
UNIT 7 DERIVATIVES MARKETS

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7.0 OBJECTIVES

After going through this unit, you will be able to:

- explain the concept of a derivative;
- describe the various types of derivatives;
- discuss various models of derivative pricing; and
- describe the evolution of Derivatives in India.

7.1 INTRODUCTION

Derivative products like futures and options on Indian stock markets have become important instruments of price discovery, portfolio diversification and risk hedging in recent times. The volumes in derivative markets, especially in the case of National Stock Exchange (NSE), have shown a tremendous increase and presently the turnover in derivative markets is much higher than the turnover in spot markets. At the NSE, the total turnover in the cash segment was Rs.6, 95,049 crore during April-September 2005. The turnover in the NSE's derivative segment continued to be higher than in the cash segment. It increased by 59.2 per cent to Rs.17, 55,790 crore during April-September 2005 (Chart 53). Hence, it becomes increasingly important to know its intricacies.

7.2 DERIVATIVE: BASIC CONCEPTS

The term “Derivative” indicates that it has no independent value, i.e., its value is entirely “derived” from the value of the underlying asset. The underlying asset can be

securities, commodities, bullion, currency, live stock or anything else. In other words, Derivative means a forward, future, option or any other hybrid contract of pre determined fixed duration, linked for the purpose of contract fulfillment to the value of a specified real or financial asset or to an index of securities. Derivatives are also known as “deferred delivery of deferred payment instruments”. In a sense, they are similar to securitised assets, but unlike the later they are not the obligations, which are backed by the original issuer of the underlying assets or security.

With Securities Laws (Second Amendment) Act, 1999, Derivatives has been included in the definition of Securities. The term Derivative has been defined in Securities Contracts (Regulations) Act, as:

A Derivative includes:

- a) *a security derived from a debt instrument, share, loan, whether secured or unsecured, risk instrument or contract for differences or any other form of security;*
- b) *a contract which derives its value from the prices, or index of prices, of underlying securities.*

Eg. A stock option is a derivative whose value is dependent on a price of a stock. However, derivative can be dependent on almost every variable from price of hogs to the amount of snow falling at a certain ski resort.

The derivative contract also has a fixed expiry period mostly in the range of 3 to 12 months, from the date of commencement of the contract. The value of the contract depends on the expiry period and also on the price of the underlying asset.

For example, a farmer fears that the price of wheat (underlying), when his crop is ready for delivery will be lower than his cost of production. Let's say the cost of production is Rs 8,000 per ton. In order to overcome this uncertainty in the selling price of his crop, he enters into a contract (derivative) with a merchant, who agrees to buy the crop at a certain price (exercise price), when the crop is ready in three months time (expiry period). In this case, say the merchant agrees to buy the crop at Rs 9,000 per ton. Now, the value of this derivative contract will increase as the price of wheat decreases and vice-a-versa. If the selling price of wheat goes down to Rs 7,000 per ton, the derivative contract will be more valuable for the farmer, and if the price of wheat goes down to Rs 6,000, the contract becomes even more valuable. This is because the farmer can sell the wheat he has produced at Rs .9000 per tonne even though the market price is much less. Thus, the value of the derivative is dependent on the value of the underlying.

If the underlying asset of the derivative contract is coffee, wheat, pepper, cotton, gold, silver, precious stone or for that matter even weather, then the derivative is known as a commodity derivative.

If the underlying is a financial asset like debt instruments, currency, share price index, equity shares, etc, the derivative is known as a financial derivative.

Derivative contracts can be standardized and traded on the stock exchange. Such derivatives are called exchange-traded derivatives. Or they can be customized as per the needs of the user by negotiating with the other party involved. Such derivatives are called over-the-counter (OTC) derivatives.

From the example above: if he thinks that the total production from his land will be around 150 quintals, he can either go to a food merchant and enter into a derivatives

contract to sell 150 quintals of soybean in three months time at Rs 9,000 per ton. Or the farmer can go to a commodities exchange, like the National Commodity and Derivatives Exchange Limited, and buy a standard contract on wheat. The standard contract on wheat has a size of 100 quintals. So the farmer will be left with 50 quintals of wheat uncovered for price fluctuations.

However, exchange traded derivatives have some advantages like low transaction costs and no risk of default by the other party, which may exceed the cost associated with leaving a part of the production uncovered.

Economic Functions of Derivatives Market

- 1) They help in transferring risks from risk averse people to risk oriented people.
- 2) They help in the discovery of future as well as current prices.
- 3) They catalyze entrepreneurial activity.
- 4) They increase the volume traded in markets because of participation of risk averse people in greater numbers.
- 5) They increase savings and investment in the long run.

Check Your Progress 1

- 1) Explain the concept of a derivative.

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- 2) List the various functions of derivative markets.

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7.3 TYPES OF DERIVATIVES

In the financial markets, various derivatives are traded. To start with we need to understand the following three:

- 1) Forward
- 2) Futures
- 3) Options
- 4) Swaps

7.3.1 Forward Contract

Forward Contracts (forwards) have flourished for many centuries in many countries including India. It is the simplest of all derivatives. A forward contract is a customized contract between two entities, where settlement takes place on a specific date in the future at today's pre-agreed price.

It is a one to one bipartite contract, which is to be performed in the future at the terms decided today. One of the parties in the forward contract assumes a *long position* and agrees to buy the underlying asset on a certain specified future date for a certain specified period. The other party assumes a *short position* and agrees to sell the asset on the same date at the same price. Let us understand the concept with the help of an illustration.

Two parties enter into a contract to buy and sell 100 shares of Reliance at Rs 850 per share, two months down the line from the date of the contract. Assume A is the buyer and B is the seller. In the instant case, the product (shares of Reliance), quantity of product (100 shares), product's price (Rs. 850 per share), and time of delivery (2 months from the date of the contract) have been determined and well understood, in advance by both the parties concerned. The delivery and payment (settlement of the trade) will take place as per the terms of the contract on the designated date and place.

But there could be risk of default here, suppose the Reliance's price two months down the line goes up substantially, seller B would prefer to sell the share the markets rather than selling these shares to A as contracted; because, market would fetch him better price. Therefore he may default. Similarly, in case, price of Reliance goes down, buyer may choose to default because he may find he would find it attractive to buy Reliance share from the market at lower price, instead of honoring the contract. This way, both A and B are exposed to each other's risk of default.

Forwards markets are used in India on a large scale in foreign exchange market to hedge the currency risk. Forward contracts being negotiated by the parties on one to one basis, offer them tremendous flexibility to articulate the contract in terms of price, quantity, quality, delivery time & place. But because they are customized they are plagued with poor liquidity and default risk (credit risk) as explained in the example above.

Calculation of Payoffs from Forward Contracts

This example will help us understand it better, two parties might agree today to exchange 500,000 barrels of crude oil for US \$ 42.08 a barrel three months from today.

A forward contract is specified with four variables:

- 1) the underlier,
- 2) the notional amount n ,
- 3) the delivery price k , and
- 4) the settlement date on which the underlier and payment will be exchanged.

In our example, oil is the underlier. The notional amount is 500,000 barrels. The delivery price is US \$ 42 per barrel. The settlement date is the actual date three months from now when the oil will be delivered in exchange for a total payment of US \$ 21.04 MM.

The party who receives the underlier is said to be long the forward. The other party is short. At settlement, the forward has a market value given by

$$n(s - k) \quad [1]$$

where s is the spot price of the underlier at settlement. This formula derives from the fact that, at settlement, the long party is paying a delivery price k for an underlier

then trading at price s . The difference between those two prices, multiplied by the notional amount, is the market value of the forward.

Formula [1] tells us that forwards have linear payoffs.

A forward may be cash settled, in which case the underlier and payment never exchange hands. Instead, the contract settles with a single payment for the market value of the forward at settlement, as given by [1]. If the market value is positive, the short party pays the long party. If it is negative, the long party pays the short party.

Suppose the forward in our oil example were cash-settled. On the settlement date three months from today, no oil would change hands, and there would be no payment of US \$ 21.04MM. If the spot price at settlement were, say, US \$ 47.36, then the forward would settle with a single payment of

$$500,000(47.36 - 42.08) = \text{US } \$ 2.64\text{MM} \quad [2]$$

made by the short party to the long party.

Forwards are generally quoted as delivery prices, which are called forward prices. Forward prices fluctuate with market conditions. When a forward is entered into, the contract's delivery price is set equal to the quoted forward price. That delivery price then remains fixed until the forward settles. For example, a dealer might quote a three-month oil forward at 41.25/41.29. Those are the bid and offer forward prices. If a counterparty accepts the offer price for 500,000 barrels, then the delivery price on that contract will be USD 41.29.

Issues such as the time value of money, short-term supply and demand, market expectations of future spot prices and cash-and-carry arbitrage tend to make forward prices diverge from spot prices, but relevant factors vary from one market to the next. A graph of forward prices for different maturities is called a forward curve.

7.3.2 Futures

Futures contract means a legally binding agreement to buy or sell the underlying security on a future date. Future contracts are the organized/standardized contracts in terms of quantity, quality (in case of commodities), delivery time and place for settlement on any date in future. The contract expires on a pre-specified date which is called the expiry date of the contract. On expiry, futures can be settled by delivery of the underlying asset or cash. Cash settlement enables the settlement of obligations arising out of the future/option contract in cash.

Unlike forward contracts, futures are normally traded on an exchange. These markets being organized/standardized, are very liquid by their own nature. Therefore, the liquidity problem, which persists in the forward market, does not exist in the future market. In future markets, clearing corporation /house becomes the counter party to all the trades or provides the unconditional guarantee for their settlement i.e. assumes the financial integrity of the entire system. In futures markets, clearing corporation/house maintains the accounts of all the operations in the market. So it is in a position to tell in the last trading day of the contract, who two are the counter parties to each other and provides the solution to the settlement problem, which is very acute in the forward market.

The following example will help to understand the concept in a better way. Referring to the earlier example of A & B entered into a contract to buy and sell Reliance shares. Now, assume that this contract is taking place through the exchange, traded

on the exchange and clearing corporation/ house provides the unconditional guarantee for its settlement, it would be called a future contract.

The fundamental difference between futures and forwards is the fact that futures are traded on exchanges. Forwards trade over the counter. This has three practical implications:

- 1) Futures are standardized instruments. You can only trade the specific contracts supported by the exchange. Forwards are entirely flexible. Because they are privately negotiated between parties, they can be for any conceivable underlier and for any settlement date. Parties to the contract decide on the notional amount and whether physical or cash settlement will be used. If the underlier is for a physically settled commodity or energy, parties agree on issues such as delivery point and quality.
- 2) Forwards entail both market risk and credit risk. A counterparty may fail to perform on a forward. With futures, there is only market risk. This is because exchanges employ a system whereby counterparties exchange daily payments of profits or losses on the days they occur. Through these margin payments, a futures contract's market value is effectively reset to zero at the end of each trading day. This all but eliminates credit risk.
- 3) The daily cash flows associated with margining can skew futures prices, causing them to diverge from corresponding forward prices.

In view of the above we can say that when standardized and traded on the exchange forward contract becomes futures contract. Therefore all futures contracts are standardized forward contracts. Forward and futures do the same but futures do it better and more efficiently because of transparency and robust risk management in this case. The futures contracts may be settled through physical delivery of asset/ assets or only in cash. Settlement features of futures contracts are well defined in the contract specifications by the exchanges.

A future is transacted through brokerage firms that hold a "seat" on the exchange that trades that particular contract. Working through their respective brokers, two parties will transact a trade. *Legally*, that trade is structured as two trades both with a clearinghouse owned by or closely affiliated with the exchange. For example, suppose Party A and Party B trades five May natural gas futures at US\$ 3.24. Party A is long and Party B is short. This would be legally structured as Party A being long five May natural gas futures at US\$ 3.24 with the exchange's clearinghouse being the counterparty; and The exchange's clearinghouse being long five May natural gas futures at US\$ 3.24 with Party B being the counterparty. Party A and B then have no legal obligation to each other. Their respective legal obligations are to the exchange's clearinghouse. The clearinghouse never takes market risk because it always has offsetting positions with different counterparties.

Two parties negotiate (through their respective brokers) a futures transaction. They agree on the price and the number of contracts. Legally, the transaction is structured as two contracts, each between one of the parties and the exchange's clearinghouse. In this way, the parties are not exposed to each other's credit risk. There is credit risk between the respective parties and the clearinghouse, but that is all but eliminated through a margining process. Because the clearinghouse always takes offsetting positions, it never takes market risk.

Before you can trade a futures contract, the broker collects a deposit from you called initial margin. This may be in the form of cash or acceptable securities. The

broker holds this deposit for you in a margin account and, in the case of a cash deposit, credits interest on the balance. The amount of initial margin is determined according to a formula set by the exchange. For a single futures contract, it will be a small fraction of the *market value* of the futures' underlier. For *futures spreads*, or if you are using futures to hedge a physical position in the underlier, initial margin may be even lower. Generally, initial margin is intended to represent the maximum one-day net loss you could reasonably be expected to incur on a position.

Every day, the profit or loss is calculated on your futures position. If there is a loss, your broker transfers that amount from your margin account to the clearinghouse. If there is a profit, the clearinghouse transfers that amount to your broker who then deposits it into your margin account. This is the daily margining process. The clearinghouse's margin cash flows net to zero. For every margin payment it receives from one party, it makes an offsetting margin payment to another party.

Through the margining process, futures settle every day. Unlike a forward, where all contract obligations are satisfied at maturity, obligations under the futures contract are satisfied every day on an ongoing basis as mark-to-market profits or losses are realized. This essentially eliminates credit risk for futures.

Futures can be of many types viz., Index Futures, Stock Futures etc. Futures contract of this type are based on an index. For example, futures contract on NIFTY Index and BSE-30 Index. **An index future** is, as the name suggests, a future on the index i.e., the underlying is the index itself. There is no underlying security or a stock, which is to be delivered to fulfill the obligations, as index futures are cash settled. As other derivatives, the contract derives its value from the underlying index. The underlying indices in this case will be the various eligible indices and as permitted by the Regulator from time to time. **A stock futures** contract is a standardized contract to buy or sell a specific stock at a future date at an agreed price. A stock future is, as the name suggests, a future on a stock i.e., the underlying is a stock. The contract derives its value from the underlying stock. Single stock futures are cash settled.

7.3.3 Options

Options Contract is a type of Derivatives Contract which gives the buyer/holder of the contract the right (but not the obligation) to buy/sell the underlying asset at a predetermined price within or at end of a specified period. The buyer/holder of the option purchases the right from the seller/writer for a consideration which is called the premium. The seller/writer of an option is obligated to settle the option as per the terms of the contract when the buyer/holder exercises his right. The underlying asset could include securities, an index of prices of securities etc.

Under Securities Contracts (Regulations) Act, 1956 options on securities has been defined as "option in securities" means a contract for the purchase or sale of a right to buy or sell, or a right to buy and sell, securities in future, and includes *a teji*, *a mandi*, *a teji mandi*, *a galli*, a put, a call or a put and call in securities;

An Option to buy is called *Call option* and option to sell is called *Put option*. Puts and calls are sometimes called **vanilla options** to distinguish them from more exotic structures. Further, if an option that is exercisable on or before the expiry date is called *American option* and one that is exercisable only on expiry date, is called *European option*. The price at which the option is to be exercised is called *Strike price* or *Exercise price*. Not that whereas it costs nothing to enter into a forward or futures contract, there is a cost to acquiring an option.

As an example, consider a three-month, European exercise, strike US \$ 45 put option on 100,000 barrels of Brent oil. Such an option might trade OTC. It has:

underlier: Brent oil

Notional amount: 100,000 barrels

Expiration: in three months

Strike price: US \$ 45

It gives the holder the right, but not the obligation, to sell the issuer 100,000 barrels of Brent oil three months from today for a price of US \$ 45 per barrel.

Another Example is, assume Mr. A goes to the market likes the television worth Rs. 12,000. A doesn't have the money to make the full payment at that time and he offers Rs. 2,000 to the shopkeeper with a proposal to take the delivery within two days after paying the balance amount. Further assume that the shopkeeper makes it clear that the television is not brought within two days, contract would expire. This is typical example of a forward contract wherein shopkeeper not being confident about the counter party has taken the advance, which may be treated as margin money in the instant case. It is clear that if Mr A doesn't turn up in 2 days he would be loosing his advance paid. This would be a default situation. Another way to structure this deal is that Mr A tell the shopkeeper that you pay Rs. 200 – 300 for reserving the television for me. If I want I will come back and pay Rs. 12,000 and buy the television. Otherwise, I will lose the right to buy the same. In the case A is a option buyer and the shopkeeper is the option seller. An option buyer has a right to buy the option but not the obligation. If he comes across another shop selling the same television from the second shop but for a lesser price than he will forget the deal with the first shop and buy the television and buy it from the second shop. In other words A would let his right expire if he finds it unattractive to exercise the option. If A doesn't exercise his right, he is not going to receive back Rs. 200 – 300, paid for reserving the television for 2 days This money may be called the price of the option.

In the case of American options the buyer has the right to exercise the option at anytime on or before the expiry date. This request for exercise is submitted to the Exchange, which randomly assigns the exercise request to the sellers of the options, who are obligated to settle the terms of the contract within a specified time frame.

As in the case of futures contracts, option contracts can be also be settled by delivery of the underlying asset or cash. However, unlike futures cash settlement in option contract entails paying/receiving the difference between the strike price/exercise price and the price of the underlying asset either at the time of expiry of the contract or at the time of exercise/assignment of the option contract.

The options contracts, which are based on some index, are known as Index options contract. However, unlike Index Futures, the buyer of Index Option Contracts has only the right but not the obligation to buy/sell the underlying index on expiry. Index Option Contracts are generally European Style options i.e., they can be exercised/assigned only on the expiry date. The investment strategies with index options are similar to that of an individual stock options. Investors expecting a market rise but calls, and those expecting a market decline buy puts. Index options offer benefit of lower cost (premium cost & transaction costs) and greater efficiency than a package

of options on a multitude of individual stocks and debt securities because of diversifying effects of operating with a sort of portfolio which an index represents.

In the beginning futures and options were permitted only on S&P Nifty and BSE Sensex. Subsequently, sectoral indices were also permitted for derivatives trading subject to fulfilling the eligibility criteria. Derivative contracts may be permitted on an index if 80% of the index constituents are individually eligible for derivatives trading. However, no single ineligible stock in the index shall have a weightage of more than 5% in the index. The index is required to fulfill the eligibility criteria even after derivatives trading on the index has begun. If the index does not fulfill the criteria for 3 consecutive months, then derivative contracts on such index would be discontinued.

By its very nature, index cannot be delivered on maturity of the Index futures or Index option contracts therefore, these contracts are essentially cash settled on Expiry.

In options markets, the exercise (strike or striking) price means the price at which the option holder can buy or sell the underlying asset. If the current price of the underlying asset exceeds the exercise price of a call option, the call is said to be *in the money*. Similarly if the current price of the underlying asset is less than the exercise price of a call option, it is said to be *out of the money*. The *near the money* call options are those whose exercise price is slightly greater than current market price of the asset. *Premium* is the price paid by the buyer to the seller of the option, whether put or call. A call option when it is written against the asset owner by the option writer is called a *covered option*, and the one written without owning the asset is called *naked option*.

A complex variety of options exist in the world options markets. There are simple options such as commodity options, stock options, bond options, currency options, stock index options, and options on futures: compound options such as swaptions, flortions and captions: and synthetic options which combine options and futures.

Options Position

There are two sides of every option contract. On one side there is the investor who has taken long position (i.e., has bought the option). On the other side is the investor who has taken the short position (i.e., has sold or written the option). The writer of an option receives cash up front, but has the potential liabilities later. The writer's profit or loss is the reverse of that for the purchaser of the option. There are four type of option position:

A long position in a call option

A long position in a put option

A short position in a call option

A short position in a put option

It is often useful to characterize European option positions in terms of the terminal value or payoff to the investor at maturity. The initial cost of the option is then not included in the calculation.

If K is the strike price and S_T is the final price of the underlying asset, the payoff from a long position in a European call option is

$$\max (S_T - K, 0)$$

The option will be exercised if $S_T > K$ and not be exercised if $S_T \leq K$. The payoff to the holder of a short position in the European call option is

$$- \max (S_T - K, 0) = \min (K - S_T, 0).$$

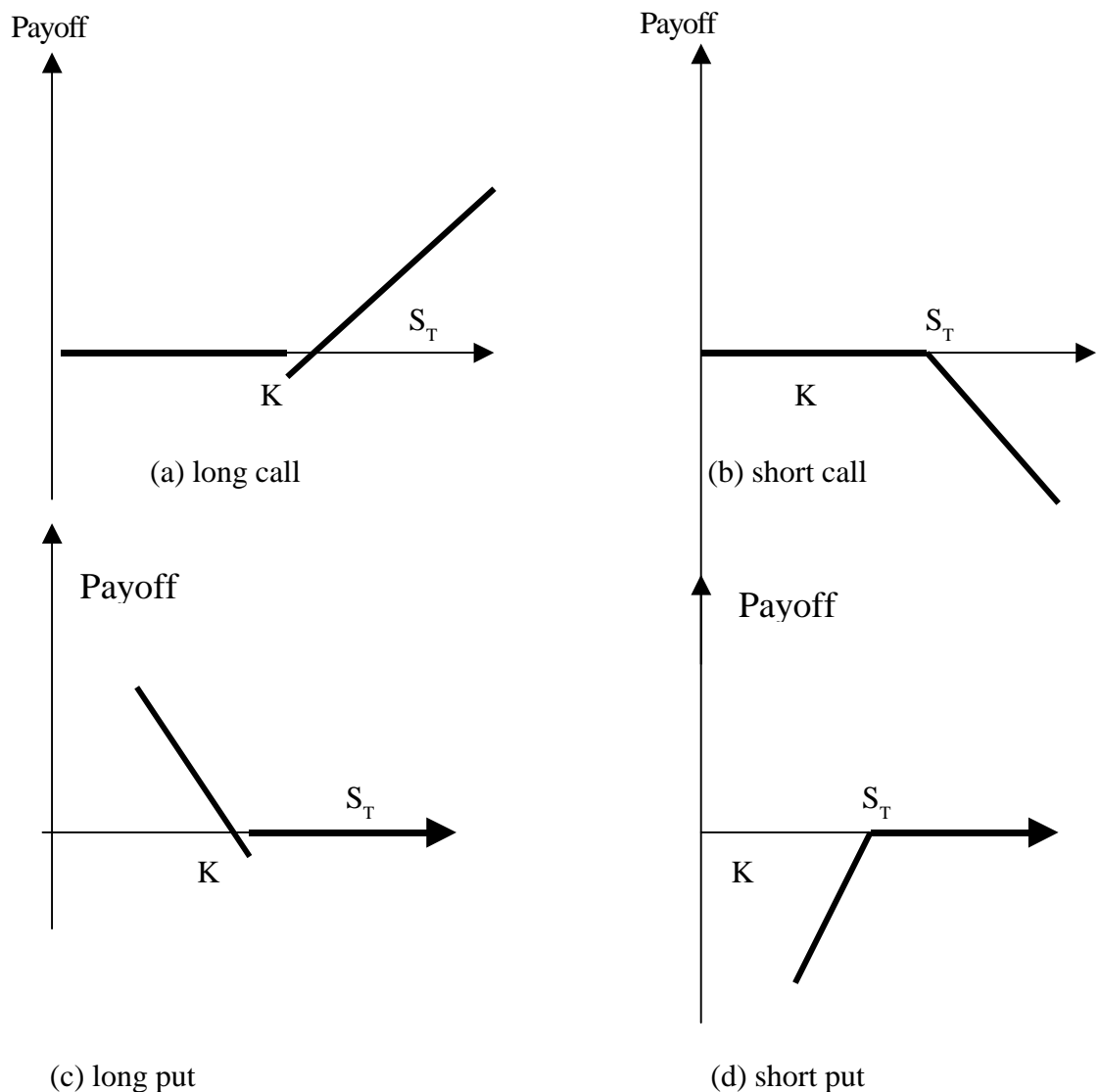
The payoff to the holder of a long position in a European put option is

$$\max (K - S_T, 0) = \min (S_T - K, 0)$$

And the payoffs from a short position in a European put option is

$$- \max (K - S_T, 0) = \min (S_T - K, 0)$$

Payoffs from positions in European options



7.3.4 Swaps

A swap is an agreement between two parties to exchange assets or sets of financial obligations or a series of cash flows for a specified period of time at predetermined intervals. They include both spot and forward transactions in one agreement, and are generally customized transactions. The corporations, banks, individual investors, etc. are now using swaps to arrange complex and innovative financing that reduces borrowing costs, and to increase control over interest rate risk and foreign currency exposure.

While swaps are used for various purposes—from hedging to speculation—their fundamental purpose is to change the character of an asset or liability without liquidating that asset or liability. For example, an investor realizing returns from an equity investment can swap those returns into less risky fixed income cash flows—without having to liquidate the equities. A corporation with floating rate debt can swap that debt into a fixed rate obligation—without having to retire and reissue debt.

The most common type of swap is a “plain vanilla” interest rate swap. In this, a company agrees to pay cash flows equal to interest at a predetermined fixed rate on a notional principal for a number of years. In return, it receives interest at a floating rate on the same notional principal for the same period of time. The floating rate in many interest rate swap agreements is the London Interbank Offer Rate (LIBOR).

The markets have developed a variety of swaps viz., interest rate swaps, coupon swaps, basis rate swaps, bond swaps, substitution swap, intermarket spread swap, swaps with timing mismatches, swaps with options like payoffs, currency swaps etc. Out of this interest rate swaps & currency swaps are the most commonly used swaps.

An interest rate swap is a transaction between two parties involving an exchange of one stream of interest obligations (payments) for another. It has specific maturity on a notional principal amount, which is simply a reference amount against which the interest is calculated; no principal amount ever really changes hands in such a transaction. Maturities range from under a year to over 15 years, but most transactions fall within a two year to 10-year period.

A currency swap is a contract exchanging foreign currency in the spot market with simultaneous agreement to reserve the transaction in the forward market. Both exchange rate and timing of the forward market transaction are specified at the time of the swap. Put differently, in a currency swap, both the principal and interest in one currency are swapped for principal and interest in another currency. On maturity the principal amount are swapped back.

For Example, a company that has borrowed rupees at a fixed interest rate can swap away the exchange rate risk by setting up a contract whereby it receives rupees at a fixed rate in return for dollar at either a fixed or floating interest rate. Currency swap can help to manage both interest rate and exchange rate risk. By setting up a contract whereby it receives Rs at a fixed rate in return for dollars at either a fixed or floating interest rate.

Type of Traders in the Market

The derivative market has been very successful because they have attracted different type of traders because of liquidity. There are three broad categories of traders: hedgers, speculators and arbitrageurs.

- Hedgers use forwards, futures and options to reduce the risk that they face from potential future movements in a market variable.
- Speculators use them to bet on the future direction of a market variable.
- Arbitrageurs take offsetting positions in two or more instruments to lock in a profit.

Put – Call parity

In financial mathematics, **put–call parity** defines a relationship between the price of a European call option and a European put option - both with the identical strike price and expiry. No assumptions other than a lack of arbitrage in the market are made in order to derive this relationship.

An example, using stock options follows, though this may be generalised.

Consider a call option and a put option with the same strike K for expiry at the same date T on some share, which pays no dividend. Let S denote the (unknown) underlier value at expiration.

$$\begin{cases} K & \text{if } S \leq K \text{ (the put has value } (K-S) \text{ and the share has value } S) \\ S & \text{if } S > K \text{ (the put has value } 0 \text{ and the share has value } S) \end{cases}$$

First consider a portfolio that consists of one put option and one share. This portfolio has value:

Now consider a portfolio that consists of one call option and K bonds that each pay 1 (with certainty) at time T . This portfolio at T has value:

$$\begin{cases} K & \text{if } S \leq K \text{ (the call has value } 0 \text{ and the bonds have value } K) \\ S & \text{if } S > K \text{ (the call has value } S-K \text{ and the bonds have value } K) \end{cases}$$

Notice that, whatever the final share price S is at time T , each portfolio is worth the same as the other. This implies that these two portfolios must have the same value at any time t before T . To prove this supposes that, at some time t , one portfolio were cheaper than the other. Then one could purchase (go long) the cheaper portfolio and sell (go short) the more expensive. Our overall portfolio would, for any value of the share price, have zero value at T . We would be left with the profit we made at time t . This is known as a risk-less profit and represents an arbitrage opportunity. Thus the following relationship exists between the value of the various instruments at a general time t :

$$C(t) + K * B(t,T) = P(t) + S(t)$$

where

$C(t)$ is the time- t value of the call

$P(t)$ is the time- t value of the put

$S(t)$ is the time- t value of the share

K is the strike price

and $B(t,T)$ is the time- t value of a bond that pays at T . If a stock pays dividends, they should be included in $B(t, T)$, because option prices are typically not adjusted for ordinary dividends.

If the bond interest rate is assumed to be constant, with value r , $B(t, T)$ is equal to $e^{-r(T-t)}$.

Using the above, and given the (fair) value of any three of the call, put, bond and stock prices one can compute the (implied) fair value of the fourth.

Put-call parity implies:

1) Equivalence of calls and puts

Parity implies that a call and a put can be used interchangeably in any delta-neutral portfolio. If d is the call's delta, then buying a call, and selling d shares of stock, is the same as buying a put and buying $1 - d$ shares of stock. Equivalence of calls and puts is very important when trading options.

2) Parity of implied volatility

In the absence of dividends or other costs of carry (such as when a stock is difficult to borrow or sell short), the implied volatility of calls and puts must be identical.

Check Your Progress 2

1) Explain what you mean by a forwards contract.

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2) Explain the workings of a futures contract.

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3) What is a swap?

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7.4 MODELS OF DERIVATIVE PRICING

7.4.1 Binomial Option Pricing Model

The **binomial options pricing model** provides a generalisable numerical method for the valuation of options. Cox, Ross and Rubinstein first proposed the binomial model in 1979. Essentially, the model uses a “discrete-time” model of the varying price over time of the underlying financial instrument. Option valuation is then via

application of the risk neutrality assumption over the life of the option, as the price of the underlying instrument evolves. The Binomial options pricing model approach is widely used as it is able to handle a variety of conditions for which other models cannot easily be applied. This is largely because the BOPM models the underlying instrument over time – as opposed to at a particular point. For example, the model is used to value American options, which can be exercised at any point, and Bermudan options, which can be exercised at various points. The model is also relatively simple, mathematically, and can therefore be readily implemented in a software (or even spreadsheet) environment. Although slower than the Black-Scholes model, it is considered more accurate, particularly for longer-dated options, and options on securities with dividend payments. For these reasons, various versions of the binomial model are widely used by practitioners in the options markets.

For options with several sources of uncertainty (e.g., real options), or for options with complicated features (e.g. Asian options), lattice methods face several difficulties and are not practical. Monte Carlo option models are generally used in these cases. Monte Carlo simulation is, however, time-consuming in terms of computation, and is not used when the Lattice approach (or a formula) will suffice.

The binomial pricing model uses a “discrete-time framework” to trace the evolution of the option’s key underlying variable via a binomial lattice (tree), for a given number of time steps between valuation date and option expiration.

Each node in the lattice, represents a *possible* price of the underlying, at a *particular* point in time. This price evolution forms the basis for the option valuation.

The valuation process is iterative, starting at each final node, and then working backwards through the tree to the first node (valuation date), where the calculated result is the value of the option.

Option valuation using this method is, as described, a three step process:

- 1) Price tree generation,
- 2) Calculation of option value at each final node,
- 3) Progressive calculation of option value at each earlier node; the value at the first node is the value of the option.

Option value at each final node

At each final node of the tree — i.e., at expiration of the option — the option value is simply its intrinsic, or exercise, value.

$\text{Max} [(S - K), 0]$, for a call option

$\text{Max} [(K - S), 0]$, for a put option:

(Where K is the Strike price and S is the spot price of the underlying asset)

Option value at earlier nodes

At each earlier node, the value of the option is calculated using the risk neutrality assumption. Under this assumption, today’s fair price of a derivative is equal to the discounted expected value of its future payoff.

Expected value is therefore calculated using the option values from the later two nodes (*Option up* and *Option down*) weighted by their respective probabilities —

“probability” p of an up move in the underlying, and “probability” $(1-p)$ of a down move.

The expected value is then discounted at r , the risk free rate corresponding to the life of the option. This result, the “Binomial Value”, is thus the fair price of the derivative at a particular point in time (i.e., at each node), given the evolution in the price of the underlying to that point.

The Binomial Value is found for each node, starting at the penultimate time step, and working back to the first node of the tree, the valuation date, where the calculated result is the value of the option. For an American option, since the option may either be held or exercised prior to expiry, the value at each node is: $\text{Max}(\text{Binomial Value}, \text{Exercise Value})$.

The Binomial Value is calculated as follows.

$$\text{Binomial Value} = [p \times \text{Option up} + (1-p) \times \text{Option down}] \times \exp(-r \times t)$$

7.4.2 The Black Scholes Formula

The **Black–Scholes model** is a model of the evolving price of financial instruments, in particular stocks. The **Black–Scholes formula** is a mathematical formula for the theoretical value of European put and call stock options derived from the assumptions of the model. The formula was derived by Fischer Black and Myron Scholes and published in 1973. They built on earlier research by Edward Thorpe, Paul Samuelson, and Robert C. Merton. The fundamental insight of Black and Scholes is that the option is implicitly priced if the stock is traded.

The Black-Scholes model is used to calculate a theoretical call price (ignoring dividends paid during the life of the option) using the five key determinants of an option's price: stock price, strike price, volatility, time to expiration, and short-term (risk free) interest rate.

The original formula for calculating the theoretical option price (OP) is as follows:

Where:

$$OP = SN(d_1) - Xe^{-rt}N(d_2)$$

$$d_1 = d_2 + \sqrt{t}$$

The variables are:

S = stock price

X = strike price

t = time remaining until expiration, expressed as a percent of a year

r = current continuously compounded risk-free interest rate

v = annual volatility of stock price (the standard deviation of the short-term returns over one year).

\ln = natural logarithm

$N(x)$ = standard normal cumulative distribution function

e = the exponential function

Advantages & Disadvantages of the Model

Advantage: The main advantage of the Black-Scholes model is speed — it lets you calculate a very large number of option prices in a very short time.

Limitation: The Black-Scholes model has one major limitation: it cannot be used to accurately price options with an American-style exercise as it only calculates the option price at one point in time — at expiration. It does not consider the steps along the way where there could be the possibility of early exercise of an American option.

This is the **Black-Scholes equation** for pricing options.

In deriving this equation, we assumed that:

- 1) There are no arbitrage opportunities (no free lunch).
- 2) Short selling of shares is possible at all times.
- 3) No transaction costs or taxes in setting up a portfolio.
- 4) All securities are perfectly divisible.
- 5) Trading can take place continuously.
- 6) The underlying share pays no dividends during the lifetime of the option.
- 7) The risk-free rate r and the share volatility σ are known over the lifetime of the option.

What is the relation between Binomial Options Pricing Model & Black Scholes Model?

The same underlying assumptions regarding stock prices underpin both the binomial and Black-Scholes models: that stock prices follow a stochastic process described by geometric brownian motion. As a result, for European options, the binomial model converges on the Black-Scholes formula as the number of binomial calculation step increases. In fact the Black-Scholes model for European options is really a special case of the binomial model where the number of binomial steps is infinite. In other words, the binomial model provides discrete approximations to the continuous process underlying the Black-Scholes model.

7.5 A BRIEF HISTORY OF DERIVATIVES IN INDIA

The history of derivatives is surprisingly longer than what most people think. Some texts even find the existence of the characteristics of derivative contracts in incidents of Mahabharata. Traces of derivative contracts can even be found in incidents that date back to the ages before Jesus Christ. However, the advent of modern day derivative contracts is attributed to the need for farmers to protect themselves from any decline in the price of their crops due to delayed monsoon, or overproduction. Derivatives have had a long presence in India. The commodity derivative market has been functioning in India since the nineteenth century with organized trading in cotton, through the establishment of Cotton Trade Association in 1875. Since then contracts on various other commodities have been introduced as well. Exchange

traded financial derivatives were introduced in India in June 2000 at the two major stock exchanges, NSE and BSE. There are various contracts currently traded on these exchanges. National Commodity & Derivatives Exchange Limited (NCDEX) started its operations in December 2003, to provide a platform for commodities trading. The derivatives market in India has grown exponentially, especially at NSE. Stock Futures are the most highly traded contracts on NSE accounting for around 55% of the total turnover of derivatives at NSE, as on April 13, 2005.

Derivative trading in India takes place either on a separate and independent Derivative Exchange or on a separate segment of an existing Stock Exchange. Derivative Exchange/Segment function as a Self-Regulatory Organisation (SRO) and SEBI acts as the oversight regulator. The clearing & settlement of all trades on the Derivative Exchange/Segment would have to be through a Clearing Corporation/House, which is independent in governance and membership from the Derivative Exchange/Segment. Derivative products have been introduced in a phased manner starting with Index Futures Contracts in June 2000. Index Options and Stock Options were introduced in June 2001 and July 2001 followed by Stock Futures in November 2001. Sectoral indices were permitted for derivatives trading in December 2002. Interest Rate Futures on a notional bond and T-bill priced off ZCYC have been introduced in June 2003 and exchange traded interest rate futures on a notional bond priced off a basket of Government Securities were permitted for trading in January 2004.

Check Your Progress 3

- 1) Compare and contrast the binomial model and the Black-Scholes formulation of derivative pricing.

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- 2) Briefly describe the evolution of derivatives in India.

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7.6 LET US SUM UP

This unit continued the discussion that was started in units 5 and 6 regarding assets and their pricing. The units discussed a type of asset whose value is determined by, and influenced by the price of an underlying asset. The unit began with the basic concept of, and idea behind, a derivative. Then the unit embarked on a long discussion of the various types of derivatives, including forward contracts, futures, and swaps.

The unit then went on to a discussion of pricing of derivatives, discussing the binomial model, and the Black-Scholes formula, and discussed the differences between the two models. The unit concluded with a brief history of derivatives in India.

7.7 KEY WORDS

Arbitrageurs	: Investors who seek discrepancy in Security prices in order to earn riskless return
Call option	: An option that gives the its holder the right to buy an asset at a fixed price during a certain period
Forward Contract	: An agreement between two parties to exchange an asset for cash at a predetermined future date for a price that is specified today
Hedge	: An investment purchased against another investment in order to counter any loss made by either
Interest Rate Future	: Future contracts on fixed income securities such as treasury bills bonds and CDs
Interest Rate Options	: Option contracts on fixed income securities such as treasury bills
Interest Rate Parity	: The concept that difference between the interest rates in two countries is equal to the difference between the forward and spot rate of the respective countries
Long Position	: A position in which the investor is entitled to receive an asset in future. It is the position of buyer in the security
Margin	: The part of the transaction value that the customer must pay to initiate transaction with the other part being borrowed from the broker. There are two types of margin initial margin & maintenance margin.
Option Contract	: An Agreement that confers the right to buy or sell an asset at a set price at some future date. The right is exercisable at the discretion of the option buyer
Put Option	: An option that gives the holder the right to sell an asset at a fixed price during a certain period
Short Position	: A position in which the investor is obliged to deliver an asset in future. In other words it is a position of a seller of a security
Spot Rate	: Exchange rate which applies to 'on the spot delivery of the currency – in practice it means delivery in two days after the day of trade
Swap Contract	: An agreement between two parties (counterparties) to exchange assets or a series of cash flows for a specified period of time at predetermined intervals

7.8 SOME USEFUL BOOKS

Bhole, L M . (2004) *Financial Institutions and Markets 4th edition*, TataMcGraw-Hill, New Delhi:

Dothan, U. (1990), *Prices in Financial Markets*, Oxford University Press, New York

Hull, John , (2002) *Options, Futures and other Derivatives* Prentice-Hall of India, New Delhi.

7.9 ANSWERS/HINTS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress 1

- 1) See section 7.2 and answer
- 2) See section 7.2 and answer

Check Your Progress 2

- 1) See section 7.3 and answer
- 2) See section 7.3 and answer
- 3) See section 7.3 and answer

Check Your Progress 3

- 1) See section 7.4 and answer
- 2) See section 7.5 and answer