

# Department of AI & DS

## CSE and CS&IT

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**COURSE NAME: PROBABILITY, STATISTICS AND QUEUING THEORY**

**COURSE CODE: 23MT2005**

**Topic**

**Expected value of a Random Variable and Variance**

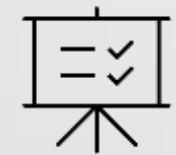
**Session - 4**

## AIM OF THE SESSION



To familiarize students with the mean and variance of a random variable.

## INSTRUCTIONAL OBJECTIVES



This Session is designed

1. Discuss the concept of expected value of a random variable/mean
2. List out the rules of determining the mean and variance of discrete and continuous random variable

## LEARNING OUTCOMES



At the end of this session, you should be able to:

1. Define probability and its axioms
2. Describe the different types of events.
3. Summarize the concept of probability with suitable example.

## CONTENTS

- ❖ Expected value of a random variable
- ❖ Properties of a Random variable
- ❖ Variance of a random variable

## Expected value of a Random Variable

Let  $X$  be a random variable with probability distribution  $f(x)$ . The mean or expected value of  $X$  is

$$\mu = E(X) = \sum_x xf(x), \quad \text{if } X \text{ is discrete and}$$

$$\mu = E(X) = \int_{-\infty}^{\infty} xf(x)dx \quad \text{If } X \text{ is continuous.}$$

Note: If  $X$  is a random variable, a function of  $X$ ,  $g(X)$  is also a random variable.

The expected value of the random variable  $g(X)$  is

$$\mu_{g(X)} = E(g(X)) = \sum_x g(x)f(x) \quad \text{if } X \text{ is discrete and}$$

$$\mu_{g(X)} = \int_{-\infty}^{\infty} g(x)f(x)dx \quad \text{If } X \text{ is continuous.}$$

Let  $X$  be a random variable with probability distribution  $f(x)$  with mean  $\mu$ . The variance of  $X$  is

$$\sigma^2 = E[(X - \mu)^2] = \sum_x (X - \mu)^2 f(x),$$

if  $X$  is discrete, and

$$\sigma^2 = E[(X - \mu)^2] = \int_{-\infty}^{\infty} (X - \mu)^2 f(x)$$

if  $X$  is continuous.

### Properties of a Random variable

1.  $E(c) = c$
2.  $E(aX + b) = aE(X) + b$
3.  $V(aX + b) = a^2V(X)$
4. If  $X$  and  $Y$  are independent random variables, then  $V(aX + bY) = a^2V(X) + b^2V(Y)$
5.  $\sigma^2 = E(X^2) - \mu^2$

## EXAMPLES

**Example 1:** The table below, adapted from a snapshot in India today, shows the probability distribution for  $x$ , the number of daily coffee breaks taken per day by coffee drinkers

$x$	0	1	2	3	4	5
$p(x)$	0.28	0.37	0.17	0.12	0.05	0.01

- Find the probability that a randomly selected coffee drinker would take more than two coffee breaks during the day
- Calculate the mean for the random variable  $x$ .

**SOLUTION:**  $X$  : NUMBER OF DAILY COFFEE BREAKS TAKEN PER DAY BY COFFEE DRINKERS

- $P(\text{Coffee drinkers would take more than two coffee breaks during the day})$

$$= P(x > 2) = 0.12 + 0.05 + 0.01 = 0.18$$

- Mean =  $\sum x f(x)$

$$= 0(0.28) + 1(0.37) + 2(0.17) + 3(0.12) + 4(0.05) + 5(0.01) = 1.32$$

## SUMMARY

In this session, determination of mean and variance of a random variable along with its properties have noted.

1. Expected Value of a Random variable
2. Variance of a Random variable
3. Properties of mean and variance.

## SELF-ASSESSMENT QUESTIONS

If  $X$  is a random variable having its probability density function  $f(x)$ , the  $E(x)$  is called:

- a) Arithmetic mean
- b) geometric mean
- c) harmonic mean
- d) first quartile

If  $X$  is a random variable and  $r$  is an integer, then  $E(x^r)$  represents:

- a)  $r^{\text{th}}$  central moment
- b)  $r^{\text{th}}$  factorial moment
- c)  $r^{\text{th}}$  raw moment
- d) none of the above



## TERMINAL QUESTIONS

1. Let  $X$  be random variable with following probability distribution:

$X$	-3	6	9
$f(x)$	$1/6$	$1/2$	$1/3$

Compute  $\mu_{g(x)}$ , where  $g(x)=(2x+1)^2$ .

2. The length of time  $Y$ , in minutes , required to generate a human reflex to tear gas has the density function

$$f(y) = \begin{cases} \frac{1}{4}e^{-\frac{y}{4}}, & 0 \leq y < \infty \\ 0, & \text{elsewhere} \end{cases}$$

- i) Obtain the mean time to reflex
- ii) Obtain the value of  $E(Y^2)$  and  $V(Y)$ .

## Reference Books:

1. Chapter 1 of TP1: William Feller, An Introduction to Probability Theory and Its Applications: Volume 1, Third Edition, 1968 by John Wiley & Sons, Inc.
2. Richard A Johnson, Miller & Freund's Probability and statistics for Engineers, PHI, New Delhi, 11th Edition (2011).

## Sites and Web links:

1. \* <https://ncert.nic.in/textbook.php?kcmh1=16-16> \*
2. Notes: sections 1 to 1.3 of <http://www.statslab.cam.ac.uk/~rrw1/prob/prob-weber.pdf>
3. [https://ocw.mit.edu/courses/res-6-012-introduction-to-probability-spring-2018/91864c7642a58e216e8baa8fcb4a5cb5/MITRES\\_6\\_012S18\\_L01.pdf](https://ocw.mit.edu/courses/res-6-012-introduction-to-probability-spring-2018/91864c7642a58e216e8baa8fcb4a5cb5/MITRES_6_012S18_L01.pdf)
4. [https://www.probabilitycourse.com/chapter3/3\\_2\\_1\\_cdf.php](https://www.probabilitycourse.com/chapter3/3_2_1_cdf.php)
5. [https://en.wikipedia.org/wiki/Cumulative\\_distribution\\_function](https://en.wikipedia.org/wiki/Cumulative_distribution_function)

THANK YOU



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