Department of AIDS

DEEP LEARNING
23AD2205A

Topic:

MARKOV MODEL

CO4-Session -30

AIM OF THE SESSION

To familiarize students with the concepts **HIDDEN MARKOV MODELS**

INSTRUCTIONAL OBJECTIVES

This Session is designed to:

1. Discussion on bayes theorem
2. Demonstrate the Markov model and Markov chain

LEARNING OUTCOMES

At the end of this session, you should be able to:

1. Able to apply hidden Markov model

Markov-Chain(Markov 1914) has been applied

- To short term market forecasting and business decision,
- Market research problems (market share predictions)
- Markov text generators
- Asset pricing and other financial predictions
- Customer journey predictions
- Population genetics
- Algorithmic music composition
- Page ranks (google results)

Bayes' Theorem

Bayes' Theorem finds the probability of an event occurring given the probability of another event that has already occurred. Bayes' theorem is stated mathematically as the following equation:

Probability of B occurring given evidence A has already occurred

Probability of A occurring

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Probability of A occurring given evidence B has already occurred

Probability of B occurring

Markov Random Processes

- ***Markov process*** is a simple stochastic process in which the distribution of future states depends only on the present state and not on how it arrived in the present state.
- A random sequence has the **Markov property if its distribution is determined solely by its current state.** Any random process having this property is called a *Markov random process*.
- For observable state sequences (state is known from data), this leads to a ***Markov chain* model.**
- For non-observable states, this leads to a ***Hidden Markov Model* (HMM).**

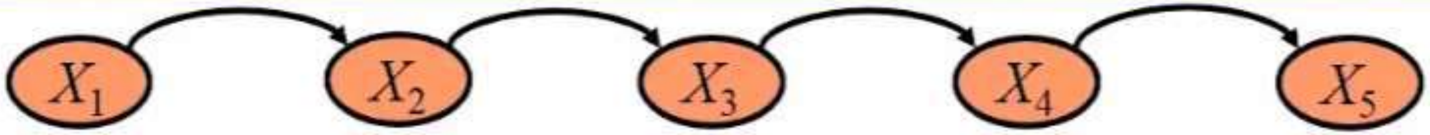
Markov Models

- A discrete (finite) system:
  - N distinct states.
  - Begins (at time t=1) in some initial state(s).
  - At each time step (t=1,2,…) the system moves from current to next state (possibly the same as the current state) according to transition probabilities associated with current state.
- This kind of system is called a **finite, or discrete Markov model.**
- **After Andrei Andreyevich Markov (1856 -1922)**

Markov Process

- **Markov Property:** The state of the system at time $t+1$ depends only on the state of the system at time t

$$P[X_{t+1} = x_{t+1} | X_1 \cdots X_t = x_1 \cdots x_t] = P[X_{t+1} = x_{t+1} | X_t = x_t]$$



- **Stationary Assumption:** In general, a process is called stationary if transition probabilities are independent of t, namely

$$\text{for all } t,\ P[X_{t+1} = x_j | X_t = x_i] = p_{ij}$$

This means that if system is in state i, the probability that the system will next move to state j is p_{ij}, no matter what the value of t is.

Markov Models

- Set of States:

$$\boxed{\{s_1, s_2, \dots, s_N\}}$$

- Process moves from one state to another generating a sequence of states :

$$\boxed{s_{i1}, s_{i2}, \dots, s_{ik}, \dots}$$

- Markov chain property: probability of each subsequent state depends only on what was the previous state:

$$\boxed{P(s_{ik} \mid s_{i1}, s_{i2}, \dots, s_{ik-1}) = P(s_{ik} \mid s_{ik-1})}$$

- To define Markov model, the following probabilities have to be specified:

Transition probabilities

$$\boxed{a_{ij} = P(s_i \mid s_j)}$$

and Initial probabilities

$$\boxed{\pi_i = P(s_i)}$$

- The output of the process is the set of states at each instant of time

Example

- **Weather:**

raining today → **40% rain tomorrow**

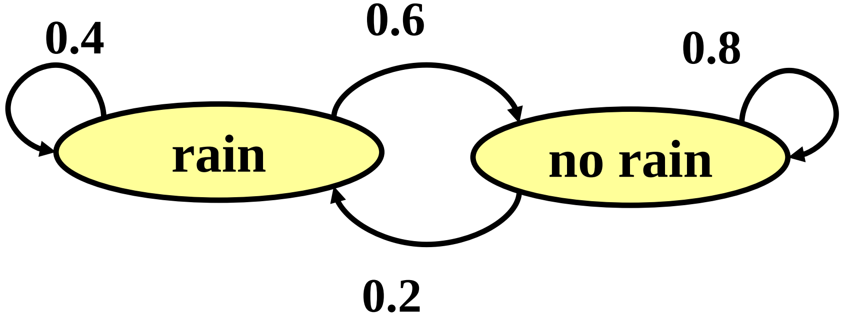
60% no rain tomorrow

not raining today → **20% rain tomorrow**

80% no rain tomorrow

Here we have two states {rain, norain}

State Transition Diagram



Transition Probability is given by

$$\mathbf{P}= \begin{array}{cc} \boldsymbol{Rain} & \boldsymbol{NoRain} \\ \end{array} \begin{bmatrix} \mathbf{0.4} & \mathbf{0.6} \\ \mathbf{0.2} & \mathbf{0.8} \end{bmatrix}$$

- Stochastic matrix:

  Rows sum up to 1

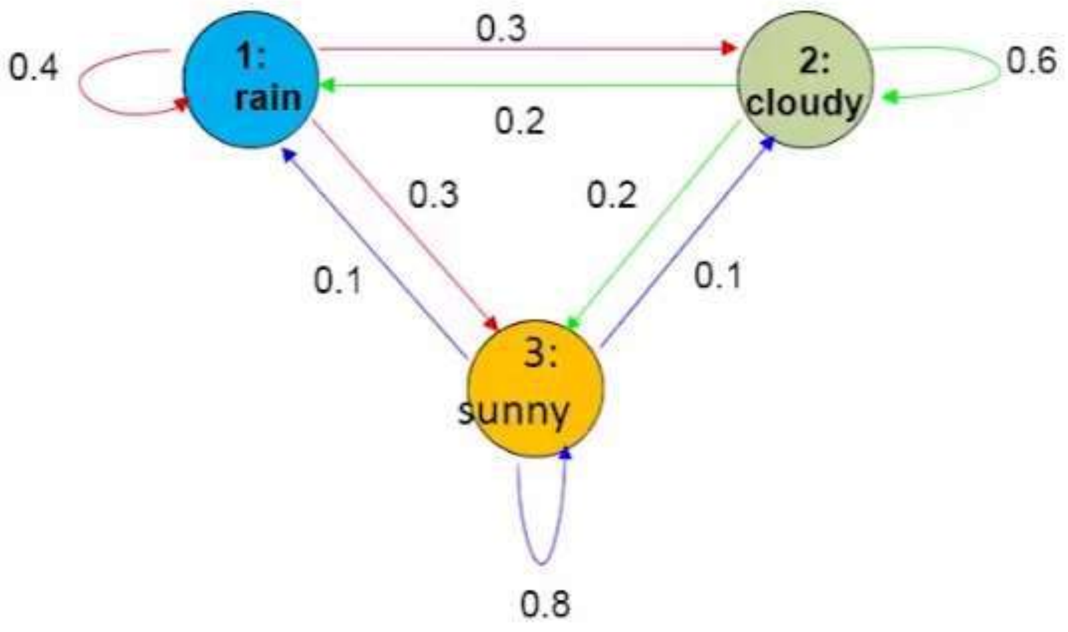
Markov Model – Example 1

- Markov Model: Scenario
- Classify a weather into three states
  - State 1: rain or snow
  - State 2: cloudy
  - State 3: sunny
- the weather of some city found following weather change pattern

		Tomorrow		
Today		Rainy	Cloudy	Sunny
	Rainy	0.4	0.3	0.3
	Cloudy	0.2	0.6	0.2
	Sunny	0.1	0.1	0.8

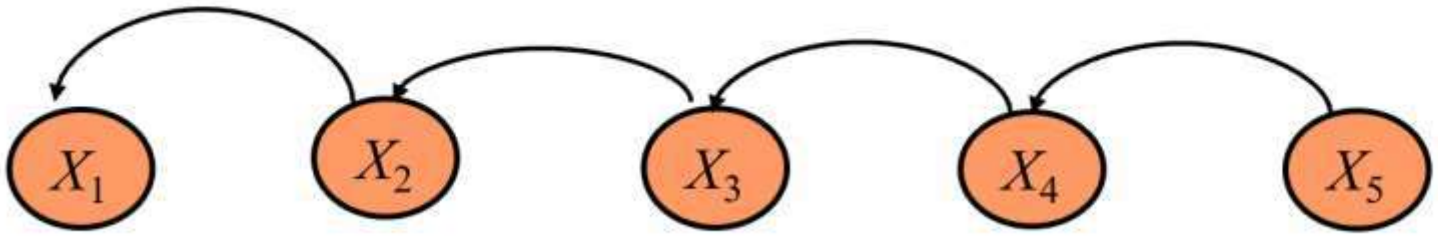
Tomorrow's Weather depends only on today's

Markov Model: Graphical Representation



Each state corresponds to one observation
- Sum of outgoing edge weights is one

Markov Chain



States: $\{X_1,X_2,X_3,X_4, X_5\}$,if x_3 is t be current state, then x_2 =t-1,x_1 =t-2,x_4 = t+1,x_5 = t+2

- Any Chain that follow Markov propery (i.e)$P(X_t \mid X_{t-1})$ then it is know as Markov chain.

Markov Model: Sequence Prob.

- Conditional probability
- $P(A,B) = P(A \mid B)P(B)$
- Sequence probability of Markov mode

Chain rule

$$
\begin{aligned}
\boldsymbol{P(q_1, q_2, \mathrm{L}, q_T)} &= P(q_1)P(q_2 \mid q_1)\mathrm{L}P(q_{T-1} \mid q_1, \mathrm{L}, q_{T-2})P(q_T \mid q_1, \mathrm{L}, q_{T-1}) \\
&= P(q_1)P(q_2 \mid q_1)\mathrm{L}P(q_{T-1} \mid q_{T-2})P(q_T \mid q_{T-1})
\end{aligned}
$$

1st order Markov assumption

Markov Model: Sequence Prob. (Cont.)

Question: What is the probability that the weather for the next 7 days will be "sun-sun-rain-rain-sun-cloudy-sun" when today is sunny?

$S_1 : rain, \quad S_2 : cloudy, \quad S_3 : sunny$

$$P(O \mid \text{model}) = P(S_3, S_3, S_3, S_1, S_1, S_3, S_2, S_3 \mid \text{model})$$

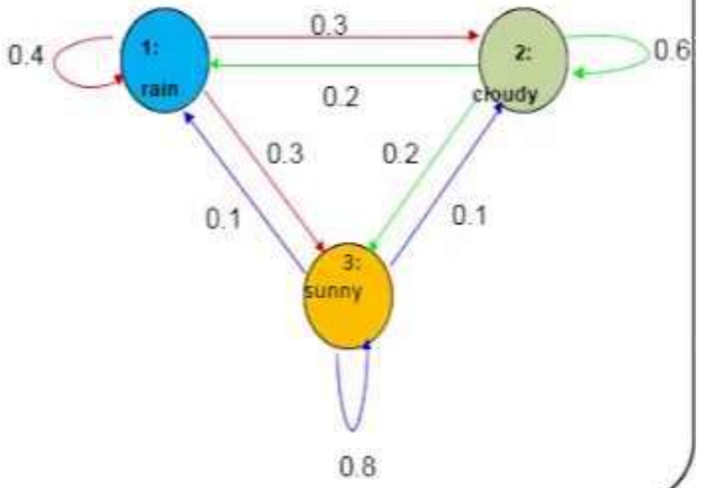
$$= P(S_3) \cdot P(S_3 \mid S_3) \cdot P(S_3 \mid S_3) \cdot P(S_1 \mid S_3)$$

$$\cdot P(S_1 \mid S_1) P(S_3 \mid S_1) P(S_2 \mid S_3) P(S_3 \mid S_2)$$

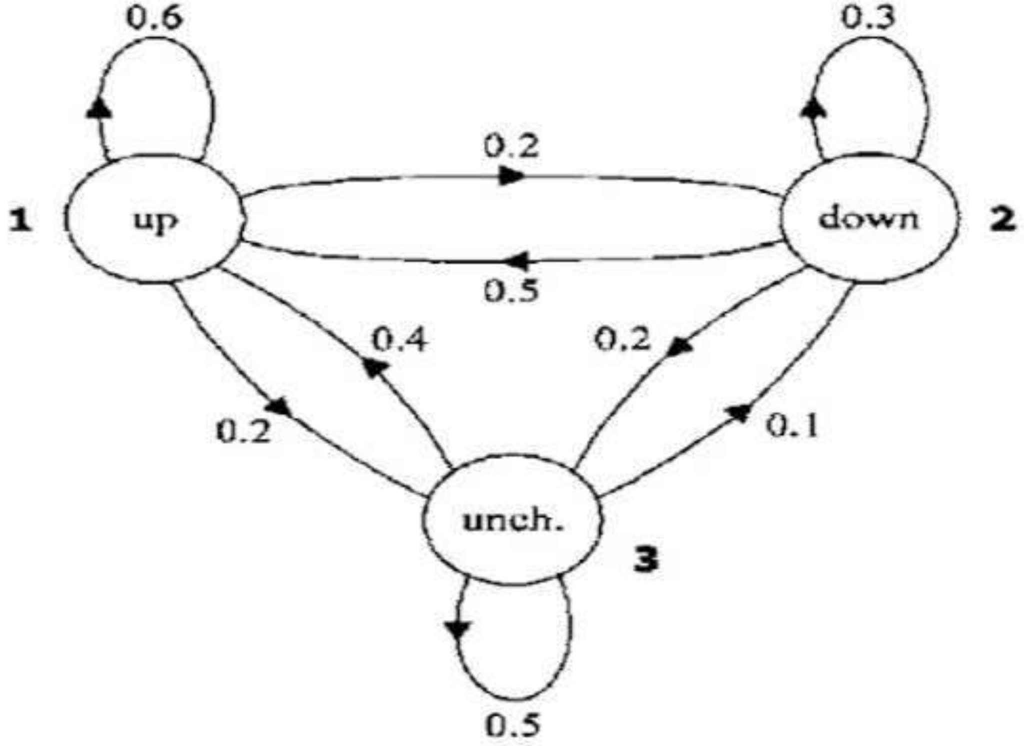
$$= \pi_3 \cdot a_{33} \cdot a_{33} \cdot a_{31} \cdot a_{11} \cdot a_{13} \cdot a_{32} \cdot a_{23}$$

$$= 1 \cdot (0.8)(0.8)(0.1)(0.4)(0.3)(0.1)(0.2)$$

$$= \quad 1.536 \text{X} 10^{-4}$$



Markov model Matrix - Example 2



Initial state probability matrix

$$\boldsymbol{\pi} = (\pi_i) = \begin{pmatrix} 0.5 \\ 0.2 \\ 0.3 \end{pmatrix}$$

State-transition probability matrix

$$\mathbf{A} = \{a_{ij}\} = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.5 & 0.3 & 0.2 \\ 0.4 & 0.1 & 0.5 \end{bmatrix}$$

Example

- What is the probability of 5 consecutive up days?
- Sequence is up-up-up-up-up
- I.e., state sequence is 1-1-1-1-1
- P(1,1,1,1,1) =
  - $\pi_1 a_{11} a_{11} a_{11} a_{11} = 0.5 \times (0.6)^4 = 0.0648$

KL (DEEMED TO BE UNIVERSITY)

SELF-ASSESSMENT QUESTIONS

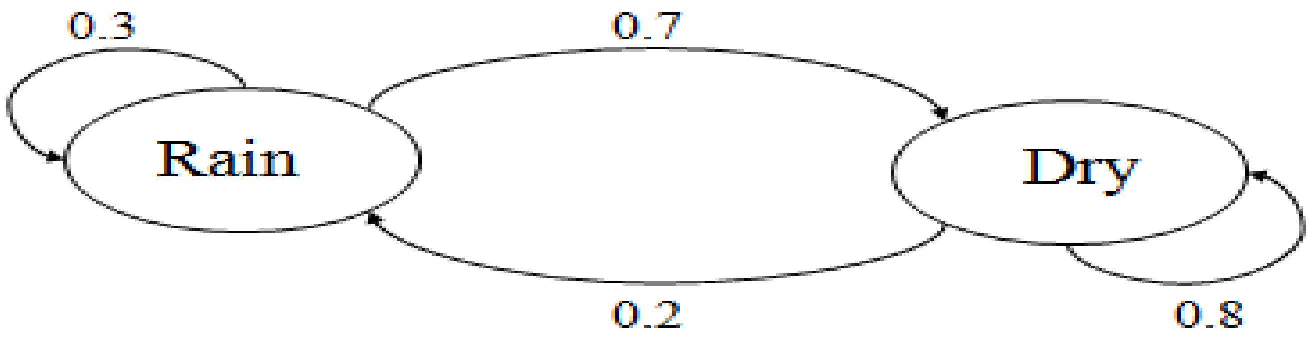
1. **Where does Markov models used**
   a. speech recognization
   b. understanding real world
   c. none mentioned

2. Which algorithm works by first running the standard forward pass to compute?
a) Smoothing
b) Modified smoothing
c) HMM
d) DFS algorithm

modified smoothing

TERMINAL QUESTIONS

- Consider the given probabilities for the two given states: Rain and Dry. The initial probability for rain and dry is [0.4,0.6]. Find the following (i)Today weather is rain and what is the probability that tomorrow will be dry and day after tomorrow will be rain.(ii) If today is dry, yesterday was rain and find the probability of tomorrow will be dry.



REFERENCES FOR FURTHER LEARNING OF THE SESSION

Books:

1 Ian Goodfellow and Yoshua Bengio and Aaron Courville (2016) Deep Learning Book

Resources:

https://towardsdatascience.com/hidden-markov-models-simply-explained-d7b4a4494c50