TRANSPORTATION PROBLEM

- **Transportation problem** is a special kind of **Linear Programming Problem (LPP)** in which goods are transported from a set of sources to a set of destinations subject to the supply and demand of the sources and destination respectively such that the total cost of transportation is minimized. It is also sometimes called as Hitchcock problem.

Types of Transportation problems:

Balanced: When both supplies and demands are equal then the problem is said to be a balanced transportation problem.

Unbalanced: When the supply and demand are not equal then it is said to be an unbalanced transportation problem. In this type of problem, either a dummy row or a dummy column is added according to the requirement to make it a balanced problem. Then it can be solved similar to the balanced problem.

BASIC STRUCTURE OF TRANSPORTATION PROBLEM:

In the given table
D1, **D2**, **D3** and **D4** are the destinations where the products/goods are to be delivered from different sources **S1**, **S2**, **S3** and **S4**.
$\mathbf{S_i}$ is the supply from the source $\mathbf{O_i}$.
$\mathbf{d_j}$ is the demand of the destination $\mathbf{D_j}$.
$\mathbf{C_{ij}}$ is the cost when the product is delivered from source $\mathbf{S_i}$ to destination $\mathbf{D_j}$.

Destination

Source	D1	D2	D3	D4	Supply(s_i)
O1	C_{11}	C_{12}	C_{13}	C_{14}	S_1
O2	C_{21}	C_{22}	C_{23}	C_{24}	S_2
O3	C_{31}	C_{32}	C_{33}	C_{34}	S_3
O4	C_{41}	C_{42}	C_{43}	C_{44}	S_4
Demand (d_j):	d_1	d_2	d_3	d_4	

METHODS TO SOLVE:

To find the initial basic feasible solution there are three methods:

- NorthWest Corner Cell Method.
- Least Call Cell Method.
- Vogel’s Approximation Method (VAM).

NORTHWEST CORNER METHOD

Given three sources **O1**, **O2** and **O3** and four destinations **D1**, **D2**, **D3** and **D4**. For the sources **O1**, **O2** and **O3**, the supplyis **300**, **400** and **500** respectively. The destinations**D1**,**D2**, **D3** and **D4** have demands **250**, **350**, **400** and **200** respectively.

Source \ Destination	D1	D2	D3	D4	Supply
O1	3	1	7	4	300
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand:	250	350	400	200	1200

NORTHWEST CORNER METHOD

(O1, D1) has to be the starting point i.e. the north-west corner of the table. Each and every value in the cell is considered as the cost per transportation. Compare the demand for column **D1** and supply from the source **O1** and allocate the minimum of two to the cell **(O1, D1)** as shown in the figure.
The demand for Column **D1** is completed so the entire column **D1** will be canceled. The supply from the source **O1** remains **300 – 250 = 50**.

Destination

Source	D1	D2	D3	D4	Supply
O1	250 3	1	7	4	~~300~~ 50
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand:	~~250~~ 0	350	400	200	1200

NORTHWEST CORNER METHOD

Destination

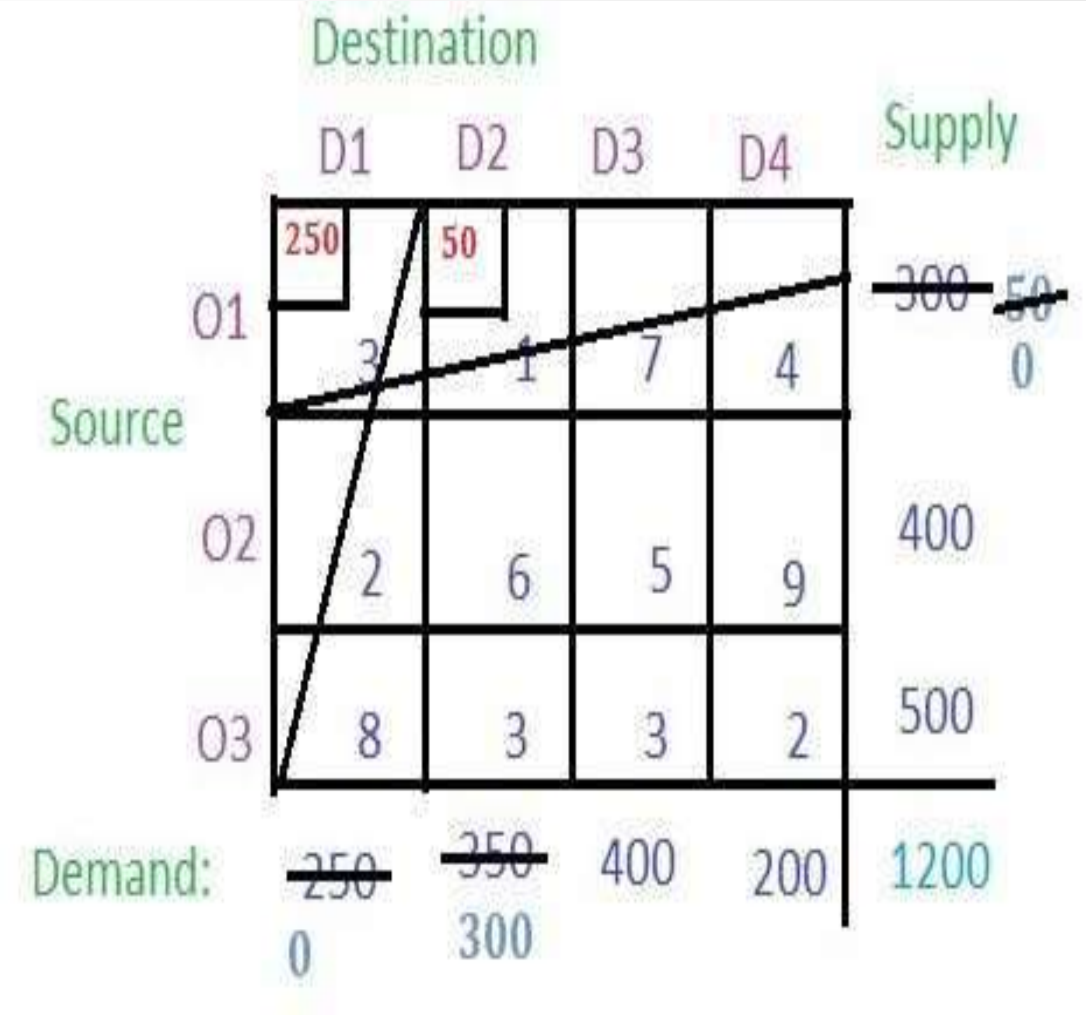
Source

	D1	D2	D3	D4	Supply
O1	250 3	1	7	4	~~300~~ 50
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand:	~~250~~ 0	350	400	200	1200

NORTHWEST CORNER METHOD

From the remaining table the north-west corner cell is **(O2, D2)**. The minimum among the supply from source **O2** (i.e 400) and demand for column **D2** (i.e 300) is **300**,

so allocate **300** to the cell **(O2, D2)**. The demand for the column **D2** is completed so cancel the column and the remaining supply from source **O2** is **400 – 300 = 100**.



NORTHWEST CORNER METHOD

Destination

Source	D1	D2	D3	D4	Supply
O1	250 / 3	50 / 1	7	4	~~300~~ ~~50~~ 0
O2	2	300 / 6	5	9	~~400~~ 100
O3	8	3	3	2	500
Demand:	~~250~~ 0	~~350~~ ~~300~~ 0	400	200	1200

NORTHWEST CORNER METHOD

Now from remaining table find the north-west corner i.e. **(O2, D3)** and compare the **O2** supply (i.e. 100) and the demand for **D2** (i.e. 400) and allocate the smaller (i.e. 100) to the cell **(O2, D2)**.

The supply from **O2** is completed so cancel the row **O2**. The remaining demand for column **D3** remains

400 – 100 = 300.

Destination

Source	D1	D2	D3	D4	Supply
O1	250 / 3	50 / 1	7	4	~~300~~ ~~50~~ 0
O2	2	300 / 6	100 / 5	9	~~400~~ ~~100~~ 0
O3	8	3	3	2	500
Demand:	~~250~~ 0	~~350~~ ~~300~~ 0	~~400~~ 300	200	1200

PROCEEDING IN THE SAME WAY, THE FINAL VALUES OF THE CELLS WILL BE :

Destination

Source	D1	D2	D3	D4	Supply
O1	250 (3)	50 (1)	7	4	~~300~~ ~~50~~ 0
O2	2	300 (6)	100 (5)	9	~~400~~ ~~100~~ 0
O3	8	3	300 (3)	200 (2)	~~500~~ ~~200~~ 0
Demand:	~~250~~ 0	~~350~~ ~~300~~ 0	~~400~~ ~~300~~ 0	~~200~~ 0	1200

NORTHWEST CORNER METHOD

In the last remaining cell the demand for the respective columns and rows are equal which was cell **(O3, D4)**. In this case, the supply from **O3** and the demand for **D4** was **200** which was allocated to this cell. At last, nothing remained for any row or column.

Now just multiply the allocated value with the respective cell value (i.e. the cost) and add all of them to get the basic solution i.e. **(250 * 3) + (50 * 1) + (300 * 6) + (100 * 5) + (300 * 3) + (200 * 2) = 4400**

LEAST COST CELL METHOD

Destination

Source	D1	D2	D3	D4	Supply
O1	3	1	7	4	300
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand:	250	350	400	200	1200

LEAST COST CELL METHOD

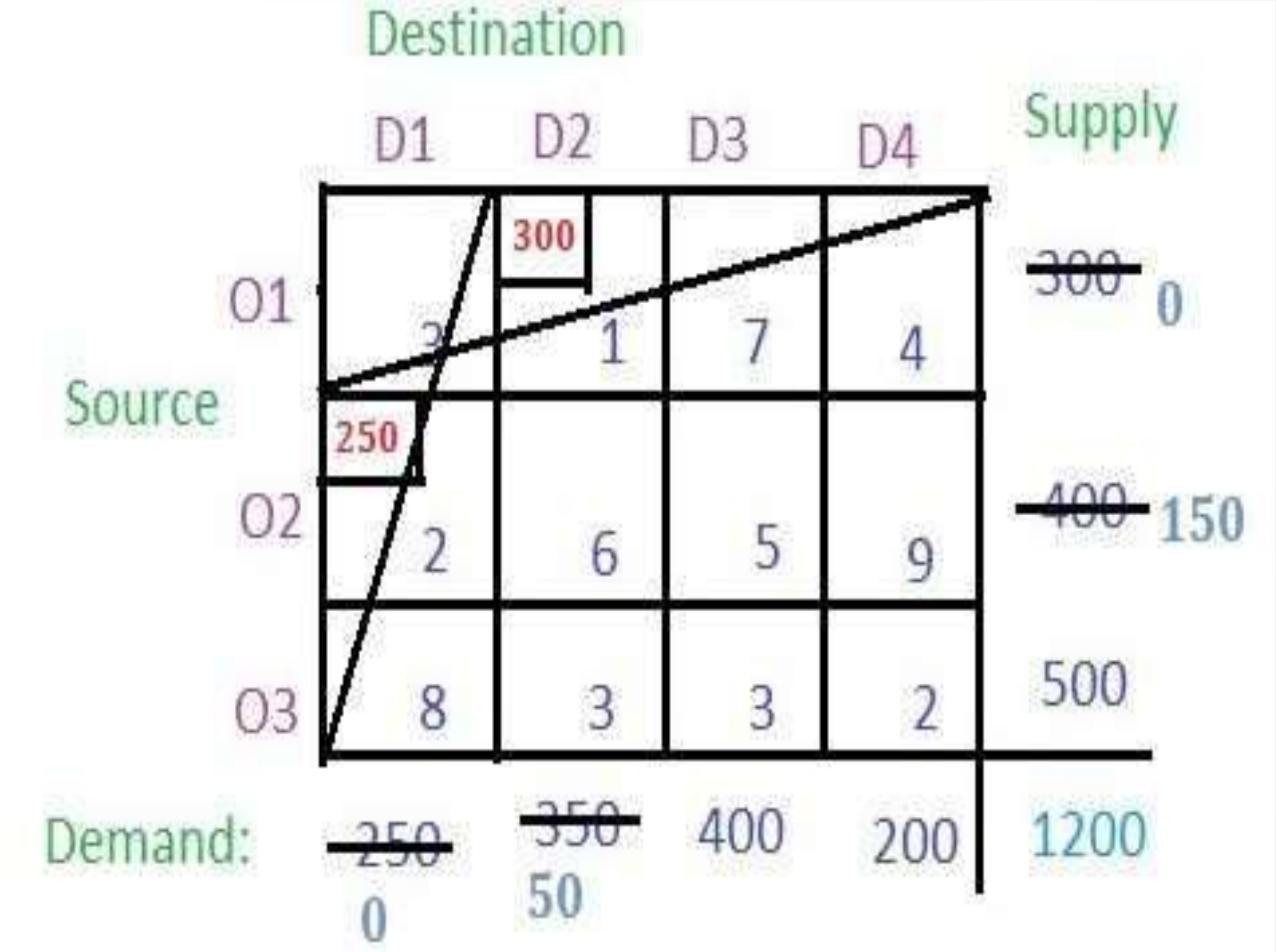
- According to the Least Cost Cell method, the least cost among all the cells in the table has to be found which is **1** (i.e. cell **(O1, D2)**).
  Now check the supply from the row **O1** and demand for column **D2** and allocate the smaller value to the cell. The smaller value is **300** so allocate this to the cell.
- The supply from **O1** is completed so cancel this row and the remaining demand for the column **D2** is **350 – 300 = 50**.

Destination

Source	D1	D2	D3	D4	Supply
O1	3	300 1	7	4	~~300~~ 0
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand:	250	~~350~~ 50	400	200	1200

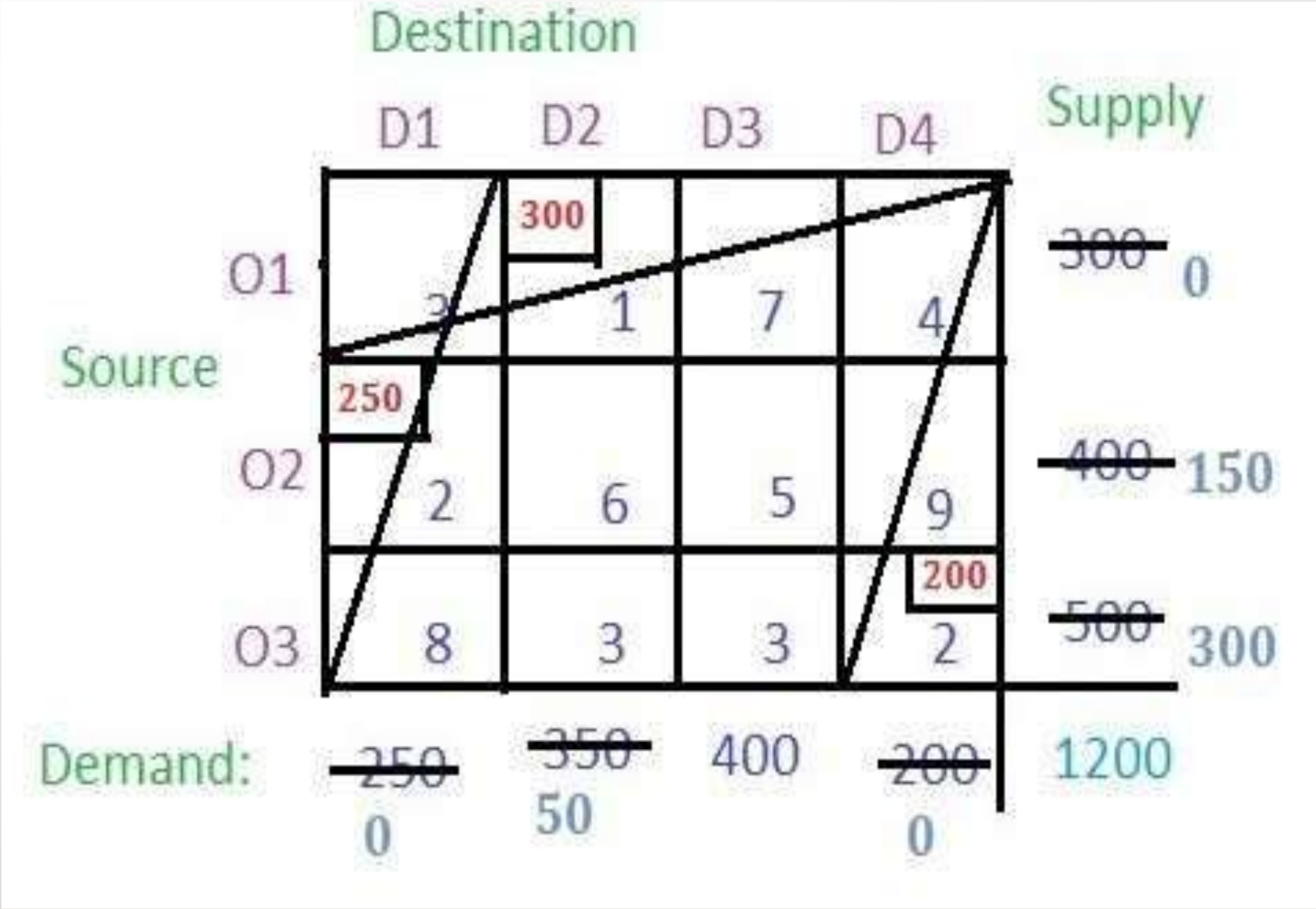
LEAST COST CELL METHOD

Now find the cell with the least cost among the remaining cells. There are two cells with the least cost i.e. **(O2, D1)** and **(O3, D4)** with cost **2**. Lets select **(O2, D1)**. Now find the demand and supply for the respective cell and allocate the minimum among them to the cell and cancel the row or column whose supply or demand becomes **0** after allocation.



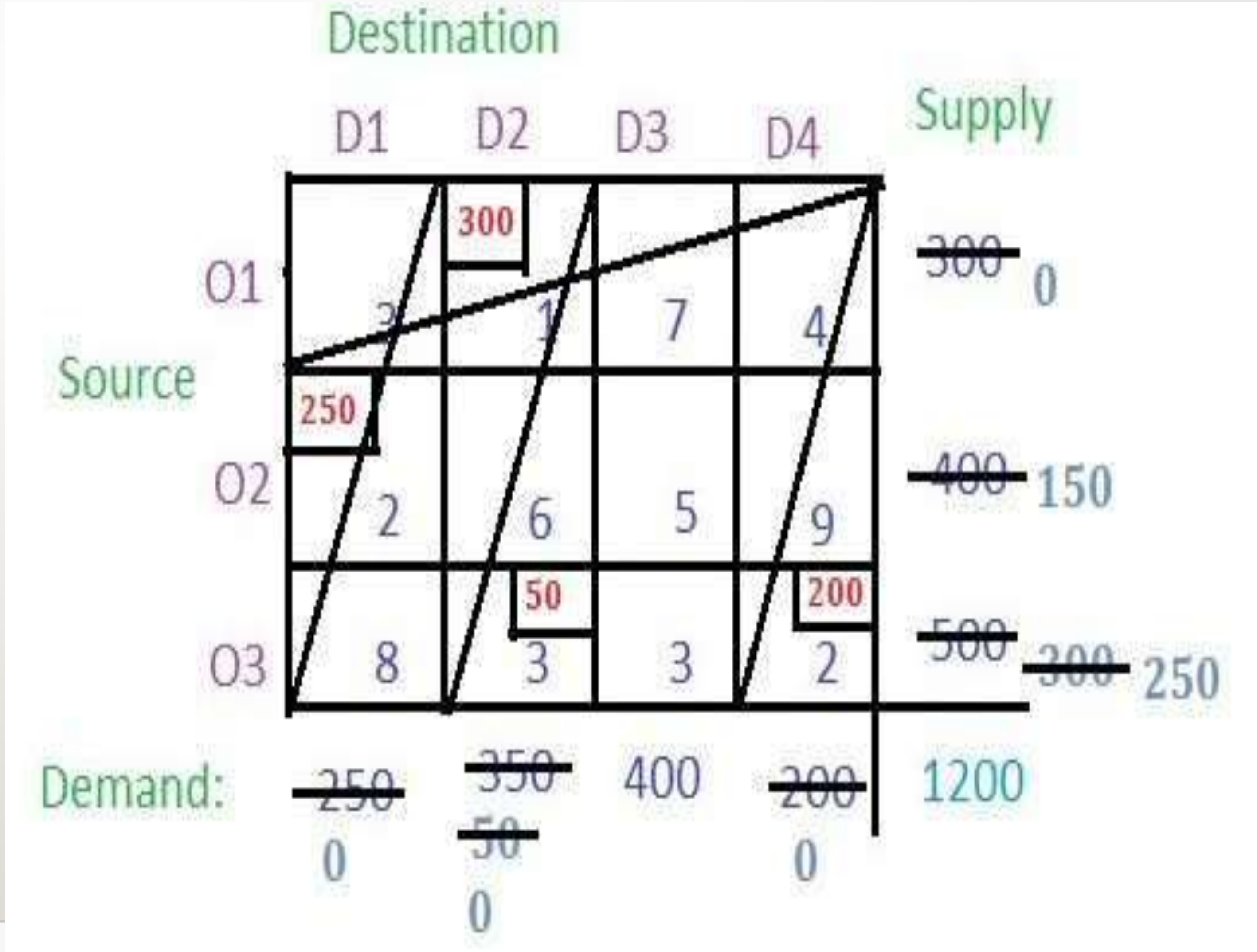
LEAST COST CELL METHOD

Now the cell with the least cost is **(O3, D4)** with cost **2**. Allocate this cell with **200** as the demand is smaller than the supply. So the column gets cancelled.

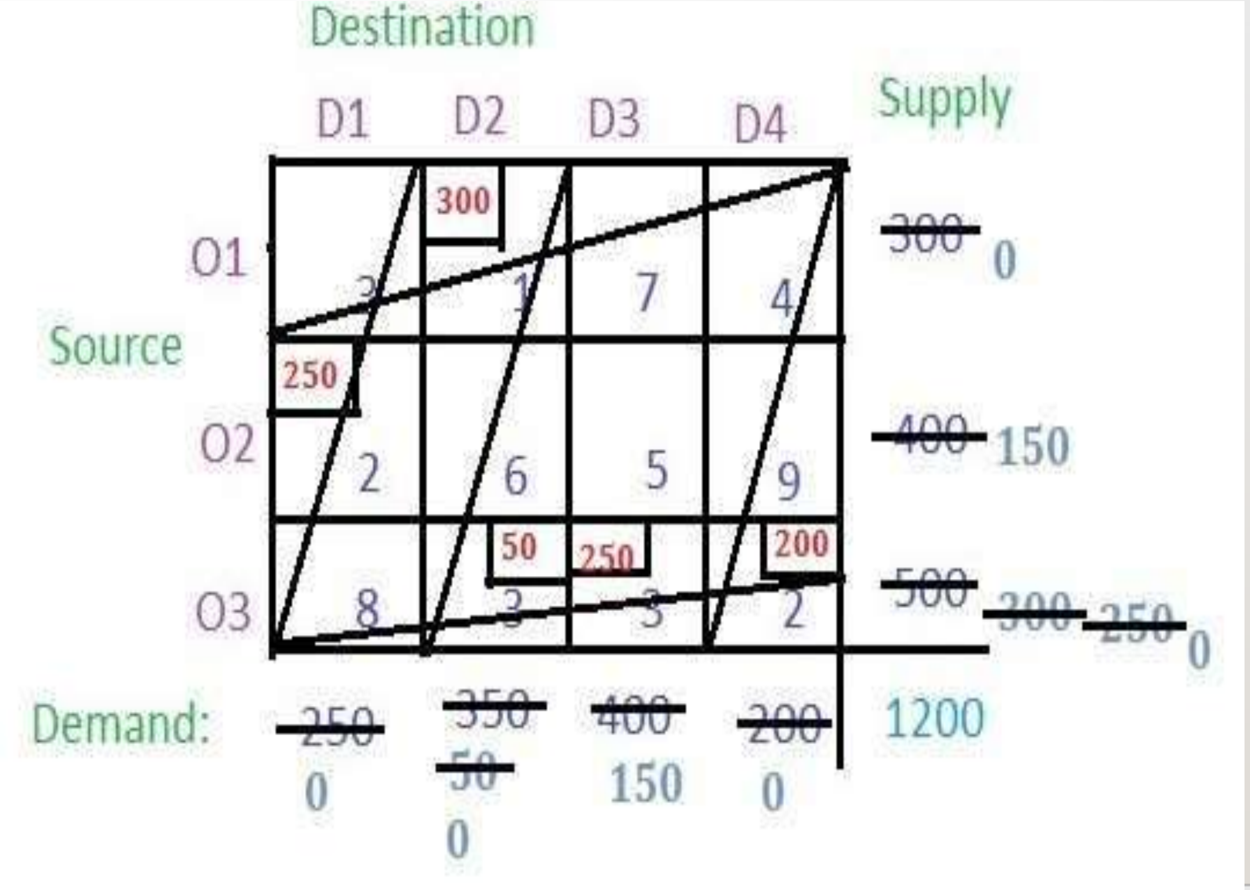


LEAST COST CELL METHOD

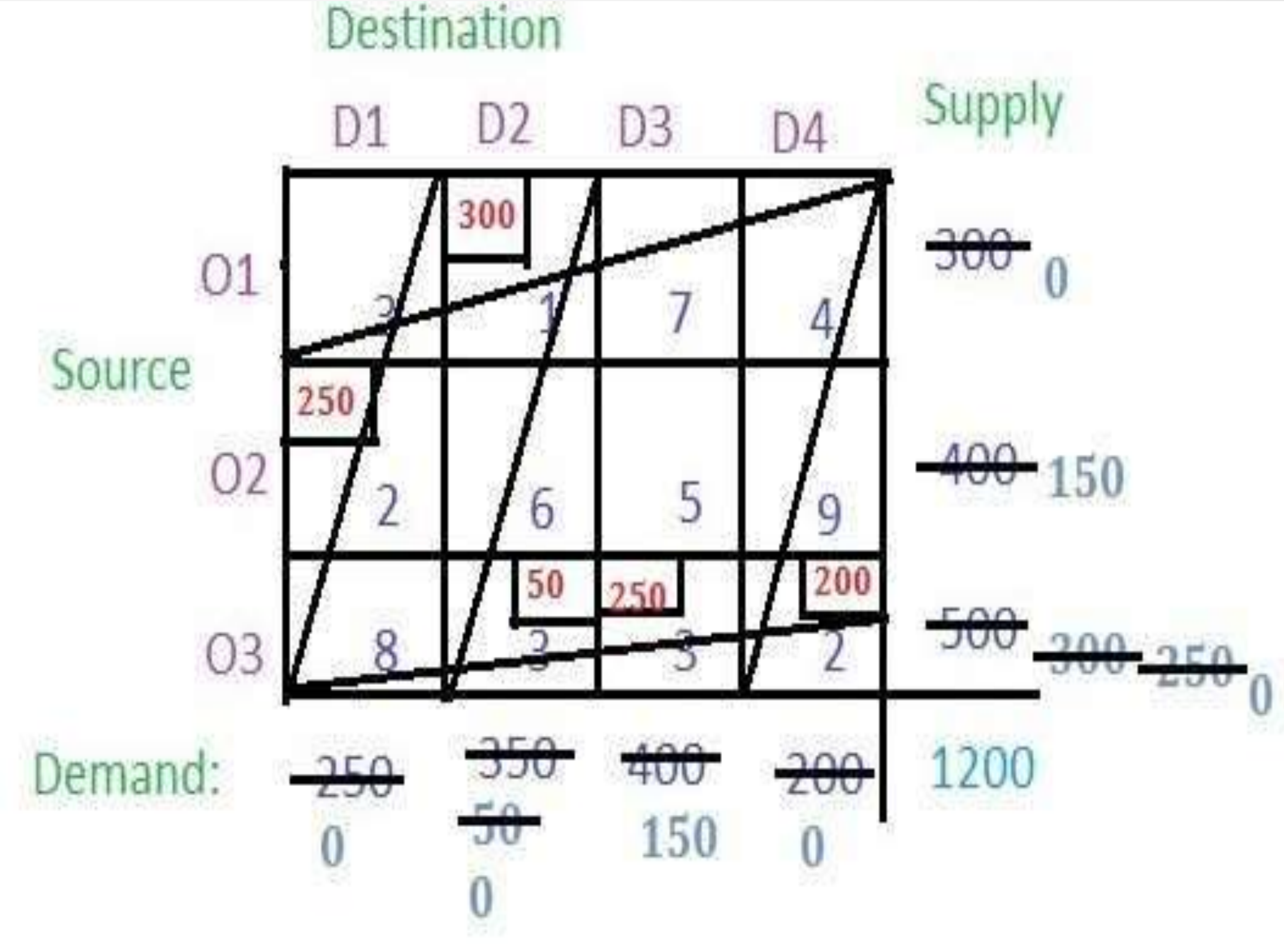
- There are two cells among the unallocated cells that have the least cost. Choose any at random say **(O3, D2)**. Allocate this cell with a minimum among the supply from the respective row and the demand of the respective column. Cancel the row or column with zero value.



Now the cell with the least cost is **(O3, D3)**. Allocate the minimum of supply and demand and cancel the row or column with zero value.



The only remaining cell is **(O2, D3)** with cost **5** and its supply is **150** and demand is **150** i.e. demand and supply both are equal. Allocate it to this cell.



LEAST COST CELL METHOD

Destination

Source	D1	D2	D3	D4	Supply
O1	3	300 / 1	7	4	~~300~~ 0
O2	250 / 2	6	150 / 5	9	~~400~~ ~~150~~ 0
O3	8	50 / 3	250 / 3	200 / 2	~~500~~ ~~300~~ ~~250~~ 0
Demand:	~~250~~ 0	~~350~~ ~~50~~ 0	~~400~~ ~~150~~ 0	~~200~~ 0	1200

LEAST COST CELL METHOD

- Now multiply the cost of the cell with their respective allocated values and add all of them to get the basic solution

i.e. **(300 * 1) + (250 * 2) + (150 * 5) + (50 * 3) + (250 * 3) + (200 * 2) = 2850**

VOGEL'S APPROXIMATION METHOD

Source	D1	D2	D3	D4	Supply	Row Difference
O1	3	1	7	4	300	**2**
O2	2	6	5	9	400	**3**
O3	8	3	3	2	500	**1**
Demand:	250	350	400	200	1200	
Column Difference:	**1**	**2**	**2**	**2**		

Destination: D1–D4

VOGEL'S APPROXIMATION METHOD

Destination

Source	D1	D2	D3	D4	Supply	Row Difference
O1	~~3~~	1	7	4	300	2
O2	$\boxed{250}$ 2	6	5	9	~~400~~ 150	3
O3	~~8~~	3	3	2	500	1
Demand:	~~250~~ 0	350	400	200	1200	
Column Difference:	1	2	2	2		

VOGEL'S APPROXIMATION METHOD

Destination

Source	D1	D2	D3	D4	Supply	Row Difference	
O1	3	1	7	4	300	2	3
O2	$\boxed{250}$ 2	6	5	9	~~400~~ 150	3	1
O3	8	3	3	2	500	1	1
Demand:	~~250~~ 0	350	400	200	1200		
Column Difference:	1	2	2	2			
	-	2	2	2			

VOGEL’S APPROXIMATION METHOD

Destination

Source	D1	D2	D3	D4	Supply	Row Difference		
O1	3	300 1	7	4	~~300~~ 0	2	3	-
O2	250 2	6	5	9	~~400~~ 150	3	1	1
O3	8	3	3	200 2	~~500~~ 300	1	1	1
Demand:	~~250~~ 0	~~350~~ 50	400	~~200~~ 0	1200			
Column Difference:	1	2	2	2				
	-	2	2	2				
	-	3	2	7				

VOGEL'S APPROXIMATION METHOD

Destination

Source	D1	D2	D3	D4	Supply	Row Difference	
O1	2	300 1	7	4	~~300~~ 0	2	3
O2	250 2	6	5	9	~~400~~ 150	3	1
O3	8	3	3	2	500	1	1
Demand:	~~250~~ 0	~~350~~ 50	400	200	1200		
Column Difference:	1	2	2	2			
	-	2	2	2			

VOGEL'S APPROXIMATION METHOD

Destination

Source	D1	D2	D3	D4	Supply	Row Difference			
O1	2	300 1	7	4	~~300~~ 0	2	3	-	-
O2	250 2	6	5	9	~~400~~ 150	3	1	1	1
O3	8	50 3	3	200 2	~~500~~ ~~300~~ 250	1	1	1	0
Demand:	~~250~~ 0	~~350~~ ~~50~~ 0	400	~~200~~ 0	1200				
Column Difference:	1	2	2	2					
	-	2	2	2					
	-	3	2	7					
	-	3	2	-					

VOGEL'S APPROXIMATION METHOD

Destination

Source	D1	D2	D3	D4	Supply	Row Difference			
O1	3	300 1	7	4	~~300~~ 0	2	3	-	-
O2	250 2	6	5	9	~~400~~ 150	3	1	1	1
O3	8	50 3	250 3	200 2	~~500~~ ~~300~~ ~~250~~ 0	1	1	1	0
Demand:	~~250~~ 0	~~350~~ ~~50~~ 0	~~400~~ 150	~~200~~ 0	1200				
Column Difference:	1	2	2	2					
	-	2	2	2					
	-	3	2	7					
	-	3	2	-					

VOGEL’S APPROXIMATION METHOD

Destination

Source	D1	D2	D3	D4	Supply	Row Difference			
O1	3	300 1	7	4	~~300~~ 0	2	3	-	-
O2	250 2	6	150 5	9	~~400~~ ~~150~~ 0	3	1	1	1
O3	8	50 3	250 3	200 2	~~500~~ ~~300~~ ~~250~~ 0	1	1	1	0
Demand:	~~250~~ 0	~~350~~ ~~50~~ 0	~~400~~ ~~150~~ 0	~~200~~ 0	1200				
Column Difference:	1	2	2	2					
	-	2	2	2					
	-	3	2	7					
	-	3	2	-					

VOGEL'S APPROXIMATION METHOD

- No balance remains. So multiply the allocated value of the cells with their corresponding cell cost and add all to get the final cost

- i.e. **(300 * 1) + (250 * 2) + (50 * 3) + (250 * 3) + (200 * 2) + (150 * 5) = 2850**