23MT2014

THEORY OF COMPUTATION

Topic:

PROPERTIES OF REGULAR LANGUAGES

Session - 9

AIM OF THE SESSION

To gain an understanding of the fundamental characteristics and limitations of regular languages. This knowledge is essential for analyzing and manipulating regular languages in various computational contexts, including pattern matching, parsing, and formal language processing.

INSTRUCTIONAL OBJECTIVES

This Session is designed to:

1. To introduce students to key properties of regular languages, such as closure properties, determinism, and regular operations (union, concatenation, and Kleene closure).
2. To enable students to identify and prove properties of regular languages using formal methods, including regular expressions, finite automata, and regular grammars.

LEARNING OUTCOMES

At the end of this session, you should be able to:

1. By the end of the instruction, students will be able to describe and apply closure properties of regular languages, such as the closure under union, concatenation, and Kleene closure, to generate new regular languages.
2. Students will be able to analyze regular languages using formal methods, including regular expressions, finite automata, and regular grammars, to determine and prove properties of regular languages.

For regular languages L_1 and L_2 we will prove that:

Union: $L_1 \cup L_2$

Concatenation: $L_1 L_2$

Star: $L_1{}^*$

Reversal: $L_1{}^R$

Complement: $\overline{L_1}$

Intersection: $L_1 \cap L_2$

Are regular Languages

We say: Regular languages are **closed under**

Union: $L_1 \cup L_2$

Concatenation: $L_1 L_2$

Star: $L_1{}^*$

Reversal: $L_1{}^R$

Complement: $\overline{L_1}$

Intersection: $L_1 \cap L_2$

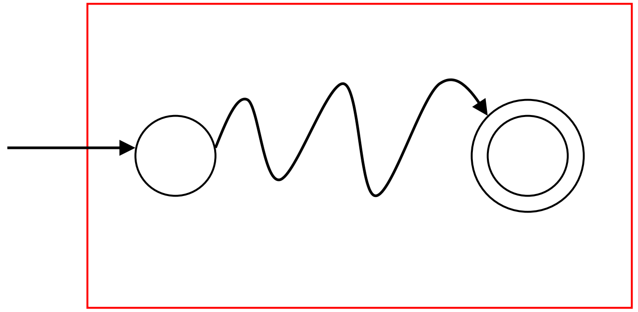
Regular language L_1

Regular language L_2

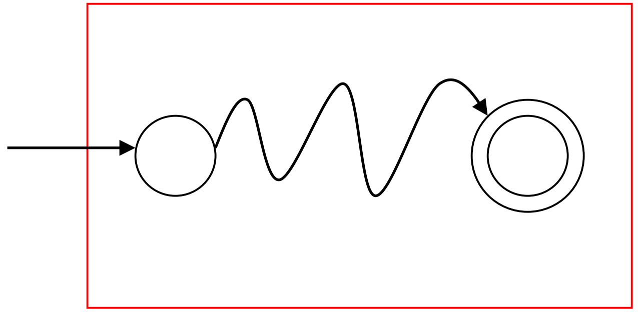
$$L(M_1) = L_1$$

$$L(M_2) = L_2$$

NFA M_1



NFA M_2



Single final state

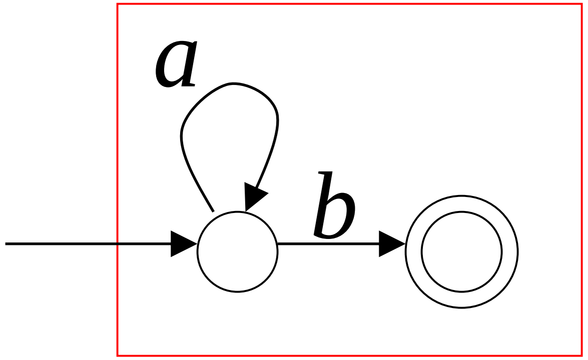
Single final state

Example

$n \geq 0$

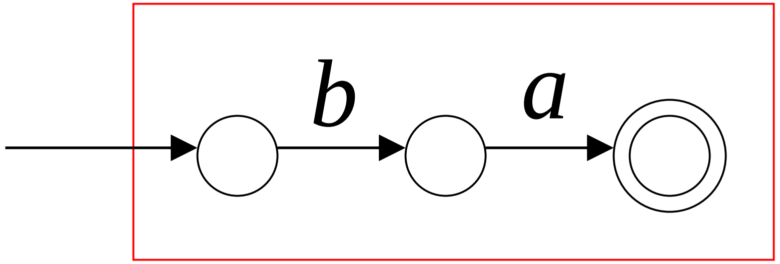
$L_1 = \{a^n b\}$

M_1



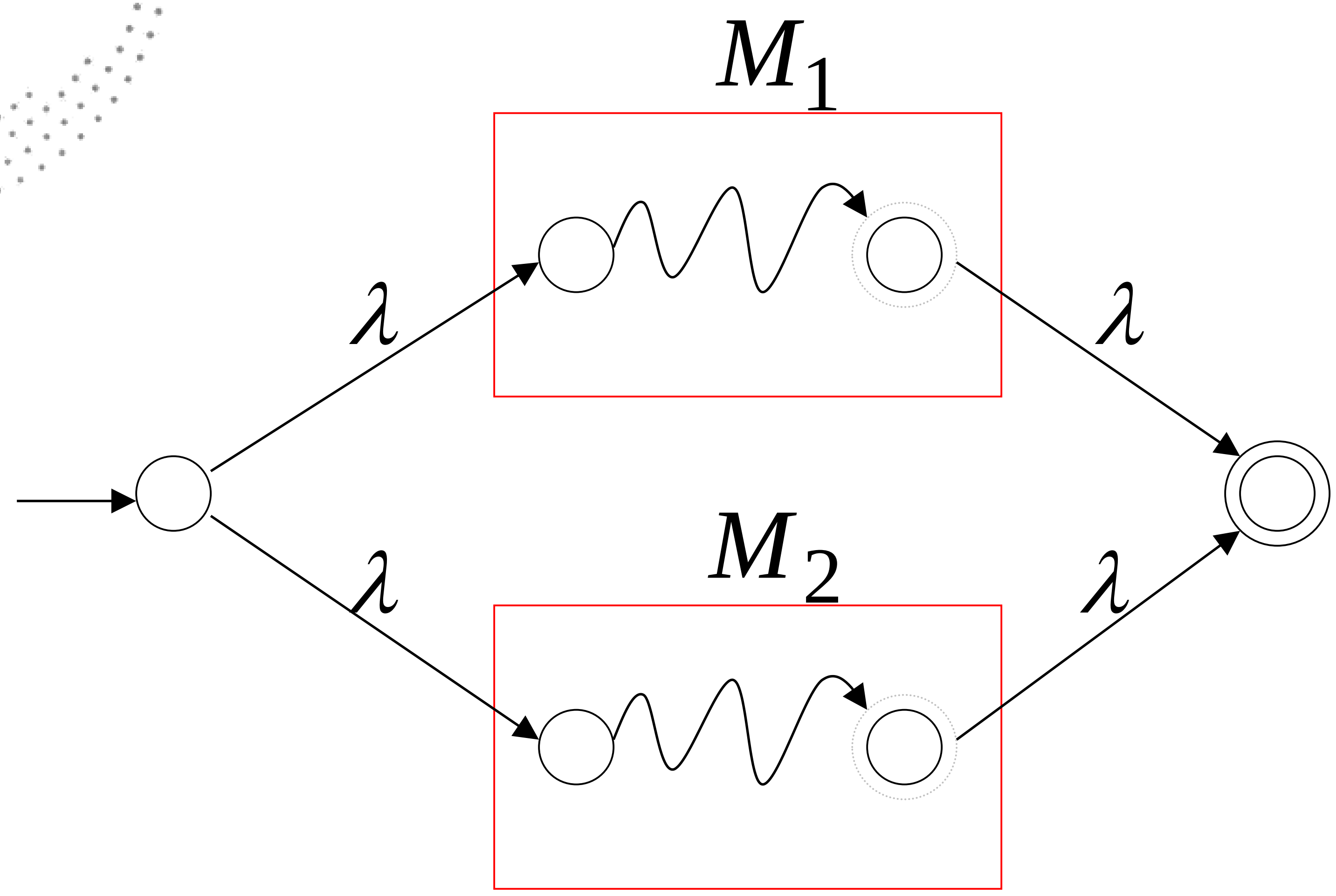
$L_2 = \{ba\}$

M_2



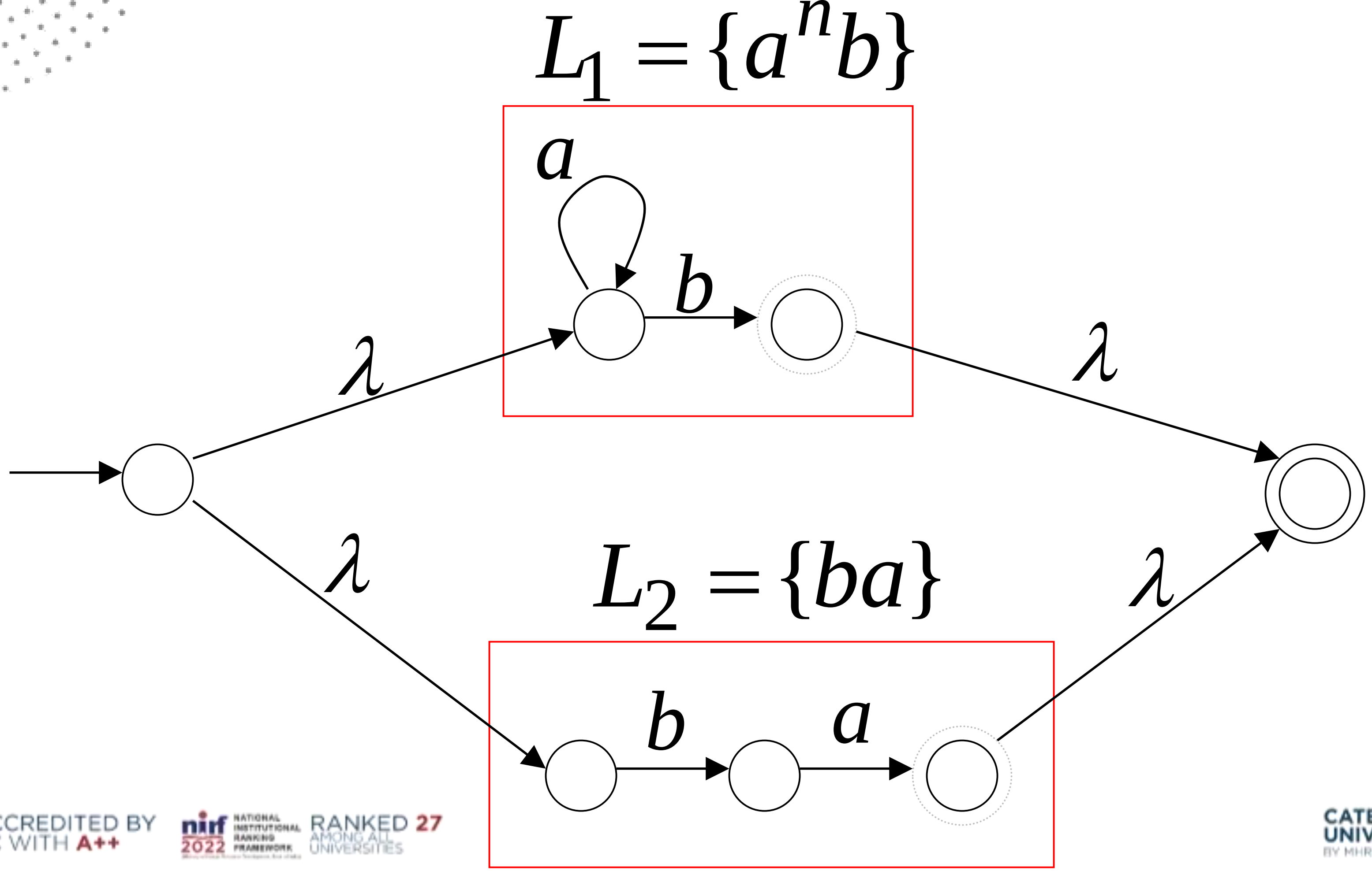
Union

- NFA for $L_1 \cup L_2$



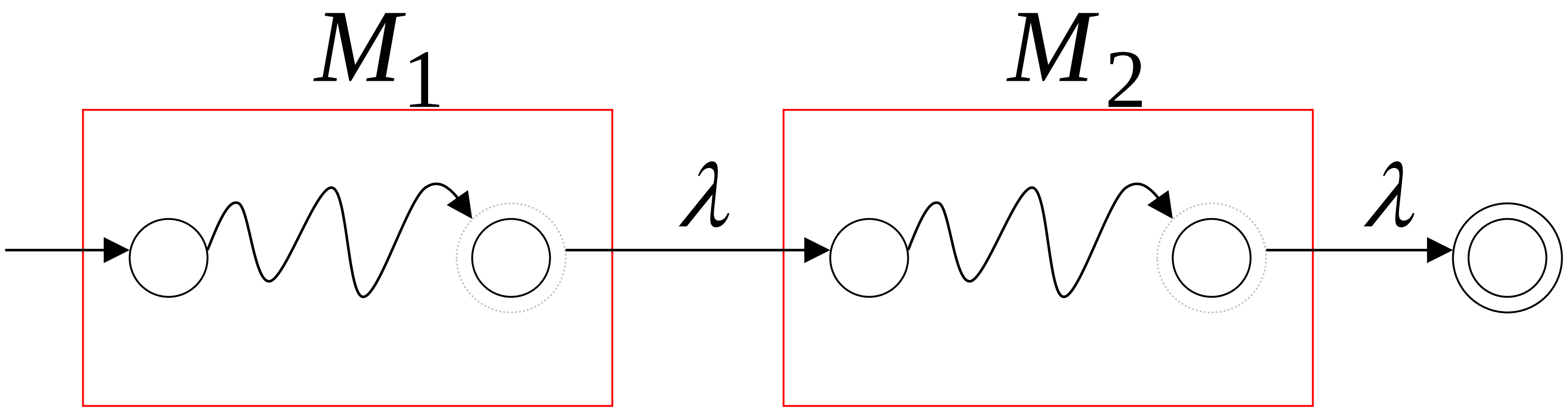
Example

- NFA for $L_1 \cup L_2 = \{a^n b\} \cup \{ba\}$



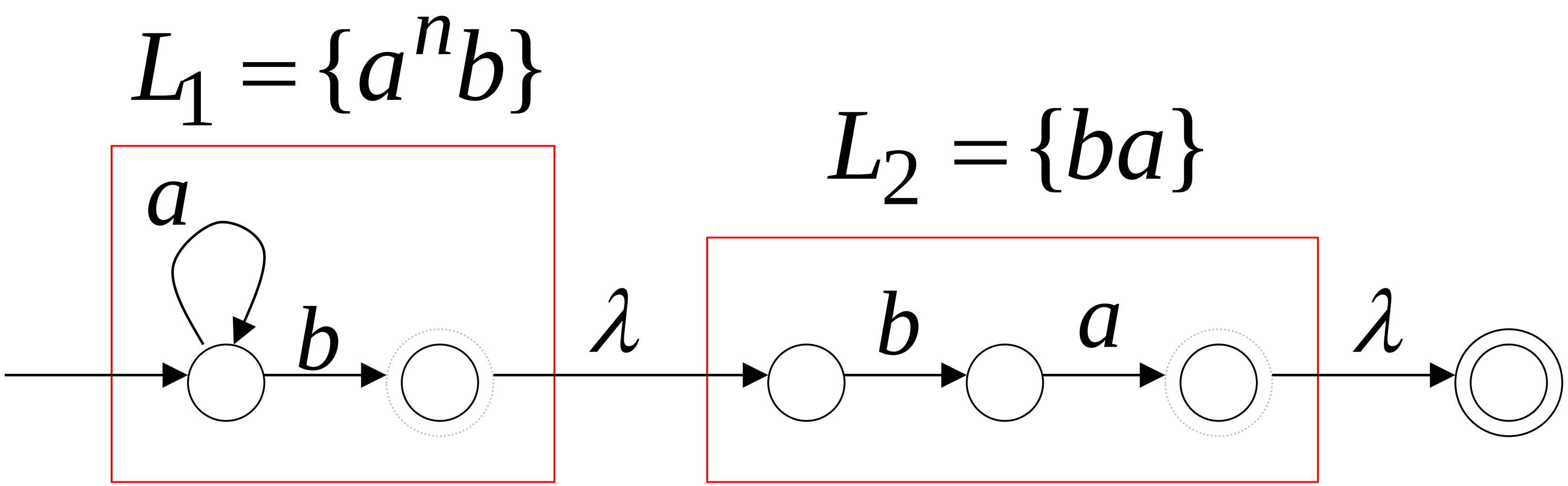
Concatenation

- NFA for $L_1 L_2$



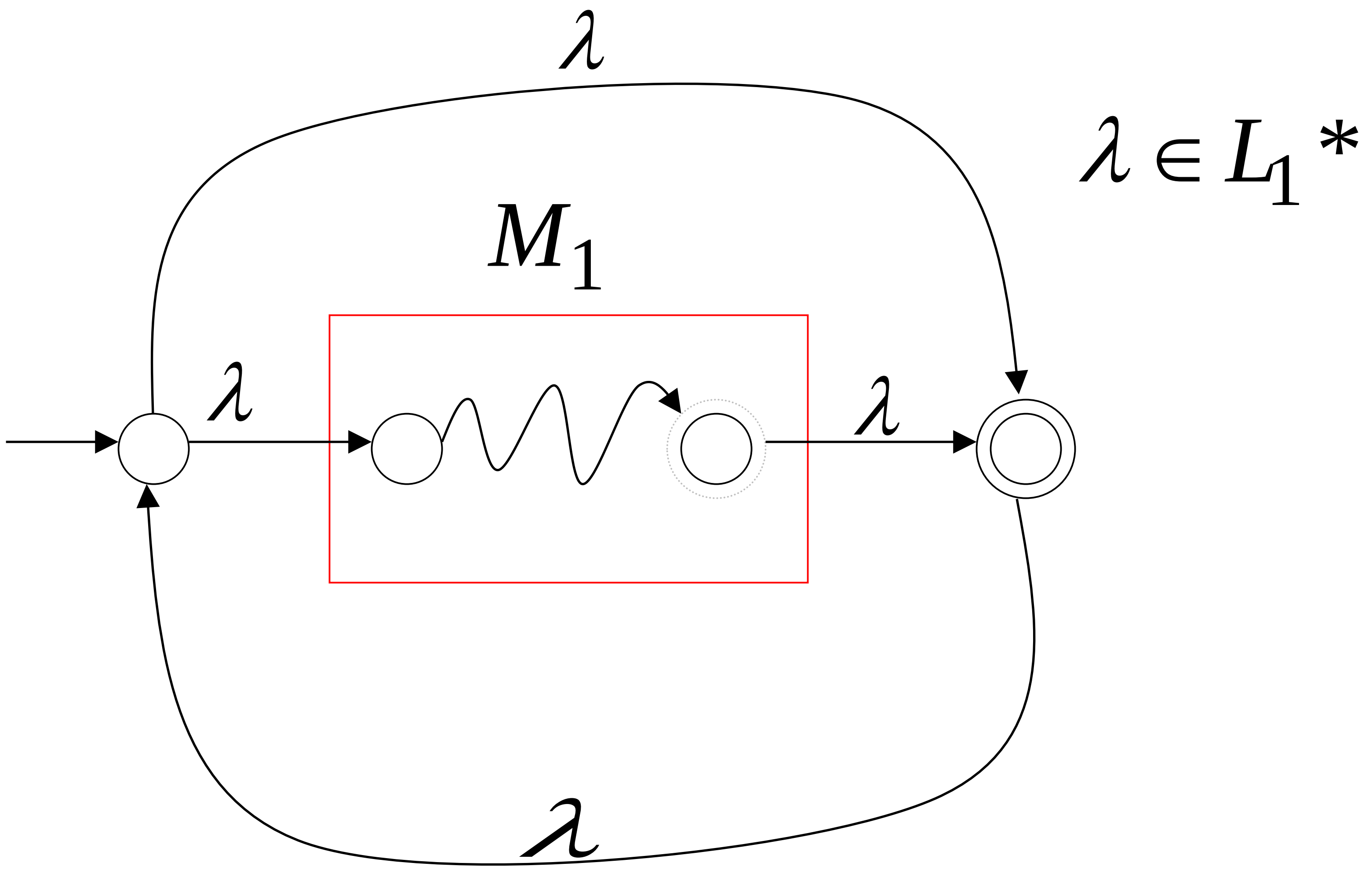
Example

-
- NFA for $L_1 L_2 = \{a^n b\}\{ba\} = \{a^n bba\}$



Star Operation

- NFA for $L_1{}^*$

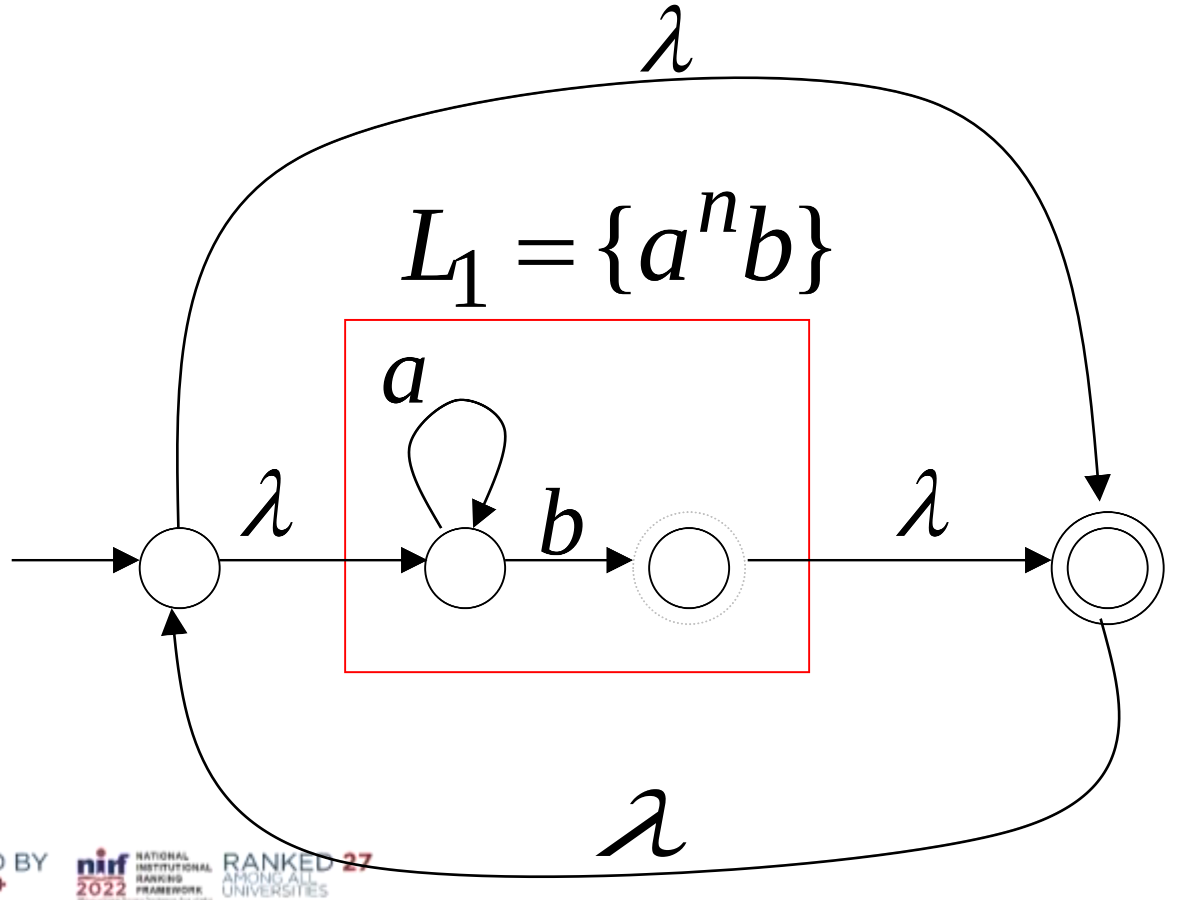


Example

- NFA for $L_1{}^* = \{a^n b\}^*$

$$w = w_1 w_2 \cdots w_k$$

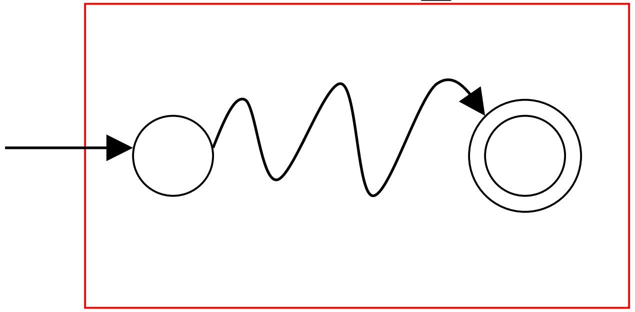
$$w_i \in L_1$$



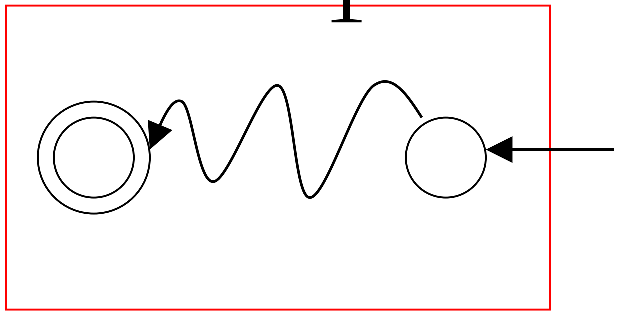
Reverse

NFA for L_1^R

L_1 M_1



M_1'



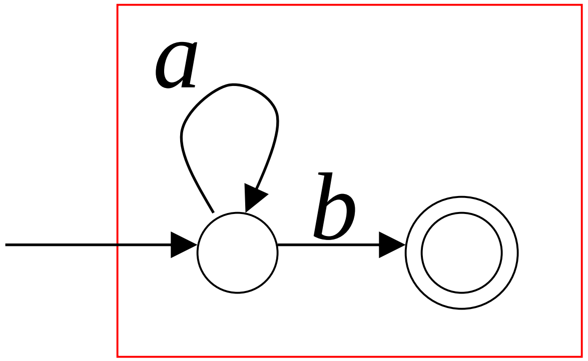
1. Reverse all transitions

2. Make initial state final state and vice versa

Example

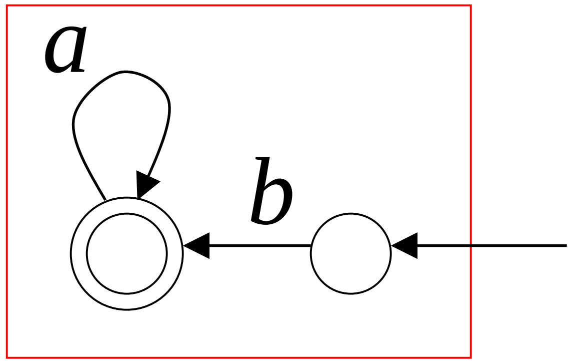
$$M_1$$

$$L_1 = \{a^n b\}$$

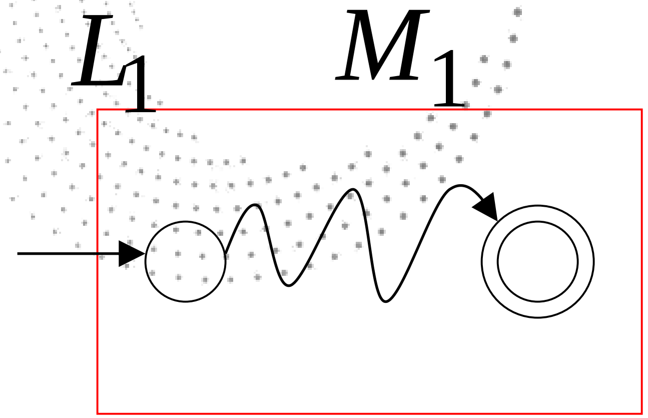


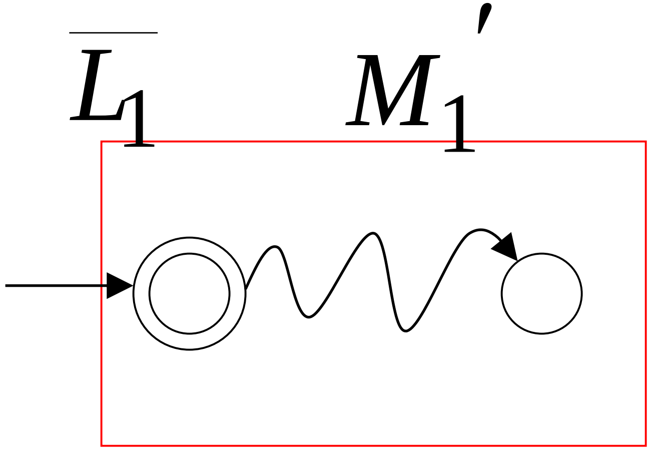
$$M_1'$$

$$L_1^R = \{ba^n\}$$



Complement





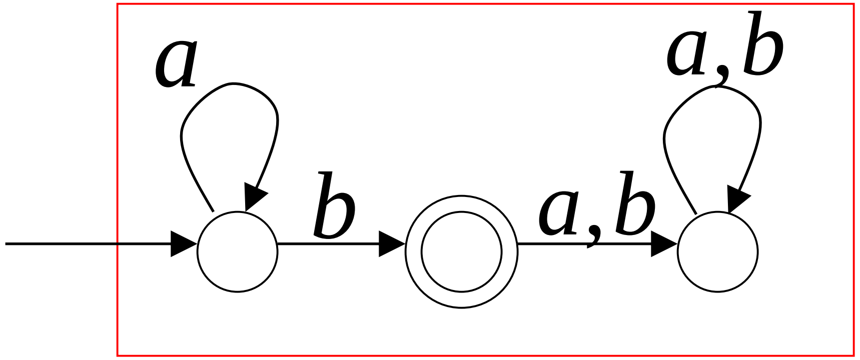
1. Take the **DFA** that accepts L_1

2. Make final states non-final,
   and vice-versa

Example

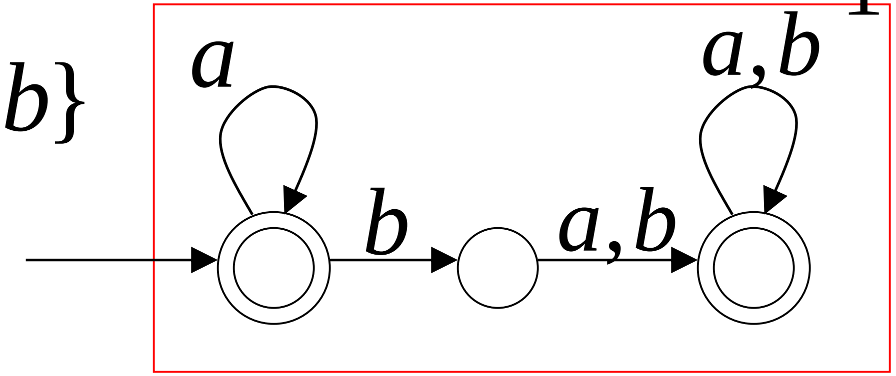
$$L_1 = \{a^n b\}$$

M_1



$$\overline{L_1} = \{a,b\}^* - \{a^n b\}$$

M_1'



Intersection

DeMorgan's Law: $$L_1 \cap L_2 = \overline{\overline{L_1} \cup \overline{L_2}}$$

L_1 , L_2 regular

$\Rightarrow$ $\overline{L_1}$, $\overline{L_2}$ regular

$\Rightarrow$ $\overline{L_1} \cup \overline{L_2}$ regular

$\Rightarrow$ $\overline{\overline{L_1} \cup \overline{L_2}}$ regular

$\Rightarrow$ $L_1 \cap L_2$ regular

Example

$L_1 = \{a^n b\}$ regular

$L_2 = \{ab, ba\}$ regular

$\Rightarrow$ $L_1 \cap L_2 = \{ab\}$

regular

QUIZ TIME

Which of the following properties is true for the intersection of two regular languages?

a) The intersection of two regular languages is always regular.

b) The intersection of two regular languages is always non-regular.

c) The intersection of two regular languages can be regular or non-regular.

d) The intersection of two regular languages cannot be determined.

Answer: a) The intersection of two regular languages is always regular.

QUIZ TIME

Which of the following operations is closed under regular languages?

a) Union

b) Intersection

c) Complementation

d) Concatenation

Answer: d) Concatenation

QUIZ TIME

Which of the following properties is true for the complement of a regular language?

a) The complement of a regular language is always regular.

b) The complement of a regular language is always non-regular.

c) The complement of a regular language can be regular or non-regular.

d) The complement of a regular language cannot be determined.

Answer: a) The complement of a regular language is always regular.

QUIZ TIME

Which of the following properties is true for the union of two regular languages?

a) The union of two regular languages is always regular.

b) The union of two regular languages is always non-regular.

c) The union of two regular languages can be regular or non-regular.

d) The union of two regular languages cannot be determined.

Answer: a) The union of two regular languages is always regular.

QUIZ TIME

Which of the following properties is true for the Kleene closure (star) of a regular language?

a) The Kleene closure of a regular language is always regular.

b) The Kleene closure of a regular language is always non-regular.

c) The Kleene closure of a regular language can be regular or non-regular.

d) The Kleene closure of a regular language cannot be determined.

Answer: a) The Kleene closure of a regular language is always regular.

Terminal question

Explain the main idea behind the pumping lemma for regular languages.

How can the pumping lemma be used to prove that a language is not regular?

What are the main steps involved in using the pumping lemma to prove non-regularity?

(DEEMED TO BE UNIVERSITY)

THANK YOU

(DEEMED TO BE UNIVERSITY)

Team - TOC