
UNIT 3 RISK AND FINANCIAL ASSETS

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3.0 OBJECTIVES

After going through this unit, you will be able to:

- Define financial assets;
- Explain the impact of uncertainty on financial decision-making;
- State the measures of risk of financial assets; and
- Discuss and critically examine the theories of pricing of assets.

3.1 INTRODUCTION

In the previous unit, you were acquainted with the meaning of financial markets, institutions and instruments. You have come to learn about financial flows and equilibrium. In this unit, we look theoretically at the concept of an asset, and see how it differs from an ordinary consumption good. We understand what a security is, and how these are traded in situations of risk.

Two defining characteristics of financial markets are the time dimension enters explicitly into the picture, and there is a varying degree of risk present. In the market for many financial assets, risk is pervasive. This unit concentrates on the analysis of uncertainty and risk. Of course, the time dimension is occasionally brought in, but that is only central in a few theories. The next unit will leave out uncertainty and risk

from the picture, and discuss the time dimension, the time value of money and the determination of interest rates. This unit is primarily about risky assets.

To set the ball rolling, in this section, we look at the concept of an asset and find out how it differs from an ordinary good; this gives us an idea of the concept of investing in an asset.

Investment, in general terms, can be defined as sacrifice of current money or resources for benefits in the future. Very simply stated, **investment** is deferred consumption. Investment is done in assets. What are assets? **Assets** are claims to resources. These can be real or financial. **Real assets** are assets that are physical in nature, like real estate while financial assets are claims. Physical assets are also called **tangible assets** while **intangible assets** are legal claims to some future benefits. The entity that has agreed to make future cash payments is known as the issuer of the financial asset, while the owner of the financial asset is called the investor.

The claim that the holder of a financial asset has may be either a fixed amount or a varying (or residual) amount. In the first case, the financial asset is called a **debt instrument**, while in the latter case it is called an **equity claim**. Some financial assets display characteristics of both types. Those among these that pay a fixed amount are called **fixed-income instruments**. Debt instruments, equity claims, and instruments whose prices are derived from that of some other underlying assets (these are called derivatives, are together called **securities**.

The main roles of financial assets are: first, to transfer funds from those who have surplus funds to those who require funds to invest in tangible assets, and secondly, to redistribute and spread the risk associated with the cash flow generated by tangible assets among those seeking and those providing the funds. Of course, these functions are carried out through financial intermediaries and financial markets.

In the subsequent sections, the unit discusses the risk and returns associated with assets; some important theories of asset pricing; the concept of efficient financial markets; and finally, decision-making under uncertainty.

3.2 FINANCIAL ASSETS: RISK AND RETURN

We are concerned with the meaning, process and method of valuation of assets. Although we discuss theories of asset pricing in detail in the next section, in this section we set out some basic principles in this section.

Risk is one of the basic factors that affect the prices of securities. In asset markets, there are market risks. Risk is associated with a possibility of loss. It is associated with uncertainty. Earlier, some economists like Frank Knight used to distinguish between situations of uncertainty and that of risk. The latter were situations where individuals can list the possible outcomes and can assign probabilities to these outcomes. We now turn to methods that attempt to quantify risk and return to an asset.

3.2.1 Ex Post (Historical) Return

Let us begin with returns. Let R stand for the return from the asset, let P be its price, and assuming we are talking of stocks, let D be the dividend paid out on the stock. Then, the return from holding the stock in the year 2007, say, is given by

$$R_{2007} = \frac{(P_{2007} - P_{2006}) + D_{2007}}{P_{2006}}$$

Thus, for stocks, the return for a particular time period is equal to the sum of the price change plus dividends received, divided by the price at the beginning of the time period. Assuming there are many stocks, we can have the general measure of returns for the i^{th} stock, for the time period $t-1$ to t :

$$R_{it} = \frac{(P_{it} - P_{i,t-1}) + D_i}{P_{i,t-1}}$$

Suppose we are concerned only with the i^{th} stock and are interested in obtaining a measure of historical performance of his stock, that is a measure of average returns on this stock over the time period $t = 1, 2, \dots, T$. We get is straightforward arithmetic mean:

$$\bar{R}_i = \frac{1}{T} (R_{i1} + R_{i2} + R_{i3} + \dots R_{iT})$$

This can be written more compactly as:

$$\bar{R}_i = \frac{1}{T} \sum_{t=1}^T R_{it}$$

Of course, other than finding out the average returns over time for a single stock, we can as well obtain the average returns for several stock for a *single* time period. The method is the same, except that we aggregate over the number of shares rather than number of time periods. Let there be n shares: $i = 1, 2, 3, \dots, n$. Then

$$\bar{R} = \frac{1}{n} \sum_{i=1}^n R_i$$

When we consider the average returns of a single share over several time periods it is better to use the geometric mean:

$$GM = \left[(1 + R_1)(1 + R_2) \dots (1 + R_T) \right]^{1/T} - 1$$

The concept of geometric mean is very closely related to that of compound growth rate, as for example, compound interest. This will be explained in greater detail in the next unit.

3.2.2 Ex Post (Historical) Risk

In investment analysis, basically risk is associated with variability of rates of return. Variability is usually measured as individual returns in relation to the average. In statistics, one of the basic measures of variability is the variance. The positive square root of the variance is the standard deviation, usually denoted by the lower-case Greek letter sigma (σ). The variance (square of standard deviation) is defined as:

$$\sigma^2 = \frac{1}{T-1} \sum_{t=1}^T (R_{it} - \bar{R}_i)^2$$

Thus the variance can be considered as the average square deviation from the mean return. To calculate the variance, we first calculate the mean return. Then the difference between the return for each period and the mean return is obtained. These deviations from the mean are squared and added together. This sum is divided by $T - 1$ (the total number of time periods minus one).

The standard deviation is the positive square root of the variance:

$$\sigma = +\sqrt{\sigma^2}$$

We can think of the standard deviation as the average deviation from the mean.

3.2.3 Ex Ante (Expected) Risk and Return

We have discussed the return and risk of securities *after* the event (actual). But actually when an investor is contemplating whether to invest in a stock, she is certainly interested in the historical track record of the risk and return of that stock. But the investor is also interested in the future return of a stock. Thus expectations about future return and risk is very important for the investor. We describe the expectation about the future in terms of probability distributions. A probability distribution is just a listing of the various alternatives and the probability of each alternative occurring. if we roll a die, any one of the numbers 1 to 6 can turn up, each with a probability of $1/6$. this is an example of discrete probability distribution. Probability distributions can also be continuous, as the normal distribution. In this case, we speak of probability of occurrence of a certain range of values, as the probability that a single value occurs is zero.

We use the concept of expected value when using probability distributions. Let us suppose there are different states of the world that affect the return of a stock (for example, booms or depressions, different economic policies etc) and that there is a certain probability of each state occurring. Let there be S states, indexed by s , that is, $s = 1, 2, 3, \dots, S$. Let π_s be the probability of state s occurring. (probabilities of the different states sum to 1 since one of the states is certain to occur). Then if R_{is} is the return on the i th stock when state s occurs, the expected value of the i th share is:

$$E(R_i) = \sum_{s=1}^S R_{is} \pi_s$$

Thus the mean (expected value) of ex-ante returns is just a weighted average of the conditional returns where the weights are the probabilities of the occurrence of the states of the world.

The variance of ex-ante data is computed according to the following formula:

$$\sigma_i^2 = \sum_{s=1}^S [R_{is} - E(R_i)]^2 \pi_s$$

The standard deviation is $\sqrt{\sigma^2}$. Thus the standard deviation can be thought of as the weighted average of the potential deviations from the expected return.

3.2.4 Risk and Return of a Portfolio

We have been discussing the ex-ante and ex-post risk and return associated with a

single stock. However, investors often hold a portfolio of assets. We shall be discussing the portfolio theory of asset pricing in greater detail in the next section. However, here we set out the basics of risk and return associated with a portfolio of assets. This type of analysis was pioneered by Harry Markowitz. Markowitz observed that investors do not always try to maximize returns. If they wanted to do so, they would simply hold only that security which they expected would give the highest returns. Thus investors are concerned both with return and risk, and since they hold a portfolio of assets, it showed that diversification can lower risk without adversely affecting returns.

The return for a portfolio is simply a weighted average of the returns of the securities in the portfolio. For a single time period t , the portfolio return is calculated as:

$$R_{pt} = \sum_{i=1}^n R_{it} W_{it} \quad \text{where } W_{it} \text{ is the market value of the } i^{\text{th}} \text{ asset divided by the}$$

market value of the entire portfolio.

For ex-ante calculations:

$$E(R_p) = \sum_{i=1}^n R_i W_i$$

the variance of a portfolio is a little complicated because we also have to consider any two assets of a portfolio together. The general formula for variance of a portfolio is

$$\sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n \text{Cov}_{ij} W_i W_j$$

Where Cov_{ij} represents the covariance between any two assets i and j .

We can calculate the correlation coefficient :

$$\rho_{ij} = \frac{\text{Cov}_{ij}}{\sigma_i \sigma_j}$$

The correlation coefficient always lies between -1 and $+1$ and is a measure of the strength of the linear association between assets i and j . A value of -1 or $+1$ shows perfect linear relation (the former an inverse relation) while a value of 0 shows no relationship. You have studied about these concepts in the course on quantitative methods in the first year.

Check Your Progress 1

- 1) Describe how ex-post return and average return on a security is calculated.

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- 2) What is the basic measure of ex-post risk?

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- 3) How would you find ex-ante risk of an asset, and how would you find the risk of a portfolio?

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3.3 THEORIES OF ASSET PRICING

Having grasped the concept of risks and returns associated with financial assets, let us now look at some theories that have been put forward to explain how prices of financial assets are formed and determined. We discuss three main theories, mean-variance portfolio theory mainly associated with Harry Markowitz, who put it forward in the 1950s; the Capital Asset Pricing Model, the basis of which was laid by William Sharpe in 1964, by John Linter in 1965, and Jan Mossin in 1966; and finally, the Arbitrage Pricing Theory developed by S.A. Ross in 1976. Markowitz and Sharpe have received the Nobel Prize in economics for their pioneering theories.

3.3.1 Portfolio Theory

Markowitz's work on portfolio theory considers how an optimizing investor would behave. This theory asserts that in constructing a portfolio of assets, investors seek to maximize the expected return from their investment given some level of risk they are willing take. Those portfolios that satisfy this requirement are called efficient portfolios. Markowitz's theory explains how this should be done.

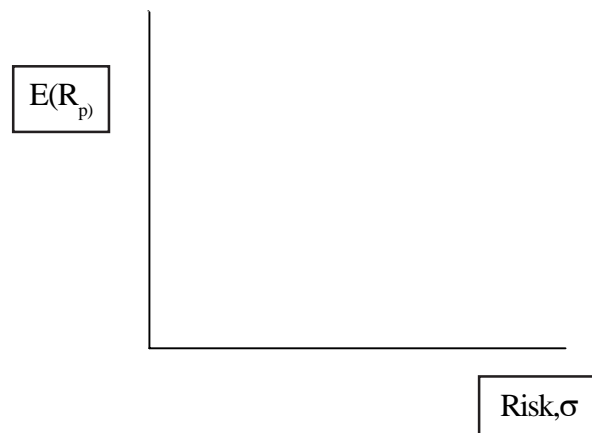
Markowitz made certain assumptions regarding the behaviour of investors. These are as follows:

- 1) Investors choose portfolios on the basis of their expected mean and variance of return.
- 2) Investors are risk-averse expected utility maximisers (we shall study more about risk aversion and expected utility maximization in the next section)
- 3) Investors have a single-period time horizon, and it is the same for all investors.
- 4) Investors have identical expectations about expected returns, variances, and covariances for all risky assets.

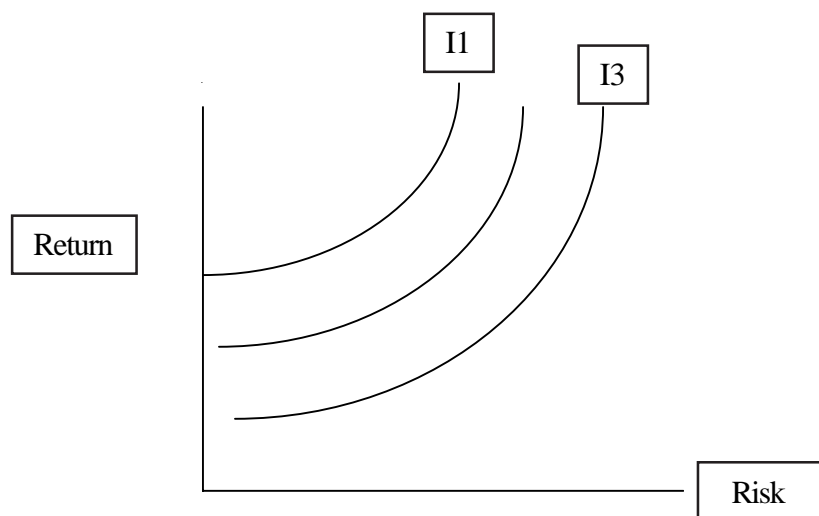
Basically, Markowitz suggested the following procedure for choosing optimal portfolio of risky assets: mark out the set of efficient portfolio; specify the return-risk indifference

curves; and choose the optimal portfolio. This is elaborated below.

The first thing to do is visualise a diagram where expected returns are depicted on the vertical axis and risk (variance on the horizontal axis):



Now the thing to notice is that if we drew indifference curves on this space, (the thing on the horizontal axis is a ‘bad’ (a ‘bad’ is something the more you consume of, the less utility you get—like pollution. Here risk is a ‘bad’). Thus if we were to draw indifference curves on the return-risk space, they will be convex –but upward sloping! They would look like the following:



All points on an indifference curve provides the same level of satisfaction. The steeper the slope of the indifference curve, the greater the degree of risk aversion. As one moves *leftward* across different indifference curves, the utility increases. Indifference curve I1 gives a higher level of satisfaction than indifference curve I3.

Now let us see how the set of efficient portfolio (what is called the efficient frontier) is delineated. Given the same level of risk, different portfolios will have different returns. The investor will select the portfolio with the with the greatest expected return for a given level of risk. Usually the procedure for obtaining the highest expected return for a given level of risk is found by using a complicated mathematical technique called quadratic programming, which is an optimisation technique. It is beyond the scope of our discussion. We can give an intuitive explanation.

Given the assets that are available, an investor can create many possible portfolios. Any portfolio that can be created from the available assets is called a feasible portfolio.

The collection of all feasible portfolios is called the feasible set of portfolios. However only a subset of the frontier of the feasible set of portfolios will be the Markowitz efficient set of portfolios. Finally to choose a single portfolio, the investor will choose that point on the return-risk space where the Markowitz efficient set of portfolios is tangent to an indifference curve. This is the optimal portfolio.

3.3.2 Capital Asset Pricing Model

The works by Sharpe by Lintner and by Mossin on the Capital Asset Pricing Model (CAPM for short) is concerned with economic equilibrium assuming all investors optimize in the particular manner that Harry Markowitz proposed. The CAPM is an equilibrium model of asset pricing. As such the CAPM provides an understanding of the behavior of security prices, the risk-return relationship, and the appropriate measure of risk for securities.

In the CAPM some additional assumptions over and above those made for Markowitz's portfolio theory, are made. These are, first, that unrestricted borrowing and lending can take place at the risk free rate; secondly, that investors have homogeneous expectations regarding the means, variances, and covariances of security returns; and finally, that there are no imperfections in the capital market such as transaction costs, and also that there are no taxes.

Given these assumptions, the implications are that there is a capital asset pricing model that consists of a capital market line and a security market line. To understand this, we must first remember that there are some assets that are risk free. In Markowitz's theory, no risk-free asset was considered: portfolio theory suggests that efficient portfolios can be constructed using expected returns and variance. Once a risk-free asset is brought into the picture, and assuming the investor can borrow and lend at the risk-free rate (an assumption explicitly made in the CAPM) the picture changes. It can be shown that the investor can reach a point with higher expected returns than with merely the Markowitz efficient frontier. The investor will select a portfolio on an upward sloping line in the (expected) return-risk plane [the line starts on the y-axis at a point on the expected-return axis (y axis) that depicts the risk-free return and slopes upward, tangent to the Markowitz efficient curve at a certain point. This line is called the capital market line. To the left of the point of tangency of the capital market line to the Markowitz efficient portfolio curve, there will be points on the line vertically above point on the Markov efficient frontier. This shows that with risk-free asset in the portfolio, the investor will select a portfolio on the line, representing a combination of borrowing or lending at the risk-free rate and purchases of Markowitz efficient portfolio. There is a theorem that says that all risk-averse investors will hold a combination of the risk-free asset and market portfolio, known as the two-fund separation theorem.

3.3.3 The Arbitrage Pricing Theory

The CAPM has the limitations that it is based on certain restrictive assumptions and that market factors are not the sole factor influencing stock returns. Stephen Ross put forward the Arbitrage Pricing Theory (APT) in 1976 to address the shortcomings of the CAPM. It is a unique approach to determining asset prices.

The CAPM assumes that investors decide within a mean-variance framework. The APT is a more general approach in that it assumes that asset prices can be influenced by factors other than mean and variances. APT makes certain assumptions that are the same as those made by CAPM. These are: investors have homogeneous beliefs;

that investors are risk-averse utility maximisers; and that markets are perfect so that there are no transactions cost. However, unlike the CAPM, the APT does not assume a single-period investment horizon; it does not assume there are no taxes; it does not assume that investors can freely borrow and lend at the risk-free rate; and finally, it does not assume that investors select portfolio on the basis of mean and variance of a return.

APT makes an additional assumption which is not made under CAPM: it assumes that security returns are generated according to what is called a factor model. This means that there are underlying factors that give rise to returns on stocks. These may include the inflation rate, the rate of growth of GNP, or financial variables like capital structure and dividends. The theory assumes that the return on any stock is a linear function of these factors that are also called systematic factors or risk factors.

Ross employed an arbitrage argument to develop a model of equilibrium asset pricing. The arguments and mathematics are quite complex and advanced and beyond the scope of our discussion. We give the basic form of equations in the APT model. The equation depicts return on a stock as a linear function of the underlying factors:

$$R_i = \alpha_i + \gamma_{i1}F_1 + \gamma_{i2}F_2 + \dots + \gamma_{ij}F_j + \dots + \gamma_{in} + \varepsilon_i$$

where R_i = return on stock i

α_i is the expected return on stock I if all factors have a value of zero.

F_j is the value of jth factor which influences the return on stock I

γ_{ij} is the sensitivity of stock I's return to the jth factor

ε_i is a random error term

Given the return-generating process in the above equation, the APT derives an equilibrium risk-return relationship. The key idea underlying this derivation is the law of one price, which says that an identical good cannot sell for different prices (there should be no arbitrage). Applied to portfolios it means that two portfolios that have the same risk cannot differ in terms of expected returns. If it were so some people will take advantage of the situation and buy cheap and sell dear. In other words, arbitrage would take place. Hence in equilibrium there should be no scope of arbitrage.

The equilibrium equation according to the APT is as follows:

$$E(R_i) = \lambda_{rf} + \gamma_{i1}\lambda_1 + \dots + \gamma_{ij}\lambda_j + \dots + \gamma_{in}\lambda_n$$

where λ_{rf} is return on risk-free asset, λ_{ij} is the risk premium for the type of risk associated with factor j

$E(R_i)$ expected return on stock i

Check Your Progress 2

- 1) Explain the concept of an efficient portfolio.

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- 2) Compare the assumptions needed for the CAPM with those needed for Markowitz's portfolio theory.

- 3) Compare the advantages and disadvantages of the APT and the CAPM.

3.4 THE EFFICIENT MARKET HYPOTHESIS

Eugene Fama introduced the efficient markets hypothesis (EMH) in the finance literature in the mid-1960s. In this theory the word 'market' refers exclusively to financial markets. The basic idea behind this hypothesis is that pricing in efficient financial markets is such that owners of debt and equity securities do not reap any extra-normal returns and benefits. The concept is very similar to that of perfect competition in goods market. Of course, the efficient market hypothesis is based on situations of risk and uses probability theory.

This hypothesis states that because of intense competition in financial markets, pricing of assets would be 'fair'. The efficient market hypothesis postulates that in an efficient market, the current price of an asset reflects all available information about the value of that asset. This means that if we want to draw a trend line, say, of share prices and want to forecast the future price of a share to make some profits, we would gain nothing. The reason is that this theory suggests that others can do the same thing then no one can be persuaded to buy from us so that we gain. In general terms, the EMH suggests that the risk-adjusted expected returns on all investments would be equal. The return that we expect to earn on a particular stock exactly equals the return that we could earn on any other share with similar risk characteristics. If risk-adjusted returns on all financial assets are equal, we cannot beat the market by picking out a share that we think will do better than other stocks in the market. By chance, we may pick one that does better than average, but the probability of picking one that does worse than average is equally large. That is why the EMH suggests it is better to have a diversified portfolio of assets so that on average we have equal chances of picking a good stock and a bad one.

An important idea behind the EMH is the using of information and forming of expectations about the future value of a variable (in the present case, asset prices).

Before proceeding further, let us first clarify the notion of expectation and expectation formation.

We always want to have an idea about the future value of a variable, since there is uncertainty present and we do not have perfect foresight. For instance, we want to know if prices are going to go up, or if there is inflation, whether inflation will persist. For asset prices too, investors are keen to know how asset prices will perform. We look at three ways of expectations formation. : Markov expectations, adaptive expectations and rational expectations. Each method involves a different level of sophistication.

The simplest way to form expectations is the Markov expectations. Put baldly, the Markov expectations suggests the immediate past will continue into the next period. Suppose we denote by P_t the price of an asset at time t ; then the expected price of the asset for time $t + 1$ formed at time t will be P_t . Let us denote by P_{t+1}^e the expected value of P for time $t+1$. Then the Markov expectations hypothesis simply states that

$$P_{t+1}^e = P_t$$

In other words, we simply use the most recent value of P to forecast future values. The most obvious shortcoming of the Markov expectations hypothesis is that it does not take into account knowable events that might change the future environment.. if the price is not only rising, but rising at a rising rate each time period, then the Markov expectations hypothesis will not only make systematic error, but the forecast error will rise. However, the Markov expectations hypothesis is a reasonable method of forming expectations at a time when the forecasting environment is stable. If, for instance, there is no inflation, then it might be reasonable to expect that the general price level will stay the same.

The adaptive expectation hypothesis, as the name suggests, says that people form expectations but when forming further expectations, they revise or adapt subsequent expectations in the light of forecast errors that have taken place. Let us try to understand it as follows. Let x be a variable, say the price for a share. Let x_t^e be the expectation about the price of the share that the investor has regarding the current time. Of course this expectation was formed in the last period, at $t-1$. Suppose 2007 is the current year. In 2006 the investor formed an expectation about share price to prevail in 2007. Let x_t be the actual price of the share that prevails in 2007 (of course, prices of shares do not change once a year, and people do not form an expectation once a year, but this is merely for ease of exposition. You can easily think of the previous time as last month, last fortnight, last week or yesterday).

Now, the investor has to now make an expectation about the price of the share for next year. So in 2007 the investor makes a forecast about price of the share in 2008. Let x_{t+1}^e be the expected price of the share for 2008. The adaptive expectations hypothesis says that that will depend on the forecast error of last time, that is, on . This hypothesis says that (the difference between the expectation for $t+1$ and that for t) will be influenced by . We can write this formally as:

$$x_{t+1}^e - x_t^e = \alpha (x_t - x_t^e)$$

Here α is called the speed of adjustment, and this shows how quickly new expectations adjust to past forecast errors. The above equation can be rewritten as:

$$x_{t+1}^e = x_t^e + \alpha (x_t - x_t^e)$$

This equation states that the expectation formed now about next period equals the expectation formed in the last time period about the current time period plus a term that adjusts this old expectation in the light of past forecast.

The speed of adjustment, α , takes values between 0 and 1. If it equals 0, then adjustment never takes place, and if it equals 1, then adjustment is instantaneous and hence equals x_t from the first equation. Here expectation formation becomes Markov in nature.

We might wonder why the speed of adjustment is not always equal to 1. The reason is that not all changes are permanent. A variable may increase at a faster rate for some time, and then change slowly or not at all for some time.

Some basic points about the adaptive expectations should be noted. First, like Markov expectations, adaptive expectations relies on past actual and expected values of only one variable for forming expectations and ignores the effect of other variables on this variable. Thus all current information is not used in making expectations. Secondly, even in response to a permanent change in the value of a variable, the adjustment is gradual. Expectations take a long time to converge to actual values. When the change in actual values is very small and for a very short period, expectations overreact. Thus, both in Markov expectations and adaptive expectations, there is the possibility of systematic error.

It is these weaknesses that the hypothesis of rational expectations tries to correct. This hypothesis was first put forward by John Muth in 1961 but it was not used much in the finance and macroeconomics literature until the early 1970s. The rational expectations hypothesis is the most sophisticated method of expectations formation. The rational expectations hypothesis states that in expectations formation, all relevant information is taken into account in forming expectations. This information would include all variables that the relevant economic theory would suggest has an influence on the variable in question.

In your study of quantitative methods in MEC-003, you have learnt about probability theory. There you studied about mathematical expectation and conditional expectation. Rational expectations hypothesis says that the subjective expectations formed by say, an investor about the future price of the share, will be exactly equal to the mathematical conditional expectation suggested by economic theory. The rational expectations hypothesis says, in effect, that if we as economists are theoretically studying decision-makers in the economy, why would we ever think of modelling these decision-makers as being less intelligent than the theorist who is studying them. If the theorist knows about probability theory and about conditional expectations then economic agents ought to be modelled as though they too know these. The rational expectations hypothesis suggests that if an economic agent rolls a die, she would know that the probability of a particular number from 1 to 6 appearing is $1/6$; if she were to toss a coin then she knows that the probability of a head appearing is $1/2$. Economic agents may not know the actual value of variables but they know the probability distribution of these variables.

People sometimes have one misconception about rational expectations hypothesis. They assume that the hypothesis suggests that an economic agent who forms expectation using rational expectations can correctly forecast the actual value of the variable; the assumption is that rational expectations implies perfect foresight. This is not correct. Even under rational expectations there can be forecast errors; however, these will be purely random in nature, implied only by probability or chance.

Why have we discussed expectations formation and distinguished among these? It is because the EMH is one variant of the most sophisticated among the different expectation formation hypotheses, namely, the rational expectations hypothesis. Let us now discuss EMH in greater detail, armed as we are with the theory of expectations formation. There are three variants of the EMH that we discuss in some detail now.

The EMH studies how expectations of investors are translated into security prices. Are investor expectations about future cash flow accurately and quickly reflected in prices of securities? In an efficient market, the current price of securities should reflect the unbiased estimates of the intrinsic prices of the securities. If all securities are fairly valued, investors will earn a return on their investment that is appropriate for the level of the risk assumed, regardless of which security they invest in. the assumption is that there are lots of well-informed market analysts who make all relevant information correctly and costlessly to investors. But the paradox is that since there is too much competition among these analysts, no one is able to beat the market. This paradox is somewhat like perfect competition of microeconomic theory where competition is so 'perfect' that no seller is able to make extra-normal profits.

There are three versions of the EMH. These have been suggested by Fama. The first is the Weak-form EMH. In this the only type of information being considered are the historical prices. If the weak-form EMH holds, investors should not be able to earn supernormal profits just by observing historical prices of securities. Time-series analysis of security prices is vulnerable to weak form EMH. Also technical analysis, which consists of plotting charts of security prices over the recent historical past, cannot much help in making extra profits in security markets.

The second category of EMH is the so-called semi-strong version of the EMH. This asserts that security prices adjust quickly and accurately to all *publicly* available information. This information pertains not only to security prices, but also about items like dividend payouts, inflation rates, earnings reports of firms, and so on. Any information that is publicly available should be quickly reflected in security prices so that investors cannot consistently make extra-normal profits. The semi-strong form EMH suggests that investors cannot gain much from the services of security analysts. But paradoxically, it is their presence and intense competition among them that makes the market efficient.

The final category of EMH, the strong-form EMH represents the most extreme case of market efficiency. Under the strong EMH form it is argued that security prices reflect all information, both public and private (monopolistic) information. This is an extreme form. In reality this form is hardly satisfied. There is often 'insider trading' as for example Ivan Boesky in the USA earned millions of dollars each year for several years in the 1980s by cashing in on inside information about corporate takeovers.

An interesting feature of the EMH, particularly in the strong form, is that use some version of Markov expectations for predicting asset prices. In fact, in some version, the Markov expectation becomes rational expectations.

Check Your Progress 3

- 1) What is an efficient financial market?

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- 2) Distinguish among Markov, adaptive, and rational expectations, providing an intuitive explanation of each.

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- 3) Distinguish among the three levels of market efficiency.

3.5 DECISION-MAKING UNDER UNCERTAINTY

We just saw what an asset means. We have also discussed asset pricing theories and the features of efficient markets. Markets of several types of assets are characterised by uncertainty. In this section, optimal decision-making in the presence of uncertainty is explained.

We had mentioned earlier that some economists in the days gone by had made a distinction between situations of risk and that of uncertainty, with the former being situations where the individual could list the outcomes and assign probabilities to these, while uncertainty denoted situations where no such outcomes could be listed and moreover, probabilities could not be assigned. Nowadays such distinctions are not made because, if followed, it would imply that no theoretical analysis would be possible of situations of uncertainty. So what was earlier called situations of risk is now routinely called situations of risk and situation of uncertainty interchangeably. So we basically set out the theory of decision-making under situations of risk; in many expositions these are also called decision-making under situations of uncertainty.

What this section does is to take the theory of individual decision-making as set out in microeconomics and extend it to situations of uncertainty. The basic point about decision-making in microeconomic environments is that it assumes that the decision-maker is rational in the sense that the decision-maker wants to optimise (maximise or minimise) certain objective function subject to constraints. For example, the consumer maximises utility subject to the budget constraint while the producer maximises profits or minimises costs. We assume the individual investors, like the consumer wants to maximise their utility. But while in basic theory of the consumer it is assumed that the consumer's utility function is a function of the goods consumed (more the goods consumed, greater the utility—"more is preferred to less") in situations of uncertainty, utility is a function of something more complex. This is explained below.

While discussing asset pricing theories and mean-variance portfolio analysis, we have discussed decision-making under uncertainty. While discussing portfolio selection we mentioned that the investor looks at both the mean returns and risk of the asset,

where the variance was taken as a measure of risk. In this section we look at the decision-making by an individual under uncertainty, while keeping utility maximisation as the basic behavioural characteristic.

We can think of the different outcomes of some random event as being different states of nature. We just modelled states of nature in the mean- variance portfolio choice. We next think of a contingent consumption plan as being a listing of what the consumer will consume in the different states of nature each of which is a different outcome of the random process. Consumer/investors have preferences over different plans of contingent consumption, just as they have over actual consumption.

Since the consumer has preferences about consumption in different states, we can use a utility function to describe these preferences. The key here is to frame an appropriate concept of utility function, which will depict correctly the decision-making under uncertainty. We will presently see that the expected utility is the appropriate concept. Presently we will discuss the properties of the expected utility and also certain axioms that expected utility should satisfy. Subsequently, we will develop the concept of attitude towards risk—that is, whether the individual is risk-averse or risk lover or risk-neutral on the basis of the curvature of the expected utility function.

Let us now develop the notion of expected utility. For this, let us observe the basic thing that investor/consumer is concerned with probability distribution of getting different consumption bundles of goods. We say utility is a function of wealth, but wealth or consumption has probability distribution, and the consumer has to choose a pattern of probability distribution. People have different preferences over probability distribution the same way that they have preferences over goods. In situations of uncertainty, we can think of monetary rewards in different situations as different goods.

For simplicity, consider two mutually exclusive states s_1 and s_2 and let π_1 and π_2 be probabilities of the occurrences of these states. Since there are only two states, .

We can write the utility function as:

$$U = f(C_1, C_2, \pi_1, \pi_2)$$

Since the c 's are consumption, and the π s are probability, a function such as gives utility as a function of the expected, or average, value of consumption.

If on the other hand we have a utility function like

$$U = \pi_1 v(C_1) + \pi_2 v(C_2)$$

we have the right-hand side expressing the average utility, or expected utility function. This is also known as the von Neumann-Morgenstern utility function after the mathematician von Neumann and economist Oskar Morgenstern, who suggested this form. The expected utility function is a reasonable depiction of utility from consumption under uncertainty because only one of the states s_1 or s_2 will occur and if s_1 occurs, π_2 will be zero and $U = v(C_1)$ and similarly for state 2. A strong assumption is made about the utility function: it is that the choice individuals make in one state of nature is independent of the choices they make in another state of nature. This is called the **independence assumption** and it implies that the utility function is additive separable.

There are certain properties of the expected utility function. The **first** is that more is preferred to less, that is, the utility function is an upward sloping function of wealth. The first derivative of utility with respect to wealth is positive. The **second property**

of a utility function is an assumption regarding an investor's taste for risk. An investor can be risk averse, be neutral to risk or a risk lover, that is, risk seeker. These can be explained in terms of a fair gamble. A fair gamble is one where the expected returns are equal to the cost of undertaking the gamble. For example, let the cost of playing the gamble be rupee 1. let there be a 0.5 chance of winning rupee 2 and 0.5 chance of winning nothing. We can see that the expected value of winning is $0.5 \times 2 + 0.5 \times 0 = 1$, which is equal to the cost of playing. If the person does not undertake the gamble he retains rupee 1 with certainty. Since the expected value of winning equals the cost, it is called a fair gamble.

Risk aversion means that an investor will reject a fair gamble because the disutility of loss is greater than the utility of an equivalent win. Utility functions that exhibit this property have a negative second derivative of utility with respect to wealth. This means the utility function is upward sloping but concave to the origin.

The **third property** of utility functions is an assumption about how the investor's preferences change with a change in wealth. We ask whether, as the investor's wealth increases, will she invest less or more of that wealth in risky assets? If the investor increases the *absolute* amount invested in risky assets as her wealth increases, then the investor is said to exhibit **decreasing absolute risk aversion**. If the absolute amount invested in risky assets remains constant as wealth increases the investor is said to display **constant absolute risk aversion**. Similarly for **increasing absolute risk aversion**. If wealth is shown by W , and U' and U'' are first and second derivatives of utility with respect to wealth, then a measure of absolute risk aversion is given by

$$A(W) = \frac{-U''(W)}{U'(W)}$$

The **final property** of the characteristic of the utility function or you may say the final restriction on the investor's utility function is to see how the *percentage or proportion* of wealth invested in risky assets changes as wealth increases. If this percentage increases as wealth of the investor increase, the investor is said to display **decreasing relative risk aversion**. You can work out the cases of increasing and constant relative risk aversion. Relative risk aversion is closely related to absolute risk aversion. The measure of relative risk aversion is

$$R(W) = \frac{-WU''(W)}{U'(W)} = WA(W)$$

If $R'(W)$ is negative it indicates the utility function shows decreasing relative risk aversion. Similarly for $R'(W) > 0$ and $R'(W) = 0$.

We have mentioned about decision-making under uncertainty. We have also seen in the previous section the uses of diversification. Here we just touch upon, as a way of closing our discussion, the benefits of diversification and risk spreading, and briefly mention an institution that helps in spreading and transferring risks: the stock market. The stock market plays a role similar to insurance. The stock market (primary market) allows the original owners of a firm to convert their stream of returns over time to a lump sum. The original owners of a firm can spread their risks by issuing shares to a very large number of people. Moreover, all the wealth of the original owners need not be tied up in a single enterprise: once the stock market has allowed them to convert their stream of returns to a lump sum, they can use it to invest in a variety of assets.

Check Your Progress 4

- 1) Explain the concept of expected utility.

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- 2) What do you understand by risk aversion?

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- 3) State the restrictions placed on the utility function in the case of decision-making under uncertainty.

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3.6 LET US SUM UP

This unit began the theoretical analysis of financial assets and prices in this course. In the introductory section, some concepts relating to financial assets were presented. The subsequent section went on to give a systematic analysis of returns and risk of assets the variance was shown to be an appropriate measure of risk. Both ex-ante (expected) as well as ex-post (historical returns and risk were discussed, both for a single asset as well as for a portfolio.

The next section discussed in considerable detail three important theories of asset pricing: the portfolio theory, the capital-asset pricing theory and the arbitrage pricing theory. The next section discussed the important idea of market efficiency. To do this, the concepts of Markov, adaptive and rational expectations were discussed. The three forms of market efficiency – weak, semi-strong and strong—were discussed.

Finally, the unit took up for discussion the theory of decision-making under uncertainty. In the course of this, the reason why expected utility would be reasonable as a depiction of choosing under uncertainty was discussed. Some properties of the utility function were presented, and absolute and relative risk aversion were explained, and the measures of these were explained.

3.7 KEY WORDS

Adaptive Expectations	: Expectations about future value of some variable such that these expectations evolve very slowly over time, and are revised in light of <i>past</i> experience only. These can give rise to systematic errors.
Arbitrage	: Buying a security in one market and immediately selling it in another at a higher price to gain advantage from the price differential.
Beta	: A symbol (the second letter in the Greek alphabet), and although used merely as a notation to represent the riskiness or volatility of a security or portfolio relative to the stock market riskiness, it has come to be used as a synonym for the <i>measure</i> of that volatility.
Capital Asset Pricing Model	: A theory which shows and explains the relationship between risk and return for efficient and inefficient portfolios.
Capital Market Line	: The relationship between expected rate of return and risk (measured by standard deviation) for efficient portfolios in the presence of some risk-free asset.
Efficient Market Hypothesis	: This hypothesis states that the current price of an asset reflects all available information about the value of that asset.
Financial Asset	: A paper representing claims on physical asset.
Portfolio	: A set or combination of assets held by an investor
Premium	: The difference between the face value of a security and the price at which it is actually sold (issued) in the market.
Rational Expectations	: Expectations about the future value of a variable such as price of an asset or the general price level in the economy, such that these expectations are formed taking into account all available information about the variable; these expectations are revised not only in light of past experience, but also current and new information that keeps being available or changes in the structure of the economy, or even a change in economic policy.

- Risk** : A situation where economic agents know the various possible outcomes, and can assign probabilities to these outcomes
- Risk Aversion** : A situation where a sure thing is preferred to a risky alternative with identical expected value.

3.8 SOME USEFUL BOOKS

Alexander, G.J., Sharpe, W.F., *Fundamentals of Investment*, Second Edition, Prentice-Hall, Englewood Cliffs, New Jersey. and Bailey, V.J. (1993)

Elton, E.J., and Gruber, M.J. (1997) [(2001) Asian Reprint] *Modern Portfolio Theory and Investment Analysis*, Fifth Edition, John Wiley and Sons, Singapore

Fama, E.F. (1972) *Foundations of Finance*, Basic Books, New York.

Markowitz, H.M. (1987) *Mean-Variance Analysis in Portfolio Choice and Capital Markets*, Basil Blackwell, New York.

3.9 ANSWERS/HINTS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress 1

- 1) See sub-section 3.2.1 and answer.
- 2) See sub-section 3.2.2 and answer
- 3) See sub-section 3.2.3 and answer

Check Your Progress 2

- 1) See section 3.3 and answer.
- 2) See section 3.3 and answer.
- 3) See section 3.3 and answer.

Check Your Progress 3

- 1) See sub-section 3.4.1 and answer.
- 2) See sub-section 3.4.2 and answer.
- 3) See sub-section 3.4.3 and answer

Check Your Progress 4

- 1) See section 3.5 and answer.
- 2) See section 3.5 and answer.
- 3) See section 3.5 and answer.