Department of AI & DS
CSE and CS&IT

COURSE NAME: PROBABILITY, STATISTICS AND QUEUING THEORY

COURSE CODE: 23MT2005

Topic

BASIC CONCEPTS OF PROBABILITY

Session - 1

AIM OF THE SESSION

To familiarize students with the basic concepts of probability,

INSTRUCTIONAL OBJECTIVES

This Session is designed

1. Define the concept of probability
2. List out the different approaches of probability
3. Discuss he importance of probability in real life applications.

LEARNING OUTCOMES

At the end of this session, you should be able to:

1. Define probability and its axioms
2. Describe the different types of events.
3. Summarize the concept of probability with suitable example.

CONTENTS

- Basic concepts of Probability
- Different approaches of probability
- Addition theorem for two and three events

Basic Concepts of Probability

Deterministic Experiments: Everyone will get exact result.

Random Experiments: In life, we perform many experimental activities, where the result may not be same, when the experiments are repeated under identical conditions. We are not sure which one of many possible results will actually be obtained. Such experiments are called random experiments.

Probability is a measure of uncertainty of various phenomenon.

The role of probability theory is to provide a framework for analyzing phenomena with uncertain outcomes.

Different approaches of probability

There are three approaches:

1. **The classical theory of proba**bility: The probability of an event is computed as the ratio of the number of outcomes favorable to the event, to the total number of equally likely outcomes. This could be a thought experiment; example: tossing a coin; outcomes: head or tail

2. **The statistical approach of probability**: the probability on the basis of observations and collected data. The above two approaches assume that all outcomes are equally likely.

3. **The axiomatic approach of probability**: Here, the outcomes need not have equal chances of occurrence. We may have reason to believe that one outcome is more likely to occur than the other. In this approach, some axioms are stated to interpret probability of events.

To understand this approach, let us learn a few basic terms viz. random experiment, sample space, events

Random Experiment

Random experiment: In life, we perform many experimental activities, where the result may not be same, when the experiments are repeated under identical conditions. We are not sure which one of many possible results will actually be obtained. Such experiments are called random experiments.

A possible result of a random experiment is called its outcome.

An experiment is called random experiment if it satisfies the following two conditions:

(i) It has more than one possible outcome.

(ii) It is not possible to predict the outcome in advance.

Two steps in description of a random experiment:

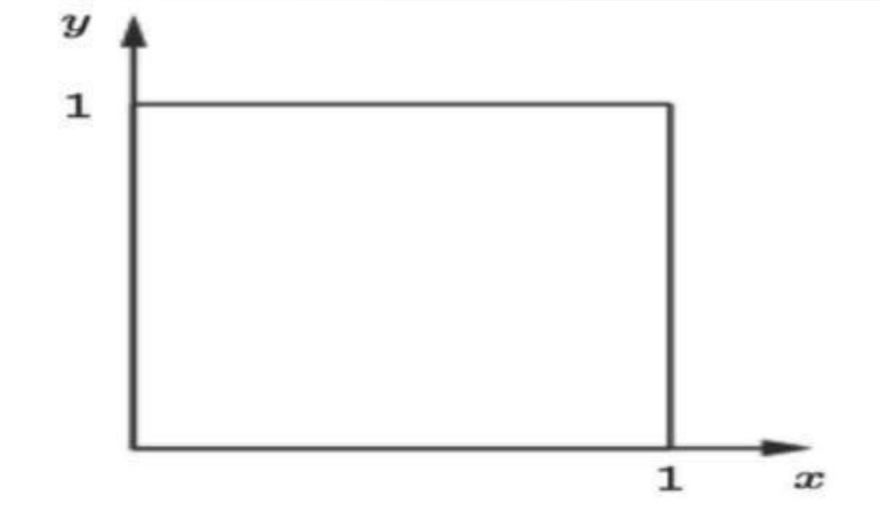
1. Describe possible outcomes of a random experiment
2. Describe beliefs about likelihood (chance) of outcomes

Sample Space

The sample space S is a list (set) of possible outcomes.

The list must be

- Mutually exclusive, and
- Collectively exhaustive



Types of outcomes:

1. Discrete
2. Continuous

Examples of experiments with discrete outcomes:

Tossing a coin: sample space, S = {H, T}

Tossing a dice: sample space, S = {1,2,3,4,5,6} Each element of the sample space is called a sample point.

In other words, each outcome of the random experiment is also called sample point.

Sample space Continuous example: (x,y) suchthat $0 \leq x, y \leq 1$

Events

Any subset E of a sample space S is called an event.

Consider the experiment of throwing a dice.

Description of events	Corresponding subset of S
the number is exactly 2	A = {2}
the number is an even integer	B = {2,4,6}
the number is greater than 6	φ = {}

Occurrence of an event: The event E of a sample space S is said to have occurred if the outcome ω of the experiment is such that $\omega \in E$. If the outcome ω is such that $\omega \notin E$, we say that the event E has not occurred.

In the above example, if the outcome is 6, event B has occurred, and event A has not occurred. On the other hand, if the outcome is 2, both events A and B have occurred.

Types of Events

Impossible event : The null set $\varphi=\{\}$ is called an impossible event

Sure event : S, i.e., the whole sample space is called the sure event.

Simple Event : If an event E has only one sample point of a sample space, it is called a simple (or elementary) event.

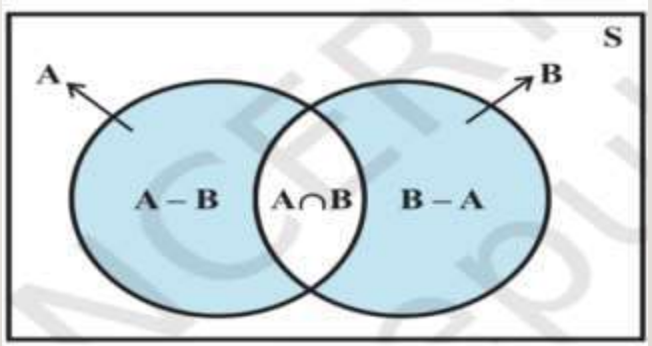
Compound Event : If an event has more than one sample point, it is called a Compound event.

We can combine two or more events to form new events.

Let A, B, C be events associated with an experiment whose sample space is S.

Complementary event : For every event A, there corresponds another event A′ called the complementary event to A. It is also called the event 'not A'.

$$\mathbf{A' = \{\omega : \omega \in S \text{ and } \omega \notin A\} = S - A}$$



A or B $\quad A \cup B = \{\omega : \omega \in A \text{ or } \omega \in B\}$

A and B $\quad A \cap B = \{\omega : \omega \in A \text{ and } \omega \in B\}$

A but not B $\quad A - B = A \cap B'$

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Types of Events

Mutually exclusive events : Two events A and B are called mutually exclusive events if the occurrence of any one of them excludes the occurrence of the other event, i.e., if they can not occur simultaneously. In this case the sets A and B are disjoint.

For example, If A = {2,4,6} and B = {1,3},

A and B are mutually exclusive events.

Exhaustive Events : if $E1$, $E2$, ..., En are n events of a sample space S and

if $E1 \cup E2 \cup E3 \cup \ldots \cup En = \cup Ei = S$ i = 1 to n then $E1$, $E2$, ..., En are called exhaustive events.

In other words, events $E1$, $E2$, ..., En are said to be exhaustive if at least one of them necessarily occurs whenever the experiment is performed.

A partition of a set S is a set of nonempty subsets of S, such that every element x in S is in exactly one of these subsets.

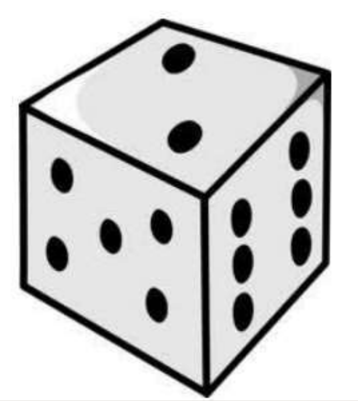
EXAMPLES

1: Tossing a fair coin Two outcomes: Head (H) and Tail (T) Both are equally likely.

2: Tossing a fair dice Set of outcomes = {1,2,3,4,5,6} = sample space All 6 outcomes are equally likely.

3: Mathematics examination grade The likelihood of a student getting first class is smaller than the likelihood of getting pass class.

4: Winner of world cup The likelihood of India winning the next world cup in cricket is …



5: Getting a Tail when tossing a coin is an event

6. Rolling a "5" is an event.

7. An event can include one or more possible outcomes:

8. Choosing a "King" from a deck of cards (any of the 4) is an event

9. Rolling an "even number" (2, 4 or 6) is also an event

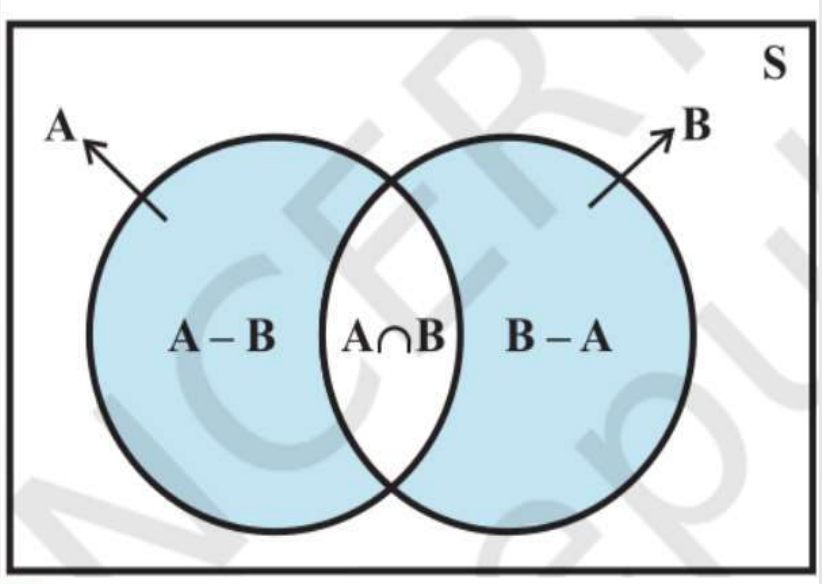
ADDITION THEOREM ON PROBABILITY

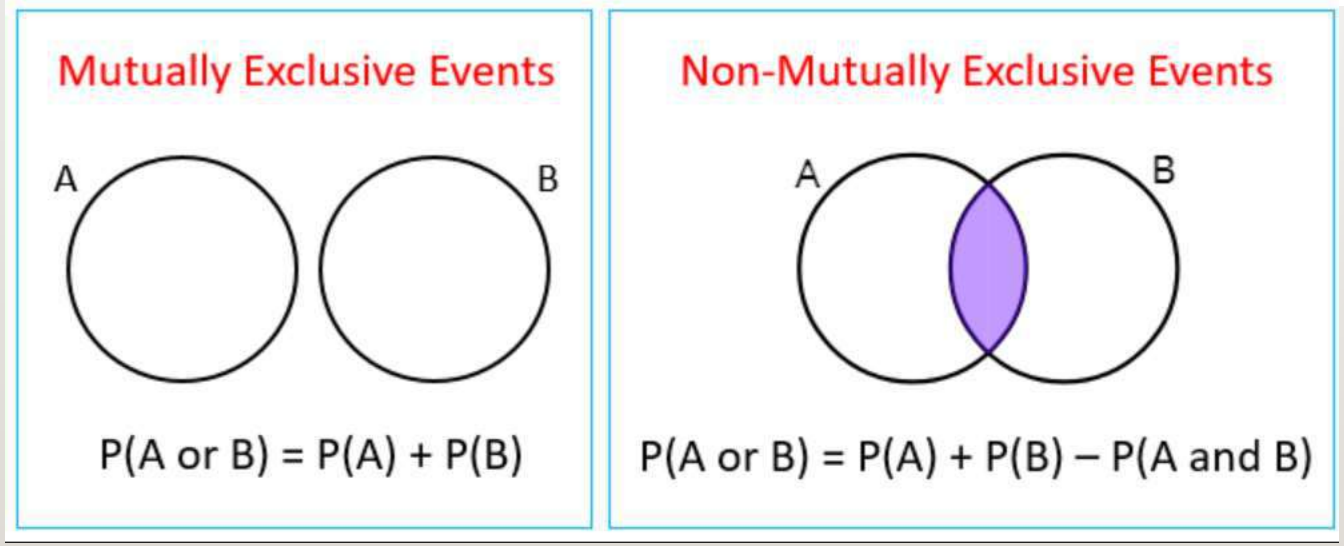
If A and B are any two events then

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

If A and B are two mutually exclusive events (disjoint events) then

$P(A\cup B)=P(A)+P(B)$





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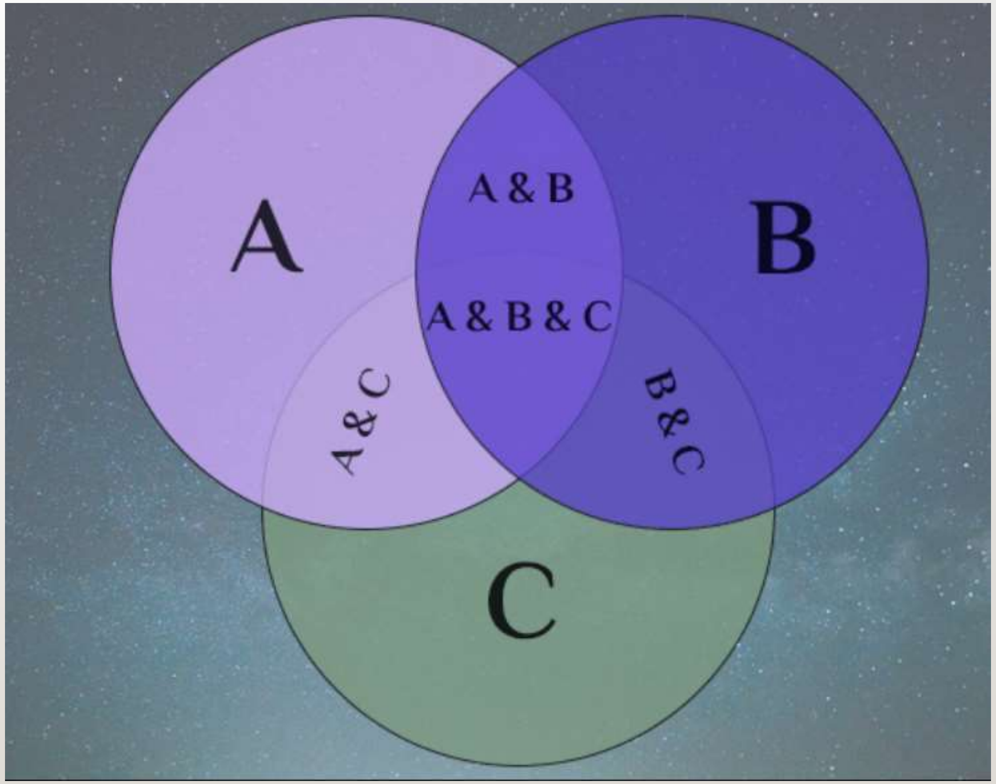
ADDITION THEOREM ON PROBABILITY

For three events A, B and C

$$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$$

If A, B and C three mutually exclusive events then

$$P(A\cup B\cup C) = P(A) + P(B) + P(C)$$



Some Properties based on Addition Rule

If A and B are any two events then

1. P(A and B)=P(Both A and B)=$P(A\cap B)$

2. P(at least one)=P(Either A or B)= $P(A\cup B)=P(A)+P(B)-P(A\cap B)$=P(A or B)

3. P(only A)=$P(A)-P(A\cap B)=P(A\cap \bar{B})$

4. P(only B)=$P(B)-P(A\cap B)=P(\bar{A}\cap B)$

5. P(Anyone)=P(only one)=$[P(A\cap \bar{B})\cup P(\bar{A}\cap B)]= P(A\cap \bar{B}) + P(\bar{A}\cap B)$
$= P(A)-P(A\cap B) + P(B)-P(A\cap B)$

EXAMPLE

The route used by a certain motorist in commuting to work contains two intersections with traffic signals. The probability that he must stop at the first signal is 0.4, the analogous probability for the second signal is 0.5, and the probability that he must stop at at least one of the two signals is 0.6. Obtain the probability that he must stop i) At both signals ii) At the first signal but not at the second one iii) At exactly one signal

Solution: Given that

Let A be the probability tha the person stops at first signal

Let B be the analogous probability for the second signal

$P(A)=0.4$, $P(B)=0.5$, $P(AUB)=0.6$

i) $P(A\cap B)=P(A)+P(B)-P(AUB)=0.4+0.5-0.6=0.3$

ii) $P(A\cap \bar{B})=P(A)-P(A\cap B)=0.4-0.3=0.1$

iii) $P(A\cap \bar{B})UP(\bar{A}\cap B)=P(A)-P(A\cap B)+P(B)-P(A\cap B)=0.4+0.5-0.3-0.3=0.3$

SUMMARY

In this session, the basic concepts of probability and its importance have described with the following topics

1. Difference between Deterministic and Random experiments
2. Sample space, Sample points, events
3. Mutually exclusive events, exhaustive event, equally likely events
4. Different approaches of probability .
5. Addition Rule

SELF-ASSESSMENT QUESTIONS

1. We are told that in a random experiment there are five possible outcomes. Which of the following statements is true ?

(a) If, after 20 trials, one outcome has not been observed then the probability that it will occur in the next trial is increased.
(b)nIf, after 20 trials, one outcome has been observed then the probability that it will not occur in the next trial is increased.
(c) If, after 20 trials, one outcome has not been observed then the probability that it will occur in the next trial is unchanged.
(d) If the outcomes are equally likely then the trials are independent.

2. A coin is tossed 6 times. What is the probability of exactly 2 heads occurring in the 6 tosses.

$(a)\,(6_{c_2})(\frac{1}{2})^6$
$(b)\,(\frac{1}{2})^6$
$(c)\,(\frac{1}{3})^6$
$(d)\,(6_{c_2})(\frac{1}{3})^6$

SELF-ASSESSMENT QUESTIONS

In a standard deck of 52 cards there are 13 diamonds and 13 hearts (red) and 13 spades and 13 clubs (black). Find the probability of choosing a card at random that is a spade OR a 7

(a) 1/52
(b) 1/13
(c) 4/13
(d) 17/52

2. If you draw one card from a standard deck, what is the probability of drawing a 5 or a diamond?

(a) 2/52
(b) 4/52
(c) 16/52
(d) 26/52

TERMINAL QUESTIONS

1. If 3 books are picked at random from a shelf containing 5 novels, 3 books of poems, and a dictionary what is the probability that

(a) the dictionary is selected

(b) 2 novels and 1 book of poems are selected

(c) a novel, a book of poems and the dictionary is selected

(d) all three books are novels.

2. The probability that a new airport will get an award for its design is 0.16, the probability that it will get an award for the efficient use of materials is 0.24, and the probability that it will get both awards is 0.11.

a) what is the probability that it will get at least one of the two awards?
b) what is the probability that it will get only one of two awards?
c) what is the probability that it will get neither award
d) what is the probability that it will get award for its design only?

3. Describe the concept of Probability and its importance in the various fields with suitable examples.

4. List out the different approaches of Probability.

REFERENCES FOR FURTHER LEARNING OF THE SESSION

Reference Books:

1. Chapter 1 of TP1: William Feller, An Introduction to Probability Theory and Its Applications: Volume 1, Third Edition, 1968 by John Wiley & Sons,Inc.

2. Richard A Johnson, Miller& Freund's Probability and statistics for Engineers, PHI, New Delhi, 11th Edition (2011).

Sites and Web links:

1. * https://ncert.nic.in/textbook.php?kemh1=16-16 *
2. Notes: sections 1 to 1.3 of http://www.statslab.cam.ac.uk/~rrw1/prob/prob-weber.pdf
3. https://ocw.mit.edu/courses/res - 6 -012 -introduction -to -probability - spring - 2018/91864c7642a58e216e8baa8fcb4a5cb5_MITRES_6_012S18_L01.pd f 9
4. https://www.probabilitycourse.com/chapter3/3_2_1_cdf.php
5. https://en.wikipedia.org/wiki/Cumulative_distribution_function

THANK YOU

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