

# SUM OF SUBSETS PROBLEM BY BACKTRACKING

# SUM OF SUBSETS

Problem Definition: Given  $n$  distinct positive numbers  $w_i$ , and  $m$ , find all subsets whose sums are  $m$ .

- **Explicit Constraints :**
- $X_i = \{ j / j \text{ is an integer and } 1 \leq j \leq n \}$
- **Implicit Constraints :**
  1. No two  $x_i$ 's are same
  2. Sum of the chosen weights must be equal to  $m$ .
  3. To avoid generation of multiple instances of the same subsets)

We can formulate this problem using Fixed tuple or Variable tuples.

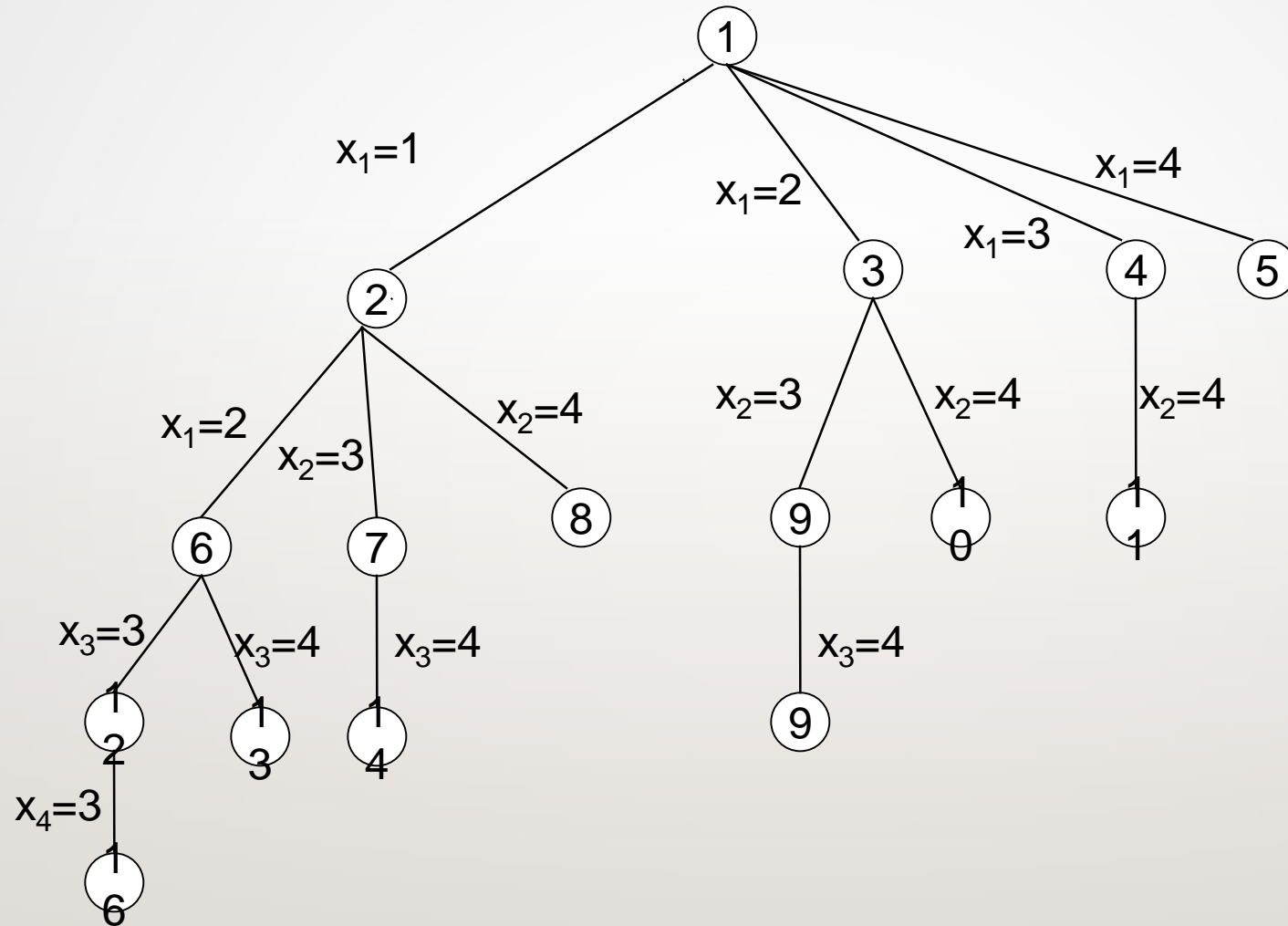
## VARIABLE- SIZED TUPLE

Ex:-  $n=4$ ,  $(w_1, w_2, w_3, w_4) = (11, 13, 24, 7)$ ,  $m=31$ .

- Solutions are  $(11, 13, 7)$  and  $(24, 7)$
- Rather than representing the solution by  $w_i$  's, we can represent by giving the indices of these  $w_i$
- Now the solutions are  $(1, 2, 4)$  and  $(3, 4)$ .
- Different solutions may have different-sized tuples.
- We use the following condition to avoid generating multiple instances of the same subset ( e.g.,  $(1, 2, 4)$  and  $(1, 4, 2)$  )

$$x_i < x_{i+1}$$

# Subset Tree for $n=4$ (variable- sized tuple )



Nodes are numbered as in Breadth first search.

# FIXED- SIZED TUPLE

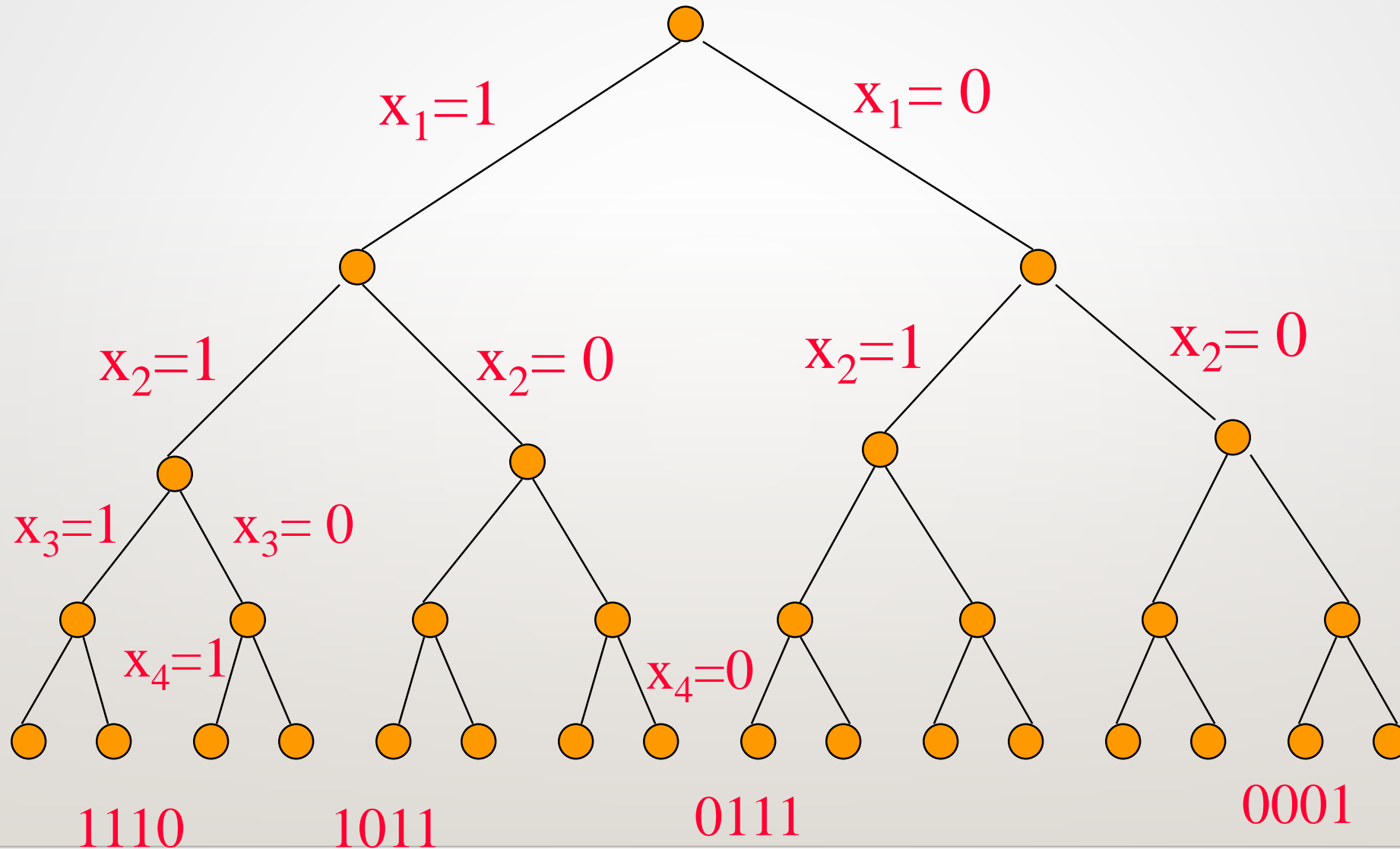
- In this method, each solution subset is represented by an n-tuple  $(x_1, x_2, \dots, x_n)$ .
- $x_i = 0$  if  $w_i$  is not chosen and  $x_i = 1$  if  $w_i$  is chosen

Ex:-  $n=4$ ,  $(w_1, w_2, w_3, w_4) = (11, 13, 24, 7)$ ,  $m=31$ .

- Solutions are  $(11, 13, 7)$  and  $(24, 7)$

- Solutions are:
- $X=(1,1,0,1)$  and  $X=(0,0,1,1)$

# SUBSET TREE FOR N = 4 ( FIXED – SIZED TUPLE )



## BOUNDARY CONDITIONS OF SUM OF SUBSETS

Simple choice for the bounding Function is  $B_k (X_1 \dots X_k) = \text{true}$  iff

$$\sum_{i=1}^k w_i x_i + \sum_{i=k+1}^n w_i \geq m$$

Clearly  $x_1 \dots x_k$  can not lead to an answer node if this condition is not satisfied.

$$\sum_{i=1}^k w_i x_i + w_{k+1} > m$$

Assuming  $w_i$ 's in non decreasing order,  $(x_1 \dots x_k)$  cannot lead to an answer node if

So, the bounding functions we use are therefore

$$B_k (x_1, \dots, x_k) = \text{true iff } \sum_{i=1}^k w_i x_i + \sum_{i=k+1}^n w_i \geq m$$

and

$$\sum_{i=1}^k w_i x_i + w_{k+1} \leq m$$



Algorithm **SumOfSub**( s, k, r )

// Find all subsets of  $w[1:n]$  that sum to m

// It is assumed that  $w[1] \leq m$  and  $\sum w_i \geq m$

{

$x[k]=1$ ; // left child

    if (  $s + w[k] = m$  ) then write(  $x[1:k]$  ); // Subset found

    else if (  $s + s[k] + s[k+1] \leq m$  )

        then SumOfSub(  $s + w[k]$ ,  $k+1$ ,  $r - w[k]$  )

        // Generate right child and evaluate Bk

    if ( (  $s + r - w[k] \geq m$  ) and (  $s + w[k+1] \leq m$  ) ) then

    {

$x[k] = 0$ ;

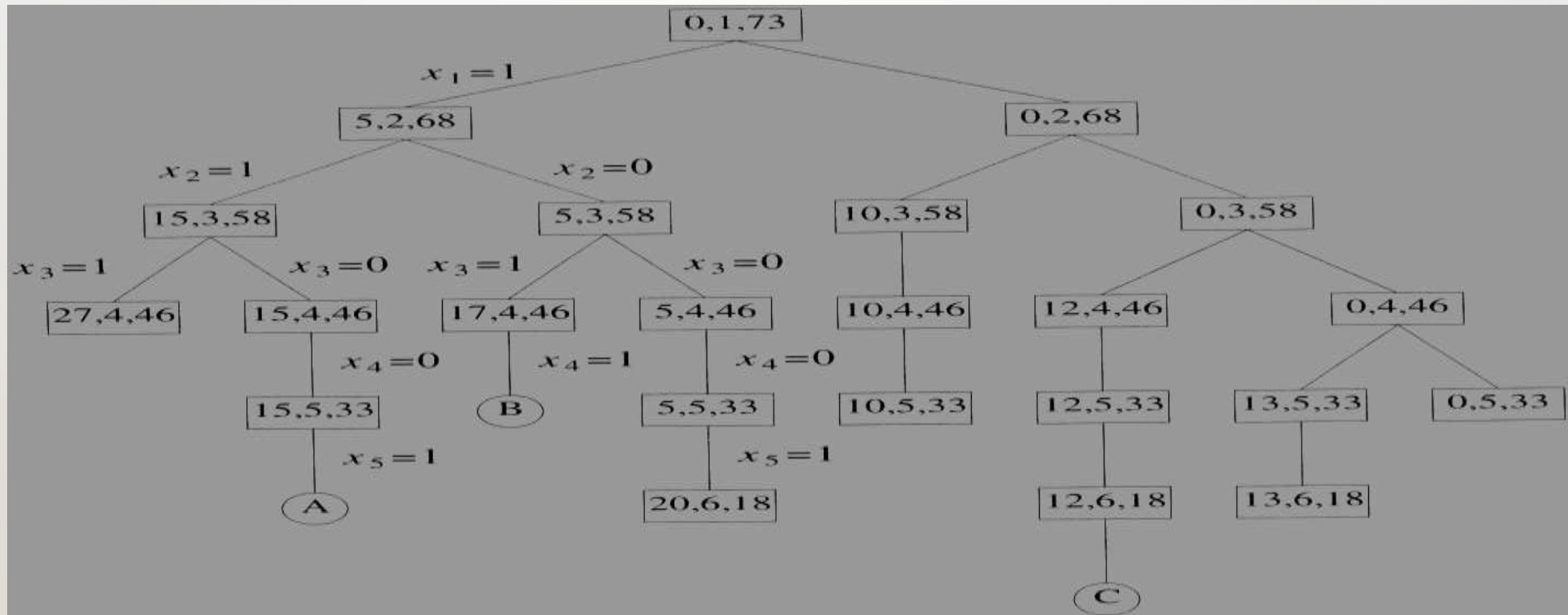
    }

        SumOfSub( s,  $k+1$ ,  $r - w[k]$  );

}

EX:-  $N=6$ ,  $M=30$ ,  $W [ 1:6 ] = \{ 5,10,12,13,15,18 \}$   
 PORTION OF STATE SPACE TREE GENERATED BY SUMOFSUB.  
 CIRCULAR NODES INDICATE SUBSETS WITH SUMS EQUAL TO  $M$ .

- Example 7.6 (Figure 7.10)
- $n=6$ ,  $w[1:6]=\{5,10,12,13,15,18\}$ ,  $m=30$



# SAMPLE QUESTIONS

---

- What is the Sum of Subsets problem
- Describe the problem statement of the Sum of Subsets. What are the inputs and outputs of the problem
- Explain about state space tree in sum of subsets problem
- Solve sum of subsets by using Input: set[] = {4, 16, 5, 23, 12}, sum = 9