Tutorial 1

Formulation of Linear Programming Problem (LPP) and Graphical method

Date of the Session: . . . .

Learning outcomes:

- Understanding the process of formulating a given problem in linear form.
- This requires defining the decision variables of the problem, establishing inter-relationship between them and formulating the objective function and constraints.
- Understanding the process of solving a given LPP using graphs will be discussed.

1.1 PRE-TUTORIAL

1. What is Linear Programming Problem (LPP)?

An LPP is a mathematical problem where an objective function is maximized or minimized subject to linear constraints.

2. Define the following terminology in LPP:

   (a) Objective function

   (b) Constraints

(c) Decision variables

(d) Non-Negativity Conditions

- **Objective Function:** A linear function representing the goal (maximize or minimize), e.g., $Z = c_1x_1 + c_2x_2$.
- **Constraints:** Linear inequalities or equations that limit the values of decision variables, e.g., $a_1x_1 + a_2x_2 \leq b$.
- **Decision Variables:** Variables that represent choices to optimize, e.g., x_1, x_2.
- **Non-Negativity Conditions:** Constraints ensuring variables cannot be negative, e.g., $x_1 \geq 0$.

3. Enumerate the steps involved in Formulation of LPP.

1. Identify decision variables.
2. Formulate the objective function.
3. Define the constraints.
4. Apply non-negativity conditions.
5. Write the model mathematically.

4. Which type of L.P.P. can be solved using graphical method?

The graphical method applies to LPPs with **two decision variables**.

1.2 IN-TUTORIAL

1. A hotel has requested a manufacturer to produce pants and jackets for their boys. For materials, the manufacturer has $750m^2$ of cotton textile and $1000m^2$ of silk. Every pair of pants (1 unit) needs $2m^2$ of silk and $1m^2$ of cotton. Every jacket needs $1.5m^2$ of cotton and $1m^2$ of silk. The price of the pants is fixed at $50 and the jacket, $40. What is the number of pants and jackets that the manufacturer must give to the hotel so that these items obtain a maximum sale? Formulate the problem using mathematical modeling of LPP and define the objective function?

   Solution:

1. Decision Variables:

Let:

- x: the number of pants to be produced.
- y: the number of jackets to be produced.

2. Objective Function:

The objective is to **maximize the total revenue**, which can be expressed as:

$$Z = 50x + 40y$$

Where:

- $50x$: Revenue from selling x pants.
- $40y$: Revenue from selling y jackets.

3. Constraints:

The constraints are based on the availability of resources (cotton and silk):

1. **Cotton constraint:** Each pair of pants requires $1\,m^2$ of cotton, and each jacket requires $1.5\,m^2$ of cotton. The total cotton available is $750\,m^2$.

$$x + 1.5y \leq 750$$

2. **Silk constraint:** Each pair of pants requires $2\,m^2$ of silk, and each jacket requires $1\,m^2$ of silk. The total silk available is $1000\,m^2$.

$$2x + y \leq 1000$$

3. **Non-negativity constraint:** The number of pants and jackets produced cannot be negative:

$$x \geq 0,\ y \geq 0$$

Mathematical Model:

$$\text{Maximize } Z = 50x + 40y$$

Subject to:

$$x + 1.5y \leq 750$$

$$2x + y \leq 1000$$

$$x \geq 0,\ y \geq 0$$

2. A transport company has two types of trucks, Type A and Type B. Type A has refrigerated capacity of $20m^3$ and a non-refrigerated capacity of $40m^3$ while Type B has refrigerated capacity of $30m^3$ and non-refrigerated capacity of $30m^3$. A grocer needs to hire trucks for the transport of $3000m^3$ of refrigerated stock and $4000m^3$ of non-refrigerated stock. The cost per kilometer of a Type A is $30 and $40 for Type B. How many trucks of each type should the grocer rent to achieve the minimum total cost? Formulate the problem using mathematical modeling of LPP and define the objective function?

   Solution:

1. Decision Variables

Let:

- x: Number of Type A trucks rented.
- y: Number of Type B trucks rented.

---

2. Objective Function

The goal is to **minimize the total cost**. The cost per kilometer is $30 for Type A trucks and $40 for Type B trucks. Hence, the objective function is:

$$\text{Minimize } Z = 30x + 40y$$

3. Constraints

Capacity Constraints

1. The **total refrigerated capacity** provided by the trucks must be at least 3000 m^3:

$$20x + 30y \geq 3000$$

2. The **total non-refrigerated capacity** provided by the trucks must be at least 4000 m^3:

$$40x + 30y \geq 4000$$

Non-Negativity Constraints

Both x and y must be non-negative:

$$x \geq 0,\ y \geq 0$$

4. LPP Formulation

The mathematical model for the problem is:

$$\text{Minimize } Z = 30x + 40y$$

Subject to:

$$20x + 30y \geq 3000$$

$$40x + 30y \geq 4000$$

$$x \geq 0,\ y \geq 0$$

3. A hotel has requested a manufacturer to produce pillows and blankets for their room service. For materials, the manufacturer has $750m^2$ of cotton textile and $1000m^2$ of silk. Every pillow needs $2m^2$ of cotton and $1m^2$ of silk. Every blanket needs $2m^2$ of cotton and $5m^2$ of silk. The price of the pillow is fixed at $5 and the blanket is fixed at $10.What is the number of pillows and blankets that the manufacturer must give to the hotel so that these items obtain a maximum sale? Formulate the LPP and solve the LPP using Python PuLP.

Solution:

Define the decision variables:

- Let x be the number of pillows to be produced.
- Let y be the number of blankets to be produced.

Objective Function:

The objective is to maximize the total sales revenue. The revenue from each pillow is $5, and the revenue from each blanket is $10. Therefore, the objective function is:

$$\text{Maximize } Z = 5x + 10y$$

Constraints:

1. **Cotton Constraint:**
   Every pillow needs 2 m² of cotton, and every blanket needs 2 m² of cotton. The total amount of cotton available is 750 m². Therefore, the constraint for cotton is:
   $$2x + 2y \leq 750$$
2. **Silk Constraint:**
   Every pillow needs 1 m² of silk, and every blanket needs 5 m² of silk. The total amount of silk available is 1000 m². Therefore, the constraint for silk is:
   $$x + 5y \leq 1000$$
3. **Non-negativity Constraints:**
   The number of pillows and blankets produced cannot be negative. Therefore, we have:
   $$x \geq 0 \quad \text{and} \quad y \geq 0$$

The Linear Programming Problem (LPP) is:

$$\text{Maximize } Z = 5x + 10y$$

Subject to:

$$2x + 2y \leq 750$$

$$x + 5y \leq 1000$$

$$x \geq 0, \quad y \geq 0$$

CODE

```
from pulp import LpMaximize, LpProblem, LpVariable, lpSum

model = LpProblem(name="maximize_revenue", sense=LpMaximize)

x = LpVariable(name="pillows", lowBound=0, cat="Integer")
y = LpVariable(name="blankets", lowBound=0, cat="Integer")

model += 5 * x + 10 * y, "Total Revenue"

model += (2 * x + 2 * y <= 750), "Cotton Constraint"
model += (x + 5 * y <= 1000), "Silk Constraint"

status = model.solve()

print(f"Status: {model.status}, {model.solver.name}")
print(f"Number of pillows to produce: {x.value()}")
print(f"Number of blankets to produce: {y.value()}")
print(f"Maximum Revenue: ${model.objective.value()}")
```

OUTPUT

Status: 1, PULP_CBC_CMD
Number of pillows to produce: 219.0
Number of blankets to produce: 156.0
Maximum Revenue: $2655.0

4. An Industry makes two items of P and Q by using two devices X and Y. Processing time requires 50 hrs for item P on device X and 30 hrs requires on device Y. Processing time requires 24 hrs for item Q on device X and 33 hrs requires on device Y. At starting of the current week, 30 pieces of A and 90 pieces of B are available. Processing time that is available on device X is predict to be 40 hrs and on device Y is predict to be 35 hrs. Demand for P in the current week is predict to be 75 pieces and for Q is predict to be 95 pieces. Industry policy is to maximize the combined sum of the pieces of P and the pieces of Q in stock at the end of the week. Formulate the problem of deciding how much of each item to make in the current week as a linear program. Obtain the solution using graphical method. Also, solve the LPP using Python PuLP

Solution:

Objective Function:

Maximize $Z = x + y$ (where x is the number of items P and y is the number of items Q).

Constraints:

1. $50x + 24y \leq 40$ (Device X time constraint).
2. $30x + 33y \leq 35$ (Device Y time constraint).
3. $x \leq 75$ (Demand for P).
4. $y \leq 95$ (Demand for Q).
5. $x \geq 0,\ y \geq 0$ (Non-negativity constraints).

Step 2: Identify the feasible region

The region satisfying the constraints $50x + 24y \leq 40$, $30x + 33y \leq 35$, $x \leq 75$, and $y \leq 95$ lies in the first quadrant and is bounded by these lines.

The **corner points** of the feasible region are:

1. $(0, 1.67)$
2. $(0.8, 0)$
3. $(1.17, 0)$
4. $(0, 1.06)$

CALCULATION STEPS FOR ABOVE IMAGE 1,2,3,4

For $50x + 24y = 40$:

1. **When $x = 0$:** Substitute $x = 0$ into the equation:

$$50(0) + 24y = 40 \implies 24y = 40 \implies y = \frac{40}{24} = 1.67.$$

   So, the point is $A(0, 1.67)$.

2. **When $y = 0$:** Substitute $y = 0$ into the equation:

$$50x + 24(0) = 40 \implies 50x = 40 \implies x = \frac{40}{50} = 0.8.$$

   So, the point is $B(0.8, 0)$.

For $30x + 33y = 35$:

1. **When $x = 0$:** Substitute $x = 0$ into the equation:

$$30(0) + 33y = 35 \implies 33y = 35 \implies y = \frac{35}{33} \approx 1.06.$$

   So, the point is $C(0, 1.06)$.

2. **When $y = 0$:** Substitute $y = 0$ into the equation:

$$30x + 33(0) = 35 \implies 30x = 35 \implies x = \frac{35}{30} \approx 1.17.$$

   So, the point is $D(1.17, 0)$.

Step 3: Solve the intersection of $50x + 24y = 40$ and $30x + 33y = 35$

To find the intersection point, solve the system of equations:

1. $50x + 24y = 40$
2. $30x + 33y = 35$

Multiply the first equation by 33 and the second by 24 to eliminate y:

$$33(50x + 24y) = 33(40) \implies 1650x + 792y = 1320$$

$$24(30x + 33y) = 24(35) \implies 720x + 792y = 840$$

Subtract the second equation from the first:

$$(1650x + 792y) - (720x + 792y) = 1320 - 840 \implies 930x = 480 \implies x = \frac{480}{930} \approx 0.515$$

Substitute $x = 0.515$ into $50x + 24y = 40$:

$$50(0.515) + 24y = 40 \implies 25.75 + 24y = 40 \implies 24y = 14.25 \implies y = \frac{14.25}{24} \approx 0.594$$

So, the intersection point is $(0.515, 0.594)$.

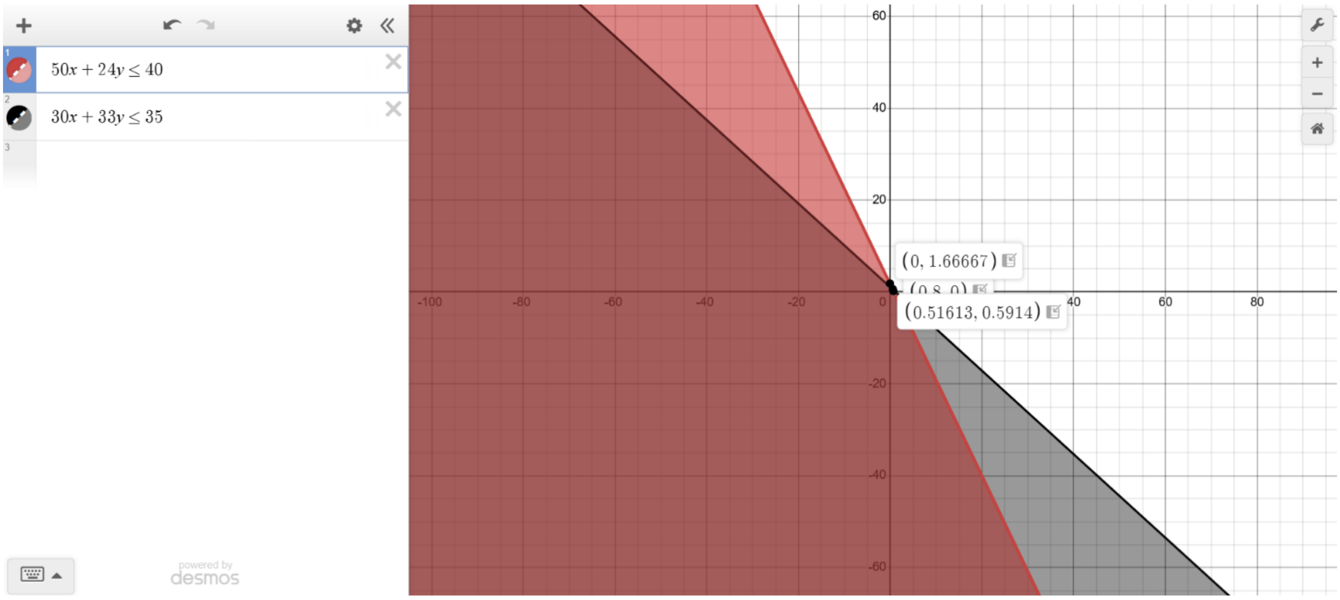
Step 4: Evaluate $Z = x + y$ at all corner points

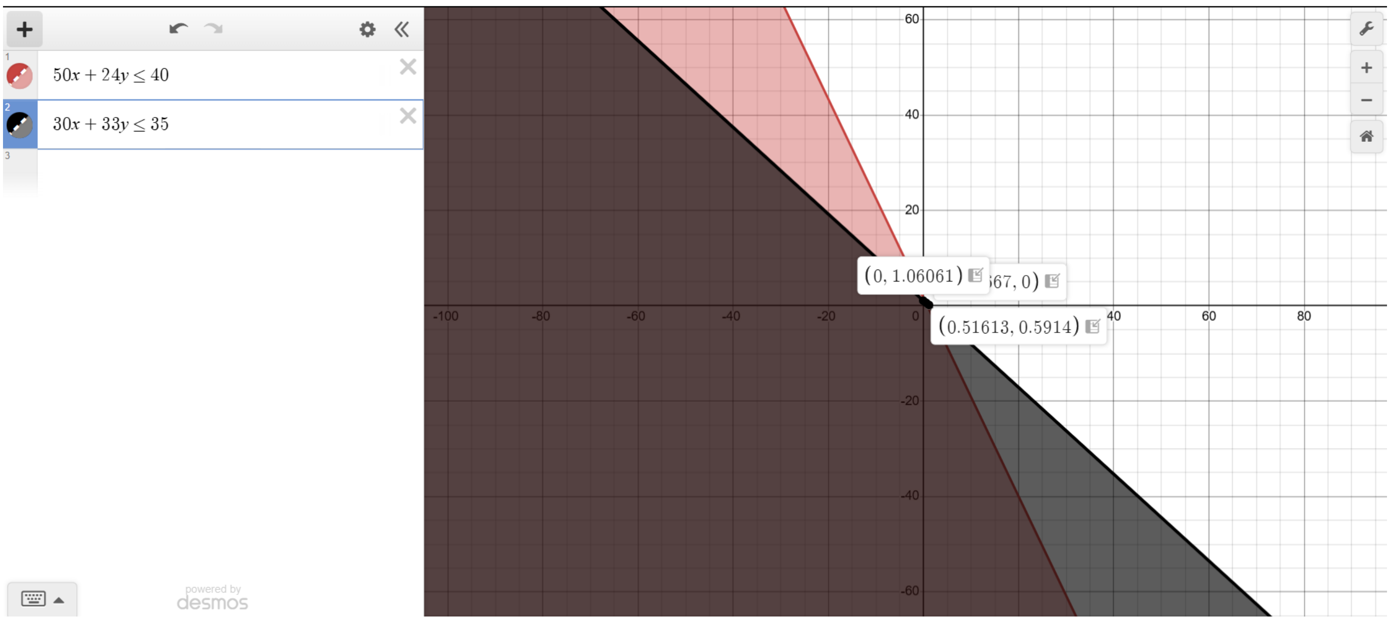
1. **At** $(0, 1.67)$:
   $Z = 0 + 1.67 = 1.67.$
2. **At** $(0.8, 0)$:
   $Z = 0.8 + 0 = 0.8.$
3. **At** $(1.17, 0)$:
   $Z = 1.17 + 0 = 1.17.$
4. **At** $(0.515, 0.594)$:
   $Z = 0.515 + 0.594 = 1.109.$
5. **At** $(0, 1.06)$:
   $Z = 0 + 1.06 = 1.06.$

Step 5: Conclusion

- The maximum value of $Z = x + y$ is **1.67**, achieved at $(0, 1.67)$.

GRAPH





"Note: Draw a graph in the tutorial book, then draw two lines and locate the points as shown in the images above. If you can't understand the points, please click the link above."

https://www.desmos.com/calculator/0igegbmtox

CODE

```
from pulp import LpMaximize, LpProblem, LpVariable, LpStatus

problem = LpProblem("Maximize_Items", LpMaximize)

x = LpVariable("x", lowBound=0)
y = LpVariable("y", lowBound=0)

problem += x + y, "Total_Items"
problem += 50 * x + 24 * y <= 40
problem += 30 * x + 33 * y <= 35
problem += x <= 75
problem += y <= 95

status = problem.solve()

print(f"Status: {LpStatus[status]}")
print(f"Optimal value of x: {x.value()}")
print(f"Optimal value of y: {y.value()}")
print(f"Maximum Z = x + y: {problem.objective.value()}")
```

OUTPUT

```
Status: Optimal
Optimal value of x: 0.51612903
Optimal value of y: 0.59139785
Maximum Z = x + y: 1.10752688
```

5. Consider the following linear programming problem

$$\text{Maximize: } P = 7x + 12y$$
$$2x + 3y \leq 6$$
$$3x + 7y \leq 12$$

Obtain the solution using graphical method.

Solution:

Objective Function:

Maximize $P = 7x + 12y$.

Constraints:

1. $2x + 3y \leq 6$.
2. $3x + 7y \leq 12$.
3. $x \geq 0,\ y \geq 0$ (non-negativity constraints).

Step 2: Identify the feasible region

- The region satisfying $2x + 3y \leq 6$, $3x + 7y \leq 12$, $x \geq 0$, and $y \geq 0$ lies in the first quadrant and is bounded by these lines.
- The **corner points** of the feasible region are:
  1. $(0, 2)$
  2. $(3, 0)$
  3. $(4, 0)$
  4. $(1.2, 1.2)$ (intersection of $2x + 3y = 6$ and $3x + 7y = 12$)
  5. $(0, 1.71)$.

CALCULATION STEPS FOR ABOVE IMAGE 1,2,3,5

First Equation: $2x + 3y = 6$

To find the points where this line intersects the axes:

1. When $x = 0$:

   Substitute $x = 0$ into $2x + 3y = 6$:

   $$2(0) + 3y = 6 \implies 3y = 6 \implies y = 2.$$

   So, the point is $A(0, 2)$.

2. When $y = 0$:

   Substitute $y = 0$ into $2x + 3y = 6$:

   $$2x + 3(0) = 6 \implies 2x = 6 \implies x = 3.$$

   So, the point is $B(3, 0)$.

Thus, the line $2x + 3y = 6$ passes through $A(0, 2)$ and $B(3, 0)$.

Second Equation: $3x + 7y = 12$

To find the points where this line intersects the axes:

1. When $x = 0$:

   Substitute $x = 0$ into $3x + 7y = 12$:

   $$3(0) + 7y = 12 \implies 7y = 12 \implies y = \frac{12}{7} \approx 1.71.$$

   So, the point is $C(0, 1.71)$.

2. When $y = 0$:

   Substitute $y = 0$ into $3x + 7y = 12$:

   $$3x + 7(0) = 12 \implies 3x = 12 \implies x = 4.$$

   So, the point is $D(4, 0)$.

Thus, the line $3x + 7y = 12$ passes through $C(0, 1.71)$ and $D(4, 0)$.

Step 3: Solve the intersection of $2x + 3y = 6$ and $3x + 7y = 12$

To find the intersection point, solve:

1. $2x + 3y = 6$
2. $3x + 7y = 12$.

Multiply the first equation by 7 and the second by 3 to eliminate y:

$$7(2x + 3y) = 7(6) \implies 14x + 21y = 42, 3(3x + 7y) = 3(12) \implies 9x + 21y = 36.$$

Subtract the second equation from the first:

$$14x + 21y - 9x - 21y = 42 - 36 \implies 5x = 6 \implies x = 1.2.$$

Substitute $x = 1.2$ into $2x + 3y = 6$:

$$2(1.2) + 3y = 6 \implies 2.4 + 3y = 6 \implies 3y = 3.6 \implies y = 1.2.$$

So, the intersection point is $(1.2, 1.2)$.

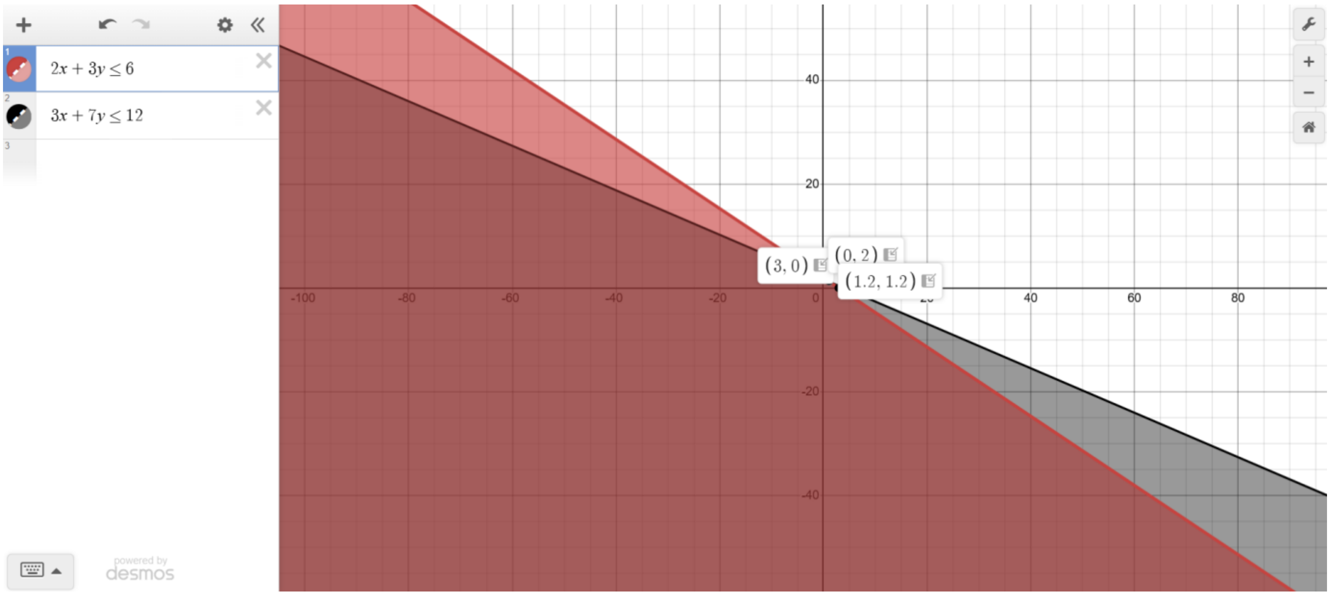
Step 4: Evaluate $P = 7x + 12y$ at all corner points

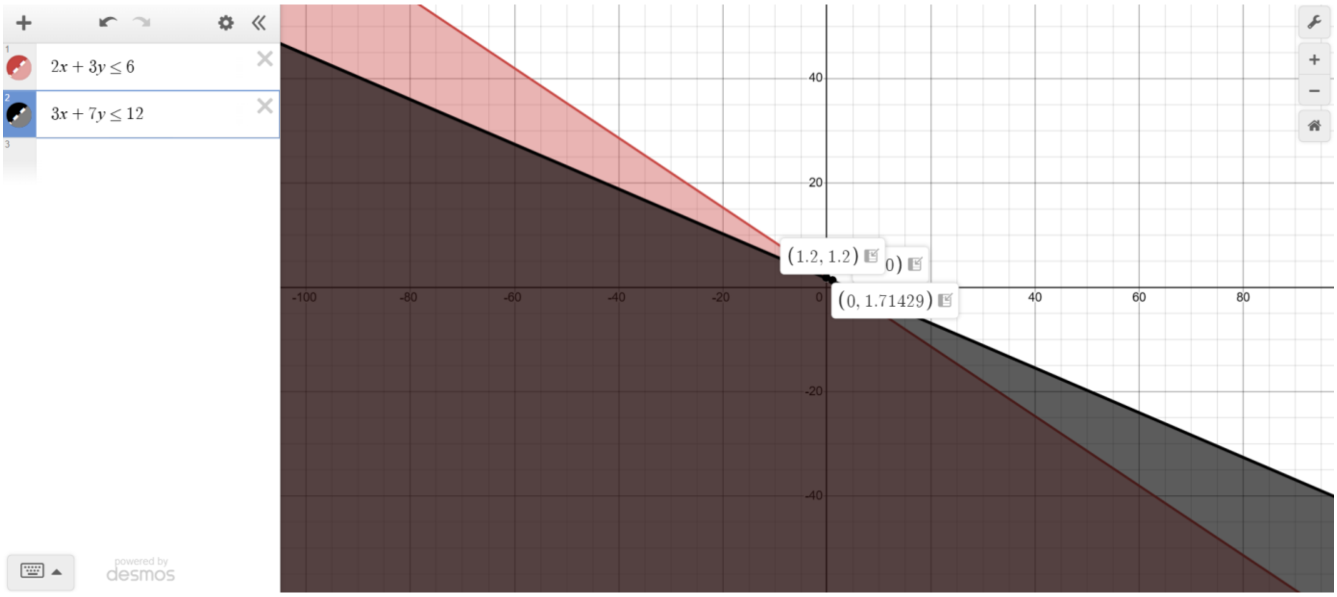
1. **At** $(0, 2)$:
   $P = 7(0) + 12(2) = 24.$
2. **At** $(3, 0)$:
   $P = 7(3) + 12(0) = 21.$
3. **At** $(4, 0)$:
   $P = 7(4) + 12(0) = 28.$
4. **At** $(1.2, 1.2)$:
   $P = 7(1.2) + 12(1.2) = 8.4 + 14.4 = 22.8.$
5. **At** $(0, 1.71)$:
   $P = 7(0) + 12(1.71) = 20.52.$

Step 5: Conclusion

- The maximum value of $P = 7x + 12y$ is **28**, achieved at $(4, 0)$.

GRAPH





"Note: Draw a graph in the tutorial book, then draw two lines and locate the points as shown in the images above. If you can't understand the points, please click the link above."

https://www.desmos.com/calculator/odacjijkeg

1.3 POST-TUTORIAL

1. What are the Advantages of Linear Programming Problem?
   Solution:

- **Optimal Solutions:** Helps find the best solution within given constraints.
- **Decision Support:** Aids in making complex decisions efficiently.
- **Flexibility:** Applicable in various industries.
- **Resource Optimization:** Maximizes resource usage (time, materials, etc.).
- **Clear Formulation:** Easy to define and apply.

2. Applications of Linear Programming.
   Solution:

- **Production Planning:** Optimizing product mix.
- **Transportation:** Minimizing delivery costs.
- **Diet:** Finding the cheapest food combination to meet nutritional needs.
- **Scheduling:** Optimizing workforce shifts.
- **Portfolio Optimization:** Balancing risk and return in investments.
- **Manufacturing:** Efficient allocation of materials and machine time.

3. A company owns two flour mills viz. A and B, which have different production capacities for high, medium and low-quality flour. The company has entered a contract to supply flour to a firm every month with at least 8, 12 and 24 quintals of high, medium and low quality respectively. It costs the company Rs.2000 and Rs.1500 per day to run mill A and B respectively. On a day, Mill A produces 6, 2 and 4 quintals of high, medium and low-quality flour, Mill B produces 2, 4 and 12 quintals of high, medium and low-quality flour respectively. How many days per month should each mill be operated in order to meet the contract order most economically? Formulate the LPP and solve the LPP using Python PuLP.

Solution:

Decision Variables

Let:

- x = number of days Mill A operates in a month
- y = number of days Mill B operates in a month

Objective Function

The total cost of operation is given by:

$$Z = 2000x + 1500y$$

Constraints

1. **High-Quality Flour:**

$$6x + 2y \geq 8$$

(Mill A produces 6 quintals/day, Mill B produces 2 quintals/day, and at least 8 quintals are required.)

2. **Medium-Quality Flour:**

$$2x + 4y \geq 12$$

(Mill A produces 2 quintals/day, Mill B produces 4 quintals/day, and at least 12 quintals are required.)

3. **Low-Quality Flour:**

$$4x + 12y \geq 24$$

(Mill A produces 4 quintals/day, Mill B produces 12 quintals/day, and at least 24 quintals are required.)

4. **Non-Negativity Constraints:**

$$x \geq 0, \ y \geq 0$$

(The number of days must be non-negative.)

Complete LPP Formulation

$$\text{Minimize } Z = 2000x + 1500y$$

Subject to:

$$6x + 2y \geq 8$$

$$2x + 4y \geq 12$$

$$4x + 12y \geq 24$$

$$x \geq 0, \ y \geq 0$$

CODE

```
from pulp import LpProblem, LpVariable, LpMinimize, lpSum

problem = LpProblem("Flour_Mill_Optimization", LpMinimize)

x = LpVariable("Mill_A_days", lowBound=0, cat="Continuous")
y = LpVariable("Mill_B_days", lowBound=0, cat="Continuous")

problem += 2000 * x + 1500 * y, "Total Cost"

problem += 6 * x + 2 * y >= 8, "High_Quality_Flour"
problem += 2 * x + 4 * y >= 12, "Medium_Quality_Flour"
problem += 4 * x + 12 * y >= 24, "Low_Quality_Flour"

problem.solve()

print("Status:", problem.status)
print("Optimal Solution:")
print(f"Mill A (x) days: {x.varValue}")
print(f"Mill B (y) days: {y.varValue}")
print(f"Minimum Total Cost: Rs. {problem.objective.value()}")
```

OUTPUT

```
Status: 1
Optimal Solution:
Mill A (x) days: 0.4
Mill B (y) days: 2.8
Minimum Total Cost: Rs. 5000.0
```

4. An advertising company plans its advertising strategy in three different media- television, radio and magazines. Following data have been obtained from market survey: The com-

	Television	Radio	Magazine I	Magazine II
Cost of an advertising unit	Rs. 30,000	Rs. 20,000	Rs. 15,000	Rs. 10,000
No. of potential customers reached per unit	2,00,000	6,00,000	1,50,000	1,00,000
No. of female customers reached per unit	1,50,000	4,00,000	70,000	50,000

pany wants to spend no more than Rs. 4,50,000 on advertising. Following are the set of requirements that must be met:

(a) At least 1 million exposures take place among female customers.

(b) Advertising on magazines be limited to Rs. 1,50,000.

(c) The number of advertising units on television and radio should each be between 5 and 10.

Formulate the LPP.

Solution:

Decision Variables:

Let:

- x_1 = number of advertising units on television.
- x_2 = number of advertising units on radio.
- x_3 = number of advertising units on Magazine I.
- x_4 = number of advertising units on Magazine II.

Objective Function:

Maximize the total number of potential customers reached:

$$\text{Maximize } Z = 200,000x_1 + 600,000x_2 + 150,000x_3 + 100,000x_4$$

Constraints:

1. **Cost Constraint:** The total expenditure should not exceed Rs. 4,50,000. The cost per unit of each medium is given:
   - Television: Rs. 30,000 per unit.
   - Radio: Rs. 20,000 per unit.
   - Magazine I: Rs. 15,000 per unit.
   - Magazine II: Rs. 10,000 per unit.

   Therefore, the cost constraint is:

$$30,000x_1 + 20,000x_2 + 15,000x_3 + 10,000x_4 \leq 450,000$$

2. **Female Customer Exposure Constraint:** At least 1 million exposures should be reached among female customers. The number of female customers reached by each medium is:
   - Television: 1,50,000 female customers per unit.
   - Radio: 4,00,000 female customers per unit.
   - Magazine I: 70,000 female customers per unit.
   - Magazine II: 50,000 female customers per unit.

   Therefore, the constraint for female customers is:

$$150,000x_1 + 400,000x_2 + 70,000x_3 + 50,000x_4 \geq 1,000,000$$

3. **Magazine Advertising Cost Constraint:** Advertising on magazines should be limited to Rs. 1,50,000. Since the costs for Magazine I and Magazine II are Rs. 15,000 and Rs. 10,000 per unit respectively, the constraint is:

$$15,000x_3 + 10,000x_4 \leq 150,000$$

4. **Television and Radio Advertising Unit Constraints:** The number of advertising units for television and radio should each be between 5 and 10. Therefore, the constraints are:

$$5 \leq x_1 \leq 10$$

$$5 \leq x_2 \leq 10$$

5. **Non-Negativity Constraints:** All decision variables should be non-negative integers since they represent the number of advertising units.

$$x_1, x_2, x_3, x_4 \geq 0$$

Summary of the LPP:

Objective:

$$\text{Maximize } Z = 200,000x_1 + 600,000x_2 + 150,000x_3 + 100,000x_4$$

Subject to:

$$30,000x_1 + 20,000x_2 + 15,000x_3 + 10,000x_4 \leq 450,000$$

$$150,000x_1 + 400,000x_2 + 70,000x_3 + 50,000x_4 \geq 1,000,000$$

$$15,000x_3 + 10,000x_4 \leq 150,000$$

$$5 \leq x_1 \leq 10$$

$$5 \leq x_2 \leq 10$$

$$x_1, x_2, x_3, x_4 \geq 0$$

5. A cabinet maker makes benches and desks. Each bench can be sold for a profit of $30 and each desk for a profit of $10. The cabinetmaker can afford to spend up to 40 hrs per week working and takes 6 hrs to make a bench and 3 hrs to make a desk. Customer demand requires that he makes at least 3 times as many desks as benches. Benches take up 4 times as much storage space as desks and there is room for at most four benches each week. Formulate this problem as a linear programming problem and solve it graphically. Also, solve the LPP using Python PuLP

Solution:

Objective Function:

Maximize $P = 30x + 10y$ (where x is the number of benches and y is the number of desks).

Constraints:

1. $6x + 3y \leq 40$ (Time constraint: 6 hours for a bench, 3 hours for a desk, and 40 hours available).
2. $y \geq 3x$ (Demand constraint: At least 3 times as many desks as benches).
3. $x \leq 4$ (Storage constraint: At most 4 benches).
4. $x \geq 0, y \geq 0$ (Non-negativity constraints).

Step 1: Convert inequalities to equalities to plot the lines

Let's work through the constraints, starting with the first two inequalities:

1. **For** $6x + 3y = 40$**:**
   - When $x = 0$, $y = \frac{40}{3} \approx 13.33$. This gives point $A(0, 13.33)$.
   - When $y = 0$, $x = \frac{40}{6} = 6.67$. This gives point $B(6.67, 0)$.
   - Line equation: $6x + 3y = 40$.
2. **For** $y = 3x$**:**
   - This is a simple line with a slope of 3. It goes through the origin (0, 0) and has the equation $y = 3x$.

Step 2: Identify the feasible region

The feasible region is bounded by the following:

- The region satisfying $6x + 3y \leq 40$,
- The line $y = 3x$,
- The condition $x \leq 4$ (which limits the number of benches),
- The non-negativity constraints $x \geq 0$ and $y \geq 0$.

Step 3: Find the corner points (intersection points)

Let's solve the system of equations formed by the constraints:

1. **Intersection of $6x + 3y = 40$ and $y = 3x$:**
   - Substitute $y = 3x$ into $6x + 3y = 40$:
   $$6x + 3(3x) = 40 \implies 6x + 9x = 40 \implies 15x = 40 \implies x = \frac{40}{15} = 2.67.$$
   - Substitute $x = 2.67$ into $y = 3x$:
   $$y = 3(2.67) = 8.$$

   So the intersection point is $(2.67, 8)$.

2. **Intersection of $6x + 3y = 40$ and $x = 4$:**
   - Substitute $x = 4$ into $6x + 3y = 40$:
   $$6(4) + 3y = 40 \implies 24 + 3y = 40 \implies 3y = 16 \implies y = \frac{16}{3} \approx 5.33.$$

   So the intersection point is $(4, 5.33)$.

3. **Intersection of $y = 3x$ and the x-axis $y = 0$:**
   - For $y = 0$, from $y = 3x$, we get $x = 0$. So the intersection point is $(0, 0)$.

Step 4: Evaluate $P = 30x + 10y$ at all corner points

1. **At** $(0, 0)$:

$$P = 30(0) + 10(0) = 0.$$

2. **At** $(0, 13.33)$:

$$P = 30(0) + 10(13.33) = 133.33.$$

3. **At** $(4, 12)$:

$$P = 30(4) + 10(12) = 120 + 120 = 240.$$

4. **At** $(4, 5.33)$:

$$P = 30(4) + 10(5.33) = 120 + 53.33 = 173.33.$$

5. **At** $(2.67, 8)$:

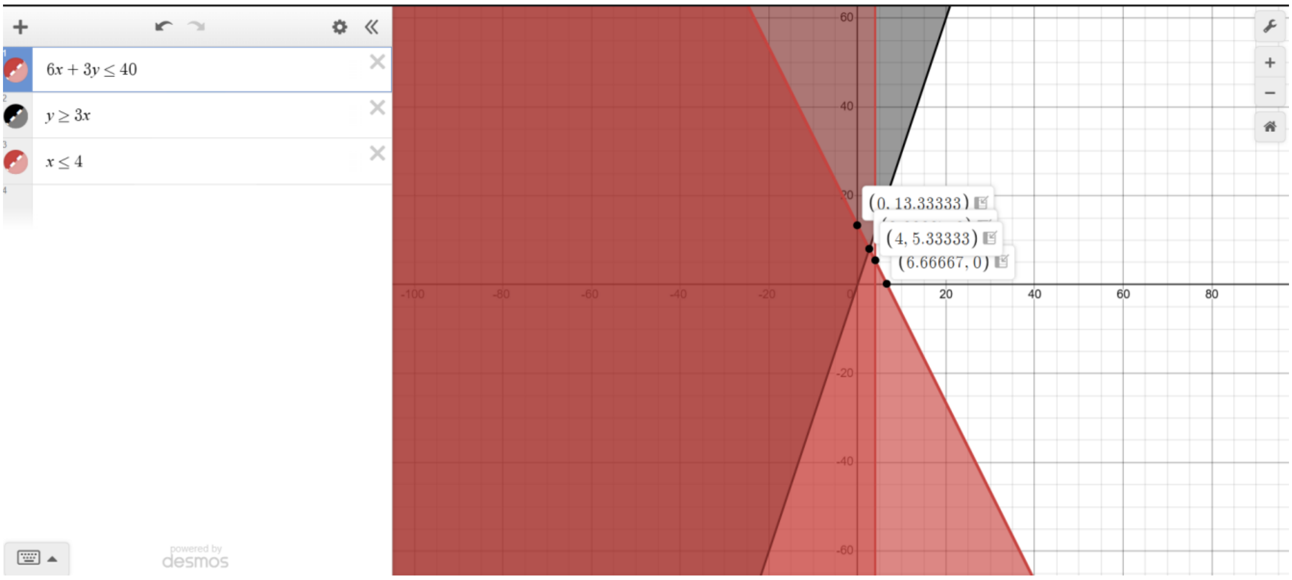
$$P = 30(2.67) + 10(8) = 80.1 + 80 = 160.1.$$

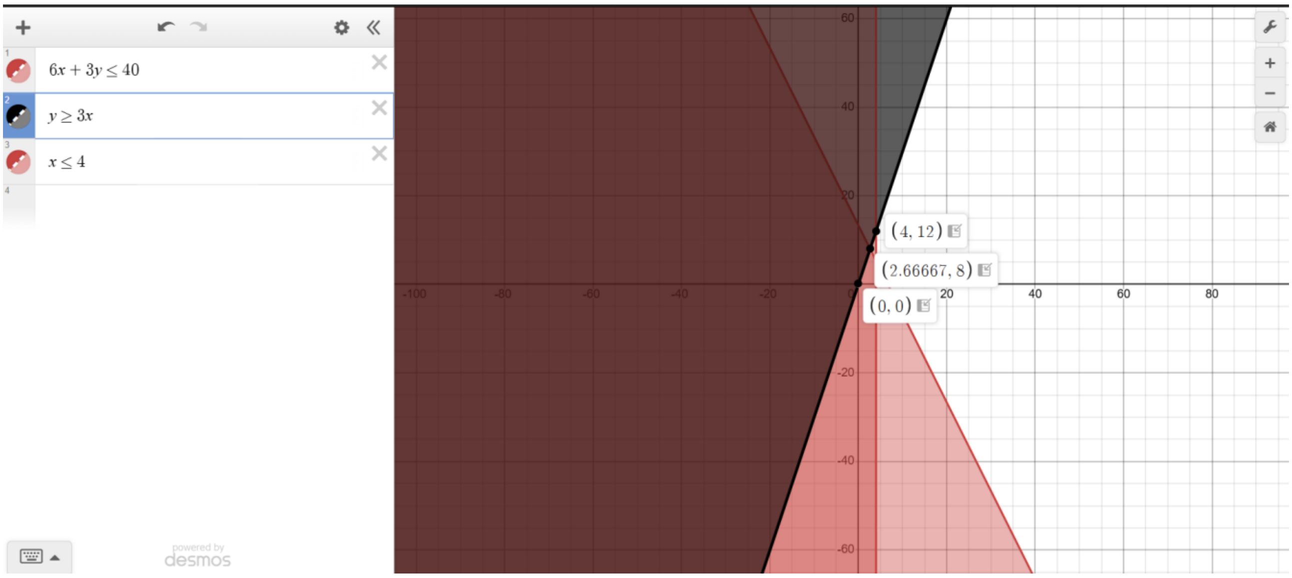
Conclusion:

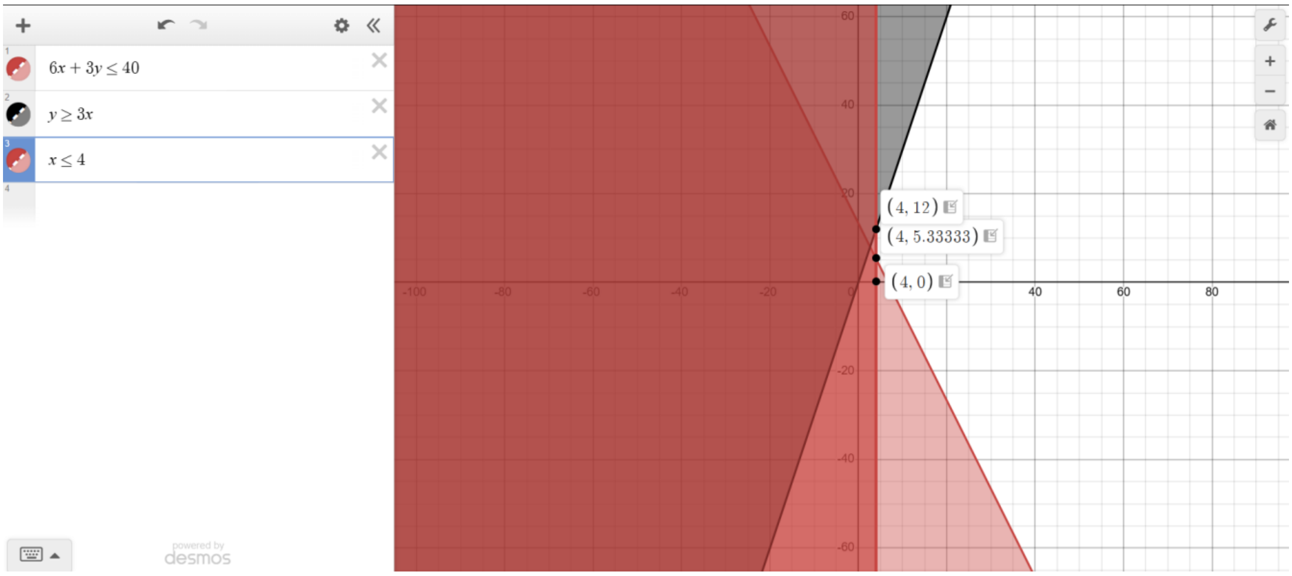
The maximum value of $P = 30x + 10y$ is **240**, which occurs at $(4, 12)$.

This corresponds to making 4 benches and 12 desks, maximizing the profit under the given constraints.

GRAPH


$6x + 3y \leq 40$
$y \geq 3x$
$x \leq 4$
(0, 13.33333)
(4, 5.33333)
(6.66667, 0)
powered by desmos


$6x + 3y \leq 40$
$y \geq 3x$
$x \leq 4$
(4, 12)
(2.66667, 8)
(0, 0)
powered by desmos



"Note: Draw a graph in the tutorial book, then draw two lines and locate the points as shown in the images above. If you can't understand the points, please click the link above."

https://www.desmos.com/calculator/otky2crgd8

CODE

```
from pulp import LpMaximize, LpProblem, LpVariable

prob = LpProblem("Maximize_Profit", LpMaximize)

x = LpVariable("x", lowBound=0, cat="Continuous")
y = LpVariable("y", lowBound=0, cat="Continuous")

prob += 30 * x + 10 * y, "Profit"

prob += 6 * x + 3 * y <= 40, "Work hours constraint"
prob += y >= 3 * x, "Demand constraint"
prob += 4 * x + y <= 40, "Storage space constraint"

prob.solve()

result_x = x.varValue
result_y = y.varValue
result_profit = prob.objective.value()

print(f"Number of benches (x): {result_x}")
print(f"Number of desks (y): {result_y}")
print(f"Maximum profit: {result_profit}")
```

OUTPUT

```
Number of benches (x): 2.6666667
Number of desks (y): 8.0
Maximum profit: 160.000001
```

For Evaluator's Use only

Evaluators Comments	Evaluator's Observation
	Marks Secured .... out of 50
	Full Name of the Evaluator:
	Signature of the Evaluator:
	Date of Evaluation: