Pre-Tutorial (To be completed by student before attending tutorial session)

1. Convert the following CFG to a PDA:

$$S \rightarrow aSb \mid \varepsilon$$

Solution: $Z \rightarrow$ stack symbol

$A \rightarrow a$ $\qquad q_0 \rightarrow$ initial state

$(q_0, a, Z) \rightarrow (q_0, AZ)$

$(q_0, a, A) \rightarrow (q_0, AA)$

$(q_0, b, A) \rightarrow (q_0, \epsilon)$

$(q_0, \epsilon, Z) \rightarrow (\underline{q_f}, Z) \rightarrow$ acceptance

$q_f \rightarrow$ final state

PDA accepts empty ~~space~~ stack, mean has no remaining unmatched 'a's in the stack.

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Date	<TO BE FILLED BY STUDENT>	Student Name	<TO BE FILLED BY STUDENT>

2. Prove that the context-free language L = {$a^n b^n$ | n ≥ 0} is closed under concatenation.

Solution:

$L = \{ \epsilon, ab, aabb, \_\_, \}$

$x = a^{n_1} b^{n_1}$ $y = a^{n_2} b^{n_2}$ $\rightarrow n_1, n_2 \geq 0$

$\downarrow$ $\downarrow$

$x \in L$ $y \in L$

concatenation of $xy = a^{n_1}, b^{n_1}, a^{n_2}, b^{n_2}$

$xy = a^{n_1} b^{n_2}, b^{n_1} b^{n_2}$ $\rightarrow$ a's $= n_1 + n_2$

$\rightarrow$ b's $= n_1 + n_2$

$a^{(n_1+n_2)} b^{(n_1+n_2)} \rightarrow a^m b^m \; (m = n_1 + n_2)$

$xy = a^m b^m$ for some m, $m \geq 0$

hence $xy \in L$

3. Use the Pumping Lemma to prove that the language $L = \{a^n b^n c^n \mid n \geq 0\}$ is not a context free language.

Solution:

$L = \{\epsilon, abc, aabbcc, aaabbbccc, \ldots\}$

S1: Assume L is CFL $\boxed{n=4}$ $k=2$

S2: Choose a string z from L, $|z| \geq n$

$6 \geq 4$

S3: $z = uvwxy$, $|vx| \geq 1$

$|vwx| \leq n$

$z = aabbcc$ $|vx| \geq 1$ $|vwx| \leq n$

$2 \geq 1$ $4 \leq 4$

S4: Find a integer $i = 0, 1, 2, 3, 4, \ldots,$

$uv^iwx^iy \notin L$

$i = 2$ $a(a)^2 bb (c)^2 c$

$= a^3 b^2 c^3 \notin L$

Our Assumption is wrong.

$\therefore$ The given language is not CFL

TUTORIAL (To be carried out in presence of faculty in classroom)

Given the CFG:

$$S \rightarrow aA \mid bB$$

$$A \rightarrow aS \mid bAA \mid \varepsilon$$

$$B \rightarrow bS \mid aBB \mid \varepsilon$$

Construct the equivalent PDA.

Solution:

$\Sigma = \{a, b\}$ $\Gamma = \{S, A, B, Z\}$

$(q_0, \epsilon, Z) \rightarrow (q_0, SZ)$

S: $(q_0, \epsilon, S) \rightarrow (q_0, A)$
$(q_0, \epsilon, S) = (q_0, B)$

A: $(q_0, \epsilon, A) \rightarrow (q_0, aS)$
$(q_0, \epsilon, A) \rightarrow (q_0, bAA)$
$(q_0, \epsilon, A) \rightarrow (q_0, \epsilon)$

S: $(q_0, \epsilon, B) \rightarrow (q_0, bS)$
$(q_0, \epsilon, B) \rightarrow (q_0, aBB)$
$(q_0, \epsilon, B) \rightarrow (q_0, \epsilon)$
$(q_0, a, a) \rightarrow (q_0, \epsilon)$
$(q_0, b, b) \rightarrow (q_0, \epsilon)$

PDA recognizes the language generated by given CFG.

2. Show that the language L = {$a^n b^m$ | n ≠ m} is context-free.

Solution:

$L = \{a^n b^m \mid n \neq m\}$

$S \to A / B'$

$A \to aA / aB / bB / b \quad (n > m)$

$B \to bB / aB / aA / a \quad (m > n)$

$S \to A$ (String with more a's than b's)

$S \to B$ (String with more b's than a's)

a's ≠ b's

So, L is context free.

... Consider the CFL L1 = $\{a^n b^n \mid n \geq 0\}$ and L2 = $\{b^n a^n \mid n \geq 0\}$. Prove that L1 U L2 is also a context-free language.

...ution:

$L_1 = \{a^n b^n / n \geq 0\}$ $\quad L_2 \{b^n a^n / n \geq 0\}$

$L_1 \cup L_2$

$S \rightarrow A/B$

$A \rightarrow aAb/\epsilon$ (generates strings $a^n b^n$)

$B \rightarrow bBa/\epsilon$ (generates strings $b^n a^n$)

CFG generates all strings in L_1 and L_2 thus proving $L_1 \cup L_2$ is a content - free language.

$L_1 = \{a^n b^n / n \geq 0\}$

$G_1 = (\{S_1\}, \{a, b\}, P_1, S_1)$

$P_1 = \{S_1 \rightarrow aS_1 b / ab\}$

$\therefore L_1$ is CFL

$L_2 = \{ww^R / w \in \{0,1\}^*\}$

$G_2 = (\{S_2\}, \{0,1\}, P_2, S_2)$

$P_2 = \{S_2 \rightarrow 0S_20 / \{S_2\} / \epsilon\}$

$\therefore L_2$ is CFL

$G = (\{S, S_1, S_2\}, \{a, b, 0, 1\}, P, S)$

$P = \{S \rightarrow S_1 / S_2$

$S_1 \rightarrow aS_1 b / ab$

$S_2 \rightarrow 0S_20 / 1S_21 / \epsilon\}$

$\therefore L$ is CFL

4. If L is a CFL and R is a regular language, then what can be L ∩ R? Will it be CFL or Re... Why?

Solution:

$L_1 = \{a^i b^i c^j \mid i, j \geq 1\}$ $L_2 = \{a^i b^j c^j \mid i, j \geq 1\}$

$P_1 = \{S_1 \rightarrow AB$

$A \rightarrow aAb \mid ab$

$B \rightarrow cB \mid c\}$

$P = \{S_2 \rightarrow MN$

$M \rightarrow aM \mid a$

$N \rightarrow bNc \mid bc\}$

$L = L_1 \cap L_2 \rightarrow$ CFL ??

$= \{a^n b^n c^n \mid n \geq 1\}$

$\therefore L$ is not CFL

d on your answer for Q#4 consider the languages:

$$L1 = \{0^n1^n2^i : n \geq 1, i \geq 1\}$$

$$L2 = \{0^i1^n2^n : n \geq 1, i \geq 1\}$$

se context-free languages? If yes, will the language L = L1 ∩ L2 be context-free?
ne language L.

$L_1 = \{0^n 1^n 2^i \,[n \geq 1, i \geq 1\}$

$L_2 = \{0^i 1^n 2^n \,/ n \geq 1, i \geq 1\}$

$S \rightarrow 0S \,/\, 1A$

$A \rightarrow 1A2 \,/\, 12$

$S \rightarrow 0S \,/\, 1A$

$A \rightarrow 1A2 \,/\, 12$

$0^n 1^n 2^i (n \geq 1, i \geq 1)$

$0^n 1^n 2^n (n \geq 1, i \geq 1)$

$L = \{0^n, 1^n, 2^n \,/ n \geq 1\}$

$\therefore$ L' is not CFl

Post-Tutorial (To be carried out by student after attending tutorial session)

1. Convert the following CFG into an equivalent PDA:

$$S \rightarrow aS \mid bA$$

$$A \rightarrow bS \mid \varepsilon$$

Solution:

$\Sigma = \{a, b\}$

$\Gamma = \{S, A, Z\}$

$(q_0, \epsilon, Z) \rightarrow (q_0, SZ)$

$(q_0, \epsilon, S) \rightarrow (q_0, aS)$

$(q_0, \epsilon, S) \rightarrow (q_0, bA)$

$(q_0, \epsilon, A) \rightarrow (q_0, bS)$

$(q_0, \epsilon, A) \rightarrow (q_0, bS)$

$(q_0, \epsilon, A) \rightarrow (q_0, \epsilon)$

$\rightarrow (q_0, a, a) \rightarrow (q_0, \epsilon)$

$(q_0, b, b) \rightarrow (q_0, \epsilon)$

PDA recognizes the language generated by the given CFG.

...ermine if the language $L = \{ww^R \mid w \in \{a, b\}^*\}$ is a context-free language and justify ...nswer.

...:

$L = \{ww^R / w \in \{a,b\}^*\}$

$G = \{\{S\}, \{a,b\}, P, S\}$

$P = \{S \rightarrow 0S0 / 1S1 / \epsilon\}$

$\therefore$ L is CFL

3. Use the Pumping Lemma to prove that the language $L = \{a^n b^n c^m \mid n \neq m\}$ is not a context-free language.

Solution:

$s = a^p b^p c^{p+1}$ $(n = p, m = p+1)$

$s = uvwxy$ where $|vwx| \leq p$ and

$|vx| \geq 1$

vwx include a's/only b's.

$n \neq m$

- v has b's & x has b's

$n = m$, violating $n \neq m$

we cannot maintain $n \neq m$

'L' is not context free

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...Questions

Can a language be generated by a context-free grammar but not recognized by a pushdown automaton?
...y your answer.

...ion:

No, a language cannot be generated by a CFG but not recognized by a pushdown automata (PDA). Thus, if a language is generated by a CFG it is guaranteed to be recognized by a PDA

What is the role of the stack in a pushdown automaton, and how does it relate to context-free grammars?

...lution:

The stack in a PDA serves as memory for tracking symbols, enabling the PDA to handle nested structures.

...r Evaluator's use only)

Comment of the Evaluator (if Any)	Evaluator's Observation
	Marks Secured: out of **50**
	Full Name of the Evaluator:
	Signature of the Evaluator Date of Evaluation: