2024-25 EVEN SEMESTER TUTORIAL

**SUBJECT CODE: 23MT2005**

**PROBABILITY STATISTICS AND QUEUING THEORY**

**Tutorial 1:**

- Demonstrate Probability: Sample Space and Events
- Make use of Addition, Conditional and Multiplicative Rule
- Bayes Rule

**Date of the Session: //______________________________Time of the Session:______to________**

**Learning outcomes:**

- Defining the events
- Make use of Different rules of probability in engineering problems

1. A fair die is thrown. Write the set of outcomes associated with the following events:

a. E1: a number less than 7

b. E2: a number greater than 7

c. E3: a multiple of 3

d. E4: a number less than 4

e. E5: an even number greater than 4

f. E6: a number not less than 3

**Solution:**

a. **E1**: A number less than 7

$$E1 = \{1, 2, 3, 4, 5, 6\}$$

b. **E2**: A number greater than 7

$E2 = \{\}$ (Empty set, as no outcome is greater than 7)

c. **E3**: A multiple of 3

$$E3 = \{3, 6\}$$

d. **E4**: A number less than 4

$$E4 = \{1, 2, 3\}$$

e. **E5**: An even number greater than 4

$$E5 = \{6\}$$

f. **E6**: A number not less than 3

$$E6 = \{3, 4, 5, 6\}$$

2. On a weekend Rani was playing cards with her family. The deck consists of 52 cards, If her brother drew one card.

   a. Find the probability of getting a king of red color.

   b. Find the probability of getting a face card.

   c. Find the probability of getting a jack of hearts.

   d. Find the probability of getting a red face card.

   e. Find the probability of getting a spade.

**Solution:**

a. **Probability of getting a king of red color**:

- There are 2 red kings (King of Hearts and King of Diamonds).
- Probability = $\frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{2}{52} = \frac{1}{26}$.

b. **Probability of getting a face card**:

- There are 12 face cards (3 face cards per suit × 4 suits).
- Probability = $\frac{12}{52} = \frac{3}{13}$.

c. **Probability of getting a jack of hearts**:

- There is only 1 Jack of Hearts in the deck.
- Probability = $\frac{1}{52}$.

d. **Probability of getting a red face card**:

- There are 6 red face cards (3 from Hearts and 3 from Diamonds).
- Probability = $\frac{6}{52} = \frac{3}{26}$.

e. **Probability of getting a spade**:

- There are 13 spades in the deck.
- Probability = $\frac{13}{52} = \frac{1}{4}$.

3. From experience a stockbroker believes that under present economic conditions a customer will invest in tax-free bonds with a probability of 0.6 will invest mutual funds with a probability of 0.3 and will invest in both tax-free bonds and mutual funds with probability of 0.15. At this time, find the probability that a customer will invest

i) In either tax-free bonds or mutual funds  ii) in neither tax-free bounds nor mutual funds

iii) in only one investment  iv) in tax free bonds only

v) in mutual funds only

**Solution:**

- $A$: Customer invests in tax-free bonds ($P(A) = 0.6$).
- $B$: Customer invests in mutual funds ($P(B) = 0.3$).
- $A \cap B$: Customer invests in both tax-free bonds and mutual funds ($P(A \cap B) = 0.15$).

The calculations are as follows:

**i) In either tax-free bonds or mutual funds:**

Use the formula for the union of two events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = 0.6 + 0.3 - 0.15 = 0.75$$

**ii) In neither tax-free bonds nor mutual funds:**

Complement of $A \cup B$:

$$P(\text{neither}) = 1 - P(A \cup B) = 1 - 0.75 = 0.25$$

iii) **In only one investment:**

Subtract the probability of investing in both from the individual probabilities:

$$P(\text{only one}) = P(A \setminus B) + P(B \setminus A) = P(A) - P(A \cap B) + P(B) - P(A \cap B)$$

$$P(\text{only one}) = (0.6 - 0.15) + (0.3 - 0.15) = 0.45 + 0.15 = 0.6$$

iv) **In tax-free bonds only:**

$$P(A \setminus B) = P(A) - P(A \cap B) = 0.6 - 0.15 = 0.45$$

v) **In mutual funds only:**

$$P(B \setminus A) = P(B) - P(A \cap B) = 0.3 - 0.15 = 0.15$$

## Summary:

- i) $P(\text{either}) = 0.75$
- ii) $P(\text{neither}) = 0.25$
- iii) $P(\text{only one}) = 0.6$
- iv) $P(\text{tax-free bonds only}) = 0.45$
- v) $P(\text{mutual funds only}) = 0.15$

4. Three cards are drawn in succession, without replacement, from an ordinary deck of playing cards. Find the probability that the events $A_1 \cap A_2 \cap A_3$ occurs where $A_1$ is the event that the first card is red ace, $A_2$ is the event that the second card is a 10 or a jack, and $A_3$ is the event that the third card is greater than 3 but less than 7.

**Solution:**

### Step-by-step calculation:

1. **Event** $A_1$: The first card is a red ace.
   - There are 2 red aces (Ace of Hearts and Ace of Diamonds) in the deck of 52 cards.
   - Probability:

$$P(A_1) = \frac{2}{52} = \frac{1}{26}$$

2. **Event** $A_2$: The second card is a 10 or a jack.
   - After removing 1 card (the red ace), 51 cards remain.
   - There are 4 tens and 4 jacks in the deck.
   - Probability:

$$P(A_2|A_1) = \frac{4+4}{51} = \frac{8}{51}$$

3. **Event $A_3$**: The third card is greater than 3 but less than 7.
   - Numbers between 3 and 7 are 4, 5, and 6, with each number having 4 cards (one per suit).
   - Total = $4 + 4 + 4 = 12$.
   - After removing 2 cards (red ace and the 10 or jack), 50 cards remain.
   - Probability:

$$P(A_3|A_1 \cap A_2) = \frac{12}{50} = \frac{6}{25}$$

**Overall probability:**

$$P(A_1 \cap A_2 \cap A_3) = P(A_1) \cdot P(A_2|A_1) \cdot P(A_3|A_1 \cap A_2)$$

$$P(A_1 \cap A_2 \cap A_3) = \frac{1}{26} \cdot \frac{8}{51} \cdot \frac{6}{25}$$

$$P(A_1 \cap A_2 \cap A_3) = \frac{48}{33150} = \frac{8}{5525}$$

**Final Answer:**

The probability is $\frac{8}{5525}$.

**5.** A committee of 5 is chosen from a group of 8 men and 4 women. What is the probability that the group contains a majority of women?

**Solution:**

## Step 1: Total number of ways to form a committee of 5

$$\text{Total ways} = \binom{12}{5} = \frac{12!}{5!(12-5)!} = 792$$

## Step 2: Favorable outcomes (committee with a majority of women)

### Case 1: 3 women and 2 men

- Number of ways to choose 3 women from 4: $\binom{4}{3} = 4$
- Number of ways to choose 2 men from 8: $\binom{8}{2} = 28$
- Total ways for this case:

$$\binom{4}{3} \cdot \binom{8}{2} = 4 \cdot 28 = 112$$

### Case 2: 4 women and 1 man

- Number of ways to choose 4 women from 4: $\binom{4}{4} = 1$
- Number of ways to choose 1 man from 8: $\binom{8}{1} = 8$
- Total ways for this case:

$$\binom{4}{4} \cdot \binom{8}{1} = 1 \cdot 8 = 8$$

### Case 3: 5 women

- Number of ways to choose 5 women from 4: $\binom{4}{5} = 0$ (Not possible).

## Step 3: Total favorable outcomes

$$\text{Favorable outcomes} = 112 + 8 = 120$$

## Step 4: Probability of majority women

$$P(\text{majority women}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{120}{792} = \frac{10}{66} = \frac{5}{33}$$

**Final Answer:**

The probability is $\frac{5}{33}$.

6. A data science team is working on a model to predict whether an email is spam or not. They are using two independent features: The presence of the word "offer "(Feature A) and the presence of a suspicious link (Feature B). The probability that an email contains the word "offer "(Feature A) is 0.6.

The probability that an email contains a suspicious link (Feature B) is 0.4.

a) What is the probability that an email contains both the word "offer" and a suspicious link?

b) What is the probability that an email contains either the word "offer" or a suspicious link or both?

c) What is the probability that an email contains neither the word "offer" nor a suspicious link?

**Solution:**

Given:

- $P(A) = 0.6$ (Probability that an email contains the word "offer").
- $P(B) = 0.4$ (Probability that an email contains a suspicious link).
- $A$ and $B$ are independent.
- $P(A \cap B) = P(A) \cdot P(B)$ because $A$ and $B$ are independent.

**a) Probability that an email contains both the word "offer" and a suspicious link:**

$$P(A \cap B) = P(A) \cdot P(B) = 0.6 \cdot 0.4 = 0.24$$

**b) Probability that an email contains either the word "offer," a suspicious link, or both:**

Use the formula for the union of two events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = 0.6 + 0.4 - 0.24 = 0.76$$

**c) Probability that an email contains neither the word "offer" nor a suspicious link:**

Complement of $A \cup B$:

$$P(\text{neither}) = 1 - P(A \cup B) = 1 - 0.76 = 0.24$$

**Final Answers:**

- a) $P(A \cap B) = 0.24$
- b) $P(A \cup B) = 0.76$
- c) $P(\text{neither}) = 0.24$

7. Police plan to enforce speed limits by using radar traps at 4 different locations within the city limits. The radar traps at each of the location $L_1, L_2, L_3, and\ L_4$ are operated 40%, 30%, 20% and 10% of the time and if a person who is speeding on his way to work has probabilities of 0.2, 0.1, 0.5 and 0.2 respectively, of passing through these locations, what is the probability that he will receive a speeding ticket?

**Solution:**

$$P(\text{Ticket}) = P(\text{Ticket at } L1) + P(\text{Ticket at } L2) + P(\text{Ticket at } L3) + P(\text{Ticket at } L4)$$

Where:

$$P(\text{Ticket at } Li) = P(\text{Trap active at } Li) \cdot P(\text{Passing through } Li)$$

**Probabilities given:**

- $P(\text{Trap active at } L1) = 0.4,\ P(\text{Passing through } L1) = 0.2$
- $P(\text{Trap active at } L2) = 0.3,\ P(\text{Passing through } L2) = 0.1$
- $P(\text{Trap active at } L3) = 0.2,\ P(\text{Passing through } L3) = 0.5$
- $P(\text{Trap active at } L4) = 0.1,\ P(\text{Passing through } L4) = 0.2$

**Calculation:**

$$P(\text{Ticket at } L1) = 0.4 \cdot 0.2 = 0.08$$

$$P(\text{Ticket at } L2) = 0.3 \cdot 0.1 = 0.03$$

$$P(\text{Ticket at } L3) = 0.2 \cdot 0.5 = 0.10$$

$$P(\text{Ticket at } L4) = 0.1 \cdot 0.2 = 0.02$$

**Total probability:**

$$P(\text{Ticket}) = 0.08 + 0.03 + 0.10 + 0.02 = 0.23$$

**Final Answer:**

The probability that the person will receive a speeding ticket is $\mathbf{0.23}$ or $23\%$.

8. A data science team is developing a predictive model to determine whether a user will click on an online advertisement. The likelihood of a user clicking on an ad depends on the type of device they are using. The team has categorized the devices into three types: desktop, tablet, and mobile. The probabilities of a user using each type of device are as follows:

The probability that a user uses a desktop is 0.5.

The probability that a user uses a tablet is 0.2.

The probability that a user uses a mobile device is 0.3.

Also,

• The probability of clicking on an ad given that the user is on a desktop is 0.04.

• The probability of clicking on an ad given that the user is on a tablet is 0.06.

• The probability of clicking on an ad given that the user is on a mobile device is 0.1.

What is the overall probability that a user will click on an ad?

**Solution:**

$$P(\text{Click}) = P(\text{Click} \mid \text{Desktop})P(\text{Desktop}) + P(\text{Click} \mid \text{Tablet})P(\text{Tablet}) + P(\text{Click} \mid \text{Mobile})P(\text{Mobile})$$

**Given Data:**

- $P(\text{Desktop}) = 0.5,\ P(\text{Click} \mid \text{Desktop}) = 0.04$
- $P(\text{Tablet}) = 0.2,\ P(\text{Click} \mid \text{Tablet}) = 0.06$
- $P(\text{Mobile}) = 0.3,\ P(\text{Click} \mid \text{Mobile}) = 0.1$

**Calculation:**

$$P(\text{Click}) = (0.04 \cdot 0.5) + (0.06 \cdot 0.2) + (0.1 \cdot 0.3)$$

1. $0.04 \cdot 0.5 = 0.02$
2. $0.06 \cdot 0.2 = 0.012$
3. $0.1 \cdot 0.3 = 0.03$

$$P(\text{Click}) = 0.02 + 0.012 + 0.03 = 0.062$$

1. $0.04 \cdot 0.5 = 0.02$
2. $0.06 \cdot 0.2 = 0.012$
3. $0.1 \cdot 0.3 = 0.03$

$$P(\text{Click}) = 0.02 + 0.012 + 0.03 = 0.062$$

### Final Answer:

The overall probability that a user will click on an ad is $\mathbf{0.062}$ or $\mathbf{6.2\%}$.

**VIVA QUESTIONS**

1. What is a sample space in probability theory?

The set of all possible outcomes of an experiment.

2. How do you calculate the number of outcomes in a sample space?

Use combinations, permutations, or direct enumeration based on the experiment.

3. Define a complementary event in probability.

An event consisting of all outcomes not in the given event.

4. Define different types of events in the context of probability theory.

- **Independent events:** Do not affect each other.
- **Mutually exclusive events:** Cannot happen together.
- **Dependent events:** Affect each other.
- **Exhaustive events:** Cover all possibilities.

5. State the importance of Baye’s Rule.

It calculates conditional probabilities using prior knowledge.

*(For Evaluators use only)*

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| <u>Comment of the Evaluator (if Any)</u> | <u>Evaluator’s Observation</u><br>Marks Secured:___________out of___________<br>Full Name of the Evaluator:<br>Signature of the Evaluator:<br>Date of Evaluation |