Department of AI & DS
CSE and CS&IT

COURSE NAME: PROBABILITY, STATISTICS AND QUEUING THEORY

COURSE CODE: 23MT2005

Topic

Bernoulli and Binomial distributions

Session - 5

AIM OF THE SESSION

To familiarize students with discrete probability distributions and its applications

INSTRUCTIONAL OBJECTIVES

This Session is designed to:

1. Demonstrate the Bernoulli trial with suitable examples
2. List out the rules of Bernoulli and Binomial distributions
3. Solving the problems of Binomial distribution
4. Discuss he importance of Binomial distribution and its applications in real world problems

LEARNING OUTCOMES

At the end of this session, you should be able to:

1. Define Bernoulli trial
2. Describe the rules of discrete probability distributions
3. Summarize the concepts of Binomial distribution and its applications

SESSION INTRODUCTION

CONTENTS

- Bernoulli distribution
- Binomial distribution

Discrete Probability distributions

Bernoulli Trial: A Bernoulli random variable is a random variable that can only take two possible values, usually 0 and 1. This random variable models random experiments that have two possible outcomes, sometimes referred to as "success" and "failure."

Examples:

You take a pass-fail exam. You either pass (resulting in X=1) or fail (resulting in X=0).

- The outcome of flipping a coin is either H or T.
- A child is born. The gender is either male or female.

Values of a Bernoulli random variable	
1	0
success	failure
H	T

If probability of "success" is **p**, the probability of "failure" is **1-p**.

Bernoulli distribution

Definition: A random variable X is said to be a **Bernoulli random variable** with parameter p, shown as X~Bernoulli(p), if its PMF is given by

$$P_X(x) = \begin{cases} p & \text{for } x = 1 \\ 1-p & \text{for } x = 0 \\ 0 & \text{otherwise} \end{cases}$$

where 0 < p < 1.

Bernoulli Distribution Graph

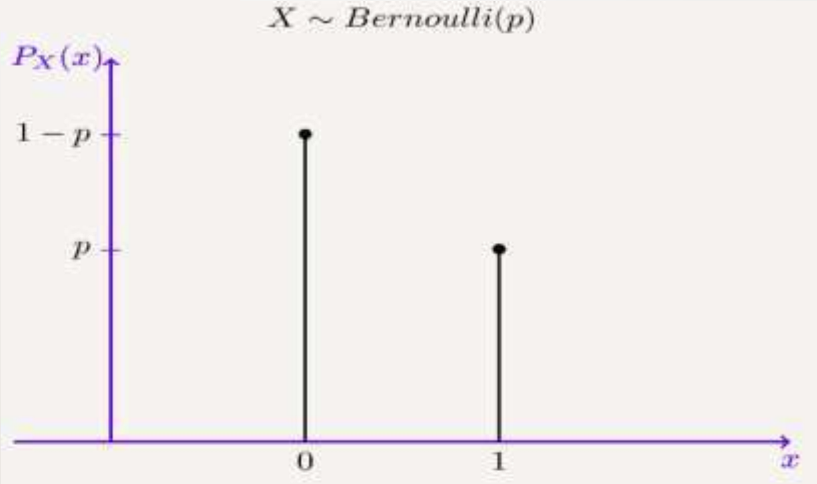


Fig.3.2 - PMF of a $Bernoulli(p)$ random variable.

Example

A basketball player can shoot a ball into the basket with a probability of 0.6. What is the probability that he misses the shot?

Solution:

We know that success probability $P(X = 1) = p = 0.6$

Thus, probability of failure is $P(X = 0) = 1 - p = 1 - 0.6 = 0.4$

Answer:

The probability of failure of the Bernoulli distribution is 0.4

Binomial distribution

A binomial random variable is random variable that represents the number of successes in n successive independent trials of a Bernoulli experiment. If X is a Binomial random variable, we denote this X ~Bin(n, p), where p is the probability of success in a given trial.

The binomial distribution formula is for any random variable X, given by;

The function for computing the probability for the binomial probability distribution is given by

$$f(x) = \frac{n!}{x!\,(n-x)!} p^x (1-p)^{n-x}$$

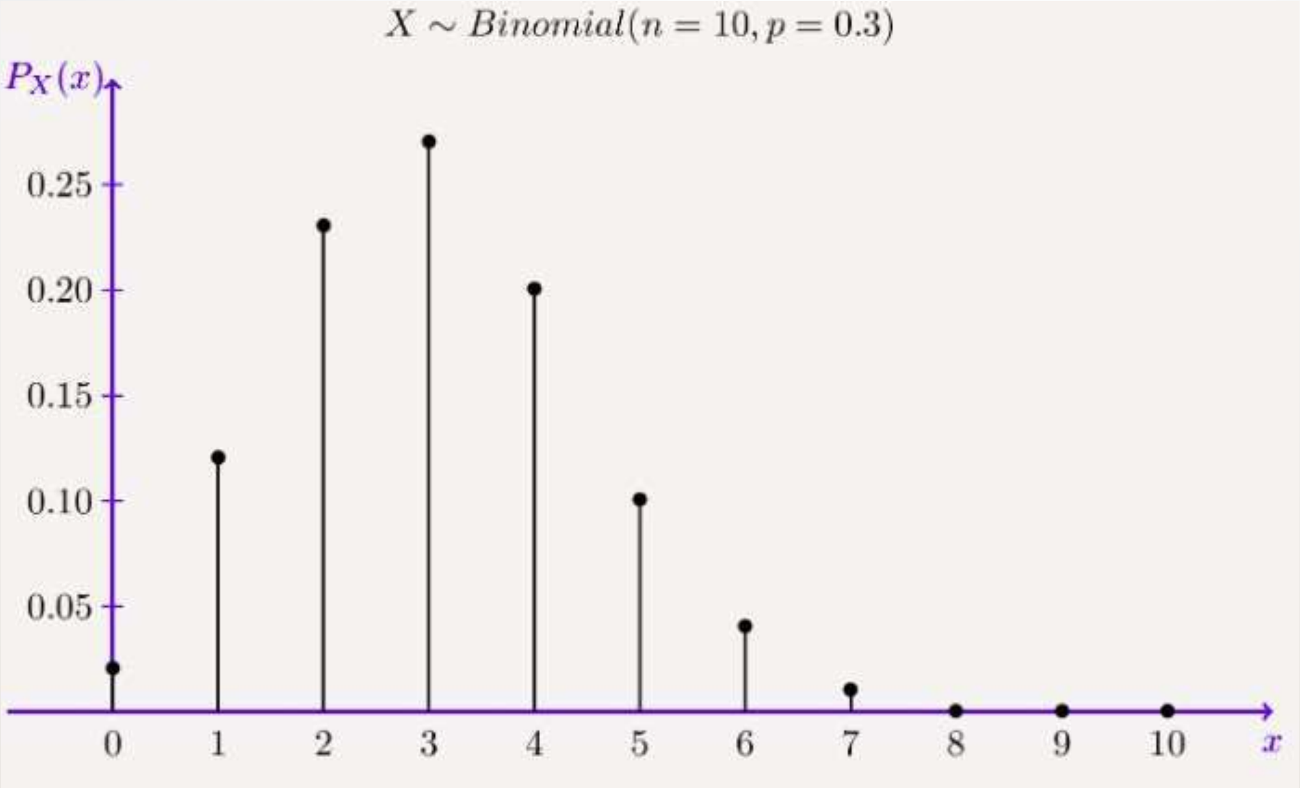
$$\text{for } x = 0, 1, 2, \ldots, n$$

Here, $f(x) = P(X = x)$, where X denotes "the number of success" and $X = x$ denotes the number of success in x trials.

Note

1) We write, X ~ b(n, p) to denote that X follows binomial distribution with parameters n and p
2) The mean of the binomial distribution is 'np'
3) The variance of the binomial distribution is 'npq'
4) Standard deviation is the square root of variance
5) In Binomial distribution, Mean is always greater than variance

Example PMF: PMF of a Binomial(10,0.3) random variable



Example 1: If a coin is tossed 5 times, using binomial distribution find the probability of:

(a) Exactly 2 heads

(b) At least 4 heads.

Solution:

(a)The repeated tossing of the coin is an example of a Bernoulli trial. According to the problem:

Number of trials: $n=5$

Probability of head: $p= 1/2$ and hence the probability of tail, $q =1/2$

For exactly two heads:

$x=2$

$P(x=2) = {}^5C2\ p^2\ q^{5-2} = 5!\ /\ 2!\ 3! \times (½)^2 \times (½)^3$ $P(x=2) = 5/16$

(b) For at least four heads,

$x \geq 4$, $P(x \geq 4) = P(x = 4) + P(x=5)$

Hence,

$P(x = 4) = {}^5C4\ p^4\ q^{5-4} = 5!/4!\ 1! \times (½)^4 \times (½)^1 = 5/32$

$P(x = 5) = {}^5C5\ p^5\ q^{5-5} = (½)^5 = 1/32$

Answer: Therefore, $P(x \geq 4) = 5/32 + 1/32 = 6/32 = 3/16$

Example

Example 2: A quality assurance inspector tests 20 circuit boards a day. If 10% of the boards have defects, what is the probability that the inspector will find

1. No defective boards on any given day
2. P(exactly 2 defectives)
3. P(between 1 and 4 defectives)

Solution:

Here n=20, p=0.10, q=0.90

$$P_X(k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k} & \text{for } k = 0, 1, 2, \cdots, n \\ 0 & \text{otherwise} \end{cases}$$

$$P(X=x) = 20_{c_x} (0.10)^x (0.90)^{20-x}, \quad x = 0,1,2,3,4,5,6,7,8,\ldots 20$$

1. P(X=0)=0.1216
2. P(exactly 2 defectives)=P(x=2)=0.2852
3. P(1<x<4)=P(between 1 and 4 defectives)= 0.5651.

Facts related to Bernoulli and Binomial

Bernoulli Distribution	**Binomial Distribtuion**
Bernoulli distribution is used when we want to model the outcome of a single trial of an event.	If we want to model the outcome of multiple trials of an event, Binomial distribution is used.
It is represented as X ~~Bernoulli (p). Here, p is the probability of success.	It is denoted as X ~~Binomial (n, p). Where n is the number of trials.
Mean, E[X] = p	Mean, E[X] = np
Variance, Var[X] = p(1-p)	Variance, Var[X]= np(1-p)
Example: Suppose the probability of passing an exam is 80% and failing is 20%. Then the Bernoulli distribution can be used to model the passing or failing in such an exam.	Example: Suppose the probability of passing an exam is 80% and failing is 20%. Then if we want to find the probability that a student will pass in exactly 4 out of 5 exams, we use the Binomial Distribution.

Applications

Business Applications

- Banks and other financial Distribution to determine the likelihood borrowers defaulting, and apply the number towards pricing insurance, and figuring out how much money to keep in reserve, or how much to loan.
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SUMMARY

In this session, the concept of Bernoulli and Applications of Binomial distribution described

1. Define Bernoulli and Binomial distribution
2. Applications of Binomial distribution

SELF-ASSESSMENT QUESTIONS

For Bernoulli distribution with probability p of success and q of a failure, the relation between mean and variance that holds is:

a) mean<variance
b) mean>variance
c) mean=variance
d) mean≤variance

X is a binomial variate with parameters n and p. if n=1, the distribution of X reduces to:

a) Poisson distribution
b) Binomial distribution itself
c) Bernoulli distribution
d) Discrete uniform distribution

TERMINAL QUESTIONS

1. A traffic control engineer reports that 75% of the vehicles passing through a checkpoint are from within the state. What is the probability that fewer than 4 of the next 9 vehicles are from out of state?

2. The probability that a patient recovers from a Corona virus is 0.4. If 15 people are known to have contracted this disease,

a) Find the mean and variance of the number of patients who recovered from the Corona
b) Find the probability that exactly 5 survive from the disease
c) Find the probability that atleast 10 survive from the disease
d) Find the probability that 3 to 8 survive form the disease
e) Find the probability that at most 3 survive the disease
f) Find the probability that none will survive the disease

3. According to a study published by a group of University of Massachusetts sociologists, approximately 60% of the Valium users in the state of Massachusetts first took Valium for psychological problems. Find the probability that among the next 8 users interviewed from this state,

(a) None took Valium first for psychological problems

(b) exactly 3 began taking Valium for psychological problems.

REFERENCES FOR FURTHER LEARNING OF THE SESSION

Reference Books:

1. Chapter 1 of TP1: William Feller, An Introduction to Probability Theory and Its Applications: Volume 1, Third Edition, 1968 by John Wiley & Sons, Inc.

2. Richard A Johnson, Miller& Freund's Probability and statistics for Engineers, PHI, New Delhi, 11th Edition (2011).

Sites and Web links:

1. * https://ncert.nic.in/textbook.php?kemh1=16-16 *
2. Notes: sections 1 to 1.3 of http://www.statslab.cam.ac.uk/~rrw1/prob/prob-weber.pdf
3. https://ocw.mit.edu/courses/res - 6 -012 -introduction -to -probability - spring - 2018/91864c7642a58e216e8baa8fcb4a5cb5_MITRES_6_012S18_L01.pd f 9

THANK YOU

Team – PSQT EVEN SEM 2024-25