TUTORIAL-8

1.

Solution:

Given:

- Manufacturer's claimed mean (μ_0) = 130°F
- Sample mean ($\bar{x}$) = 131.08°F
- Sample size (n) = 9
- Population standard deviation (σ) = 1.50°F
- Significance level (α) = 0.01

Hypotheses:

- **Null hypothesis (H_0):** $\mu = 130$ (The mean activation temperature is 130°F).
- **Alternative hypothesis (H_1):** $\mu \neq 130$ (The mean activation temperature is not 130°F).

Test Statistic:

Since the population standard deviation is known, we use the **z-test**. The formula for the z-test is:

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

Substitute the values:

$$z = \frac{131.08 - 130}{\frac{1.50}{\sqrt{9}}} = \frac{1.08}{\frac{1.50}{3}} = \frac{1.08}{0.5} = 2.16$$

Critical Value:

For $\alpha = 0.01$, the critical z-value for a two-tailed test is **±2.576** (from the standard normal distribution).

Decision:

- If the calculated z-value is greater than 2.576 or less than -2.576, reject H_0.
- Here, $z = 2.16$, which is **less** than 2.576.

Conclusion:

Since the z-value (2.16) does **not** exceed the critical value (2.576), we **fail to reject the null hypothesis**. The data does **not** contradict the manufacturer's claim at the 0.01 significance level.

2.

a) Hypothesis Test

Given:

- Sample mean ($\bar{x}$) = 1798.4 tons
- Sample variance (s^2) = 671,330.9 tons
- Sample size (n) = 30
- Significance level (α) = 0.05

We are testing if the mean salt usage is less than 2000 tons.

Hypotheses:

- **Null hypothesis (H_0):** $\mu = 2000$ tons (mean salt usage is 2000 tons).
- **Alternative hypothesis (H_1):** $\mu < 2000$ tons (mean salt usage is less than 2000 tons).

Test Statistic:

We use the t-test since the population variance is unknown. The formula is:

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

Substitute the values:

$$t = \frac{1798.4 - 2000}{\frac{\sqrt{671,330.9}}{\sqrt{30}}} = \frac{-201.6}{\frac{819.46}{5.477}} = \frac{-201.6}{149.6} = -1.35$$

Critical Value:

For $\alpha = 0.05$ and $df = 29$, the critical t-value from the t-distribution is approximately **-1.699** (one-tailed).

Decision:

- If t-value is less than -1.699, reject H_0.
- Here, $t = -1.35$, which is **greater** than -1.699.

Conclusion:

Since $t = -1.35$ does not exceed the critical value of -1.699, we **fail to reject** the null hypothesis. There is **no significant evidence** that the mean salt usage is less than 2000 tons at the 0.05 significance level.

b) Error Explanation

- **Type II error:** We **fail to reject H_0** when the true mean is actually less than 2000 tons. In this case, we might incorrectly conclude that the mean salt usage is not less than 2000 tons when it actually is, leading to the potential for overestimating the required salt for future snowstorms.

3.

i) Hypothesis Test

Given:

- Data: 123, 106, 114, 128, 113, 109, 120, 102, 111
- $n = 9$
- Significance level $\alpha = 0.05$
- Null hypothesis: $H_0 : \mu = 107$
- Alternative hypothesis: $H_1 : \mu \neq 107$ (two-tailed test)

Step 1: Calculate sample mean and sample standard deviation

- Sample mean ($\bar{x}$):

$$\bar{x} = \frac{123 + 106 + 114 + 128 + 113 + 109 + 120 + 102 + 111}{9} = \frac{1126}{9} = 125.11$$

- Sample standard deviation (s):

First, calculate the squared differences from the mean:

$$(123 - 125.11)^2 = 4.45, (106 - 125.11)^2 = 361.61, (114 - 125.11)^2 = 123.61, \ldots$$

The sum of squared differences is 1172.22. Now calculate the standard deviation:

$$s = \sqrt{\frac{1172.22}{8}} = \sqrt{146.53} = 12.12$$

Step 2: Calculate the t-statistic

The formula for the t-statistic is:

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

Substitute the values:

$$t = \frac{125.11 - 107}{\frac{12.12}{\sqrt{9}}} = \frac{18.11}{\frac{12.12}{3}} = \frac{18.11}{4.04} = 4.48$$

Step 3: Find the critical t-value

For $\alpha = 0.05$ and $df = 8$, the critical t-value for a two-tailed test is approximately ±2.306.

Step 4: Decision

Since $t = 4.48$ is greater than 2.306, we reject the null hypothesis.

Conclusion:

There is significant evidence at the 0.05 level to conclude that the mean key performance indicator is different from 107.

ii) Error Explanation

- **Type I error:** We reject the null hypothesis when it is actually true. In this case, we might incorrectly conclude that the mean key performance indicator is different from 107 when it is actually 107. This could lead to unnecessary changes or adjustments based on incorrect conclusioNS.

4.

Hypothesis Test

Given:

- Null hypothesis (H_0): $\mu = 8$ kilograms (mean breaking strength is 8 kg)
- Alternative hypothesis (H_1): $\mu \neq 8$ kilograms (mean breaking strength is not 8 kg)
- Sample mean ($\bar{x}$) = 7.8 kg
- Population standard deviation (σ) = 0.5 kg
- Sample size (n) = 50
- Significance level (α) = 0.01

Step 1: Test Statistic

Since the population standard deviation is known, we use the **z-test**. The formula is:

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

Substitute the values:

$$z = \frac{7.8 - 8}{\frac{0.5}{\sqrt{50}}} = \frac{-0.2}{0.0707} = -2.83$$

Step 2: Critical Value

For a two-tailed test at $\alpha = 0.01$, the critical z-value is **±2.576**.

Step 3: Decision

- If $|z| > 2.576$, reject H_0.
- Here, $|z| = 2.83$, which is **greater** than 2.576.

Conclusion:

Since $|z| = 2.83 > 2.576$, we **reject the null hypothesis**. There is **sufficient evidence** at the 0.01 significance level to conclude that the mean breaking strength is not 8 kilograms.

5.

Solution:

Given:

- Sample mean ($\bar{x}$) = 26.8
- Sample standard deviation (s) = 6.5
- Sample size (n) = 100
- Hypothesized population mean (μ_0) = 28
- Significance level (α) = 0.05

Hypotheses:

- **Null hypothesis (H_0):** $\mu = 28$ (The population mean is 28)
- **Alternative hypothesis (H_1):** $\mu \neq 28$ (The population mean is not 28)

Step 1: Calculate the test statistic (t-statistic)

Since the population standard deviation is unknown, we use the **t-test**:

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

Substitute the values:

$$t = \frac{26.8 - 28}{\frac{6.5}{\sqrt{100}}} = \frac{-1.2}{0.65} = -1.846$$

Step 2: Determine the p-value

For a two-tailed test with 99 degrees of freedom ($n - 1 = 100 - 1 = 99$), we find the p-value for $t = -1.846$.

Using a t-distribution table or calculator, the p-value is approximately **0.068**.

Step 3: Compare p-value with α

- p-value = 0.068
- Significance level α = 0.05

Since the **p-value (0.068) is greater than $\alpha = 0.05$, we fail to reject** the null hypothesis.

Conclusion:

There is **insufficient evidence** at the 0.05 significance level to conclude that the population mean is different from 28.

6.

Solution:

Given:

- $n_1 = 40$ (Sample size for first intersection)
- $\bar{x}_1 = 247.3$ (Sample mean for first intersection)
- $s_1 = 15.2$ (Sample standard deviation for first intersection)
- $n_2 = 30$ (Sample size for second intersection)
- $\bar{x}_2 = 254.1$ (Sample mean for second intersection)
- $s_2 = 18.7$ (Sample standard deviation for second intersection)
- Significance level (α) = 0.01

Hypotheses:

- **Null hypothesis (H_0):** $\mu_1 - \mu_2 = 0$ (No difference in the mean number of left turns at both intersections)
- **Alternative hypothesis (H_1):** $\mu_1 - \mu_2 \neq 0$ (There is a difference in the mean number of left turns)

Step 1: Calculate the test statistic

The formula for the two-sample t-test is:

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Substitute the values:

$$t = \frac{247.3 - 254.1}{\sqrt{\frac{15.2^2}{40} + \frac{18.7^2}{30}}}$$

$$t = \frac{-6.8}{\sqrt{\frac{231.04}{40} + \frac{349.69}{30}}}$$

$$t = \frac{-6.8}{\sqrt{5.776 + 11.657}} = \frac{-6.8}{\sqrt{17.433}} = \frac{-6.8}{4.17} = -1.63$$

Step 2: Find the degrees of freedom

We approximate the degrees of freedom using the formula:

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$$

Using the values:

$$df \approx \frac{(5.776 + 11.657)^2}{\frac{(5.776)^2}{39} + \frac{(11.657)^2}{29}} = \frac{17.433^2}{\frac{33.37}{39} + \frac{135.86}{29}} \approx 62.9 \approx 63$$

Step 3: Determine the critical t-value

For $\alpha = 0.01$ and $df = 63$, the critical t-value for a two-tailed test is approximately **±2.660** (from the t-distribution table).

Step 4: Decision

- If $|t| > 2.660$, reject H_0.
- Here, $|t| = 1.63$, which is **less** than 2.660.

Conclusion:

Since $|t| = 1.63$ is less than 2.660, we **fail to reject** the null hypothesis. There is **insufficient evidence** at the 0.01 significance level to conclude that there is a difference in the mean number of left turns at the two intersections.

7.

Solution:

Given:

- **Process 1:** Sample size $n_1 = 250$, sample mean $\bar{x}_1 = 120$, standard deviation $s_1 = 12$
- **Process 2:** Sample size $n_2 = 400$, sample mean $\bar{x}_2 = 124$, standard deviation $s_2 = 14$

Step 1: Standard Error of the Difference Between the Two Sample Means

The formula for the standard error of the difference is:

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Substitute the values:

$$SE = \sqrt{\frac{12^2}{250} + \frac{14^2}{400}} = \sqrt{\frac{144}{250} + \frac{196}{400}} = \sqrt{0.576 + 0.49} = \sqrt{1.066} = 1.033$$

Step 2: Test for Significance

We will test if the difference between the means is significant. The difference in sample means is:

$$\bar{x}_1 - \bar{x}_2 = 120 - 124 = -4$$

Now calculate the **z-test** for the difference:

$$z = \frac{\text{Difference in means}}{SE} = \frac{-4}{1.033} = -3.87$$

For a significance level of $\alpha = 0.05$, the critical z-value for a two-tailed test is ± 1.96.

Since $|z| = 3.87$ is **greater** than 1.96, the difference is **significant.**

Step 3: Confidence Limits

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99% Confidence Interval for the Difference in Means:

For a 99% confidence level, the critical z-value is **2.576**.

The confidence interval is calculated as:

$$(\bar{x}_1 - \bar{x}_2) \pm (z_{\alpha/2} \times SE)$$

Substitute the values:

$$-4 \pm (2.576 \times 1.033) = -4 \pm 2.664$$

So the 99% confidence interval is:

$$(-4 - 2.664, -4 + 2.664) = (-6.664, -1.336)$$

95% Confidence Interval for the Difference in Means:

For a 95% confidence level, the critical z-value is **1.96**.

The confidence interval is:

$$-4 \pm (1.96 \times 1.033) = -4 \pm 2.026$$

So the 95% confidence interval is:

$$(-4 - 2.026, -4 + 2.026) = (-6.026, -1.974)$$

Conclusion:

- The difference in the average weights between the two processes is **significant**.
- The 99% confidence interval for the difference is **(-6.664, -1.336)**.
- The 95% confidence interval for the difference is **(-6.026, -1.974)**.

8.

Solution:

Given:

- **Brand A:** Sample size $n_1 = 12$, sample mean $\bar{x}_1 = 37,900$ km, standard deviation $s_1 = 5,100$ km
- **Brand B:** Sample size $n_2 = 12$, sample mean $\bar{x}_2 = 39,800$ km, standard deviation $s_2 = 5,900$ km
- Assume equal variances.

Hypotheses:

- **Null hypothesis (H_0):** $\mu_1 - \mu_2 = 0$ (No difference in average wear between Brand A and Brand B)
- **Alternative hypothesis (H_1):** $\mu_1 - \mu_2 \neq 0$ (There is a difference in average wear)

Step 1: Pooled Variance

Since the population variances are assumed equal, we compute the **pooled variance**:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

Substitute the values:

$$s_p^2 = \frac{(12-1)(5,100^2) + (12-1)(5,900^2)}{12 + 12 - 2}$$

$$s_p^2 = \frac{11 \times 26,010,000 + 11 \times 34,810,000}{22} = \frac{286,110,000 + 382,910,000}{22} = \frac{669,020,000}{22} = 30,387,273$$

Now calculate the **pooled standard deviation:**

$$s_p = \sqrt{30,387,273} \approx 5,514.47$$

Step 2: Test Statistic

The formula for the t-statistic is:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Substitute the values:

$$t = \frac{37,900 - 39,800}{5,514.47 \times \sqrt{\frac{1}{12} + \frac{1}{12}}}$$

$$t = \frac{-1,900}{5,514.47 \times \sqrt{\frac{2}{12}}} = \frac{-1,900}{5,514.47 \times 0.577} = \frac{-1,900}{3,183.66} \approx -0.597$$

Step 3: Degrees of Freedom

The degrees of freedom for the t-test is:

$$df = n_1 + n_2 - 2 = 12 + 12 - 2 = 22$$

Step 4: P-value

Using a t-distribution table or calculator, find the p-value for $t = -0.597$ with $df = 22$.

The **two-tailed p-value** is approximately **0.559**.

Step 5: Conclusion

Since the **p-value (0.559)** is greater than the significance level ($\alpha = 0.05$), we **fail to reject** the null hypothesis.

Final Conclusion:

There is **no significant difference** in the average wear between Brand A and Brand B tires.

VIVA:

1. **t-test**: A statistical test used to compare the means of two groups or to compare a sample mean with a known population mean. It is used when the sample size is small (usually n<30n < 30) or the population standard deviation is unknown.
2. **p-value**: The probability of observing the test results, or something more extreme, if the null hypothesis is true. A small p-value (typically < 0.05) suggests strong evidence against the null hypothesis.
3. **Large vs. Small Samples**:
   - **Large sample**: Typically n≥30n \geq 30, where the Central Limit Theorem applies, and normal distribution assumptions can be made.
   - **Small sample**: Typically n<30n < 30, where the sample may not approximate a normal distribution, and t-tests are used instead of z-tests.
4. **Standard error of the mean**: A measure of how much the sample mean deviates from the true population mean. It is calculated as sn\frac{s}{\sqrt{n}}, where ss is the sample standard deviation and nn is the sample size.