Department of AI & DS
CSE and CS&IT

COURSE NAME: PROBABILITY, STATISTICS AND QUEUING THEORY

COURSE CODE: 23MT2005

Topic

Conditional Probabilities - Independent Events, Bayes Formula.

Session - 2

AIM OF THE SESSION

To familiarize students with the rules of conditional probability and independence of events

INSTRUCTIONAL OBJECTIVES

This Session is designed

1. Demonstrate the concept of conditional probability with examples
2. List out the rules of dependent and independent events
3. Describe the Bayes rule
4. Discuss he importance of Bayes rule and its applications.

LEARNING OUTCOMES

At the end of this session, you should be able to:

1. Define conditional probability
2. Describe the rules of dependent and independent events.
3. Summarize the concepts of Bayes rule and its applications

CONTENTS

- Conditional Probability
- Multiplicative Rule
- Independent events
- Bayes Rule

Conditional Probability

The probability of happening of an event 'A when the event 'B' has already happened is called conditional probability of A/B.

$$P(A/B)=\frac{P(A\cap B)}{P(B)},\ P(B)\neq 0$$

The probability of happening of an event 'B when the event 'A' has already happened is called conditional probability of B/A.

$$P(B/A)=\frac{P(A\cap B)}{P(A)},\ P(A)\neq 0$$

Conditional probability can be contrasted with unconditional probability. Unconditional probability refers to the likelihood that an event will take place irrespective of whether any other events have taken place or any other conditions are present.

One of the objectives of calculating conditional probability is to determine whether two events are related.

Dependent events

Two events are dependent when the outcome of the first event influences the outcome of the second event

An event is deemed dependent if it provides information about another event. An event is deemed independent if it offers no information about other events.

If A and B are dependent events then the probability of both occurring is

$P(A \cap B) = P(A).P(B/A)$

$P(A \cap B) = P(B).P(A/B)$

Example:

1.Boarding a plane first and finding a good seat

2.Getting into a traffic accident is dependent upon driving or riding in a vehicle.

Independent events

Two events A and B are said to be independent if

$P(A \cap B) = P(A).P(B)$

$P(A/B) = P(A)$

$P(B/A) = P(B)$

Example

Taking an Uber ride and getting a free meal at your favorite restaurant

Winning a card game and running out of bread

Growing the perfect tomato and owning a cat

Independent events do not affect one another and do not increase or decrease the probability of another event happening.

CASE STUDIES

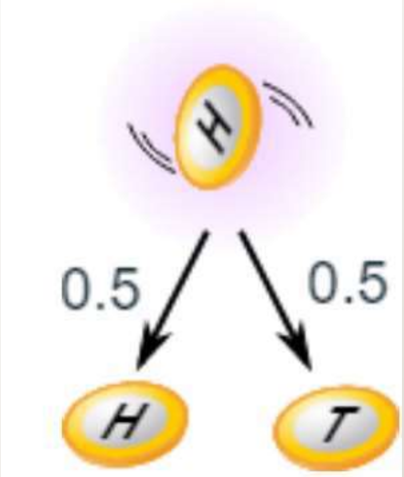
Case Study 1: Independent events

You toss a coin and it comes up "Heads" three times ... what is the chance that the next toss will also be a "Head"?

The chance is simply ½ (or 0.5) just like ANY toss of the coin.

What it did in the past will not affect the current toss!

Some people think "it is overdue for a Tail", but really truly the next toss of the coin is totally independent of any previous tosses.



EXAMPLES

Example 1: In a group of 100 computer buyers, 40 bought CPU, 30 purchased monitor, and 20 purchased CPU and monitors. If a computer buyer chose at random and bought a CPU, what is the probability they also bought a Monitor?

Solution: As per the first event, 40 out of 100 bought CPU, So, $P(A) = 40\%$ or 0.4

Now, according to the question, 20 buyers purchased both CPU and monitors.

So, this is the intersection of the happening of two events. Hence, $P(A\cap B) = 20\%$ or 0.2

By the formula of conditional probability we know;

$P(B|A) = P(A\cap B)/P(B)$

$P(B|A) = 0.2/0.4 = 2/4 = ½ = 0.5$

The probability that a buyer bought a monitor, given that they purchased a CPU, is 50%.

EXAMPLES

In a batch, there are 80% C programmers, and 40% are Java and C programmers. What is the probability that a C programmer is also Java programmer?

Let A --> Event that a student is Java programmer

B --> Event that a student is C programmer

$P(A|B) = P(A \cap B) / P(B) = (0.4) / (0.8) = 0.5$

So there are 50% chances that student that knows C also knows Java.

Dependent or Independent?

Method to Identify Independent Events Before applying probability formulas, one needs to identify an independent event. Few steps for checking whether the probability belongs to a dependent or independent events:

Step 1: Check if it possible for the events to happen in any order? If yes, go to

Step 2, or else go to Step 3

Step 2: Check if one event affects the outcome of the other event? If yes, go to step 4, or else go to Step 3

Step 3: The event is independent. Use the formula of independent events and get the answer.

Step 4: The event is dependent. Use the formula of dependent event and get the answer.

Bayes Rule

Hypotheses: The events E1 , E2 ,… En is called the hypotheses

Priori Probability: The probability P(Ei) is considered as the priori probability of hypothesis Ei

Likelihood Probability: The Probability of $P(A/E_i)$ is considered as the event likely to occur.

Posteriori Probability: The probability P(Ei |A) is considered as the posteriori probability of hypothesis Ei

Bayes' theorem is also called the formula for the probability of "causes".

Since the Ei 's are a partition of the sample space S, one and only one of the events Ei occurs (i.e. one of the events Ei must occur and the only one can occur). Hence, the above formula gives us the probability of a particular Ei (i.e. a "Cause"), given that the event A has occurred.

Applications of Baye's Rule

If the events $B_1, B_2,\ldots,B_k$ constitute a partition of the sample space S such that $P(B_i \neq 0)$ for $i = 1,2,\ldots,k$ $then\ for\ any\ event\ A$ in S such that $P(A) \neq 0$

$$P(B_r/A) = \frac{P(B_r \cap A)}{\sum_{i=1}^{K} P(B_i \cap A)} = \frac{P(B_r)P((A/B_r)}{\sum_{i=1}^{k} P(B_i)P(A/B_i)}, for = 1,2,\ldots,k$$

One of the many applications of Bayes' theorem is Bayesian inference, a particular approach to statistical inference. Bayesian inference has found application in various activities, including medicine, science, philosophy, engineering, sports, law, etc.

Example: Bayes' theorem to define the accuracy of medical test results by considering how likely any given person is to have a disease and the test's overall accuracy.

Bayes' theorem relies on consolidating prior, Likelihood probability distributions to generate posterior probabilities. In Bayesian statistical inference, prior probability is the probability of an event before new data is collected.

Applications of Baye's Rule

A Particular football team is known to run 30% of its plays to the left and 70% to the right. A linebacker on an opposing team note that the right guard shifts his stance most of the time (80%) when plays go to the right and he uses a balanced stance the remainder of the time. When plays go to the left, the guard takes a balanced stance 90% of the time and the shift stance the remaining 10%. On a particular play, the linebacker notes that the guard takes a balanced stance.

i) Find the probability that the play will go the left

ii) Find the probability that the play will go the right

iii) If you were the linebacker, which direction would you prepare to defend if you saw the balanced stance?

Applications of Baye’s Rule

Solution:

Given, P(L) = P(Plays to left)=0.30

P(R) = P (Plays to right)=0.70

P(Shifts stance/ plays to right)=0.80

P(Balanced stance/ plays to right)=0.20

P(Balanced stance / plays to the left)=0.90

P(Shits stance/ plays to the left)=0.10

P(balanced stance)=P(L).P(Balanced stance/ plays to the left) + P(R).P(Balanced stance/Place to the right)

41% of the time the guard takes a balanced stance

Applications of Baye's Rule

$$\text{(i)P(Play will go to left/balanced stance} = \frac{\text{P(L). P(Balance stance/plays to the left)}}{\text{P(Balanced stancec)}}$$

=(0.30)*(0.90)/(0.41) = 0.27/0.41 =0.6585

65.85% of the time he play to left

$$\text{ii) P(Play will go to the right)/Balanced stanc} = \frac{\text{P(R). P(Balance stance/plays to the right)}}{\text{P(Balanced stancec)}}$$

=(0.70)*(0.20)/(0.41) = 0.3415

34.15% of the time he play to the right

iii) If I am a line backer, I would prepare **left** direction to defend .

SUMMARY

In this session, the concept of conditional probability, Applications of Baye's rule have discussed.

1. How to find conditional probability.
2. Difference between Dependent and independent events
3. State the importance of Baye's Rule
4. Baye's Rule and its applications.

SELF-ASSESSMENT QUESTIONS

Let P(E) denote the probability of the event E. Given P(A) = 1, P(B) = 1/2, the values of P(A | B) and P(B | A) respectively are:

(a) 1/4 ,1/2

(b) 1/2, 1/14

(c) 1/2, 1

(d) 1, 1/2

TERMINAL QUESTIONS

1. Describe in detail about the dependent and independent events

2. List out different type of probabilities used in Baye's Rule

3. How do you find the conditional probability.

4. Summarize the Baye's rule and its applications

5. A small town has one fire engine and one ambulance available for emergencies. The probability that the fire engine is available when needed is 0.98, and the probability that the ambulance is available when called is 0.92. In the event of an injury resulting from a burning building, find the probability that

a) Both the ambulance and the fire engine will be available.

b) Ambulance or fire engine available.

6. The odds that a book will be reviewed favorably by three independent critics are 5 to 2, 4 to 3, and 3 to 4. Find the probability that of the three reviews, a majority will be favorable.

TERMINAL QUESTIONS

7. The Probability that a regularly scheduled flight departs on time is $P(D)=0.83$; the probability that it arrives on time is $P(A)=0.82$; and the probability that it departs and arrives on time is $P(D \cap A)=0.78$

Find the probability that a plane

(a) arrives on time given that it departed on time.

(b) departed on time given that it has arrived on time,

(c) neither departed on time nor arrived on time.

8. Two firms V and W consider bidding on a road building job, which may or may not be awarded depending on the amounts of the bids. Firm V submits a bid and the probability is 3/4 that it will get the job provided firm W does not bid. The probability is 3/4 that W will bid, and if it does, the probability that V will get the job is only 1/3.

i) What is the probability that V will get the job?

ii) If V gets the job, what is the probability that W did not bid?

REFERENCES FOR FURTHER LEARNING OF THE SESSION

Reference Books:

1. Chapter 1 of TP1: William Feller, An Introduction to Probability Theory and Its Applications: Volume 1, Third Edition, 1968 by John Wiley & Sons,Inc.

2. Richard A Johnson, Miller& Freund's Probability and statistics for Engineers, PHI, New Delhi, 11th Edition (2011).

Sites and Web links:

1. * https://ncert.nic.in/textbook.php?kemh1=16-16 *
2. Notes: sections 1 to 1.3 of http://www.statslab.cam.ac.uk/~rrw1/prob/prob-weber.pdf
3. https://ocw.mit.edu/courses/res - 6 -012 -introduction -to -probability - spring - 2018/91864c7642a58e216e8baa8fcb4a5cb5_MITRES_6_012S18_L01.pd f 9
4. https://www.probabilitycourse.com/chapter3/3_2_1_cdf.php
5. https://en.wikipedia.org/wiki/Cumulative_distribution_function

THANK YOU

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