Department of AI & DS
CSE and CS&IT

COURSE NAME: PROBABILITY, STATISTICS AND QUEUING THEORY

COURSE CODE: 23MT2005

Topic

Random Variables and their probability functions

Session - 3

AIM OF THE SESSION

To familiarize students with the rules of different probability distribution functions

INSTRUCTIONAL OBJECTIVES

This Session is designed

1. Demonstrate the concept of Random variables and its types
2. List out the rules of discrete probability and continuous probability functions
3. Describe the Cumulative distribution function

LEARNING OUTCOMES

At the end of this session, you should be able to:

1. Identify the different types of random variables .
2. Write the rules of Probability functions and its properties
3. Differentiate the Cumulative distribution function from other functions.

CONTENTS

- Random Variables
- Different types of Random Variables
- Probability Mass function
- Probability density function
- Cumulative distribution function

Random Variables

Random In an experiment of chance, outcomes occur randomly. We often summarize the outcome from a random experiment by a simple number.

Variable is a symbol such as X or Y that assumes values for different elements. If the variable can assume only one value, it is called a constant.

Random variable: A function that assigns a real number to each outcome in the sample space of a random experiment.

- Denote by an uppercase letter : X, Y, Z etc.,

Example: A balanced coin is tossed two times. List the elements of the sample space, the corresponding probabilities and the corresponding values X, where X is the number of getting head.

Let X be a random variable that the number of getting heads

X:	HH	HT	TH	TT
X=x:	2	1	1	0
P(X=x)	1/4	1/4	1/4	1/4

Types of Random Variables

Discrete Random Variables: A random variable is discrete if its set of possible values consist of discrete points on the number line.

Example

number of defective parts among 1000 tested

number of transmitted bits received error

number of scratches on a surface

Continuous Random Variables : A random variable is continuous if its set of possible values consist of an entire interval on the number line.

Example:

Time, Temperature, Height, Weight, Length, Electrical current

Discrete Probability distributions

If X is a discrete random variable , the function given by

$$f(x)=P(X=x)=P)=P_X(x)=P(x)$$

for each x within the range of X is called the probability distribution of X.

Properties

1. Probability of each value of discrete random variable is between 0 and 1, inclusive.

$$0\leq P(X=x)\leq 1$$

2. Total probability is equal to 1.

$$\sum_{x \epsilon S} P(X=x)$$

Activities

Check whether the given function can serve as the probability distribution random variable

f(x)=(x+2)/25; for x=1,2,3,4,5

Solution:

$$\sum_{1}^{5} f(x) = \sum_{1}^{5} \frac{x+2}{25}$$

$$= f(1)+f(2)+f(3)+f(4)+f(5)$$

$$= \frac{1+2}{25}+\frac{2+2}{25}+\frac{3+2}{25}+\frac{4+2}{25}+\frac{5+2}{25}$$

$$= \frac{3}{25}+\frac{4}{25}+\frac{5}{25}+\frac{6}{25}+\frac{7}{25}$$

$$= \frac{25}{25}$$

$$= 1$$

1. P(x)≥ 0
2. Total probability is 1.

Hence, the given function is a probability distribution of a discrete random variable.

Activities

Check whether the distribution is a probability distribution.

X	0	1	2	3	4
$P(X=x)$	0.125	0.375	0.025	0.375	0.125

$$\sum_{0} P(X=x) = P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)$$

$$= 0.125+0.375+0.025+0.375+0.125$$

$$= 1.025$$

$$\neq 1$$

Since the summation of all probabilities is not equal to 1, so the distribution is not a probability distribution

CONTINUOUS PROBABILITY DISTRIBUTIONS

Definition: In dealing with continuous variables, f(x) is usually called the probability density function or simply the density function of X. The function f(x) is a probability density function for the continuous random variable X, defined over the set of real numbers R, if

1. f(x)$\geq 0, for\ all\ x \epsilon R$
2. $\int_{-\infty}^{\infty} f(x)dx = 1$, Total area under the curve is 1
3. $$P(a < x < b) = \int_a^b f(x)dx.$$

CASE STUDY

A College professor never finishes his lecture before the end of the hour and always finishes his lectures within 2 min after the hour. Let X =the time that elapses between the end of the hour and the end of the lecture and suppose the pdf of X is

$$f(x) = \begin{cases} kx^2 & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

i) Find the value of K

ii) Obtain the probability that the lecture ends within 1 min of the end of the hour.

Solution:

Given

$$f(x) = \begin{cases} kx^2 & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

we know that $\int_{-\infty}^{\infty} f(x)dx = 1$

$$i) \int_0^2 f(x)dx = \int\limits_0^2 kx^2 = 1$$

$$= k\int_0^2 x^2 dx = 1 => k = 3/8$$

ii) P(x<1)= $\int_0^1 f(x)dx = \int_0^1 (3/8)x^2 = 1/8$

CUMULATIVE DISTRIBUTION FUNCTION

- The cumulative distribution function of a **discrete random variable *X***, denoted as ***F(x)***, is

$$F(x) = P(X \le x) = \sum_{t \le x} f(t) \quad \text{for } -\infty < x < \infty$$

- For a discrete random variable *X*, *F(x)* satisfies the following

$$1)\ 0 \le F(x) \le 1$$

$$2)\ \text{If } x \le y, \text{ then } F(x) \le F(y)$$

SUMMARY

In this session, identify the different types of random variables, probability functions and their properties have discussed.

1. Difference between discrete and continuous random variables
2. Probability Mass and Probability density function and their properties
3. Cumulative distribution function and its properties.

SELF-ASSESSMENT QUESTIONS

The milk produce by a cow is

a) discrete random variable
b) continuous random variable
c) neither discrete nor continuous random variable
d) continuous as well as discrete random variable.

The probability of all possible outcomes of a random experiment is always equal to:

a) Infinity
b) zero
c) one
d) none of the above

TERMINAL QUESTIONS

1. A Random variable X can assume 0,1,2,3,4. A Probability distribution is shown here

X	0	1	2	3	4
P(X)	0.1	0.3	0.3	?	0.1

a. Find $P(X=3)$

b. Find $P(X \geq 2)$

2. Given that $f(x)=k/2^x$ is a probability distribution for a random variable that can take on the values x=0, 1, 2, 3 and 4. Find K.

a) Find K

b) Find the Cumulative probability distribution F(x)

TERMINAL QUESTIONS

3. Given that

$$f(x)=\begin{cases} k\sqrt{x}, & 0<x<1 \\ 0, & elsewhere \end{cases}$$

a) Evaluate k.

b) Evaluate P(0.3<X<0.6) using the density function.

REFERENCES FOR FURTHER LEARNING OF THE SESSION

Reference Books:

1. Chapter 1 of TP1: William Feller, An Introduction to Probability Theory and Its Applications: Volume 1, Third Edition, 1968 by John Wiley & Sons,Inc.

2. Richard A Johnson, Miller& Freund’s Probability and statistics for Engineers, PHI, New Delhi, 11th Edition (2011).

Sites and Web links:

1. * https://ncert.nic.in/textbook.php?kemh1=16-16 *
2. Notes: sections 1 to 1.3 of http://www.statslab.cam.ac.uk/~rrw1/prob/prob-weber.pdf
3. https://ocw.mit.edu/courses/res - 6 -012 -introduction -to -probability - spring - 2018/91864c7642a58e216e8baa8fcb4a5cb5_MITRES_6_012S18_L01.pd f 9
4. https://www.probabilitycourse.com/chapter3/3_2_1_cdf.php
5. https://en.wikipedia.org/wiki/Cumulative_distribution_function

THANK YOU

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