H-atom: RK4

In this problem, you will solve the hydrogen atom problem using RK4.

- **The Equations**

  The radial Schrödinger equation for the central potential $V(r)$ is given by

$$\left[\frac{d^2}{dr^2}+\frac{2}{r}\right]R(r)+\frac{2\mu}{\hbar^2}\left[E+V(r)-\frac{l(l+1)\hbar^2}{2\mu r^2}\right]R(r)=0.$$

  Here, μ is the reduced mass of the system, l is the orbital-angular momentum quantum number, and $R(r)$ is the radial wave function. The above equation, in atomic units, for the ground state (l=0) of the hydrogen atom can be written as

$$\left[\frac{d^2}{dr^2}+\frac{2}{r}\right]R+2\left[E_0+\frac{1}{r}\right]R=0.$$

  Write a Fortran program to solve the above equation using RK4 to find E_0 with following starting values: $R(r=0.0005)=0.000001, R'(r=0.0005)=-1000.0$. The r grid will be from 0.0005 unit to 5 unit with 10000 points.

  The code will be for a range of E values, $-0.6 \leq E \leq -0.4$, with ΔE=0.01. For finding the correct value of E, plot $R(r)$ and the radial distribution function, $|rR(r)|^2$, against r and check their convergence with respect to E.