		STUDENT ID:	
Experiment #		STUDENT NAME:	
Date:			

SUBJECTCODE: 23MT2005
PROBABILITY STATISTICS AND QUEUING THEORY

Date of the Session: //________________ Time of the Session:______ to______

Tutorial 11:

- Demonstrate Birth-death processes, Poisson process and exponential distribution
- Apply multi server queuing model to the real-world applications.

Learning outcomes:

1. For the Queueing Model M /M/s/∞/FCFS (Bulk Arrival Queues), derive the formula "Ls" which is expected number of customers in a system.

Solution:

M/M/s:- Poission arrivals, exponential service times

∞:- Infinite queue capacity

FCFS:- first come, first served

Parameters:- i) λ
ii) μ
iii) s
iv) ρ (rho)

i) $\rho = \frac{\lambda}{s\mu}$.

ii) $P(n) = \frac{(s\rho)^n}{n!} \cdot \left(\sum_{k=0}^{s} \frac{(s\rho)^k}{k!} \right)^{-1}$

iii) $Ls = \lambda \cdot Ws$

iv) $Ws = \frac{1}{\mu} \cdot \frac{1}{1-\rho}$

2. The Golden Muffler Shop has decided to open a second garage bay and hire a second mechanic to handle installations. Customers, who arrive at the rate of about $\lambda = 2$ per hour, will wait in a single line until 1 of the 2 mechanics is free. Each mechanic installs mufflers at the rate of about $\mu = 3$ per hour. To find out how this system compares with the old single-channel waiting-line system, we will compute several operating characteristics for the M = 2 channel system and compare the results with those found in single-channel.

Solution:

Given $\lambda = 2$

$\mu = 3$

$M = 2$

i.) $\rho = \dfrac{\lambda}{M\mu}$

$= 0.3333$

ii) $P_0 = \dfrac{1}{\sum_{n=0}^{M} \dfrac{(\lambda/\mu)^n}{n!}}$

$= 1.8889 = \dfrac{1}{1.8889} \simeq 0.5294$

iii) $P_{wait} = 1 - P_0 = 1 - 0.5294 = 0.4706$

iv.) $L = \dfrac{\lambda}{\mu - \lambda/M} = 1$

v) $W = \dfrac{L}{\lambda} = 0.5\,hrs$

vi) $L_q = L - \dfrac{\lambda}{\mu}$

$= 0.3333$

vii) $W_q = \dfrac{L_q}{\lambda} = 10min$

3. A computer center is equipped with four identical mainframe computers. The number of users at any time is 25. Each user is capable of submitting a job through a terminal every 15 minutes. On the average, but the actual time between submissions is exponential. Arriving jobs will automatically go to the first available computer. The execution time per submission is exponential with mean 2 minutes. Compute the following:

i) Probability that a job is not executed immediately on submission.

ii) Average time until the output of a job is returned to the user.

iii) Average number of jobs waiting for execution.

iv) Average number of idle computers.

Solution: Given

$\lambda = 1.67$.

$\mu = 0.5$.

$s = 4$

i) $$P_{busy} = \frac{\frac{(\lambda/\mu)^s}{s!}}{\sum_{k=0}^{s} \frac{(\lambda/\mu)^k}{k!}} = 0.2437.$$

$\boxed{\because \rho = \frac{\lambda}{s\mu}}$

ii) $$W = \frac{1}{\mu} \cdot \frac{1}{1-\rho} = 0.835.$$

iii) $$L_q = \frac{(\lambda/\mu)^s \cdot \frac{\lambda}{\mu}}{s! \cdot (1-\rho)^2} = 25.98$$

iv) $$I = s - \frac{\lambda}{\mu} \cdot \left(\sum_{k=0}^{s} \frac{(\lambda/\mu)^k}{k!} \right) = -67.34$$

4. A car service station has two bays where service can be offered simultaneously. Due to space limitation, only four cars are accepted for service. The arrival pattern is Poisson with 12 cars per day. The service time in both the bays is exponentially distributed with $\mu = 8$ cars per day per bar. Find the average number of cars in the service station, the average number of cars waiting to be serviced and the average time a car spends in the system.

Solution:

Given $\lambda = 12$

$\mu = 8$

$s = 12$

$$\rho = \frac{\lambda}{s \times \mu} = 0.75$$

$$L = \frac{\lambda}{\mu - \lambda/s} = 6$$

$$L_q = \frac{(\lambda/\mu)^s \cdot \frac{\lambda}{\mu}}{s! \cdot (1-\rho)^2} = 27$$

$$W = \frac{L}{\lambda} = 0.5$$

Avg cars in Service station = 6.

waiting to be serviced :- 27.

Spends is system = 0.5 days (12 hrs).

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VIVA QUESTIONS

1. What distinguishes Non-birth-death Markovian queueing systems from Birth-death queueing systems?

It do not have simple structure transcition b/w adjacente, transction between neighbouring states

2. Ho does the rate of arrivals and departures affect the steady-state behavior of a Birth-death queueing system?

It influence the system stability.

3. Give an example of a Non-birth-death Markovian queueing system.

In M/M/c/c there are no waiting rooms & customers are either saved (or) lost.

4. What are the challenges of analyzing complex queueing networks?

networks, modeling, large state spaces

(For Evaluators use only)

Comment of the Evaluator (if Any)	Evaluator's Observation Marks Secured: out of