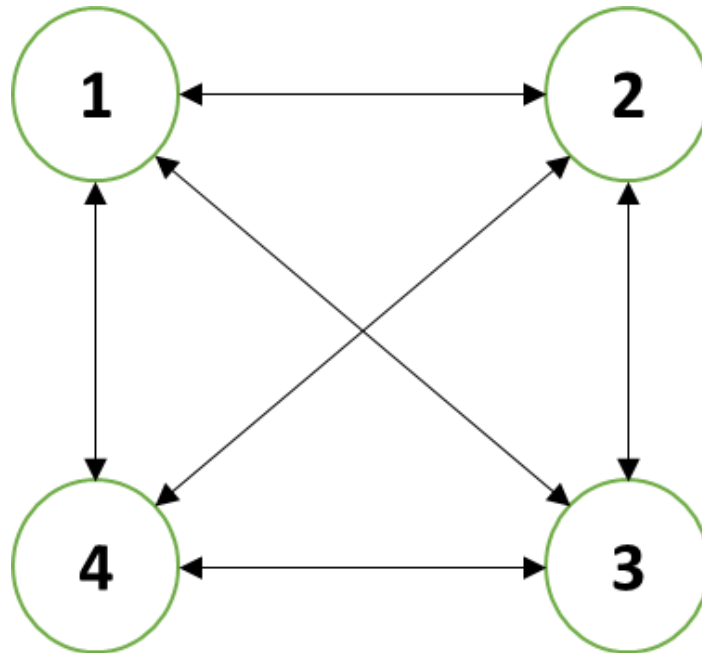


## DAA ALM – 3

Solve the following travelling sales person problem using dynamic programming technique.



**ANSWERS:**

CSE GURU

### Travelling Salesman Problem using Dynamic Programming

➤ Solve the following TSP which is represented as a directed graph and whose edge lengths are given by cost adjacency matrix

*starting city.*

	1	2	3	4
1	0	10	15	20
2	5	0	9	10
3	6	13	0	12
4	8	8	9	0

## Travelling Salesman Problem using Dynamic Programming Solution

No. of nodes = 4

Case 1  $S = \emptyset$

$$g(2, \emptyset, 1) = c_{21} = 5$$

$$g(3, \emptyset, 1) = c_{31} = 6$$

$$g(4, \emptyset, 1) = c_{41} = 8$$

Case 2  $|S| = 1$   $g(i, S) = \min_{j \in S} \{c_{ij} + g(i, S - \{i\})\}$

When  $i = 2$ :  $g(2, \{3\}) = \min_{j \in \{3\}} \{c_{23} + g(2, \emptyset)\} = 9 + 6 = 15$

$$g(2, \{4\}) = \min_{j \in \{4\}} \{c_{24} + g(2, \emptyset)\} = 10 + 8 = 18$$

When  $i = 3$ :  $g(3, \{2\}) = \min_{j \in \{2\}} \{c_{32} + g(3, \emptyset)\} = 13 + 5 = 18$

$$g(3, \{4\}) = \min_{j \in \{4\}} \{c_{34} + g(3, \emptyset)\} = 12 + 8 = 20$$

When  $i = 4$

$$g(4, \{2, 3\}) = \min_{j \in \{2, 3\}} \{c_{42} + g(4, \emptyset)\} = 8 + 5 = 13$$

$$g(4, \{3, 4\}) = \min_{j \in \{3, 4\}} \{c_{43} + g(4, \emptyset)\} = 9 + 6 = 15$$

	1	2	3	4
1	0	10	15	20
2	5	0	9	10
3	6	13	0	12
4	8	8	9	0

## Travelling Salesman Problem using Dynamic Programming Solution

Case 3  $|S| = 2$   $g(i, S) = \min_{j \in S} \{c_{ij} + g(i, S - \{i\})\}$

When  $i = 2$ :  $g(2, \{3, 4\}) = \min_{j \in \{3, 4\}} \{c_{23} + g(2, \{4\}), c_{24} + g(2, \{3\})\}$

$$= \min \{9 + 20, 10 + 15\}$$

$$= \min \{29, 25\} = 25$$

When  $i = 3$ :  $g(3, \{2, 4\}) = \min_{j \in \{2, 4\}} \{c_{32} + g(3, \{4\}), c_{34} + g(3, \{2\})\}$

$$= \min \{13 + 18, 12 + 13\}$$

$$= \min \{31, 25\} = 25$$

When  $i = 4$ :  $g(4, \{2, 3\}) = \min_{j \in \{2, 3\}} \{c_{42} + g(4, \{3\}), c_{43} + g(4, \{2\})\}$

$$= \min \{8 + 15, 9 + 18\}$$

$$= \min \{23, 27\} = 23$$

	1	2	3	4
1	0	10	15	20
2	5	0	9	10
3	6	13	0	12
4	8	8	9	0

Case 1

$$g(2, \emptyset, 1) = 5$$

$$g(3, \emptyset, 1) = 6$$

$$g(4, \emptyset, 1) = 8$$

Case 2

$$g(2, \{3\}) = 15 \quad g(4, \{2\}) = 13$$

$$g(2, \{4\}) = 18 \quad g(4, \{3\}) = 15$$

$$g(3, \{2\}) = 18$$

$$g(3, \{4\}) = 20$$

## Travelling Salesman Problem using Dynamic Programming Solution

Case 4  $|S|=3$   $g(1, \{2,3,4\} - \{1\})$

$$\min \begin{cases} C_{12} = c_{12} + g(2, \{3,4\}) = 10 + 25 = 35 \\ C_{13} = c_{13} + g(3, \{2,4\}) = 15 + 25 = 40 \\ C_{14} = c_{14} + g(4, \{2,3\}) = 20 + 23 = 43 \end{cases}$$

Optimal path  $\Rightarrow 1 \rightarrow 2 \rightarrow 4 \rightarrow 3 \rightarrow 1$   
 $10 + 10 + 9 + 6$

$35 \leftarrow$  Optimal cost

	1	2	3	4
1	0	10	15	20
2	5	0	9	10
3	6	13	0	12
4	8	8	9	0

Case 3

$$\begin{aligned} g(2, \{3,4\}) &= 25 \\ g(3, \{2,4\}) &= 25 \\ g(4, \{2,3\}) &= 23 \end{aligned}$$

ANSWER LINK [https://youtu.be/iB-UGKyTBOc?si=c\\_RZP-sgFB49RkV1](https://youtu.be/iB-UGKyTBOc?si=c_RZP-sgFB49RkV1)