MATHEMATICAL PROGRAMMING

SWARM INTELLIGENCE -2

(PARTICLE SWARM OPTIMIZATION)

CO - 4

Session - 24

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AIM`

1. To familiarise students to the fundamental principles of Particle Swarm Optimization and algorithm.

INSTRUCTIONAL OBJECTIVES

This session is designed to deliver:

1. Particle Swarm Optimization
   a. Origin, Concept
   b. Algorithm
   c. Example

LEARNING OUTCOMES

At the end of this session, students will be able to know and apply:

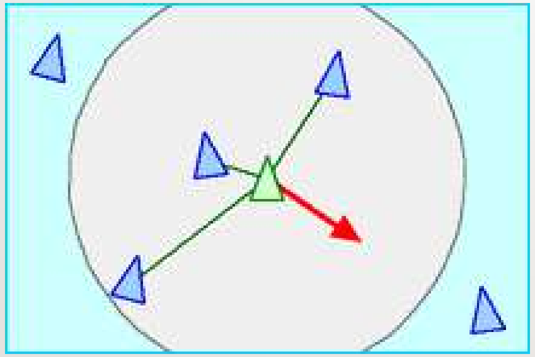
1. Particle Optimization Algorithm to different applications

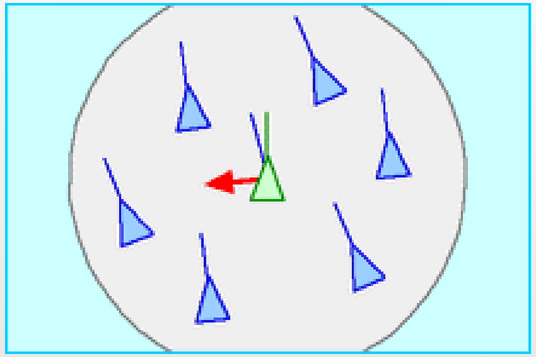
INTRODUCTION TO THE PSO: **ORIGINS**

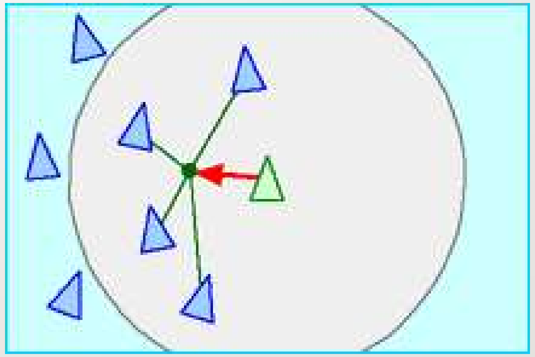
- Inspired from the nature social behavior and dynamic movements with communications of insects, birds and fish

INTRODUCTION TO THE PSO: **ORIGINS**

- In 1986, Craig Reynolds described this process in 3 simple behaviors:







Separation

avoid crowding local flockmates

Alignment

move towards the average heading of local flockmates

Cohesion

move toward the average position of local flockmates

INTRODUCTION TO THE PSO: <u>**ORIGINS**</u>

- Application to optimization: <u>Particle Swarm Optimization</u>
- Proposed by James Kennedy & Russell Eberhart (1995)
- Combines <u>self-experiences</u> with <u>social experiences</u>

INTRODUCTION TO THE PSO: **CONCEPT**

- Uses a number of agents (**particles**) that constitute a swarm moving around in the search space looking for the best solution.

- Each particle in search space adjusts its "flying" according to its own flying experience as well as the flying experience of other particles.

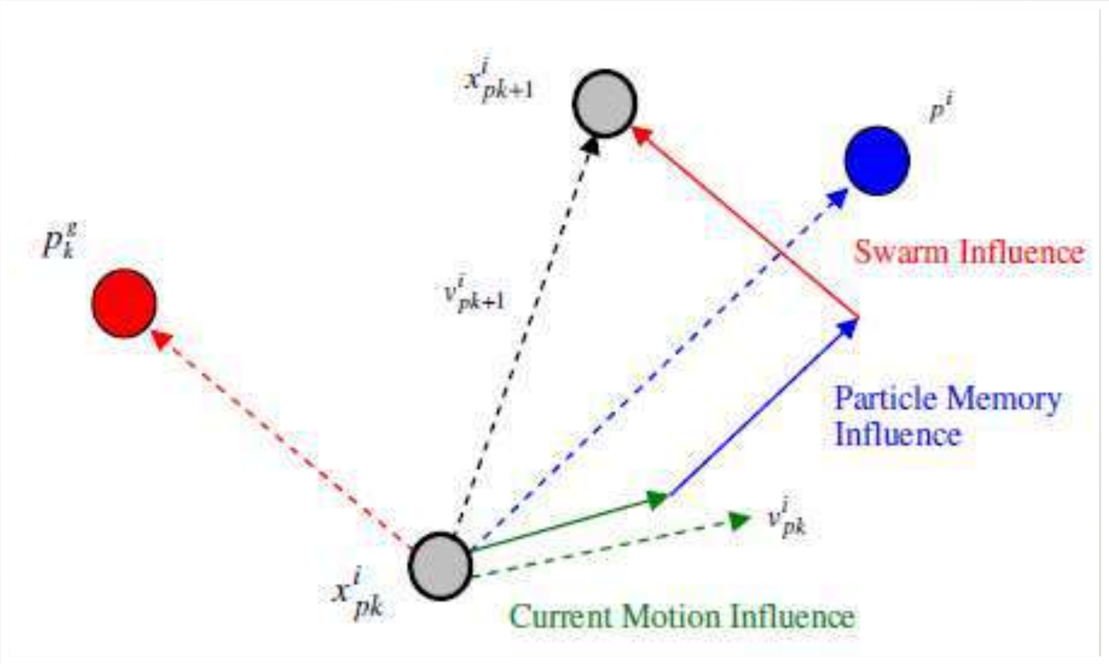
CONTD....

- Collection of flying particles (swarm) - Changing solutions
- Search area - Possible solutions
- Movement towards a promising area to get the global optimum
- Each particle keeps track:
  - its best solution, personal best, *pbest*
  - the best value of any particle, global best, *gbest*

CONTD...

- Each particle adjusts its travelling speed dynamically corresponding to the flying experiences of itself and its colleagues

- Each particle modifies its position according to:
  - its current position
  - its current velocity
  - the distance between its current position and p_{best}
  - the distance between its current position and g_{best}



PSO ALGORITHM

Basic Algorithm of PSO

1. Initialize the swarm from the solution space.

2 Evaluate fitness of each particle.

3 Update individual and global bests.

4 Update velocity and position of each particle.

5 Go to step 2, and repeat until termination condition.

UPDATE VELOCITY AND POSITION OF EACH PARTICLE.

- Velocity of particle

$$v(t+1) = \{V(t) + c_1 * r_1 * (P_{best} - x) + c_2 * r_2 * (G_{best} - x)\}$$

Where

x : particle's position, v: path direction

r_1, r_2 are the random numbers in the range of (0, 1)

c_1: weight of local information, c_2: weight of global information

p_{best}: best position of the particle

g_{best}: best position of the swarm

- Position of particle

$$x(t+1) = x(t) + v(t+1)$$

PSO ALGORITHM - PARAMETERS

- Number of particles usually between 10 and 50
- C_1 is the importance of personal best value
- C_2 is the importance of neighborhood best value
- Usually $C_1 + C_2 = 4$ (empirically chosen value)
- If velocity is too low → algorithm too slow
- If velocity is too high → algorithm too unstable

DEFINE THE PROBLEM

Find the maximum of the function

$$f(x) = -x^2 + 5x + 20$$

- with $-10 \leq x \leq 10$ using the PSO algorithm.

PROBLEM ANALYSIS

- 1 Size of a swarm.
- 2 How to generate initial particles with position and velocity.
- 3 Finding fitness function.
- 4 Finding P_{best} and G_{best}.
- 5 Updating Velocity. (values of C_1,C_2,W, etc.)
- 6 limits for velocity (Vmax,Vmin)
- 7 Updating position.
- 8 Terminating condition.
- 9 ....

INITIALIZATION

- First we generate swarm of size 5 randomly using uniform distribution in the range (-10, 10).

Particle Number	Position Vector (x)
1	-6.2
2	8.3
3	-1.1
4	0.6
5	2.6

$$\gg x = -10 + rand(5,1) * (10 + 10)$$

x =
6.2945
8.1158
-7.4603
8.2675
2.6472

PARTICLE VELOCITY INITIALIZATION

- Similarly the velocity vector is generated uniformly in the range (0, 1)

Particle Number	Position Vector (x)	Velocity Vector (v)
1	-6.2	0.4752
2	8.3	0.7797
3	-1.1	0.4141
4	0.6	0.6183
5	2.6	0.2530

$\gg v = rand(5, 1)$

v =

0.0975

0.2785

0.5469

0.9575

0.9649

EVALUATE FITNESS OF EACH PARTICLE

Particle Number	Position Vector (x)	Velocity Vector (v)	Fitness Vector (f(x))
1	-6.2	0.4752	-49.44
2	8.3	0.7797	-7.39
3	-1.1	0.4141	13.29
4	0.6	0.6183	22.64
5	2.6	0.2530	26.24

$\gg f = -(x. * x) + 5 * x + 20$

f =

11.8518

-5.2872

-72.9576

-7.0141

26.2283

UPDATE INDIVIDUAL AND GLOBAL BESTS

Particle Number	Position Vector (x)	Velocity Vector (v)	Fitness Vector (f(x))	Pbest
1	-6.2	0.4752	-49.44	-6.2
2	8.3	0.7797	-7.39	8.3
3	-1.1	0.4141	13.29	-1.1
4	0.6	0.6183	22.64	0.6
5	2.6	0.2530	26.24	2.6

Particle Number	Position Vector (x)	Velocity Vector (v)	Fitness Vector (f(x))	Pbest	Gbest
1	-6.2	0.4752	-49.44	-6.2	
2	8.3	0.7797	-7.39	8.3	
3	-1.1	0.4141	13.29	-1.1	
4	0.6	0.6183	22.64	0.6	
5	2.6	0.2530	26.24	2.6	Gbest

$\gg [v \quad p] = sort(f)$

v =

-72.9576
-7.0141
-5.2872
11.8518
26.2283

p =

3
4
2
1
5

$\gg Pbest\_value = v(p(5), :)$

$Pbest\_value =$

26.2283

$\gg Pbest = x(p(5), :)$

Pbest =

2.6472

UPDATE VELOCITY AND POSITION OF EACH PARTICLE

Particle Number	Position Vector (x)	Velocity Vector (v)	Fitness Vector (f(x))	Pbest	Gbest
1	-6.2	0.4752	-49.44	-6.2	
2	8.3	0.7797	-7.39	8.3	
3	-1.1	0.4141	13.29	-1.1	
4	0.6	0.6183	22.64	0.6	
5	2.6	0.2530	26.24	2.6	Gbest

$v(t+1) = \{V(t) + c_1 * r1 * (P_{best} - x) + c_2 * r_2 * (G_{best} - x)\}$

$v_1 = \{0.4752 + 2 * 0.6669 * (-6.2 - (-6.2)) + 2 * 0.4547 * (2.6 - (-6.2))\} =$ **8.4779**

$x(t+1) = x(t) + v(t+1)$

$x_1 = -6.2 + 8.4779 =$ **2.2779**

Particle Number	Position Vector (x)	Velocity Vector (v)	Fitness Vector (f(x))	Pbest	Gbest
1	-6.2	0.4752	-49.44	-6.2	
2	8.3	0.7797	-7.39	8.3	
3	-1.1	0.4141	13.29	-1.1	
4	0.6	0.6183	22.64	0.6	
5	2.6	0.2530	26.24	2.6	Gbest

$v_2 = \{0.7797 + 2 * 0.8109 * (8.3 - 8.3) + 2 * 0.4845 * (2.6 - 8.3)\} =$ **-4.7436**

$x_2 = 8.3 - 4.7436 =$ **3.5564**

$v_3 = \{0.4141 + 2 * 0.7567 * (-1.1 - (-1.1)) + 2 * 0.4170 * (2.6 - (-1.1))\} =$ **3.4999**

$x_3 = -1.1 + 3.4999 =$ **2.3999**

Particle Number	Position Vector (x)	Velocity Vector (v)	Fitness Vector (f(x))	Pbest	Gbest
1	-6.2	0.4752	-49.44	-6.2	
2	8.3	0.7797	-7.39	8.3	
3	-1.1	0.4141	13.29	-1.1	
4	0.6	0.6183	22.64	0.6	
5	2.6	0.2530	26.24	2.6	Gbest

$$v_4 = \{0.6183 + 2 * 0.9717 * (0.6 - 0.6) + 2 * 0.9879 * (2.6 - 0.6)\} = \mathbf{4.5699}$$
$$x_4 = 0.6 + 4.5699 = \mathbf{5.1699}$$
$$v_5 = \{0.2530 + 2 * 0.8641 * (2.6 - 2.6) + 2 * 0.3888 * (2.6 - 2.6)\} = \mathbf{0.2530}$$
$$x_5 = 2.6 + 0.2530 = \mathbf{2.853}$$

Particle Number	(x)	(v)	(f(x))	Pbest	Gbest	New (x)	New (v)
1	-6.2	0.4752	-49.44	-6.2		2.2779	8.4779
2	8.3	0.7797	-7.39	8.3		3.5564	-4.7436
3	-1.1	0.4141	13.29	-1.1		2.3999	3.4999
4	0.6	0.6183	22.64	0.6		5.1699	4.5699
5	2.6	0.2530	26.24	2.6	Gbest	2.853	0.2530

EVALUATE FITNESS OF EACH PARTICLE

Fitness function is $f(x) = -x^2 + 5x + 20$

(x)	(v)	(f(x))	Pbest	Gbest	New (x)	New (v)	New f(x)
-6.2	0.4752	-49.44	-6.2		2.2779	8.4779	26.20
8.3	0.7797	-7.39	8.3		3.5564	-4.7436	25.13
-1.1	0.4141	13.29	-1.1		2.3999	3.4999	26.24
0.6	0.6183	22.64	0.6		5.1699	4.5699	19.12
2.6	0.2530	26.24	2.6	Gbest	2.853	0.2530	26.12

UPDATE INDIVIDUAL AND GLOBAL BESTS.

(x)	(v)	f(x)	P best	G best	New (x)	New (v)	New f(x)	P best	G best
-6.2	0.47	-49.44	-6.2		2.27	8.48	26.20	2.27	
8.3	0.77	-7.39	8.3		3.55	-4.74	25.13	3.55	
-1.1	0.41	13.29	-1.1		2.39	3.5	26.24	2.39	G best
0.6	0.61	22.64	0.6		5.17	4.57	19.12	0.6	
2.6	0.25	26.24	2.6	G best	2.85	0.25	26.12	2.6	

Table : Iteration number =2

Particale Number	Position Vector (x)	Velocity Vector (v)	Pbest
1	2.2779	8.4779	2.2779
2	3.5564	-4.7436	3.5564
3	2.3999	3.4999	2.3999
4	5.1699	4.5699	0.6
5	2.853	0.2530	2.6

$G_{best} = 2.3999$

$v(t+1) = \{V(t) + c_1 * r1 * (P_{best} - x) + c_2 * r_2 * (G_{best} - x)\}$

$v_1 = \{8.4779 + 2 * 0.2466 * (2.2779 - 2.2779) + 2 * 0.7844 * (2.3999 - 2.2779)\} = \mathbf{8.6692}$

$v_2 = \{-4.7436 + 2 * 0.8828 * (3.5564 - 3.5564) + 2 * 0.91375 * (2.3999 - 3.5564)\} = \mathbf{-6.8571}$

$v_3 = \{3.4999 + 2 * 0.5582 * (2.3999 - 2.3999) + 2 * 0.5988 * (2.399 - 2.399)\} = \mathbf{3.4999}$

$v_4 = \{4.5699 + 2 * 0.1488 * (0.6 - 5.1699) + 2 * 0.8997 * (2.399 - 5.1699)\} = \mathbf{-2.1342}$

$v_5 = \{2.853 + 2 * 0.4503 * (2.6 - 2.853) + 2 * 0.2056 * (2.399 - 2.6)\} = \mathbf{2.5425}$

$x(t+1) = x(t) + v(t+1)$

$x_1 = 2.2779 + 8.6692 = \mathbf{10.9471}$

$x_2 = 3.5564 - 6.8571 = \mathbf{-3.3007}$

$x_3 = 2.3999 + 3.4999 = \mathbf{5.8998}$

$x_4 = 5.1699 - 2.1342 = \mathbf{3.0357}$

$x_5 = 2.853 + 2.5425 = \mathbf{5.3955}$