Experiment #		STUDENT ID:	
Date:		STUDENT NAME:	

SUBJECTCODE: 23MT2005

PROBABILITY STATISTICS AND QUEUING THEORY

Tutorial 10:

- Demonstrate Introduction to queues, measures of system performance
- Demonstrate Characteristics of queueing systems.

Date of the Session: //____________________ Time of the Session:______to______

Learning outcomes:

Understanding the queueing theory

Demonstrate the performance measures of queueing system

1. In the production shop of a company, the breakdown of the machinists is found to be distributed with an average rate of 3 machines per hour. Breakdown time at one machine co per hour to the company. There are two choices before the company for hiring the repairm the repairmen is slow but cheap the other fast but expensive. The slow-cheap repairman de 20 per hour and will repair the breakdown machines exponentially at the rate of 4 per hou expensive repairman demands Rs. 30 per hour and will repair machines exponentially at t per hour, which repairmen should be hired?

Solution:

Fast repairman:- $\lambda = 3, \mu = 6$

$$L = \frac{3}{6-3} = 1$$

Waiting cost $= 1 \times 40 = 40$

Total cost $= 40 + 30 = \boxed{\text{Rs } 70/\text{hr}}$

slow!

$\lambda = 3, \mu = 4$

$L = \frac{3}{4-3} = 3$

waiting cost $= 3 \times 40 = 120$

Total cost $= 120 + 20 = \boxed{Rs.140/hr}$

$\therefore$ choose fast repairman

2. Arrivals of machinists at a tool crib are Poisson distributed at an average rate of 6 per hour. The length of time the machinists must remain at the tool crib is exponentially distributed with the average time being 0.05 hour.

a) What is the average number of machinists in the queue?
b) What is the average number of machinists at the tool crib?
c) What is the probability that a machinist arriving at the tool crib will have to wait?
d) What is the average length of the queue that from time to time?
e) What is the probability that there are more than 2 machinists at the tool crib?
f) What is the probability that no machinist is waiting to be served?
g) What is the expected length of a non-empty queue?
h) Estimate the fraction of time that there is no machinist at the tool crib?

$\lambda = 6$

$\mu = 1/0.05 = 20$

a) $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = 0.1286$

b) $L = \frac{\lambda}{\mu - \lambda} = 0.4286$

c) $\rho = \frac{\lambda}{\mu} = \frac{6}{20} = 0.3$

d) $L_q = 0.13$

e) $P(X > N) = 1 - \sum_{k=0}^{n} P(X = k)$

$= 0.027$

f) $P(0) = 1 - \rho = 1 - 0.3 = 0.7$

g) $\frac{\lambda}{\mu - \lambda} = \cancel{0} \frac{\lambda^2}{\mu(\mu-\lambda)} = 0.3$

h) $P(0) = 0.7$

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3. A maintenance service facility has Poisson arrival rates, negative exponential service times, and operates on a first-come first-served queue discipline. Breakdowns occur on an average of three per day, with a range of zero to eight. The maintenance crew can service on an average six machines per day with a range from zero to service. Find the

i) utilization factor of the service facility,

ii) mean time in the system,

iii) mean number in the system in Break down or repair,

iv) mean waiting time in the queue,

v) probability of finding two machines in the system.

vi) expected number in the queue.

Solution:

$\because \lambda = 3$

$\mu = 6$

i) $\rho = \frac{\lambda}{\mu} = \frac{3}{6} = 0.5$

ii) $W = \frac{1}{\mu - \lambda} = 0.3333$ days

iii) $L = \frac{\lambda}{\mu - \lambda} = 1$

vi) $L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$

$= 0.5$

iv) $W_q = \frac{\lambda}{\mu(\mu - \lambda)} = 0.1667$

v) $P(X = n) = (1 - \rho)\rho^n$

$= 0.125$

4. Customers arrive at a one-window drive-in-counter according to a Poisson distribution with mean 10 per hour. Service time per customer is exponential with the mean 5 minutes. The car space in front of the window, including that for serviced cars can accommodate a maximum of 3 cars. Other cars can wait outside this space.

i) What is the probability that a customer arriving can drive directly to the space in front of the window?

ii) What is the probability that an arriving customer will have to wait outside the indicated space?

iii) How long is an arriving customer expected to wait before starting service?

Solution: Given

$\lambda = 10$

$\mu = 1$ Per 5 minutes $= 12$ customers Per hour

i) $P(\text{no wait}) = 1 - P(\text{system is full})$.

$P(\text{system is full}) = \left(\frac{\lambda}{\mu}\right)^3 = \left(\frac{10}{12}\right)^3 = 0.5787$

$P(\text{no wait}) = 1 - 0.5787$

$= 0.4213$

ii) $P(\text{wait outside}) = P(\text{system is full})$

$= 0.5787$

iii) $W_q = \frac{P(\text{wait})}{\mu(1-\rho)} = \frac{0.5787}{12 \times 0.1667} = 0.2893$ hrs.

$= 0.2893 \times 60$

$= 17.36$ min

5. An email server processes incoming emails, and the system administrator wants to understand the average number of emails in the server queue. The server receives an average of 50 emails per minute and the average time a single email spends in the queue is 2 minutes. Calculate the mean number of emails in the server queue using Little's Law.

Solution:

~~Given~~

~~$\lambda = 50$~~

~~$w = 2$~~

~~$L = \lambda \times w = 100$~~

To calculate mean no.of emails in server queue using Littles law

formula:- $L = \lambda \times w$

Given

Arrival rate $\lambda = 50$

Average time $w = 2$

Now

$L = \lambda \times w$

$L = 50 \times 2$

$= 100.$

6. A car park contains 5 cars. The arrival of cars is Poisson at a mean rate of 10 per hour. The length of time each car spends in the car park has negative exponential distribution with mean of 5 hours.

i) Find the probability that arrival finds the car park empty

ii) Find the probability that an arrival finds the car park is full.

iii) How many cars are in the car park on average?

iv) What is the effective arrival rate?

Given $\lambda = 10$

$\mu = 1$ for 5 hrs $= \frac{1}{5}$ cars per hr.

$c = 5$ cars

i) $P_0 = \dfrac{1}{\sum_{n=0}^{c} \dfrac{(\lambda/\mu)^n}{n!}}$

$= \dfrac{1}{2863758.67} \simeq 0.000000349$

ii) $P_5 = \dfrac{\left(\frac{\lambda}{\mu}\right)^5}{5!} \times P_0 = \dfrac{50^5}{5!} \times 0.000000349$

$= 0.0905.$

iv) $\lambda_{eff} = \lambda \times (1 - P_{full})$

$= 0.95.$

iii) $L = \sum_{n=0}^{c} n \times P_n$.

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VIVA QUESTIONS

1. What are the key performance measures used to evaluate a queueing system?

Average no. of customers, Lq, W, P, Wq.

2. What is a queue in the context of computer science or operations research?

It is wait for processing in fcfs manner

3. Define the term "arrival rate" in a queueing system.

average no. of entities arriving at queue per unit of time.

4. Explain the term "utilization factor" in a queueing system.

is proportion of time the server is busy.
arrival rate to service rate.

(For Evaluators use only)

Comment of the Evaluator (if Any)	Evaluator's Observation
	Marks Secured:___________out of______ Full Name of the Evaluator: Signature of the Evaluator: Date of Evaluation