UNIT 16 CREDIT RISK AND OTHER RISKS

Structure

16.0 OBJECTIVES

After going through the unit, you should be able to:

- explain the concept of default risk:
- discuss some statistical models to help in credit risk management;
- analyse some mathematical models of credit risk which treat loans as options;
- describe KVM credit monitor; and
- explain J.P. Morgan company's CreditMetrics as a tool and technique of credit risk management.

16.1 INTRODUCTION

In recent years, credit risk measurement and management has taken centre stage in almost all discussions involving financial institutions. Financial institutions have to look at how much credit risk they face and how to manage risk. The key questions asked in credit risk are:

- What are the chances that a borrower will default on its loan obligations?
- What is the value of a risky loan?

In this Unit we will try to address the above issues using some key concepts and models.

Default risk is the uncertainty surrounding a firm's ability to service its debts and obligations. Prior to default, there is no way to discriminate unambiguously between firms that will default and those that will not. At best we can only make probabilistic assessments of the likelihood of default. In this chapter, we will study some of the techniques involved in default probability estimation.

Default is a deceptively rare event. The typical firm has a default probability of around 2% in any year. However, there is considerable variation in default probabilities across firms. A firm with a high credit rating has a much lower probability of default than a firm with a lower credit rating. In addition to knowing the default probability, the portfolio management of default risk requires the measurement of default correlations. Correlations measure the degree to which the default risks of the various borrowers and counterparties in the portfolio are related. The elements of credit risk can therefore be grouped as follows:

1. Standalone Risk
Default probability—the probability that the counterparty or borrower will fail to service obligations.
Loss given default—the extent of the loss incurred in the event the borrower or counterparty defaults.
Migration risk—the probability and value impact of changes in default probability.

2. Portfolio Risk
Default correlations—the degree to which the default risks of the borrowers and counterparties in the portfolio are related.
Exposure—the size, or proportion, of the portfolio exposed to the default risk of each counterparty and borrower.

16.2 STATISTICAL MODELS IN CREDIT RISK MEASUREMENT

We may, at the beginning, point out that you should, if necessary, look up Units in the Course on Quantitative Techniques that you studied last year. That will help you to grasp the concepts in this and the next section.

The basic question that we ask in this section is: *What is a significant relationship between default tendencies and key observables of a borrower?* Statistical techniques are often used to answer this question. In essence, a good credit assessment methodology should be able to suggest a relationship of the form,
$p_{it} = f\left(X_{i,t-k}\right)$ where p_{it} is the probability that the i^{th} borrower will default at time t and $X_{i,t-k}$ is the vector of observables at time $t-k$ for borrower i. The relationship could be linear as well as non linear. We briefly study some of the commonly used statistical techniques employed to estimate default probability.

16.2.1 Altman Z Score

The Z-Score is a measure of a company's health and utilises several key ratios for its formulation The model incorporates five weighted financial ratios into the calculations of the Z-Score. The model is continuously being updated incorporating new information. The basic description as well as further calibration on this model can be found in Altman (1993). The Altman Z-score uses the Linear Discriminant Analysis (LDA) to identify 5 key ratios that can predict bankruptcy for public and private companies. The five key ratios are

X_1 = Working Capital/ Total Assets

X_2 = Retained Earnings/ Total Assets

X_3 = Earnings before Interest and Taxes (EBIT)/ Total Assets

X_4 = Market Value of Equity/ Total Assets

X_5 = Net Sales/ Total Assets

The Z –Score for any firm can be calculated using either of the following two formulas.

$$Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5 \ldots\ldots\ldots \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(Z - Public)}$$

$$Z = 6.56X_1 + 3.26X_2 + 6.72X_3 + 1.05X_4 \quad \ldots\ldots\ldots \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(Z - Private)}$$

The Z-Score model is easy to calculate and interpret. This makes it one of the most widely used credit scoring models. Note that, higher is the calculated Z-Score of a firm, healthier it is. Once the respective ratios for a company is calculated (publicly or privately held), the respective formulas are used to calculate the Z –Score. A healthy public company has a Z >2.99; it is in the grey zone if 1.81 < Z < 2.99; it is unhealthy if it has a Z < 1. 81. A healthy private company has a Z >2.60; it is in the grey zone if 1. 1 < Z < 2.59; it is unhealthy if it has a Z <1.1.

16.2.2 Linear Probability Models (LPM)

Another approach of estimating probability of default is to estimate the multiple regression equation,

$$p = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots. + \beta_k x_k + u \quad \ldots \quad \ldots.. \quad (2.2.1)$$

where, p is the probability of default and $x_0, x_1, \ldots, x_k$ are the k explanatory variables. The above estimated equation is the Linear Probability Model (LPM). Note that the term *linear* is used to indicate that the probability of default is linearly dependent on the *parameters* $\beta_0, \beta_1, \ldots, \beta_k$.

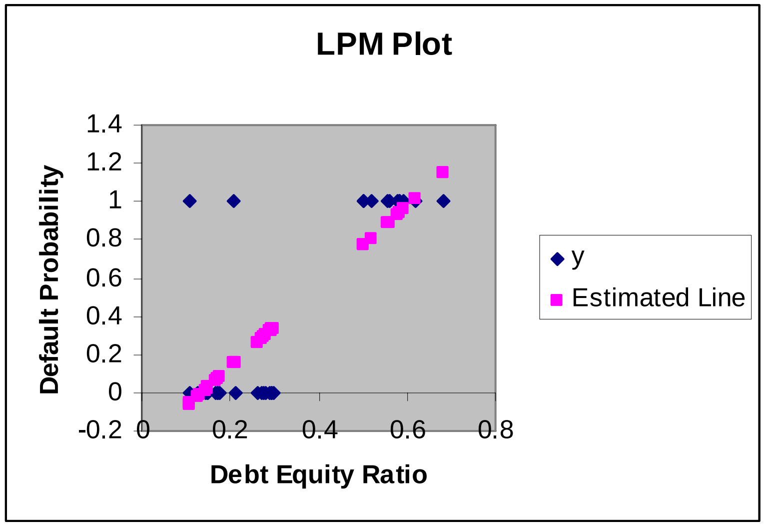
Note that while estimating (LPM 1), we will not observe the probabilities of default, instead observe whether a firm has defaulted or not. Therefore, in the regression, we will regress an indicator variable y on $x_0, x_1, \ldots, x_k$. Note that y is a dichotomous variable with possible values

$$y = \begin{cases} 1 & \text{if the firm has defaulted} \\ 0 & \text{if the firm has not defaulted} \end{cases}$$

Estimating the (LPM1) along with the indicator variable y will give us the estimated $\beta_0, \beta_1, \ldots, \beta_k$. The LPM is therefore a simple model to use. However, one immediately encounters three problems with the LPM.

- **Goodness of Fit**: Consider the scatter plot below in Figure 16.1. The plot is of 32 companies – with 13 defaulters and 19 non defaulters. The explanatory variable is debt equity ratio. Note that the scatter plots, imply that the observations either all lie on $y = 0$ or $y = 1$. Therefore, any line we wish to fit that is "closest" to all the observations, will have very low explanatory power. From the data set given in Statistical data.xls check that the $R^2 = 0.62$.

Figure 16.1:



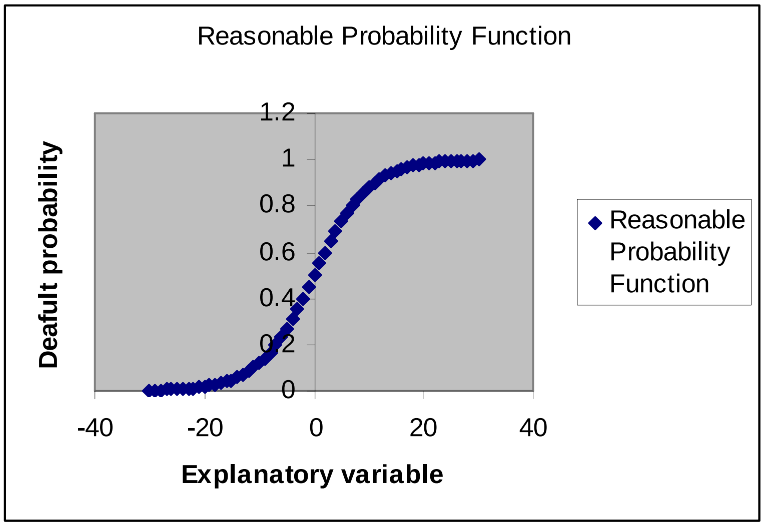
- **Improbable Probability Estimates**: In figure 16.1, the estimated default probability line as a function of debt equity ratio, is

$$p = -0.29 + 2.10.x \; \ldots \; \ldots.. \; \text{(LPM Estimated)}$$

where x, the explanatory variable, is the debt equity ratio of the borrower. The estimated LPM can be interpreted as follows. One percentage increase (or decrease) in leverage ratio increases (or decreases) default probability of a firm by approximately 2.1%. Consider, firms that have debt equity ratios either (a) less than 0.137 or (b) more than 0.62. For the first group of firms, the estimated probability of default is negative while for the second group, the estimated default probability exceeds one!

Considering the above three criticisms, it appears that a more reasonable probability function should appear as one given in Figure 16.2. The plotted probability function is non linear – both in parameters as well as the explanatory variables. This will solve, to a large extent, the problems of LPM encountered above. Note that, the cumulative probability function *asymptotically* approaches 0 and 1 when the value of explanatory variable is too small or too large. Therefore, the problem of obtaining improbable estimates are immediately done away with. The crucial question is: How do we estimate such non linear probability functions?

Figure 16.2



We discuss the intuition, methodology and estimation techniques involving non linear probability functions in the next section.

16.2.3 Non Linear Probability Models

In section 2.2, we described the basic regression technique that is employed while estimating probability models. However, three problems stand out while estimating the Linear Probability Model. These are, low goodness of fit, unreasonable probability estimates and non linear effect of variables on default probability. In this section, we will explore the possibility of estimating non linear probability estimates. In order to overcome the above difficulties, the proposed model is the logistic regression (known popularly as the *logit* model). In a logit model, the probability of default $F(Z_i)$, is estimated as a function of k explanatory variables, $Z_1, Z_2 .... Z_k$. I particular, we estimate

$$F(Z_i) = \frac{e^{Z_i}}{1+e^{Z_i}} \quad .... \quad ... \text{(2.3.1)}.$$

Note,

$$\ln \frac{F(Z_i)}{1-F(Z_i)} = Z_i$$

where, $\ln$ denotes the natural logarithm (base e). Therefore,

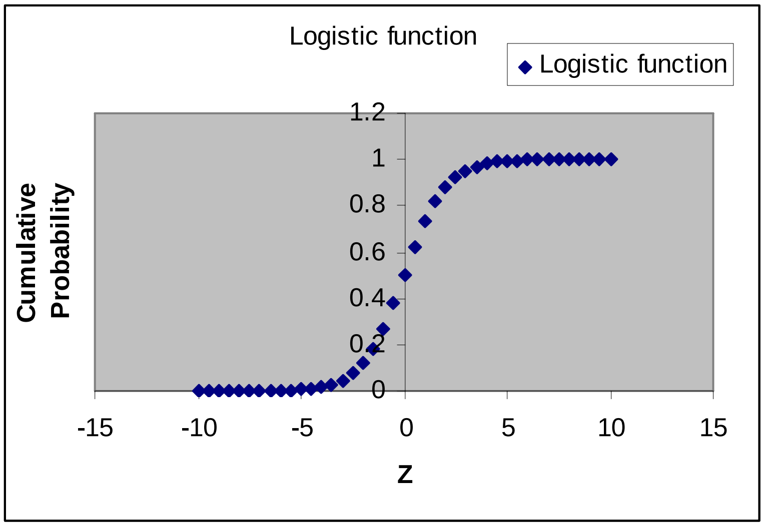
$$L_i = \ln \frac{P_i}{1-P_i} = \beta_0 + \sum_{j=1}^{k} \beta_{ij}$$

The left hand side of this equation is called the *log-odds* ratio.
Some of the interesting features of the logistic model are;

- Note that, as $Z_i \to -\infty$, the cumulative probability approaches zero, while as, $Z_i \to +\infty$ the cumulative probability approaches one.
- Although L is linear in the explanatory variables, the probabilities are not.

A simple plot of logistic distribution function is given in figure 16.3.

Figure 16.3



How to estimate the above equation? Unfortunately, the standard estimation technique like the Ordinary Least Square (OLS) is not applicable here. Instead, we employ a technique that is called *Maximum Likelihood Technique.* This is arrived at by setting up a likelihood function

$$\ell = \prod_{y_i=1} P_i \prod_{y_i=0} \left(1 - P_i\right)$$

and then choosing the parameters β_j to maximize ℓ. It also suffers from heteroscedasticity. Most advance econometric software packages will do this regularly We will report below the routine findings of a logistic regression that with the data set given in Statistical data.xls. The software package employed for the logistic regression is EViews 5.0. The estimated logistic regression equation is

$$L_i = \ln \frac{P_i}{1 - P_i} = -5.15 + 14.17\left({}^{Debt}\!/\!_{Equity} \right) \ldots \ldots. \quad (2.3.2)$$

Therefore, for a firm that has debt equity ratio of 0.5 (say), the probability of default is (approximately) 0.535.

In this section we have looked at three broad statistical techniques employed in measuring default risk. Although, Altman Z –score is still widely regarded as the base model in credit risk estimation, recent developments using non linear probability models is increasingly making it popular. In sections 3 and 4, we will consider mathematical models used for estimating default.

Check Your Progress 1

1. Critically assess the Linear Probability Model used in credit risk analysis.

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2. Explain the non-linear probability model.
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3. What is the Altman z-score? Explain how it is used to study default risk.
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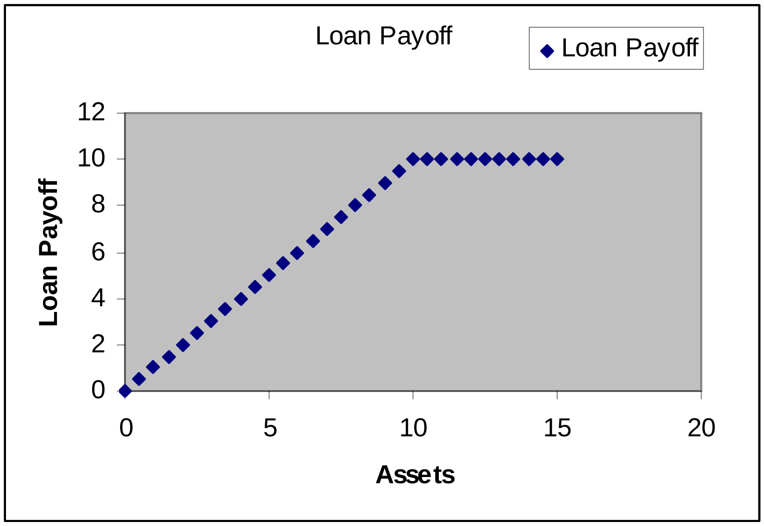
16.3 MATHEMATICAL MODELS: LOANS AS OPTIONS

In section 16.2, we have studied statistical models of predicting default. We will now discuss some mathematical models those are used to estimate default.
Consider the following example. A borrower X agrees to pay Rs 100 Crores (1 crore=100,00,000) to a bank t years from now. If the risk free rate prevailing in the market is 5%, what would be the market value of the loan? That is, at what price could the bank expect to sell the loan to another bank in the secondary loan market? In this chapter as well as the next chapter we will try to answer these questions.

The borrower can pay the bank the face value of the loan, L, if the resources available to him are sufficient to do so. Typically, the borrower is expected to meet its obligations from the total assets he has. Assuming no transaction cost (mainly cost of bankruptcy), if the total assets exceed L, then the bank can expect to receive L. However, if the monetary value of the total assets are less than L, then the bank can expect to get only an amount that is equal to the total assets. This is the classic nature of fixed income instruments. The pay off to a bank lender will appear as given in figure 16.4.

Figure 16.4



Note that, if the value of the assets available with the borrower exceeds $L = 10$, irrespective of the actual value of the assets, A, the borrower will not pay more than L to the bank. It keeps the residual, $A - L$ for itself as "profits". However, protected by *limited liability*, the borrower will only pay A to the bank in the event that $A \le L$. The future payment stream to the bank is therefore uncertain. It will depend upon the possible asset values of the borrower at time t. Denote, A_t as the value of the borrower's asset at time t. As A_t, is uncertain, we will assume that the cumulative probability distribution of A_t is $F(A_t)$ with a well defined density function, $f(A_t)$.

Therefore, to the bank, there are two types of payments it can expect from the borrower, one, when there will be no default, (occurs with a cumulative probability of $1 - F(L)$) and when there is default (occurs with a cumulative probability of $F(L)$). In the event there is no default, the present value of the loan, payable t periods from now, is therefore,

$$V_L = e^{-r.t}\left\{\int_0^L A_t.f(A_t)dA_t + \int_L^\infty L.f(A_t)dA_t\right\} \quad \ldots \ldots \qquad (3.1)$$

If the asset distributions can be estimated from data, the value of the loan V_L can easily be computed from (3.1).

Example: If the asset distribution follows *uniform distribution* over $[0,20]$, then a loan maturing 2 years from now with a face value of 10 crore and a risk free rate of 5% is valued as,

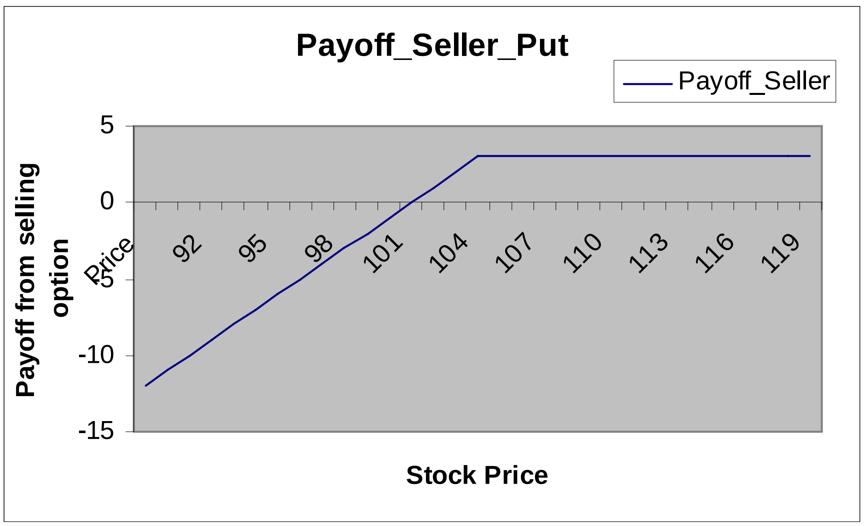
$$V_L = e^{-0.05*2}\frac{1}{20-0}\left\{\int_0^{10} A_t.dA_t + \int_{10}^{20} 10.dA_t\right\} = 6.78$$

Therefore, the "fair value" or the value at which the bank can expect to sell this loan in the secondary market is approximately Rs 6.78 crores. ■

Therefore the crucial link in this whole argument is estimation of the asset distribution function. Once, the asset distribution is properly estimated, calculation of the expected loan value is routine. A careful inspection of the above argument will reveal that there is a direct relationship between options and risky debt valuation.

An option to sell the underlying financial asset at a pre specified price on or before a pre specified date is called a *put* option. The buyer of the option pays a price upfront to the seller of the option, often called the *option premium.* Suppose a put option is written on a stock that is currently trading at $S_0 = 100$. The expiry date is 4 months hence. The strike price of the option, $X = 105$. The option premium is currently Rs 3. Finally, let the possible price the underlying stock can take 4 months from now, S_T, follows uniform distribution over $[90,120]$. We plot the payoffs to the option writer (the seller of the put option) in figure 16.5

Figure 16.5



16.3.1 Options Valuation

So far, we have explored a simple example involving *European put option*. What remains is an exercise that determines the possible premium the buyer of the option needs to pay.

The price or premium, $\pi = f(k, S_0, X, T, \sigma, r); k = C, P$ of a European option is typically defined over the following variables

$S_0 =$ Current price of the underlying financial asset

$X =$ Exercise/Strike price of the underlying financial asset

$\mathrm{T} =$ Time to expiry

$\sigma =$ Volatilty of the underlying financial asset

$r =$ Risk free rate prevailing in the market.

$C =$ If the option is a call option

$P =$ If the option is a put option

Under the assumption of stock prices following *log normal* distribution, the famous Black- Scholes (1973) results give us analytical solutions to calculating the option premium. Note that if z follows log normal distribution if $\ln z$ follows normal distribution. It is usual to assume that f S_T follows log normal distribution such that,

$$\ln S_T \approx Normal\left[\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right]$$

where, μ is the "drift" rate or the natural growth rate of the stock, and "ln" stands for natural logarithm.

Denoting π_C and π_P as the option premiums for a European call and put option, respectively, the Black- Scholes valuation of options premium (on a non dividend paying) stock is,

$$\pi_P = Xe^{-r.T}\Phi(-d_2) - S_0\Phi(-d_1) \quad \dots \quad \dots (3.2)$$

where, $\Phi(Z)$ is the cumulative normal probability corresponding at Z and,

$$d_1 = \frac{\ln\left(S_0/X\right) + \left(r + \sigma^2/2\right)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln\left(S_0/X\right) + \left(r - \sigma^2/2\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

Example: Consider a stock currently trading at $S_0 = 100$, with a strike price, $X = 105$ and expiry date of 3 months, that is, $T = 3/12 = 0.25$. Finally, let the stock volatility (annual), $\sigma = 30\%$ and the prevailing risk free rate in the market is, $r = 5\%$. Therefore,

$$d_1 = \frac{\ln\left(100/105\right) + \left(0.05 + 0.30^2/2\right)0.25}{0.30\sqrt{0.25}} = -0.167; \Phi(-0.167) = 0.434; \Phi(-(-0.167)) = 0.566$$

$$d_1 = \frac{\ln\left(100/105\right) + \left(0.05 - 0.30^2/2\right)0.25}{0.3\sqrt{0.25}} = -0.317; \Phi(\text{-} 0.317) = 0.376; \Phi(\text{-}(-0.317)) = 0.624.$$

Therefore, using (5.2) we have

$$\pi_C = 100 * 0.434 - 105e^{-(0.05*0.25)} * 0.376 = 4.42$$

$$\pi_P = 100e^{-(0.05*0.25)} * 0.624 - 100 * 0.566 = 8.11$$

This is the theoretical valuation of option premiums in the Black -Scholes world.

16.3.2 Link between Options and Loans

A closer comparison of figure 16.4 and 16.5 suggests a striking similarity between options and loans. In particular, the payoff to a loan holder is equivalent to the payoff to a put option writer. That is, when a bank makes a loan, its payoff is isomorphic to writing a put option on the assets of the borrower. Merton (1974) noted this formally. Therefore, if the assumptions on the underlying probability distribution on a stock is same as that on assets, the Black Scholes formula can be used to value the loan. Therefore, one can use the BSM (Black Scholes Merton) valuation for risky loans.

In general,

$$V_P = f(S, X, r, \sigma, T)$$
$$V_L = f(\widetilde{A}, D, r, \widetilde{\sigma}_A, T)$$

where V_P is the value of a put option on a stock and V_L is the value of a risky loan Note that, while all the variables in the right hand side of the put option valuation formula is observable, when it comes to valuing risky loans, two of the key variables are not observable. These variables are indicated by a ~ above them. The two unobservable variables are the current asset value, $\widetilde{A}$ and the volatility of the assets, $\widetilde{\sigma}_A$.

Although the current asset value and its volatility are unobservable, models and technique developed in the last decade has made significant progress to resolve this issue. In particular, the KMV Credit Monitor model (which we investigate in details in the next chapter) is designed to approximate these unobserved values. In this chapter, having established the similarities between options and loan valuation techniques, we will use our understanding to value a risky loan.

In accordance with the options model, the equation for the market value of a risky loan, V_L, maturing T period from now is

$$V_L = De^{-rT}\left[\left(\frac{1}{d}\right)\Phi(h_1) + \Phi(h_2)\right] \ldots\ldots \quad \ldots\ldots \quad (3.3)$$

where,

d = the firm's leverage ratio measured as $\left(De^{-rT}\right)\big/A$,

A = current value of the firm's assets,

D = Face value of the loan,

$$h_1 = \frac{-\left[\frac{1}{2}\sigma_A^2 T - \ln(d)\right]}{\sigma_A\sqrt{T}};$$

$$h_1 = \frac{-\left[\frac{1}{2}\sigma_A^2 T + \ln(d)\right]}{\sigma_A\sqrt{T}}$$

σ_A = asset volatility.

Example: Chipdale Ltd. Has assets currently estimated at Rs 10 crores. The asset volatility (annual) is estimated at 30%. Chipdale has a loan with face value of Rs 8 crores payable after 2 years. If the market risk free interest rate is 5%, what is the market value of the loan?

Therefore,

$$A = 10$$
$$D = 8$$
$$T = 2$$
$$r = 0.05$$
$$\sigma = 0.3$$
$$d = \frac{De^{-r*T}}{A} = 0.7238$$

Substituting these values we get,

$$h_1 = \frac{-\left[\frac{1}{2}(0.3)^2 * 2 - \ln(0.7238)\right]}{0.3 * \sqrt{2}} = -0.97379 \Rightarrow \Phi(-0.97379) = 0.165$$

$$h_1 = \frac{-\left[\frac{1}{2}(0.3)^2 * 2 + \ln(0.7238)\right]}{0.3 * \sqrt{2}} = 0.54952 \Rightarrow \Phi(0.54952) = 0.70867$$

Thus the current value of the risky loan, from equation (3.3) is,

$$V_L = 8e^{-0.05*2}\left[\left(\frac{1}{0.7238}\right) * 0.165 + 0.70867\right] = 6.780$$

From the above equation, we can also calculate the yield spread. The yield spread is the equilibrium default risk premium that the borrower should be charged.

Denote, r_p as the default risk premium. Therefore, r_p denotes the premium over and above the risk free rate that makes this risky loan entirely risk free. The present value of a risk free loan that has a face value of D, maturing T periods from now is $De^{-r.T}$. Therefore, the risk premium is,

$$V_D e^{[r+r_p]T} = D \Rightarrow r_p = -\left(\frac{1}{T}\right)\ln\left[\left(\frac{1}{d}\right)\Phi(h_1) + \Phi(h_2)\right]$$

In our example, the risk premium is,

$$r_p = -\left(\frac{1}{2}\right)\ln\left[\left(\frac{1}{0.72387}\right)0.16598 + 0.70867\right] = 0.0326 \approx 3.26\%$$

Mathematical models in credit risk serve a dual purpose of simultaneously estimating default probability as well as valuation of risky loans. This is done by establishing the similarity between risky loan and put options. However, certain unanswered questions still remain which we do next.

Check Your Progress 2

1. In what way is a loan similar to an option?

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2. How can an asset distribution function be used to analyse loan default?

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3. Briefly state the Black-Scholes result on calculation of option premium. How did Merton extend this?

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16.4 MOODY'S KMV CREDIT MONITOR

In the previous section we valued a risky loan. Recall, the valuation technique adopted there had two major parts. First, we had to identify the probability of default, and two, the repayment schedule in the event of default. In identifying the probability of default, we drew the parallel between the financial options and risky loan. In that framework, there were three main elements that determine default probability

- Value of assets
- Asset risk
- Leverage

However, as discussed earlier, the asset values and the asset volatility is not directly observable.

In this section, we deal with a specific method of estimating asset values and volatilities. It is a technique provided by the company Moody's KMV, (after Kaelhofer, McQuown and Vasicek). The KMV monitor approach suggests a methodology that allows us to estimate asset values and asset volatilities and thereafter use them in measuring the default probabilities.
Credit Monitor allows users to measure and manage credit risk with greater accuracy. It enables the world's leading financial institutions and regulators to significantly improve their credit risk process: discriminating between 1 and 3,500 basis points of risk, placing analytical resources where the real risk lies and anticipating credit events instead of reacting to them. It is a software solution that helps to find out about expected risk of default.

16.4.1 Practical Approaches towards Implementing KMV

There are three basic types of information available that are relevant to the default probability of a publicly traded firm: financial statements, market prices of the firm's debt and equity, and subjective appraisals of the firm's prospects and risk. Therefore, the most effective default measurement, therefore, derives from models that utilize both market prices and financial statements. Without getting into the debate of market efficiency, it is prudent that, where available, we want to utilize market prices in the determination of default risk because prices add considerably to the predictive power of the estimates.

What does it mean to say that a firm will default? In simple terms, this means, the total assets of the firm are less than equal to its total liabilities. In other words, this means that the net worth (NW) of the firm is non-positive. However, it is not obvious that a firm will default if and only if the NW is non-positive. For example, a firm may have currently negative NW but it could be solely due to the fact that this is because of a temporary mismatch of assets and liabilities. Therefore, the relevant net worth of the firm is therefore the market value of the firm's assets minus the firm's default point. This relevant net worth will be called the market net worth.

A firm will default when its market net worth reaches zero. Like the firm's asset value, the market measure of net worth must be considered in the context of the firm's business risk. Asset value, business risk and leverage can be combined into a single measure of default risk which compares the market net worth to the size of a one standard deviation move in the asset value. We refer to this ratio as the distance-to-default.

Suppose, a firm is expected to have total assets equal to Rs 100 crores and liabilities of Rs 80 crores with and annual asset volatility of 10%. The distance to default is measured as

$$\begin{aligned}\text{Distance to Default(DD)} &= \frac{\text{Total Assets} - \text{Total Liabilities}}{\text{(Total Assets).(Asset Volatility)}} \\ &= \frac{100-80}{(100).(0.1)} \\ &= 2 \text{ standard deviations.}\end{aligned}$$

Therefore, the borrower is 2 standard deviations away from default. What does it mean? How likely is the firm to default? If the asset distribution of the firm follows *Normal distribution*, this means there is a 2.5% chance that the firm will default. This is called the Expected Default Frequency (EDF).

Once, the EDF is calculated, a default database is used to derive an empirical distribution relating the distance-to-default to a default probability. We call this as the *Actual Default Frequency* (ADF). The ADF is calculated as the proportion of defaulting firms belonging to the same industry that have the same distance to default. In our example, if we have a default database of 1000 firms in the software industry that consists of 150 firms that have DD of 2 sigma, out of which 6 firms have defaulted.

The ADF will be calculated as

$$\text{ADF} = \frac{\text{Number of defaulters among firms with DD} = 2\text{sigma}}{\text{Number of firms with DD} = 2\text{ sigma}} = \frac{6}{150} = 0.04$$

Note that, the EDF and the ADF need not be identical. In section 4.3, we explore this issue further.

In summary, there are essentially three steps in the determination of the default probability of a firm:

- **Estimate asset value and volatility:** In this step the asset value and asset volatility of the firm is estimated from the market value and volatility of equity and the book value of liabilities.

- **Calculate the distance-to-default:** The distance-to-default (DD) is calculated from the asset value and asset volatility (estimated in the first step) and the book value of liabilities.

• **Calculate the default probability:** The default probability is determined directly from the distance-to-default and the default rate for given levels of distance-to-default.

16.4.2 Estimating Asset Value and Volatility

So far, we have calculated the distance to default and default frequency. What remains is calculating the asset value and asset volatility. If the market price of equity is available, the market value and volatility of assets can be determined directly using an options pricing based approach, which recognizes equity as a call option on the underlying assets of the firm. In particular, we solve backwards from the option price and option price volatility for the implied asset value and asset volatility.

The BS model assumes that the market value of the firm's underlying assets follows the stochastic process:

$$dV_A = \mu V_A dt + \sigma_A V_A dz \quad \ldots \quad \ldots.. \ (4.1)$$

where

$V_A =$ Value of the underlying asset

$dVA =$ Change in the value of the underlying asset

$\mu =$ Drift rate of the firm's asset value

$\sigma_A =$ Volatility of the firm's asset value

$dz =$ Wiener process, $dz = \varepsilon\sqrt{dt}$

$dt =$ Change in time period

$\varepsilon =$ White noise, ε follows Normal$(0,1)$

The simple BS model allows only two types of liabilities, a single class of debt and a single class of equity. If D is the book value of debt which is due at time T, then the market value of firm's assets and the market value of the firm's equity, V_E is related as,

$$V_E = V_A\Phi(d_1) - De^{-r.T}\Phi(d_2) \quad \ldots \qquad \ldots\ldots \quad (4.2)$$

where, $$d_1 = \frac{\ln\left(V_A/D\right) + \left(r + \sigma_A^2/2\right)T}{\sigma_A\sqrt{T}}; \; d_2 = d_1 - \sigma_A\sqrt{T}$$

Finally, the equity and the asset volatility is related as,

$$\sigma_E = \frac{V_A}{V_E}\Delta\sigma_A \quad \ldots \qquad \ldots\ldots\ldots\ldots \qquad (4.3)$$

where σ_E is equity volatility. In order to calculate the asset value and the asset volatility, equations (4.2) and (4.3) has to be solved simultaneously.

16.4.3 Reconciliation of Expected Default Frequency and Actual Default Frequency and Other Issues

The popularity of KMVs credit metrics among financial institutions in the process of implementing default measurement models, stems mainly from the fact that it uses three major data sources of default for firms. First, it uses the market valuation data linking the firms' equity value and asset value. Second, it uses the asset volatility to measure the estimated default frequency. Finally, it uses the actual default frequency of firms in similar categories to validate. However, there are certain issues that require a bit more inspection before one can implement KMV.

From the above set of calculations, the EDF can be easily calculated. The next stage is to compare the EDF and ADF. In the event the two converges, the estimated asset value and the asset volatility can be used to calculate the default probability as well as the value of the risky debt. However, if the two does not converge, the calculations need to be updated.

Note that the calculations of EDF depends upon some observed values as well as some estimated values. The estimated asset values are the asset values and the asset volatility. Therefore, one possible way to reconcile the EDF with the ADF is to update the asset volatility.

A three-step process is used to calculate MKMV's EDF credit measure:
Estimate the market value and volatility of the firm's assets, Calculate the distance-to-default, the number of standard deviations the firm is away from default, and Transform the distance-to-default into an expected default frequency (EDF) using an empirical default distribution.

EDF credit measures are an effective tool in any institution's credit process. Accurate and timely information from the equity market provides a continuous credit monitoring process that is difficult and expensive to duplicate using traditional credit analysis. However, in spite of the advantages of KMV EDF credit measure, there are certain shortcomings that the financial institution should be prepared to tackle. These are related to estimating asset values or volatilities when either (i) the market is not efficient, or (ii) when the firm is not listed.

16. 5 J P MORGAN'S CREDIT METRICS

16.5.1 Introduction to Credit Metrics

CreditMetrics is a tool for assessing portfolio risk due to changes in debt value caused by
changes in obligor credit quality. This tool includes changes in value caused not only by possible default events, but also by upgrades and downgrades in credit quality. The two major contributions of the model are:

- This is a dynamic model of credit risk management. It also integrates with the value-at-risk (VaR) – the volatility of value – not just the expected losses.

- The model assesses risk within the full context of a portfolio by incorporating the correlation of credit quality moves across obligors. This allows direct calculations of the diversification benefits or potential over-concentrations across the portfolio.

For example, suppose we invest in a bond. Credit risk arises because the bond's value in one year can vary depending on the credit quality of its issuer. The value of this bond will decline with a downgrade or default of its issuer – and appreciate if the credit quality of the obligor improves. Therefore, the holder of the bond has to appropriately value the bond anticipating the possible future upgrading or down grading.

In this section, we study the CreditMetrics methodology. Specifically, we will:
• establish the link between the process of credit quality migration and the resulting changes in debt value;
• illustrate the resulting risk assessment with the simple example of a single bond;

The key question we intend to answer here (CreditMetrics) is "If next year is a bad year, how much will I lose on my loans as well as on the loan portfolio?" Note that RiskMetrics seeks to answer "If next year is a bad year, how much will I lose on my loans as well as on the loan portfolio?" However, answering the CreditMetrics question has some additional difficulties. As loans are not publicly traded, we will neither observe the market value of the loan, nor will we observe its volatility. Therefore, the CreditMetrics approaches the problem of estimating the VaR by using the following information

- Borrower's credit rating
- The probability that the ratings will change the next year (the rating migration matrix)
- Recovery rates on defaulted loans
- Credit spreads and yields.

Therefore, CreditMetrics tries to achieve the value-at-risk (VaR) due to credit quality changes. These measures will assist in the evaluation, deployment and management of credit risk taking across both a portfolio and marginal transactions. These measures are consistent with the – perhaps more familiar – value-at-risk models which are used for market risks.

16.5.2 Rating Migration and Other Inputs

Obtaining A Distribution Of Values For Ratings

Credit ratings obtained on borrower's are not static. Rating agencies rate the same borrower over a given period of time. Therefore, a borrower with a particular rating, can be upgraded or downgraded in the subsequent ratings. This is the main idea behind dynamic valuation of loans issued by the borrower.

To begin, let us use S&P's rating categories. Standard and Poor have the following bond rating categories: AAA, AA are high grade bonds (very low rate of default), A, BBB, are what are called medium grade, BB, B are what are called speculative grade, and CCC, CC, C and D are classified as default danger. Consider a single BBB rated bond which matures in five years. For the purposes of this example, we make two choices. The first is to utilize the Standard & Poor's rating categories and corresponding transition matrices. The second is to compute risk over a one year horizon.

Our risk horizon is one year; therefore we are interested in characterizing the range of values that the bond can take at the end of that period. Let us first list all possible credit outcomes that can occur at the end of the year due to credit events. There are four such possible events:

- the issuer stays at BBB at the end of the year;
- the issuer is updated to AAA, AA, or A
- the issuer is downgraded to BB, B, or CCC; or
- the issuer defaults.

Each outcome above has a different likelihood or probability of occurring. These likelihoods can be derived from historical rating data. Throughout this chapter we will assume that the probabilities are known. That is, for a bond starting out as BBB, we know precisely the probabilities that this bond will end up in one of the seven rating categories (AAA through CCC) or defaults at the end of one year. These probabilities are shown in table 5.1

Table 5.1: One Year Transition Probabilities for BBB –Rated Borrower

Rating Migration to	Percentage	Probabilities
AAA	0.02	0.0002
AA	0.33	0.0033
A	5.95	0.0595
BBB	86.93	0.8693
BB	5.30	0.053
B	1.17	0.0117
CCC	0.12	0.0012
Default	0.18	0.0018

Source: Credit Metrics Technical Document (1997), p 9

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Note that there is a 86.93% likelihood that the bond stays at the original rating of BBB.
There is a smaller likelihood of a rating change (e.g., 5.95% for a rating change to single-A), and a 0.18% likelihood of default. Finally, note that there are eight possible ratings that any bond can migrate to after one year.

Transition probabilities can be calculated by observing the historical pattern of rating change and default. They have been published by S&P and Moody's rating agencies, and can be computed based on KMV's studies, but any provider's matrix is usable. However, the only restriction before using the transition probabilities is to ensure consistency between ratings migration horizon and our risk horizon. For instance, a semi-annual risk horizon would use a semi-annual rather than one-year transition matrix.

Similarly, there are other bonds (except those are default grade) who have transition probabilities. In table 5.2, we consider, another such bond which has an initial rating of A.

Table 5.2: One Year Transition Probabilities for A –Rated Borrower

Rating Migration to	Percentage	Probabilities
AAA	0.09	0.0009
AA	2.27	0.0227
A	91.05	0.9105
BBB	5.52	0.0552
BB	0.74	0.0074
B	0.6	0.006
CCC	0.01	0.001
Default	0.06	0.006

Source: Credit Metrics Technical Document (1997), p 13

The individual likelihoods for BBB and single-A rating are separately shown in tables 5.1 and 5.*2* respectively, but this information is more compactly represented in matrix form as shown below in Table 5.3. This table is called a *transition matrix.* The ratings in the first column are the starting or current ratings. The ratings in the first row are the ratings at the risk horizon. For example, the likelihoods in table 5.1 corresponding
to an initial rating of BBB are represented by the BBB row in the matrix. Further, note
that each row of the matrix sums to 100%.

Table 5.3: One Year Rating Transition Matrix (in %)

Initial ratings								
	AAA	**AA**	**A**	**BBB**	**BB**	**B**	**CCC**	**Default**
AAA	90.81	8.33	0.68	0.06	0.12	0	0	0
AA	0.7	90.65	7.79	0.64	0.06	0.14	0.02	0
A	0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
BBB	0.02	0.33	5.95	86.93	5.3	1.17	0.12	0.18
BB	0.03	0.14	0.67	7.73	80.53	8.84	1	1.06
B	0	0.11	0.24	0.43	6.48	83.46	4.07	5.2
CCC	0.22	0	0.22	1.3	2.38	11.24	64.86	19.79

Source: Credit Metrics Technical Document (1997), p 20

Transition matrices can be calculated by observing the historical pattern of rating change
and default. They have been published by S&P and Moody's rating agencies, and can be

computed based on KMV's studies, but any provider's matrix is welcome and usable within CreditMetrics. But as mentioned previously, the transition matrix should, however, be estimated for the same time interval as the risk horizon over which we are interested in estimating risks.

Market Value of Loans for Different Rating Classes

So far we have specified: (i) each possible outcome for the bond's year-end rating, and
(ii) the probabilities of each outcome. Now we must obtain the value of the bond under
each of the possible rating scenarios. What value will the bond have at year-end if it is
upgraded to single-A? If it is downgraded to BB?

To answer these questions, we must find the new present value of the bond's remaining
cash flows at its new rating. Note that, any bond's subsequent change in ratings alters it risk profile. Therefore, while a bond that is rated R_i and promises to pay after T periods, a face value of $F(T)$ will be valued differently than another bond having identical maturity and face value, but has a different rating, R_j. The difference is arrived after discounting the two bonds by an appropriate discounting rate. This discount rate, used to discount the payoffs at time t, is the forward zero coupon rate, $f(R_i,t)$. Therefore, the value of a bond that is rated R_i with coupon (paid annually) of $C(t)$ payable each period and a principal of $F(T)$ payable after T periods is

$$V(R_i,C,F)=\sum_{t=0}^{T}\frac{C(t)}{(1+f(t,R_i))^t}+\frac{F(T)}{(1+f(t,R_i))^T} \quad ..... \quad ...... \quad (5.1)$$

In CreditMetrics, the discount rate that enters this present value calculation is read from the forward zero curve that extends from the end of the risk horizon to the maturity of the bond. The forward zero curves are given in the following table:

Table: One Year Forward Zero Curve Rates By Rating Categories

Caetgory	Year1	Year 2	Year 3	Year 4
AAA	3.60%	4.17%	4.73%	5.12%
AA	3.65%	4.22%	4.78%	5.17%
A	3.72%	4.32%	4.93%	5.32%
BBB	4.10%	4.67%	5.25%	5.63%
BB	5.55%	6.02%	6.78%	7.27%
B	6.05%	7.02%	8.03%	8.52%
CCC	15.05%	15.02%	14.03%	13.52%

Source: Credit Metrics Technical Document (1997), p 27

This zero curve is different for each forward rating category. To illustrate, consider a five-year BBB bond. Say the face value of this bond, $F(5)$ is 100 and the coupon rate, $C(5)$ is 6%. While, $F(5)$ and $C(5)$ remains unchanged during rating migration, the forward rate, $f(t, R_i)$ at any given time, t is different across bonds with different ratings. We want to find the value of the bond at year-end if it upgrades to single-A., at the end of one year we receive a coupon payment of $6 from holding the bond. Four coupon payments ($6 each) remain, as well as the principal payment of $100 at maturity. To obtain the value of the bond assuming an upgrade to single-A, we discount these five cash flows (four coupons and one principal) with interest rates derived from the forward zero single-A curve. Therefore, the value of a loan that is updated to A is given by,

$$V(A,6,100) = 6 + \frac{6}{(1.0372)} + \frac{6}{(1.0432)^2} + \frac{6}{(1.0493)^3} + \frac{106}{(1.0532)^4} = 108.64 \qquad \ldots$$

(5.2)

Note that calculations in equations (5.1) and (5.2) can be used to value bonds that have either different ratings, coupon rates or maturity. In table 5.5 and 5.6, we give the bond valuations corresponding to different coupon rates and maturities. The bond valuations in table 5.5 is done using bonds that having different coupon rates but with maturity at 5 years.

Table 5.5: Calculation of year-end values after credit rating migration (5 years)

	Par Value =100			
Rating Categories	**Coupon**	**Total value**	**Coupon**	**Total value**
AAA	6	109.35	5	108.53
AA	6	109.17	5	108.35
A	6	108.64	5	107.83
BBB	6	107.53	5	106.72
BB	6	102.00	5	101.25
B	6	98.085	5	97.364
CCC	6	83.625	5	83.023

The bond valuations in table 5.6 is done using bonds that having different coupon rates but with maturity at 3 years.

Table 5.6: Calculation of year-end values after credit rating migration (3 years)

	Par Value =100			
Rating Categories	**Coupon**	**Total value**	**Coupon**	**Total value**
AAA	6	109.4749	5	106.5881
AA	6	109.3784	5	106.4929
A	6	109.1874	5	106.3044
BBB	6	108.516	5	105.6426
BB	6	105.9885	5	103.1515
B	6	104.2076	5	101.3915
CCC	6	91.33848	5	88.71341

Note that, tables 5.5 and 5.6 will help us valuing a bond that is upgraded or downgraded to any rating class except for default. In section 5.3, we value loans that are default grade.

Recovery During Defaults

From the discussions in section 5.2.1 it is clear that, any obligor may migrate to a different rating class (albeit with different likelihood). The probability transition matrix allows us to do that. In section 5.2.2, we develop a framework that allows us to value bonds that have migrated to different rating classes. However, the discussions in 5.3 do not allow us to value a bond that is default grade. This can be seen from table 5.4, that does not incorporate forward zero coupon rates for default grade loans. This is as if, default grade bonds have no payoffs. However, this is not the case. Typically, bonds that default, may pay still pay some cash flow to the holder. This is so because, during defaults, the creditor has all the claims on the assets of the borrower. Unless the asset values are zero, the creditor gets positive payoff from the loan. In other words, the creditor is able to *recover* even during default states.

The recovery rates for the creditor depends mainly upon the nature of debt. CreditMetrics presents recovery rates for major seniority classes. This is summarized in table 5.7.

Table 5.7: Recovery Rates By Seniority Class

Seniority Class	Mean (%)	Standard deviation (%)
Senior Secured	53.80	26.86
Senior Unsecured	51.13	25.45
Senior Subordinated	38.52	23.81
Subordinated	32.74	20.18
Junior Subordinated	17.09	10.90

Source: Credit Metrics Technical Document (1997), p 26

Therefore, according to the recovery rates given in table 5.7, a senior secured bond that is default grade at a par value of 100, can expect to give the holder 53.80 with a standard deviation of 26.86. This table now completes the possible range of valuations a bond can have in the event of its upgrading or downgrading.

Calculating Portfolio Risks of a Single Bond Portfolio

In this section we will calculate the credit risk of a portfolio that has a single bond (rated BBB) and is senior subordinated. We will use the concepts developed through sections 5.2.1 - 5.2.3. We will now use these concepts to value a portfolio that has an exposure of Rs. 1 million in a senior subordinated BBB bond with a maturity of 5 years and a coupon rate of 6%.

Denote p_i as the probability that a BBB rated bond will migrate to rating i, $i = \{AAA, AA, A, BBB, BB, B, CCC, Default\}$. Further, denote V_i denote the final value of a senior subordinated bond with an (annual) coupon of 6% and maturing 5 years from now. The valuation of the BBB bond is given in table 5.8.

Table 5.8: Valuation of a Senior Subordinated BBB Rated Bond with Possible Rating Migration

Year End Ratings	p_i	V_i	p_iV_i	V_i^2	$p_iV_i^2$
AAA	0.0002	109.35	0.02	11958.06	2.39
AA	0.0033	109.17	0.36	11918.61	39.33
A	0.0595	108.64	6.46	11803.30	702.30
BBB	0.8693	107.53	93.48	11562.90	10051.63
BB	0.053	102.01	5.41	10405.30	551.48
B	0.0117	98.09	1.15	9620.85	112.56
CCC	0.0012	83.63	0.10	6993.27	8.39
Default	0.0018	51.13	0.09	3261.98*	10.56
					11478.6
		Mean	$\bar{V} = \sum_i p_iV_i$ =107.07	**Variance**	$\sum_i p_iV_i^2 -$ 10.11

Two important statistics come out of table 5.8. First, the mean value of the bond and second the variation in the mean value. The BBB rated bond can migrate to any of the eight possibilities next year. Therefore, the expected value of the bond is, $\bar{V} = \sum_i p_iV_i$.

The row entries corresponding to BBB in table 5.3 gives us the migration likelihoods of a BBB bond to any of the eight categories. The second column in table 5.8 contains these probabilities. From table 5.5 and 5.7, we get the final value of a bond (under different ratings) with 6% annual coupon and a maturity of 5 years. The third column contains these values.

To calculate the variation in the bond value, note that for default graded bonds, while the mean recovery rate is 51.13%, this has a standard deviation of 25.45%. The variance calculations reflect this as, during default,

$$V_i^2 = 51.13^2 + 25.45^2 = 3261.98.$$

The calculations in table 8.8 shows that the mean value of a BBB rated senior subordinated bond is 107.07 while its standard deviation is 3.18. This means, holder of a par (face value = Rs 100) senior subordinated BBB rated that pays an annual coupon of 6% with a maturity of 5 years will on an average be valued at Rs. 107.07. However, there is an uncertainty, regarding this value- it is an on average value. The extent of uncertainty is given by the standard deviation of 3.18.

The credit risk of a portfolio that has an exposure of 1 million in our BBB rated bond can be calculated in either of the two ways. One, calculate it from the variance, or two, as a percentile level. Suppose we are interested in calculating the worst (99%) loss scenario. This is equivalent to calculating the 99%VaR.

If we assume the portfolio value is normally distributed, then the VaR(99%) (for a Rs 100 of exposure) is given by 2.33*3.18=7.409. Therefore, if the total exposure is Rs 1 million, then the VaR at 99% is Rs 74,095.

Check Your Progress 3

1. What are the steps in the determination of the default probability of a firm?

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2. Explain briefly the working of Moody's KMV monitor as a credit monitoring device

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3. Assess the use of J P Morgan company's solution CreditMetrics as a tool for assessing credit risks.

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16.6 LET US SUM UP

In this unit we took a brief tour of the tools and techniques in credit risk management and modeling. We covered two major approaches- the statistical modeling and the mathematical modeling approach. We saw how loans can be considered as options and assessed. The Black-Scholes-Merton formulation was discussed.

The Unit then discussed some established tools for assessing credit risks and evaluating risks of default. The unit discussed in detail two main solutions provided by two established organisations: a rating agency and a financial company. These are Moody's KMV monitor, and Moody's CreditMetrics.

16.7 KEY WORDS

Actual Default Frequency: The ADF is calculated as the proportion of defaulting firms belonging to the same industry that have the same distance to default.

Estimated Default Frequency If the asset distribution of the firm follows *Normal distribution*, the Expected Default Frequency (EDF) shows the expected value of the chance of default.

Market Net Worth the market value of the firm's assets minus the firm's default point

Option: a financial contract that gives the holder the right but not the obligation to buy the security at a specified time and price in the future.

Risk Premium is the minimum difference between the expected value of an uncertain 'lottery' that a person is willing to take and the certain value that he is indifferent to.

16.8 SOME USEFUL BOOKS

Gupton, Greg M., Christopher C. Finger, and Mickey Bhatia (1997): *Credit Metrics Technical Document*, Morgan Guarantee Trust, New York:

Hull John (2003) *Options, Futures, and Other Derivatives* 5th ed. Prentice Hall International, New Delhi

Jorion, Philippe: *Value at Risk* (1997), McGraw Hill, New York.

Mishkin, Frederic S. (1998) '*The Economics of Money, Banking and Financial Markets'* Addison-Wesley, Reading.

16.8 ANSWERS/HINTS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress 1

1. See subsection 16.2.2 and answer.
2. See subsection 16.2.3 and answer.
3. See subsection 16.2.1 and answer.

Check Your Progress 2

1. See Section 16.3 and answer.
2. See Section 16.3 and answer.
3. See Section 16.3 and answer.

Check Your Progress 3

1. See Section 16.4 and answer.
2. See Section 16.4 and answer.
3. See Section 16.5 and answer.