# Design and Analysis of Algorithms

## NODE COVER DECISION PROBLEM(NCDP)

Session -35

**Node Cover Decision Problem(NCDP):**

A set $S \subseteq V$ is a *node cover* for a graph $G = (V,E)$ if and only if all edges in E are incident to at least one vertex in S. The size $|S|$ of the cover is the number of vertices in S.

**Example 11.12** Consider the graph:

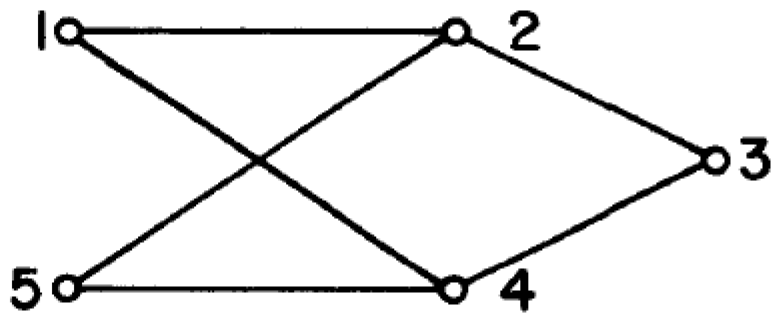


**Figure 11.2** A sample graph and node cover

S = {2,4} is a node cover of size 2.

S = { 1,3,5} is a node cover of size 3.

In a node cover decision problem we are given a graph G and an integer k.

We are required to determine whether G has a node cover of size at most k.

**Theorem: The clique decision problem $\alpha$ the node cover decision problem.**

**Proof:** let G = (V,E) and k define an instance of CDP. Assume that |V| = n.

We Construct a Graph G' such that G' has a node cover of size at most n-k if and only if G has a clique of size at least k.

Graph G'= (V,E) = ,where E = {(u,v) | u ϵ V, v ϵ V and (u,v) ∉ E}.

The set G' is known is the complement of G

Now, we shall show that G has a clique of size at least k iff G' has a node cover of size at most n - k. Let K be any clique in G.

Since there are no edges in $\bar{E}$ connecting vertices in K, the remaining n - |K| vertices in G' must cover all edges in $\bar{E}$.

Similarly, if S is a node cover of G' then V - S must form a complete subgraph in G.

Since G' can be obtained from G in polynomial time, CDP can be solved in polynomial deterministic time if we have a polynomial time deterministic algorithm for NCDP.

Note that since CNF-satisfiability $\propto$ CDP, CDP $\propto$ NCDP and $\propto$ is transitive, it follows that NCDP is NP-hard.

**Example 11.11** Consider $F = (x_1 \vee x_2 \vee x_3) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3)$. The construction of Theorem 11.2 yields the graph:

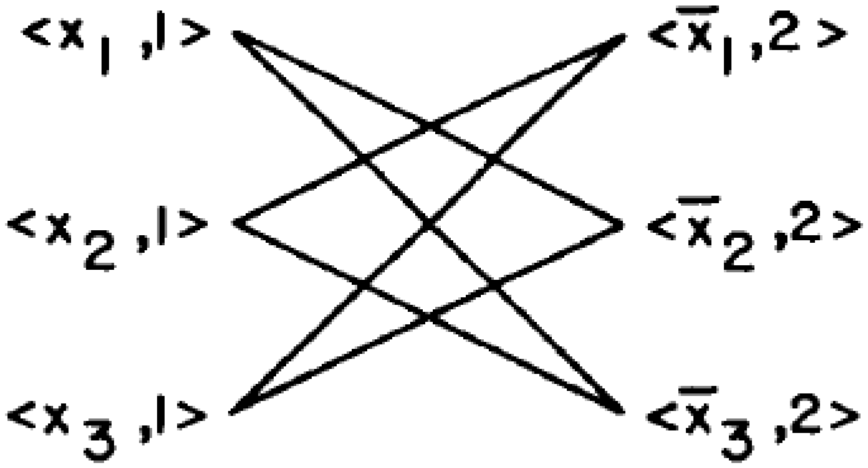


**Figure 11.1** A sample graph and satisfiability

This graph contains six cliques of size two. Consider the clique with vertices $\{\langle x_1, 1\rangle, \langle \bar{x}_2, 2\rangle\}$. By setting $x_1$ = true and $\bar{x}_2$ = true (i.e. $x_2$ = false) $F$ is satisfied. $x_3$ may be set either to true or false. □

Questions:

1. Explain in detail about NCDP?
2. Prove that The clique decision problem $\alpha$ the node cover decision problem.

# THANK YOU