1. Consider a recurrence of the form :

4 / 4 points

$$T(n) = \begin{cases} \Theta(1) & n \leq 1 \\ 3T(n/3) + \Theta(n) & \text{otherwise} \end{cases}$$

Select all the correct options from the list below.

- [x] The recurrence above can be obtained from a divide and conquer scheme that divides inputs of size n into 3 subparts of size n/3 each.

Correct
Correct.

- [ ] The overall complexity of divide + combine steps in the algorithm is $\Theta(1)$.
- [ ] Master method is applied with $a = b = 3$ and 1. Case-1 applies and the overall complexity is $T(n) = \Theta(n^{\log_3(3)}) = \Theta(n)$
- [x] Master method is applied with $a = b = 3$ and 1. Case-2 applies and the overall complexity is $T(n) = \Theta(n^{\log_3(3)} \log(n)) = \Theta(n \log(n))$

Correct

2. Consider a recurrence of the form :

4 / 4 points

$$T(n) = \begin{cases} \Theta(1) & n \leq 1 \\ 2T(n/3) + \Theta(\sqrt{n}) & \text{otherwise} \end{cases}$$

Select all the correct options from the list below.

- [ ] The recurrence could be produced by a divide and conquer algorithm that divides input of size n into three parts of size $n/2$ each.

- [x] The recurrence denotes a divide and conquer scheme wherein the divide and combine steps take $\Theta(\sqrt{n})$ time in total.

Correct
Correct.

- [x] Master method is applied with $a = 2$ and $b = 3$. We have $\log_b(a) = \log_3(2)$ which is a number between 0 and 1 but is greater than half.

Correct
Correct. Note that $\log_3(1.732..) = 0.5$.

- [x] Case-1 of the master method applies and the complexity of the algorithm is $\Theta(n^{\log_3(2)})$.

Correct
Correct.