

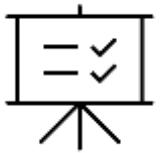
Advanced Algorithms & Data Structures

AIM OF THE SESSION



To familiarize students with the basic concept of B Tree

INSTRUCTIONAL OBJECTIVES



This Session is designed to:

1. Demonstrate The B Tree.
2. Describe the types of B Tree traversals.
3. Constructing B Tree and traversals.
4. Constructing an expression tree.
5. List out the advantages and applications of B Tree.

LEARNING OUTCOMES



At the end of this session, you should be able to:

1. Define B Tree
2. Describe the various traversals in B Tree Expression tree construction
3. Summarize definition, types and operations of B Tree and its applications

B Tree

- In search trees like binary search tree, AVL Tree, Red-Black tree, etc., every node contains only one value (key) and a maximum of two children.
- But there is a special type of search tree called B-Tree in which a node contains more than one value (key) and more than two children.
- B-Tree was developed in the year 1972 by Bayer and McCreight with the name Height Balanced m-way Search Tree. Later it was named as B-Tree.

B Tree

B-Tree can be defined as follows...

B-Tree is a self-balanced search tree in which every node contains multiple keys and has more than two children.

Here, the number of keys in a node and number of children for a node depends on the order of B-Tree. Every B-Tree has an order.

B-trees save time by using nodes with many branches (called children), compared with binary trees, in which each node has only two children, thereby speeding up the process.

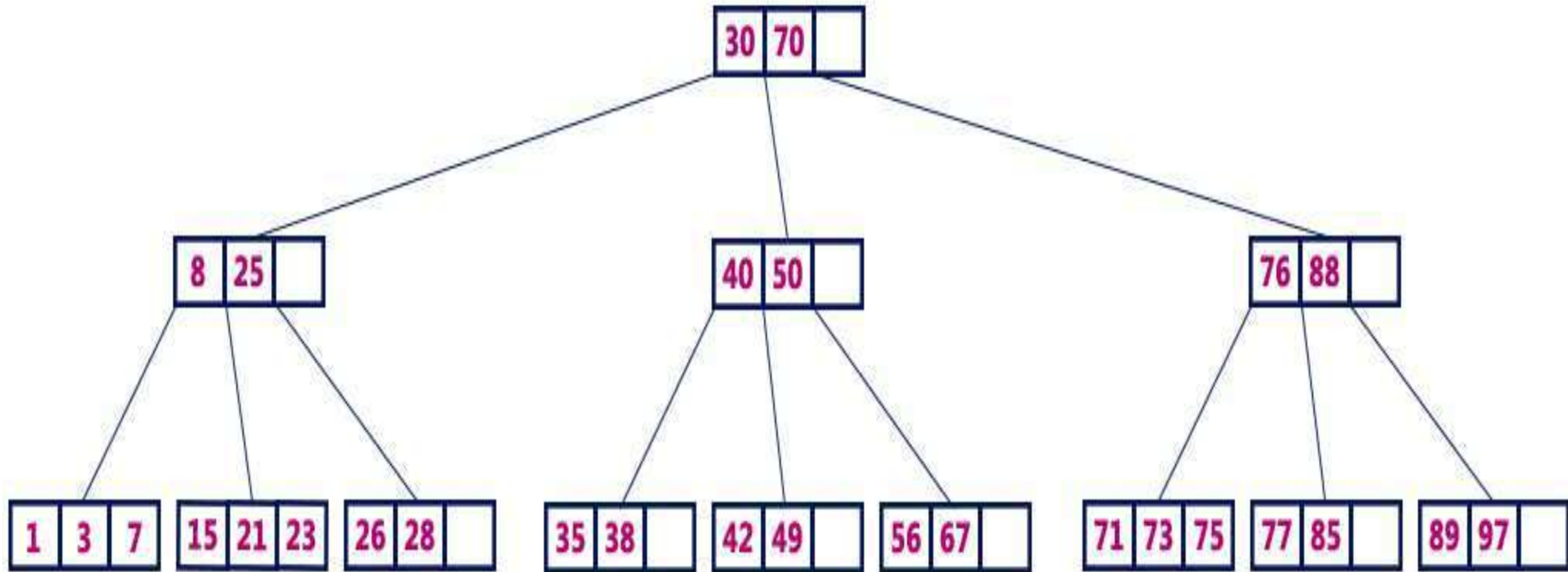
B-Tree of Order m has the following properties...

- **Property #1** - All leaf nodes must be at same level.
- **Property #2** - All nodes except root must have at least $\lceil m/2 \rceil - 1$ keys and maximum of $m-1$ keys.
- **Property #3** - All non leaf nodes except root (i.e. all internal nodes) must have at least $m/2$ children.
- **Property #4** - If the root node is a non leaf node, then it must have at least 2 children.
- **Property #5** - A non leaf node with $n-1$ keys must have n number of children.
- **Property #6** - All the key values in a node must be in Ascending Order.

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- **Property #3** - All non leaf nodes except root (i.e. all internal nodes) must have at least $m/2$ children.
- **Property #4** - If the root node is a non leaf node, then it must have **atleast 2** children.
- **Property #5** - A non leaf node with $n-1$ keys must have n number of children.
- **Property #6** - All the **key values in a node** must be in **Ascending Order**.

B-Tree of Order 4



Operations on a B-Tree

The following operations are performed on a B-Tree...

1.Search

2.Insertion

3.Deletion

Search Operation in B-Tree

The search operation in B-Tree is similar to the search operation in Binary Search Tree. In a Binary search tree, the search process starts from the root node and we make a 2-way decision every time (we go to either left subtree or right subtree). In B-Tree also search process starts from the root node but here we make an n-way decision every time. Where 'n' is the total number of children the node has. In a B-Tree, the search operation is performed with $O(\log n)$ time complexity. The search operation is performed as follows...

Step 1 - Read the search element from the user.

Step 2 - Compare the search element with first key value of root node in the tree.

Step 3 - If both are matched, then display "Given node is found!!!" and terminate the function

Step 4 - If both are not matched, then check whether search element is smaller or larger than that key value.

Step 5 - If search element is smaller, then continue the search process in left subtree.

Step 6 - If search element is larger, then compare the search element with next key value in the same node and repeat steps 3, 4, 5 and 6 until we find the exact match or until the search element is compared with last key value in the leaf node.

Step 7 - If the last key value in the leaf node is also not matched then display "Element is not found" and terminate the function.

Insertion Operation in B-Tree

In a B-Tree, a new element must be added only at the leaf node. That means, the new keyValue is always attached to the leaf node only. The insertion operation is performed as follows...

Step 1 - Check whether tree is Empty.

Step 2 - If tree is **Empty**, then create a new node with new key value and insert it into the tree as a root node.

Step 3 - If tree is **Not Empty**, then find the suitable leaf node to which the new key value is added using Binary Search Tree logic.

Step 4 - If that leaf node has empty position, add the new key value to that leaf node in ascending order of key value within the node.

Step 5 - If that leaf node is already full, **split** that leaf node by sending middle value to its parent node. Repeat the same until the sending value is fixed into a node.

Step 6 - If the spiling is performed at root node then the middle value becomes new root node for the tree and the height of the tree is increased by

Before going through the steps below, one must know these facts about a B tree of degree **m**.

1. A node can have a maximum of m children. (i.e. 3)
2. A node can contain a maximum of $m - 1$ keys. (i.e. 2)
3. A node should have a minimum of $\lceil m/2 \rceil$ children. (i.e. 2)
4. A node (except root node) should contain a minimum of $\lceil m/2 \rceil - 1$ keys. (i.e. 1)

There are three main cases for deletion operation in a B tree.

Example

Construct a B-Tree of Order 3 by inserting numbers from 1 to 10.

Construct a B-Tree of order 3 by inserting numbers from 1 to 10.

insert(1)

Since '1' is the first element into the tree that is inserted into a new node. It acts as the root node.



insert(2)

Element '2' is added to existing leaf node. Here, we have only one node and that node acts as root and also leaf. This leaf node has an empty position. So, new element (2) can be inserted at that empty position.



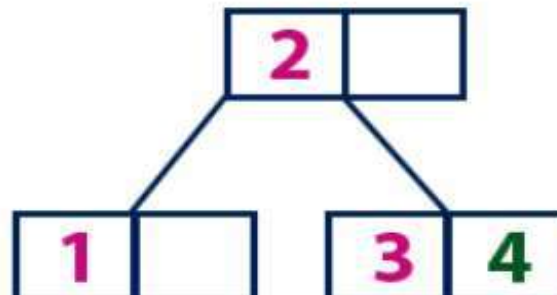
insert(3)

Element '3' is added to existing leaf node. Here, we have only one node and that node acts as root and also leaf. This leaf node doesn't have an empty position. So, we split that node by sending middle value (2) to its parent node. But here, this node doesn't have a parent. So, this middle value becomes a new root node for the tree.



insert(4)

Element '4' is larger than root node '2' and it is not a leaf node. So, we move to the right of '2'. We reach to a leaf node with value '3' and it has an empty position. So, new element (4) can be inserted at that empty position.



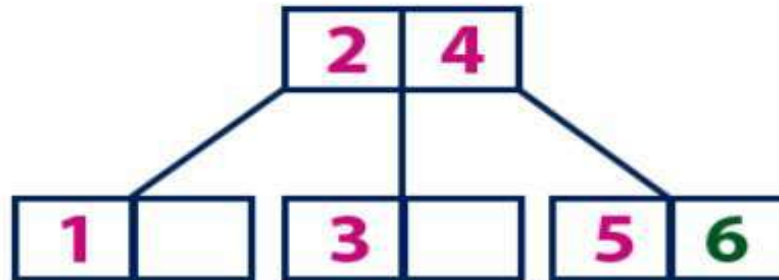
insert(5)

Element '5' is larger than root node '2' and it is not a leaf node. So, we move to the right of '2'. We reach to a leaf node and it is already full. So, we split that node by sending middle value (4) to its parent node (2). There is an empty position in its parent node. So, value '4' is added to node with value '2' and new element '5' added as new leaf node.



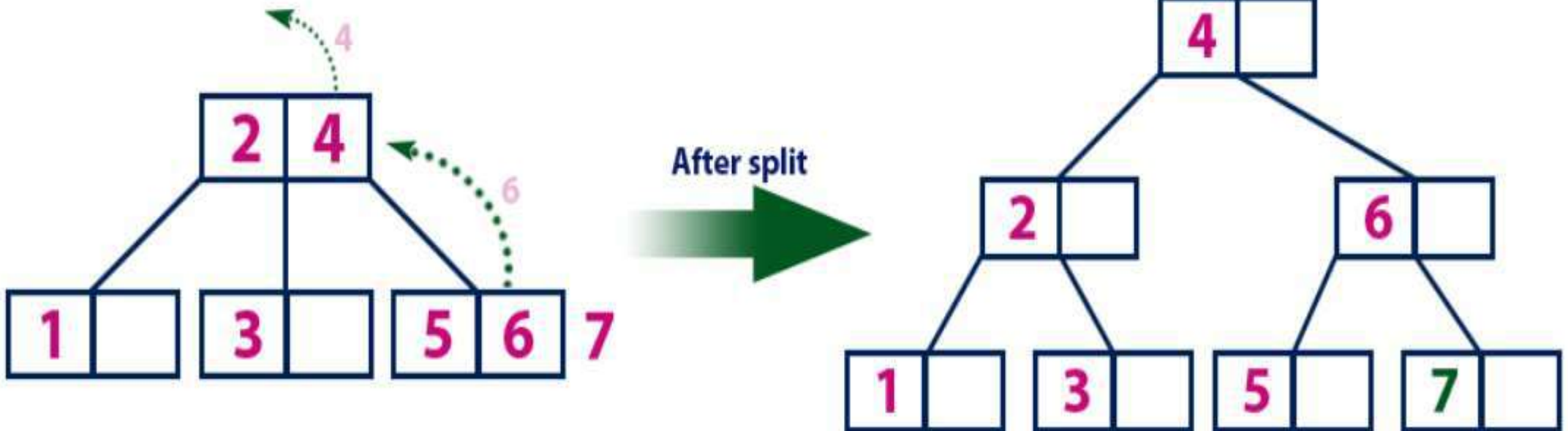
insert(6)

Element '6' is larger than root node '2' & '4' and it is not a leaf node. So, we move to the right of '4'. We reach to a leaf node with value '5' and it has an empty position. So, new element (6) can be inserted at that empty position.



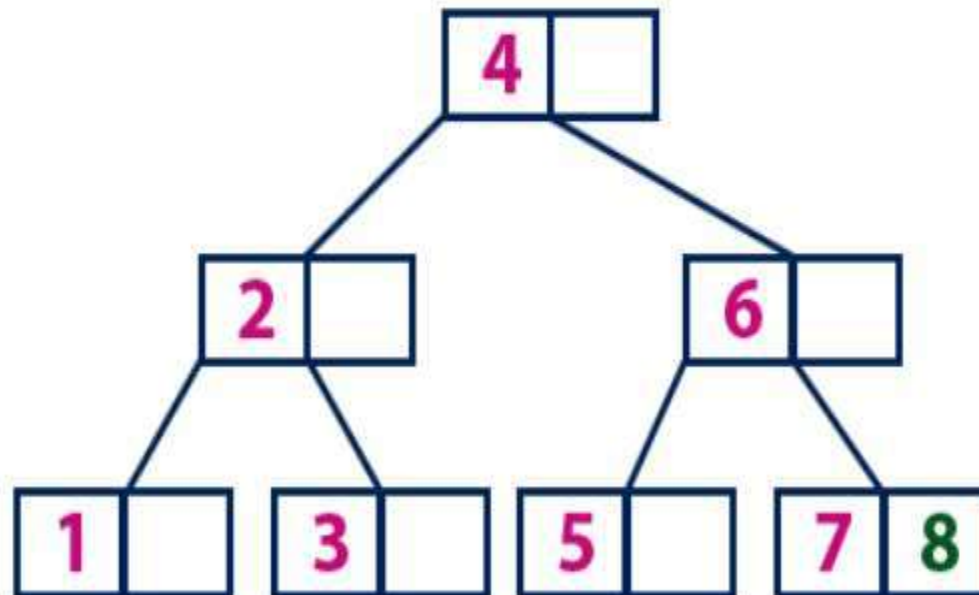
insert(7)

Element '7' is larger than root node '2' & '4' and it is not a leaf node. So, we move to the right of '4'. We reach to a leaf node and it is already full. So, we split that node by sending middle value (6) to its parent node (2&4). But the parent (2&4) is also full. So, again we split the node (2&4) by sending middle value '4' to its parent but this node doesn't have parent. So, the element '4' becomes new root node for the tree.



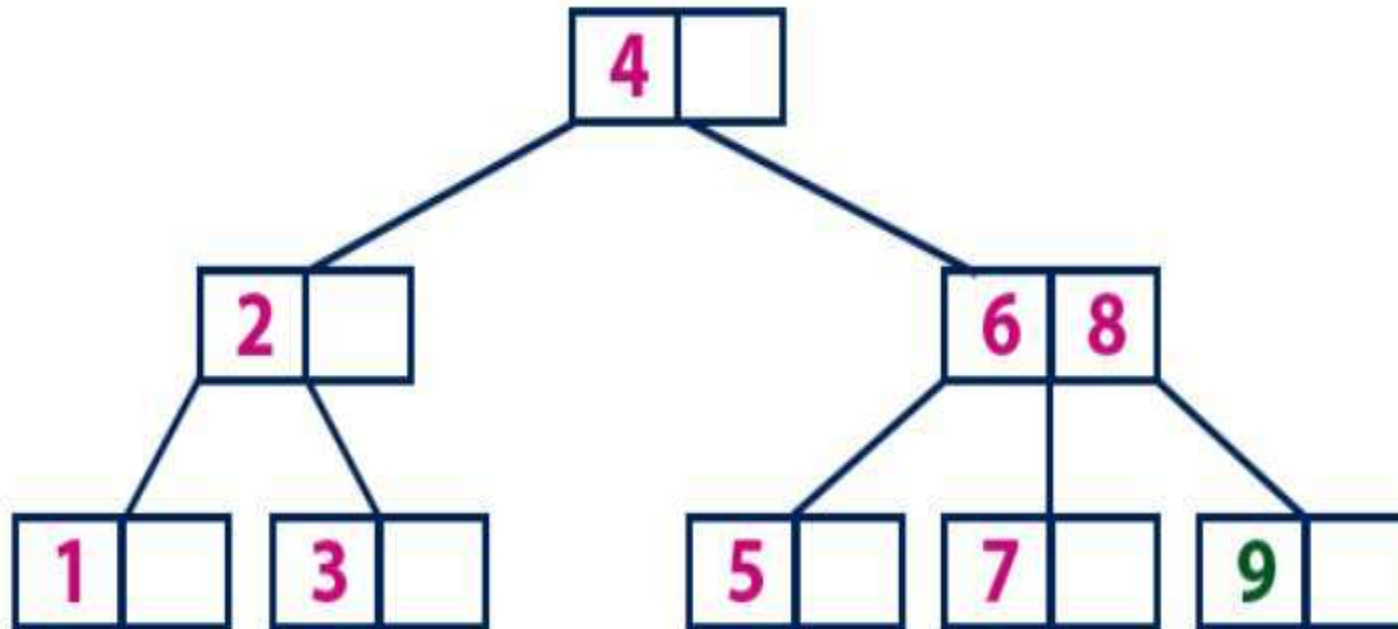
insert(8)

Element '8' is larger than root node '4' and it is not a leaf node. So, we move to the right of '4'. We reach to a node with value '6'. '8' is larger than '6' and it is also not a leaf node. So, we move to the right of '6'. We reach to a leaf node (7) and it has an empty position. So, new element (8) can be inserted at that empty position.



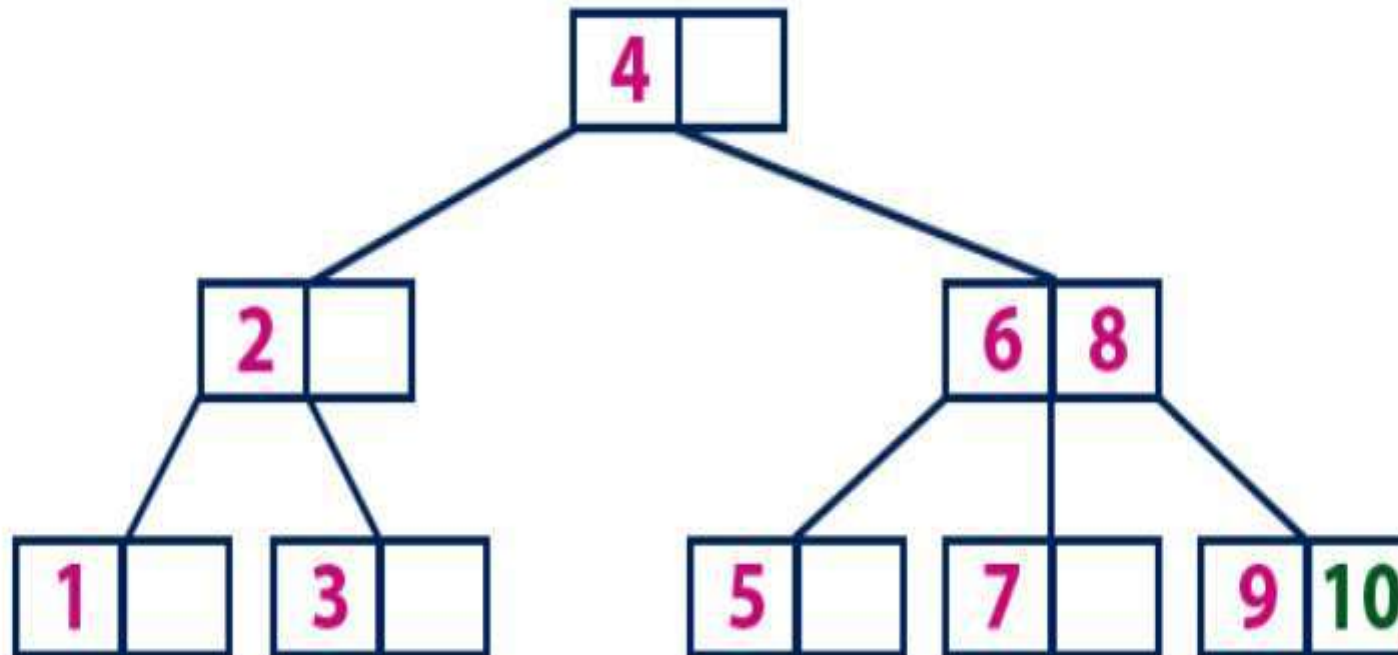
insert(9)

Element '9' is larger than root node '4' and it is not a leaf node. So, we move to the right of '4'. We reach to a node with value '6'. '9' is larger than '6' and it is also not a leaf node. So, we move to the right of '6'. We reach to a leaf node (7 & 8). This leaf node is already full. So, we split this node by sending middle value (8) to its parent node. The parent node (6) has an empty position. So, '8' is added at that position. And new element is added as a new leaf node.



insert(10)

Element '10' is larger than root node '4' and it is not a leaf node. So, we move to the right of '4'. We reach to a node with values '6 & 8'. '10' is larger than '6 & 8' and it is also not a leaf node. So, we move to the right of '8'. We reach to a leaf node (9). This leaf node has an empty position. So, new element '10' is added at that empty position.



- Example: Insert the keys 78, 52, 81, 40, 33, 90, 85, 20, and 38 in this order in an initially empty B-tree of order 3

