Digital Communication 23EC2208A

Spread-Spectrum Communications

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Generation of Pseudo Noise (PN) Sequences

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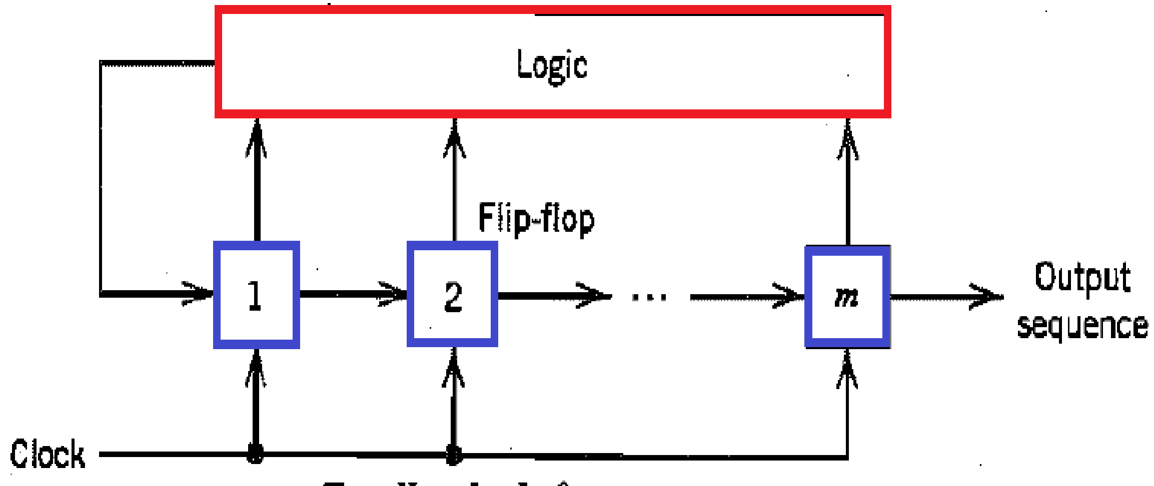
Pseudo Noise (PN) Sequence is a periodic binary sequence with noise like waveform that is usually generated by means of a feedback register.

A feedback shift register consists of

1. An ordinary shift register made up of m flip-flops, and

2. A logic circuit that are interconnected to form multi-loop feedback circuit.



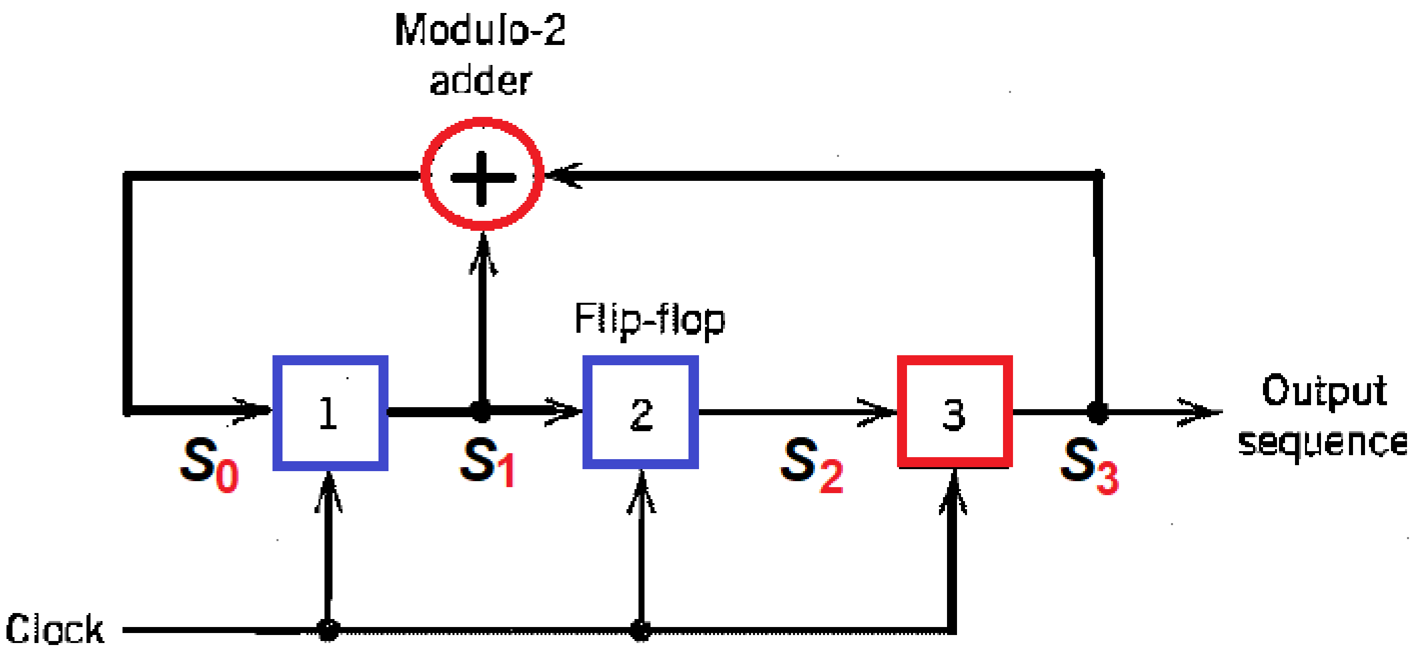
Feedback shift register.

- The flip-flops in shift register are regulated by a single timing clock.
- At each pulse (tick) of the clock the state of each flip-flop is shifted to the next one down the line.
- With each clock pulse the logic circuit computes Boolean function of the states of the flip-flops.
- The PN sequence is generated by length m of the shift register, its initial state and feed back logic.

- The period of the PN sequence produced by a linear feed back shift register with m flip-flops.

- When the PN sequence is exactly $2^m - 1$, then the PN sequence is called maximum length sequence or simply m-sequence.

Example: Maximum sequence generator for m = 3.
Assume initial state: 1 0 0



Then the successive states are:
1 0 0, 1 1 0, 1 1 1, 0 1 1, 1 0 1, 0 1 0, 0 0 1, 1 0 0

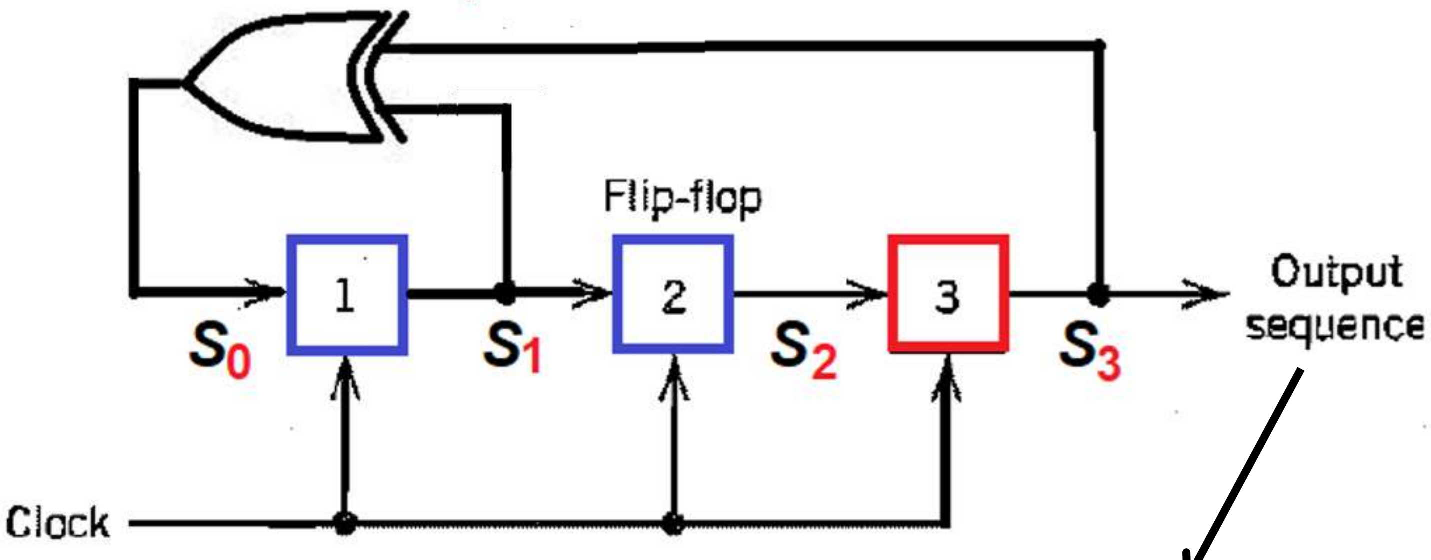
The output sequence (the last position of each state of the shift register) is therefore 0 0 1 1 1 0 1 0 ….

- That repeats with period of $2^3 - 1 = 7$.

Maximum-length sequence generator for m = 3

Assume initial state: 1 0 0

S_0	S_1	S_2	S_3
1	1	0	0
1	1	1	0
0	1	1	1
1	0	1	1
0	1	0	1
0	0	1	0
1	0	0	1
	1	0	0



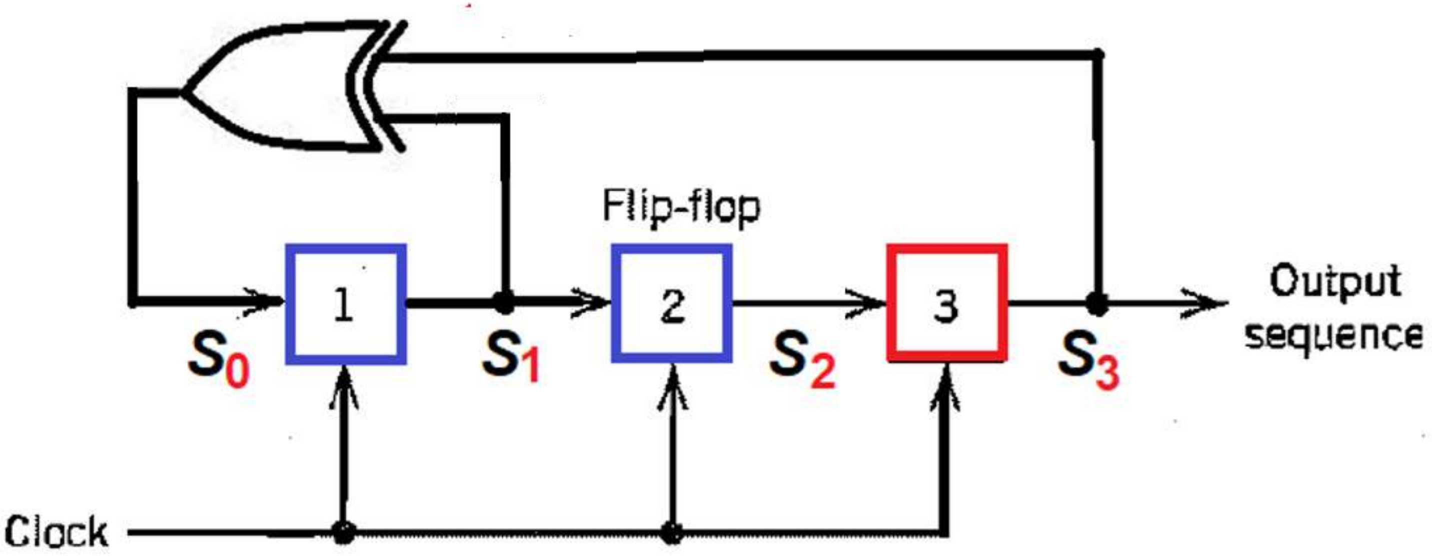
Repeated

0 0 1 1 1 0 1 . . .

Maximum-length sequence generator for m = 3

Assume initial state:

S_0	S_1	S_2	S_3



Auto-correlation and Power Spectral Density

- The autocorrelation function gives the measure of similarity between a signal and its time-delayed version.
- The autocorrelation function of power (or periodic) signal x(t) with any time period *T* is given by,

$$R(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-(T/2)}^{T/2} x(t)\,x^*(t-\tau)\;dt$$

Where, *τ* is called the *delayed parameter.*

Power Spectral Density: The distribution of average power of a signal in the frequency domain is called the power spectral density (**PSD**) or power density spectrum.

$$S(\omega) = \lim_{P\to\infty} \frac{|X(\omega)|^2}{P}$$

Relation between PSD and Autocorrelation Function $R(\tau) \xleftrightarrow{FT} S(\omega)$

Properties of m-sequence

Property1: Balance property: In each period of m-sequence, the No. of '1's is always one more than the No. of '0's.

Ex: The m-sequence 0 0 1 1 1 0 1

Property2: Runs / Run length property: Refers to the distribution of consecutive identical symbols (runs) in the sequence.

A key characteristic of PN sequences is that the number of runs of a given length follows a specific pattern.

- Approximately half of the runs are of length 1.
- Approximately a quarter of the runs are of length 2.
- Approximately an eighth of the runs are of length 3, and so on.

For m-sequence the total No. of runs is (N+1)/2, where $N = 2^m - 1$

Example on Run length property

$$\underbrace{0\ 0\ 0}\ \underbrace{1}\ \underbrace{0\ 0}\ \underbrace{1\ 1}\ \underbrace{0}\ \underbrace{1}\ \underbrace{0}\ \underbrace{1\ 1\ 1\ 1}$$

Total number of runs are given by: $\dfrac{N+1}{2} = \dfrac{(15+1)}{2} = 8$

In general run of length of n (bits) can be given as:

$$\text{run of length of n} = \frac{1}{2^n} \times \text{ total number of runs}$$

$$\text{The run of length of 1} = \frac{1}{2^1} \times 8 = 4$$

$$\text{The run of length of 2} = \frac{1}{2^2} \times 8 = 2$$

$$\text{The run of length of 3} = \frac{1}{2^3} \times 8 = 1$$

Properties of m-sequence:

Property3: Correlation Property:

The Autocorrelation function of m-sequence is periodic and binary valued.

The period of m-sequence length is given by $N = 2^m - 1$

Let the binary symbols 0 and 1 of the sequence denoted by the levels -1 and +1 respectively.

Let $c(t)$ denotes the resulting waveform of the m-sequence.

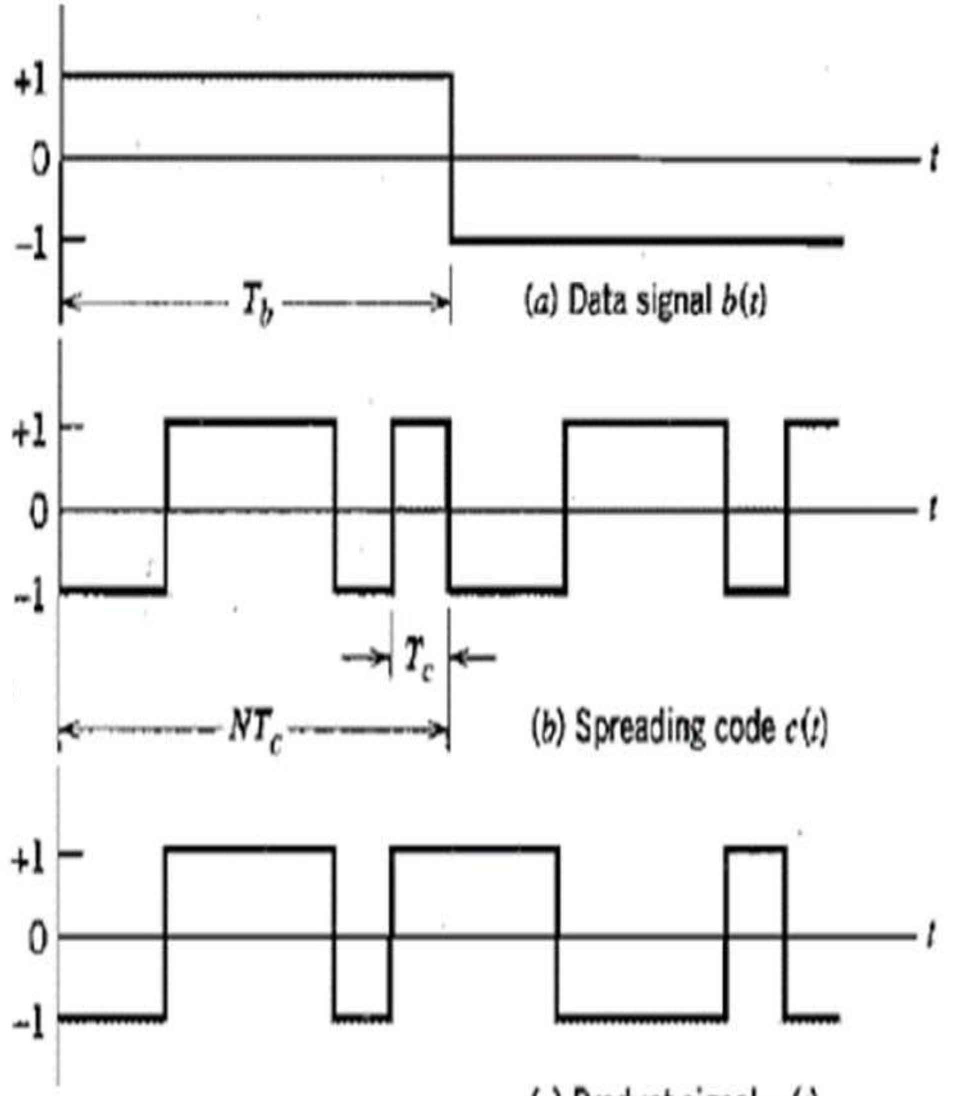
- The period of the waveform $c(t)$ is $T_b = N\,T_c$ where T_b is the data bit duration and T_c is the chip duration.

- The Autocorrelation of periodic signal $c(t)$ of period T_b

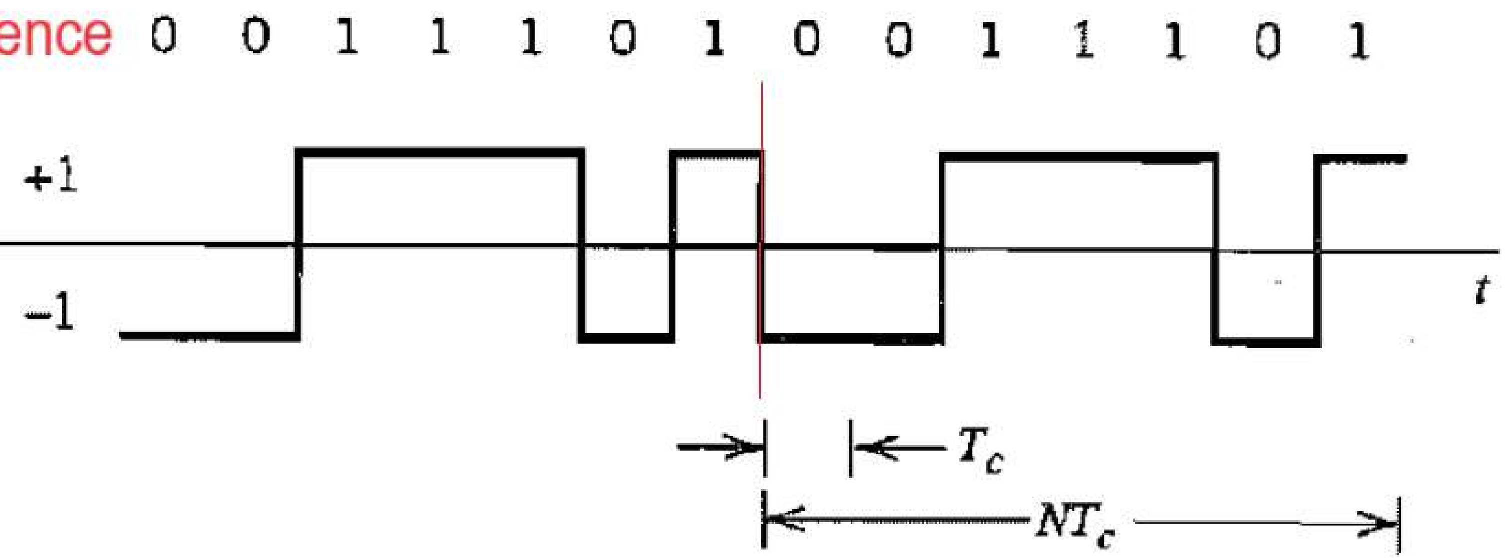
$$R_c(\tau) = \frac{1}{T_b}\int_{-T_b/2}^{T_b/2} c(t)c(t-\tau)\,dt$$

- The corresponding power spectral density

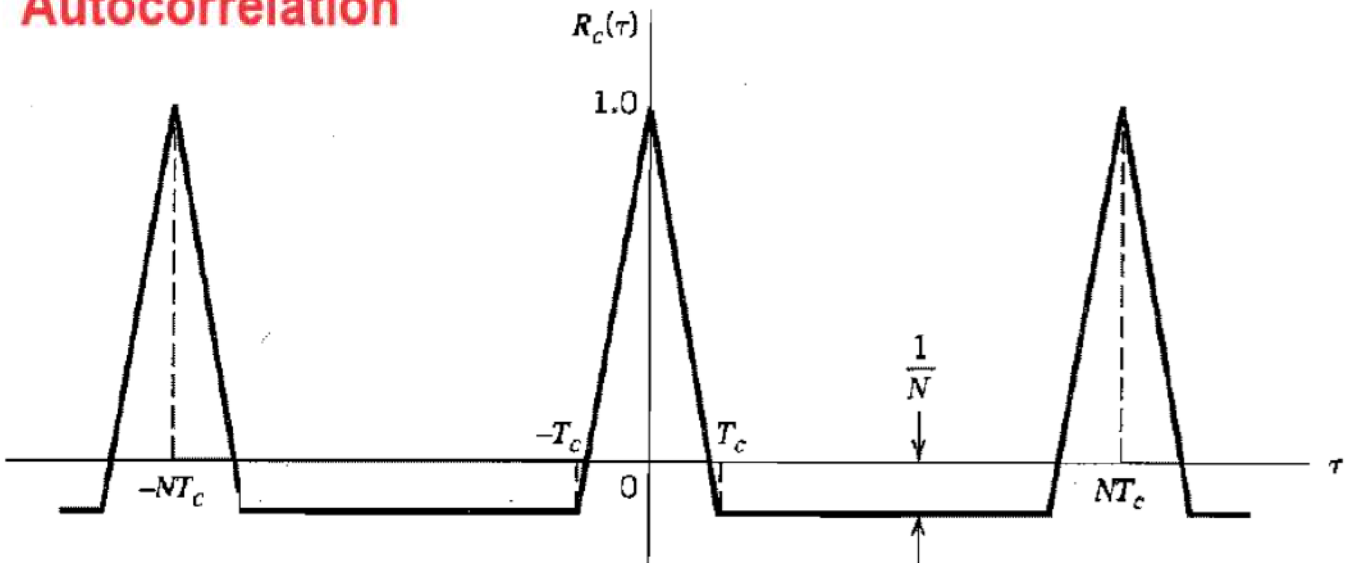
$$S_c(f) = \frac{1}{N^2}\delta(f) + \frac{1+N}{N^2}\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \operatorname{sinc}^2\left(\frac{n}{N}\right)\delta\left(f - \frac{n}{NT_c}\right)$$



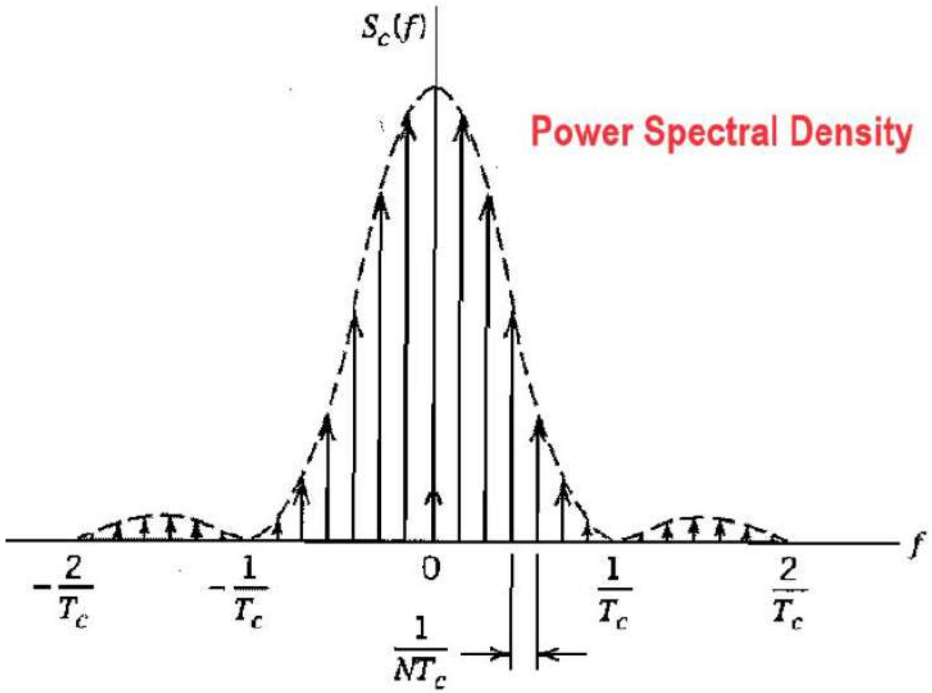
Binary Sequence



Autocorrelation



Power Spectral Density



Processing Gain

- Processing gain is the ratio of the spread RF bandwidth to the original information bandwidth.
- The effect of multipath fading as well as interference can be reduced by a factor equivalent to the processing gain.
- In fact, it quantifies the degree of interference rejection.

$$G_p = \frac{B_{ss}}{B_s} \qquad G_p = \frac{T_b}{T_c} \qquad G_p = \frac{R_c}{R_b}$$

B_s Bandwidth of input data

B_{ss} Bandwidth of spreaded signal

T_b Bit duration of data

T_c Chip duration

R_b Data bit rate

R_c Chip rate

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A direct sequence spread binary phase shift keying system uses a feedback shift register of length 19 for the generation of PN sequence. Calculate the processing gain of the system.

Ans: Given length of shift register $= m = 19$

Therefore, length of PN sequence $N = 2^m - 1 = 2^{19} - 1 = 524287$

Processing gain PG = Tb/Tc = N in dB $= 10\log_{10} N$

$$= 10 \log_{10} (2^{19}) = 57 \text{ dB}$$

A Spread spectrum communication system has the following parameters. Information bit duration Tb = 1.024 msecs and PN chip duration of 1μsecs. The average probability of error of system is not to exceed 10^-5.
Calculate (a) Length of shift register (b) Processing gain (c) jamming margin

Ans: Processing gain PG =N= Tb/Tc =1024, then length of shift register m = 10

In case of coherent BPSK For Probability of error 10^{-5}. [Referring to error function table] Eb/N0 =10.8. Therefore jamming margin

$$(jamming\ margin)_{dB} = (Processing\ gain)_{dB} - 10\log_{10}\left(\frac{E_b}{N_0}\right)_{min}$$

$$(jamming\ margin)_{dB} = 10\log_{10}PG_{dB} - 10\log_{10}\left(\frac{E_b}{N_0}\right)_{min}$$

$$(jamming\ margin)_{dB} = 10\log_{10}1024 - 10\log_{10}10.8$$

$$(jamming\ margin)_{dB} = 30.10 - 10.33 = 19.8\ dB$$

The limitations are:

- Long acquisition time due to large code length
- Fast code generator is required because the chip rate is much higher than the data rate
- Requires wideband channel with very little distortion
- Susceptible to the near-far problem

End