Department of AI & DS
CSE and CS&IT

COURSE NAME: PROBABILITY, STATISTICS AND QUEUING THEORY

COURSE CODE: 23MT2005

Topic

STOCHASTIC PROCESS

Session – 24

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CATEGORY 1 UNIVERSITY BY MHRD, Govt. of India
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AIM OF THE SESSION

To familiarize students with the concept of stochastic process

INSTRUCTIONAL OBJECTIVES

This Session is designed to:

1. Define stochastic process
2. Describe the classification of stochastic process
3. To write the transition probability matrix.

LEARNING OUTCOMES

At the end of this session, you should be able to:

1. Differentiate between one step, two step and n-step transition probability matrix.
2. Summarize the discrete time and continuous time Markov chains.

Stochastic process

Stochastic process: A family of random variables which are functions of say, time are known as stochastic process (or random process).

Eg 1. Consider the experiment of throwing an unbiased die. Suppose that X_n is the outcome of the n[th] throw, $n \geq 1$. Then $\{X_n, n \geq 1\}$ is a family of random variables such that for distinct values of n (=1, 2, 3…), one gets distinct random variable X_n ; $\{X_n, n \geq 1\}$ constitutes a stochastic process.

Eg 2. Suppose that X_n is the number of sixes in the first n throws. For a distinct value of n= 1,2,3……., we get a distinct binomial variable X_n ; $\{X_n, n \geq 1\}$ which gives a family of random variables is a stochastic process.

Eg 3. Suppose that X_n is the maximum number shown in the first 'n' throws. Here $\{X_n, n \geq 1\}$ constitutes a stochastic process.

Eg 4. Consider the number of telephone calls received at a switch board. Suppose that X(t) the random variable which represents the number of incoming calls in an interval (o, t) of duration t units. The number of calls in one unit of time is X(1). The family $\{X(t), t \in T\}$ constitutes a stochastic process $(T = [0, \infty))$.

Stochastic process

Eg 1: Suppose that X(t) is the number of telephone calls at a switch board in an interval (o, t). Here the state space of X(t) is discrete though X(t) is defined for a continuous range of time. This is a continuous time stochastic process with discrete state space.

Eg 2. Suppose that X(t) represents the maximum temperature at a particular place in (0, t), then set of possible values of X(t) is continuous. This is a continuous time stochastic process with continuous state space.

Thus the stochastic processes can be classified into the following four types of processes:

(i) Discrete time; discrete state space

(ii) Discrete time; continuous state space

(iii) Continuous time; discrete state space

(iv) Continuous time; continuous state space

Stochastic process

All the four types may be represented by $\{X(t), t \in T\}$. In case of discrete time, the parameter generally used is n, i.e., the family is represented by $\{X(n), n = 0,1,2....\}$. In case of continuous time both the symbols $\{X_t, t \in T\}$ and $\{X(t), t \in T\}$ (where T is finite or infinite interval) are used. The parameter t is usually interpreted as time, through it may represent such characters as distance, length, thickness and so on.

Markov process: If $\{X(t), t \in T\}$ is a stochastic process such that, for, $t_1 < t_2 < ... < t_n < t$

$\Pr\{a \le X(t) \le b / X(t_1) = x_1, X(t_2) = x_2, ... X(t_n) = x_n\} = \Pr\{a \le X(t) \le b / X(t_n) = x_n\}$ the process $\{X(t), t \in T\}$ is a Markov process.

Markov chain: A discrete parameter Markov process is know as a Markov chain.

Stochastic process

Transition probability: P_{jk} is called the transition probability and represents the probability of transition from state j at the n^{th} trial to the state k at the $(n+1)^{th}$ trail.

Homogeneous Markov chain: If the transition probability P_{jk} is independent of n, the Markov chain is said to be homogeneous. If it is dependent on n, the chain is said to be non-homogeneous.

One step transition probability: The transition probability P_{jk} refer to the states (j, k) at two successive trails (say n^{th} and $(n+1)^{th}$ trails); the transition is one step transition probability.

If we are concerned with the pair of states (j, k) at two non-successive trails, say, j at the n^{th} trail and k at the $(n+m)^{th}$ trail, the corresponding probability is then called m-step transition probability and is denoted by $P_{jk}^{(m)} = \Pr\{X_{n+m} = k / X_n = j\}$.

Transition probability Matrix or Matrix of transition probabilities: The transition probability P_{jk} satisfy P_{jk}>0, $\sum P_{jk} = 1$ for all j. These probabilities may be written in the matrix form

Stochastic process

Transition probability Matrix or Matrix of transition probabilities: The transition probability P_{jk} satisfy $P_{jk}>0$, $\sum P_{jk} = 1$ for all j. These probabilities may be written in the matrix form

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & \cdots \\ p_{21} & p_{22} & p_{23} & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

This is called the transition probability Matrix of the Markov chain. P is a stochastic matrix.

Thus a transition matrix is a square matrix with non-negative elements and unit-row sums.

Classification of chains: The Markov chains are of two types (i) **ergodic** (ii) **regular**

An ergodic Markov chain has the property that it is possible to pass from one state to another in a finite number of steps, regardless of present state.

A special type of ergodic Markov chain is the regular Markov chain.

A regular Markov chain is defined as a chain having a transition matrix P such that for some power of P it has only non-zero positive probability values.

Note: Thus all regular chains must be ergodic chains.

Examples

1. Customers tend to exhibit loyalty to product brands but may be persuaded through clever marketing and advertising to switch brands. Consider the case of three brands: A, B and C. Customer “unyielding” loyalty to a given brand is estimated at 75%, giving the competitors only a 25% margin to realize a switch. Competitors launch their advertising campaigns once a year. For brand A customers, the probabilities of switching to brands B and C are 0.1 and 0.15 respectively. Customers of Brand B are likely to switch to A and C with probabilities 0.2 and 0.05 respectively. Brand C customers can switch to brands A and B with equal probabilities.

Express the situation as a Markov chain

In the long run, determine the market share for each brand?

Solution:

The transition matrix for the given problem is

$$\boldsymbol{P} = \begin{bmatrix} \mathbf{0.75} & \mathbf{0.1} & \mathbf{0.15} \\ \mathbf{0.2} & \mathbf{0.75} & \mathbf{0.05} \\ \mathbf{0.125} & \mathbf{0.125} & \mathbf{0.75} \end{bmatrix}$$

Here loyalty to a given brand is estimated at 75%, giving the competitors only a 25% margin to realize a switch

Examples

In the long run, the market share for each brand is

$$[x \;\; y \;\; z]\begin{bmatrix} \mathbf{0.75} & \mathbf{0.1} & \mathbf{0.15} \\ \mathbf{0.2} & \mathbf{0.75} & \mathbf{0.05} \\ \mathbf{0.125} & \mathbf{0.125} & \mathbf{0.75} \end{bmatrix} = [\mathbf{x} \;\; \mathbf{y} \;\; \mathbf{z}]$$

0.75x+0.2y+0.125z=x; 0.1x+0.75y+0.125z=y; 0.15x+0.05y+0.75z=z; x+y+z=1

Solving the above equations we will get x=0.3947; y=0.3070; z=0.2982

SUMMARY

In this session, Stochastic process and its characteristics have discussed.

1. Different types of stochastic process.

2. Determine the one step, two step and steady state transition probabilities.

TERMINAL QUESTIONS

1. A market survey is made on two brands of breakfast food A and B. Every time a customer purchases, he my buy the same brand or switch to another brand. The transition matrix is given below:

$$\text{from}\quad \begin{array}{c} \\ \\ A \\ B \end{array}\begin{array}{c} \text{To} \\ \begin{array}{cc} A & B \end{array} \\ \begin{bmatrix} 0.8 & 0.2 \\ 0.6 & 0.4 \end{bmatrix} \end{array}$$

At present, it is estimated that 60% of the people buy brand A and 40% buy brand B. Determine the market shares of brand A and brand B

i) After two years
ii) in the steady state

2. Consider a certain community in well-defined area with three types of grocery stores; for simplicity we shall call them I, II and III. Within this community (we assume that the population is fixed) there always exists a shift of customer from one grocery store to another. A study was made on January 1 and it was found that ¼ shopped at store I, 1/3 at store II and 5/12 at store III. Each month store I retains 90% of its customers and loses 10% of them to store II. store II retains 90% of its customers and loses 5% each to store I store III. Store III retains 40% of its customers and loses 50% of them to store I and 10% to store II.

I) What proportion of customers will each store retain by February 1; March 1?
II) Assuming the same pattern continues, what will be the long run distribution of customers among the three stores?

REFERENCES FOR FURTHER LEARNING OF THE SESSION

Reference Books:

1. D. Gross, J.F.Shortle, J.M. Thompson, and C.M. Harris, Fundamentals of Queueing Theory, 4th Edition, Wiley, 2008

2.William Feller, An Introduction to Probability Theory and Its Applications:Volime 1,Third Edition, 1968 by John Wiley & Sons,Inc.

Sites and Web links:

1. https://onlinecourses.nptel.ac.in/noc22_ma17/previ ew 3.

2. https://www.youtube.com/watch?v=Wo75G99F9f M&list=PLwdnzlV3ogoX2OHyZz3QbEYFhbqM 7x275&index=3

3. J.F. Shortle, J.M. Thompson, D. Gross and C.M. Harris, Fundamentals of Queueing Theory, 5th Edition, Wiley, 2018.

4. https://onlinecourses.nptel.ac.in/noc22_ma17/previ ew 3.

5https://www.youtube.com/watch?v=Wo75G99F9f M&list=PLwdnzlV3ogoX2OHyZz3QbEYFhbqM 7x275&index=3

THANK YOU

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