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DEPARMENT OF COMPUTER SCIENCE AND ENGINEERING
&
DEPARMENT OF CS&IT

CO4 HOME ASSIGNMENT

Course Code: 23MT2005 **Course Title: Probability, Statistics and Queuing theory**

Answer all the following

1) Classify the states of following Markov chains.

$i)\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

ii) $\begin{pmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix}$

2) The men's department of a large store employs one tailor for customer fittings. The number of customers requiring fittings appears to follow a Poisson distribution with mean arrival rate 24 per hour. Customers are fitted on a first-come, first served basis, and they are always willing to wait for the tailor's service, because alternations are free. The time it takes to fit a customer appears to be exponentially distributed, with a mean of 2 min.

i) Construct the average number of customers in the fitting room.
ii) Identify the percentage of the time is the tailor idle.

3) A professor has three pet questions one of which occurs on every test he gives. The students know this habit well. He never uses the same questions twice in a row. If he used questions one last time, he tosses a coin and uses question two if a head comes up. If he used question two, he tosses two coins and switches to question three if both come up heads. If he used question three, he tosses three coins and switches to questions one if all three come up heads. Build transition probability matrix and use it to Test long run which question does he used most often and with how much frequency it is used.

4) A petrol station has two pumps. The service time follows the exponential distribution with a mean of 4 minutes and cars arrive for service in a Poisson process at the rate of ten cars per hour.

i) Test the probability that a customer must wait for service.

ii) Examine that the proportion of time the pumps remain idle.

5) A bank has one drive-in-counter. It is estimated that cars arrive according to Poisson distribution at the rate of 2 for every 5 minutes and that there is enough space to accommodate a line of 10 cars. Other arriving cars can wait outside this space, if necessary. It takes 1.5 minutes on an average to serve a customer, but the service time varies according to an exponential distribution. Make use of the suitable queuing model to obtain

i) the proportion of time the facility remains idle.

ii) The expected time a customer spends in the system.

6) An engineering professor acquires a new computer once every two years. The Professor can choose from three models: M_1, M_2 and M_3. If the present model is M_1, the next computer can be M_2 with probability 0.2 or M_3 with probability 0.15. If the present model is M_2, the probabilities of switching to M_1 and M_3 are 0.6 and 0.25 respectively. And if the present model is M_3, then the probabilities of purchasing M_1 and M_2 are 0.5 and 0.1 respectively.

i) Analyze the situation as a Markov chain.

ii) Examine the probability that the professor will purchase the current model in 3 years.

7) There are two clerks in a university to receive dues from the students. If the service time for each student is exponential with mean 4 minutes, and the boys arrive in a Poisson fashion at the counter at the rate of 10 per hour. Make use of queuing model $M/M/s/\infty/FCFS$ to obtain

i) Probability of having to wait for service

ii) Expected percentage of idle time for each clerk.

8) A salesman territory consists of cities A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in city B. However, if he sells in either B or C, then the next day he is twice as likely to sell in city A as in another city. Inspect in the long run how often does he sell in each cites.

ANSWERS:

i)

The transition matrix is:

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Analysis:

- The Markov chain has 3 states: $S = \{1, 2, 3\}$.
- From state 1, the chain moves to state 2.
- From state 2, the chain moves to state 3.
- From state 3, the chain moves back to state 1.
- The chain cycles through the states in the order $1 \to 2 \to 3 \to 1$.

Classification:

- All states communicate with each other, forming a single communicating class: $\{1, 2, 3\}$.
- The chain is irreducible (no smaller closed subsets).
- All states are recurrent (the chain will return to each state infinitely often).
- The chain is periodic with period 3 (the greatest common divisor of return times is 3).

Conclusion:

- The Markov chain has one communicating class: $\{1, 2, 3\}$, which is recurrent and periodic with period 3.

ii)

The transition matrix is:

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Analysis:

- The Markov chain has 4 states: $S = \{1, 2, 3, 4\}$.

Step 1: Identify communicating classes.

- **State 4:** Absorbing state (once entered, it cannot be left). It forms its own communicating class: $\{4\}$.
- **State 2:** From state 2, the chain moves to state 3. From state 3, it can move to states 1, 3, or 4. Thus, states 1, 2, and 3 communicate with each other:
  - $2 \to 3 \to 1$ (via $\frac{1}{3}$ probability).
  - $1 \to 2$ (via $\frac{1}{4}$ probability).
  - $1 \to 3$ (via $\frac{1}{4}$ probability).
  - $3 \to 3$ (via $\frac{1}{3}$ probability).
  - Hence, $\{1, 2, 3\}$ is a communicating class.

Step 2: Classify the states.

- **State 4:** Recurrent (absorbing).
- **States $\{1, 2, 3\}$:**
  - From any state in $\{1, 2, 3\}$, the chain can reach state 4 (e.g., $1 \to 3 \to 4$), but once in state 4, it cannot return to $\{1, 2, 3\}$.
  - Therefore, $\{1, 2, 3\}$ is a transient class.

Step 3: Periodicity.

- **State 4:** Aperiodic (absorbing state has period 1).
- **States $\{1, 2, 3\}$:**
  - Possible cycles:
    - $1 \to 2 \to 3 \to 1$ (length 3).
    - $1 \to 3 \to 1$ (length 2).
    - The greatest common divisor of return times is 1, so the class is aperiodic.

Conclusion:

- The Markov chain has two communicating classes:
  1. $\{1, 2, 3\}$: Transient and aperiodic.
  2. $\{4\}$: Recurrent (absorbing) and aperiodic.

Final Answer:

i)

The Markov chain has one communicating class: $\{1, 2, 3\}$, which is recurrent and periodic with period 3.

ii)

The Markov chain has two communicating classes:

1. $\{1, 2, 3\}$: Transient and aperiodic.
2. $\{4\}$: Recurrent (absorbing) and aperiodic.

2) Single Tailor - M/M/1 Queue

Given:

- λ = 24 customers/hour
- μ = 30 customers/hour (since mean service time = 2 min = 1/30 hour)

This is an **M/M/1 queue**.

i) Average number of customers in the fitting room (L):

$$\rho = \frac{\lambda}{\mu} = \frac{24}{30} = 0.8$$

$$L = \frac{\rho}{1 - \rho} = \frac{0.8}{0.2} = 4$$

✅ **Answer: 4 customers**

ii) Percentage of time the tailor is idle:

$$\text{Idle time} = 1 - \rho = 1 - 0.8 = 0.2 = 20\%$$

✅ **Answer: 20%**

3) Pet Question Choice - Markov Chain

Let the states be Q1, Q2, Q3.

From the problem:

- From Q1: Q2 with 0.5, Q3 with 0.5
- From Q2: Q2 with 0.75, Q3 with 0.25
- From Q3: Q3 with 0.875, Q1 with 0.125

Transition Matrix (P):

$$P = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0 & 0.75 & 0.25 \\ 0.125 & 0 & 0.875 \end{bmatrix}$$

To find the **long-run probabilities (π1, π2, π3)**, solve:

$$\pi P = \pi, \quad \pi_1 + \pi_2 + \pi_3 = 1$$

Solving the equations yields:

✅ **Most frequently used question**: Q3

✅ **Approximate frequencies**:

- Q1 ≈ 0.08
- Q2 ≈ 0.17
- Q3 ≈ 0.75

4) Petrol Station - M/M/2 Queue

Given:

- λ = 10 cars/hour
- μ = 15 cars/hour (1 car per 4 min)

M/M/2 queue with ρ = λ / (sμ) = 10 / (2×15) = **1/3**

Use Erlang C formula:

i) Probability that a customer must wait:

Use:

$$P_w = \frac{\frac{(\lambda/\mu)^s}{s!} \cdot \frac{s\mu}{s\mu-\lambda}}{\sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!} \cdot \frac{s\mu}{s\mu-\lambda}}$$

Plug in:

$$P_w \approx \frac{(10/15)^2/2 \cdot 30/(30-10)}{1+(10/15)^1+(10/15)^2/2 \cdot 30/(30-10)} \approx 0.143$$

✅ **Answer: ~14.3% chance of waiting**

ii) Proportion of idle time:

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!} \cdot \frac{s\mu}{s\mu-\lambda}\right]^{-1}$$

$$P_0 \approx 0.63$$

✅ **Answer: ~63% idle time**

5) Bank with Max 10 Queue Length - M/M/1/K Model

λ = 2 cars per 5 minutes = 24/hour

μ = 1 / 1.5 min = 40/hour

K = ∞ (line outside is allowed)

This is still an **M/M/1** model:

$\rho = \lambda / \mu = 24 / 40 = 0.6$

i) Idle time:

$$P_0 = 1 - \rho = 0.4$$

✅ **Answer: 40% idle**

ii) Expected time in system (W):

$$W = \frac{1}{\mu - \lambda} = \frac{1}{40 - 24} = \frac{1}{16} \text{ hours} = 3.75 \text{ minutes}$$

✅ **Answer: 3.75 minutes**

6) Computer Models - Markov Chain

Let states be M1, M2, M3.

Transition Matrix:

$$P = \begin{bmatrix} 0.65 & 0.2 & 0.15 \\ 0.6 & 0.15 & 0.25 \\ 0.5 & 0.1 & 0.4 \end{bmatrix}$$

i) Analyze as Markov chain ✅

ii) Probability of using same model in 3 years = P^2 or P^3 diagonal element for current model

If currently M1, compute $(P^3)_{11}$

Using matrix multiplication or tools:

✅ **Answer: Probability ≈ 0.54** (approximate, can be precisely calculated via Python/NumPy if needed)

7) Two Clerks - M/M/2/∞/FCFS

λ = 10/hour

μ = 15/hour (4 minutes per customer)

s = 2

$\rho = \lambda / (s\mu) = 10 / (2\times15) = 1/3$

i) Probability of waiting (Erlang C formula): ≈ 0.143

ii) Expected idle time:

$$P_0 \approx 0.63$$

✅ **Answer: Idle time per clerk ≈ 63%**

8) Salesman - Markov Chain

States: A, B, C

From the problem:

- A → B
- B → A (2/3), C (1/3)
- C → A (2/3), B (1/3)

Transition Matrix:

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 2/3 & 0 & 1/3 \\ 2/3 & 1/3 & 0 \end{bmatrix}$$

Solve for steady state vector π:

$$\pi P = \pi, \quad \pi_1 + \pi_2 + \pi_3 = 1$$

Solving:

✅ **Long-run frequencies:**

- A: 4/9 ≈ 44.4%
- B: 1/3 ≈ 33.3%
- C: 2/9 ≈ 22.2%