PSQT

TUTORIAL-7

1. In this situation, a hypothesis test is being conducted to determine if the mean temperature of the discharged water is above or below 150°F, which is the threshold for ensuring there are no negative effects on the river's ecosystem.

Type I Error:

A Type I error occurs when the null hypothesis is **rejected** when it is actually **true**. In the context of this problem:

- **Null Hypothesis (H_0):** The mean temperature of the discharged water is at most 150°F (the plant is following regulations).
- **Alternative Hypothesis (H_1):** The mean temperature of the discharged water is above 150°F (the plant is violating regulations).

A **Type I error** would occur if, based on the sample data, the test concludes that the mean temperature of the water is above 150°F (rejecting the null hypothesis), **even though** the true mean temperature is actually at or below 150°F (the plant is actually following regulations).

Consequences of Type I Error: The plant would be wrongly penalized for violating regulations when, in reality, it is not. This could lead to unnecessary restrictions or fines on the plant.

Type II Error:

A Type II error occurs when the null hypothesis is **not rejected** when it is actually **false**. In this case:

- **Null Hypothesis (H_0):** The mean temperature of the discharged water is at most 150°F (the plant is following regulations).
- **Alternative Hypothesis (H_1):** The mean temperature of the discharged water is above 150°F (the plant is violating regulations).

A **Type II error** would occur if the test concludes that the mean temperature of the discharged water is at or below 150°F (failing to reject the null hypothesis), **even though** the true mean temperature is actually above 150°F (the plant is violating regulations).

Consequences of Type II Error: The plant would be allowed to continue operating without corrective action, even though it is violating the regulations. This could lead to ecological harm to the river's ecosystem, as the discharged water is too hot.

Which Error is More Serious?

In this case, a **Type II error** would likely be considered **more serious** because it could lead to **ecological damage**. If the test fails to detect that the temperature is too high (when in fact it is), the plant may continue to discharge water that is harmful to the river ecosystem. This could have long-term environmental consequences, such as harming aquatic life or disrupting the local ecosystem.

On the other hand, a Type I error would result in **unnecessary consequences for the plant** (like penalties or shutdowns) even though it is complying with regulations. While this is undesirable, it is not as damaging in terms of the ecological impact as a Type II error, which could allow harmful discharges to continue.

Thus, **protecting the river's ecosystem from harm** by detecting violations of the temperature regulation would likely be the priority, making a Type II error the more serious one in this context.

2.

. **Null Hypothesis (H_0):** The average time to set up a new desktop computer is **at least 2 hours**. (μ≥2\mu \geq 2μ≥2 hours)

- **Alternative Hypothesis (H_1):** The average time to set up a new desktop computer is **less than 2 hours**. (μ<2\mu < 2μ<2 hours)

Error if μ = 1.9:

- **Type I error**: Rejecting H_0 when H_0 is true. In this case, concluding the setup time is less than 2 hours when it actually is 1.9 hours (correct decision).
- **Type II error**: Not rejecting H_0 when H_0 is false. If $\mu = 1.9$ hours, a Type II error would mean failing to detect that the time is indeed less than 2 hours.

Error if μ = 2.0:

- **Type I error**: Rejecting H_0 when H_0 is true, concluding the setup time is less than 2 hours when it is exactly 2 hours.
- **Type II error**: Failing to reject H_0 when H_0 is false. If $\mu = 2.0$ hours, there is no real difference from H_0, so a Type II error isn't particularly relevant here.

3.

- **Null Hypothesis (H_0):** The true standard deviation of sheath thickness is **at least 0.05 mm**. (σ≥0.05\sigma0.05σ≥0.05 mm)
- **Alternative Hypothesis (H_1):** The true standard deviation of sheath thickness is **less than 0.05 mm**. (σ<0.05\sigma < 0.05σ<0.05 mm)

Type I Error: Rejecting H_0 when it is true, concluding the standard deviation is less than 0.05 mm when it is actually greater or equal to 0.05 mm.

Type II Error: Failing to reject H_0 when it is false, concluding the standard deviation is at least 0.05 mm when it is actually less than 0.05 mm.

4.

Given:

- Variance (σ^2) = 6.25, so Standard deviation (σ) = $\sqrt{6.25} = 2.5$
- Sample size (n) = 90
- Population mean (μ) = 40
- Confidence level = 95%

To find the confidence interval, we use the formula for the confidence interval for the population mean when the population standard deviation is known:

$$CI = \mu \pm Z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}}$$

For a 95% confidence level, the critical value $Z_{\alpha/2}$ is 1.96 (from the standard normal distribution).

Substitute the values:

$$CI = 40 \pm 1.96 \times \frac{2.5}{\sqrt{90}}$$

$$CI = 40 \pm 1.96 \times 0.264$$

$$CI = 40 \pm 0.517$$

Thus, the 95% confidence interval is:

$$(40 - 0.517, 40 + 0.517) = (39.483, 40.517)$$

So, the 95% confidence interval for the population mean is **(39.483, 40.517)**.

5.

5. 95% and 98% Fiducial Limits for the True Mean

Given:

- Sample size (n) = 900
- Sample mean ($\bar{x}$) = 3.4 cms
- Sample standard deviation (s) = 2.61 cms
- Population mean (μ) = 3.25 cms (for comparison)
- The population is normal, and the population standard deviation is unknown.

Since the population standard deviation is unknown, we use the **t-distribution** for the confidence intervals. The formula for the confidence interval is:

$$CI = \bar{x} \pm t_{\alpha/2} \times \frac{s}{\sqrt{n}}$$

Where $t_{\alpha/2}$ is the critical value from the t-distribution with $n-1$ degrees of freedom (df = 900 - 1 = 899).

For large sample sizes like this, the **t-distribution** approaches the **z-distribution**, so we can use the **z-value** as an approximation.

Since the population standard deviation is unknown, we use the **t-distribution** for the confidence intervals. The formula for the confidence interval is:

$$CI = \bar{x} \pm t_{\alpha/2} \times \frac{s}{\sqrt{n}}$$

Where $t_{\alpha/2}$ is the critical value from the t-distribution with $n - 1$ degrees of freedom (df = 900 - 1 = 899).

For large sample sizes like this, the **t-distribution** approaches the **z-distribution**, so we can use the **z-value** as an approximation.

95% Confidence Interval:

For a 95% confidence level, $Z_{\alpha/2}$ = 1.96.

$$CI_{95\%} = 3.4 \pm 1.96 \times \frac{2.61}{\sqrt{900}}$$

$$CI_{95\%} = 3.4 \pm 1.96 \times 0.087$$

$$CI_{95\%} = 3.4 \pm 0.171$$

Thus, the 95% confidence interval is:

$$(3.4 - 0.171, 3.4 + 0.171) = (3.229, 3.571)$$

So, the 95% confidence interval for the true mean is **(3.229, 3.571)**.

98% Confidence Interval:

For a 98% confidence level, $Z_{\alpha/2}$ = 2.33.

$$CI_{98\%} = 3.4 \pm 2.33 \times \frac{2.61}{\sqrt{900}}$$

$$CI_{98\%} = 3.4 \pm 2.33 \times 0.087$$

$$CI_{98\%} = 3.4 \pm 0.202$$

Thus, the 98% confidence interval is:

$$(3.4 - 0.202, 3.4 + 0.202) = (3.198, 3.602)$$

So, the 98% confidence interval for the true mean is **(3.198, 3.602)**.

6.

i) 95% Confidence Interval for True Average Porosity

Given:

- Sample mean ($\bar{x}$) = 4.85
- Population standard deviation (σ) = 0.75
- Sample size (n) = 20
- Confidence level = 95%

The formula for the confidence interval for the population mean when the population standard deviation is known is:

$$CI = \bar{x} \pm Z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}}$$

For a 95% confidence level, $Z_{\alpha/2}$ = 1.96.

$$CI = 4.85 \pm 1.96 \times \frac{0.75}{\sqrt{20}}$$

$$CI = 4.85 \pm 1.96 \times 0.167$$

$$CI = 4.85 \pm 0.327$$

Thus, the 95% confidence interval is:

$$(4.85 - 0.327, 4.85 + 0.327) = (4.523, 5.177)$$

So, the 95% confidence interval is **(4.523, 5.177)**.

ii) 99% Confidence Interval for True Average Porosity

Given:

- Sample mean ($\bar{x}$) = 4.56
- Population standard deviation (σ) = 0.75
- Sample size (n) = 16
- Confidence level = 99%

For a 99% confidence level, $Z_{\alpha/2}$ = 2.576.

$$CI = 4.56 \pm 2.576 \times \frac{0.75}{\sqrt{16}}$$

$$CI = 4.56 \pm 2.576 \times 0.1875$$

$$CI = 4.56 \pm 0.483$$

Thus, the 99% confidence interval is:

$$(4.56 - 0.483, 4.56 + 0.483) = (4.077, 5.043)$$

So, the 99% confidence interval is **(4.077, 5.043)**.

VIVA:

1. **Importance of Standard Error of Mean:**
   The standard error of the mean (SEM) measures how much the sample mean is likely to vary from the true population mean. A smaller SEM indicates more precise estimates of the population mean, while a larger SEM suggests greater uncertainty.
2. **Confidence Intervals in Hypothesis Testing:**
   Confidence intervals are calculated by taking the sample mean and adding/subtracting a margin of error based on the standard error and the critical value from the appropriate distribution (e.g., z or t-distribution). It shows the range where the population parameter is likely to fall.
3. **P-value in Hypothesis Testing:**
   The p-value measures the probability of observing the sample data, or something more extreme, if the null hypothesis is true. A small p-value (< 0.05) indicates strong evidence against the null hypothesis, suggesting it should be rejected.