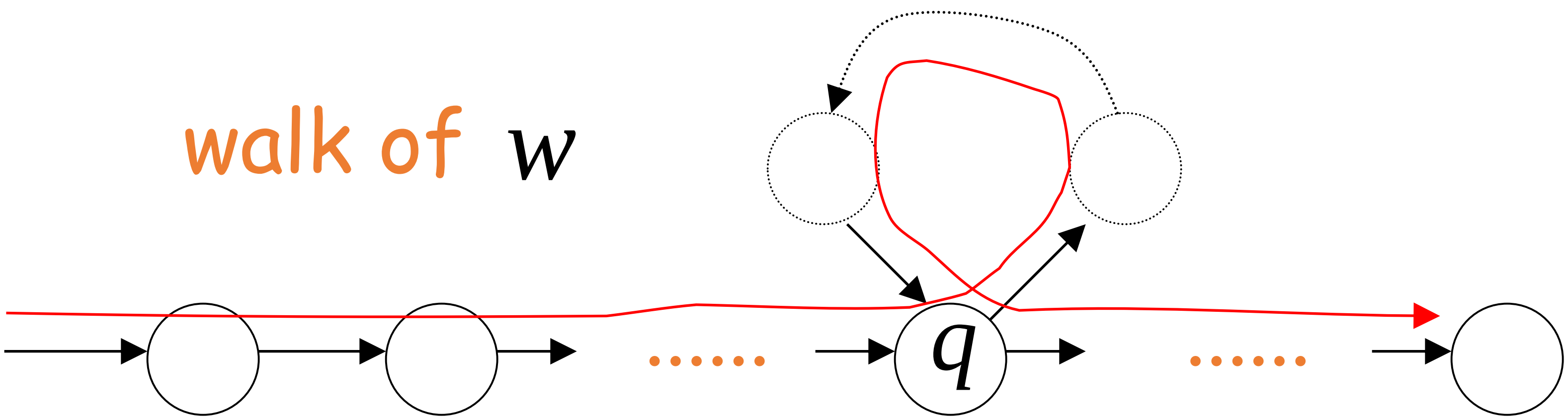
In other words for a string w:

$\xrightarrow{a}$ transitions are pigeons

q states are pigeonholes



walk of w



Repeated state

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The Pumping Lemma:

- Given a infinite regular language L
- there exists an integer m
- for any string $w \in L$ with length $|w| \geq m$
- we can write $w = x\, y\, z$
- with $|x\, y| \leq m$ and $|y| \geq 1$
- such that: $x\, y^i\, z \in L \qquad i = 0, 1, 2, \ldots$

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THANK YOU

Team – TOC

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