Course Title: **Mathematical programming (2-2-0-0)**

Topic :

Duality in LPP

CO-1

Session - 4

AIM

To familiarize students with the concept of duality in linear programming

INSTRUCTIONAL OBJECTIVES

This session is designed to:

1. Illustrate in detail of duality in linear programming
2. Demonstrate how to formulate the duality in linear programming

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LEARNING OUTCOMES

At the end of this unit, you should be able to:

1. Describe the ways to construct the duality in linear programming
2. Solve the formulation of duality in linear programming

DUALITY IN LINEAR PROGRAMMING

- For every LP problem, there is a related unique LP problem involving the same data which also describes the original problem
- The given original problem is called the primal problem
- If the problem is rewritten by transposing the rows and columns of the algebraic statement of the problem, then we get another LP problem which we call as Dual of the problem.

DUALITY IN LINEAR PROGRAMMING

- A solution to the dual problem may be found in a manner similar to that used for primal problem
- The optimal solution of the dual problem gives complete information about optimal solution of the primal problem and vice-versa

SOME INTERESTING FEATURES OF DUALITY

- If the primal problem contains a large number of rows (constraints) and smaller number of columns (variables), then the computational procedure can be considerably reduced by converting it into dual and then solving it. Hence, it offers an advantage in many applications
- It gives additional information as to how the optimal solution changes as a result of the changes in the coefficients and the formulation of the problem.

SOME INTERESTING FEATURES (CONT…)

- It helps managers answer questions about alternative courses of actions and their relative values
- Calculation of the dual checks the accuracy of the primal solution
- Duality in linear programming shows that each linear program is equivalent to Two-person-Zero sum game. This indicates that fairly close relationship exist between linear programming and the theory of games
- Duality is not restricted to linear programming problem only but finds applications in many domains like Economics, physics and other fields.

SOME INTERESTING FEATURES (CONT…)

- Economics interpretations of the dual helps the management in making future decisions
- Duality is used to solve LP problems in which the initial solution is infeasible.

RULES FOR CONVERTING THE PRIMAL PROBLEM INTO A DUAL PROBLEM

- If the primal contains n-variables and m-constraints, the dual will contain m variables and n constraints
- The maximization problem in the primal becomes the minimization problem in the dual and vice-versa
- The maximization problem has ≤ constraints while the minimization problem has ≥ constraints
- Constraints of ≤ type in the primal become ≥ type in the dual and vice-versa
- The coefficient matrix of the constraints of the dual is the transpose of the primal
- A new set of variables appear in the dual.

RULES FOR CONVERTING THE PRIMAL PROBLEM INTO A DUAL PROBLEM

- The constants $c_1, c_2, c_3 \ldots, c_n$ in the objective function of the primal appear in the constraints of the dual
- The constants $b_1, b_2, b_3 \ldots, b_n$ in the constraints of the primal appear in the objective function of the dual
- The variables in both problems are non-negative.

PROBLEM

Construct the dual to the primal problem

Maximize $Z=3x_1+5x_2$

Subject to $2x_1+6x_2\leq 50$

$3x_1+2x_2\leq 35$

$5x_1-3x_2\leq 10$

$x_2\leq 20$ where $x_1\geq 0$, $x_2\geq 0$

- The dual of the given LPP is

$W = 50y_1 + 35y_2 + 10y_3 + 20y_4$

Subject to $2y_1 + 3y_2 + 5y_3 \geq 3$

$6y_1 + 2y_2 - 3y_3 + y_4 \geq 5$

where y_1, y_2, y_3 and $y_4 \geq 0$

As the dual problem has lesser number of constraints than the primal, it requires lesser work and effort to solve it.

Note: This follows from the fact that, the computational difficulty in the LPP is mainly associated with the number of constraints rather than number of variables.

- Construct the dual of the problem

Minimize $Z=3x_1-2x_2+4x_3$ subject to the constraints

$3x_1+5x_2+4x_3 \geq 7$

$6x_1+x_2+3x_3 \geq 4$

$7x_1-2x_2-x_3 \leq 10$

$x_1-2x_2+5x_3 \geq 3$

$4x_1+7x_2-2x_3 \geq 2$

$x_1, x_2, x_3 \geq 0$

- Solution

As the given problem is minimization, All constraints should be of ≥ type.

Multiplying the third constraint by -1 on both sides, we get

$$-7x1+2x2+x3 \geq -10$$

So, the dual of the given problem will be

Maximize $W=7y1+4y2-10y3+3y4+2y3$

Subject to $3y1+6y2-7y3+y4+4y5 \leq 3$

$5y1+y2+2y3-2y4+7y5 \leq -2$

$4y1+3y2+y3+5y4-2y5 \leq 4$ where

$y1,y2,y3,y4$ and $y5 \geq 0$

ASSIGNMENT

- Construct the dual of the problem
- Maximize $z = 3x_1 + 17x_2 + 9x_3$
- Subject to $x_1 - x_2 + x_3 \geq 3$

$$-3x_1 + 2x_3 \leq 1$$

where $x_1, x_2, x_3 \geq 0$

REFERENCES

Textbook:

1. *Introduction to Mathematical Programming* by Russell C. Walker, published by Pearson Custom Publishing, 2006

2. S. P. Bradley, A. C. Hax, and T. L. Magnanti, *Applied Mathematical Programming*, Addison-Wesley Publishing Company, 1977

3. Lenstra, Rinnooy Kan, & Schrijver (eds.), *History of Mathematical Programming: A Collection of Personal Reminiscences*, Elsevier, 1991.

Web Resources:

1. https://nptel.ac.in/courses/110105096
2. https://www.coursera.org/learn/operations-research-modeling
3. https://www.semanticscholar.org/paper/Applied-Mathematical-Programming-Bradley-Hax/8a4ee083b23505df221410e6a2b41fc56fa250a6

THANK YOU

(DEEMED TO BE UNIVERSITY)

Team – Mathematical Progrmming