SUBJECTCODE: 23MT2005
PROBABILITY STATISTICS AND QUEUING THEORY

Date of the Session: //____________________ Time of the Session:_____to_____

Tutorial 12:

- Understand Discrete-time and Continuous-time Markov chain
- Understand Birth-death processes, Poisson process

1. Consider a simple weather model with three states: Sunny, Cloudy, and Rainy. The transitionprobabilities between these states are as follows:

On a Sunny day, there is a 30% chance of transitioning to a Cloudy day and a 10% chance of Rain.

On a Cloudy day, there is a 40% chance of remaining Cloudy, a 30% chance of becoming Sunny, and a 30% chance of Rain.

On a Rainy day, there is a 20% chance of transitioning to Sunny, a 40% chance of becoming Cloudy, and a 40% chance of remaining Rainy.

Simulate the weather for a period of 7 days using a discrete-time Markov chain.

Solution:

$$\begin{array}{c c} & \begin{array}{ccc} \text{sunny} & \text{cloudy} & \text{rainy.} \end{array} \\ \begin{array}{c} \text{Sunny} \\ \text{cloudy} \\ \text{rainy} \end{array} & \begin{bmatrix} 0.6 & 0.3 & 0.1. \\ 0.3 & 0.4. & 0.3. \\ 0.2. & 0.4 & 0.4. \end{bmatrix} \end{array}$$

2. Assume that a computer system is one of three states: busy, idle, or undergoing repair, respectively denoted by states 0, 1, and 2. Observing its state at 2 P. M. each day, we believe that the system approximately behaves like a homogeneous Markov chain with the transition probability matrix:

$$P = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0.0 & 0.4 \end{bmatrix}$$

i) Prove that the chain is irreducible
ii) Determine the steady-state probabilities

Solution:

i) Step I:- Direct

$P(0\to 0) = 0.6$ $\quad$ $P(1\to 0) = 0.1$ $\quad$ $P(2\to 0) = 0.6$

$P(0\to 1) = 0.2$ $\quad$ $P(1\to 1) = 0.8$ $\quad$ $P(2\to 1) = 0.0$

$P(0\to 2) = 0.2$ $\quad$ $P(1\to 2) = 0.1$ $\quad$ $P(2\to 2) = 0.4$

step-ii) indirect.

$P(0\to 0) \times P(0\to 1) = 0.12$ $\quad$ $P(1\to 2) = 0.1$

$P(0\to 2) = 0.2.$ $\quad$ $P(2\to 0) = 0.6$

$P(1\to 0) = 0.1$ $\quad$ $P(2\to 0) \times P(0\to 1) = 0.12$

$\therefore$ chain is irreducible

ii)

$$\pi_0 = 0.6\pi_0 + 0.1\pi_1 + 0.6\pi_2$$

$$\pi_1 = 0.2\pi_0 + 0.8\pi_1 + 0.0\pi_2$$

$$\pi_2 = 0.2\pi_0 + 0.1\pi_1 + 0.4\pi_2.$$

$$\boxed{\therefore \pi_0 = \frac{5}{12} \quad \pi_1 = \frac{5}{12} \quad \pi_2 = \frac{1}{6}.}$$

3.. Classify the states of following Markov chains.

a) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

b) $\begin{pmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Solution:

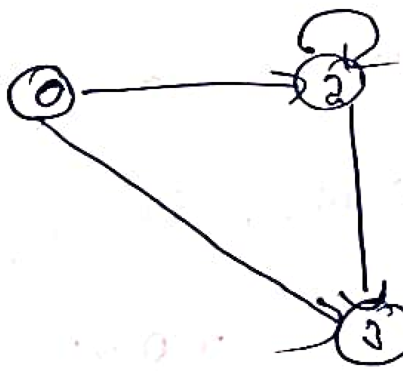
a.)

class-1:- {0,1}

class-2:- {2}, recurrent periodic.

Transient.

b)



State 2 is absorbing

State 3 is abolbing

State 1 is transient.

4. Customers tend to exhibit loyalty to product brands but may be persuaded through clever marketing and advertising to switch brands. Consider the case of three brands: A, B and C. Customer "unyielding" loyalty to a given brand is estimated at 75%, giving the competitors only a 25% margin to realize a switch. Competitors launch their advertising campaigns once a year. For brand A customers, the probabilities of switching to brands B and C are 0.1 and 0.15 respectively. Customers of Brand B are likely to switch to A and C with probability of 0.2 and 0.05 respectively. Brand C customers can switch to brands A and B with equal probabilities.

i) Express the situation as a Markov chain

ii) In the long run, determine the market share for each brand?

Solution:

i:)

$$P = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 0.75 & 0.1 & 0.15 \\ 0.2 & 0.75 & 0.05 \\ 0.5 & 0.5 & 0.75 \end{pmatrix} \end{array}$$

ii)

$$\pi_0 = 0.75\pi_0 + 0.2\pi_1 + 0.5\pi_2$$

$$\pi_1 = 0.1\pi_0 + 0.75\pi_1 + 0.5\pi_2$$

$$\pi_2 = 0.15\pi_0 + 0.05\pi_1 + 0.75\pi_2$$

$\pi_0 = 0.5263, \quad \pi_1 = 0.3158, \quad \pi_2 = 0.679$

$\therefore A = 52.63\%$

$B = 31.58\% \qquad C = 15.79\%$

5. Research analyzing brand switching between different airlines, operating on the Delhi-Mumbai route by frequent fliers. On the basis of the data collected by her, the researcher has developed the following transition probability matrix.

$$\begin{array}{cc} & \text{To airline} \\ \text{From airline} \begin{array}{c} AA \\ BB \\ CC \end{array} & \begin{bmatrix} 0.9 & .03 & 0.07 \\ 0.15 & 0.80 & 0.05 \\ 0.20 & 0.30 & 0.50 \end{bmatrix} \end{array}$$

It is found that currently the airlines AA, BB and CC have 20%, 50% and 30% of the market respectively.

i) Obtain the market share for each airline in two months time, and

ii) Calculate the long run market share for each time.

Solution: Given

$$V0 = 0.2, 0.5, 0.3$$

i) $$p^2 = \begin{matrix} 0.8379 & 0.0669 & 0.0952 \\ 0.1845 & 0.7389 & 0.0766 \\ 0.265 & 0.339 & 0.396 \end{matrix}$$

$$V_2 = V0 \cdot p^2 = \boxed{0.242, \quad 0.489, \quad 0.269}$$

AA = 24.2%

BB = 48.9%

CC = 26.9%.

ii) $$\pi \cdot P = \quad \pi_1 + \pi_2 + \pi_3 = 1$$

AA = 25.32% CC = 23.54%

BB = 51.14%

Date:		STUDENT NAME:

6.A salesman territory consists of cities A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in city B. However, if he sells in either B or C, then the next day he is twice as likely to sell in city A as in other city. In the long run how often does he sell in each cites.

Solution:

From given conditions:-

$$\begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 2/3 & 0 & 1/3 \\ 2/3 & 1/3 & 0 \end{bmatrix} \end{array}$$

long-run

$$\pi P = \pi$$

$$\pi_A + \pi_B + \pi_C = 1$$

$$\pi_A = \frac{2}{3}\pi_B + \frac{2}{3}\pi_C$$

$$\pi_B = \pi_A + \frac{1}{3}\pi_C$$

$$\pi_C = \frac{1}{3}\pi_B + \frac{1}{3}\pi_C$$

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VIVA QUESTIONS

1 What is Discrete-time Markov chain?

It is a stohastic Process that undergo transition from one state to another in discrete time steps.

2 How is the rate matrix used to describe a Continuous-time Markov chain?

The System evolves in continuous time and transistion b/w states occur at time point.

3. Define Periodic and Aperiodic Markov chains with one suitable example.

Peioidic if it exists a state in chain.

Apaiodic: if GCD no.of steps return to state i is 1.

(For Evaluators use only)

Comment of the Evaluator (if Any)	Evaluator's Observation Marks Secured:________out of________ Full Name of the Evaluator: Signature of the Evaluator: Date of Evaluation