Department of AI & DS
CSE and CS&IT

COURSE NAME: PROBABILITY, STATISTICS AND QUEUING THEORY

COURSE CODE: 23MT2005

Topic

Queuing Model-2

Session – 22

AIM OF THE SESSION

To familiarize students with the basic concept of Queuing Model 2

INSTRUCTIONAL OBJECTIVES

This Session is designed to:

1. Define Model 2
2. Describe the Characteristics of Queuing model2
3. Describe the performance measures of Model 2

LEARNING OUTCOMES

At the end of this session, you should be able to:

1. Define Queue and its characteristics
2. Describe the characteristics of queuing theory and queue discipline
3. Summarize the performance measures

Model 2- M/M/1/N/FCFS

Arrivals are Poisson, service times are exponentially distributed and single sever.

The capacity of the system is limited, say N. Any new arrivals are not allowed to join the queue, when the system is full.

Like in the M /M/1/∞/FCFS model. The steady state difference equations are given by

$$P_0 = \begin{cases} \frac{1-\rho}{1-\rho^{(N+1)}}, & \rho \neq 1 \\ \frac{1}{N+1}, & \rho = 1, \text{where } \rho = \frac{\lambda}{\mu} \end{cases}$$

$$P_n = \left(\frac{1-\rho}{1-\rho^{(N+1)}}\right)\rho^n, \ n=1,2,3\ldots,N$$

Note: In this model ρ may be greater than 1.

Model 2- M/M/1/N/FCFS

$$L_s = \left(\frac{1-\rho}{1-\rho^{(N+1)}}\right)\sum_{n=0}^{N} n\rho^n = P_0 \sum_{n=0}^{N} n\rho^n$$

Since the capacity of the system is finite, all the arrivals towards the service facility do not join the queue. A fraction P_N of the arrivals do not join the system. Thus the actual rate of arrivals to join the system is called the **effective arrival rate** and is denoted by $\boldsymbol{\lambda_{eff} = \lambda (1- P_N)}$.

$$L_q = L_s - \frac{\lambda_{eff}}{\mu}$$

$$W_s = \frac{L_s}{\lambda_{eff}}$$

$$W_q = \frac{L_q}{\lambda_{eff}} = W_s - \frac{1}{\mu}$$

Example 1

Example-1 : In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the service time distribution is also follow exponential with an average 30 minutes. Evaluate the following assuming that the capacity of the yard is to admit 9 trains

a) Probability that the yard is empty.

b) Average no. of trains waiting in the queue and in service.

c) Find the effective arrival rate.

d) Expected waiting time of a train until it leaves the yard.

What is the probability that a newly arriving train finding the yard full

Example 1

Solution:

λ=30/day and μ=40/day

$$\rho=\frac{\lambda}{\mu}=0.75$$

a) Probability that the yard is empty $P_0 = \frac{1-\rho}{1-\rho^{(N+1)}}, \qquad \rho \neq 1$

$$P_0 = \frac{1-0.75}{1-0.75^{(9+1)}}$$

b) Average number of trains waiting in the queue and in service

$$L_s = \left(\frac{1-\rho}{1-\rho^{(N+1)}}\right)\sum_{n=0}^{N} n\rho^n = P_0 \sum_{n=0}^{N} n\rho^n = 2.79 \cong 3\ trains$$

c) Effective arrival rate $\lambda_{eff} = \lambda\ (1- P_N)$=30(1- P_N)=30(1- P_9)

Example 1

Expected waiting time of a train until it leaves the yard $W_s \quad = \frac{L_s}{\lambda_{eff}}$

$$W_s \quad =\left(\frac{3}{30(1-P_9)} \times 24\right) \text{ hours}$$

Probability that the newly arriving train finds the yard full

$$P_n \quad = \left(\frac{1-\rho}{1-\rho^{(N+1)}}\right)\rho^9 = P_0\,\rho^9 = (0.28)(0.75)^9$$

SUMMARY

In this session, Queuing models and its performance measures have discussed.

1. Performance measures of Model 2 using the notations.

2. Difference between effective arrival rate and arrival rate.

TERMINAL QUESTIONS

1. Patients arrive at a clinic according to a Poisson distribution at the rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate 20 per hour.

   (i). Find the effective arrival rate at the clinic.

   (ii). What is the probability that an arriving patient will not wait? Will he find a vacant seat in the room?

   iii) What is the expected waiting time until a patient is discharged from the clinic?

[**Hint :** Here N = 14, $\lambda = 30/60$, $\mu = 20/60$, $\rho = 3/2$. Find $1 - P_n$, P_0 and $W_s = \frac{P_0}{\lambda} \sum_{n=0}^{N} n\,\rho^n$]

2. A car park contains 5 cars. The arrival of cars is Poisson at a mean rate of 10 per hour. The length of time each car spends in the car park has negative exponential distribution with mean of 5 hours.

a) Find the probability that an arrival finds the car park empty
b) Find the probability that an arrival finds the car park is full.
c) How many cars are in the car park on an average?
d) What is the effective arrival rate?

REFERENCES FOR FURTHER LEARNING OF THE SESSION

Reference Books:

1. D. Gross, J.F.Shortle, J.M. Thompson, and C.M. Harris, Fundamentals of Queueing Theory, 4th Edition, Wiley, 2008

2. William Feller, An Introduction to Probability Theory and Its Applications: Volime 1, Third Edition, 1968 by John Wiley & Sons,Inc.

Sites and Web links:

1. https://www.khanacademy.org/math/statistics-probability/significance-tests-one-sample/more-significance-testing-videos/v/small-sample-hypothesis-test

2. J.F. Shortle, J.M. Thompson, D. Gross and C.M. Harris, Fundamentals of Queueing Theory, 5th Edition, Wiley, 2018.

3. https://onlinecourses.nptel.ac.in/noc22_ma17/previ ew 3.

4. https://www.youtube.com/watch?v=Wo75G99F9f M&list=PLwdnzlV3ogoX2OHyZz3QbEYFhbqM 7x275&index=3

THANK YOU

Team – PSQT EVEN SEMESTER 2024-25