Tutorial 2

Simplex method and Principle of Duality

Date of the Session: ....

Learning outcomes:

- Understanding the key terms: feasible solution, feasible region and optimal solution.
- Understanding the limitations of graphical method and introduce the Simplex algorithm.
- Understand that an LPP consists of more number of constraints as compared to number of decision variables.
- Understanding the computational procedure can be considerably reduced by converting the LPP into a form called as DUAL and then solving it.

2.1 PRE-TUTORIAL

1. Which type of L.P.P. can be solved using Simplex method?

A) i) Linear

ii) Decision Variables

iii) Equality Constraints

iv) Feasible Starting Point

v) Bounded Feasible Region

vi) Non-negative Constraints

2. What do you mean by feasible region, feasible solution and optimal solution?

i) Feasible Region means also known as the feasible set; is the set of all possible Combinations of values for decision variables that satisfy all of problems constraints.

ii) Feasible Solution means is a specific ~~set of~~ values for decision variables that satisfies all the constraints of the linear programming problem, it is a point.

iii) Optimal Solution means the best possible solution within the feasible region, as determined by the obj function.

3. State the general rules for formulating a dual LPP from its primal?

i) Primal Problem

ii) Dual Variables

iii) Dual PF

iv) Dual Constraints

v) Dual Constraints computed variables

vi) Complete Dual CP problem.

2.2 IN-TUTORIAL

1. Consider the following linear programming problem

$$\begin{aligned} \text{Maximize} \quad & P = 7x + 12y \\ & 2x + 3y \leq 6 \\ & 3x + 7y \leq 12 \end{aligned}$$

Set up the Initial Simplex Tableau and obtain the solution.

Solution:

Let S_1, S_2 are slack variables.

becomes,

$\text{Max } P = 7x + 12y + 0S_1 + 0S_2$

subject to constraints:

$2x + 3y + S_1 + 0S_2 = 6$

$3x + 7y + 0S_1 + S_2 = 12$

where

$x_1, y_1, S_1, S_2 \geq 0$.

Initial Simplex Table:

		C_j	7	12	0	0	
C_B	B_v	X_B	x	y	S1	S2	Ratio
0	S1	6	2	3	1	0	6/3 = 2
0	S2	12	3	7	0	1	12/7 = 1.2
		Z_j	0	0	0	0	
		$C_j - Z_j$	−7	−12	0	0	
	S1	8/7	5/7	0	1	−3/7	
	x_2	12/7	5/7	1	0	1/7	
A_j			17/7	0	0	12/7	

$x_1 = 0$

$x_1 = 12/7 = 1.7$

$S1 = 6/7$

$S2 = 0$

$Z = \cancel{7x} + 2y$

$= 7(0) + 2 \times \frac{12}{7}$

$= \frac{24}{7}$

$=$

2. Find the dual problem for the given LPP model

$$\begin{aligned} \text{Minimize } & C = 5x_1 + 2x_2 \\ \text{Subject to } & x_1 + 3x_2 \geq 15, \\ & 2x_1 + x_2 \geq 20, \\ & x_1, x_2 \geq 0 \end{aligned}$$

Solution:

$$A = \begin{bmatrix} 1 & 5 \\ 2 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$$

The Dual Problem:

$$\text{Max } z = 15y_1 + 20y_2$$

$$y_1 + 2y_2 \leq 5$$

$$3y_1 + y_2 \leq 2$$

$$\text{where } y_1, y_2 \geq 0$$

3. State the dual for the following LPP and hence solve LPP.

$$\begin{aligned} \text{Minimize : } & C = 21x_1 + 50x_2 \\ \text{Subject to : } & 2x_1 + 5x_2 \geq 12, \\ & 3x_1 + 7x_2 \geq 17, \\ & x_1, x_2 \geq 0 \end{aligned}$$

Solution:

$$A = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix} \qquad A^T = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$$

The Dual Function:

$$\text{Max } z = 12y_1 + 17y_2$$

$$2y_1 + 3y_2 \leq 21$$

$$5y_1 + 7y_2 \leq 50$$

$$\text{where } y_1, y_2 \geq 0$$

Bv	C_B	X_B	x_1	x_2	S1	S2	Ratio
S1	0	21	2	3	1	0	21/3
S2	0	50	5	7	0	1	50/7
x_2	0	7	-12 2/3 1/3	-17 1 0	0 1/3 -7/3	0 0 1	10 5 1/3 = 3
	A_j		-2/3	0	17/3	0	
x_2	0	5	0	1	5/3	-2	
x	0	3	1	0	-7	3	
			0	0	1	2	

$x_1 = 3$

$x = 5$

$Z = 12y_1 + 17y_2$

$= 12(3) + 17(5)$

$= 36 + 85$

$= 121$

1. A XYZ company is hired by a retailer to transport goods from its store rooms in A and B to its outlet stores in C and D. The XYZ company is contracted to deliver 30 vehicles each month to deliver goods. The company determines that it will need to send at least 12 of the vehicles to the 'C' location and at least 13 vehicles to the 'D' location. At least 15 vehicles can come from the A storeroom and at least 20 vehicles can come from the 'B'. The truck company wants to minimize the number of miles placed on its trucks. How many trucks should the send out from each location and to which outlets should they send them?

	A	B
C	24 ml	31 ml
D	20 ml	38 ml

Formulate its dual and solve the LPP.

Solution:

w is dual variable of Room A.

x Room B

y Room C

z Room D

of,

$$\text{Min } z = 15w + 20x + 10y + 13z$$

Constraints:

$$w + y \geq 2$$

$$x + z \geq 20$$

where $w, x, y, z \geq 0$

Let

x_{AC}: A to C = 24

x_{AD}: A to D = 20

x_{BC}: B to C = 31

x_{BD}: B to D = 38

Primal:

$$\text{Min } z = 24x_{AC} + 20x_{AD} + 31x_{BC} + 38x_{BD}$$

$$x_{AC} + x_{BC} \geq 12$$

$$x_{AD} + x_{BD} \geq 13$$

$$x_{AC} + x_{AD} \geq 15$$

$$x_{BC} + x_{BD} \geq 20$$

$$x_{AC} + x_{AD} + x_{BC} + x_{BD} \geq 30$$

$$x_{AC}, x_{AD}, x_{BC}, x_{BD} \geq 0$$

Dual:

$$\text{Max } z = 12y_{AC} + 13y_{AD} + 15y_{BC} + 20y_{BD}$$

Let $U_1 = x_{AC} + x_{BC}$

$U_2 = x_{AD} + x_{BD}$

$V_1 = x_{AC} + x_{AD}$

$V_2 = x_{BC} + x_{BD}$

$W = x_{AC} + x_{AD} + x_{BC} + x_{BD}$

$\max z = 12u_1 + 13u_2 + 15v_1 + 20v_2 - 30w$

$u_1 + v_1 \leq 24$

$u_1 + v_2 \leq 31$

$u_2 + v_1 \leq 20$

$u_2 + v_2 \leq 38$

$w, u_1, u_2, v_1, v_2 \geq 0$

Graphical method is not applicable

Simplex method is applicable

	C_j	12	13	15	20	-30		
B.V	CV	u_1	u_2	v_1	v_2	w	Sol^n	Ratio
0	S1	1	0	1	0	0	24	24
0	S2	1	0	0	1	0	31	31
0	S3	0	1	1	0	0	20	20
0	S4	0	1	0	1	0	38	38
	Z_j	0	0	0	0	0		
	$C_j - Z_j$	12	13	15	20	30		

Entering variable: W

Leaving variable: S3

2.3 POST-TUTORIAL

1. Solve LPP using the simplex method

$$
\begin{aligned}
&\text{Maximize } Z = 3x_1 + 5x_2 \\
&\text{Subject to} \\
&\quad 3x_1 + 2x_2 \leq 18 \\
&\quad x_1 \leq 4 \\
&\quad 2x_2 \leq 12 \\
&\quad x_1, x_2 \geq 0
\end{aligned}
$$

Solution:

$Z = 3x_1 + 5x_2 + 0S_1 + 0S_2 + 0S_3$

Subject to :

$3x_1 + 2x_2 + S1 = 18$

$x_1 + S2 = 4$

$2x_2 + S_3 = 12$

$x_1, x_2, S1, S2, S3 \geq 0$

Bv	CB	XB	x1	x2	S1	S2	S3	θ
S1	0	18	3	2	1	0	0	18/2 = 9
S2	0	14	1	0	0	1	0	12/2 = 6
S3	0	12	0	2	0	0	1	12/2 = 6
	Δ_j	-3	-5	0	0	0		
S1	0	6	5	0	1	0	-1	
S2	0	7	1	0	0	1	0	
S3	0	6	0	1	0	0	0	
	Δ_j		3	0	0	0	5/2	

$x_1 = 0$

$x_2 = 6$

$S_1 = 6$

$S_2 = 0$

$S_3 = 0$

$Z = 0$

2. Find its dual and obtain the optimal solution for minimization problem

$$\begin{aligned} \text{Minimize} \quad & C = 16x_1 + 8x_2 + 4x_3 \\ \text{Subject to} \quad & 3x_1 + 2x_2 + 2x_3 \geq 16 \\ & 4x_1 + 3x_2 + x_3 \geq 14 \\ & 5x_1 + 3x_2 + x_3 \geq 12 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Solution

Primal Problem:

$\text{Min } C = 16x_1 + 8x_2 + 4x_3$

Subject to:

$3x_1 + 2x_2 + 2x_3 \geq 16$

$4x_1 + 3x_2 + x_3 \geq 14$

$5x_1 + 3x_2 + x_3 \geq 12$

$x_1, x_2, x_3 \geq 0.$

Dual Problem:

$\text{Max } C = 16y_1 + 14y_2 + 12y_3$

Subject to

$3y_1 + 4y_2 + 5y_3 \leq 16$

$2y_1 + 3y_2 + 3y_3 \leq 8$

$2y_1 + y_2 + y_3 \leq 4$

$x_1, x_2, x_3 \geq 0.$

mplex Method:

	C_j	16	8	4	0	0	0		
BV	BV	x_1	x_2	x_3	S1	S2	S3	Soln	Ratio
0	S1	3	2	2	1	0	0	16	16/3
0	S2	4	3	1	0	1	0	14	14/4
0	S3	5	3	1	0	0	1	12	12/5
	Z_j	0	0	0	0	0	0	0	
	C_j-Z_j	16	8	4	0	0	0		

	C_j	16	8	4	0	0	0		
BV	BV	x_1	x_2	x_3	S1	S2	S3	Soln	Ratio
0	S1	0	1/5	7/5	1	0	-3/5	44/5	6.2
0	S2	0	3/5	1/5	0	1	-4/5	22/5	22
6	x_1	1	3/5	1/5	0	0	1/5	12/5	12
	Z_j	16	48/5	16/5	0	0	16/5	192/5	
	C_j-Z_j	0	-1.6	0.8	0	0	-16/5		

	C_j	16	8	4	0	0	0		
CBV	B.V	x_1	x_2	x_3	S1	S2	S3	Soln	Ratio
4	x_3	0	0.1	1	5/7	0	0.4	6.2	
0	S2	0	0.01	0	0.14	1	0.1	1.8	
16	x_1	1	0.4	1	0.14	0	0.5	0.4	
	Z_j	16	8	20	5	0	2.4	31.2	
	C_j-Z_j	0	0	-16	-5	0	-2.4		

∴ The values are

$x_3 : 6.2$

$s_2 : 1.8$

$x_1 : 0.4$

$C_j : 31.2$

3. A producer of Healthy food makes two important and secret ingredients that goes into their human food, named as a Healthy Man and Common Man. Each kg of Healthy Man contains 300 g of vitamins, 400 g of protein, and 100 g of carbs. Each kg of common man contains 100 g of vitamins, 300 g of protein, and 200 g of carbs. Guidelines for minimum nutritional that require a mixture made from these ingredients contain at least 900 g of vitamins, 2400 g of protein, and 800 g of carbs. Healthy Man costs \$2 per kg to produce and Common Man costs \$1.25 per kg to produce. Find the number of kgs of each ingredient that should be produced in order to minimize cost. Obtain its Dual and solve.

Solution:

Primal:

$$\text{Min } C = 2x_1 + 1.25x_2$$

Subject to:

$$300x_1 + 100x_2 \geq 900$$

$$400x_1 + 300x_2 \geq 2200$$

$$100x_1 + 200x_2 \geq 800$$

where $x_1, x_2 \geq 0$.

Dual:

$$\text{Max } C = 900y_1 + 2200y_2 + 800y_3$$

Subject to:

$$300y_1 + 400y_2 + 100y_3 \leq 2$$

$$100y_1 + 300y_2 + 200y_3 \leq 1.25$$

where $y_1, y_2, y_3 \geq 0$.

Simplex Method:

	C_j	300	2400	300	0	0		
CBV	BV	y_1	y_2	y_3	S1	S2	Sol^n	Ratio
0	S1	300	400	100	1	0	2	2/400
0	S2	100	300	200	0	1	1.25	1.25/300
	Z_j	0	0	0	0	0	0	
	$C_j - Z_j$	300	2400	300	0	0		

	C_j	300	2400	300	0	0		
CBV	B.V	Y_1	Y_2	y_3	S1	S2	Sol^n	Ratio
0	S1	500/300	0	500/300	1	-1/30	0.4	
2400	y_2	1/3	1	2/3	0	1/3	0.004	
	Z_j	2400/3	2400	4800/3	0	2400/3	9.6	
	$C_j - Z_j$	-500	0	-1300	0	-2400/3		

∴ The values of S1 : 0.4 & y_2 : 0.004 & Z_j : 9.6

For Evaluator's Use only

Evaluators Comments	Evaluator's Observation
	Marks Secured ....48.... out of 50
	Full Name of the Evaluator:
	Signature of the Evaluator:
	Date of Evaluation: