(DEEMED TO BE UNIVERSITY)

23MT2014

THEORY OF COMPUTATION

Topic:

COMPUTING FUNCTIONS WITH TM, UNDECIDABILITY, UTM

Session – 18-19

AIM OF THE SESSION

The aim of this session is to provide an understanding of recursively enumerable and recursive languages, the computation of functions using Turing Machines, and the concept of combining multiple Turing Machines.

INSTRUCTIONAL OBJECTIVES

This Session is designed to:

- To define and differentiate recursively enumerable and recursive languages.
- To explain how Turing Machines can be used to compute functions and solve computational problems.
- To demonstrate the process of combining multiple Turing Machines to perform complex computations.

LEARNING OUTCOMES

At the end of this session, you should be able to:

1. Differentiate between recursively enumerable and recursive languages.
2. Analyze the computational complexity of functions and evaluate their computability using Turing Machines.
3. Design and implement a combined Turing Machine to solve a given computational problem.

Recursively Enumerable and Recursive Languages

Definition:

A language is **recursively enumerable** if some Turing machine accepts it

Let L be a recursively enumerable language

and M the Turing Machine that accepts it

For string w :

if $w \in L$ then M halts in a final state

if $w \notin L$ then M halts in a non-final state

or loops forever

Definition:

A language is **recursive**
if some Turing machine accepts it
and halts on any input string

In other words:

A language is recursive if there is
a **membership algorithm** for it

Let L be a recursive language

and M the Turing Machine that accepts it

For string w :

if $w \in L$ then M halts in a final state

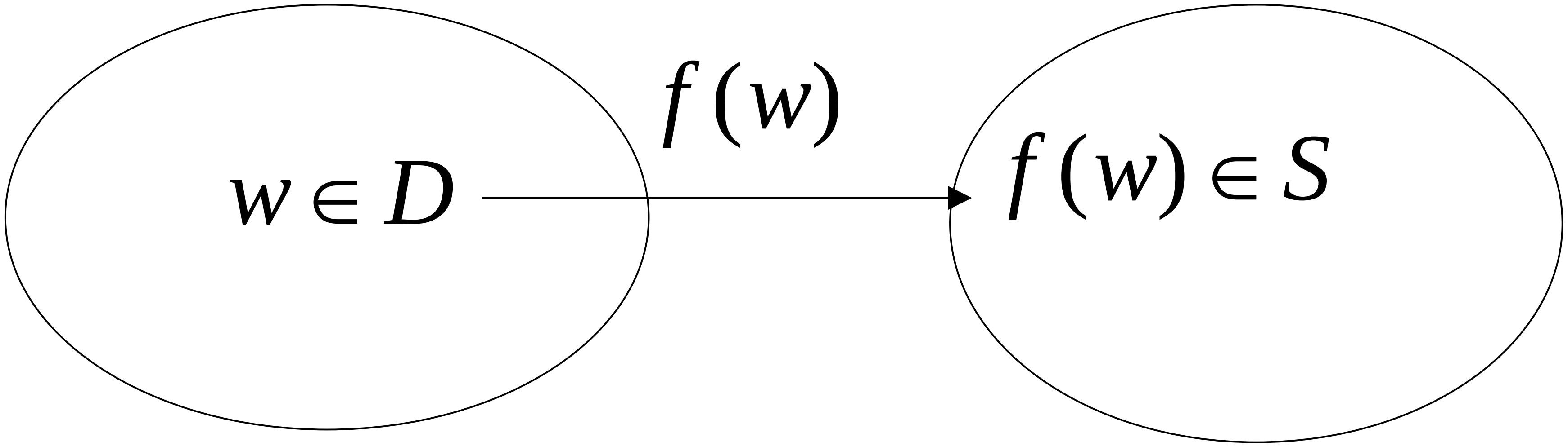
if $w \notin L$ then M halts in a non-final state

Computing Functions with Turing Machines

A function $f(w)$ has:

Domain: D

Result Region: S



A function may have many parameters:

Example: Addition function

$$f(x, y) = x + y$$

Integer Domain

Decimal: 5

Binary: 101

Unary: 11111

We prefer **unary** representation:

easier to manipulate with Turing machines

Definition:

A function f is computable if there is a Turing Machine M such that:

Initial configuration

$\Diamond$	w	$\Diamond$	

q_0 initial state

Final configuration

$\Diamond$	$f(w)$	$\Diamond$	

q_f final state

For all $w \in D$ Domain

In other words:

A function f is computable if there is a Turing Machine M such that:

$$q_0\, w \overset{*}{\succ} q_f\, f(w)$$

Initial Configuration

Final Configuration

For all $w \in D$ Domain

Example

The function $f(x, y) = x + y$ is computable

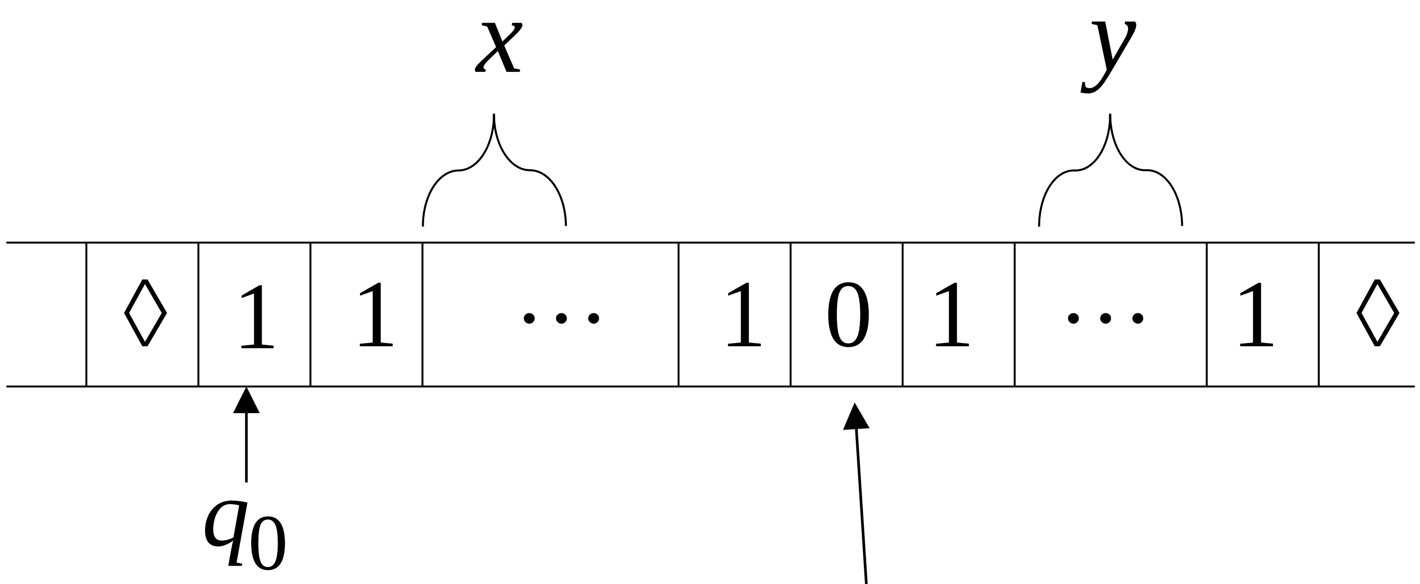
x, y are integers

Turing Machine:

Input string: $x0y$ unary

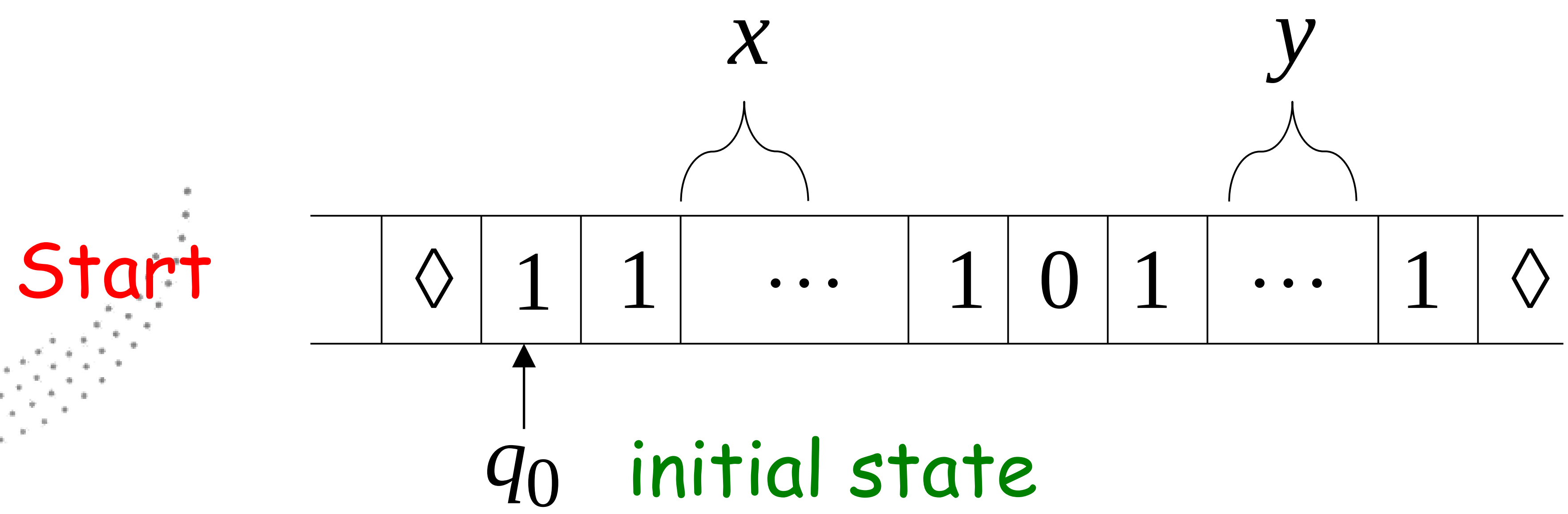
Output string: $xy0$ unary

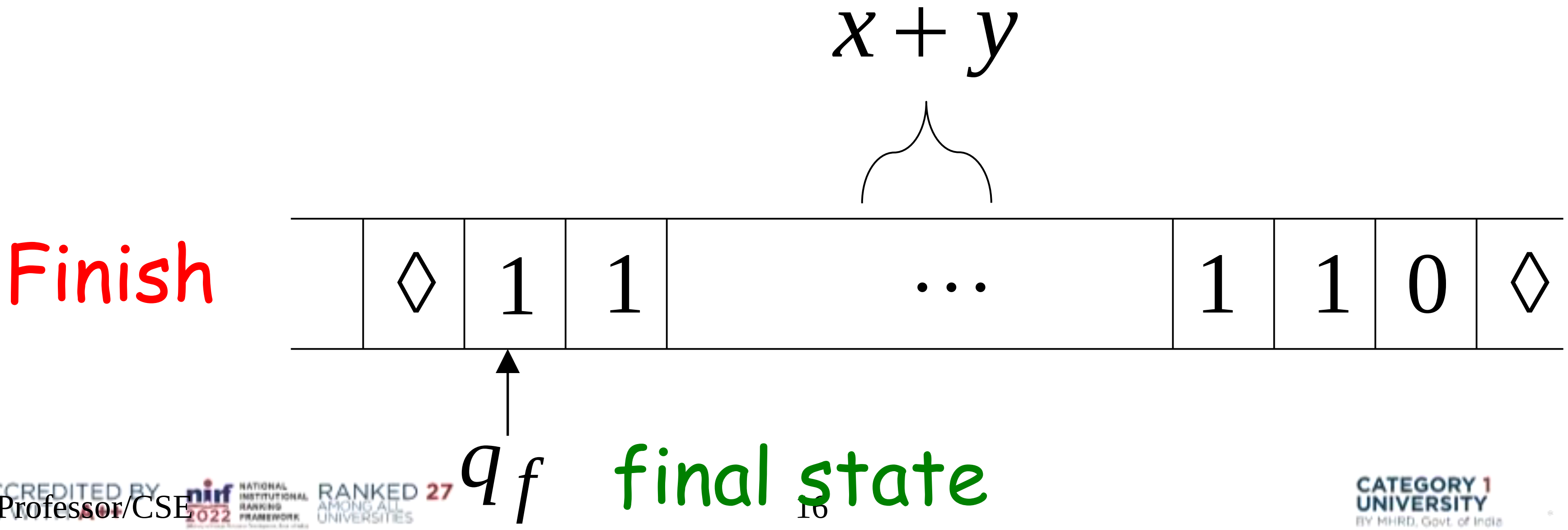
Start



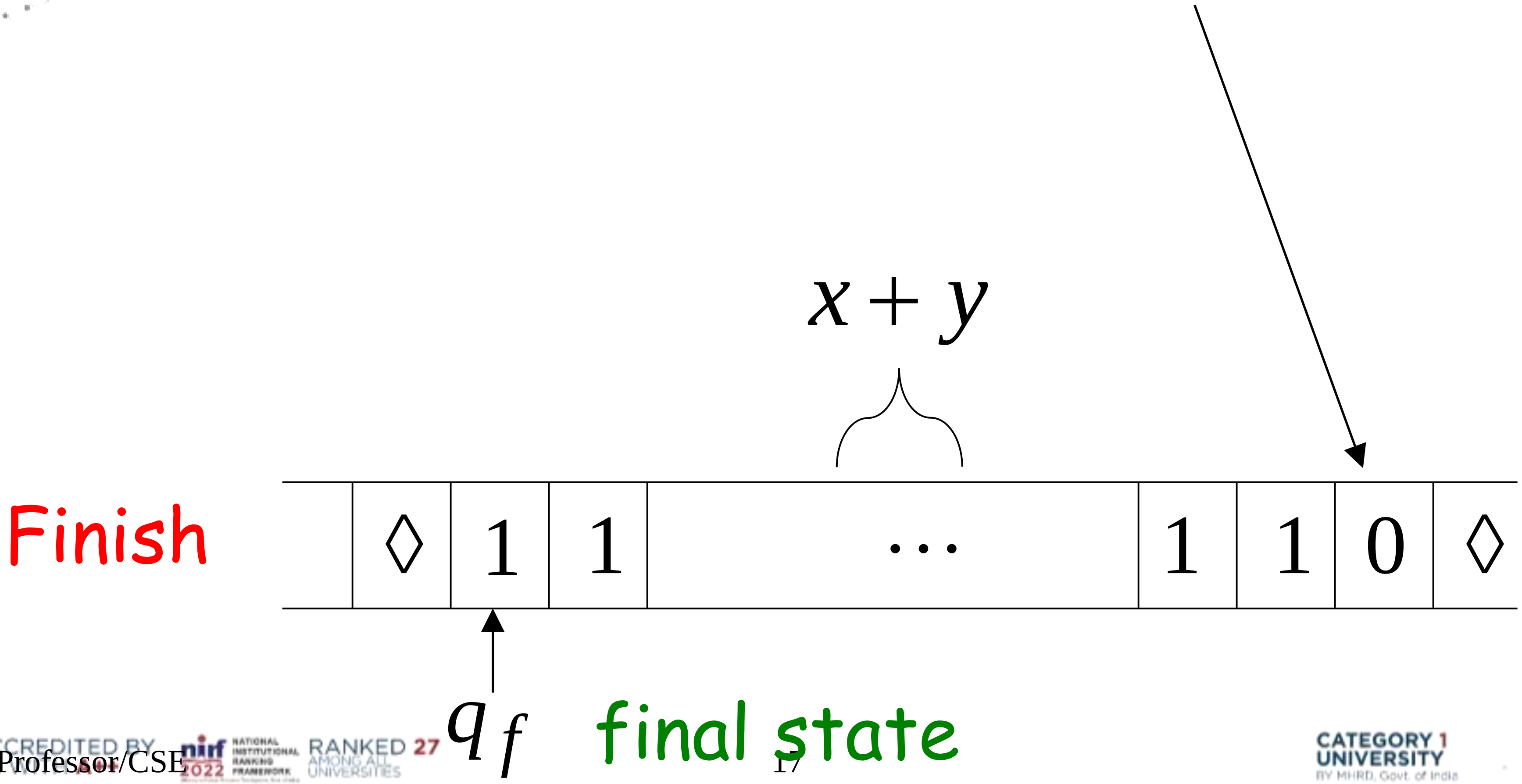
initial state

The 0 is the delimiter that separates the two numbers

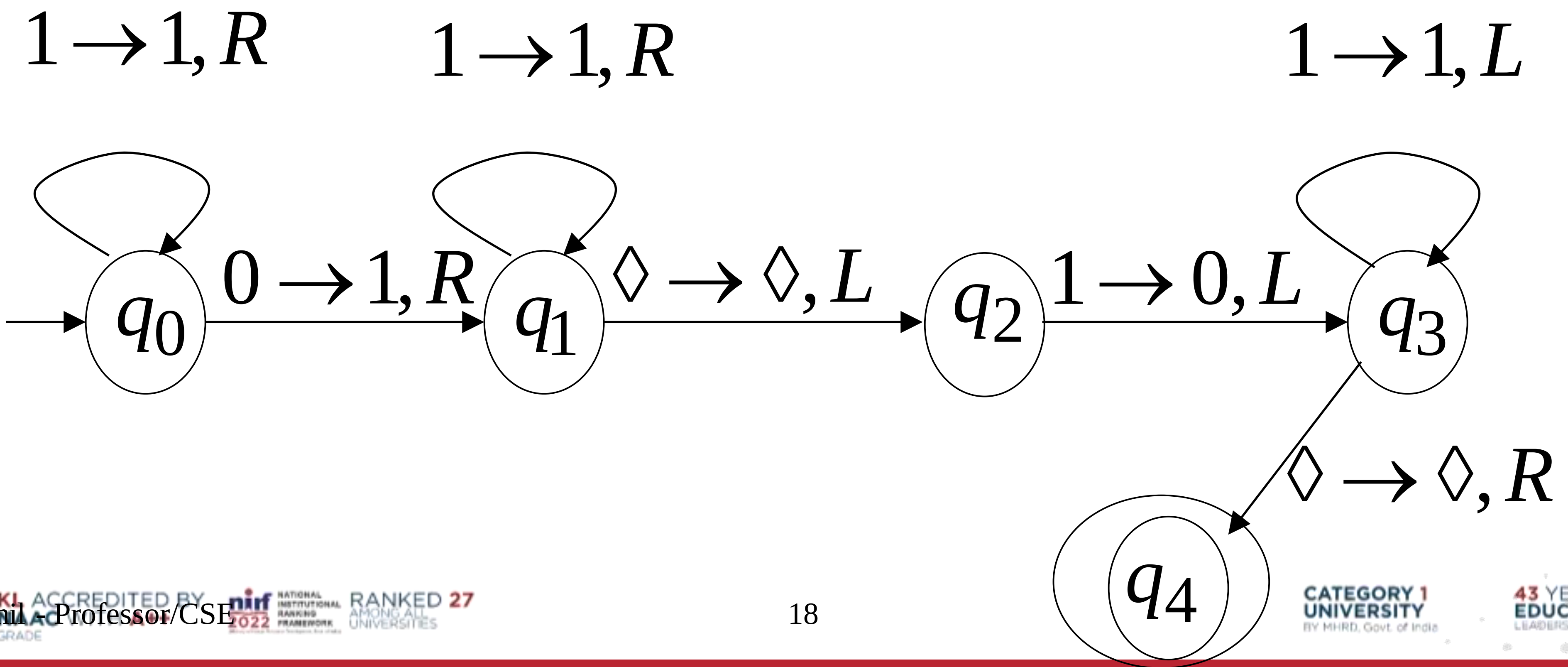

Start
x
y
◊ 1 1 ... 1 0 1 ... 1 ◊
q_0
initial state


Finish
$x+y$
◊ 1 1 ... 1 1 0 ◊
q_f
final state

The 0 helps when we use
the result for other operations



Turing machine for function $f(x,y) = x + y$



Execution Example:

$x = 11$ (2)

$y = 11$ (2)

Time 0

x y

	$\Diamond$	1	1	0	1	1	$\Diamond$

q_0

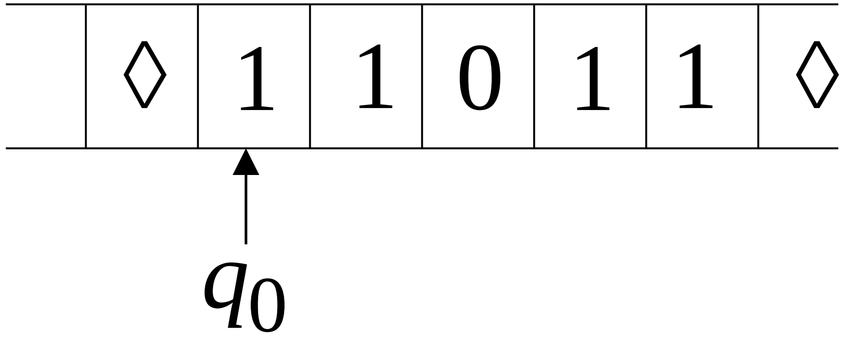
Final Result

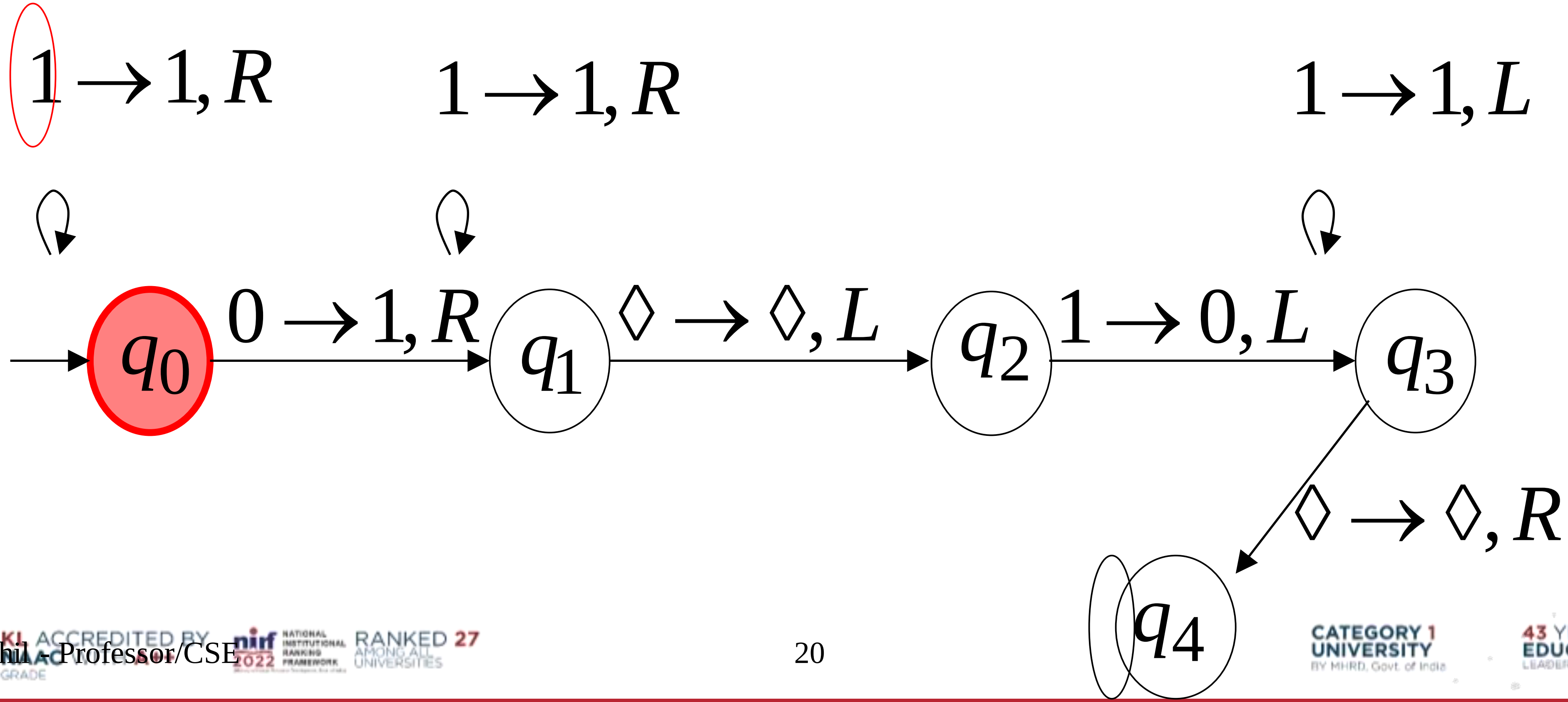
$x + y$

	$\Diamond$	1	1	1	1	0	$\Diamond$

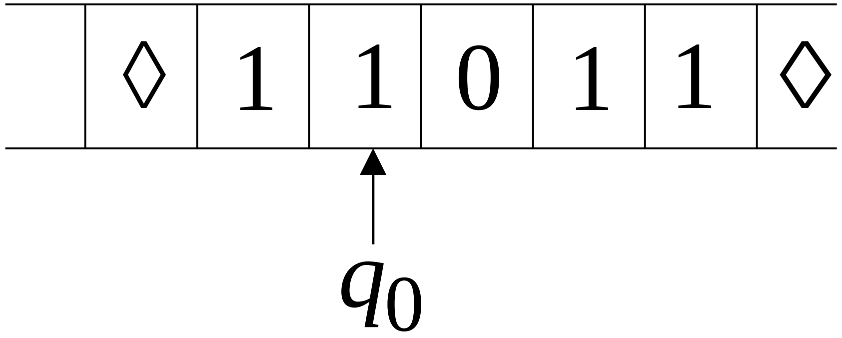
q_4

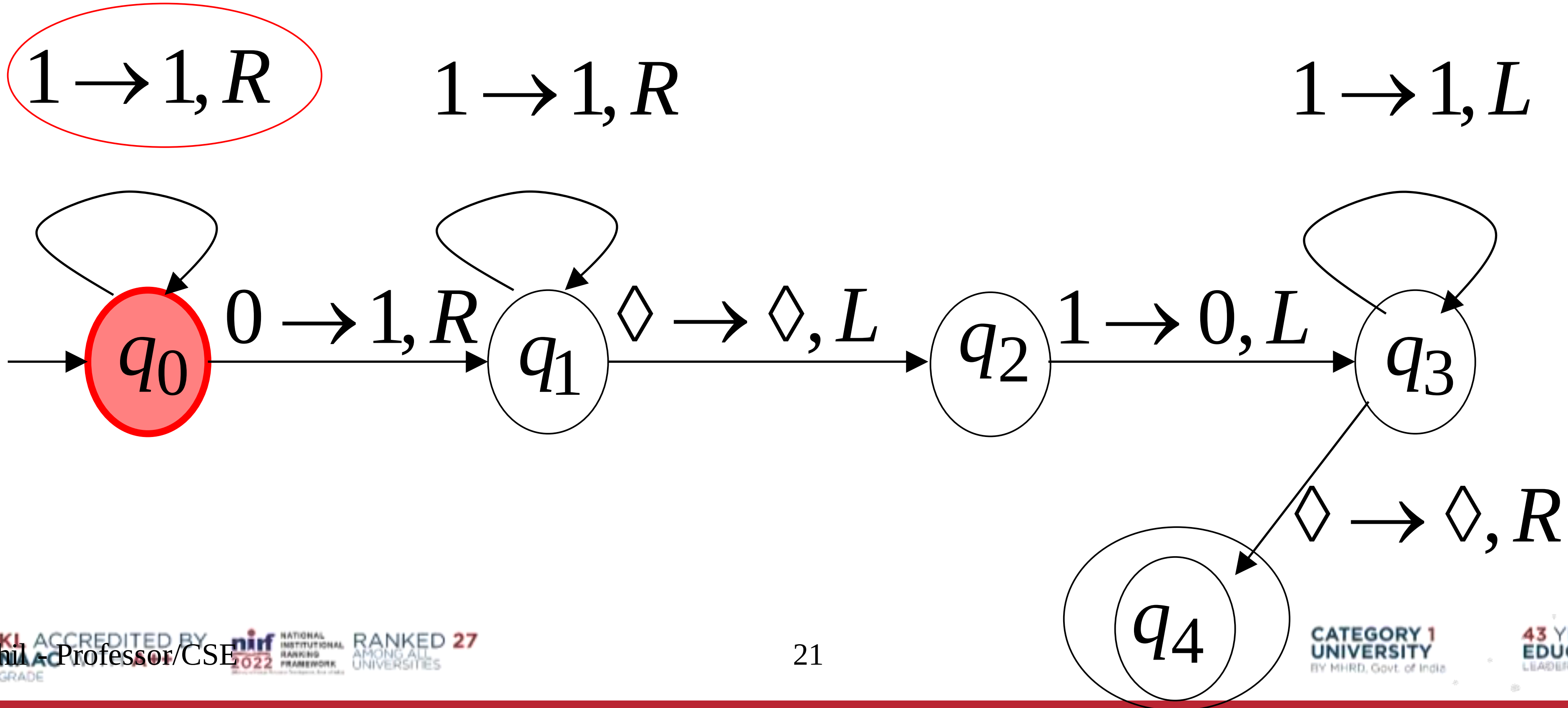
Time 0


◊
1
1
0
1
1
◊
q_0

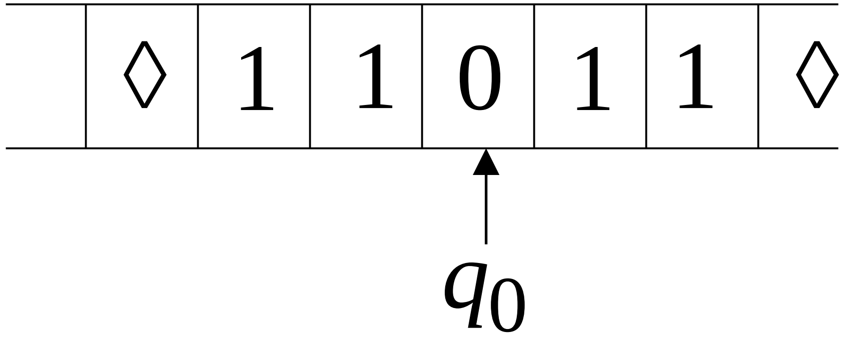

$1 \rightarrow 1, R$
$1 \rightarrow 1, R$
$1 \rightarrow 1, L$
q_0
$0 \rightarrow 1, R$
q_1
$◊ \rightarrow ◊, L$
q_2
$1 \rightarrow 0, L$
q_3
$◊ \rightarrow ◊, R$
q_4

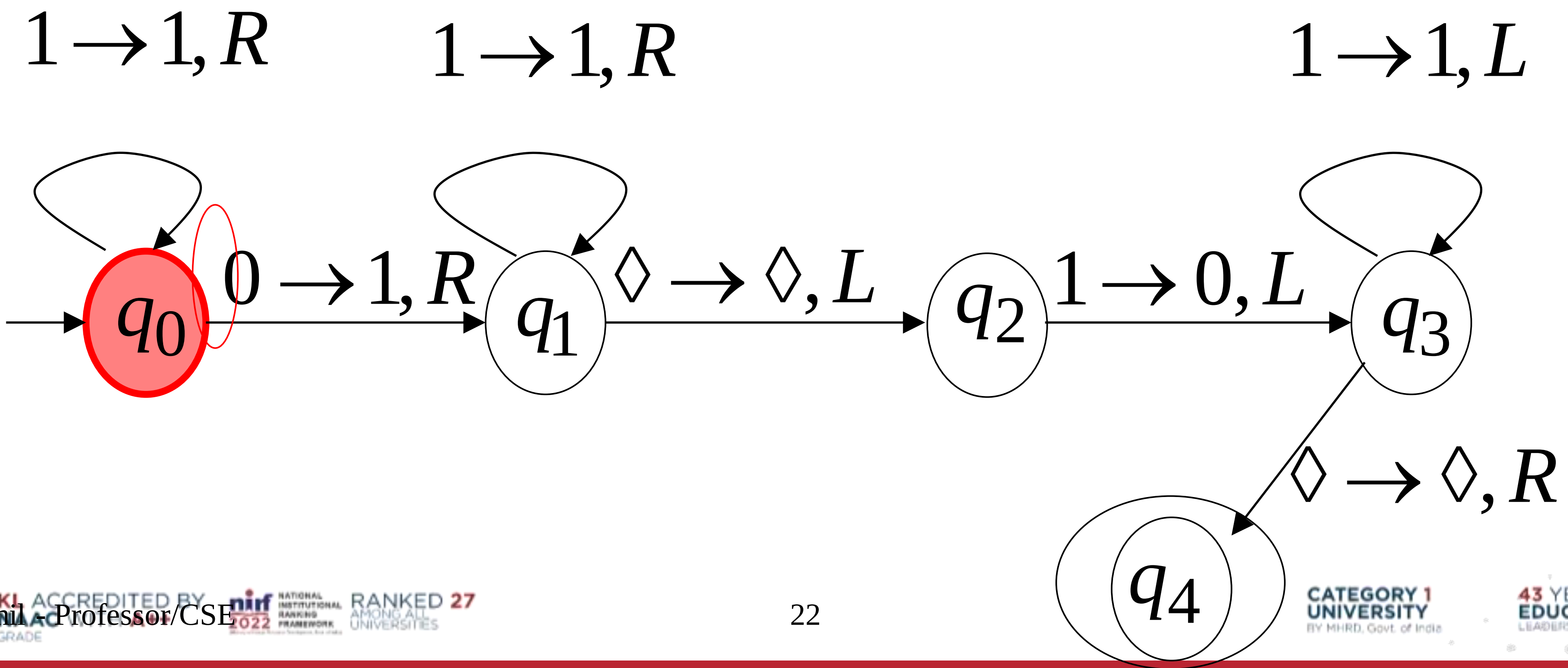
Time 1


◊ 1 1 0 1 1 ◊
q_0

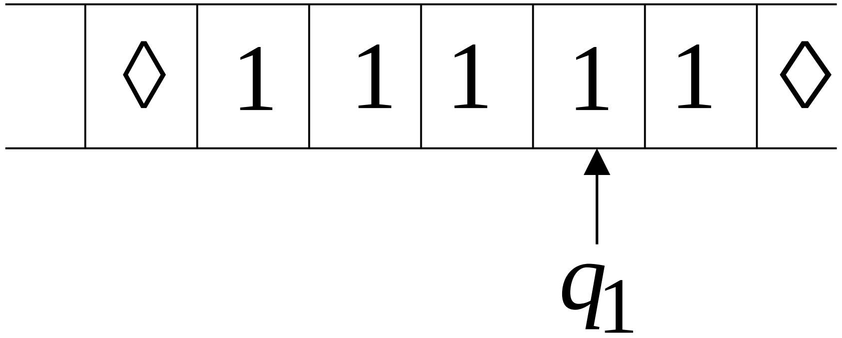

$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$0 \to 1, R$
$\lozenge \to \lozenge, L$
$1 \to 0, L$
$\lozenge \to \lozenge, R$
q_0
q_1
q_2
q_3
q_4

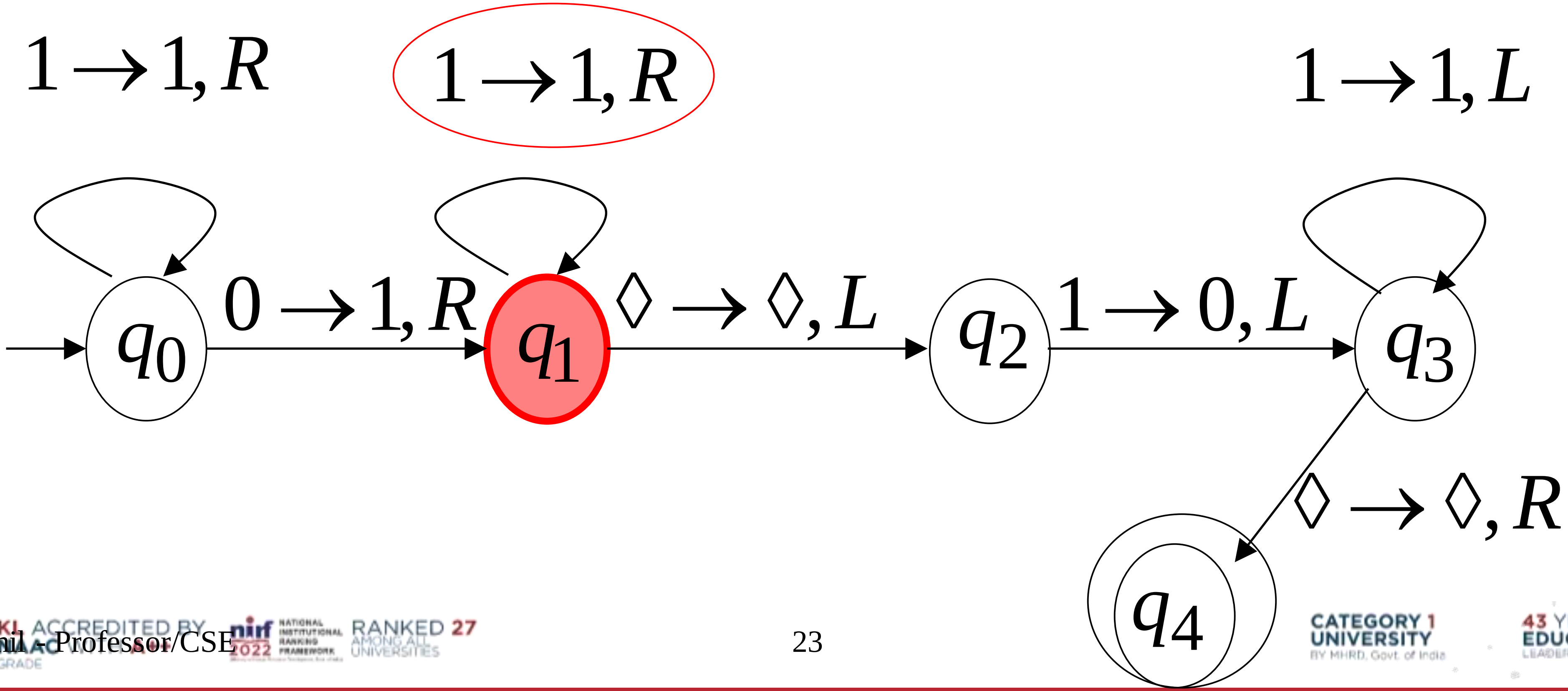
Time 2


◊ 1 1 0 1 1 ◊
q_0

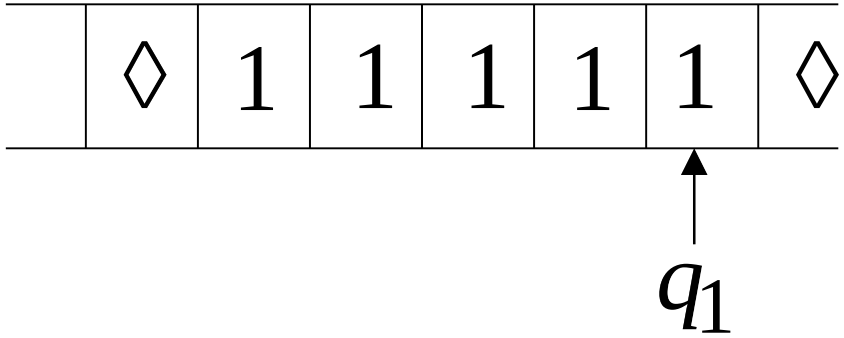

$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
q_0
$0 \to 1, R$
q_1
$\Diamond \to \Diamond, L$
q_2
$1 \to 0, L$
q_3
$\Diamond \to \Diamond, R$
q_4

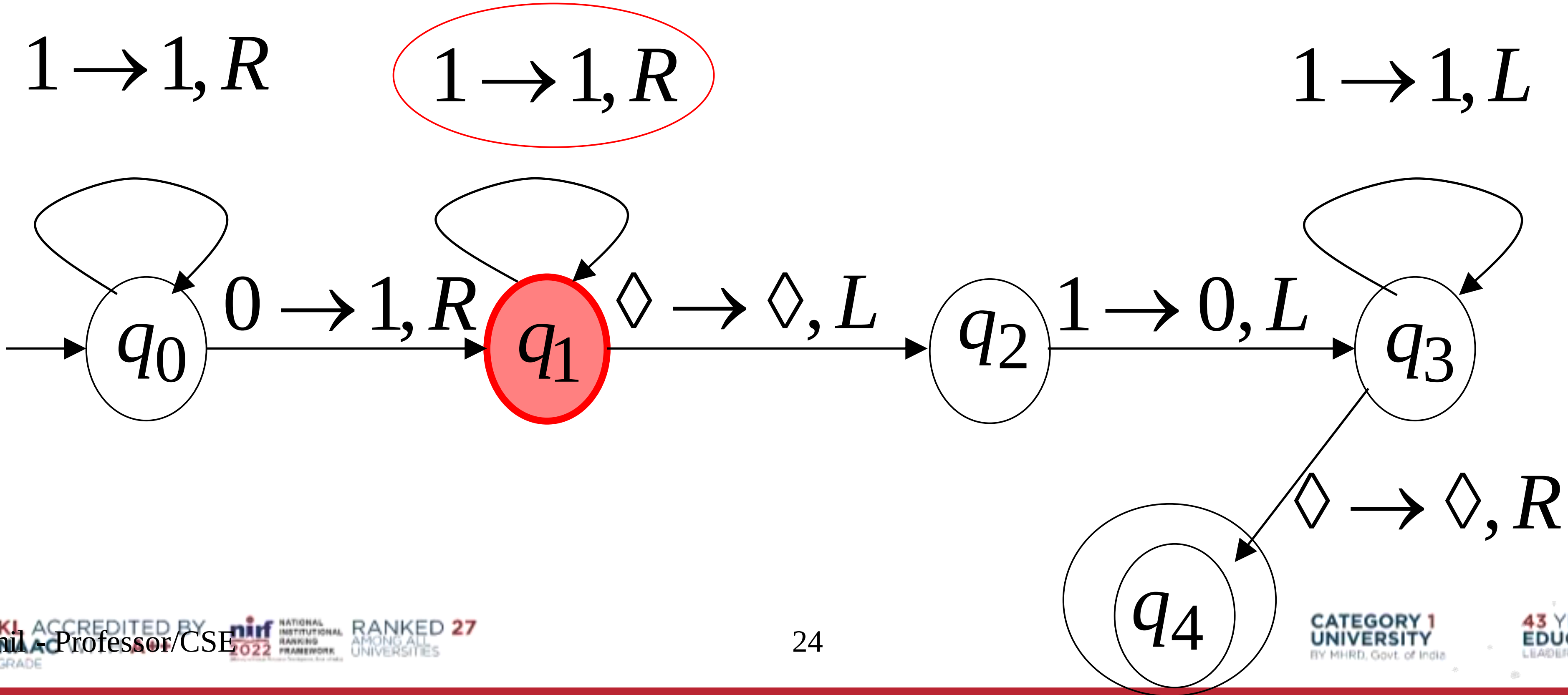
Time 3



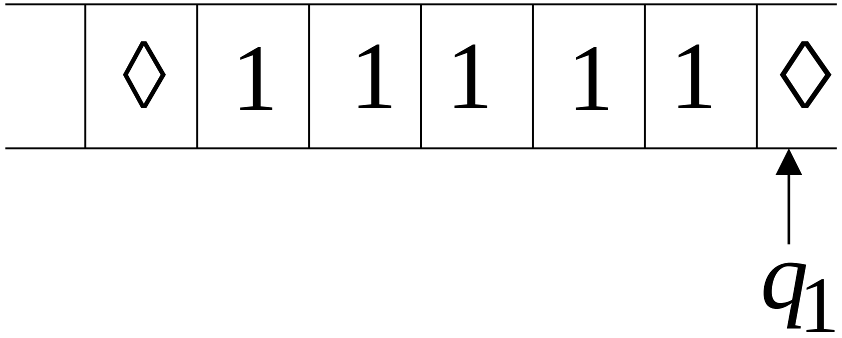


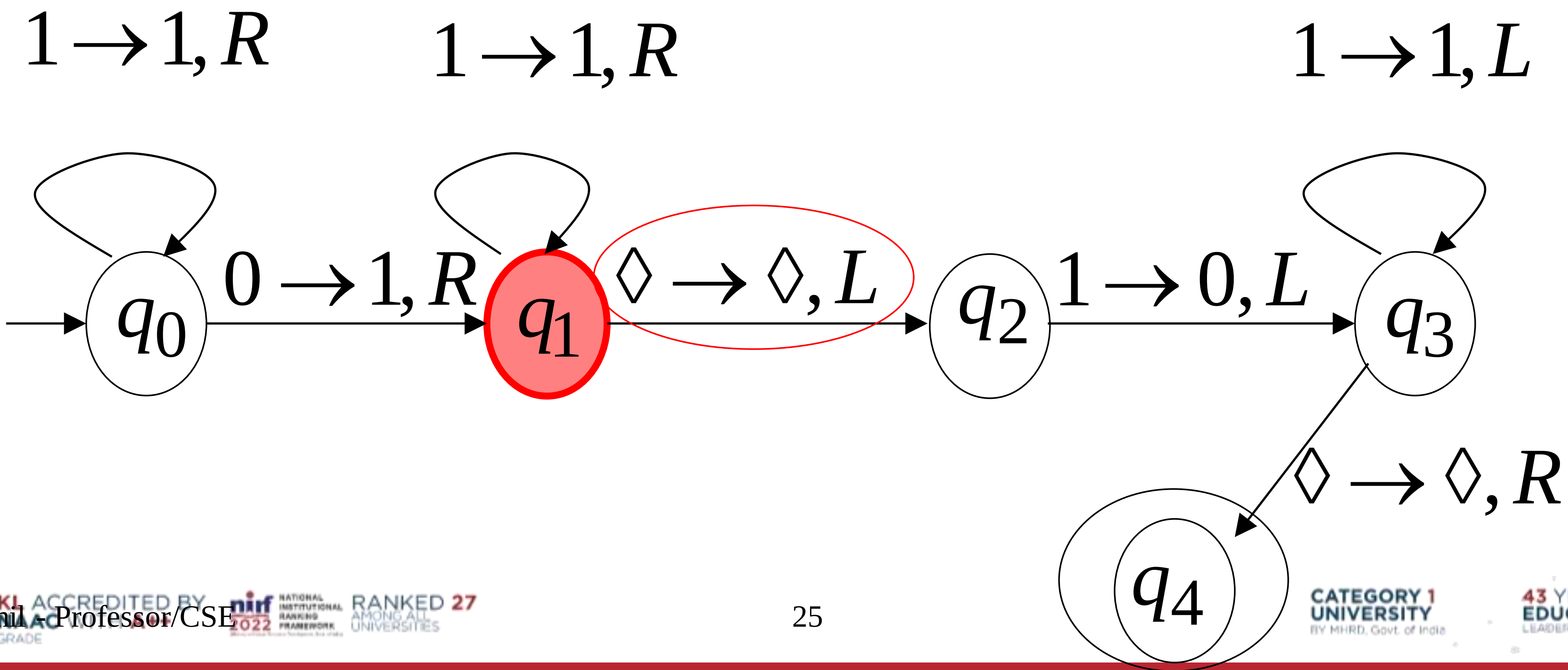
Time 4


◊
1
1
1
1
1
◊
q_1

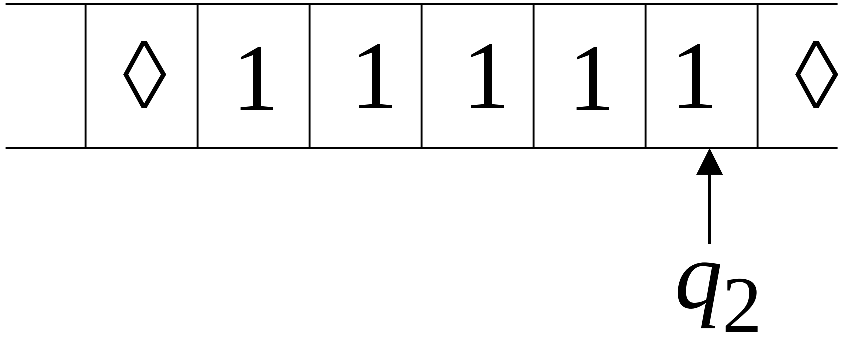

$1 \rightarrow 1, R$
$1 \rightarrow 1, R$
$1 \rightarrow 1, L$
$0 \rightarrow 1, R$
$\Diamond \rightarrow \Diamond, L$
$1 \rightarrow 0, L$
$\Diamond \rightarrow \Diamond, R$
q_0
q_1
q_2
q_3
q_4

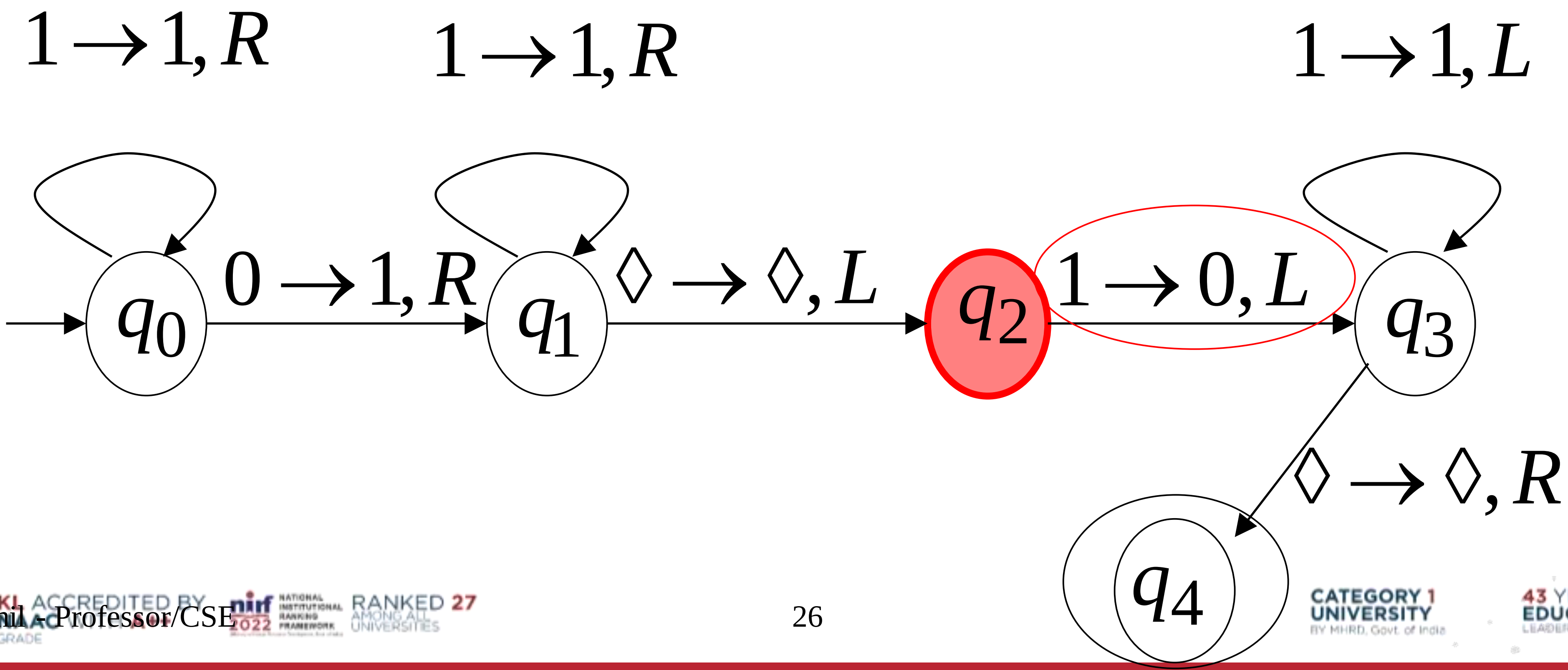
Time 5


◊
1
1
1
1
1
◊
q_1

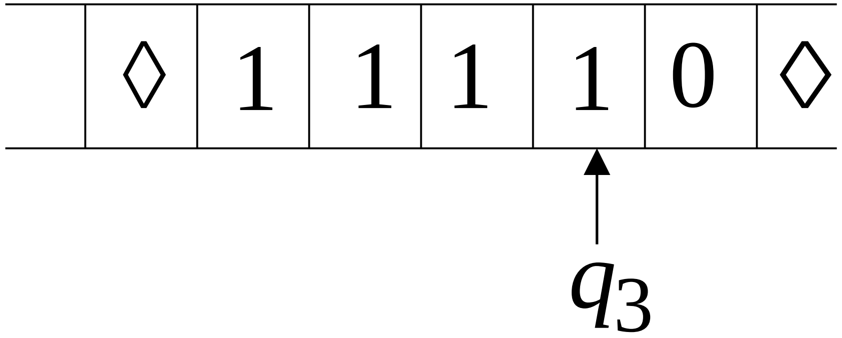

$1 \rightarrow 1, R$
$1 \rightarrow 1, R$
$1 \rightarrow 1, L$
q_0
$0 \rightarrow 1, R$
q_1
$\Diamond \rightarrow \Diamond, L$
q_2
$1 \rightarrow 0, L$
q_3
$\Diamond \rightarrow \Diamond, R$
q_4

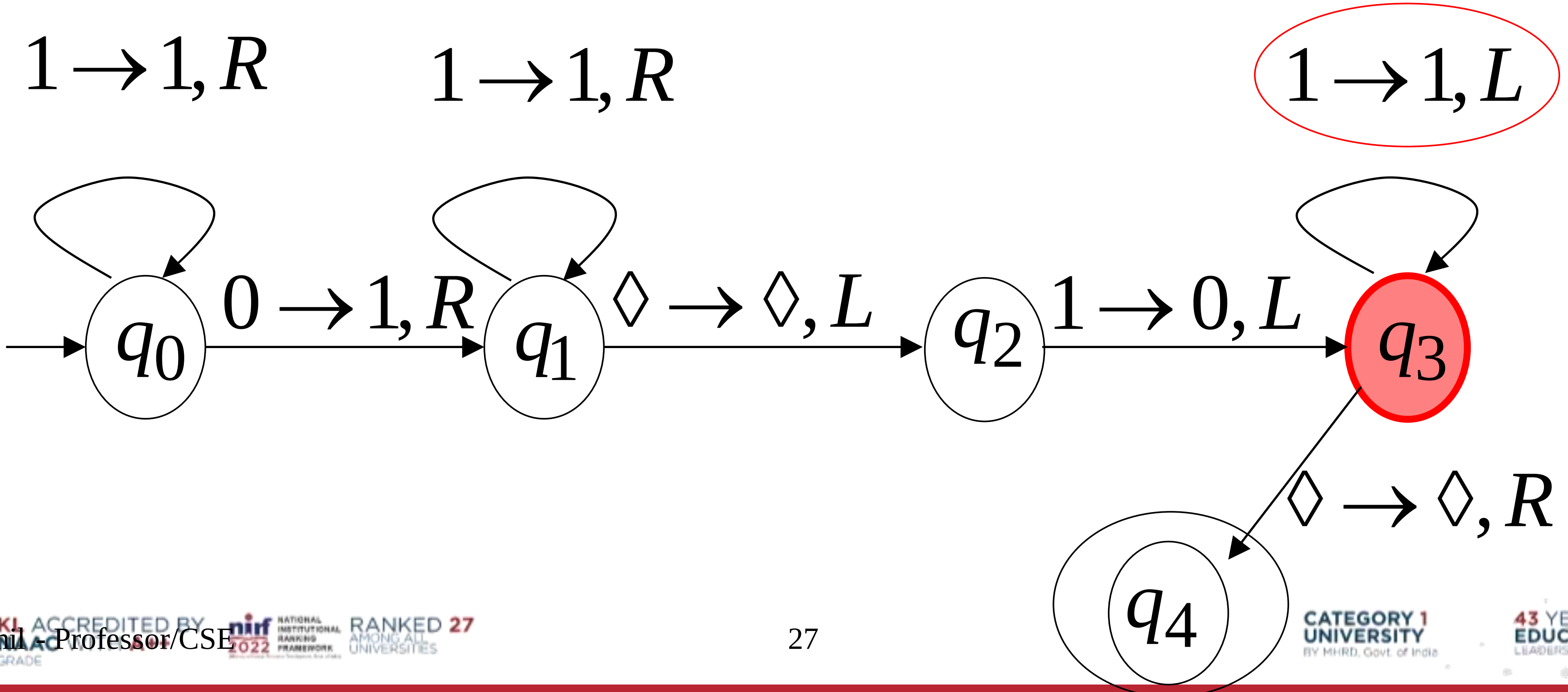
Time 6



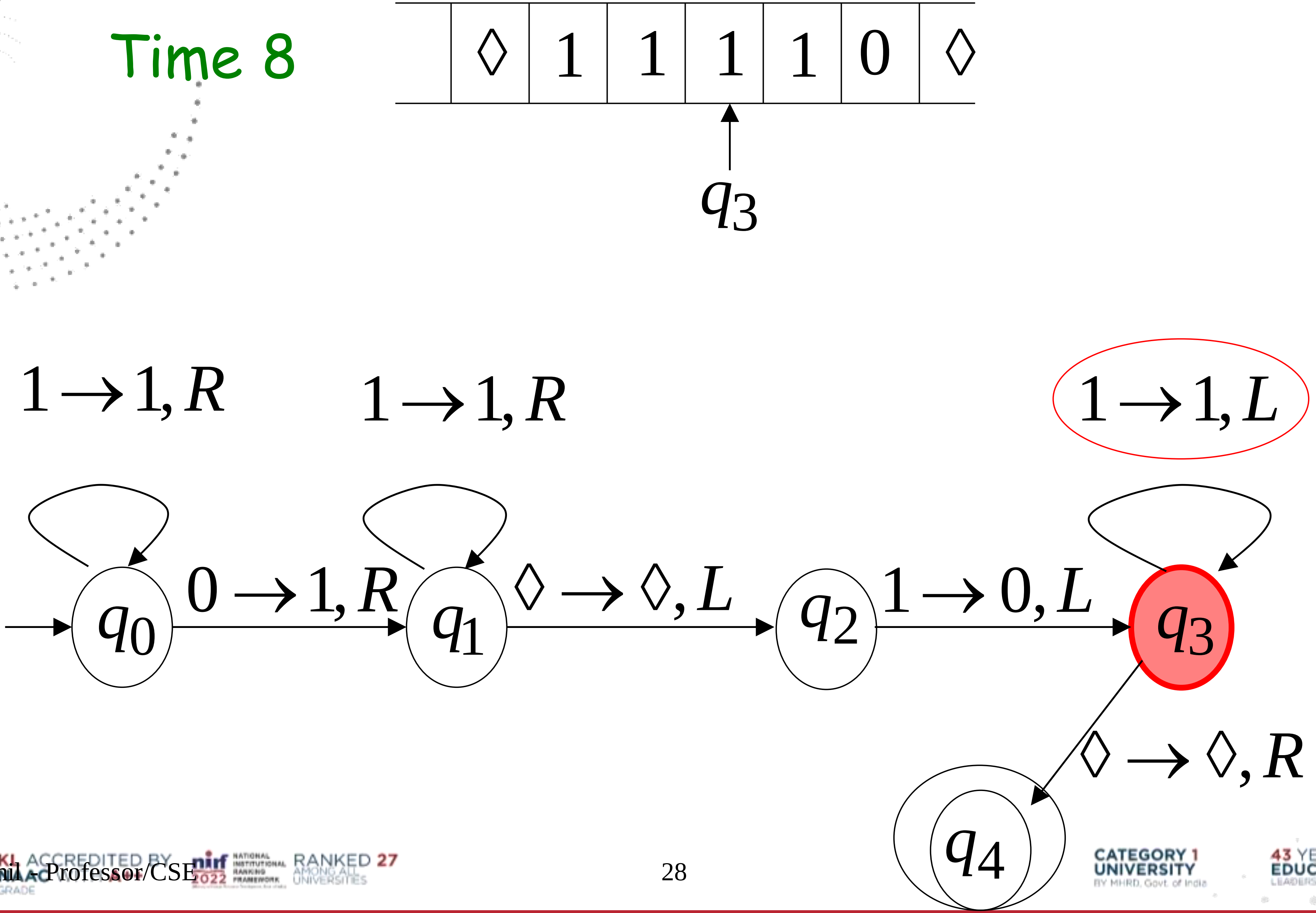


Time 7



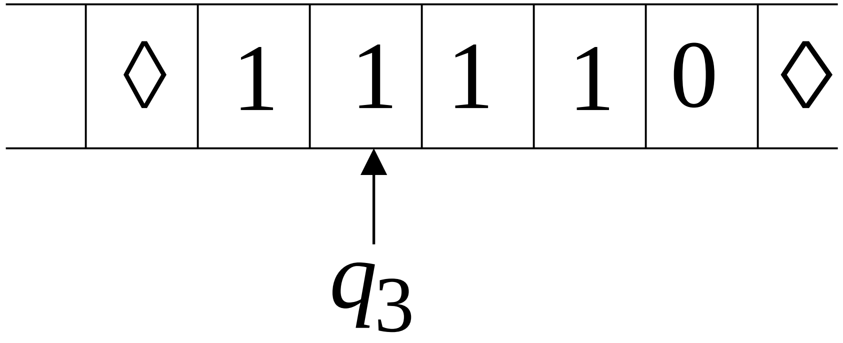


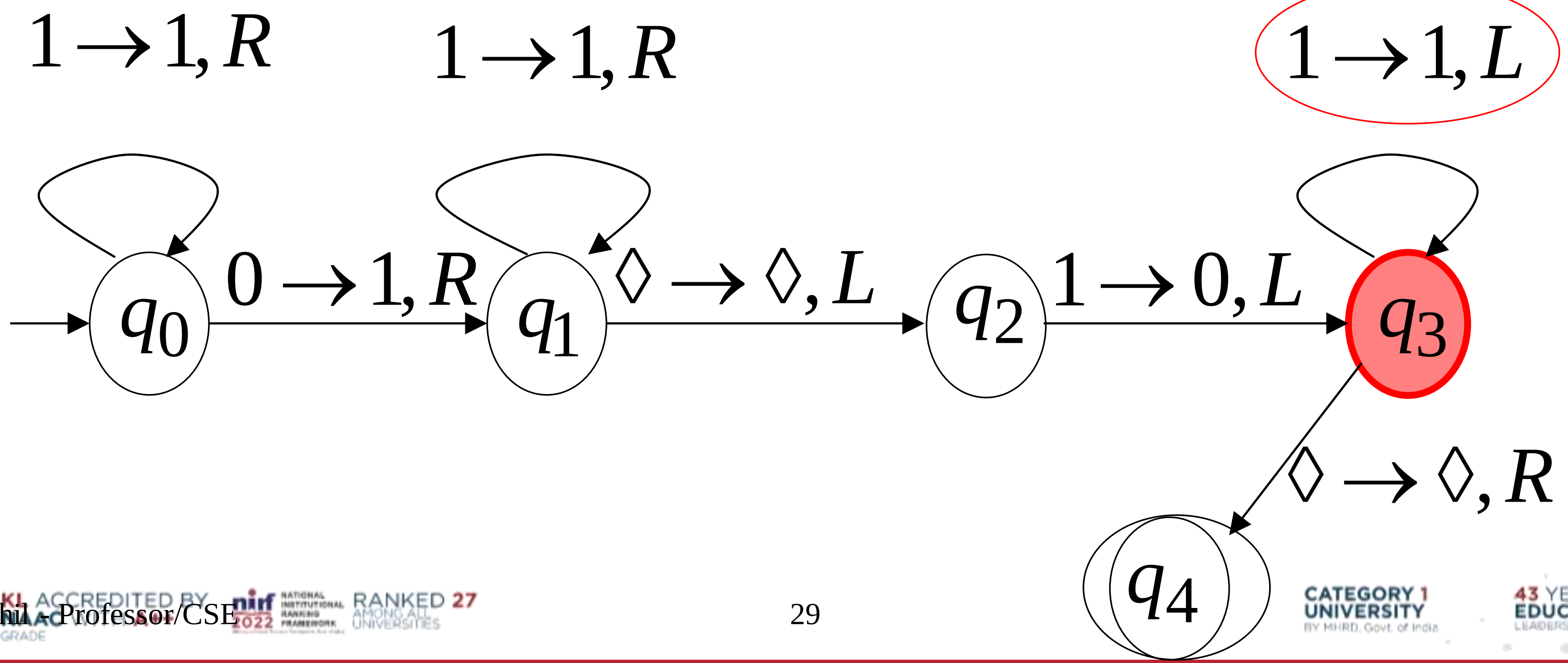
Time 8



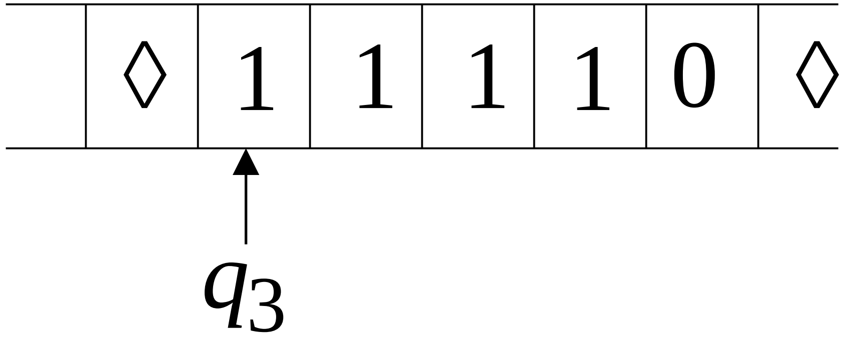
◊
1
1
1
1
0
◊
q_3
$1 \rightarrow 1, R$
$1 \rightarrow 1, R$
$1 \rightarrow 1, L$
q_0
$0 \rightarrow 1, R$
q_1
$◊ \rightarrow ◊, L$
q_2
$1 \rightarrow 0, L$
q_3
$◊ \rightarrow ◊, R$
q_4

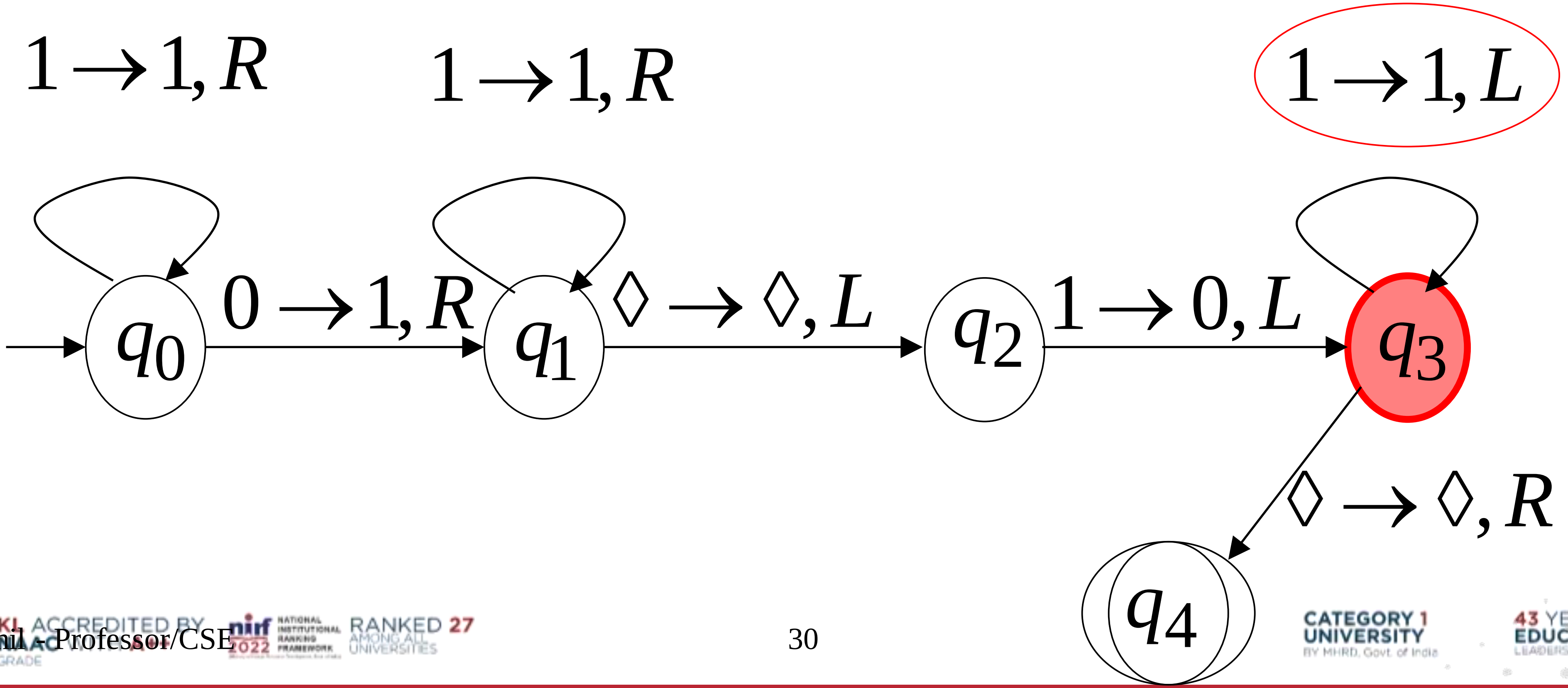
Time 9


◊
1
1
1
1
0
◊
q_3

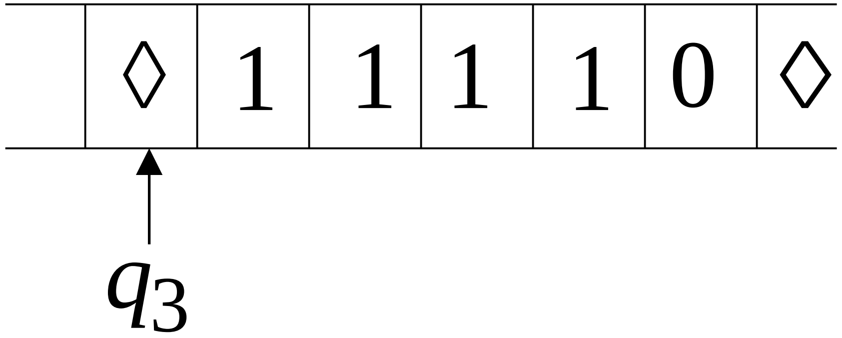

$1 \rightarrow 1, R$
$1 \rightarrow 1, R$
$1 \rightarrow 1, L$
q_0
$0 \rightarrow 1, R$
q_1
$◊ \rightarrow ◊, L$
q_2
$1 \rightarrow 0, L$
q_3
$◊ \rightarrow ◊, R$
q_4

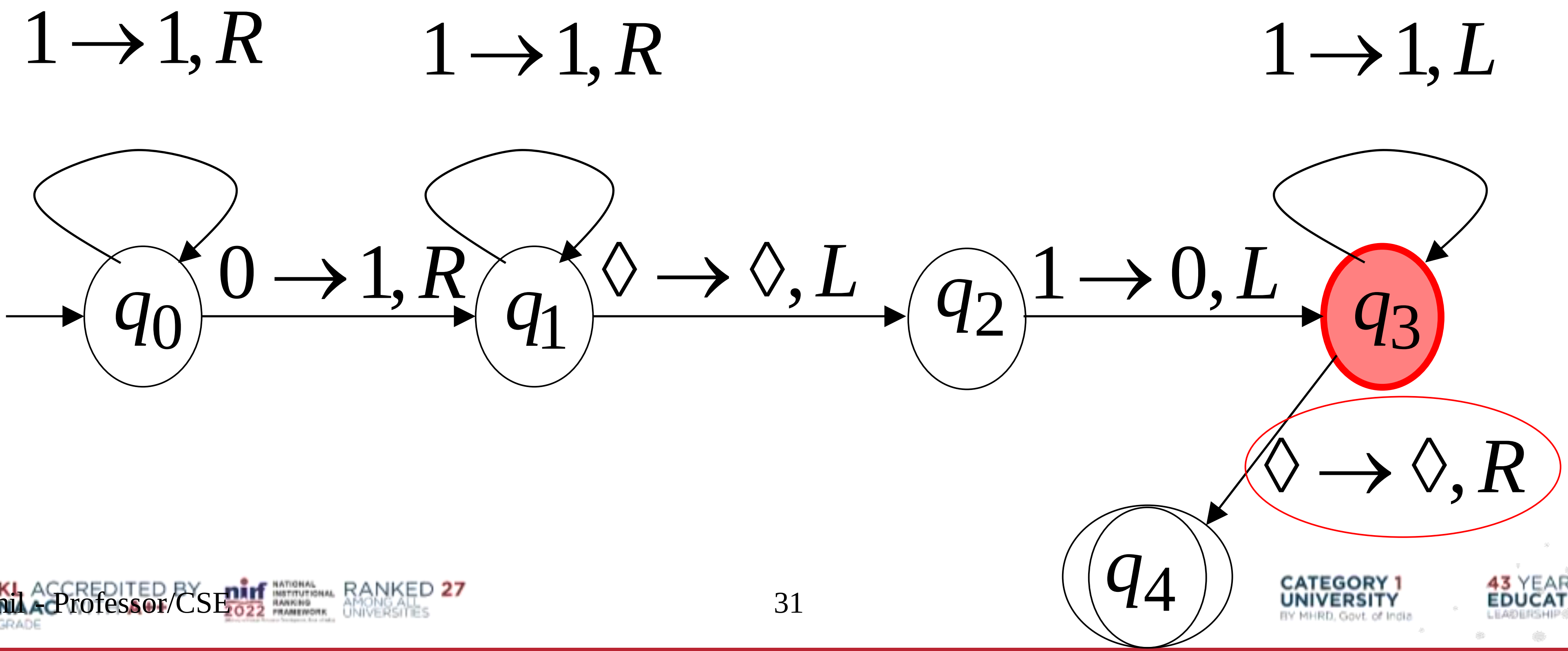
Time 10


◊ 1 1 1 1 0 ◊
q_3

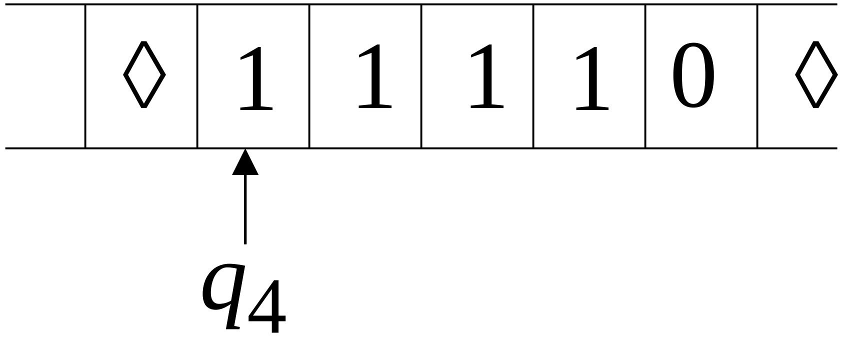

$1 \rightarrow 1, R$
$1 \rightarrow 1, R$
$1 \rightarrow 1, L$
q_0
$0 \rightarrow 1, R$
q_1
$\lozenge \rightarrow \lozenge, L$
q_2
$1 \rightarrow 0, L$
q_3
$\lozenge \rightarrow \lozenge, R$
q_4

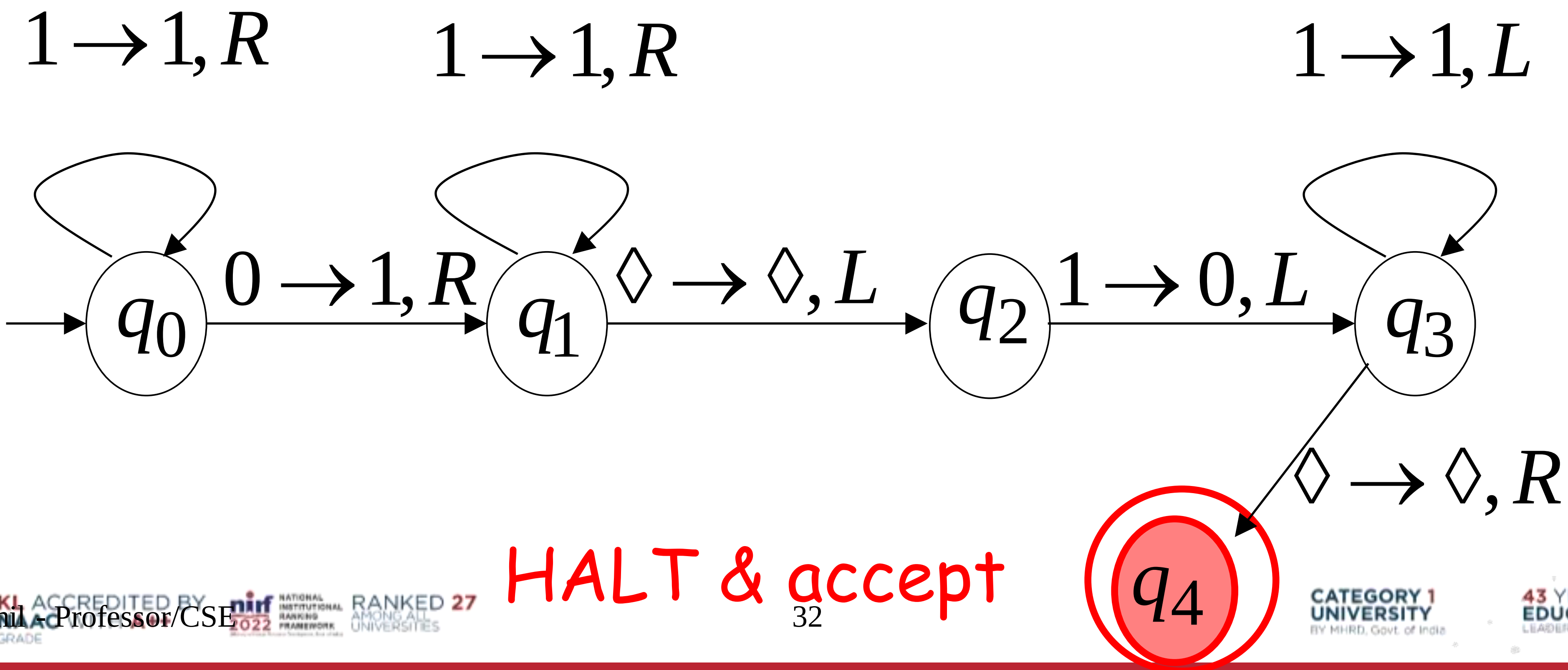
Time 11


◊
1
1
1
1
0
◊
q_3


$1 \rightarrow 1, R$
$1 \rightarrow 1, R$
$1 \rightarrow 1, L$
q_0
$0 \rightarrow 1, R$
q_1
$◊ \rightarrow ◊, L$
q_2
$1 \rightarrow 0, L$
q_3
$◊ \rightarrow ◊, R$
q_4

Time 12





HALT & accept

Another Example

The function $f(x) = 2x$ is computable

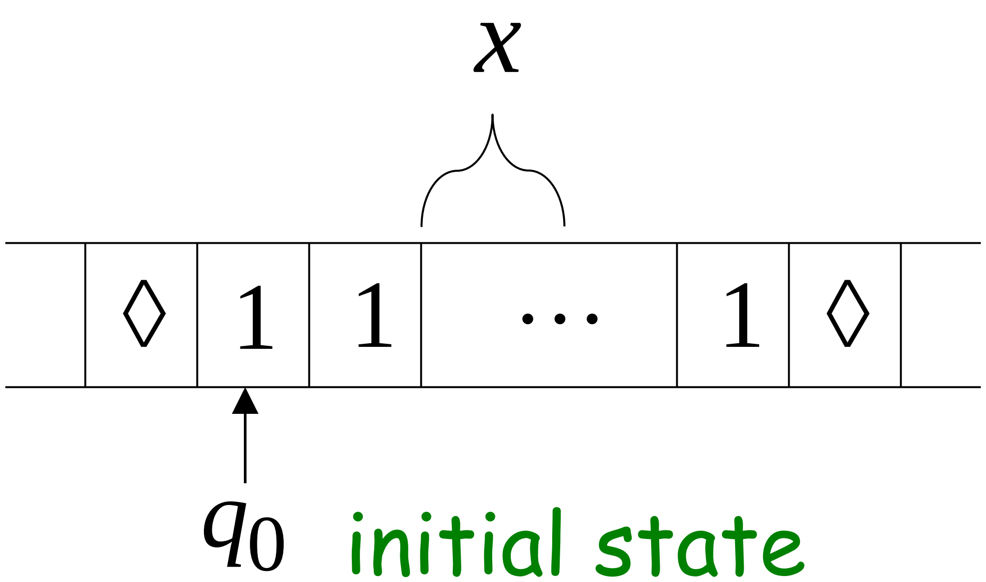
x is integer

Turing Machine:

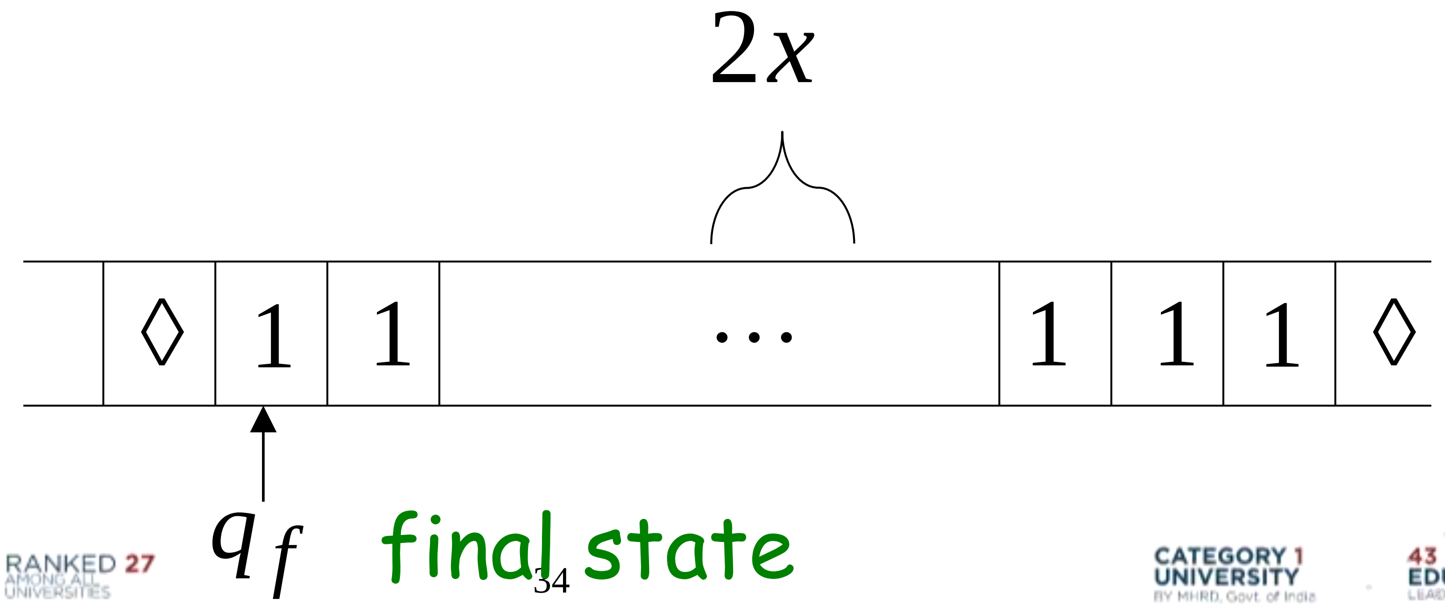
Input string: x unary

Output string: xx unary

Start



Finish

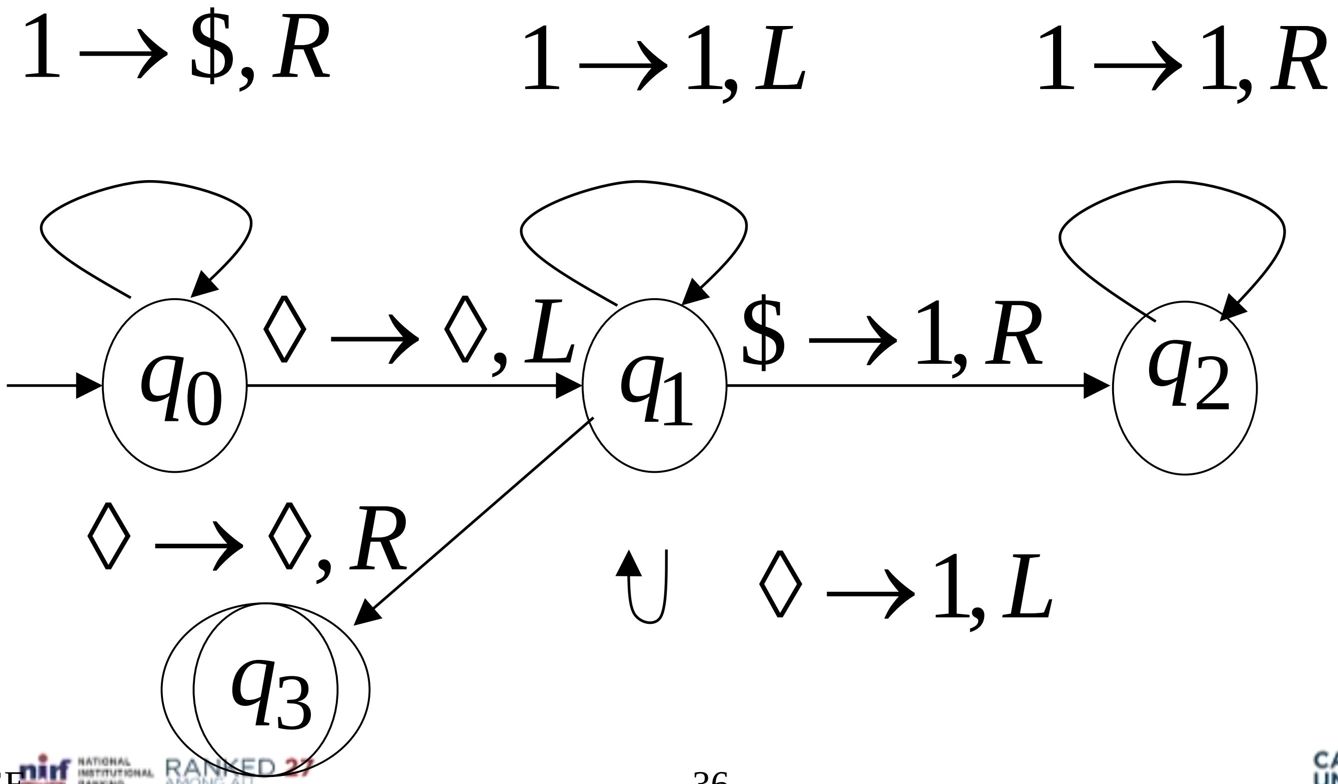


Turing Machine Pseudocode for $f(x) = 2x$

- Replace every 1 with \$
- Repeat:
  - Find rightmost \$, replace it with 1
  - Go to right end, insert 1

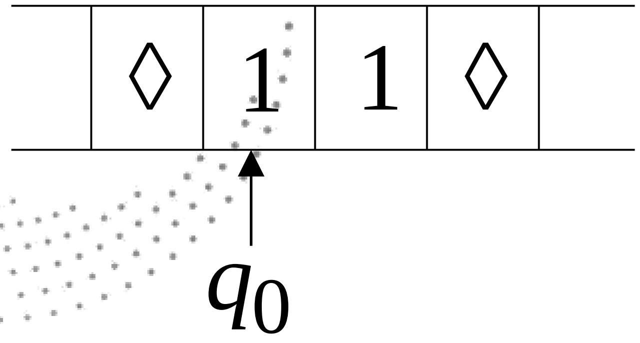
Until no more \$ remain

Turing Machine for $f(x) = 2x$

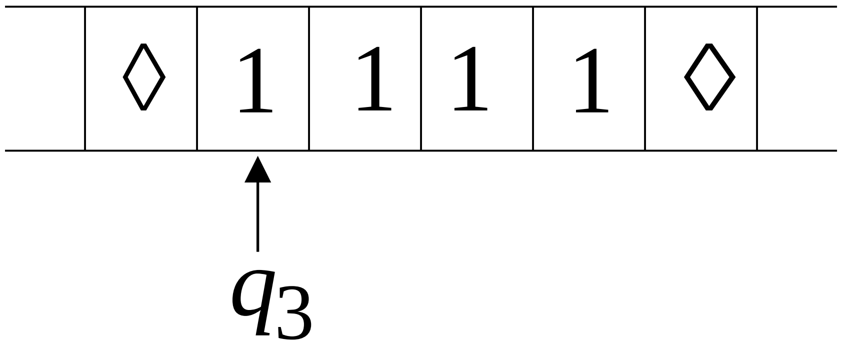


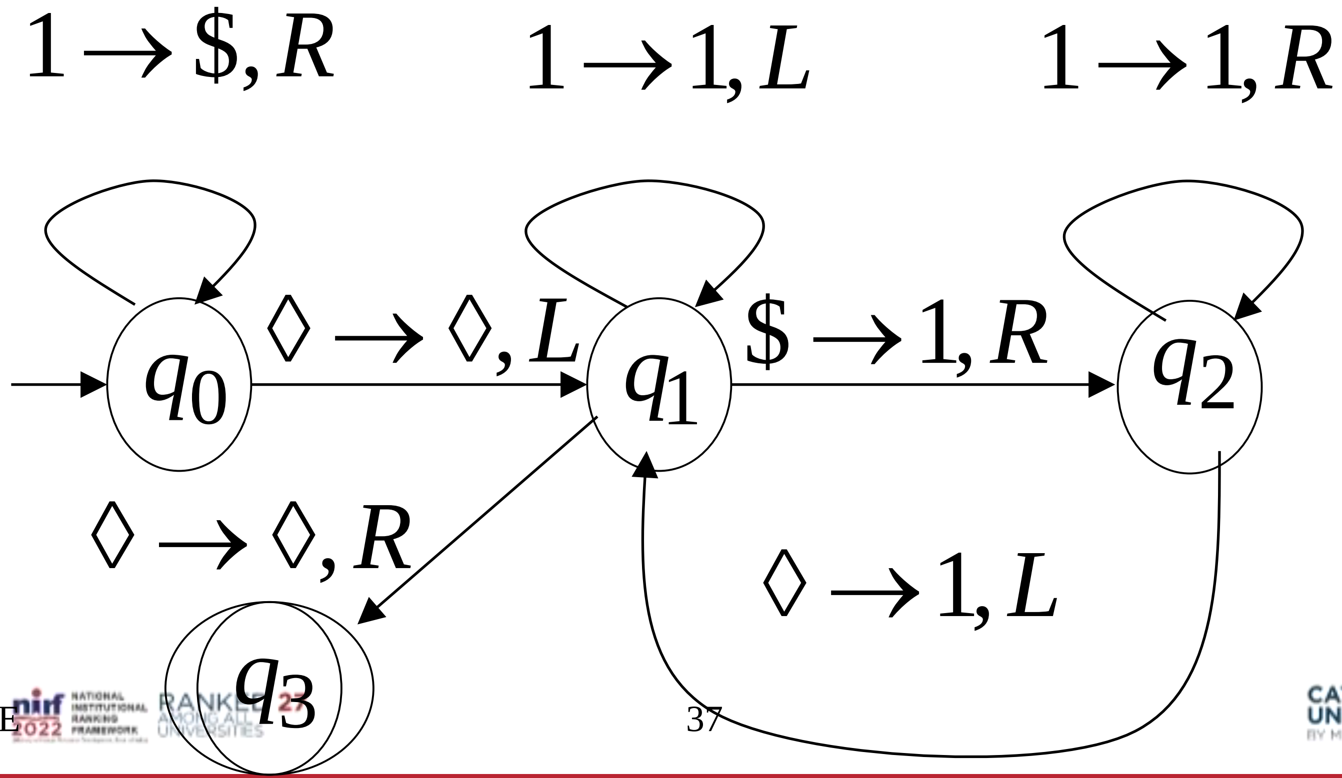
Example

Start



Finish





Another Example

The function

is computable

$$f(x,y) = \begin{cases} 1 & \text{if} \quad x > y \\ 0 & \text{if} \quad x \le y \end{cases}$$

Turing Machine for

$$f(x, y) = \begin{cases} 1 & \text{if } x > y \\ 0 & \text{if } x \leq y \end{cases}$$

Input: $x0y$

Output: 1 or 0

Turing Machine Pseudocode:

- Repeat

  Match a 1 from x with a 1 from y

  Until all of x or y is matched

- If a 1 from x is not matched

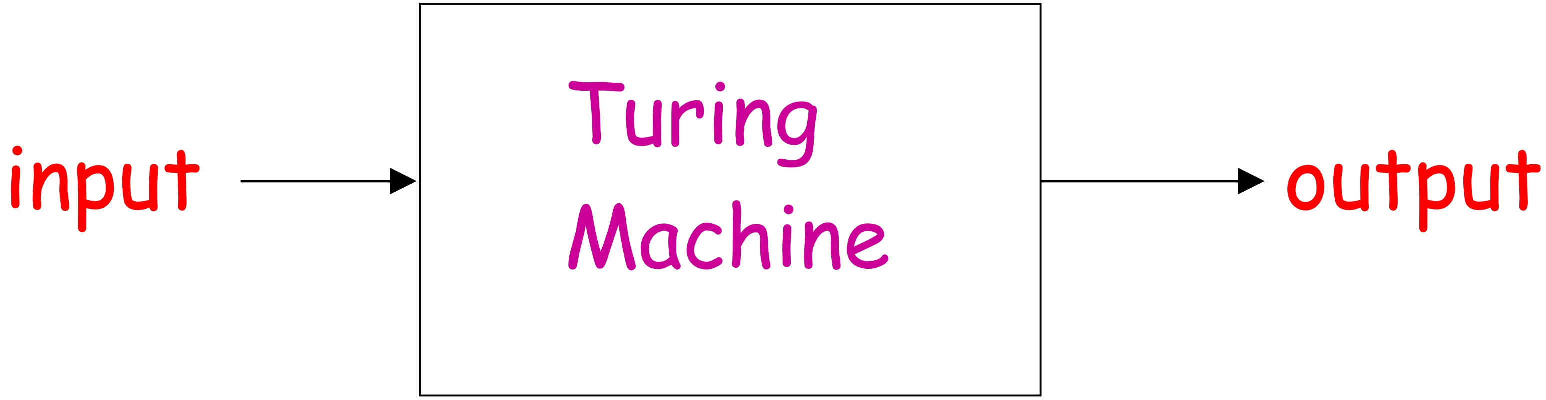
  erase tape, write 1 $\quad (x > y)$

  else

  erase tape, write 0 $\quad (x \leq y)$

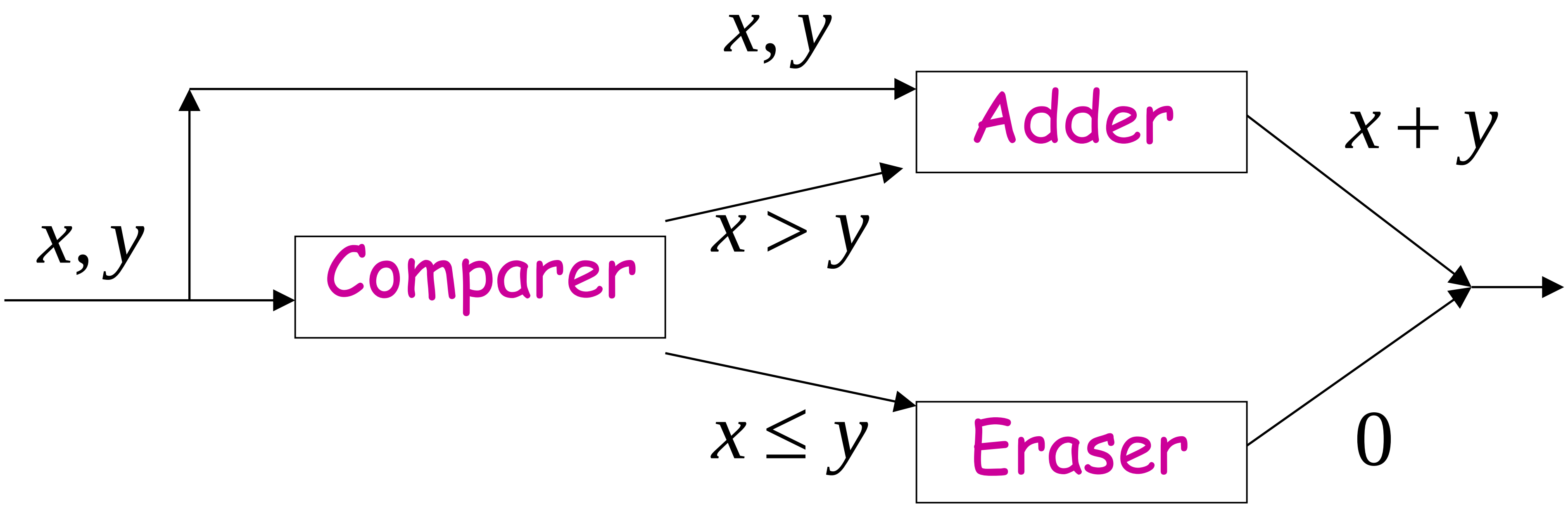
Combining Turing Machines

Block Diagram



Example:

$$f(x,y) = \begin{cases} x+y & \text{if } x > y \\ 0 & \text{if } x \le y \end{cases}$$



THANK YOU

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Team - TOC