

# Department of AI & DS

## CSE and CS&IT

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**COURSE NAME: PROBABILITY, STATISTICS AND QUEUING THEORY**

**COURSE CODE: 23MT2005**

**Topic**

**Two types of Errors, Level of Confidence**

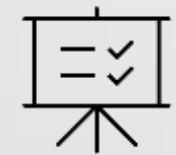
**Session - 15**

## AIM OF THE SESSION



To familiarize students with the basic concept of testing of hypothesis

## INSTRUCTIONAL OBJECTIVES



This Session is designed to:

1. Identify two types of errors
2. Describe Level of Confidence

## LEARNING OUTCOMES



At the end of this session, you should be able to:

1. Define Type I and Type II errors.
2. State the meaning of level of confidence and level of significance.
3. Summarize the testing of hypothesis with defining right and wrong decisions.

There is a probability of committing an error in making a decision about a hypothesis. Hence, two types of errors are defined as follows:

**Type I Error:** In a hypothesis test, a type-I error occurs when the null hypothesis is rejected when it is in fact true. i.e.,  $H_0$  is wrongly rejected.

e.g. in a clinical trial of a new drug, the null hypothesis might be that the new drug is no better, on average, than the current drug. That is, there is no difference between the two drugs on average.

Reject the null hypothesis ( $H_0$ ) when it is true. First kind of error is also named as Type I error or rejection error. Probability of type I error is denoted by ' $\alpha$ ' (level of significance). It is also named as producer's risk.

The error of rejecting  $H_0$  (accepting  $H_1$ ) when  $H_0$  is true is called Type I error.

$\alpha$  = Probability of type I error = Probability of rejecting  $H_0$  when  $H_0$  is true. ( $P(\text{reject } H_0 / H_0 \text{ is true})$ )

$P(\text{Type I error}) = \alpha = P(\text{we are rejecting } H_0 \text{ when it is true}) = (P(\text{reject } H_0 / H_0 \text{ is true}))$

**Type II Error:** In a hypothesis test, a type –II error occurs when the null hypothesis,  $H_0$ , is not rejected when it is in fact false.

e.g., in a clinical trial of a new drug, the null hypothesis might be that the new drug is no better, on average, than the current drug.

Accepting null hypothesis ( $H_0$ ), when alternative hypothesis ( $H_1$ ) is true.

Second kind of error named as acceptance error. Probability of type II error is denoted by ' $\beta$ '. It is also named as consumer's risk. Type II error is more dangerous than type I error.

The error of accepting  $H_0$  (rejecting  $H_1$ ) when  $H_1$  is true is called Type II error.

$\beta$  = Probability of type II error

= Probability of accepting  $H_0$  when  $H_1$  is true. ( $P(\text{accept } H_0 / H_1 \text{ is true})$ )

**Example:** If a medicine is administered to a few patients of a particular disease to cure them and the medicine is curing the disease, but it is claimed that it has no effect or has an adverse effect, and hence it is discontinued. This is type I error. On the contrary, the medicine has adverse effect and is claimed to have good effect and the treatment is continued. This is type II error.

## LEVEL OF CONFIDENCE

**Level of Confidence:** Experimenters and researchers have selected some arbitrary standards – called levels of significance to serve as the cut-off points or critical points along the probability scale. Generally, the 0.05 and the 0.01 levels of significance are the most popular in social sciences research.

The confidence with which an experimenter rejects a null hypothesis depends upon the level of significance adopted. These may, hence, sometime be termed as **levels of confidence**.

Level	Amount of Confidence	Interpretation
0.05	95%	If the experiment is repeated a 100 times, only on five occasions the obtained mean will fall outside the limited $\mu \pm 1.96$ SE
0.01	99%	If the experiment is repeated a 100 times, only on one occasions the obtained mean will fall outside the limited $\mu \pm 2.58$ SE

## Level of significance

The level of significance is that probability of chance occurrence of observed results up to and below which the probability 'p' of the null hypothesis being correct is considered too low and the results of the experiment are considered significant.

The selection of level of significance depends on the choice of the researcher. Generally level of significance is taken to be 5% or 1%, i.e.,  $=0.05, 0.01$ ).

$P(x \in W/H_0) = \alpha$  = Level of significance of the test

$P(x \in W/H_1) = 1 - \beta$  is called the power of the test

$P(x \in \bar{W}/H_0) = 1 - \alpha$

$P(x \in \bar{W}/H_1) = \beta$

**Example: 1:** Let  $p$  be the probability that a coin will fall head in a single toss in order to test  $H_0: p=1/2$  against  $H_1: p=3/4$ . The coin is tossed 5 times and  $H_0$  is rejected if more than 3 heads are obtained. Obtain the probability of type I and type II error and power of the test.

**Solution:** Here  $H_0: p=1/2$  against  $H_1: p=3/4$

If the random variable  $X$  denotes the number of heads in  $n$  tosses of a coin then  $X \sim B(n, p)$

$$\text{So that } P(X=x) = n C_x p^x q^{n-x} = 5 C_x p^x q^{5-x}$$

The critical region is given by :  $W = \{x: x \geq 4\} \implies \bar{W} = \{x: x \leq 3\}$

$$\alpha = \text{probability of the type I error} = P[X \geq 4 / H_0] = P(x=4/p=1/2) + P(x=5/p=1/2) = 5 C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{5-4} + 5 C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^{5-5} = 3/16$$

$$\beta = \text{probability of the type II error} = P(x \in \bar{W} / H_1) = 1 - P(x \in W / H_1)$$

$$= P[X \leq 3 / H_1] = 1 - P[X \geq 4 / H_1] = 1 - P(x=4/p=3/4) - P(x=5/p=3/4) = 47/128,$$

$$\text{Power of the test} = 1 - \beta = 81/128$$

In this session, the basic concepts of testing of hypothesis have described

1. Define two types of errors
2. Discuss in detail about the level of confidence and level of significance



## SELF-ASSESSMENT QUESTIONS

A wrong decision about  $H_0$  leads to

- A: one kind of error
- B: Two kinds of error
- C: Three kinds of error
- D: four kinds of error

Level of significance is the probability of

- A: type I error
- B: type II error
- C: not committing error
- D: any of the above

1. Identify the two types of errors of decision that arise in testing of hypothesis.
2. A computer manufacturer wants to establish that the average time to set up a new desktop computer is less than 2 hours.
  - i) Formulate the null and alternative hypothesis
  - ii) Determine the error could be made if  $\mu=1.9$ . Explain in the context of the problem
  - iii) Determine the error could be made if  $\mu=2.0$ . Explain in the context of the problem
3. Describe level of confidence and level of significance and Critical region.

## REFERENCES FOR FURTHER LEARNING OF THE SESSION

### Reference Books:

1. William Feller, An Introduction to Probability Theory and Its Applications: Volume I, Third Edition, 1968 by John Wiley & Sons, Inc.
2. Alex Tsun, Probability & Statistics with Applications to Computing (Available at: [http://www.alextsun.com/files/Prob\\_Stat\\_for\\_CS\\_Book.pdf](http://www.alextsun.com/files/Prob_Stat_for_CS_Book.pdf))
3. Richard A Johnson, Miller & Freund's Probability and statistics for Engineers, PHI, New Delhi, 11th Edition (2011).

### Sites and Web links:

1. <https://www.khanacademy.org/math/statistics-probability/significance-tests-one-sample/more-significance-testing-videos/v/small-sample-hypothesis-test>

THANK YOU



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