Department of AI & DS
CSE and CS&IT

COURSE NAME: PROBABILITY, STATISTICS AND QUEUING THEORY

COURSE CODE: 23MT2005

Topic

Poisson distribution

Session - 6

AIM OF THE SESSION

To familiarize students with discrete probability distributions and its applications

INSTRUCTIONAL OBJECTIVES

This Session is designed to:

1. Demonstrate the special case of Binomial tends to Poisson distribution
2. List out the rules of Poisson distributions
3. Solving the problems of Poisson distribution

LEARNING OUTCOMES

At the end of this session, you should be able to:

1. Define Poisson distribution
2. Describe the rules associated with the Poisson distribution
3. Summarize the concepts of Poisson distribution and its applications

CONTENTS

- Poisson distribution

- Applications of Poisson distribution

Poisson distribution

The Poisson distribution is one of the most widely used probability distributions. It is usually used in scenarios **where we are counting the occurrences of certain events in an interval of time or space**. In practice, it is often an approximation of a real-life random variable. A Poisson random variable can be used to answer real-life questions such as

- How many babies born in the next minute?
- How many car crashes happen per hour?
- How many customers visit a store in an hour?

Suppose that we are counting the number of customers who visit a certain store from 1pm to 2pm. Based on data from previous days, we know that on average $\lambda=15$ customers visit the store. Of course, there will be more customers some days and fewer on others. Here, we may model the random variable X showing the number customers as a **Poisson random variable** with parameter $\lambda=15$.

Poisson distribution

There are some experiments, which involve the occurring of the number of outcomes during a given time interval (or in a region of space). Such a process is called Poisson process.

Example:

Number of clients visiting a ticket selling counter in a metro station.

Poisson distribution

Definition: The probability distribution of the Poisson random variable X, representing the number of outcomes occurring in a given time interval or specified region denoted by t, is

$$P(x;\lambda t) = \frac{e^{-\lambda t}(\lambda t)^x}{x!}, \qquad x = 0,1,2,\dots..$$

Note: Both the mean and variance of the Poisson distribution P(x; λt) are λt.

1) If the time t is one unit then

$$P(x;\lambda) = \frac{e^{-\lambda}(\lambda)^x}{x!}, \qquad x = 0,1,2,\dots..$$

Note

When n is large and p is small, binomial probabilities are often approximated by means of the Poisson distribution with the parameter λ equal to the product np i.e., Poisson distribution is used in case of rare events.

Examples

Suppose you work at a call center, approximately how many calls do you get in a day? It can be any number. Now, the entire number of calls at a call center in a day is modeled by Poisson distribution. Some more examples are

- The number of emergency calls recorded at a hospital in a day.
- The number of thefts reported in an area on a day.
- The number of customers arriving at a salon in an hour.
- The number of suicides reported in a particular city.
- The number of printing errors at each page of the book.

- Poisson Distribution is applicable in situations where events occur at random points of time and space wherein our interest lies only in the number of occurrences of the event.

Case Study I

Example 1: For a certain type of copper wire, it is known that, on the average, 1.5 flaws occur per millimeter. Assuming that the number of flaws is a Poisson random variable, find the probability that no flaws occur in a certain portion of wire of length 5 millimeters? Also find the mean number of flaws in a portion of length 5 millimeters

Solution:

Given that on the average, λ=1.5 flaws occur per millimeter.

Here the number of flaws is a poisson random variable.

Now, we want to find that no flaws occur in a certain portion of wire of length 5 millimeters.

Hence, the probability mass function of Poisson distribution is

$$P(X = x) = \frac{e^{-\lambda t}\lambda t^{x}}{x!}, \qquad x = 0,1,2, \dots, \infty$$

$=0$ otherwise

Case Study 1

$$P(X=0) = \frac{e^{-(1.5)(5)}(1.5*5)^0}{0!}, \qquad x = 0,1,2,\dots,\infty$$

$$P(X=0) = \frac{e^{-7.5}(1.5*5)^0}{0!}, \qquad x = 0,1,2,\dots,\infty$$

$P(X=0) =$0.0006

Mean number of flaws in a portion of length 5 millimeters=(5*1.5)=λ=7.5.

Applications

- Predicting customer sales on particular days/times of the year.
- Supply and demand estimations to help with stocking products.
- Service industries can prepare for an influx of customers, hire temporary help, order additional supplies, and make alternative plans to reroute customers if needed.

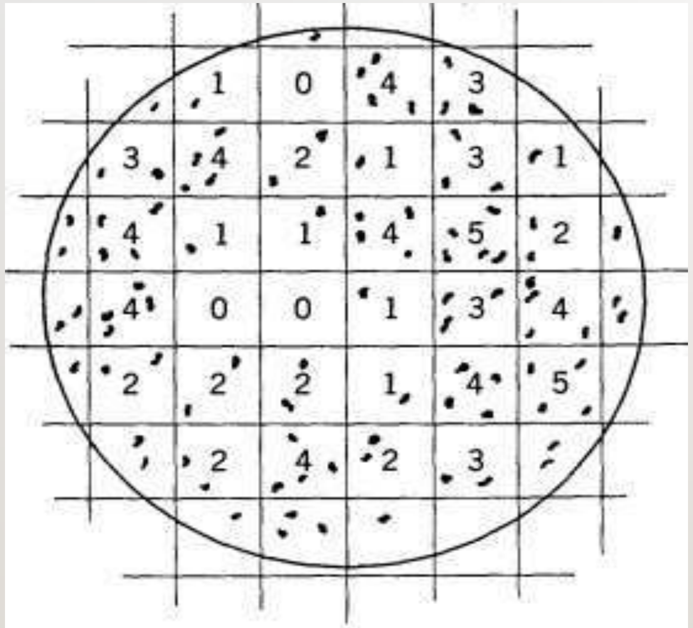

Figure 1. Bacteria on a Petri plate.

Figure 1 reproduces a photograph of a Petri plate with **bacterial colonies**, which are visible under the microscope as dark spots. The plate is divided into small squares. The observed numbers of squares with exactly k dark spots follow a Poisson distribution.

SUMMARY

In this session, the concept of Poisson distribution has described

1. Define Poisson distribution
2. Applications of Poisson distribution

SELF-ASSESSMENT QUESTIONS

A family of parametric distribution in which mean is equal to variance is

a) Binomial distribution
b) Gamma distribution
c) normal distribution
d) Poisson

Binomial distribution tends to Poisson distribution when

a) $n \to \infty$, $p \to 0$ and $np = \mu$ (finite)
b) b) $n \to \infty$, $p \to (1/2)$ and $np = \mu$ (finite)
c) $n \to 0$, $p \to 0$ and $np \to 0$
d) $n \to 15$, $p \to 0$ and $np \to 0$

TERMINAL QUESTIONS

1. If a bank received on the average 6 bad checks per day, what are the probabilities that it will receive

i) 4 bad checks on any given day?

ii) 10 bad checks over any 2 consecutive days

iii) No bad check on any given day

iv) What are the mean and variance of the number of bad check per day?

2. On an Average a certain intersection results in 3 Traffic accidents per month. What is the probability that for any given month at this intersection

a) Exactly 5 accidents will occur?
b) Less than 3 accidents will occur?
c) at least 2 accidents will occur?

3. The probability that a person will die from certain respiratory infection is 0.002.Find the probability that fewer than 5 of the next 2000 so infected will die. Also find mean and Variance.

REFERENCES FOR FURTHER LEARNING OF THE SESSION

Reference Books:

1. Chapter 1 of TP1: William Feller, An Introduction to Probability Theory and Its Applications: Volume 1, Third Edition, 1968 by John Wiley & Sons,Inc.

2. Richard A Johnson, Miller& Freund's Probability and statistics for Engineers, PHI, New Delhi, 11th Edition (2011).

Sites and Web links:

1. * https://ncert.nic.in/textbook.php?kemh1=16-16 *
2. Notes: sections 1 to 1.3 of http://www.statslab.cam.ac.uk/~rrw1/prob/prob-weber.pdf
3. https://ocw.mit.edu/courses/res - 6 -012 -introduction -to -probability - spring - 2018/91864c7642a58e216e8baa8fcb4a5cb5_MITRES_6_012S18_L01.pd f 9

THANK YOU

Team – PSQT EVEN SEM 2024-25