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## UNIT 15    INTEREST RATE RISK

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### 15.0 OBJECTIVES

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After going through the unit, you will be able to:

- explain the concept of interest-rate risk;
- describe the pricing, returns and yields related to a fixed-income securities;
- define the concept of duration and explain its link to the maturity of the asset; and
- discuss the strategies of risk management.

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### 15.1 INTRODUCTION

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In this unit, we discuss some risks arising from changes in the in the interest rates of fixed income securities. Fixed income securities are usually securities that are issued by agencies or organisations that are going in for debt. To proceed with the discussion, we continue where we had left off in Unit 4 in Block 1 about the time value of money and cash flows. We look at some features and characteristics of bonds; however, bonds are but one example. Our discussion in this unit pertains to individuals and organisations that hold fixed-income securities and face risks. These may be individuals who hold bonds, these may be banks as they manage their own assets which include loans that they give out, these may be firms and so on. We have to understand the types of risks that may arise. In the present unit, we discuss interest rate risks. The next unit deals primarily with credit risks.

We begin our discussion in this unit with the features of bonds and their pricing. We develop some concepts related to the main features of bonds and see what kind of risks bondholders can face. We then go on to explain the concept of market risk, or as it is also called, interest rate risk. We see what it means and what its implications are. Following that, the unit discusses important issues regarding risk strategies and in this context discusses repricing models and what is called GAP analysis. The unit subsequently goes on to deal with aspects of measurement of risk and the attempt to

quantify it into a single index. The tool to do this is what is called duration analysis. Finally, the unit discusses the very important issue of Value at Risk, which is a way of assessing risks by looking at the probability of losses that securities may make.

## 15.2 MARKET RISK

We focus in this unit on debt instruments and hence on the interest rates associated with these. We largely concentrate on debt instruments about which we make the assumption that these are free of risk of default by the issuer. We also assume away complications arising from taxation. For the moment we assume also that there are no embedded option features associated with the bond. Before proceeding further towards the pricing of the bonds let us familiarise ourselves with some basic concepts.

The term to maturity or maturity of the debt contract is the number of years during which the issuer (borrower) has promised to meet the conditions of the debt instrument. The amount that will be repaid by the borrower is called the principal. A bond is a special type of debt contract, and the amount paid at maturity is called the **par value, maturity value or face value**. If the price at which a bond is sold is exactly the same as its nominal value, we say the bond is **sold at par**. If the price is higher than the nominal value, we say the bond is sold **above par**, and if it is sold below the nominal value then we say it is sold **below par**. The financial contract between the issuer of the bonds and the purchaser of the bond is called the **bond indenture**.

A debt contract's **coupon** is the periodic interest payment that is paid to the holders of the debt instrument. The name comes from the fact that in olden there would actually be coupons handed out to show the payments. The **coupon rate**, multiplied by the unpaid outstanding principal gives us the rupee amount of the coupon payment. There are certain debt instruments for which no periodic coupon interest is paid over the life of contract. Instead, the principal along with the interest is paid at the maturity date. Such debt instruments are called **zero-coupon instruments**.

Now a discussion of bond valuation is in order. The value of a bond is equal to the present value of the cash flows expected from it. For simplicity, let us assume that the coupon interest rate stays constant for the term of the bond. Let us also assume that coupon payments are paid only once a year, every year on an annual basis for the life of the bond and that the bond will be redeemed at par on maturity. In this case, the value of the bond of the bond is given by:

$$P = \sum_{t=1}^T \frac{C}{(1+r)^t} + \frac{M}{(1+r)^T}$$

where

$P$  = value (in rupees)

$T$  = number of years

$C$  = annual coupon payment (in rupees)

$r$  = periodic required return

$M$  = maturity value

$t$  = time period when the payment is received

There are bonds that pay interest semi-annually, that is, twice a year. In this case, the unit period is six months, rather than a year. In this case, interest payment becomes  $C/2$ . The total time period becomes  $2T$ , and in this case  $2T$  shows the number of *half-yearly* periods, and the discount rate becomes  $r/2$  to get the discount rate applicable to half-yearly periods. With these modifications, the basic bond valuation becomes

$$P = \sum_{t=1}^{2T} \frac{C/2}{(1 + r/2)^t} + \frac{M}{(1 + r/2)^{2T}}$$

Now let us define yield of a bond. A bond's yield is the interest payment implied by the payment structure. The **yield** of a bond is that value of the rate of return (shown by  $r$  in our formulation—in the context of yield, we will later in the unit, on occasion denote it by ' $y$ ') for which the current price of the bond is equal to the present value of the bond's stream of payments (coupon and face value). This yield is also called **yield to maturity**. In the context of investment in general, yield is also known as the **internal rate of return (IRR)**. The IRR of a bond is its yield. A basic property of a bond is that its price varies inversely with yield. This happens because as the required yield increases, the present value of cash flow decreases. There is another type of yield called the current yield, which relates the annual coupon interest to the market price. It is expressed as equal to annual interest/price.

Let us give a formulation for the yield to maturity that will help us in understanding the concepts of maturity and duration that we will come across later in the unit. This is just an extension of the second equation above. Let there be a bond with a face value  $M$  that makes  $n$  coupon payments of  $C/n$  each year, and let there be  $T$  periods remaining. The coupon payments sum to  $C$  in a year. Let  $P$  be the current price of the bond. Then the yield to maturity is the value of  $y$  such that

$$P = \frac{M}{[1 + (y/n)]^T} + \frac{C}{y} \left\{ 1 - \frac{1}{[1 + (y/n)]^T} \right\}$$

This equation must be solved for  $y$  to determine the yield. Usually for  $T > 2$ , it is difficult to solve explicitly for  $y$  and it is solved through trial and error.

Let us explore at this stage the concept of interest rate risk. Even though bonds are called risk-free assets, there are several reasons why bonds might actually involve risk. First of all, debtors may fail to meet the payment obligation embedded in the bond. If the debtor fails to meet such payment we say the debtor has defaulted. Risks of this type are called **default risk** or **credit risk**. Secondly, there may be **inflation risk** if, even though the debtor has met the payment obligation, the general price level has gone up. There may be other types of risk as well. Suppose the creditor (purchaser of the bond) requires money before the maturity period and tries to sell the bond. However, there is no price guarantee before maturity. The creditor can sell the bond, but the price that will prevail is unpredictable. The risk of having to sell at a given time at low prices is called **liquidity risk**. In this unit, however, we do not consider these risks, although the next unit will deal with credit risk and other risks in greater detail. For the present, we are concerned with interest rate risk, and we turn to that now.

An **interest rate risk** is the fluctuation in bond prices that may come about due to the variations in interest rates over time. A rise in interest rates will lower the market price of the bond whereas a fall in interest rates will push the prices up. Interest rate risk is also called market risk, since there is a risk regarding the *price* of bonds. It is measured by the percentage change in the value of the bond in response to a given change in interest rates. It depends on the maturity period of the bond and the coupon interest rate associated with it. It can be shown from the general formula for the current price of a bond that:

- the longer the maturity period, the greater the sensitivity of bond prices to changes in interest rates;
- the larger the coupon payment, the lesser the sensitivity of bond prices to changes in the interest rates

We know that bond prices and yields are inversely related. As interest rates fluctuate, there occur capital losses and gains for bondholders. The reason can be explained with the following illustration. Suppose the market demands a 5.5 per cent yield on a 10-year bond. If the coupon rate is also 5.5 per cent, the bond will trade at par. But what if the market demands a yield of 6 per cent on a 10-year bond? There will be no demand for the 5.5 per cent 10-year bond. So, the price of the 5.5 per cent coupon 10-year bond will decline in value such that it yields 6 per cent. Those who hold bonds, therefore, run the risk of interest rate increase. This risk is called the interest rate risk. Now, interest rate risk is dependent on the maturity of the bond; longer the bond maturity, higher the interest rate risk. So, the 5.5 per cent 10-year bond will decline more to yield 6 per cent compared with a 5.5 per cent five-year bond. When you buy a bond, you receive interest every year and the redemption price on maturity. The bond price is, hence, a function of the present value of these cash flows. When the market demands higher yield, we have to use a higher discount factor to calculate the present value of cash flows. In the case of a 10-year bond, we have to use the higher discount factor for 10 years. For a five-year bond, a higher discount factor has to be used only for five years. But higher discount factor means lower price. That is why longer maturity bonds are more sensitive to increase in interest rates.

The price of a bond as we have seen depends mainly on three things: its coupon rate, its maturity, and the prevailing rate of interest. Hence over time, the price of a bond can change due to any of the following reasons: a change in the level of interest rate in the economy; a change in the price of a bond selling at a price other than par as it moves towards maturity without any change in the required yield; also, though we are not much concerned with it in the present unit, a change in the perceived credit quality of the issuer. The following relations exist between bond prices, interest and yields:

(1) There is an inverse relation between relationship bond prices and yields. (2) An increase in yield causes a proportionately smaller price change than a decrease in yield of the same magnitude. (3) Prices of long-term bonds are more sensitive to interest rate changes than prices of short-term bonds. (4) As maturity increases, interest rate increases, but at a decreasing rate. (5) Prices of low-coupon bonds are more sensitive to interest rate changes than prices of high-coupon bonds. (6) Bond prices are more sensitive to yield changes when the bond is initially selling at a lower yield.

Another significant risk that may arise is what is called **reinvestment risk**.

Reinvestment risk generally arises in context of financial contracts involving more than one cash flow. The yield on such securities will be influenced by the reinvestment opportunities available for cash flows accruing before the maturity of the contract. Thus investment in coupon bonds would involve some degree of reinvestment risk. Zero coupon bonds and single payment loans; on the other hand involve no reinvestment risk. The nature of reinvestment risk is such that risk associated with market value is negatively correlated with risk associated with reinvestment. Finally, we may mention **prepayment risk**, which may arise in those borrowings where the borrower has the option of repaying the loan before the maturity period. In the context of bonds, this is generally referred to as the option to call the bond. What makes prepayment risk problematic for an investor in either mortgages (housing loans) or bonds is that prepayments tend to increase when interest rates are declining.

### Check Your Progress 1

- 1) Define yield to maturity and current yield.

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- 2) Explain the concept of market risk.

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- 3) List the factors that influence the price of bonds.

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## 15.3 RISK MEASUREMENT MODELS

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Generally, a **measure** is an operation for assigning a number to something. A **metric** is our interpretation of the assigned number. When we apply a measure, the number obtained is a **measurement**.

There are many metrics of risk—volatility, duration, etc. We call these **risk metrics**. A measure that supports a risk metric is called a **risk measure**. The value obtained from applying a risk measure is a **risk measurement**.

We have already learnt that prices of bonds with longer maturity are more sensitive to interest rate changes than bonds with shorter maturity period. But this is only a rule of thumb. Maturity by itself is not a complete indicator of interest rate sensitivity. A different measure of time-length called duration does give a direct measure of interest rate sensitivity. We discuss duration as a measure of interest rate sensitivity in this section.

**Duration** is a term used in the analysis of fixed-income instruments. At its most basic level, duration is a measurement of how long in years it takes for the price of a bond to be repaid by its internal cash flows. It is an important measure for investors to consider, as bonds with higher durations (given equal credit, inflation and reinvestment risk) may have greater price volatility than bonds with lower durations. It is an important tool in structuring and managing a fixed-income portfolio based on selected investment objectives.

The price of a bond or any fixed-income investment is determined by summing the cash flows discounted by a rate of return. The rate of return can change at any time period and will be reflected in the calculation of an investment's market price. The sensitivity of a bond's value to changing interest rates depends on both the length of time to maturity and on the pattern of cash flows provided by the bond.

There are many variables associated with pricing a bond. Changes in each of these variables, taken separately and in combination, can have a significant effect on price.

Duration is a calculation that brings all these factors together in one number, allowing a measurement of a bond's price sensitivity to changes in maturity and interest rates.

The common objective behind the different definitions of duration is to measure the price sensitivity or market risk of a fixed-income security to changes in its yield. Bonds of similar duration will have similar price movements for a given movement in interest rates.

In general terms, we may say that duration of a fixed-income security like a bond is the weighted average of the times that payments, that is, cash flows, are made. The weights are the present values of the individual cash flows. Duration is the weighted average maturity of its cash flow stream, where the weights are proportional to the present value of cash flows. Formally, it is defined as:

$$\text{Duration} = [PV(C_1) \times 1 + PV(C_2) \times 2 + PV(C_3) \times 3 + \dots + PV(C_T) \times T] / V_0$$

Where  $PV(C_t)$  = present value of the cash flow receivable at the end of year  $t$

( $t = 1, 2, 3, \dots, T$ ),  $V_0$  is the current value of the bond.

Let  $V$  be the price of the bond, and  $y$  the yield, then the proportional change in the price of a bond in response to the change in its yield is as follows

$$\frac{\Delta V}{V} = -D \times \left( \frac{\Delta(1+y)}{1+y} \right)$$

where  $\Delta V/V$  is proportional change in price,  $D$  is duration, and  $y$  is yield.

Duration measures are of three types:

- 1) **Macaulay Duration:** Developed in 1938 by Frederick Macaulay, this form of duration measures the number of years required to recover the true cost of



a bond, considering the present value of all coupon and principal payments received in the future (which is why it is the only type of duration quoted in “years”). It assumes interest rates are continuously compounded. In 1938, Macaulay suggested a method for determining price volatility of bonds. He gave the name duration to the measure, but it is now often called Macaulay duration. The formula that he gave is:

$$D_{Mac} = \frac{\sum \frac{t_j C_j}{(1+r)^{t_j}}}{\sum \frac{C_j}{(1+r)^{t_j}}}$$

where:  $D_{Mac}$  is the Macaulay Duration;  $r$  is the periodic yield (yield for one period);  $t_j$  is the time until the  $j$ th cash flow;  $C_j$  is the  $j$ th cash flow;  $k$  is the total number of cash flows.

Because of very little volatility of interest rates, as these were regulated, little attention was paid to duration until the 1970s. In the 1980s interest rates started to rise dramatically in the United States. Investors and traders started to look for any tool that would help them to know how much the prices of bonds would change for a given change in yields. Here the concept of duration came in handy.

Macaulay duration can be used to determine the following:

- The duration of a zero coupon bond is equal to its time to maturity.
- The duration of a coupon bearing bond is less than its time to maturity.
- If two bonds have the same coupon rate and yield, then the bond with the greater maturity has the greater duration.
- If two bonds have the same yield and maturity, then the one with the lower coupon rate has the greater duration.

2) **Modified Duration:** While all of this is useful, it does not tell investors exactly how much a bond’s price changes given a change in yield. However, it was noticed that there is a relationship between Macaulay duration and the first derivative of the price/yield function. This relationship led to the definition of modified duration:

$$D^* = \frac{D_{Mac}}{(1+r)} = \frac{\frac{\partial P(r)}{\partial r}}{P(r)}$$

where

- $D^*$  is the modified duration;
- $D_{Mac}$  is the Macaulay duration;
- $r$  is the periodic yield;
- $P(r)$  is the price of the bond at yield  $r$ .

This measure expands or *modifies* Macaulay duration to measure the responsiveness of a bond’s price to interest rate changes. It is defined as the percentage change in

price for a 100 basis point change in interest rates. The formula assumes that the cash flows of the bond do not change as interest rates change, which is not the case for most callable bonds.

- 3) **Effective Duration:** Effective duration (sometimes called option-adjusted duration) further refines the modified duration calculation. Effective duration requires the use of a model for pricing bonds that adjusts the price of the bond to reflect changes in the value of the bond's "embedded options" (e.g., call options or a sinking fund schedule) based on the probability that the option will be exercised. All things being equal, as interest rates fall, bonds with embedded call options are exercised and the "in-the-money" bond is repaid. If interest rates rise, embedded options will not be exercised and the "out-of-the-money" bond will continue to maturity. Effective duration will shorten and be closer to the call date for "in-the-money" bonds, while lengthening and being closer to the maturity date for "out-of-the-money" bonds.

Three factors that influence the duration calculation are *coupon rate* (which determines the size of the periodic cash-flow), *interest rates* (which determines the *present value* of the periodic cash flow), and *maturity* (which weights each cash flow) all contribute to the above duration measures. As a result, the two main principals of duration are:

As maturity increases, duration increases and the bond's price becomes more sensitive to interest rate changes.

- A decrease in maturity decreases duration and renders the bond less sensitive to changes in market yield.

Therefore, duration varies directly with maturity.

As the bond coupon increases, its duration decreases and the bond's price becomes less sensitive to interest rate changes.

- Increases in coupon rates raise the present value of each periodic cash flow and therefore the market price.

This higher market price lowers the duration.

Let us now mention some properties of duration. First, the duration of a zero-coupon bond is the same as its maturity. Second, for a given maturity, a bond's duration is higher when its coupon rate is lower. Third, for a given coupon rate, a bond's duration generally increases with maturity. Fourth, other things remaining the same, the duration of a coupon bond varies inversely with its yield to maturity. Finally, the duration of a coupon bond is approximately

$$\frac{1+y}{y} - \frac{(1+y) + T(r-y)}{r[(1+y)^T - 1] + y}$$

where  $y$  is the bond's yield per time period,  $T$  is the number of payment periods, and  $r$  is the coupon rate per payment period.

### Implications

Duration allows bonds of different maturities and coupon rates to be directly compared.

The higher the duration, the higher the risk of price changes as interest rates change.



- Constructing a bond portfolio based on weighted average duration provides the ability to determine value changes based on forecasted changes in interest rates.
- The interest rate sensitivity of a bond portfolio can only be estimated if there is a change in interest rates that leads to a parallel shift in the yield curve. In general, bonds of different maturities seldom experience the same change in rates (i.e., parallel shift in the yield curve). Thus, two bond portfolios that may have the same duration at the beginning of the investment horizon may end up being affected differently by interest rate changes one period later, depending on how the yield curve has moved.
- Neither basic duration formula can be used to estimate interest rate sensitivity of callable bonds because changes in interest rates may affect not only the prices of the bonds but the receipt of the cash flows as well.

This is because the bonds may be called as a result of changes in interest rates. The use of *effective duration* modelling can significantly improve these limitations.

### Check Your Progress 2

- 1) Explain Macaulay's measure of duration.

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- 2) What are the factors that affect duration?

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- 3) In what way is the so-called 'modified' duration measure a modification of the Macaulay measure?

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## 15.4 REPRICING AND MATURITY MODELS

We have seen that not only individuals holding fixed-income securities like bonds, but also banks and firms face the consequences of interest rate risk. In this section we look at some strategies to manage interest rate risks, which can be employed by organisations whose assets and liabilities both involve interest rates. The liabilities may be deposits of banks or borrowings if it is a firm. On the other hand, banks may provide loans to others, which are assets of the banks. So banks are concerned with interest rates affecting both their assets as well as liabilities.

There are two approaches towards understanding the effects of exposure of a bank or company to interest rate risk. One approach focuses on the effects of interest rate changes on future financial flows such as cash flows, net interest incomes, or earnings. The second approach looks at the effect of interest rate changes on market value. In the case, the main concern is not with how rate changes will affect future financial flows, *per se*. Rather, the emphasis is laid on how changes in interest rates will affect the *current value* of future cash flows or profits, usually measured as the market value of the firm, or the value of owners' equity. Here the notion of duration that we studied in the previous section is used to quantify the sensitivity of assets and liabilities to interest rate changes. Clearly, these two approaches to studying the effect of interest rate changes on assets and liabilities are related to each other, since the market value of the firm can be seen as the present value of its future cash flows. However, there may be differences in these approaches and we have to see what these differences are.

Now we come to the important topics of repricing of securities and GAP analysis with regard to these securities. We understand that interest is the price of loans. Now sometimes these are repriced, that is, their prices are adjusted and revised. For a bank, for example, both its assets and liabilities may come to be repriced. Repricing can occur either after the maturity of the loan, or it may be repriced, if the contract rate is reset prior to maturity. Suppose you have gone in for a home loan on a floating interest rate term. Before you entirely repay the money, the contract for your loan may be repriced.

The interest rate GAP (it is actually gap but usually denoted GAP) is defined by classifying financial assets and liabilities into two types: those that will be repriced within a specified interval called the gap interval or gap maturity and those that will be repriced later. Assets and liabilities that are repriced within the gap interval are called "rate sensitive assets" and "rate sensitive liabilities", or RSA and RSL respectively. The difference between RSA and RSL, or RSA minus RSL is equal to the GAP.

The GAP is the money amount of assets and liabilities that are mismatched. If there are more RSL than RSA, the GAP will be negative. Basically, the GAP measures the volume of fixed rate assets that are financed with variable rates liabilities. If rates of interest rise, it will increase the cost of interest on that volume while the interest incomes will not. This will reduce the net interest margin. The magnitude of these effects will depend on the magnitude of the GAP.

Many a time, companies restructure their balance sheets in order to rearrange or redistribute interest rate risk exposure or to change the timing of interest payments. Companies can do this through cash or spot market transactions. This usually involves selling off an existing asset, paying off debt and replacing it with some other obligation. But sometimes companies do not take this route; it may be because they want to

reduce transaction costs of issuing new securities or retiring existing ones, or because they use off-balance sheet methods to hide the actual nature of the intended changes.

Now we come to a very important concept that has to do with protection against interest rate risk. This is the concept of **immunisation**, and this is what we turn to now. The name indicates that it is a process that 'immunises' the value of the fixed-income securities portfolio against interest rate changes. It is a method to protect an investment with a specific time horizon from interest rate changes. An investor who invests in fixed-income securities may hold these securities with a specific purpose in mind. For example, suppose a person will retire in 10 years and wants the proceeds from her investment to finance her retirement. She could invest in a zero-coupon bond that will mature after 10 years so that her invested amount is locked in for 10 years. Another way could be to invest in a portfolio non-zero coupon bond but make the portfolio behave as though it were a zero-coupon one. Designing a portfolio of coupon bonds so that it behaves like a zero-coupon bond with a maturity equal to the investor's investment horizon is called immunisation.

We know that a coupon bond involves two kinds of risks. First, there is the risk of changes in price that will be realised if it is liquidated before maturity. Secondly, it has the risk of changes in the interest rate at which coupon payments can be reinvested. These risks move in the direction opposite to changes in the market interest rates. The aim of immunisation of a portfolio is to make these risks offsetting so that the bond value plus the accumulation of reinvested coupon payments will be the same at the end of the investment period, regardless of the level of interest rates. The way to do this is to choose the portfolio's duration to equal the length of the investment horizon. It can be shown that when the duration is equal to the length of the investment horizon, then changes in reinvestment income caused by changes in interest rates will offset changes in the price of the portfolio in a manner such that the total accrual at the end of the investment horizon cannot decrease. Thus immunisation reduces and can potentially eliminate interest rate risks for portfolios by equating the duration and intended holding period of the portfolio.

Immunisation matches duration as well as present values. Immunisation provides protection against changes in yield. The basic shortcoming of immunisation is that it assumes that all yields are equal.

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## 15.5 VALUE AT RISK

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In this section, we deal with an important concept related to market risk, called the value at risk.. **Value at risk**, usually denoted by VaR (not to be confused with variance in statistics!) is a measure of how the market value of an asset or portfolio of assets is likely to decrease over time under usual circumstances. Value-at-risk (VaR) is a category of risk metrics that describe probabilistically the market risk of a trading portfolio. We shall discuss it and see how it can be employed as a strategy to guard against market risk.

We have seen in an earlier unit (Unit 3) that there are some widely used measure of risk, for example, in the case of a portfolio of stocks, the variance of this portfolio. However, in many contexts it helps to have a measure of risk that is easily understandable and is a single index. Moreover, if the measure is such that it can widely applied in a large number of cases like firms, banks etc, many organisations will benefit. The VaR is such a measure. Moreover, the variance or standard deviation do not specify the direction of deviation from the mean. Volatility can be in either

direction. But sometimes investors or organisations like banks may be specifically interested in knowing the maximum loss they can possibly make in a particular period; they may be concerned with the worst-case scenario about their portfolio. This is where VaR comes in.

This measure is related to the probability of making large losses. Its power is its generality. Unlike other market risk metrics like duration, VaR is general. It is based on the probability distribution of a security's or portfolio's market value. The uncertain market values have a probability distribution, and all the types of risks that affect the market value have an impact on that probability distribution. Being applicable to all liquid assets and encompassing, at least in theory, all sources of market risk, VaR is an all-encompassing measure of this risk. In order to measure market risk in a portfolio using VaR, some means must be found for determining the probability distribution of that portfolio's market value.

VaR has two parameters: the time horizon for which the estimate is made because the asset is typically held for that 'holding period', usually being one day: and the confidence level at which the estimate is made (this is the usual confidence level used in statistical estimation). VaR, with the parameters: holding period  $w$  days; confidence level  $x\%$ , defines the likelihood that a given portfolio's losses will exceed a certain amount on a normal market conditions over a given period. Value at Risk (VAR) calculates the maximum loss expected (or worst-case scenario) on an investment, over a given time period and given a specified degree of confidence. Suppose we are interested in large losses of a given portfolio, and specifically, in those losses that are not likely to occur not more than once in hundred trading days. We say that we are interested in the 99% confidence level and in a daily value at risk. Daily value at risk at a 99% confidence level is the smallest number  $x$  for which the probability that the next day's portfolio loss will exceed  $x$  is not more than 1%.

The VaR metric is a real-valued function of the distribution of  $P_1$  conditional upon information available at time 0 and  $P_0$ . An example of a risk metric that is not a VaR metric is standard deviation of cash flow because this generally cannot be expressed as a function of  $P_0$  and the conditional distribution of  $P_1$ . VaR cannot anticipate *changes* in the composition of the portfolio during the day. Instead, it reflects the riskiness of the portfolio based on the portfolio's current composition.

Let us see sketchily how VaR is used. Let 0 be the current time. We know a portfolio's current market value, say  $P_0$ . We do not know its value at the end of one trading day,  $P_1$ . It is a random variable. The task is to assign  $P_1$  a probability distribution. Sometimes a normal distribution is assigned, and its properties are used to study risk. Occasionally, time-series analysis is used to study historical patterns of data related to the securities.

Another approach is to model the portfolio's behavior, not in terms of individual assets, but in terms of specific risk factors. Depending upon the composition of the portfolio, risk factors might include exchange rates, interest rates, commodity prices, spreads, implied volatilities, etc. The modeled risk factors are called key factors. The idea is to list these factors into a vector, and then devise a valuation function that is a pricing formula which expresses prices of these assets as a function of the key factors. A linear function is occasionally used. Then the vector of prices (or the list of prices may be indexed into a scalar) of the assets is expressed as a function of vector of key factors. This function is called a portfolio mapping function. The vector (or scalar) of prices, however, is random. This involves mathematics that is beyond

the scope of our discussion. There are three methods of calculating VAR: the historical method, the variance-covariance method, and monte-carlo simulation. We do not discuss these. You can look up your course on quantitative methods to understand these concepts in general terms. We may just mention that the historical method simply re-organizes actual historical returns, putting them in order from worst to best. It then assumes that history will repeat itself, from a risk perspective. **The Variance-Covariance** method assumes that stock returns are normally distributed. In other words, it requires that we estimate only two parameters, the mean and standard deviation of the returns. The third method involves developing a model for future stock price returns and running multiple *hypothetical* trials through the model. A Monte Carlo simulation refers to any method that randomly generates trials, but by itself does not tell us anything about the underlying methodology.

There are some disadvantages of using VaR. one disadvantage is that, if a large loss occurs, the VaR does not tell us much about the actual size of the loss. Moreover, while calculating VaR, it is usually assumed that normal market conditions prevail. But sometimes markets can crash, or there may be extreme moves of market variables. Moreover, since VaR is a single number, does not say anything about which of the portfolio components is responsible for the largest risk exposure. Yet another problem with VaR is that the VaR of a combination of two positions may be larger than the sum of the VaRs of the individual positions. This goes against the diversification principle, that the risk of a diversified portfolio is no larger than the combined risk of the portfolio components.

### Check Your Progress 3

- 1) Explain the concept of interest rate GAP.  
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- 2) What do you understand by immunisation? How can it be used as protection against interest rate risk?  
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- 3) Explain the concept of value-at risk as a measure of market risk.  
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## 15.6 LET US SUM UP

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In this unit, we discussed certain aspects of financial risk. We discussed in particular market risk, or interest risk, and discussed how to deal with these types of risk.

We began the unit with a discussion of the returns from, and duration regarding, fixed- income securities like bonds. We explained the various concepts related to bonds and their returns, like yield, coupon rate, maturity etc. we discussed various

aspects of maturity and duration, like Macaulay's formulation of duration. We also touched upon a modified concept of duration.

The unit then moved on to a discussion of repricing and maturity issues and discussed various types of strategy towards risk, and different aspects of a viable risk management process. The unit went on to discuss various methods of quantifying and measuring interest-risk, and to several risk metrics. Finally, the unit had a detailed discussion regarding Value at Risk. In this regard, the definition of this concept was provided, and the three main ways to measure VaR were stated.

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## 15.7 KEY WORDS

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<b>Coupon rate</b>	: it is the regular payment made by the issuer of the bond to the purchaser
<b>Face Value</b>	: the principal or par value of the debt.
<b>Maturity</b>	: the maturity date is the date on which payments stop on a fixed-income security such as a debt. It is the date when the bond is redeemed.
<b>Yield to Maturity</b>	: the internal rate of return for a bond.
<b>Zero coupon Bond</b>	: a bond that makes no periodic interest payment.

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## 15.8 SOME USEFUL BOOKS

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Cuthbertson, Keith, (1996), *Quantitative Financial Economics: Stocks, Bonds and Foreign Exchange*, John Wiley & Sons, New York.

Crouhy, M., D. Galai, and R. Mark (2001). *Risk Management*, McGraw-Hill, Singapore.

Fabozzi, F. (2005) *The Handbook of Fixed Income Securities*, 7th edition. McGraw-Hill, New York.

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## 15.9 ANSWERS/HINTS TO CHECK YOUR PROGRESS EXERCISES

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### Check Your Progress 1

- 1) See Section 5.2 and answer.
- 2) See Section 5.2 and answer.
- 3) See Section 5.2 and answer.

### Check Your Progress 2

- 1) See Section 5.3 and answer.
- 2) See Section 5.3 and answer.
- 3) See Section 5.3 and answer.

### Check Your Progress 3

- 1) See Section 5.4 and answer.
- 2) See Section 5.4 and answer.
- 3) See Section 5.5 and answer.