Tutorial 5

Branch and Bound Method

Date of the Session: .....

Learning outcomes:

- Understanding to do the process of solving branch and bound method.
- Understanding to do the process of Gomory's cutting plane method.

5.1 PRE-TUTORIAL

1. What is branch and bound Technique?

The branch and bound Technique systematically explains solution spaces by branching into sub-problems & pruning paths that exceed known bounds to find optimal solution efficiently.

2. Which strategy can be used to solve branch and bound problem?

We can use Simplex method & graphical method. Graphical method is suitable for only two variables. But where as simplex-method for 2 & more variables.

5.2 IN-TUTORIAL

1 Discrete Optimization using Cutting Plane method Solve the integer programming problem

$$\text{Maximize } Z = 3x_1 + x_2 + 3x_3$$

Subject to

$$x_1 + 2x_2 + x_3 \leq 4$$

$$2x_2 - \frac{3}{2}x_3 \leq 1$$

$$x_1 - 3x_2 + 2x_3 \leq 3$$

Where $x_1, x_2, x_3 \geq 0$ and integer. Get the optimal solution as an integer value using Gomory's cutting plane method.

Solution:

		C_j	3	1	3	0	0	0	
C_B	B.V	X_B	x_1	x_2	x_3	S1	S2	S3	Ratio
0	S1	7.5	0	0	2.25	1	0.5	1	3.35
1	x_2	0.5	0	1	-0.75	0	0.5	0	–
3	x_1	4.5	1	0	-0.25	0	1.5	1	–
	Z_j		3	1	1.5	0	5	3	
	$C_j - Z_j$		0	0	-4.5	0	5	3	

		C_j	3	1	3	0	0	0	
C_B	B.V	X_B	x_1	x_2	x_3	S1	S2	S3	Ratio
3	x_3	3.33	0	0	1	0.44	0.22	0.44	
1	x_2	3	0	1	0	0.33	0.66	0.33	
3	x_1	5.33	1	0	0	0.11	1.55	0.11	
	Z_j		3	1	3	2	6	5	
	$C_j - Z_j$		0	0	0	2	6	5	

C_B	B.V	C_j / X_B	3 x_1	1 x_2	3 x_3	0 S_1	0 S_2	0 S_3	Ratio
3	x_3	3.33	0	0	1	0.44	0.22	0.44	
1	x_2	3	0	1	0	0.33	0.66	0.33	
3	x_1	5.33	1	0	0	0.11	1.56	1.11	
		Z_j	3	1	3	2	6	5	
		$C_j - Z_j$	0	0	0	2	6	5	

C_B	B.V	X_B	x_1	x_2	x_3	S_1	S_2	S_3	G_1	G_2
3	x_3	3	0	0	1	0	0	0	1	0
1	x_2	2.75	0	1	0	0	0.5	0	0.75	0
3	x_1	5.25	1	0	0	0	1.5	1	0.25	0
0	S_1	0.75	0	0	0	1	0.5	1	-2.25	0
0	S_2	-0.75	0	0	0	0	-0.5	0	-0.75	1
		Z_j	3	1	3	0	5	3	4.5	0
		$C_j - Z_j$	0	0	0	0	5	3	4.5	0
		Ratio	-	-	-	-	-	-	-	-

C_B	B.V	X_B	x_1	x_2	x_3	S_1	S_2	S_3	G_1
3	x_3	2	0	0	1	0	-0.667	0	0
1	x_2	2	0	1	0	0	0	0	0
3	x_1	5	1	0	0	0	1.33	1	0
0	S_1	3	0	0	0	1	2	1	0
0	G_1	1	0	0	0	1	0.66	0	1
		Z_j	3	1	03	0	2	3	0
		$C_j - Z_j$	0	0	0	0	2	3	0
		Ratio	-	-	-	-	-	-	-

Since all $Z_j - C_j = 0$

$x_1 = 5,\ x_2 = 2,\ x_3 = 2$ Max $Z = 23$

2. Use Branch and Bound method to

$$\text{Maximize} \quad Z = 3x_1 + 5x_2$$

Subject to

$$2x_1 + 4x_2 \le 25$$
$$x_1 \le 8$$
$$2x_2 \le 10$$

Where x_1, x_2 are non-negative integers.

Solution:

Given,

$2x_1 + 4x_2 \le 25$

$x_1 + 2x_2 \le 18$

& $\max Z = 3x_1 + 5x_2$

$2x_1 + 4x_2 = 25$

$x_1 = 0, x_2 = \frac{25}{4}$

$4x_2 = 25$

$x_2 = \frac{25}{4}$

$x_1 = \frac{25}{2}, x_2 = 0$

$2x_1 = 25$

$x_1 = \frac{25}{2}$

Points are

$(0, \frac{25}{4})$ $(\frac{25}{2}, 0)$

$x_1 + 2x_2 = 18$

$x_1 = 0, x_2 = 9$

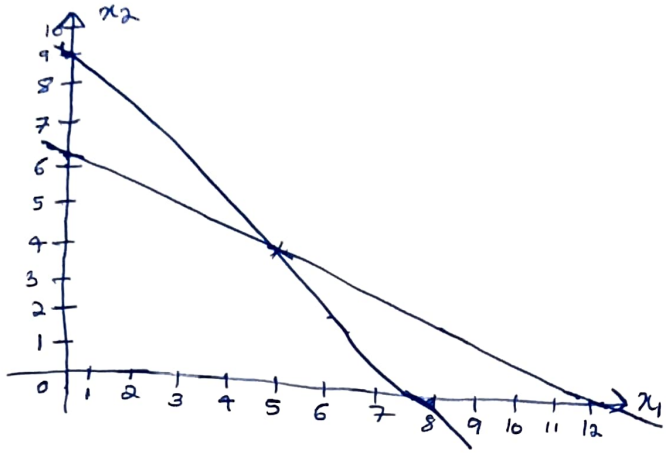
$2x_2 = 18$

$x_2 = 9$

$x_1 = 18, x_2 = 0$

$x_1 = 8$

$(0, 9), (18, 0)$



By graphical method

$\therefore$ Intersection point is $(5,4)$.

$$\text{Max } z = 3(5) + 5(4)$$

$$= 35$$

$\therefore$ **Max $z = 35$**

5.3 POST-TUTORIAL

1. Explain Gomory's method for solving an Integer Programming Problem and hence solve the following

$$\text{Maximize } Z = 2x_1 + 6x_2$$
$$\text{Subject to:}$$
$$3x_1 + x_2 \leq 5$$
$$4x_1 + x_2 \leq 9$$

Where $x_1, x_2 \geq 0$ and are integers.

Solution:

$Z = 2x_1 + 6x_2 + 0s1 + 0s2$

$3x_1 + x_2 + s1 = 5,\ 4x_1 + x_2 + s2 = 9$

C_B	C_j / B.V	2 x_1	6 x_2	0 s1	0 s2	Sol	Ratio
0	s1	3	1	1	0	5	$5/1 = 5$
0	s2	4	1	0	1	9	$9/1 = 9$
	Z_j	0	0	0	0		
	$C_j - Z_j$	2	6	0	0		

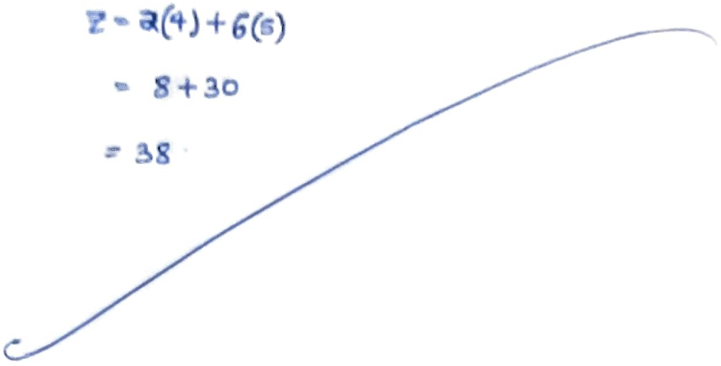
C_B	C_j / B.V	2 x_1	6 x_2	0 s1	0 s2	Sol	Ratio
6	x_2	3	1	1	0	5	
0	s2	1	0	-1	1	7	
	Z_j	18	6	6	0		
	$C_j - Z_j$	-16	0	-6	0		

$\therefore x_1 = 4,\ x_2 = 5$

$Z = 2(4) + 6(5)$

$= 8 + 30$

$= 38$



For Evaluator's Use only

Evaluators Comments

Evaluator's Observation

Marks Secured ..49..... **out of 50**

Full Name of the Evaluator:

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