BER problems and Solutions:

1. Find $\frac{E_b}{N_0}$ required to give $P_e = 10^{-5}$ for the following coherent digital modulation schemes: (a) BASK (b) BPSK (c) BFSK.

Use $Q(x) = 10^{-5} \Rightarrow x = 4.26$

Ans: (a) For BASK $Q\left(\sqrt{\frac{E_b}{N_0}}\right) = 10^{-5} \Rightarrow \frac{E_b}{N_0} = 18.19 \text{ or } 12.6\text{ dB}$

(b) For BPSK $Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = 10^{-5} \Rightarrow \frac{E_b}{N_0} = 9.1 \text{ or } 9.59\text{ dB}$

(c) For BFSK $Q\left(\sqrt{\frac{E_b}{N_0}}\right) = 10^{-5} \Rightarrow \frac{E_b}{N_0} = 18.19 \text{ or } 12.6\text{ dB}$

2. BPSK is used for data transmission over an AWGN channel with power spectral density $\frac{N_0}{2} = 10^{-10}$ W/Hz. The transmitted signal energy is $E_b = A^2 T_b / 2$, where T_b is the bit duration and A is the signal amplitude. Determine the value of A needed to achieve an error probability of 10^{-6}, if the data rate is: (a) 10 kbps (b) 100 kbps (c) 1Mbps.

Use $Q(x) = 10^{-6} \Rightarrow x = 4.75$

Ans: $Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = 10^{-6} \Rightarrow \frac{E_b}{N_0} = \frac{A^2 T_b}{N_0} = 4.75 \Rightarrow A^2 T_b = 45.125 \text{ x } 10^{-10}$

(a) For data rate 10kbps, $A = \sqrt{45.125 \text{ x } 10^{-10} \text{ x } 10^4} = 6.6\text{ mV}$

(b) For data rate 100kbps $A = \sqrt{45.125 \text{ x } 10^{-10} \text{ x } 10^5} = 0.0210\text{V}.$

(c) For data rate 100kbps $A = \sqrt{45.125 \text{ x } 10^{-10} \text{ x } 10^5} = 0.0664\text{ V}.$

3. Compare the SNR / bit and average power required at the demodulator to maintain a BER = 10^{-6} using **<u>BPSK</u>** and **<u>BPSK</u>** for data transmission over a radio channel at 56 kbps. Assume that the channel adds white Gaussian noise with power spectral density $N_0 = 10^{-10}$. Use $Q(x) = 10^{-6} \Rightarrow x = 4.75$

Ans: (a) For BASK $Q\left(\sqrt{\frac{E_b}{N_0}}\right) = 10^{-6} \Rightarrow \frac{E_b}{N_0} = 4.75 \Rightarrow E_b = 4.75^2 \text{ x } 10^{-10} = -2.256\text{x } 10^{-9}$

$$P_{av} = \frac{E_b}{T_b} = E_b R_b = 2.256\text{x}\,10^{-9}\,\text{x}\,4800 = 10.84\ \mu\text{W} = -19.65\ \text{dBm}$$

(b) For BPSK $Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = 10^{-6} \Rightarrow \frac{2E_b}{N_0} = 4.75 \Rightarrow E_b = \frac{1}{2} 4.75^2\ \text{x}\,10^{-10} = 1.128\text{x}\,10^{-9}$

$$P_{av} = \frac{E_b}{T_b} = E_b R_b = 1.128\text{x}\,10^{-9}\,\text{x}\,4800 = 5.415\ \mu\text{W} = -22.66\ \text{dBm}$$