# 23MT2014

# THEORY OF COMPUTATION

Topic:

# PUMPING LEMMA FOR REGULAR LANGUAGES

Session – 9-B

# The Pumping Lemma

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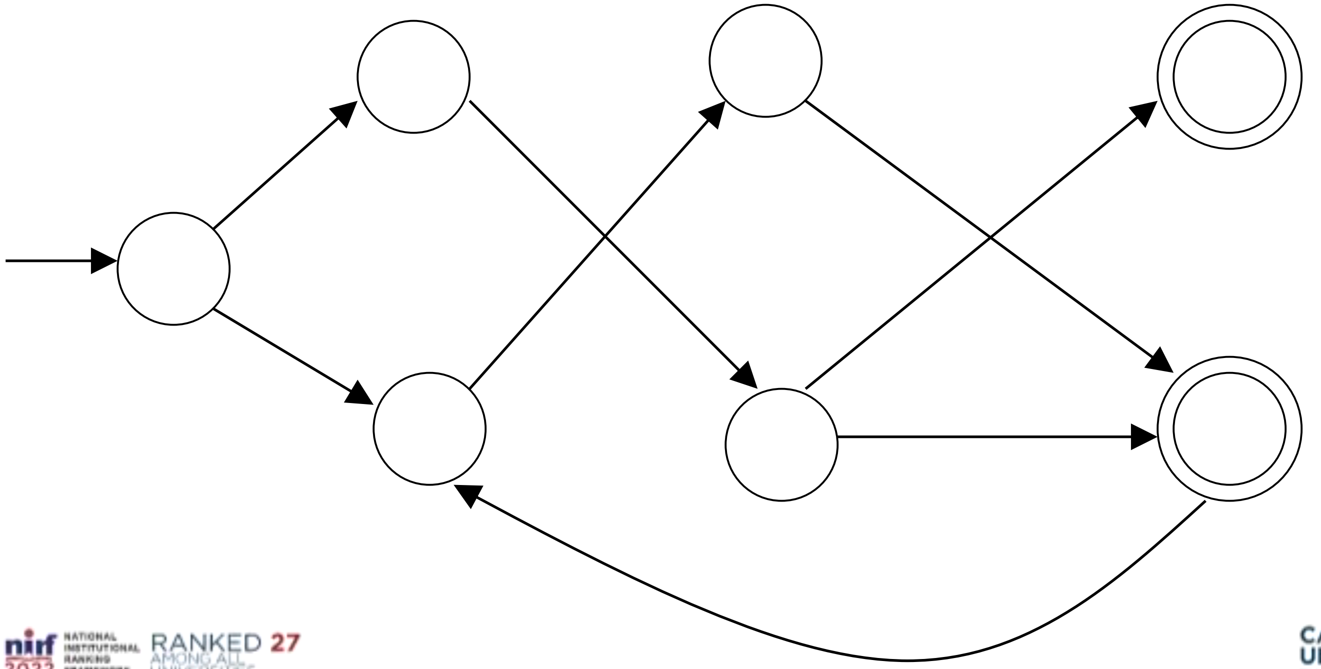
Take an infinite regular language $L$

There exists a DFA that accepts $L$



$m$ states

Take string $w$ with $w \in L$

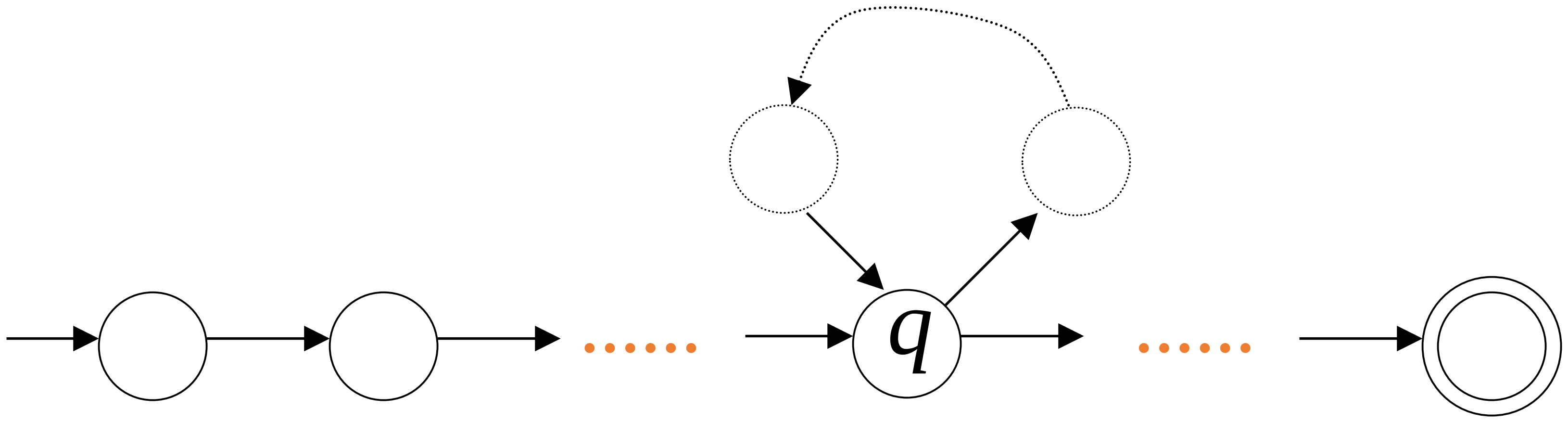
There is a walk with label $w$:

walk $w$

If string $w$ has length $|w| \geq m$ (number of states of DFA)

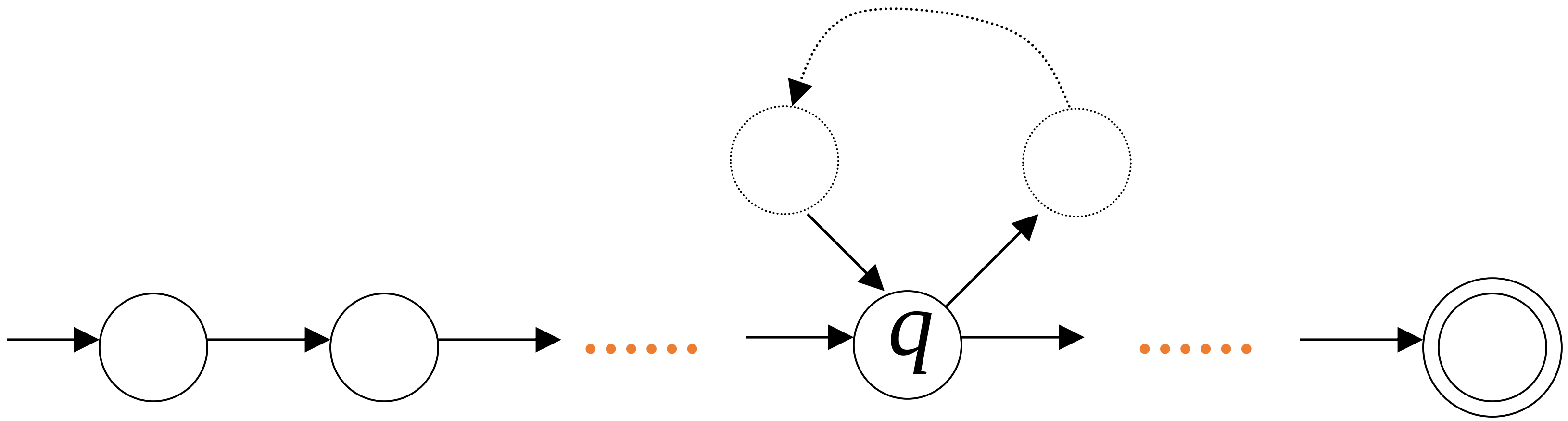
then, from the pigeonhole principle:

a state is repeated in the walk $w$



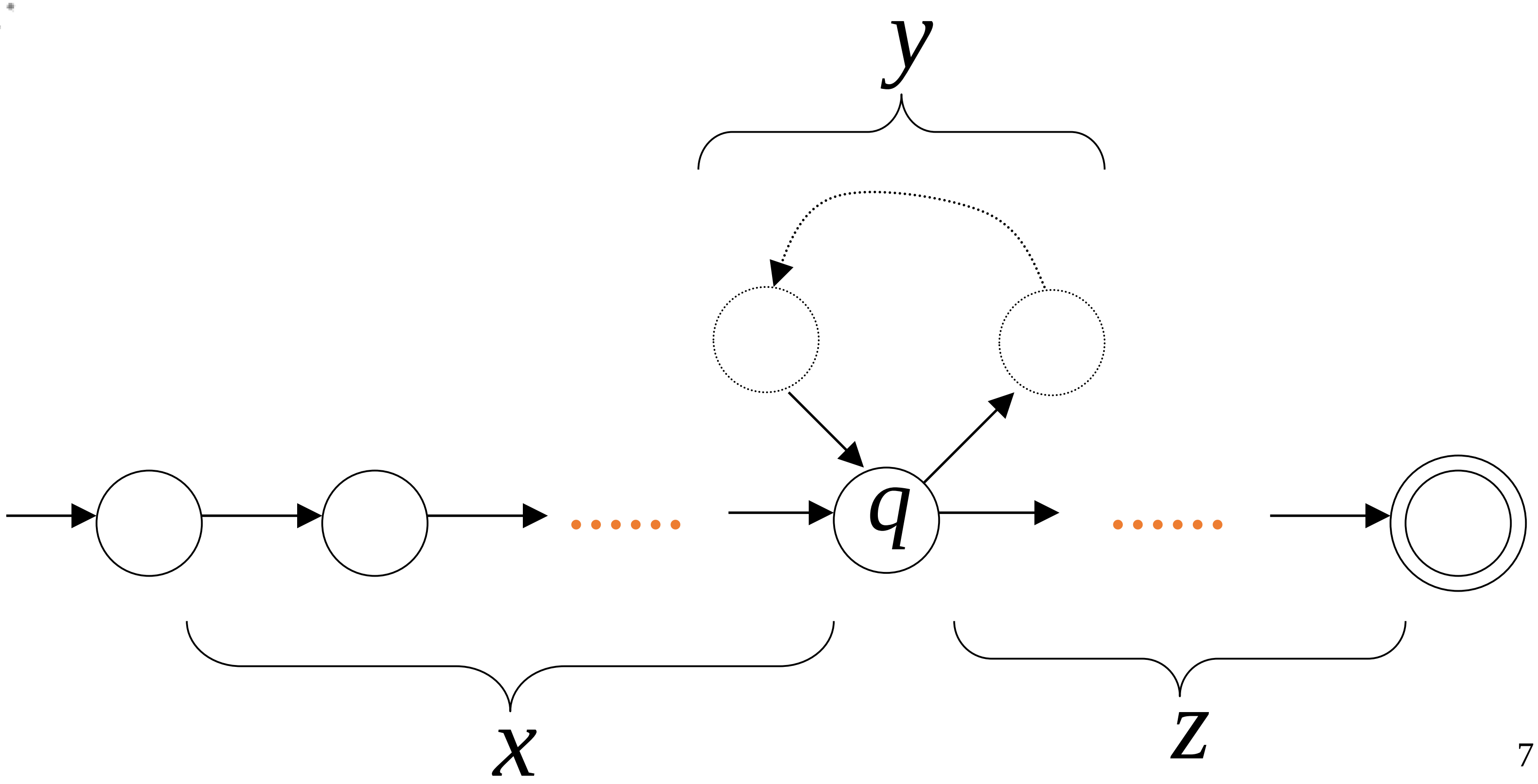

walk $w$

Let $q$ be the first state repeated in the walk of $w$




walk $w$

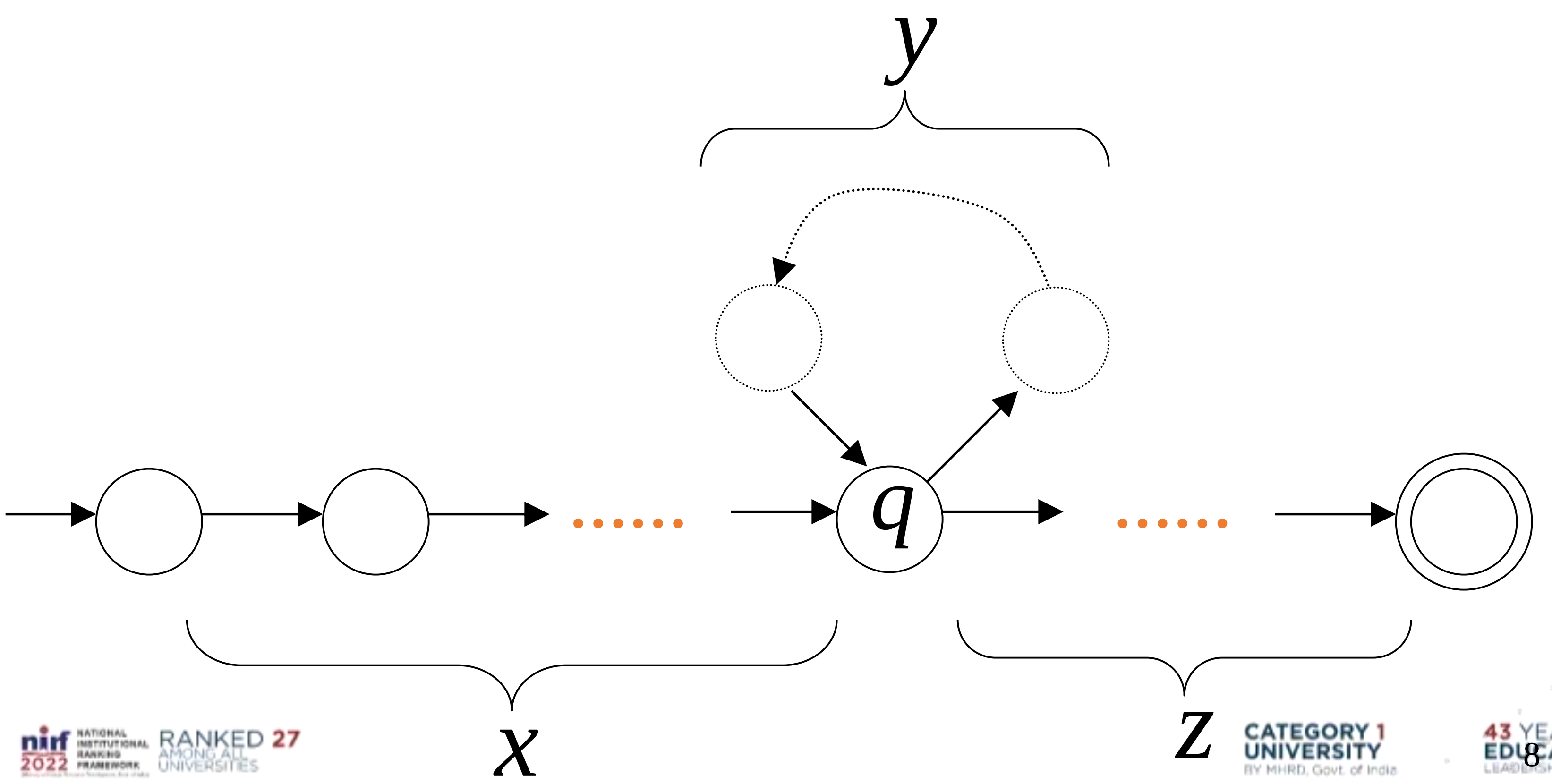
Write $w = x\ y\ z$



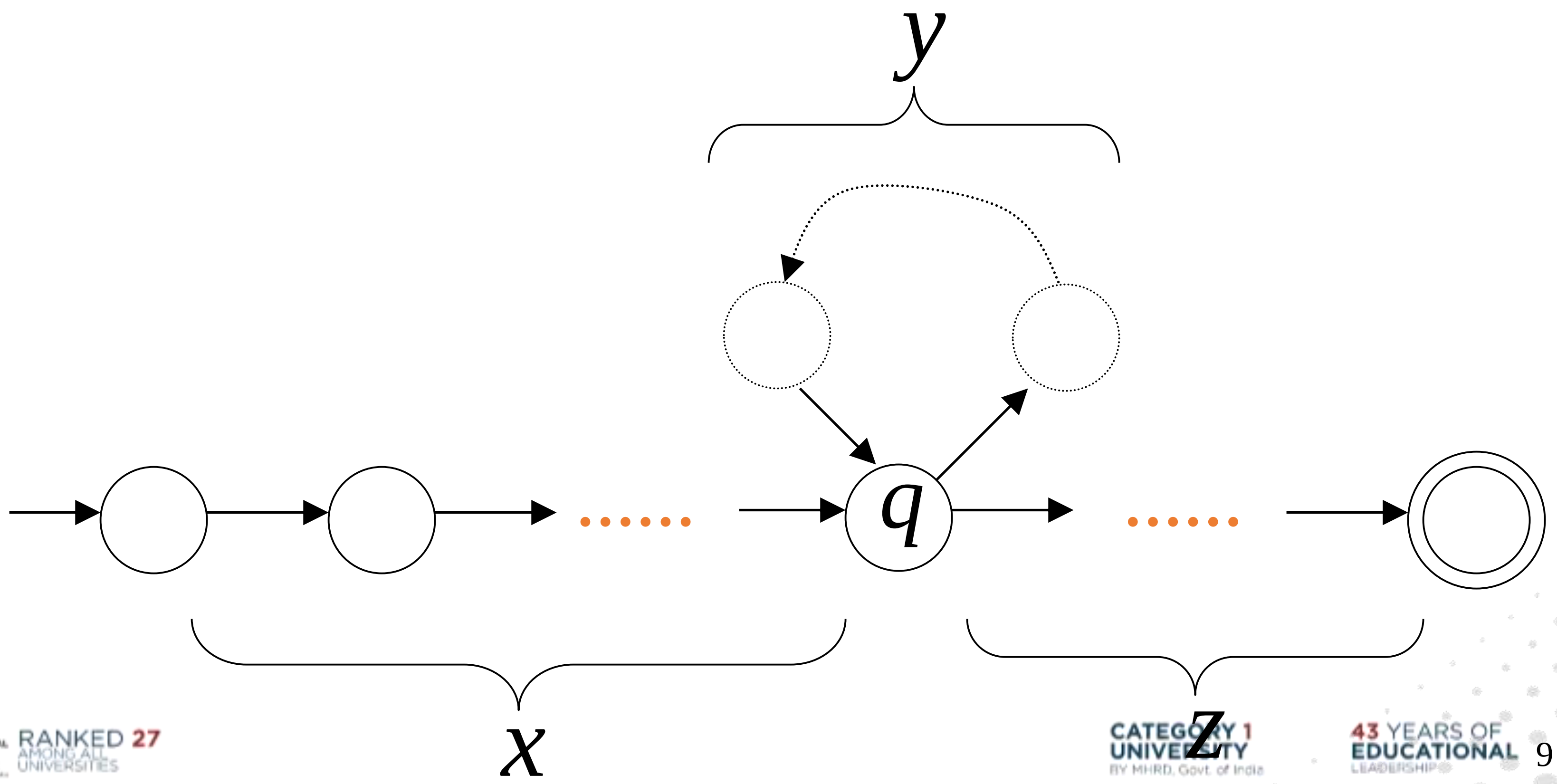
## Observations:

length $|x\,y| \leq m$ number of states of DFA

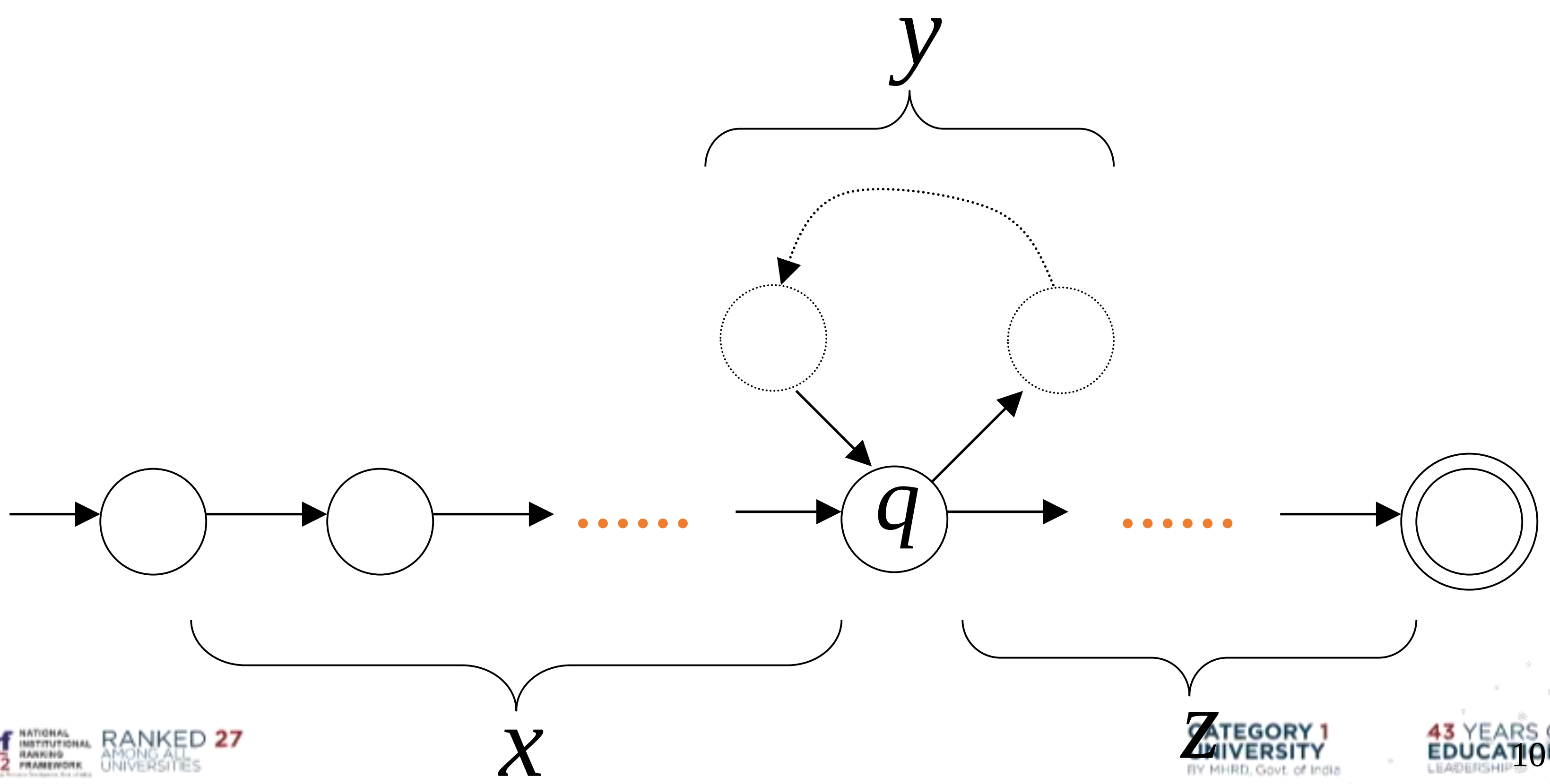
length $|y| \geq 1$



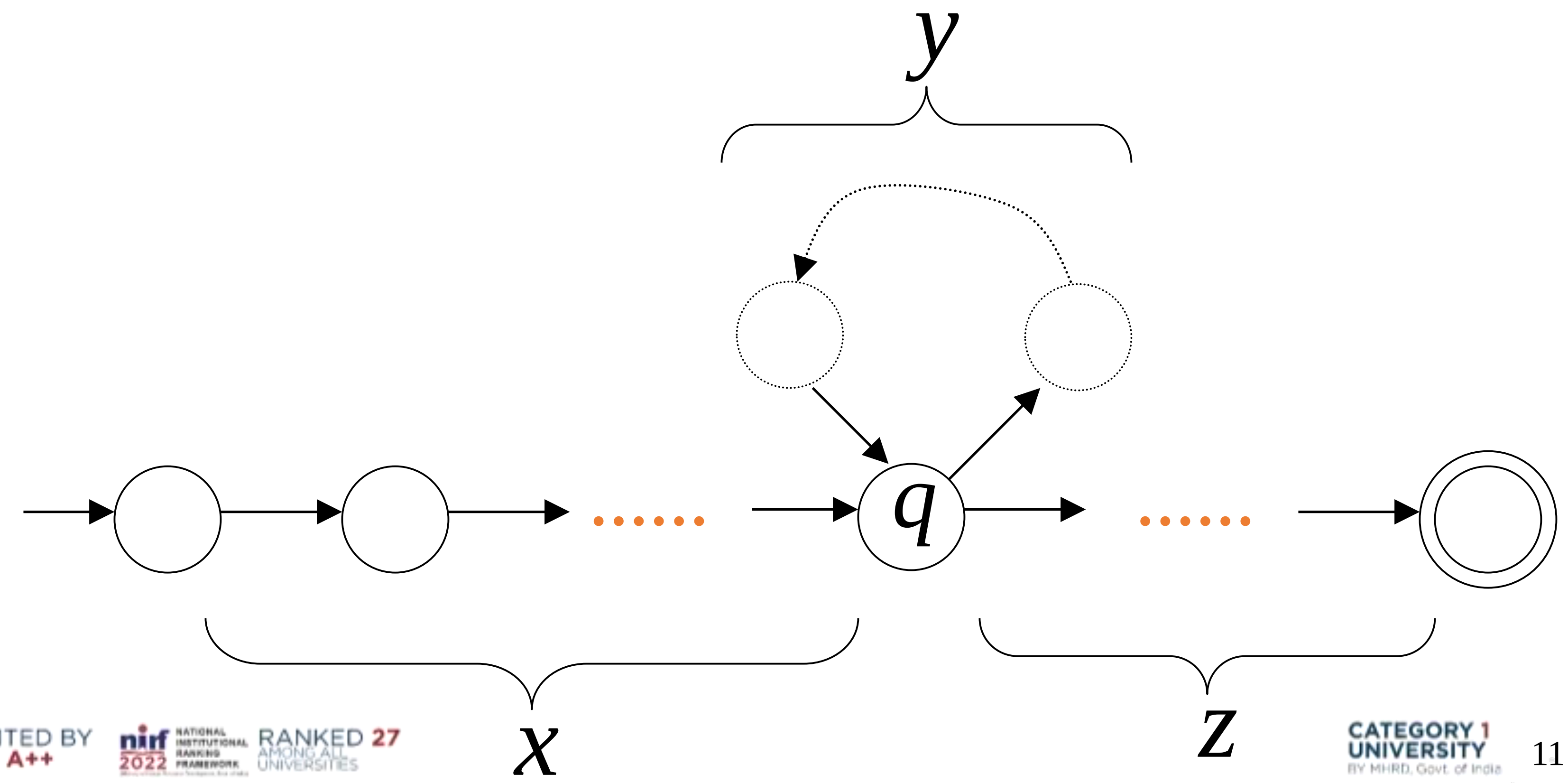
Observation: The string $x\ z$ is accepted



Observation: The string $x\ y\ y\ z$ is accepted



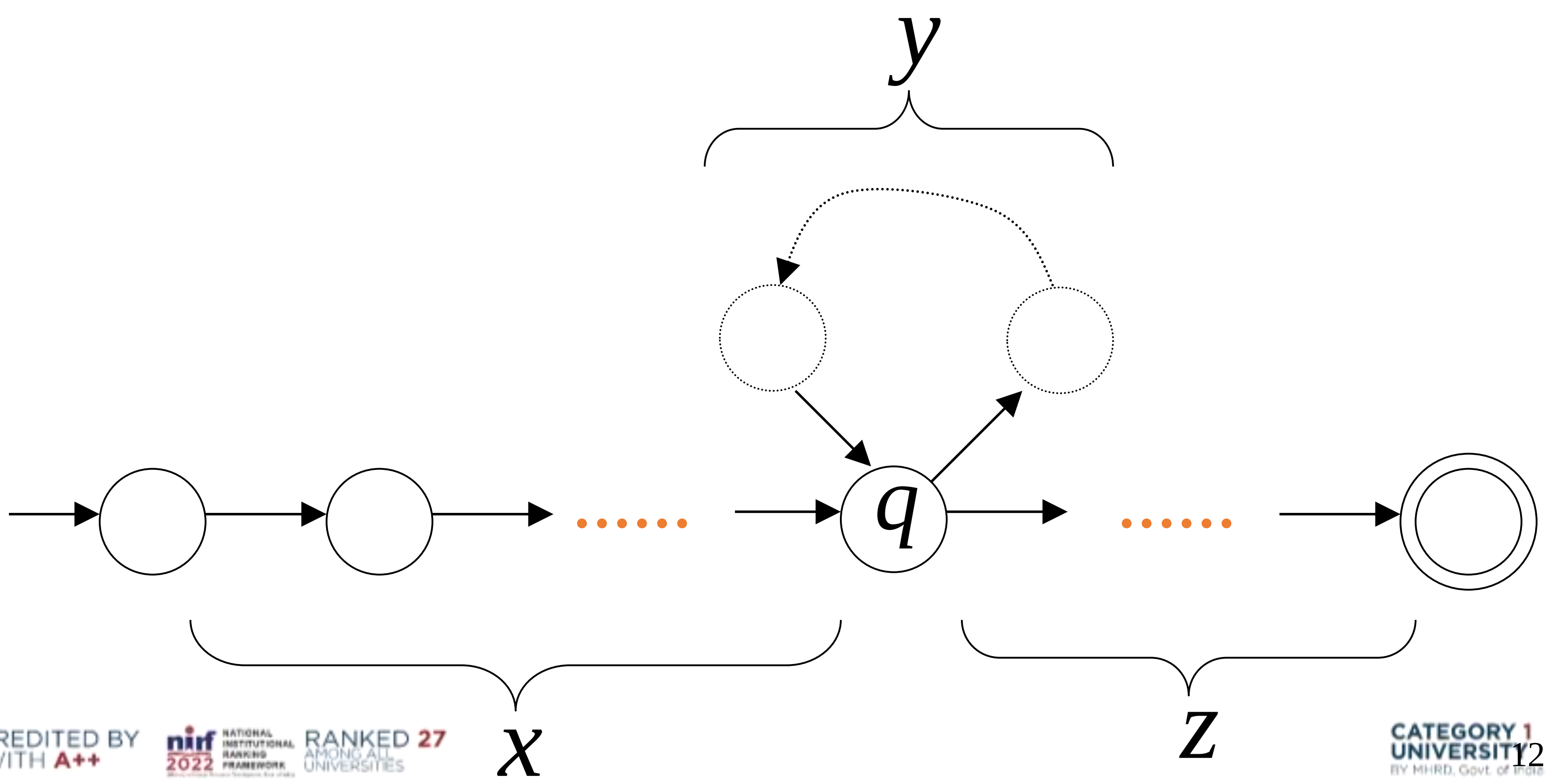
## Observation: The string $x\ y\ y\ y\ z$ is accepted



## In General:

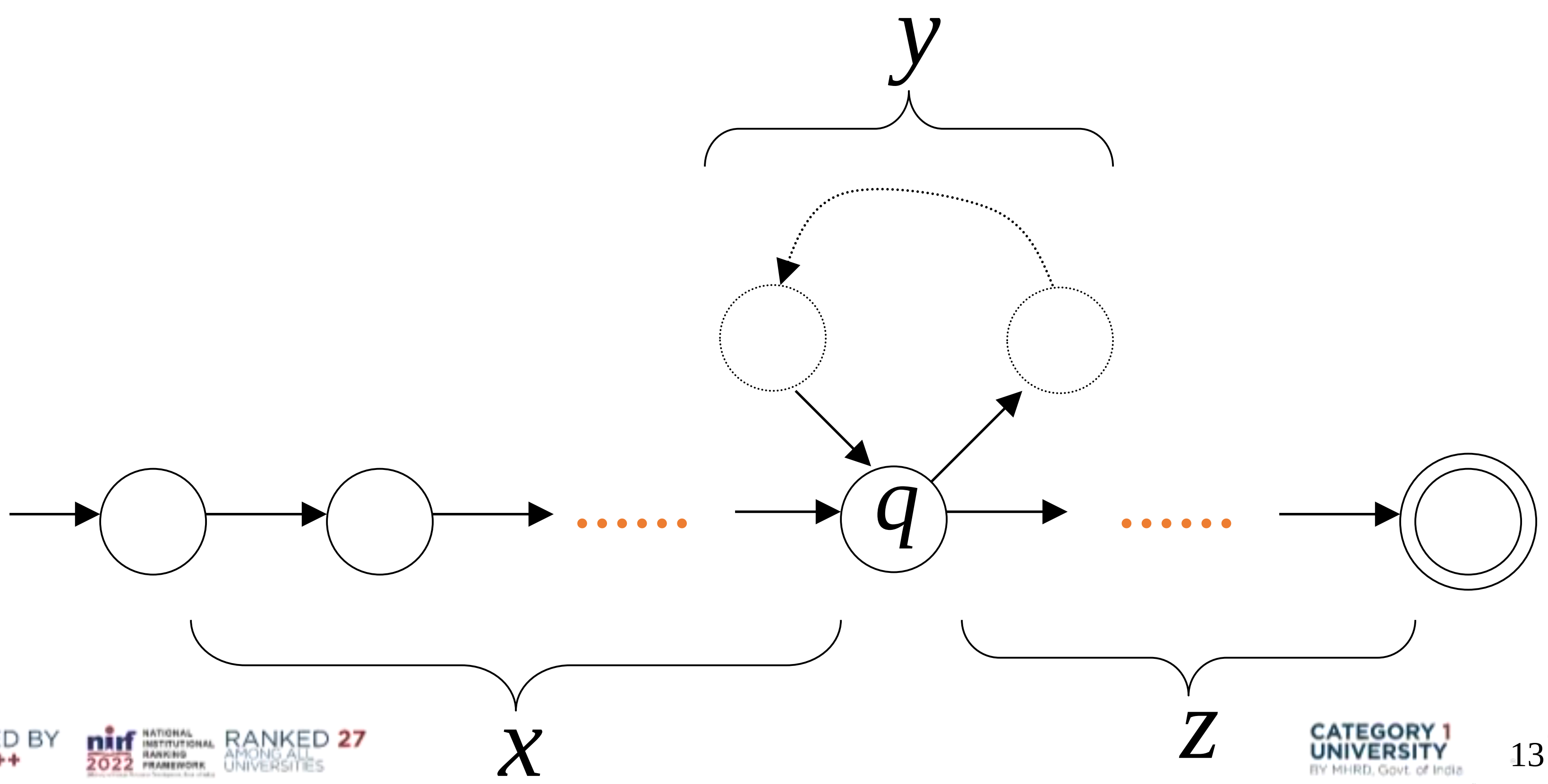
The string $x\,y^i\,z$

is accepted $i = 0, 1, 2, \ldots$



In General: $x\,y^i\,z \in L \qquad i = 0, 1, 2, \ldots$

Language accepted by the DFA



# The Pumping Lemma:

- Given a infinite regular language $L$
- there exists an integer $m$
- for any string $w \in L$ with length $|w| \geq m$
- we can write $w = x\,y\,z$
- with $|x\,y| \leq m$ and $|y| \geq 1$
- such that: $x\,y^i\,z \in L \qquad i = 0, 1, 2, \ldots$

# Applications

# of

# the Pumping Lemma

**Theorem:** The language $L = \{a^n b^n : n \geq 0\}$

is not regular

**Proof:** Use the Pumping Lemma

$$L = \{a^n b^n : n \geq 0\}$$

Assume for contradiction
that $L$ is a regular language

Since $L$ is infinite
we can apply the Pumping Lemma

$$L = \{a^n b^n : n \geq 0\}$$

Let $m$ be the integer in the Pumping Lemma

Pick a string $w$ such that: $w \in L$

length $|w| \geq m$

We pick $w = a^m b^m$

Write: $a^m b^m = x\ y\ z$

From the Pumping Lemma

it must be that length $|x\ y| \le m, \quad |y| \ge 1$

$$xyz = a^m b^m = \underbrace{a...a}_{x}\underbrace{a...a}_{y}\underbrace{a...ab...b}_{z}$$

$$\overbrace{a...aa...aa...a}^{m}\overbrace{b...b}^{m}$$

Thus: $y = a^k, \quad k \ge 1$

$$x\,y\,z = a^m b^m \qquad y = a^k, \quad k \geq 1$$

From the Pumping Lemma: $x\,y^i\,z \in L$

$$i = 0, 1, 2, \ldots$$

Thus: $x\,y^2\,z \in L$

$$x\,y\,z = a^m b^m \qquad y = a^k, \quad k \geq 1$$

From the Pumping Lemma: $x\,y^2\,z \in L$

$$xy^2z = \overbrace{\underbrace{a...a}_{x}\underbrace{a...a}_{y}\underbrace{a...a}_{y}a...a}^{m+k}\overbrace{b...b}^{m} \in L$$

Thus: $a^{m+k}b^m \in L$

$$a^{m+k}b^{m} \in L \qquad k \geq 1$$

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**BUT:** $$L = \{a^n b^n : n \geq 0\}$$

$$\Downarrow$$

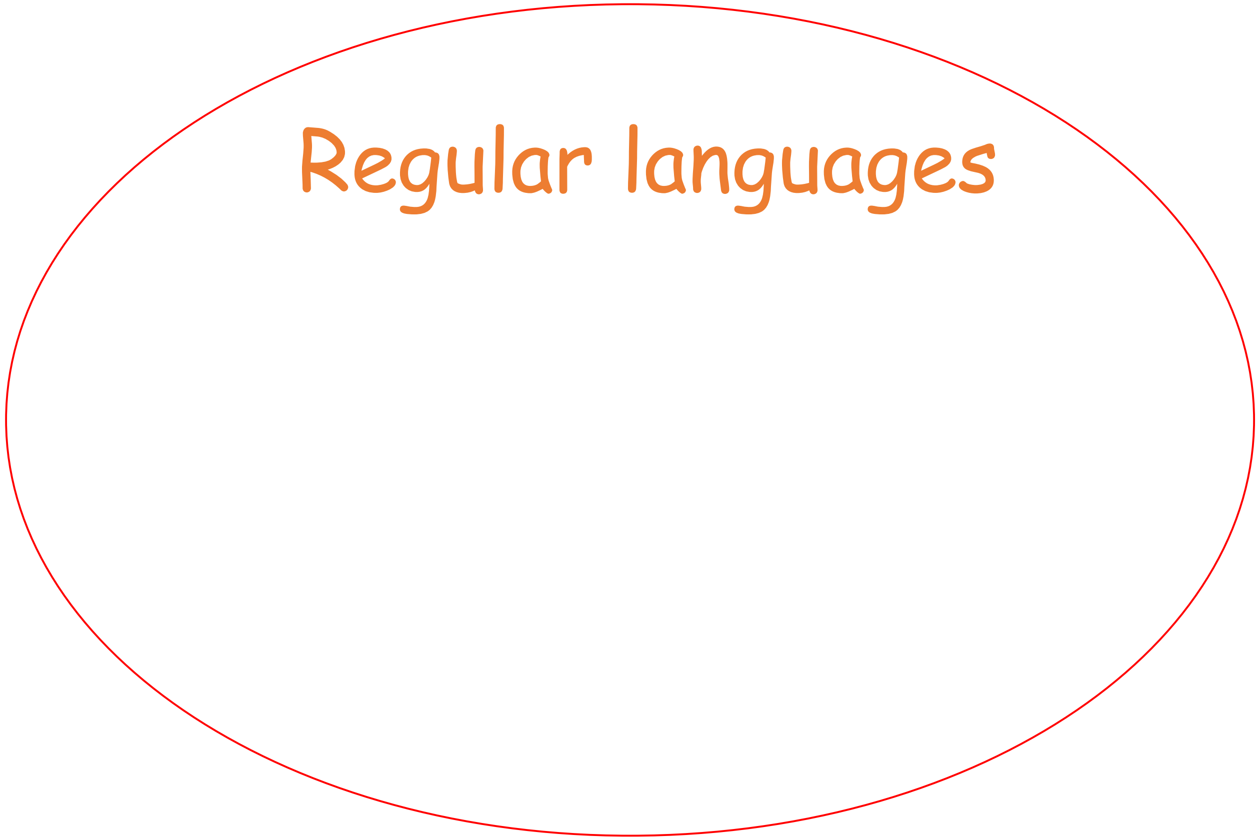
$$a^{m+k}b^{m} \notin L$$

**CONTRADICTION!!!**

Therefore: Our assumption that $L$ is a regular language is not true

**Conclusion:** $L$ is not a regular language

Non-regular languages $\{a^n b^n : \quad n \geq 0\}$



# THANK YOU

(DEEMED TO BE UNIVERSITY)

## Team – TOC