NP-Hard absolute Approximation

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• Absolute approximation and NP-hardness deal with the difficulty of finding optimal solutions to optimization problems:

NP-Hard Problems:

• These are a class of optimization problems that are believed to be difficult to solve efficiently (i.e., in polynomial time) with respect to the input size.

• Informally, solving an NP-hard problem in polynomial time would imply that a whole range of other problems can also be solved efficiently, which is considered unlikely by computer scientists.

• NP-hardness refers to the difficulty of finding the optimal solution, not necessarily approximating it.

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Approximation Algorithms:

- Since finding the optimal solution for NP-hard problems might be intractable, we often resort to approximation algorithms.
- These algorithms aim to find a "good enough" solution, guaranteed to be within a certain factor of the optimal solution, in polynomial time.
- The approximation factor can be relative (a percentage of the optimal value) or absolute (a constant difference from the optimal value).

Absolute Approximation and NP-Hardness:

- For some NP-hard problems, it can be proven that achieving a good absolute approximation is also difficult. This means there's no known polynomial-time algorithm that guarantees a solution within a small constant difference of the optimal value (assuming $P \neq NP$).
- There's a distinction between rescalable and strongly NP-hard problems:
  - Rescalable NP-hard problems might have polynomial-time approximation schemes with small absolute errors, but not constant absolute errors. (A polynomial-time approximation scheme guarantees increasingly better approximations as the input size grows).
  - Strongly NP-hard problems are unlikely to have such schemes entirely.

In summary:

- NP-hardness tells us about the inherent difficulty of finding the optimal solution for a specific problem. Absolute approximation hardness tells us that even finding a solution close to the optimal value in terms of a constant difference might be difficult for some NP-hard problems.