Advanced Algorithms & Data Structures

Department of CSE

ADVANCED ALGORITHMS AND DATA STRUCTURES
23CS03HF

Topic:

Max Flow Min Cut Theorem

Session - 28

AIM OF THE SESSION

To familiarize students with the concept of the Max Flow Min Cut Theorem.

INSTRUCTIONAL OBJECTIVES

This Session is designed to:

1. Demonstrate :- Max Flow Min Cut Theorem.
2. Describe :- Theorem.

LEARNING OUTCOMES

At the end of this session, you should be able to:

1. Define :- Max Flow Min Cut Theorem.
2. Describe :- Theorem
3. Summarize:- The **maximum value of the flow** from a source to a sink is equal to the **minimum capacity of a cut** that separates the source and the sink..

Maximum Flow

It is defined as the maximum amount of flow that the network would allow to flow from source to sink.

Multiple algorithms exist in solving the maximum flow problem.

Two major algorithms to solve these kind of problems are Ford-Fulkerson algorithm and Dinic's Algorithm.

Max Flow Min Cut Theorem

Theorem: (Optimality conditions for max flows). The following are equivalent.

1. A flow x is maximum.
2. There is no augmenting path in G(x).
3. There is an s-t cutset (S, T) whose capacity is the flow value of x.

Corollary: (Max-flow Min-Cut). The maximum flow value is the minimum value of a cut.

Proof of Theorem:

$1 \Rightarrow 2$. (not $2 \Rightarrow$ not 1)

Suppose that there is an augmenting path in G(x). Then x is not maximum.

Max Flow Min Cut Theorem Contd.

$3 \Rightarrow 1.$

Let v = F_x(S, T) be the flow from s to t. By assumption, v = CAP(S, T). By weak duality, the maximum flow is at most CAP(S, T). Thus the flow is maximum.

$2 \Rightarrow 3.$

Suppose there is no augmenting path in G(x).

Claim:

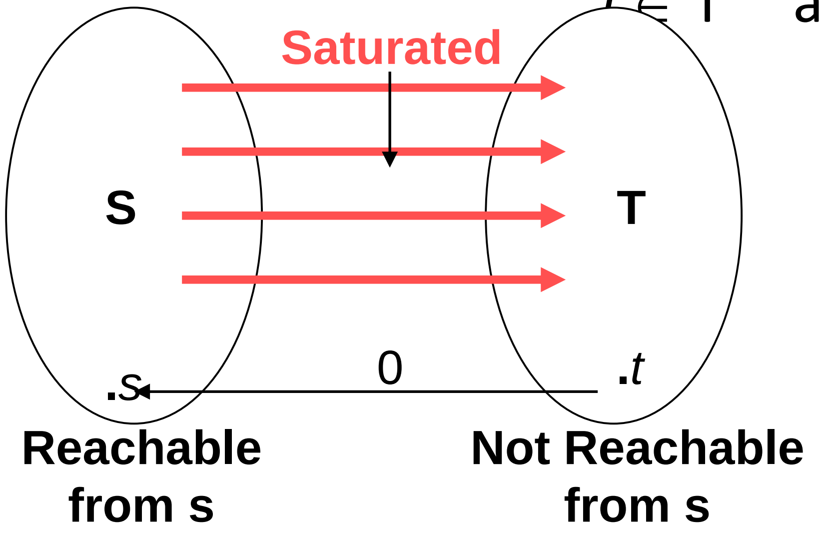
Let S be the set of nodes reachable from s in G(x).

Let T = N\S. Then there is no arc in G(x) from S to T.

Max Flow Min Cut Theorem Contd.

Thus $i \in S$ and $j \in T$ $\Rightarrow$ $f(i, j) = c(i, j)$

$i \in T$ and $j \in S$ $\Rightarrow$ $f(i, j) = 0$.



There is no arc from S to T in G(x)

It follows that

$$F_x(S,T) = \Sigma_{i\in S}\Sigma_{j\in T}\, f(i, j) - \Sigma_{i\in S}\Sigma_{j\in T}\, f(j, i)$$

$$= \Sigma_{i\in S}\Sigma_{j\in T}\, c(i, j) - \Sigma_{i\in S}\Sigma_{j\in T}\, 0 = CAP(S,T)$$

SUMMARY

- The maximum value of the flow in a flow network from a source s to a sink t is equal to the capacity of the minimum cut that separates the source and sink.

SELF-ASSESSMENT QUESTIONS

- What does the Max-Flow Min-Cut Theorem state?

A. The minimum flow in a network is equal to the maximum cut capacity.

B. The maximum flow in a network is always equal to the sum of edge capacities.

C. The maximum flow in a network is equal to the capacity of the minimum cut.

D. The maximum flow is always greater than the minimum cut.

- Which algorithm is commonly used to find the maximum flow in a network?

A. Dijkstra's Algorithm

B. Kruskal's Algorithm

C. Ford-Fulkerson Algorithm

D. Bellman-Ford Algorithm

TERMINAL QUESTIONS

1. What is a flow network, and how is the concept of a cut defined in the context of the Max-Flow Min-Cut Theorem? Provide an example of how a minimum cut is found in a simple network.

REFERENCES FOR FURTHER LEARNING OF THE SESSION

Reference Books :

1. Introduction to Algorithms, Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein., 3rd, 2009, The MIT Press.
2 Algorithm Design Manual, Steven S. Skiena., 2nd, 2008, Springer.
3 Data Structures and Algorithms in Python, Michael T. Goodrich, Roberto Tamassia, and Michael H. Goldwasser., 2nd, 2013, Wiley.
4 The Art of Computer Programming, Donald E. Knuth, 3rd, 1997, Addison-Wesley Professiona.

MOOCS :

1. https://www.coursera.org/specializations/algorithms?=
2.https://www.coursera.org/learn/dynamic-programming-greedy-algorithms#modules

THANK YOU