Tutorial 4

# Gomory's Cut-Plane Method

Date of the Session: ..................

<u>**Learning outcomes**</u>:

- If the optimal solution is integers, then problem is solved. Otherwise, add Gomory's constraint (cut) is added to optimal solution Now new problem is solved using dual simplex method The method terminates as soon as optimal solution become integers.

## 4.1 PRE-TUTORIAL

1. Enumerate the steps involved in Gomory's cutting plane method.

Step 1: Build the Simplex Table

Step 2: check out the conditions & build it.

Step 3: Initially follow the simplex.

Step 4: Get the corresponding values.

Step 5: If values are integer then stop next all steps.

Step 6: If not by considering the max fractional value from it.

Step 7: Generate the source row.

Step 8: Update the table.

Step 9: Solve it.

Step 10: We'll get corresponding values.

## 4.2 IN-TUTORIAL

1. Find solution using integer simplex method (Gomory's cutting plane method)

$$\text{Maximize}Z = x_1 + x_2$$
subject to
$$3x_1 + 2x_2 \leq 5$$
$$x_2 \leq 2$$
and $x_1, x_2$ are non-negative integers

**Solution:**

$\text{Max } z = x_1 + x_2$

$3x_1 + 2x_2 \leq 5$

$x_2 \leq 2.$

$\text{Max}(z) = x_1 + x_2 + 0S1 + 0S2.$

$\Rightarrow 3x_1 + 2x_2 + S1 = 5$

$0x_1 + x_2 + S2 = 2$

Initial Simplex Table

| $C_B$ | $C_j$ | 1 | 1 | 0 | 0 | Sol | Ratio |
|---|---|---|---|---|---|---|---|
| | B.V | $x_1$ | $x_2$ | S1 | S2 | | |
| 0 | S1 | 3 | 2 | 1 | 0 | 5 | 5/3 |
| 0 | S2 | 0 | 1 | 0 | 0 | 2 | $2/0 = \infty$ |
| | $Z_j$ | 0 | 0 | 0 | 0 | | |
| | $C_j - Z_j$ | 1 | 1 | 0 | 0 | | |

| $C_B$ | $C_j$ | 1 | 1 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|
| | B.V | $x_1$ | $x_2$ | $S1$ | $S2$ | Sol | Ratio |
| 1 | $x_1$ | 1 | 2/3 | 1/3 | 0 | 5/3 | $\frac{5}{3} \times \frac{3}{2} = \frac{5}{2}$ |
| 0 | $S2$ | 0 | 1 | 0 | 0 | 2 | 2 |
| | $Z_j$ | 1 | 2/3 | 1/3 | 0 | | |
| | $C_j - Z_j$ | 0 | 1/3 | −1/3 | 0 | | |

| $C_B$ | $C_j$ | 1 | 1 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|
| | B.V | $x_1$ | $x_2$ | $S1$ | $S2$ | Sol | Ratio |
| 1 | $x1$ | 1 | 0 | $\frac{1}{3}$ | 0 | 1/3 | |
| 1 | $x2$ | 0 | 1 | 0 | 0 | 2 | |
| | $Z_j$ | 1 | 1 | $\frac{1}{3}$ | 0 | | |
| | $C_j - Z_j$ | 0 | 0 | $\frac{-1}{3}$ | 0 | | |



$x_1 = 0 + \frac{1}{3} = \frac{1}{3}, \quad x_2 = 2 + 0 = 2$

To obtain an optimum integer we have to add gomory constant.

$\text{Max}\left(\frac{1}{3}, 0\right) = \frac{1}{3}$

$x$ is source row.

$\frac{1}{3} = x_1 + 0x_2 + \frac{1}{3}s1 + \frac{0}{3}s2.$

Fractional cut constraint

$$\frac{s1}{3} + \frac{0s2}{3} \geq \frac{1}{3}$$

$$-\frac{s1}{3} - \frac{0s2}{3} + G_1 = \frac{-1}{3}$$

| $C_B$ | $C_j$ | 1 | 1 | 0 | 0 | 0 | Sol | Ratio |
|---|---|---|---|---|---|---|---|---|
| | BV | $x_1$ | $x_2$ | $S1$ | $S2$ | $G1$ | | |
| 1 | $x_1$ | 1 | 0 | $\frac{1}{3}$ | 0 | 0 | $1/3$ | 1 |
| 1 | $x_2$ | 0 | 1 | 0 | 0 | 0 | 2 | $\infty$ |
| 0 | $G1$ | 0 | 0 | $\frac{-1}{3}$ | 0 | 1 | $-1/3$ | 1 |
| | $Z_j$ | 1 | 1 | $\frac{1}{3}$ | 0 | 0 | | |
| | $C_j - Z_j$ | 0 | 0 | $\frac{-1}{3}$ | 0 | 0 | | |

| $C_B$ | $C_j$ | 1 | 1 | 0 | 0 | 0 | Sol |
|---|---|---|---|---|---|---|---|
| | B.V | $x_1$ | $x_2$ | $S1$ | $S2$ | $G1$ | |
| 1 | $x1$ | 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | $x2$ | 0 | 1 | 0 | 0 | 0 | 2 |
| 0 | $S1$ | 0 | 0 | 1 | 0 | $-3$ | 1 |
| | $Z_j$ | 1 | 1 | 0 | 0 | 1 | |
| | $C_j - Z_j$ | 0 | 0 | 0 | 0 | $-1$ | |

$\therefore x_1 = 0, x_2 = 2 \quad Max = 2$

## 4.3 POST TUTORIAL

1. Consider the following pure integer programming problem

$$\text{Maximize } Z = 14x_1 + 16x_2$$

Subject to the constraints

$$4x_1 + 3x_2 \le 12$$
$$6x_1 + 8x_2 \le 24$$
$$\text{and } x_1, x_2 \ge 0 \text{ and are integers}$$

Solve the problem by Gomory's cutting plane method

$Z = 14x_1 + 16x_2 + 0s_1 + 0s_2$

$4x_1 + 3x_2 + s_1 = 12$

$6x_1 + 8x_2 + s_2 = 12$

| $C_B$ | $C_j$ | 14 | 16 | 0 | 0 | Sol | Ratio |
|---|---|---|---|---|---|---|---|
| | B.V | $x_1$ | $x_2$ | $s_1$ | $s_2$ | | |
| 0 | $s_1$ | 4 | 3 | 1 | 0 | 12 | 4 |
| 0 | $s_2$ | 3 | 4 | 0 | 1 | 12 | 3 |
| | $Z_j$ | 0 | 0 | 0 | 0 | | |
| | $C_j - Z_j$ | 14 | 16 | 0 | 0 | | |

| $C_B$ | $C_j$ | 14 | 16 | 0 | 0 | Sol | Ratio |
|---|---|---|---|---|---|---|---|
| | B.V | $x_1$ | $x_2$ | $s_1$ | $s_2$ | | |
| 0 | $s_1$ | 7/4 | 0 | 1 | -3/4 | 3 | 12/7 |
| 16 | $x_2$ | 3/4 | 1 | 0 | 1/4 | 3 | 12/3 |
| | $Z_j$ | 12 | 16 | 0 | 4 | | |
| | $C_j - Z_j$ | 2 | 0 | 0 | -4 | | |

| $C_B$ | $C_j$ | 14 | 16 | 0 | 0 | Sol | Ratio |
|---|---|---|---|---|---|---|---|
| | B.V | $x_1$ | $x_2$ | $S_1$ | $S_2$ | | |
| 14 | $x_1$ | 1 | 0 | 4/7 | -3/7 | 12/7 | |
| 16 | $x_2$ | 0 | 1 | -3/7 | 4/7 | 12/7 | |
| | $Z_j$ | 14 | 16 | 2 | 3 | | |
| | $C_j - Z_j$ | 0 | 0 | -2 | -3 | | |

$x_1 = \frac{12}{7}$ , $x_2 = \frac{12}{7}$

$Max\left(\frac{12}{7}, \frac{12}{7}\right)$ = I'm considering $x_2$ is the source row.

$$\frac{-3}{7} = -z + \frac{11}{7}$$

$$\frac{12}{7} = x_2 - 2s_1 + \frac{11}{7} s_1 + \frac{4}{7} s_2$$

fractional cut constraint

$$\frac{11}{7} s_1 + \frac{4}{7} s_2 \geq \frac{12}{7}$$

$$-\frac{11}{7} s_1 - \frac{4}{7} s_2 \leq -\frac{12}{7}$$

$$-\frac{11}{7} s_1 - \frac{4}{7} s_2 + G_1 = \frac{-12}{7}$$

| $C_B$ | $C_j$ | 14 | 16 | 0 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|---|
| | B.V | $x_1$ | $x_2$ | S1 | S2 | G1 | Sol | Ratio |
| 14 | $x_1$ | 1 | 0 | $-4/7$ | $-3/7$ | 0 | $12/7$ | 3 |
| 16 | $x_2$ | 0 | 1 | $-3/7$ | $4/7$ | 0 | $12/7$ | $-4$ |
| 0 | G1 | 0 | 0 | $-11/7$ | $-11/7$ | 1 | $12/7$ | |
| | $Z_j$ | 14 | 16 | 2 | 3 | 0 | | |
| | $C_j - Z_j$ | 0 | 0 | $-2$ | $-3$ | 0 | | |

| $C_B$ | $C_j$ | 14 | 16 | 0 | 0 | 0 | |
|---|---|---|---|---|---|---|---|
| | B.V | $x_1$ | $x_2$ | S1 | S2 | G1 | Sol |
| 14 | $x_1$ | 1 | 0 | 0 | $-\frac{7}{11}$ | $\frac{21}{11}$ | 2 |
| 16 | $x_2$ | 0 | 0 | 0 | $\frac{4}{11}$ | $\frac{7}{11}$ | 1 |
| 0 | G1 | 0 | 0 | 1 | $9/11$ | $-7/11$ | $12/11$ |

Max. $x_1 = 2,\ x_2 = 1$

$14x_1 + 16x_2$

$28 + 16 = 44$

**For Evaluator's Use only**

**Evaluators Comments**

**Evaluator's Observation**

**Marks Secured** ....49.... **out of 50**

**Full Name of the Evaluator:**

**Signature of the Evaluator:**

**Date of Evaluation:**