(DEEMED TO BE UNIVERSITY)

Digital Communication 23EC2208A

Digital Carrier Modulation

Dr. M. Venu Gopala Rao

A.M.I.E.T.E, M.Tech, Ph.D(Engg)

Cert. in R.S.T (City & Guild's London Institute, London)

F.I.E.T.E, L.M.I.S.T.E, I.S.O.I., S.S.I., M.I.A.E.

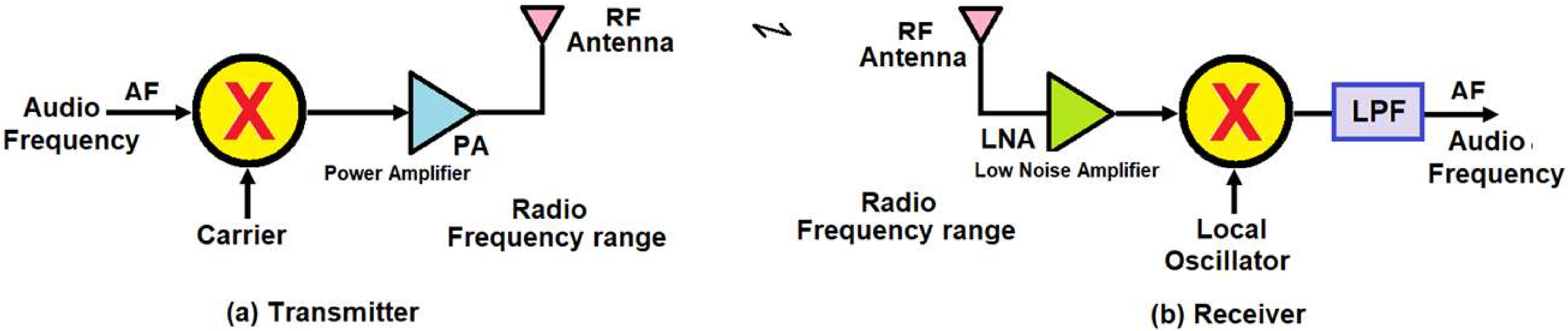
Professor, Dept. of ECE, K L University

mvgr03@kluniversity.in

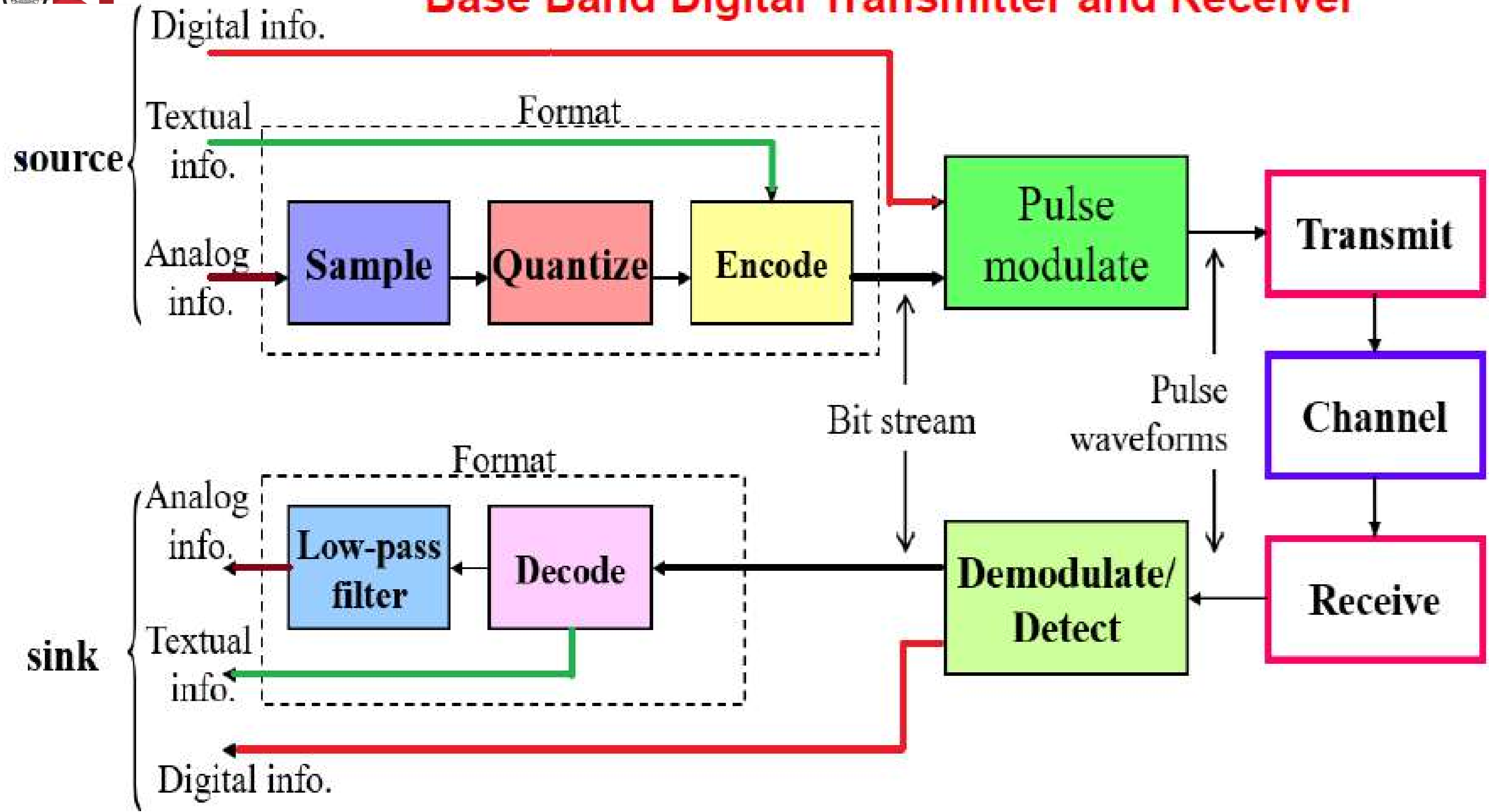
Digital Pass Band Communication

Base Band Transmission

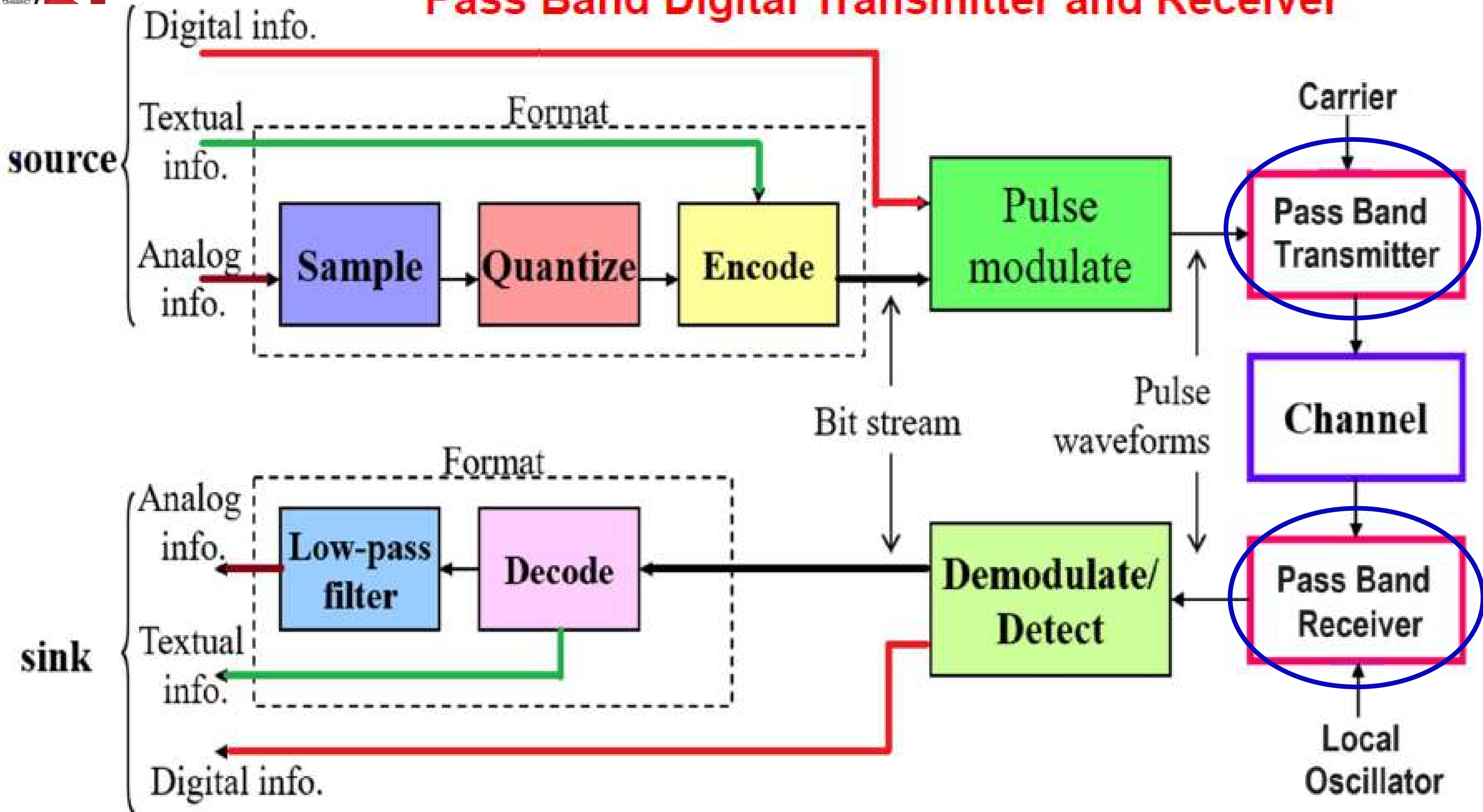
Pass Band Transmission



Base Band Digital Transmitter and Receiver

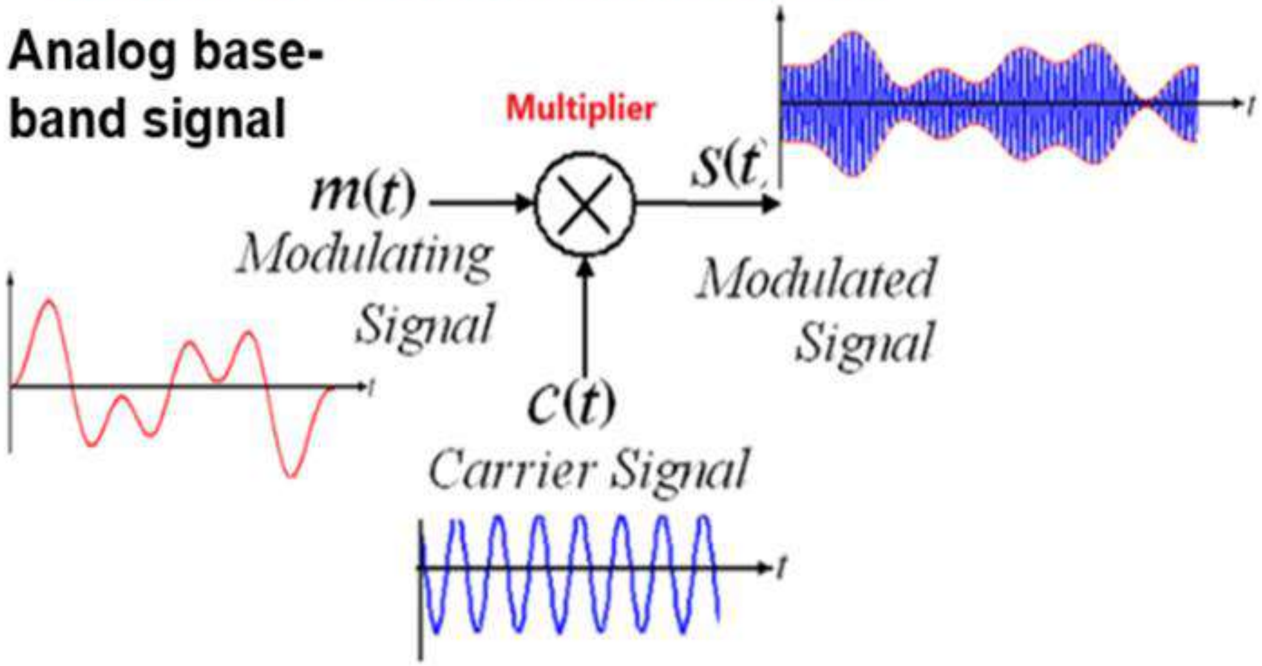


Pass Band Digital Transmitter and Receiver

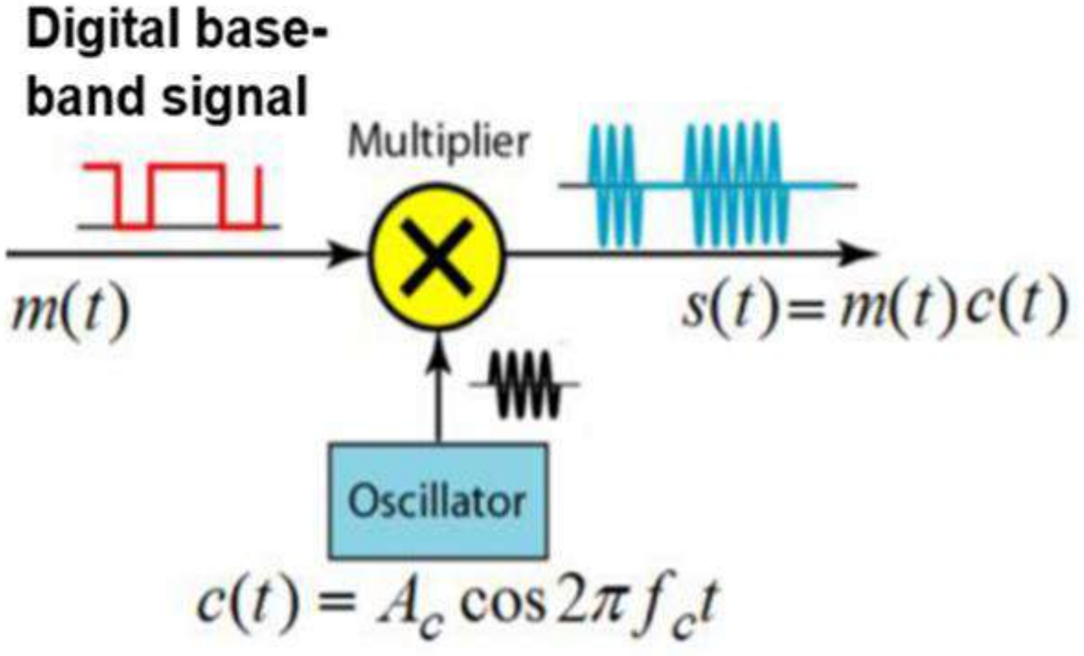


Pass-Band Communication

Analog Communication

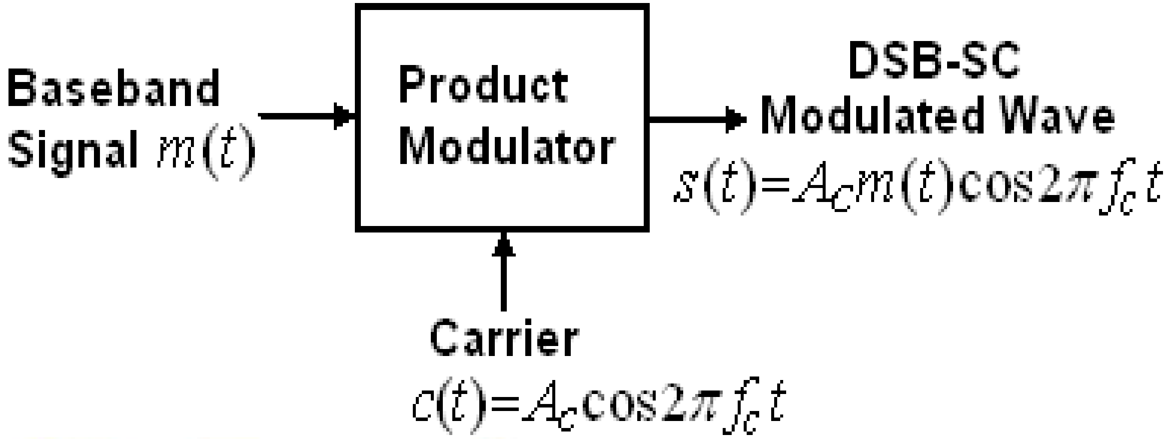


Digital Communication

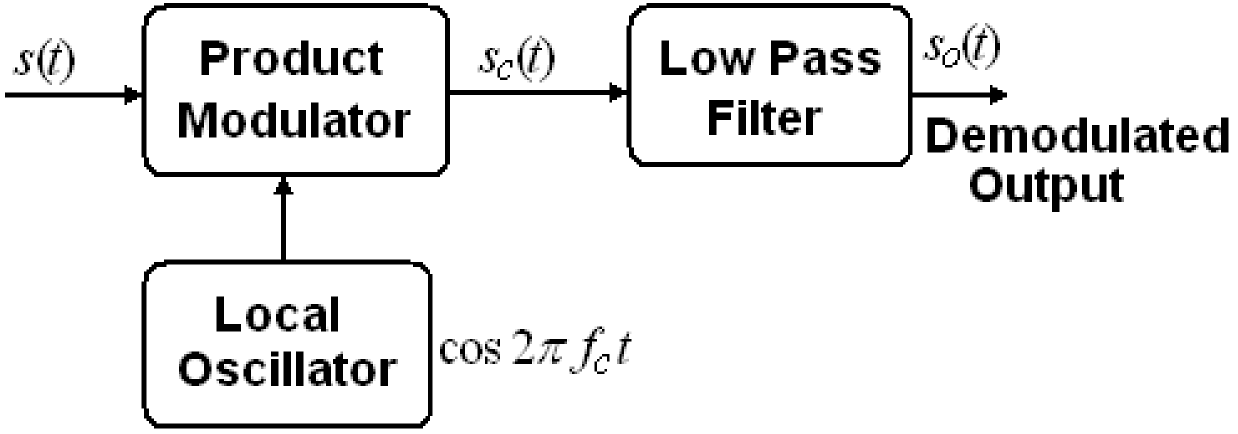


	Analog Communication	Digital Communication
Modulating Signal	Analog	Digital
Carrier Signal	Analog	Analog

DSB-SC Modulation



Demodulation



DSB-SC signal is $s(t)=A_c m(t)\cos 2\pi f_c t$

Then the product modulator output $s_c(t)$ is given by

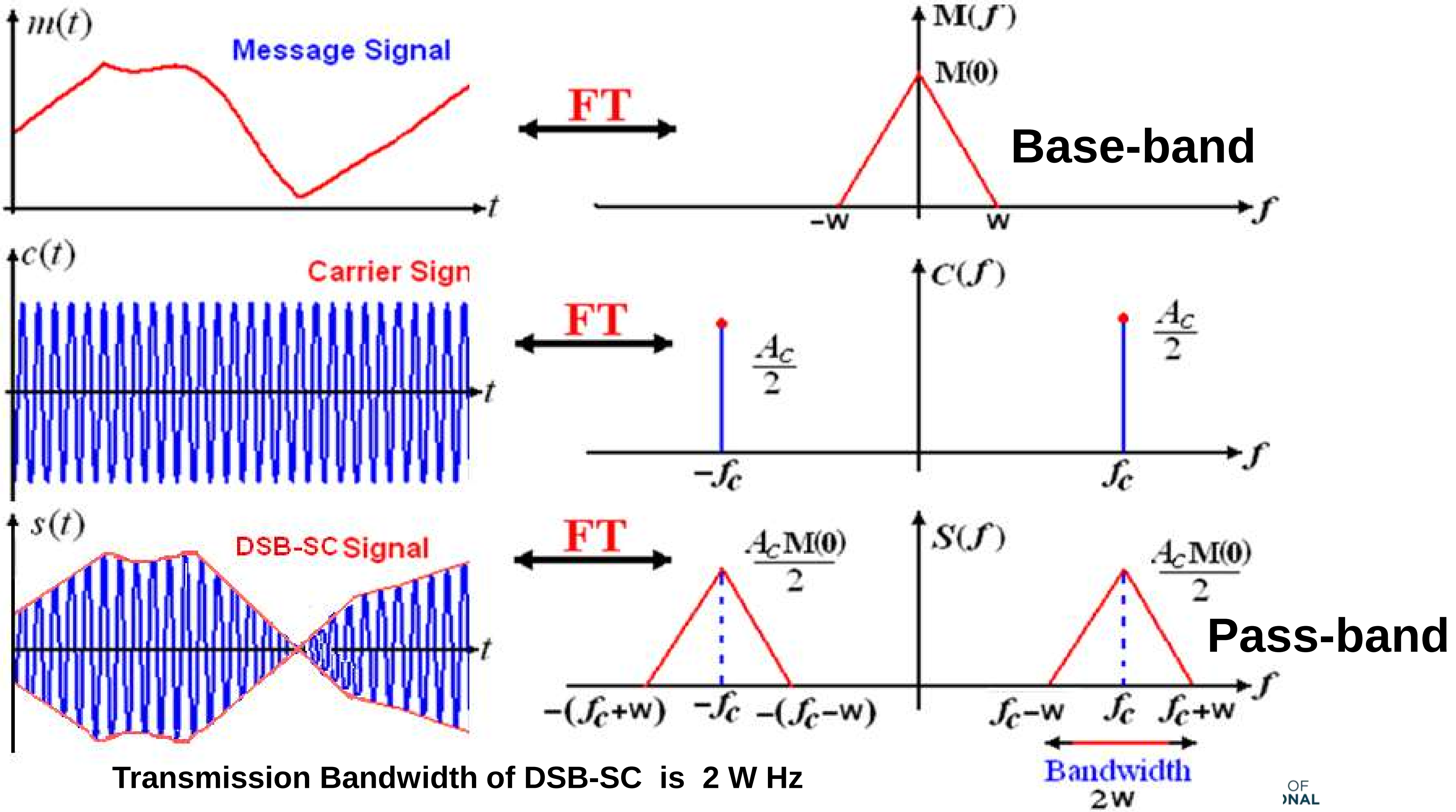
$$s_C(t)=s(t)\cos 2\pi f_C t=[A_C m(t)\cos 2\pi f_C t]\cos 2\pi f_C t$$

$$=A_C m(t)\cos^2 2\pi f_C t=A_C m(t)\frac{1}{2}[1+\cos 4\pi f_C t]$$

$$=\frac{1}{2}A_C m(t)+\frac{1}{2}A_C m(t)\cos 4\pi f_C t$$

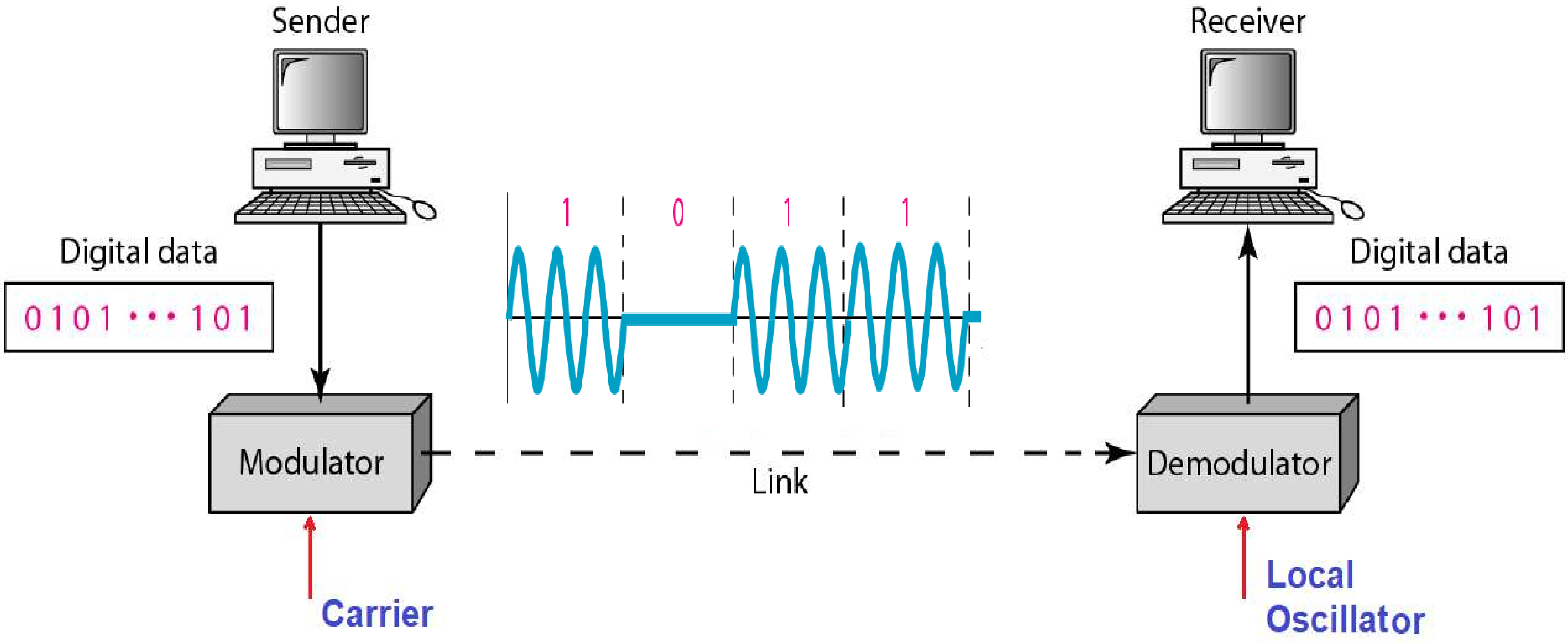
$$s_O(t)=\frac{1}{2}A_C m(t)$$

Baseband AM(DSB-SC) Spectrum

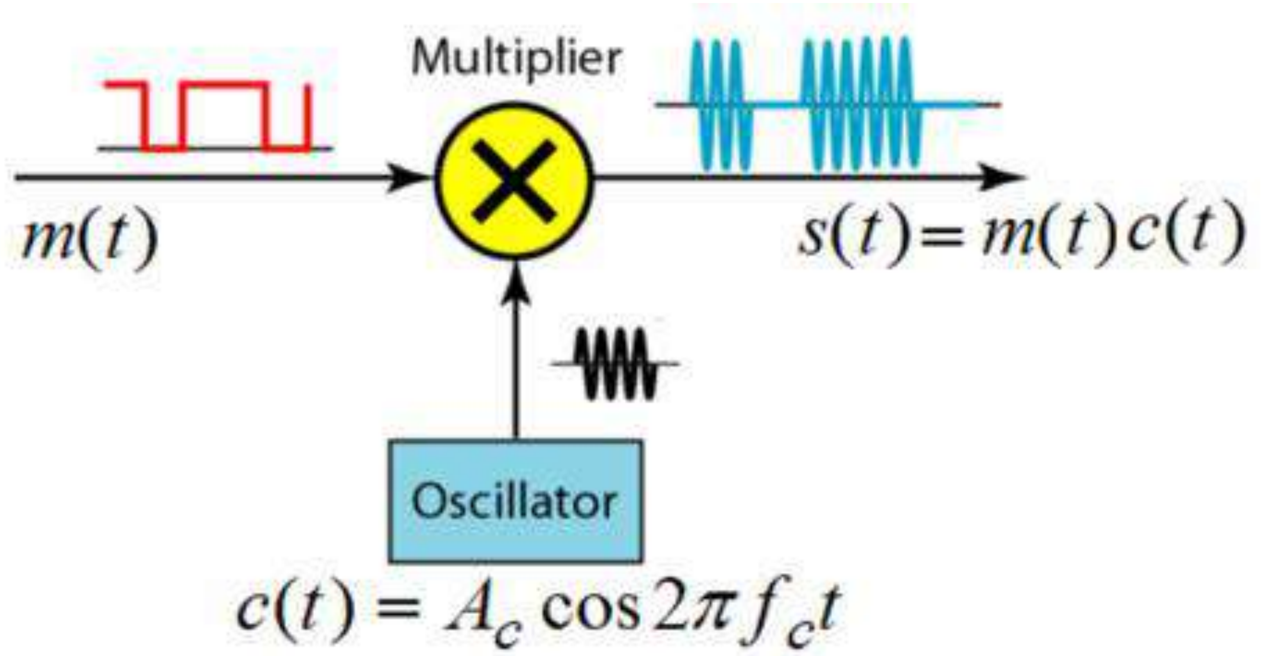


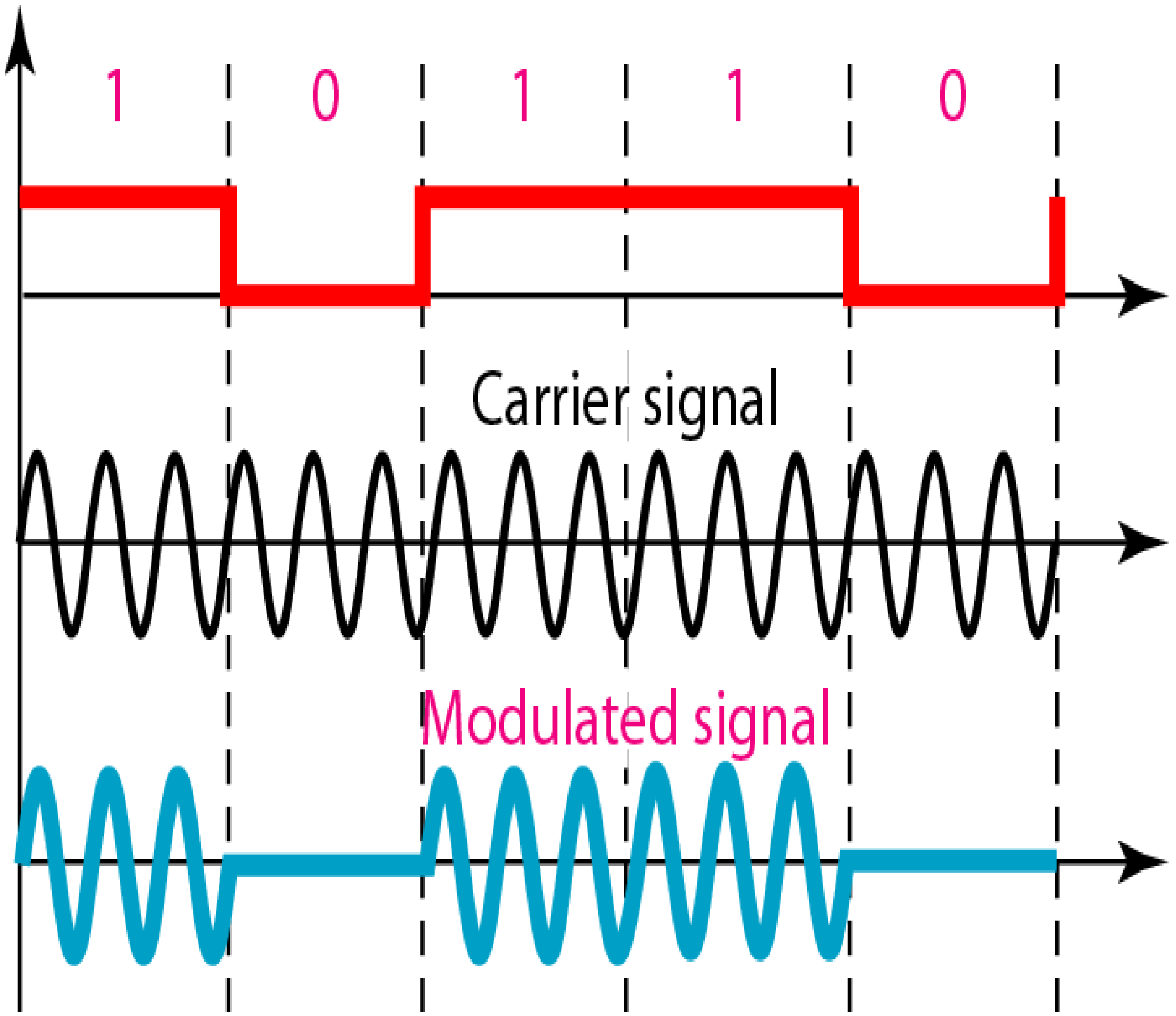
Transmission Bandwidth of DSB-SC is 2 W Hz

Pass-band Digital Communication



Pass-band Digital Transmitter





Digital Signals

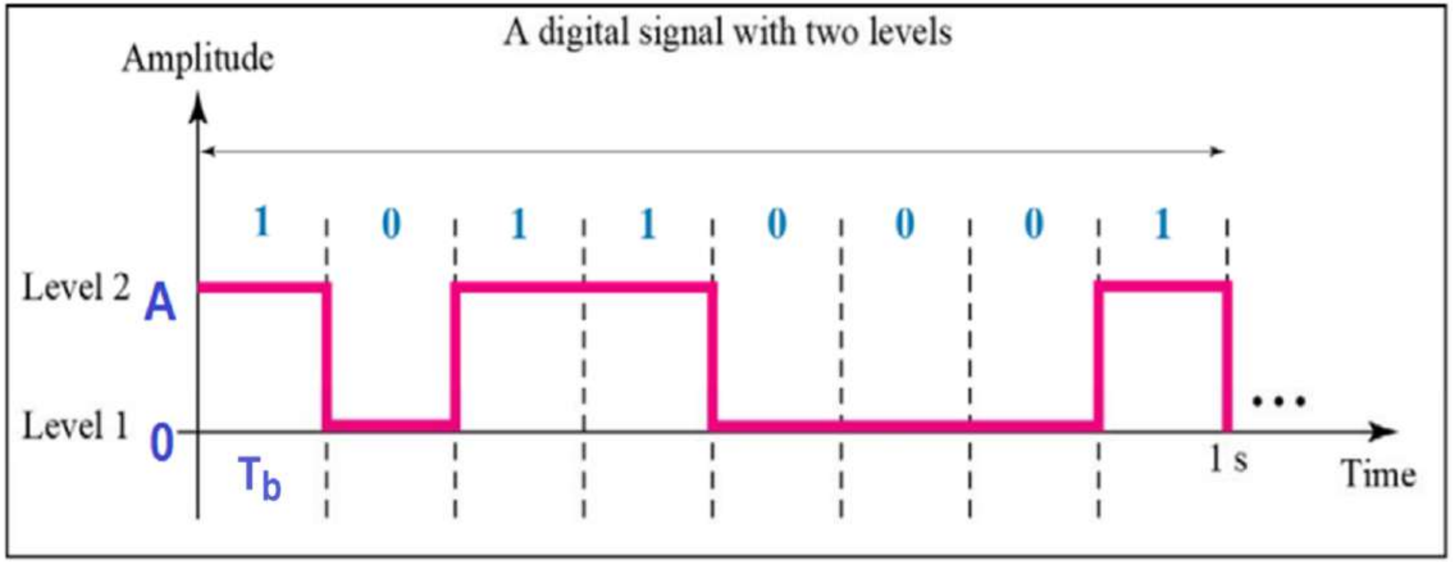
In addition to being represented by an analog signal, information can also be represented by a digital signal.

- For example, a 1 can be encoded as a positive voltage and a 0 as zero voltage as shown below:
- Bit rate is defined as the number of bits transmitted over a second.

What is the bit rate for this digital signal?

8 bits sent in 1 sec.

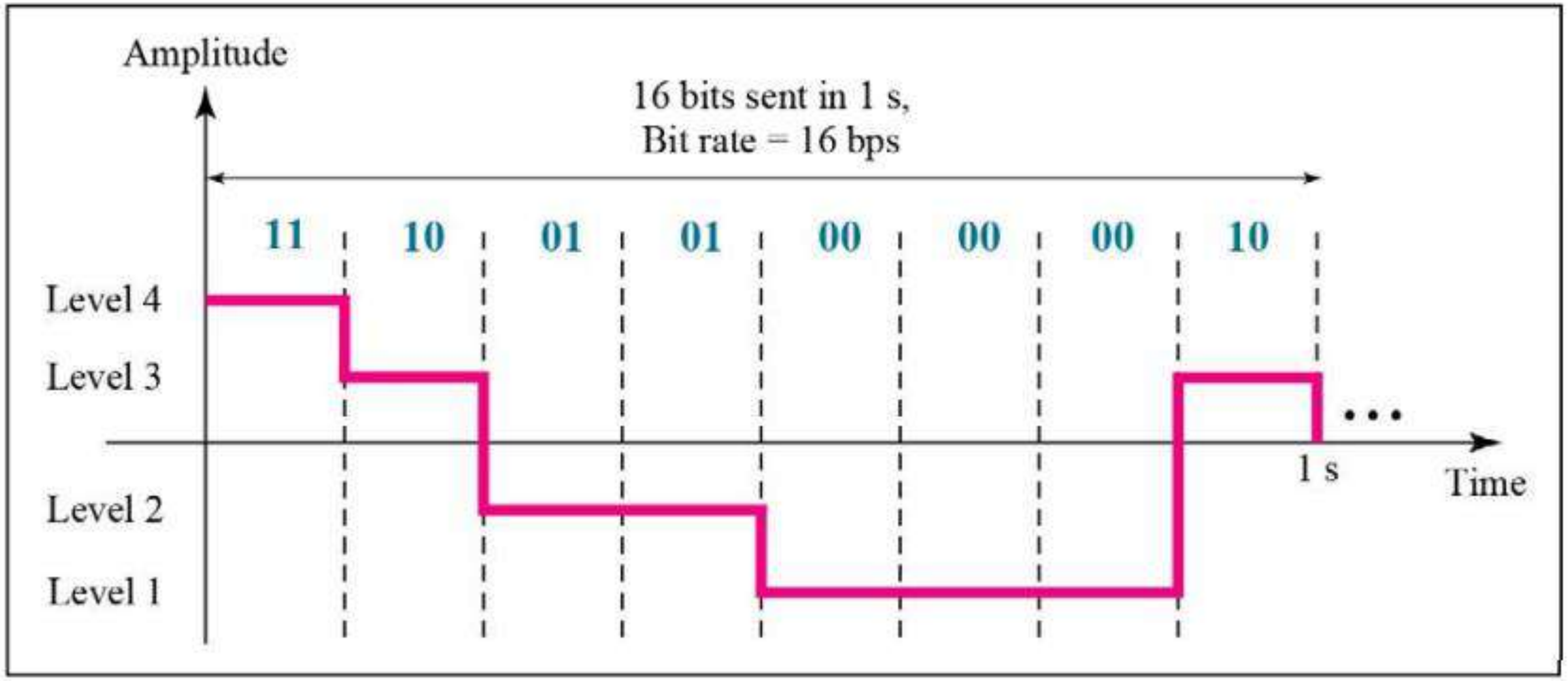
Bit rate = 8 bps



- Hence the above digital data bit rate is 8 bits per second or bps.
- What is the bit period / duration ? $\frac{1}{8}$ = 0.125 sec

Digital Signals

A digital signal can have more than two levels as shown below. In this case, we can send more than 1 bit for each level.

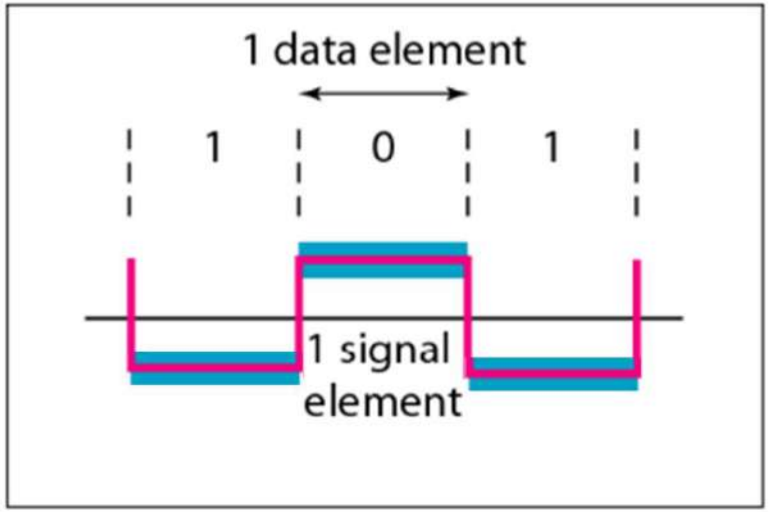


b. A digital signal with four levels

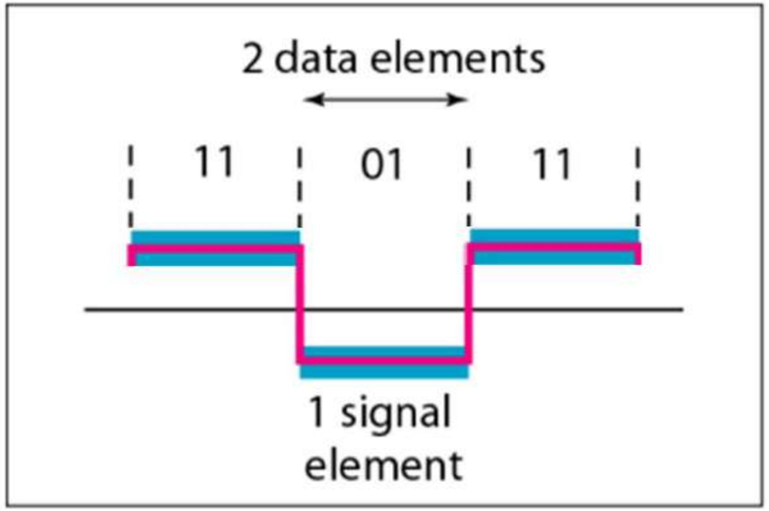
Each pulse is represented by two bits. Hence in this case the bit-rate is 8 x 2 = 16 bps

Signal element versus data element

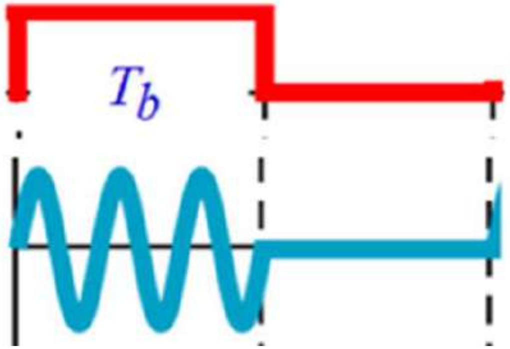
- A data element is the smallest entity that represent a piece of information; this is the bit.
- A signal elements is the shortest unit (timewise) of a digital signal.
- In other words, data elements are what we need to send; signal element are what we can send.

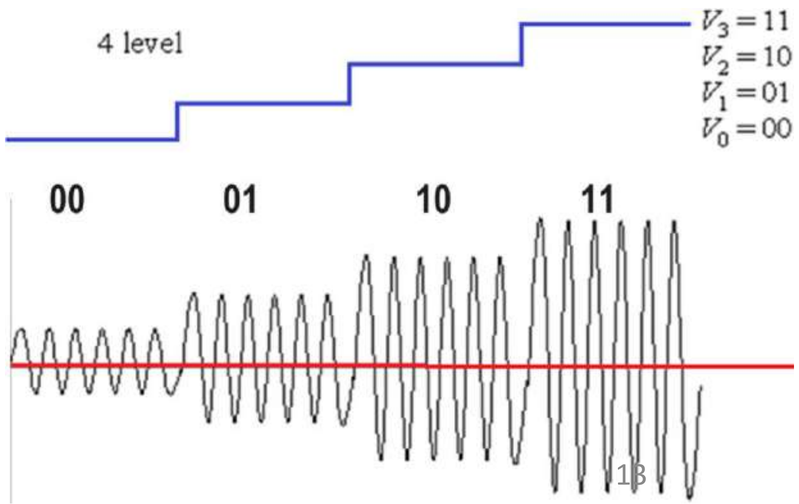


One data element per one signal element



Two data elements per one signal element

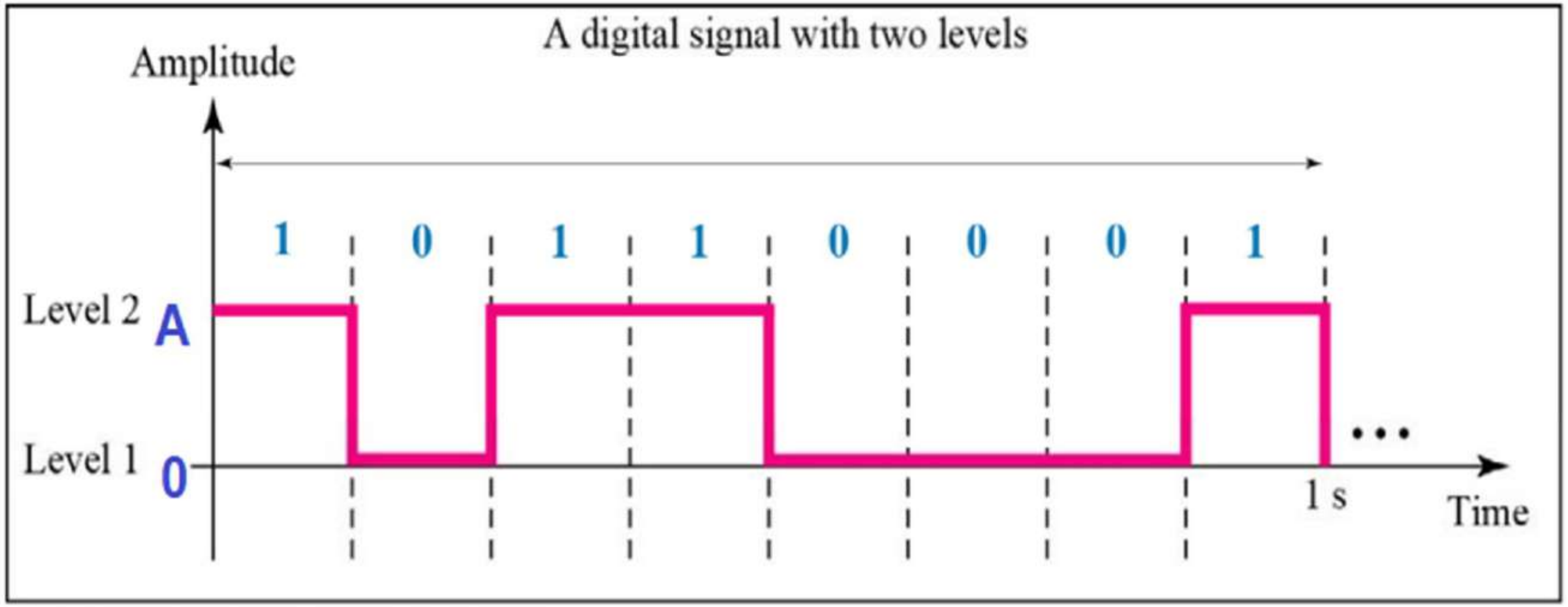




Bit rate and Baud rate

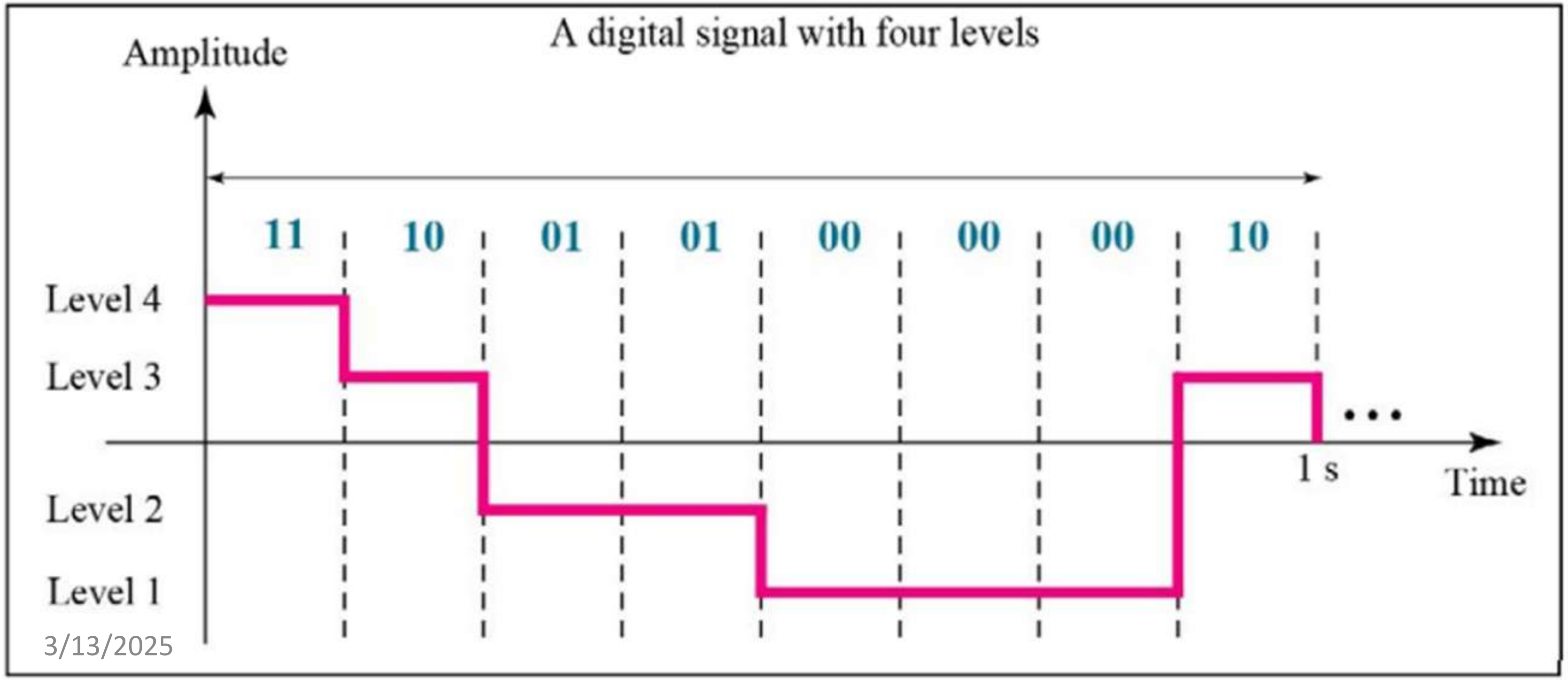
- Bit rate is the number of bits transmitted over a second.
- Baud rate is the number of symbols / signal units transmitted over a second.
- Baud rate is less than or equal to the bit rate.
- Baud rate = Bit rate / no. of bits per signal unit

$$= \frac{f_b}{n}$$



Bit rate: 8 bps

Baud rate: 8 bps



Bit rate: 16 bps

Baud rate: 8 bps

Ex1: An analog signal carries 4 bits per signal element. If 1000 signal elements are sent per second, find the bit rate.

Ans: Baud rate = 1000, No. of elements $N = 4$;

Bit rate = 1000 X 4 = 4000 bps.

Ex2: An analog signal has a bit rate of 8000 bps and a baud rate of 1000 baud. How many data elements are carried by each signal element? How many signal elements do we need?

Data elements = Bit rate / baud rate = 8 bits / baud

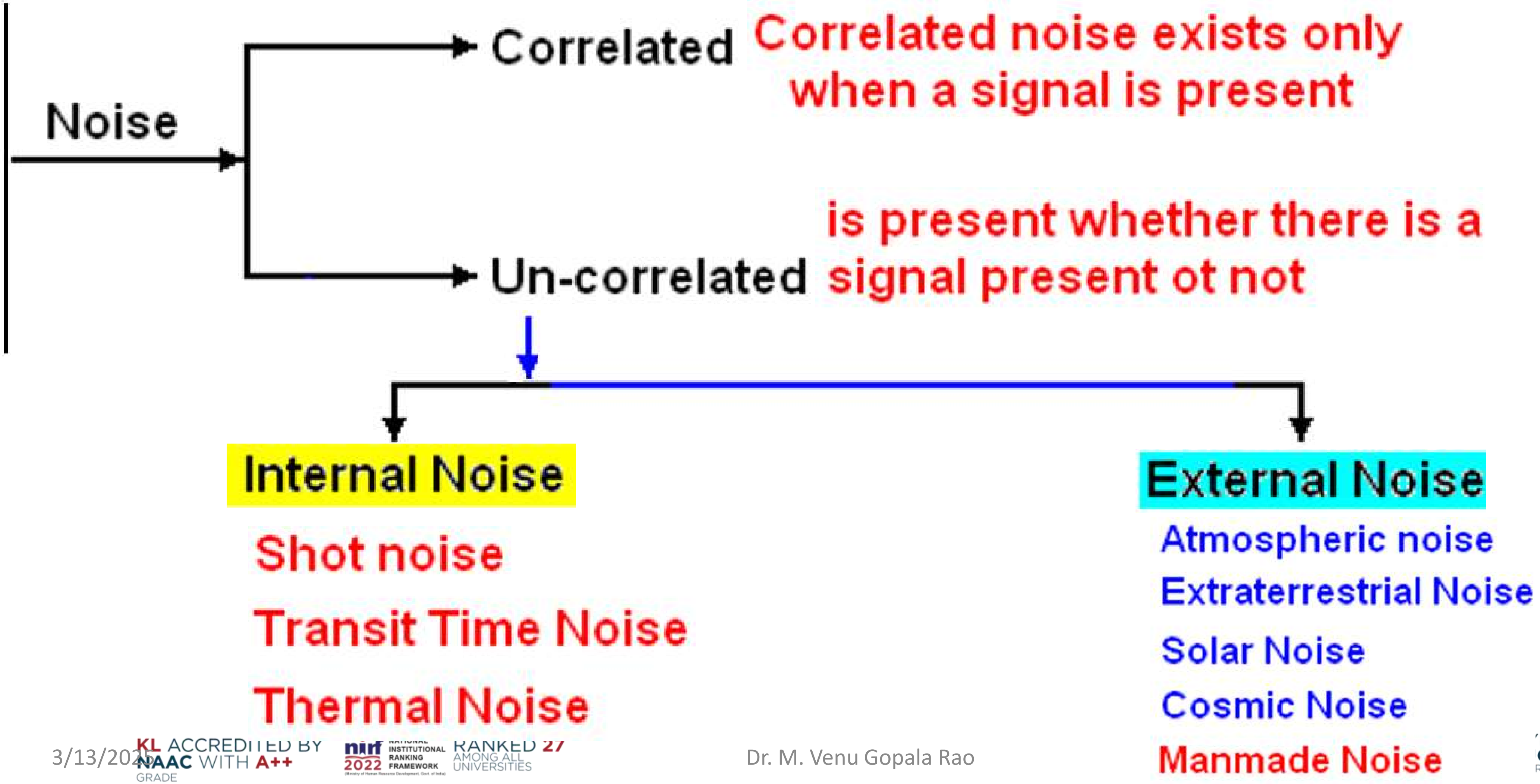
Then the number of signal elements $L = 2^8 = 256$ levels.

Bandwidth

- The actual bandwidth necessary to propagate a given bit rate depends on several factors,
  - including the type of encoding and modulation used,
  - the types of filters used, system noise, and
  - desired error performance.
- The ideal bandwidth is generally used for comparison purposes only.
- The relationship between bandwidth and bit rate also applies to the opposite situation. For a given bandwidth (*B*), *the highest theoretical bit rate is* 2*B*.

Introduction to Noise

Electrical noise is defined as undesirable electrical energy that falls within the passband of the signal.



Gaussian Noise

- Let $\eta(t)$ denote a noisy signal.
- The values of noisy signal are ***unpredictable*** and only a probability can be associated to them,

$$\Pr\{n_1 < \eta(t) \le n_2\}$$

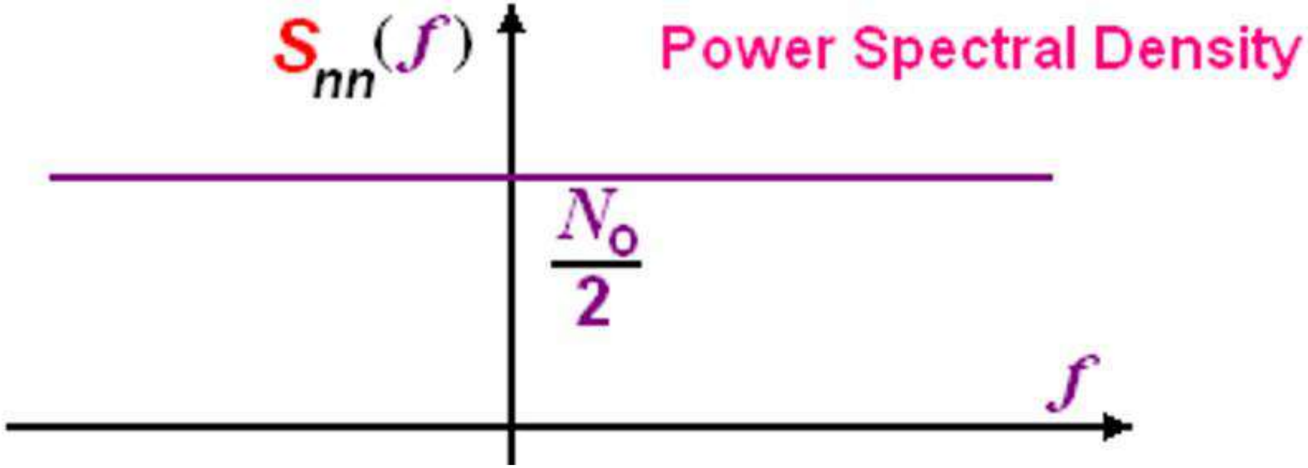
- The most common type of noise found in a communication system is ***Gaussian*** with PDF

$$P_{\eta(t)}(n) = \frac{1}{\sqrt{2\pi\sigma_\eta^2}} e^{-\frac{1}{2}\left(\frac{n-\mu_\eta}{\sigma_\eta}\right)^2}$$

where $\mu_\eta = E\{\eta(t)\}$: Mean value , and

$$\sigma_\eta^2 = E\left\{\left(\eta(t)-\mu_\eta\right)^2\right\}:\ \text{Variance}$$

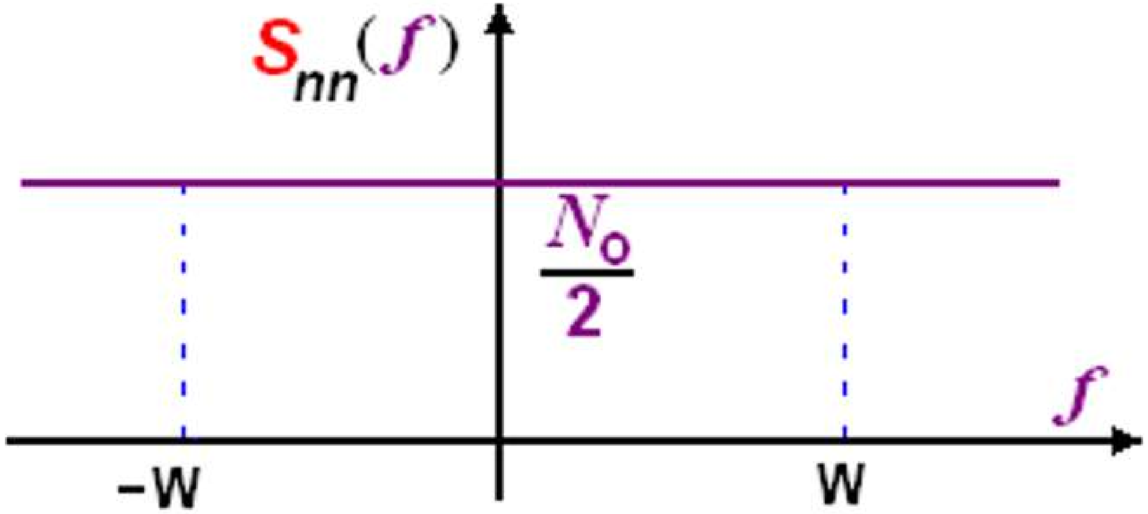
Additive White Noise



Average Power of Additive White Noise

$$P_n = \int_{-W}^{W} S_{nn}(f)\, df$$

$$= \frac{N_0}{2} 2W = N_0 W$$



Noise is assumed to be uncorrelated with the signal.

Performance Measures

1. Signal to Noise Power Ratio

$$\frac{S}{N} = \frac{P_s}{P_n}$$

Where P_s Signal Power

P_n Noise Power

The SNR is often expressed as a logarithmic function with the decibel unit

$$\frac{\tilde{S}}{N}(\text{dB}) = 10\log_{10}\frac{P_s}{P_n}$$

$$SNR(\text{dB}) = 20\log_{10}\frac{V_s}{V_n} = 20\log_{10}\frac{I_s}{I_n}$$

2. Figure of Merit

$$\text{Figure of Merit} = \frac{\text{Output SNR}}{\text{Input SNR}}$$

$$\text{Noise Figure} = \frac{\text{Input SNR}}{\text{Output SNR}}$$

Performance Measures

- *Two key performance measures of a modulation scheme are power efficiency and bandwidth efficiency*

- ***Power efficiency*** is a measure of how favorably the tradeoff between fidelity and signal power is made, and is expressed as *the ratio of the signal energy per bit (E_b) to the noise PSD (N_0) required to achieve a given probability of error (say 10^{-5}):*

$$\eta_p = \frac{E_b}{N_0}$$

Small η_p is preferred

- ***Bandwidth efficiency*** describes the ability of a modulation scheme to accommodate data within a limited bandwidth, In general, it is defined as *the ratio of the data bit rate R to the required RF bandwidth B*:

$$\eta_B = \frac{R}{B} \text{(bps/Hz)}$$

Large η_B is preferred

Error Function

- Error function $\mathrm{erf}(u) = \frac{2}{\sqrt{\pi}} \int_0^u \exp(-z^2)dz$
- Complementary error function $\mathrm{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^\infty \exp(-z^2)dz$
- Q-function $Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^\infty \exp\left(-\frac{z^2}{2}\right)dz$

$$\begin{cases} \mathrm{erf}(-u) = -\mathrm{erf}(u) \\ \mathrm{erfc}(u) = 1 - \mathrm{erf}(u) \\ Q(u) = \frac{1}{2}\mathrm{erfc}\left(\frac{u}{\sqrt{2}}\right) \end{cases}$$

Error Function

- Bounds for error function

$$\operatorname{erfc}(x) = \frac{1}{x\sqrt{\pi}} e^{-x^2} \left(1 - \frac{1}{2x^2} + \frac{1\cdot 3}{2^2 x^4} - \frac{1\cdot 3\cdot 5}{2^3 x^6} + \cdots\right)$$

$$\text{For } x > 0,\ \frac{1}{x\sqrt{\pi}} e^{-x^2}\left(1 - \frac{1}{2x^2}\right) < \operatorname{erfc}(x) < \frac{1}{x\sqrt{\pi}} e^{-x^2}$$

(The bound is good when x is large.)

Error rate due to noise

- The optimal BER formula is important in communications:

$$BER_{\text{opt}} = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_g}{N_0}}\right) = Q\left(\sqrt{\frac{2E_g}{N_0}}\right)$$

- The best decision is $\boxed{y \underset{-1}{\overset{+1}{\gtrless}} 0.}$

Orthogonal Properties of Sine and Cosine Terms

(i) $\int_0^T \sin n\omega_0 t\ dt = \int_0^T \cos n\omega_0 t\ dt = \int_0^{2\pi} \sin nx\ dx = \int_0^{2\pi} \cos nx\ dx = 0$

for all integral values of 'n'

(ii) $\int_0^T \sin m\omega_0 t \cos n\omega_0 t\ dt = \int_0^T \cos n\omega_0 t \cos m\omega_0 t\ dt = 0\ ,\ m \neq n$

(iii) $\int_0^T \sin^2 n\omega_0 t\ dt = \int_0^T \cos^2 n\omega_0 t\ dt = \frac{T}{2}$, and

(iv) $\int_0^{2\pi} \sin^2 nx\ dx = \int_0^{2\pi} \cos^2 nx\ dx = \pi \qquad m = n$

- Any element of set S, $S=\{s_1(t), s_2(t), \ldots, s_M(t)\}$, can be represented as *a point in a vector space* whose coordinates are basis signals $\phi_j(t)$, $j=1,2,\ldots,N$, such that

$$\int_{-\infty}^{\infty} \phi_i(t)\phi_j(t)\,dt = 0, i \neq j; (\rightarrow \text{orthogonal})$$

$$E = \int_{-\infty}^{\infty} [\phi_i(t)]^2\,dt = 1; (\rightarrow \text{normalization})$$

$s_i(t)$ can be represented as a linear combination of the basis signals.

$$\boxed{s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t),\ i = 1,2,\cdots,M}$$

End