"Tarea 4. Integral impropia"

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Grupo: 23.

$$\int_0^1 \frac{x-1}{\sqrt{x}}\,dx \Rightarrow \lim_{\varepsilon\to 0}\int_{0+\varepsilon}^{1}\frac{x-1}{\sqrt{x}}\,dx \Rightarrow \lim_{\varepsilon\to 0}\int_{\varepsilon}^{1}\frac{x-1}{\sqrt{x}}\,dx$$

Calculamos la integral indefinida:

$$\int \frac{x-1}{\sqrt{x}}\,dx = \int \frac{x-1}{x^{1/2}}\,dx = \int \frac{x}{x^{1/2}}\,dx - \int \frac{1}{x^{1/2}}\,dx$$

$$\left(\int x^n dx = \frac{x^{n+1}}{n+1}\right) = \int x^{1/2}\,dx - \int \frac{1}{x^{1/2}}\,dx \quad \left(\int \frac{1}{x^n}\,dx = -\frac{1}{(n-1)x^{n-1}}\right)$$

$$= \frac{x^{3/2}}{\frac{3}{2}} - \left(-\frac{1}{(-\frac{1}{2})x^{-1/2}}\right)$$

$$= \frac{2x\sqrt{x}}{3} - \left(-\frac{1}{-\frac{1}{2\sqrt{x}}}\right) = \frac{2x\sqrt{x}}{3} - 2\sqrt{x}$$

Calculamos la integral definida:

$$\left(\frac{2x\sqrt{x}}{3} - 2\sqrt{x}\right)\Big|_{\varepsilon}^{1} = \left(\frac{2(1)\sqrt{1}}{3} - 2\sqrt{1}\right) - \left(\frac{2(\varepsilon)(\sqrt{\varepsilon})}{3} - 2\sqrt{\varepsilon}\right)$$

$$= -\frac{4}{3} - \frac{2\varepsilon\sqrt{\varepsilon}}{3} - 2\sqrt{\varepsilon}$$

$$= -\frac{4+2\varepsilon\sqrt{\varepsilon}}{3} - 2\sqrt{\varepsilon}$$

Calculamos el límite:

$$\lim_{\varepsilon\to 0}\left(-\frac{4+2\varepsilon\sqrt{\varepsilon}}{3} - 2\sqrt{\varepsilon}\right) = -\frac{4+2(0)\sqrt{0}}{3} - 2\sqrt{0} = -\frac{4}{3}$$

Resultado:

$$\int_0^1 \frac{x-1}{\sqrt{x}}\,dx = -\frac{4}{3}$$