Pre-Tutorial (To be completed by student before attending tutorial session)

1. **Consider the following PDA**

   δ(q0, a, z0) = (q0, Xz0)
   δ(q0, a, X) = (q0, XX)
   δ(q0, b, X) = (q1, ε)
   δ(q1, b, X) = (q1, ε)
   δ(q1, ε, z0) = (q1, ε)

   Convert this PDA to CFG.

Solution:

Define each variable Aq_iq_j, representing strings that have the PDA from state q_i with an empty stack to q_j

$Aq_0q_0 \longrightarrow aAq_0q_0X$

$Aq_0q_0 \longrightarrow \epsilon$

$Aq_0q_1 \longrightarrow aAq_0q_0b$

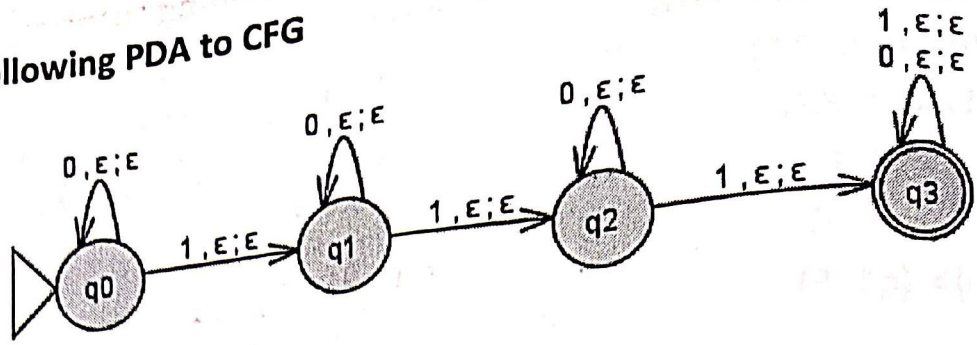
$Aq_1q_1 \longrightarrow bAq_1q_1$

$Aq_1q_1 \longrightarrow \epsilon$

CFG for given PDA is

$S \longrightarrow aSb \mid \epsilon$

3. Convert the following PDA to CFG



Solution:

① $\delta(q_0, 0, \epsilon) = (q_0, \epsilon)$

$\delta(q_0, 1, \epsilon) = (q_1, \epsilon)$

$\delta(q_0, 0, \epsilon) = (q_1, \epsilon)$

$\delta(q_1, 1, \epsilon) = (q_2, \epsilon)$

$\delta(q_2, 0, \epsilon) = (q_2, \epsilon)$

$\delta(q_2, 1, \epsilon) = (q_3, \epsilon)$

$\delta(q_3, 0, \epsilon) = (q_3, \epsilon)$

$\delta(q_3, 1, \epsilon) = (q_3, \epsilon)$

② Aq_0q_1 = q_0 to q_1

Aq_1q_2 = q_1 to q_2

Aq_2q_3 = q_2 to q_3

$Aq_0q_0 \neq Aq_1q_1$: Self loop

Aq_0q_0, Aq_1q_1 : self loops

③ Create CFG Rules:

$Aq_0q_1 \rightarrow 1$

$Aq_1q_2 \rightarrow 1$

$Aq_2q_3 \rightarrow 1$

$Aq_0q_0 \rightarrow 0Aq_0q_0 / \epsilon$

$Aq_1q_1 \rightarrow 0Aq_1q_1 / \epsilon$

$Aq_2q_2 \rightarrow 0Aq_2q_2 / \epsilon$

④ $S = Aq_0q_3$

⑤ $S \rightarrow Aq_0q_1, Aq_1q_2, Aq_2q_3$

$Aq_0q_0 \rightarrow 0Aq_0q_0 / \epsilon$

$Aq_1q_1 \rightarrow 0Aq_1q_1 / \epsilon$

$Aq_2q_2 \rightarrow 0Aq_2q_2 / \epsilon$

$Aq_3q_3 \rightarrow 0Aq_3q_3 / 1Aq_3q_3 / \epsilon$

$Aq_0q_3 \rightarrow 1$

$\Leftrightarrow Aq_0q_1 \rightarrow 1$

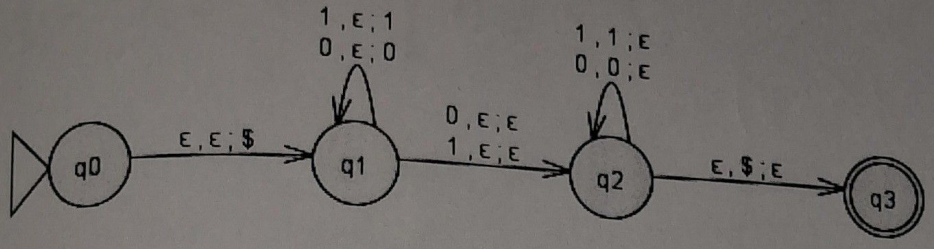
$- Aq_1q_2 \rightarrow 1$

$- Aq_2q_3 \rightarrow 1$

Final CFG: $S \rightarrow 111 / 0S1 / 0 / \epsilon$

(N-TUTORIAL (To be carried out in presence of faculty in classroom)

1. Convert the following PDA to CFG.



Solution:

$(q_1, 1, \epsilon) \rightarrow (q_1, 1)$

$(q_1, 0, \epsilon) \rightarrow (q_1, 0)$

$(q_1, 0, 0) \rightarrow (q_2, \epsilon)$

$(q_1, 1, 1) \rightarrow (q_2, \epsilon)$

$(q_2, 0, 1) \rightarrow (q_2, \epsilon)$

$(q_2, \epsilon, \$) \rightarrow (q_3, \epsilon)$

$S \rightarrow A0_3$

Date

2. Let L be the language $\{0^n1^{2n} \mid n \geq 1\}$. For the given language ... the PDA.

Solution:

$L = \{0^n 1^{2n} / n \geq 1\}$

$= \{011, 001111, 000111111, 00001111111, \ldots\}$

1) $\delta(q_0, 0, Z_0) = (q_0, 0Z_0)$ } 0's
2) $\delta(q_0, 0, 0) = (q_0, 00)$
3) $\delta(q_0, 1, 0) = (q_1, 0)$ } rejected
4) $\delta(q_1, 1, 0) = (q_2, \epsilon)$ }
5) $\delta(q_2, 1, 0) = (q_1, 0)$
6. $\delta(q_1, \epsilon, Z_0) = (q_3, Z_0)$

$(q_1, \epsilon, Z_0) = (q_3, Z_0)$ ⟶ accepted

$\therefore q_3 \in F$

0 — push '0'

1 = { q_1 - '1' - no. op - change 'q_2' }

$M = (\{q_0, q_1, q_2, q_3\}, \{0,1\}, \{0, Z_0\}, \delta, q_0, Z_0, \{q_3\})$

3.Construct a PDA for language L = {0^n1^m | n >= 1, m >= 1, m > n+2}

Solution:

States :—

q_0 → Initial states

q_1 → (read 0s and push)

q_2 → (read 1s and pop)

q_3 (ensure extra 1s)

q_4 → final state

Transitions :

Reading 0's :

$\delta(q_0, 0, \epsilon) = (q_1, x)$ — push x for each x & move to q_1 0's

$\delta(q_1, 0, \epsilon) \rightarrow (q_1, x)$

$\delta(q_1, 1, x) = (q_2, \epsilon)$ — pop 1 x for each 1

$\delta(q_2, 1, x) = (q_2, \epsilon)$ — pop 1 x for each 1

$\delta(q_2, 1, x) = (q_2, \epsilon)$ — pop 1 x for each 1 } is repeat

$\delta(q_2, 1, x) = (q_2, \epsilon)$ — pop x for each 1

$\delta(q_2, 1, \epsilon) \rightarrow$ if stack is empty move to q_3 & read additional acceptance.

$\delta(q_3, 1, \epsilon) = (q_3, \epsilon)$

$\delta(q_3, \epsilon, \epsilon) = (q_4, \epsilon)$

Initial state → q_0

final state → q_4

4. Design a PDA for accepting a language $\{a^n b^m c^{(m+n)} \mid m, n >= 1\}$.

Solution:

{ abcc , aaabbbb ccccccc , ---- }

push (aaa) pop (bbbb) ccc ccc c

$\delta(q_0, a, \epsilon) = (q_1, A)$
$\delta(q_1, a, \epsilon) = (q_1, A)$ } a's
$\delta(q_1, b, \epsilon) = (q_2, B)$

$\delta(q_2, b, \epsilon) = (q_2, B)$
$\delta(q_2, c, A) = (q_3, \epsilon)$ } b's
$\delta(q_2, c, B) = (q_3, \epsilon)$

$\delta(q_3, c, A) = (q_3, \epsilon)$
$\delta(q_3, \epsilon, B) = (q_3, \epsilon)$ } c's
$\delta(q_3, \epsilon, \epsilon) = (q_4, \epsilon) \Rightarrow$ acceptance

Initial state : q_0
final state : q_4

Date

Post-Tutorial (To be carried out by student after attending tutorial session)

1. Construct a PDA that accept the language $L = \{0^n1^n \mid n > 0\}$

Solution:

$\delta(q_0, 0, \epsilon) = (q_0, x)$ — 0's

$\delta(q_0, 1, x) = (q_1, \epsilon)$

$\delta(q_1, 1, x) = (q_1, \epsilon)$

$\delta(q_1, 1, x) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, \epsilon) = (q_2, \epsilon)$ - acceptance

Initial state = q_0

final state = q_2

$L = \{01, 0011, \ldots\}$

~~0~~ ~~0~~ 0011 ~~1~~ ~~1~~

for every 0 pop 1

2. Construct a PDA from the following CFG. G = ({S, X}, {a, b}, P, S) where the productions are –

S → XS | ε ,

A → aXb | Ab | ab

Solution:

$G = (\{S, X\}, \{a, b\}, P, S)$

$S \rightarrow XS \mid \epsilon$

$A \rightarrow aXb \mid Ab \mid ab$

$\delta(q_0, \epsilon, \epsilon) = (q_1, S)$ — push

$S \rightarrow XS$:

$\delta(q_1, \epsilon, \delta) = (q, XS)$

$S \rightarrow \epsilon$: $\delta(q_1, \epsilon, d) = (q_1, \epsilon)$

$X \rightarrow aXb$: $\partial(q, \epsilon, X) = (q_1, \epsilon)$

$X \rightarrow Ab$: $\partial(q_1, \epsilon, X) = (q_1, Ab)$

$X \rightarrow ab$: $\partial(q, \epsilon, X) = (q, ab)$

→ $\delta(q_1, a, a) = (q_1, \epsilon)$

– $\delta(q_1, b, b) = (q_1, \epsilon)$

← $\delta(q_1, \epsilon, \epsilon) = (q_2, \epsilon)$ ⇒ Acceptance

q_0 → Initial State

q_2 → final State.

3.Construct a PDA for language L = {$0^n1^m2^m3^n$ | n>=1, m>=1}

Solution:

L = {0123, 00112233, ...}.

$\delta(q_0, 0, \epsilon) = (q_0, x)$

$\delta(q_0, 1, \epsilon) = (q_1, \epsilon)$ } 0's

$\delta(q_1, 1, \epsilon) = (q_1, y)$

$\delta(q_1, 2, Y) = (q_2, \epsilon$ } 1's

$\delta(q_2, 2, 4) = (q_2, \epsilon)$

$\delta(q_2, 3, x) = (q_3, \epsilon)$ } 2's

$\delta(q_3, 3, x) = (q_3, \epsilon)$ — 3's

$\delta(q_3, \epsilon, \epsilon) = (q_4, \epsilon)$ — acceptance.

Initial state — q_0

final state — q_4

Experiment #	<TO BE FILLED BY STUDENT>	Student ID	<TO BE FILLED BY STUDENT>
Date	<TO BE FILLED BY STUDENT>	Student Name	<TO BE FILLED BY STUDENT>

Viva Question:

1.Write the instantaneous Description (ID) of PDA?

Solution:

The instantaneous Description (ID) of a PDA provides a snap shot its current Configuration.

2.What are the components of PDA?

Solution:

I/p tape: A sequence of symbols from a finite alphabet

Stack: A storage structure that allows the PDA to store an Arbitrary amount of information

4.Write the applications where the PDA used in real time?

Solution:

Syntax parsing: used in compilers & Interpreters for programming language to parse inputs & syntax trees.

(For Evaluator's use only)

Comment of the Evaluator (if Any)	Evaluator's Observation Marks Secured: out of **50** Full Name of the Evaluator: Signature of the Evaluator Date of Evaluation:

Course Title	AUTOMATA THEORY AND FORMAL LANGAUGES	ACADEMIC YEAR 2023-24