Advanced Algorithms & Data Structures

Department of CSE

ADVANCED ALGORITHMS AND DATA STRUCTURES
23CS03HF

Topic:

Single Source Shortest Path Problem

Session - 23

AIM OF THE SESSION

To familiarize students with the concept of Single Source Shortest Path Problem

INSTRUCTIONAL OBJECTIVES

This Session is designed to:

1.Demonstrate :- Single Source Shortest Path Problem.

2.Describe :- solve single-source shortest path problem using Dijkstra's Algorithm .

LEARNING OUTCOMES

At the end of this session, you should be able to:

1. Define :- Single Source Shortest path Problem.
2. Describe :- solve single-source shortest path problem using Dijkstra's Algorithm
3. Summarize:- Finding the shortest paths from a single source vertex to all other vertices in a weighted graph.

Shortest Path Problem

- Shortest path problem is a problem of finding the shortest path(s) between vertices of a given graph.
- Shortest path between two vertices is a path that has the least cost as compared to all other existing paths.

Applications

- Google Maps
- Road Networks
- Logistics Research

Types of Shortest Path Problem-

Shortest Path Problems

- Single-pair Shortest Path Problem
- Single-source Shortest Path Problem
- Single-destination Shortest Path Problem
- All pairs Shortest Path Problem

Single Source Shprtest Path Problem

- It is a shortest path problem where the shortest path from a given source vertex to all other remaining vertices is computed.
- **Dijkstra's Algorithm** and Bellman Ford Algorithm are the famous algorithms used for solving single-source shortest path problem.

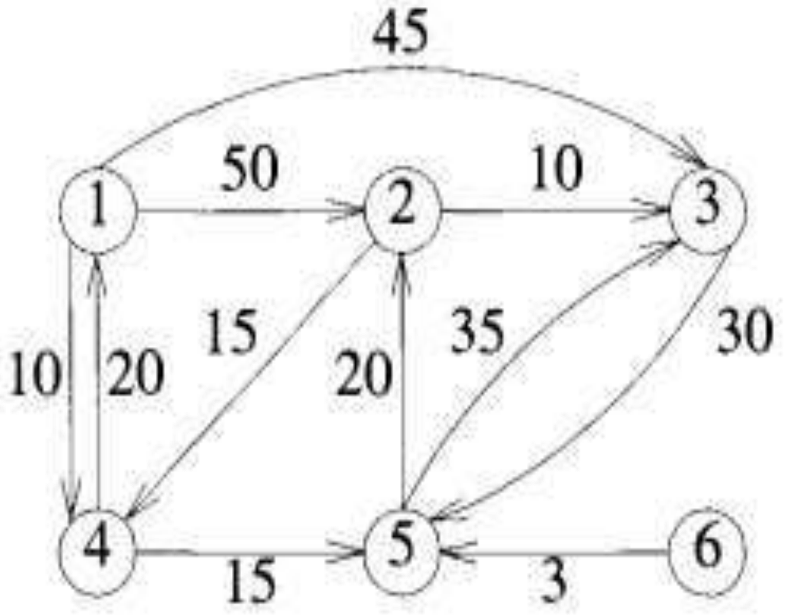
Dijkstra Algorithm

- Dijkstra Algorithm is a very famous greedy algorithm.
- It is used for solving the single source shortest path problem.
- It computes the shortest path from one particular source node to all other remaining nodes of the graph.

Conditions

It is important to note the following points regarding Dijkstra Algorithm-

- Dijkstra algorithm works only for **connected graphs**.
- Dijkstra algorithm works only for those graphs that **do not contain any negative weight edge**.
- The actual Dijkstra algorithm **does not output the shortest paths**.
- It only provides the **value or cost of the shortest paths**.
- Dijkstra algorithm works for **directed as well as undirected graphs**.



(a) Graph

	Path	*Length*
1)	1, 4	10
2)	1, 4, 5	25
3)	1, 4, 5, 2	45
4)	1, 3	45

(b) Shortest paths from 1

Implementation

Source vertex	intermediate Vertex	dist[2]	dist[3]	dist[4]	dist[5]	dist[6]	Path	Distance
1	–	50	45	10	∞	∞	1→4	10
	4	50	45	–	25	∞	1→4→5	25
	5	45	45	–	–	∞	1→4→5→2	45
	2	–	45	–	–	∞	1→3	45
	3	–	–	–	–	∞	No path to 6	

```
Algorithm ShortestPaths(v, cost, dist, n)
// dist[j], 1 ≤ j ≤ n, is set to the length of the shortest
// path from vertex v to vertex j in a digraph G with n
// vertices. dist[v] is set to zero. G is represented by its
// cost adjacency matrix cost[1 : n, 1 : n].
{
    for i := 1 to n do
    { // Initialize S.
        S[i] := false; dist[i] := cost[v, i];
    }
    S[v] := true; dist[v] := 0.0; // Put v in S.
    for num := 2 to n − 1 do
    {
        // Determine n − 1 paths from v.
        Choose u from among those vertices not
        in S such that dist[u] is minimum;
        S[u] := true; // Put u in S.
        for (each w adjacent to u with S[w] = false) do
            // Update distances.
            if (dist[w] > dist[u] + cost[u, w])) then
                    dist[w] := dist[u] + cost[u, w];
    }
}
```

Time Complexity Analysis-

- The given graph G is represented as an adjacency matrix.
- Priority queue Q is represented as an unordered list.

Here,

- A[i,j] stores the information about edge (i,j).
- Time taken for selecting i with the smallest dist is O(V).
- For each neighbor of i, time taken for updating dist[j] is O(1) and there will be maximum V neighbors.
- Time taken for each iteration of the loop is O(V) and one vertex is deleted from Q.
- Thus, total time complexity becomes $O(V^2)$.

SUMMARY

- **Objective**: Find the shortest paths from a single source vertex to all other vertices in a weighted graph.

- **Approach**: Initialize distances from the source to all vertices as infinity, update distances using relaxation to minimize path weights, and repeat until all shortest paths are found.

- **Algorithmic Solutions**: Algorithms like Dijkstra's (for non-negative weights) used to solve this problem efficiently.

SELF-ASSESSMENT QUESTIONS

Which of the following conditions must be true for Dijkstra’s algorithm to work correctly?

(a) The graph must be directed.

(b) The graph must be undirected.

(c) The graph must have non-negative edge weights.

(d) The graph must have no cycles.

What is the main application of single source shortest path algorithms?

(a) Finding Minimum Spanning Tree

(b) Finding shortest paths from a single source node to all other nodes

(c) Detecting cycles in a graph

(d) Sorting nodes in topological order

TERMINAL QUESTIONS

1. Can you explain the basic architecture of Dijkstra’s algorithm along with an example?
2. What are some real-world scenarios where Dijkstra’s algorithm can be applied effectively?

REFERENCES FOR FURTHER LEARNING OF THE SESSION

Reference Books :

1. Introduction to Algorithms, Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein., 3rd, 2009, The MIT Press.
2 Algorithm Design Manual, Steven S. Skiena., 2nd, 2008, Springer.
3 Data Structures and Algorithms in Python, Michael T. Goodrich, Roberto Tamassia, and Michael H. Goldwasser., 2nd, 2013, Wiley.
4 The Art of Computer Programming, Donald E. Knuth, 3rd, 1997, Addison-Wesley Professiona.

MOOCS :

1. https://www.coursera.org/specializations/algorithms?=
2.https://www.coursera.org/learn/dynamic-programming-greedy-algorithms#modules

THANK YOU