Department of AI & DS
CSE and CS&IT

COURSE NAME: PROBABILITY, STATISTICS AND QUEUING THEORY

COURSE CODE: 23MT2005

Topic

STOCHASTIC PROCESS

Session – 25

AIM OF THE SESSION

To familiarize students with the concept of stochastic process

INSTRUCTIONAL OBJECTIVES

This Session is designed to:

1. Define stochastic process
2. Describe the classification of stochastic process
3. Ergodic and irregular period in stochastic process

LEARNING OUTCOMES

At the end of this session, you should be able to:

1. Differentiate between Transient, Recurrent and absorbing states
2. Summarize the discrete time and continuous time Markov chains.

Stochastic process

Classification of states: Transient and persistent (Recurrent) let $f_{jk}^{(n)}$ be the probability that the system starts with state j and reaches the state k for the first time after n transitions. Let $p_{jk}^{(n)}$ be the probability that it reaches state k after n transitions(not necessarily first time).

Let F_{jk} denote the probability that starting with state j ever reach state k. Clearly

$$F_{jk} = \sum_{n=1}^{\infty} f_{jk}^{(n)}$$

When $F_{jk} = 1$, it is certain that the system starting with state j will reach state k.

The mean (first passes) time from state j to state k is given by $\mu_{jk} = \sum_{n=1}^{\infty} n f_{jk}^{(n)}$.

$\mu_{jj} = \sum_{n=1}^{\infty} n f_{jj}^{(n)}$ is known as the mean recurrence time for the state j.

Stochastic process

Persistent (recurrent) state: A state j is said to be persistent if $F_{jj} = 1$ and transient if $F_{jj} < 1$. A persistent state is said to be null persistent if $\mu_{jj} = \infty$, i.e, if the mean recurrence time is infinite and is said to be non-null(positive) persistent if $\mu_{jj} < \infty$.

Thus the states of Markov chain can be classified as transient and persistent, and persistent states can be subdivided as non-null and null persistent.

A non-null and periodic state of Markov chain is said to be ergodic.

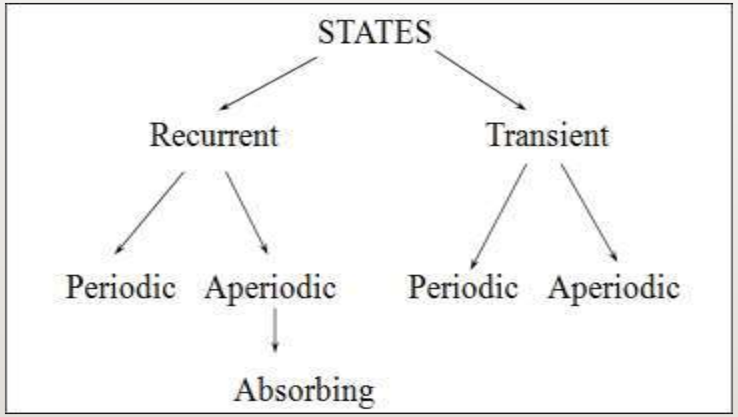
Stochastic process

Periodic and Aperiodic: A recurrent state is said to be aperiodic if its period $d_i=1$, and periodic if $d_i>1$.

Communicating Classes: States i and j communicate if each is accessible from the other.

Transient state: Once the process is in state i, there is a positive probability that it will never return to state i,

Absorbing state: A state i is said to be an absorbing state if the (one step) transition probability $P_{ii} = 1$.



Note:

1. if j is transient $P(X_n = j \mid X_0 = i) = P_{ij}^{(n)} \to 0$

2. If chain is irreducible: $\frac{1}{n}\sum_{k=1}^{n} P_{ij}^{(k)} \to \Pi_j$ **as n** $\to \infty$

3. If chain is irreducible and aperiodic: $P_{ij}^{(n)} \to \pi_j$ as $n \to \infty$

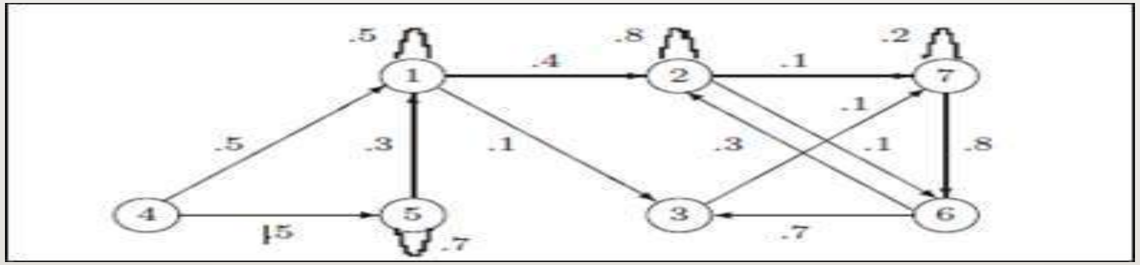
$$P^{(n)} = \begin{bmatrix} \Pi_0 & \Pi_1 & \cdots & \Pi_j \\ \Pi_0 & \Pi_1 & \cdots & \Pi_j \\ \Pi_0 & \Pi_1 & \cdots & \Pi_j \\ \vdots & \vdots & \ddots & \vdots \\ \Pi_0 & \Pi_1 & \cdots & \Pi_j \end{bmatrix}$$

Example

Example: Consider a Markov chain on $S = \{1, 2, . . . , 7\}$ with transition probabilities

$$P = \begin{bmatrix} 0.5 & 0.4 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 & 0 & 0.1 & 0.1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0.5 & 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0.3 & 0 & 0 & 0 & 0.7 & 0 & 0 \\ 0 & 0.3 & 0.7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.8 & 0.2 \end{bmatrix}$$

From the transition graph , one can see that $C = \{2, 3, 6, 7\}$ and S are the only two closed sets. Also, C is a communication class and $T = \{1, 4, 5\}$ are communication classes. From results below, it follows that C is a class of positive recurrent states and T is a class of transient states. Furthermore, the states in these classes are aperiodic.



We now establish the major result that all of the states in an irreducible set are of the same type, and they have the same period.

Terminal questions

Classify the states of the following Markov chains. If a state is periodic, determine its period.

$$a)\ \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \qquad \text{b)}\ \begin{pmatrix} 0.5 & 0.25 & 0.25 & 0 \\ 0 & 0 & 1 & 0 \\ 1/3 & 0 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

REFERENCES FOR FURTHER LEARNING OF THE SESSION

Reference Books:

1. D. Gross, J.F.Shortle, J.M. Thompson, and C.M. Harris, Fundamentals of Queueing Theory, 4th Edition, Wiley, 2008

2.William Feller, An Introduction to Probability Theory and Its Applications:Volime 1,Third Edition, 1968 by John Wiley & Sons,Inc.

Sites and Web links:

1. https://onlinecourses.nptel.ac.in/noc22_ma17/previ ew 3.

2. https://www.youtube.com/watch?v=Wo75G99F9f M&list=PLwdnzlV3ogoX2OHyZz3QbEYFhbqM 7x275&index=3

3. J.F. Shortle, J.M. Thompson, D. Gross and C.M. Harris, Fundamentals of Queueing Theory, 5th Edition, Wiley, 2018.

4. https://onlinecourses.nptel.ac.in/noc22_ma17/previ ew 3.

5https://www.youtube.com/watch?v=Wo75G99F9f M&list=PLwdnzlV3ogoX2OHyZz3QbEYFhbqM 7x275&index=3

THANK YOU

Team – PSQT EVEN SEMESTER 2024-25