Department of AI & DS
CSE and CS&IT

COURSE NAME: PROBABILITY, STATISTICS AND QUEUING THEORY

COURSE CODE: 23MT2005

Topic

Expected value of a Random Variable and Variance

Session - 4

AIM OF THE SESSION

To familiarize students with the mean and variance of a random variable.

INSTRUCTIONAL OBJECTIVES

This Session is designed

1. Discuss the concept of expected value of a random variable/mean
2. List out the rules of determining the mean and variance of discrete and continuous random variable

LEARNING OUTCOMES

At the end of this session, you should be able to:

1. Define probability and its axioms
2. Describe the different types of events.
3. Summarize the concept of probability with suitable example.

CONTENTS

- Expected value of a random variable
- Properties of a Random variable
- Variance of a random variable

Expected value of a Random Variable

Let X be a random variable with probability distribution f(x). The mean or expected value of X is

$$\mu = E(X) = \sum_x x f(x),$$ if X is discrete and

$$\mu = E(X) = \int_{-\infty}^{\infty} x f(x) dx$$ If X is continuous.

Note: If X is a random variable, a function of X, g(X) is also a random variable.

The expected value of the random variable g(X) is

$$\mu_{g(X)} = E(g(X)) = \sum_x g(x) f(x)$$ if X is discrete and

$$\mu_{g(X)} = \int_{-\infty}^{\infty} g(x) f(x) dx$$ If X is continuous.

Variance of Random Variable

Let X be a random variable with probability distribution f(x) with mean μ. The variance of X is

$$\sigma^2 = E\left[(X-\mu)^2\right] = \sum_x (X-\mu)^2 \ f(x),$$ if X is discrete, and

$$\sigma^2 = E\left[(X-\mu)^2\right] = \int_{-\infty}^{\infty} (X-\mu)^2 \ f(x)$$ if X is continuous.

Properties of a Random variable

1. E(c)=c
2. E(aX+b)=aE(X)+b
3. V(aX+b)=a^2V(X)
4. If X and Y are independent random variables, then V(aX+bY)=a^2V(X)+b^2V(Y)
5. $\sigma^2 = E(X^2) - \mu^2$

EXAMPLES

Example 1: The table below, adapted from a snapshot in India today, shows the probability distribution for x, the number of daily coffee breaks taken per day by coffee drinkers

x	0	1	2	3	4	5
p(x)	0.28	0.37	0.17	0.12	0.05	0.01

i) Find the probability that a randomly selected coffee drinker would take more than two coffee breaks during the day

ii) Calculate the mean for the random variable x.

SOLUTION: X : NUMBER OF DAILY COFFEE BREAKS TAKEN PER DAY BY COFFEE DRINKERS

i) P(Coffee drinkers would take more than two coffee breaks during the day)

$= P(x>2)=0.12+0.05+0.01=0.18$

ii)Mean$=\sum x\, f(x)$

$=0(0.28)+1(0.37)+2(0.17)+3(0.12)+4(0.05)+5(0.01)=1.32$

SUMMARY

In this session, determination of mean and variance of a random variable along with its properties have noted.

1. Expected Value of a Random variable
2. Variance of a Random variable
3. Properties of mean and variance.

SELF-ASSESSMENT QUESTIONS

If X is a random variable having its probability density function f(x), the E(x) is called:

a) Arithmetic mean
b) geometric mean
c) harmonic mean
d) first quartile

If X is a random variable and r is an integer, then $E(x^r)$ represents:

a) r^{th} central moment
b) r^{th} factorial moment
c) r^{th} raw moment
d) none of the above

TERMINAL QUESTIONS

1. Let X be random variable with following probability distribution:

X	-3	6	9
f(x)	1/6	1/2	1/3

Compute $\mu_{g(x)}$, where g(x)=$(2x+1)^2$.

2. The length of time Y, in minutes , required to generate a human reflex to tear gas has the density function

$$f(y) = \begin{cases} \frac{1}{4} e^{-\frac{y}{4}}, & 0 \leq y < \infty \\ 0, & \text{elsewhere} \end{cases}$$

i) Obtain the mean time to reflex

ii) Obtain the value of $E(Y^2)$ and $V(Y)$.

REFERENCES FOR FURTHER LEARNING OF THE SESSION

Reference Books:

1. Chapter 1 of TP1: William Feller, An Introduction to Probability Theory and Its Applications: Volume 1, Third Edition, 1968 by John Wiley & Sons,Inc.

2. Richard A Johnson, Miller& Freund’s Probability and statistics for Engineers, PHI, New Delhi, 11th Edition (2011).

Sites and Web links:

1. * https://ncert.nic.in/textbook.php?kemh1=16-16 *
2. Notes: sections 1 to 1.3 of http://www.statslab.cam.ac.uk/~rrw1/prob/prob-weber.pdf
3. https://ocw.mit.edu/courses/res - 6 -012 -introduction -to -probability - spring - 2018/91864c7642a58e216e8baa8fcb4a5cb5_MITRES_6_012S18_L01.pd f 9
4. https://www.probabilitycourse.com/chapter3/3_2_1_cdf.php
5. https://en.wikipedia.org/wiki/Cumulative_distribution_function

THANK YOU

(DEEMED TO BE UNIVERSITY)

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