NODE COVER DECISION PROBLEM(NCDP)

Node Cover Decision Problem(NCDP):

A set $S \subseteq V$ is a *node cover* for a graph $G = (V,E)$ if and only if all edges in E are incident to at least one vertex in S. The size $|S|$ of the cover is the number of vertices in S.

Example 11.12 Consider the graph:

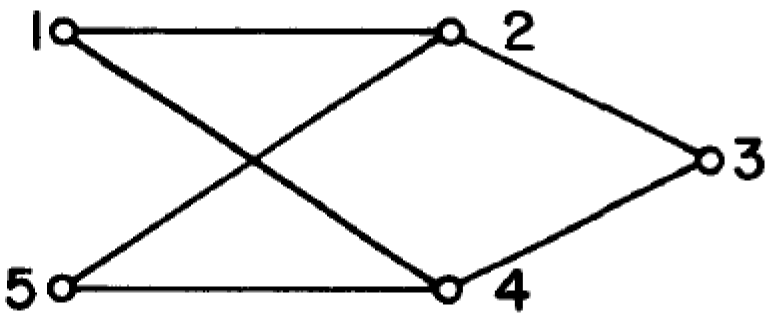


Figure 11.2 A sample graph and node cover

S = {2,4} is a node cover of size 2.

S = { 1,3,5} is a node cover of size 3.

In a node cover decision problem we are given a graph G and an integer k.

We are required to determine whether G has a node cover of size at most k.

Theorem: The clique decision problem α the node cover decision problem.

Proof: let G = (V,E) and k define an instance of CDP. Assume that |V| = n.

We Construct a Graph G' such that G' has a node cover of size at most n-k if and only if G has a clique of size at least k.

Graph G'= (V,E) = ,where E = $\{(u,v) \mid u \in V, v \in V \text{ and } (u,v) \notin E\}$.

The set G' is known is the complement of G

Now, we shall show that G has a clique of size at least k iff G' has a node cover of size at most n - k. Let K be any clique in G.

Since there are no edges in $\bar{E}$ connecting vertices in K, the remaining n - |K| vertices in G' must cover all edges in $\bar{E}$.

Similarly, if S is a node cover of G' then V - S must form a complete subgraph in G.

Since G' can be obtained from G in polynomial time, CDP can be solved in polynomial deterministic time if we have a polynomial time deterministic algorithm for NCDP.

Note that since CNF-satisfiability $\propto$ CDP, CDP $\propto$ NCDP and $\propto$ is transitive, it follows that NCDP is NP-hard.

Example 11.11 Consider $F = (x_1 \vee x_2 \vee x_3) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3)$. The construction of Theorem 11.2 yields the graph:

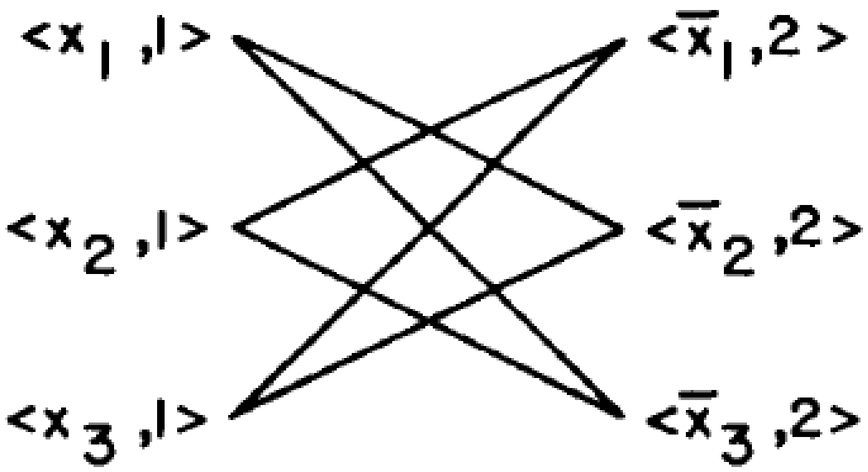


Figure 11.1 A sample graph and satisfiability

This graph contains six cliques of size two. Consider the clique with vertices $\{\langle x_1, 1\rangle, \langle \bar{x}_2, 2\rangle\}$. By setting x_1 = true and $\bar{x}_2$ = true (i.e. x_2 = false) F is satisfied. x_3 may be set either to true or false. □

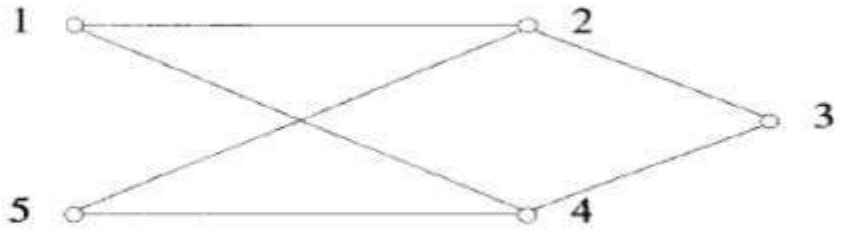
Questions:

1. Explain in detail about NCDP?
2. Prove that The clique decision problem α the node cover decision problem.

THANK YOU

Node Cover Decision Problem (NCDP)

- Node Cover Decision Problem (NCDP)
  - A set $S \subseteq V$ is a node cover for a graph $G = (V, E)$ if and only if all edges in E are incident to at least one vertex in S. The size $|S|$ of the cover is the number of vertices in S.
- Example:
  - S={2,4} is a node cover of size 2, and S={1,3,5} is a node cover of size 3.



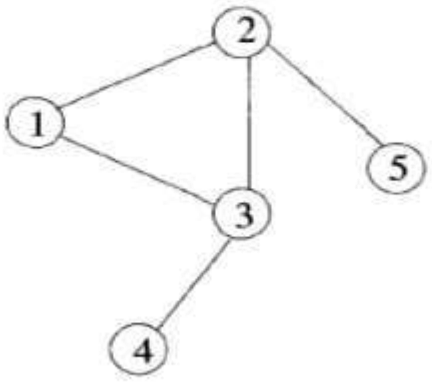
Node Cover Decision Problem (NCDP)

- **Theorem:**
  - The clique decision problem ≤ the node cover decision problem.
- **Proof:**
  - Let G = (V,E) and k define an instance of CDP. Assume that |V| = n. We construct a graph G′ such that G′ has a node cover of size at most n — k if and only if G has a clique of size at least k.
  - Graph G′ is the complement of G.

Node Cover Decision Problem (NCDP)

- Proof:
  - Let K be any clique in G.
  - Since there are no edges in $\overline{E}$ connecting vertices in K, the remaining n — |K| vertices in G' must cover all edges in $\overline{E}$.
  - Similarly, if S is a node cover of G', then V-S must form a complete subgraph in G.
- Since G' can be obtained from G in polynomial time, CDP can be solved in polynomial deterministic time if we have a polynomial time deterministic algorithm for NCDP.

Node Cover Decision Problem (NCDP)



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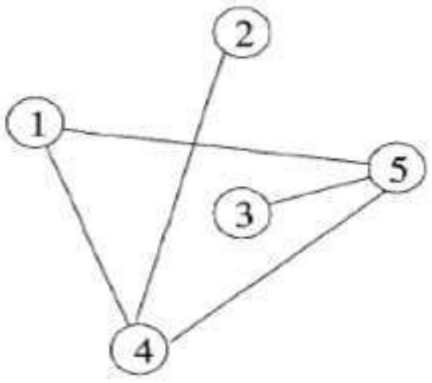

G'