

Comparisons

Persistent Array
Persistent LinkedList
Persistent Stack
Persistent Queue
Persistent Search Tree

We have done BenchMarking using: [GoogleBenchMark Tool](https://github.com/google/benchmark)
<https://github.com/google/benchmark>

Persistent Array

Persistent Array

Time Complexity

Here, V = Total No. Of Version, N = Average Length Of The Array

Strategies	Update(index, version, newVal)	RetrieveData(index, version)
Copy_On_Write	$O(\text{length_of_array_at_version}) \sim O(N)$	$O(1)$
Fat_Node	$O(1)$	Between $O(1)$ and $O(V)$ [Worst Case] V = number of versions $O(1)$ (Average)
Log_log_time method (Deitz, 1989)	$O(\log(\log(\min(n, v))))$, n = size of array v = number of versions	$O(\log(\log(\min(n, v))))$, n = size of array v = number of versions

Persistent Array

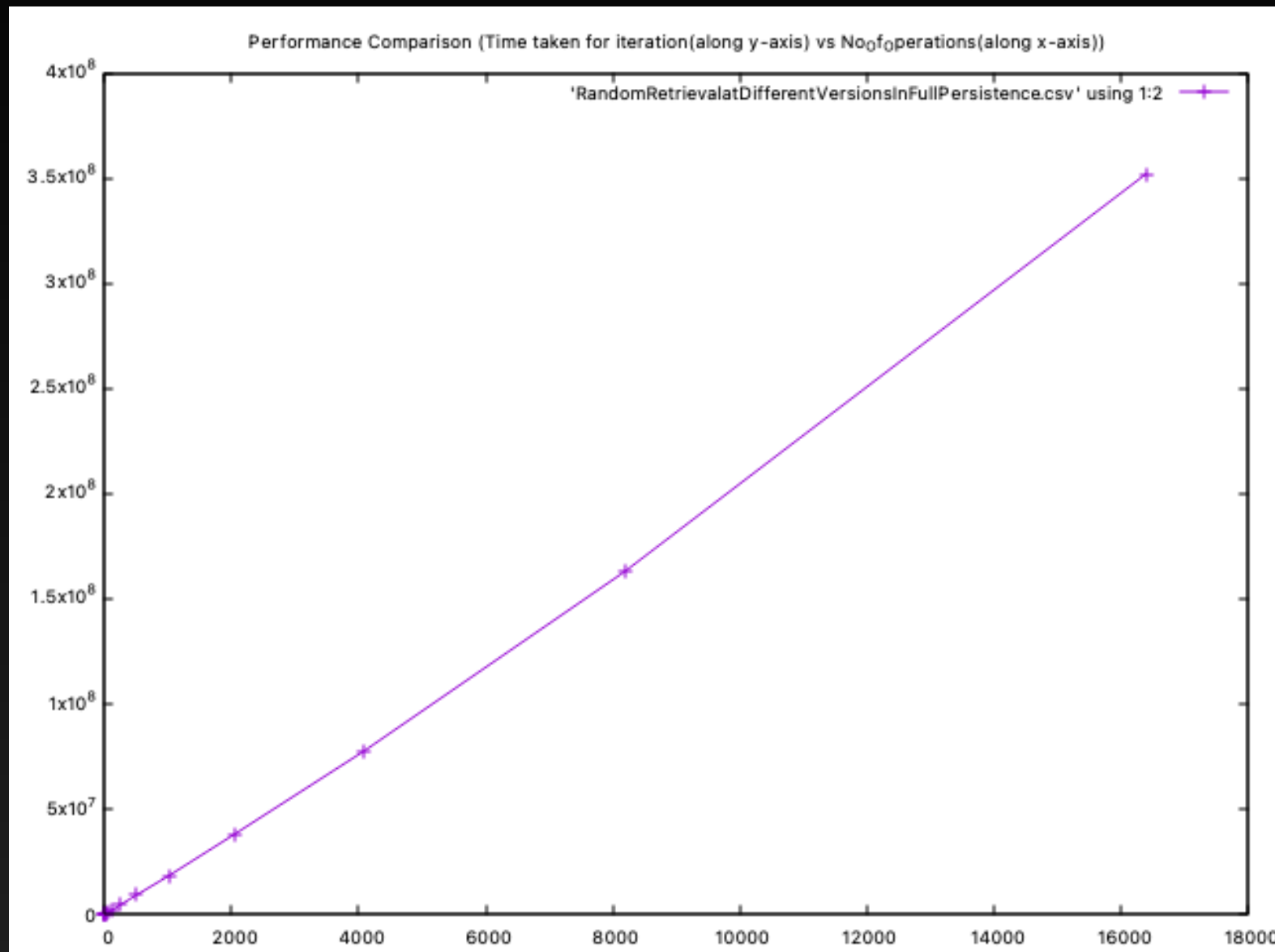
Auxiliary Space

Here, V = Total No. Of Version, N = Average Length Of The Array

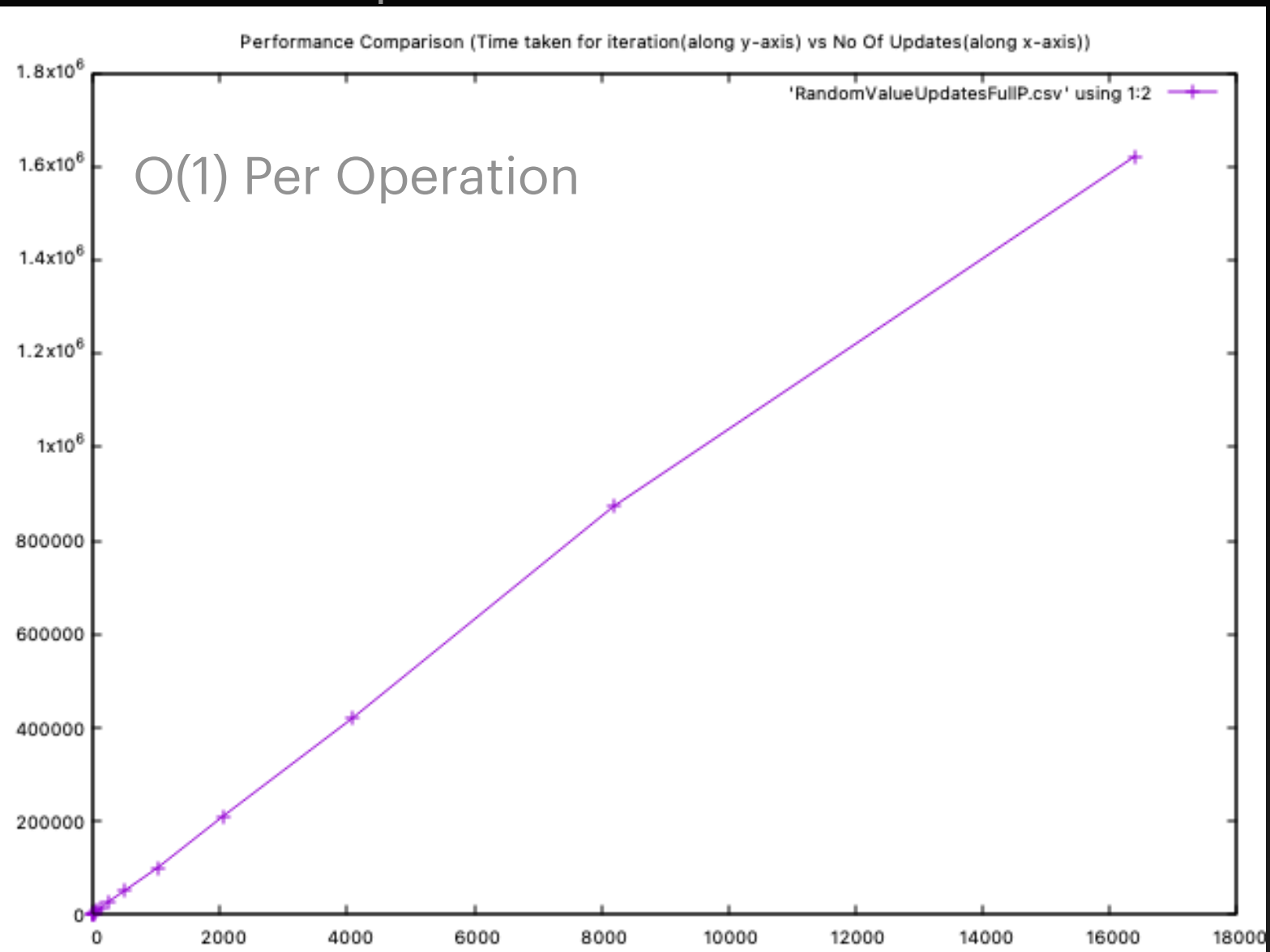
Strategies	Category1	Category2
Copy_On_Write	$\sim O(N^2)$ To Hold All The Copies of Array At Different Versions	$O(V)$ To Hold The Mapping From Version To Array
Fat_Node	$O(N + V)$ N = size of initial array, V = number of version	$O(V)$ To Hold The Mapping From Version To immediate ancestor version
Log_log_time method (Deitz, 1989)	$O(N + V)$ N = size of initial array, V = number of version	$O(V)$ to arrange the versions in form of list order maintenance tree

Benchmarking Of FatNode Model

RandomRetrievalatDifferentVersionsinFullPersistence

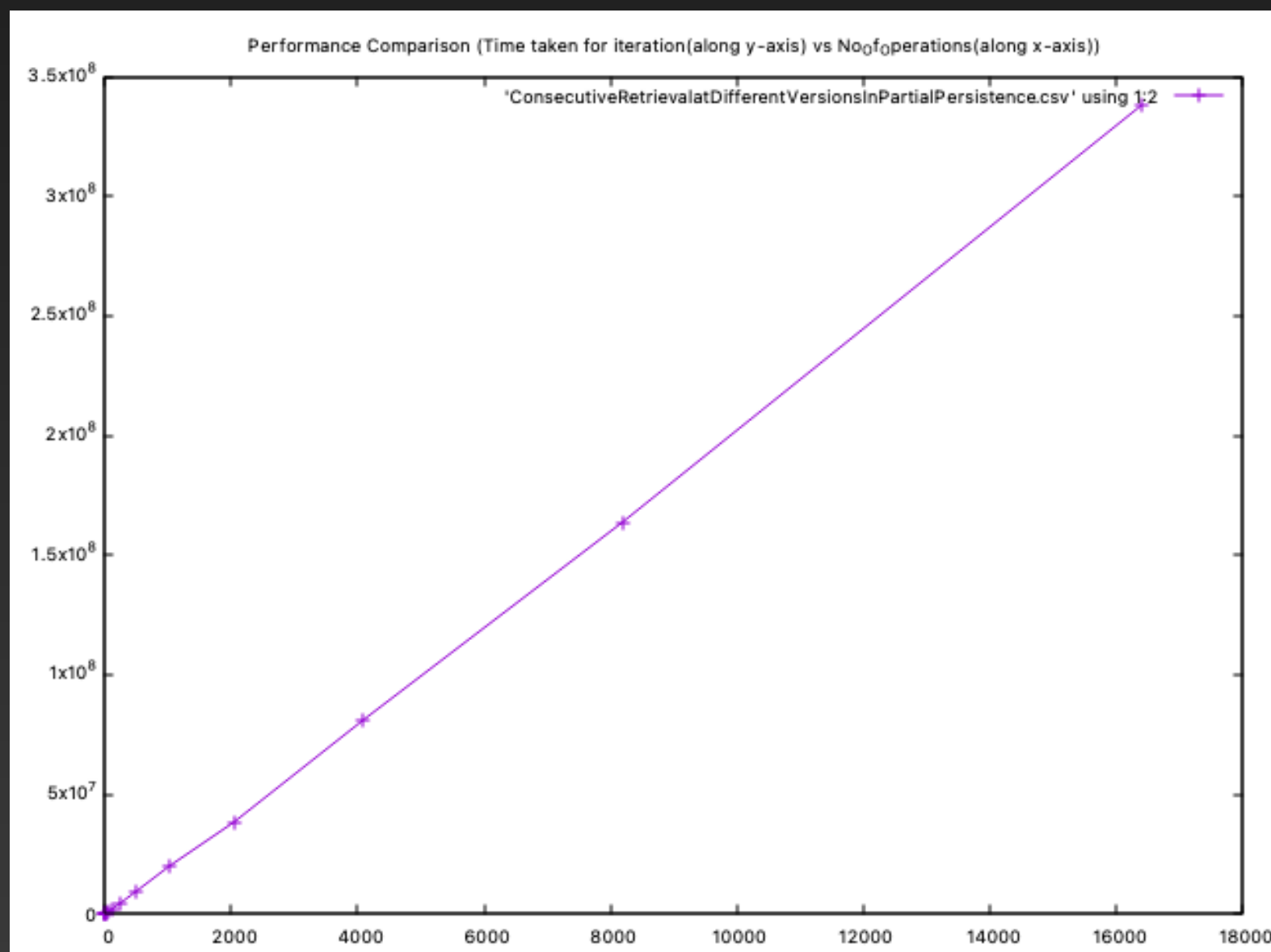


RandomValueUpdatesFullPersistence

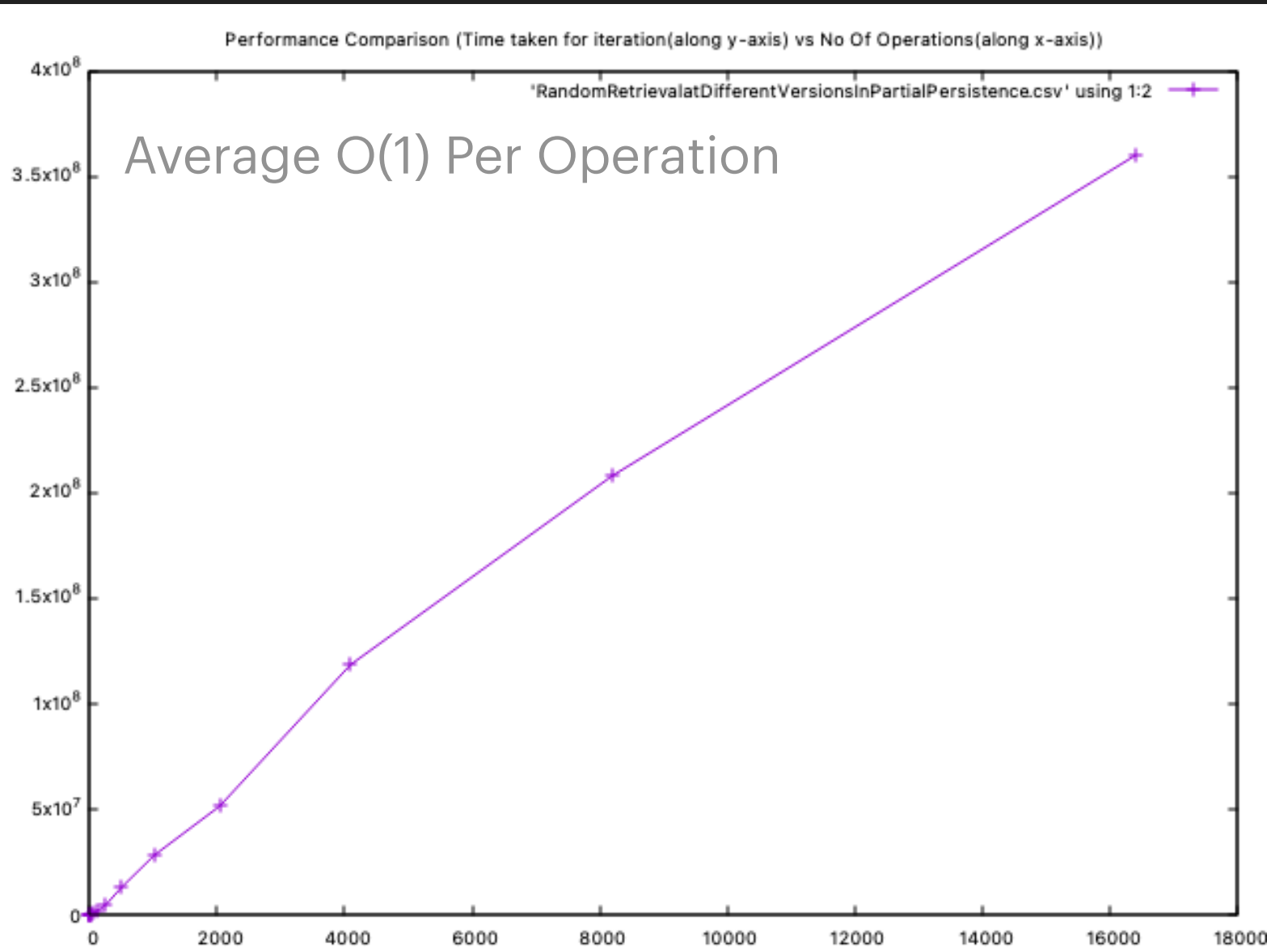


Average O(1) Per Operation

ConsecutiveRetrievalatDifferentVersionsinPartialPersistence



RandomRetrievalatDifferentVersionsinPartialPersistence



Partial Persistent Linked List

Partial Persistent Linked List

Time Complexity

Here, V = Total No. Of Version, N = Average Length Of The Linked List Considering All The Versions,
 m = no. of modifications in a particular position

Strategies	InsertAfter(position) #	DeleteAfter(position) #	UpdateData (position) #	RetrieveData (position,version)	traverseWholeLL atVer(version)
Fat Node	$O(1)$	$O(1)$	$O(1)$	$O(\log_x m)$ [If the versions are stored in a Balanced x_array Tree/ Trie] $O(1)$ [Amortised]	$O(N * \log_x m)$
Path Copying	$O(1)$ for at extreme position $O(N)$ In Average	$O(1)$ for at extreme position $O(N)$ for at Rear	$O(1)$ for at extreme position $O(N)$ for at Rear	$O(N)$	$O(N)$
Pointer Machine	$O(1)$ [Amortised]	$O(1)$ [Amortised]	$O(1)$ [Amortised]	$O(N)$	$O(N)$
Ephemeral Linked List	$O(1)$	$O(1)$	$O(1)$	$O(N)$ [Only Current Version Supported]	$O(N)$ [Only Current Version Supported]

time to reach the node at that position is not considered

Partial Persistent Linked List

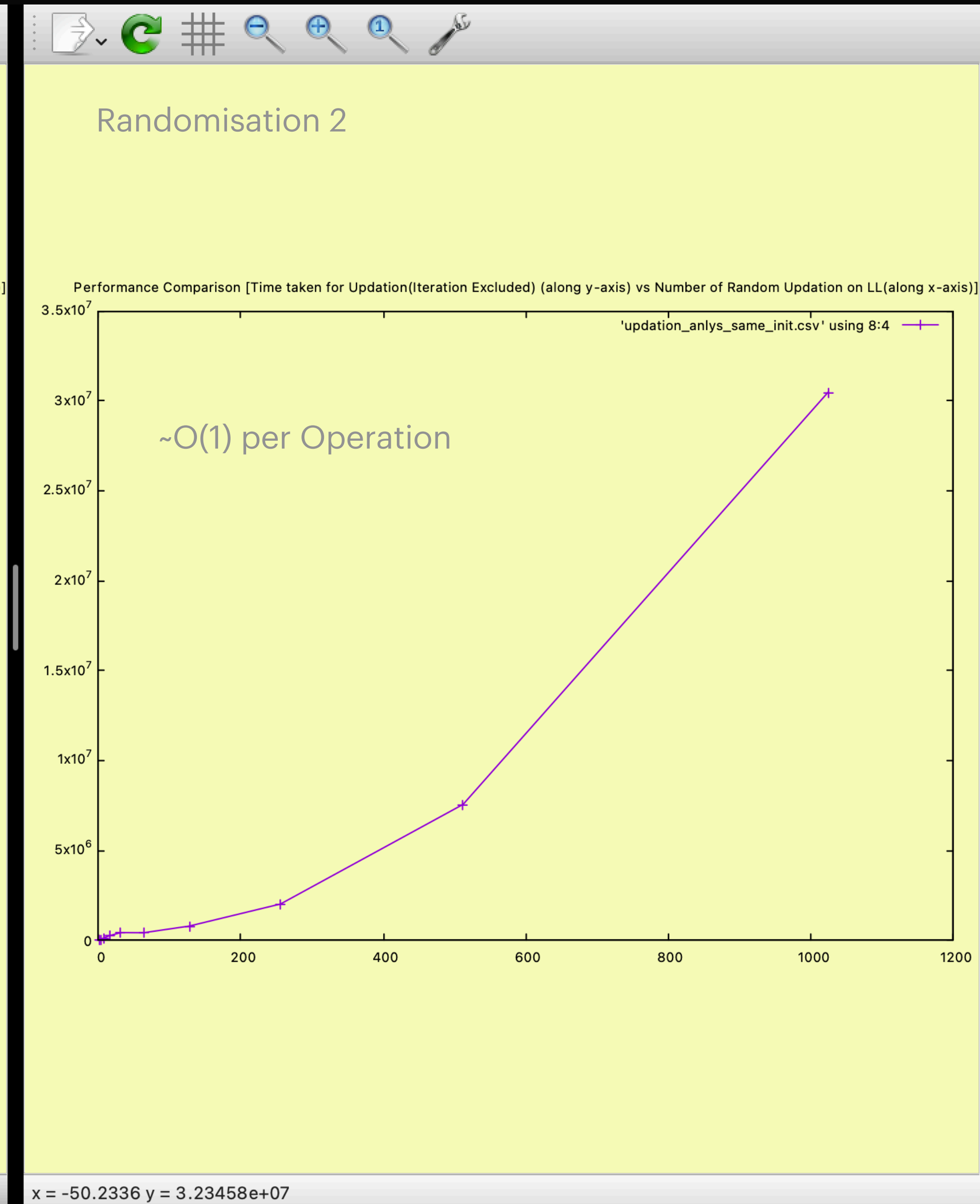
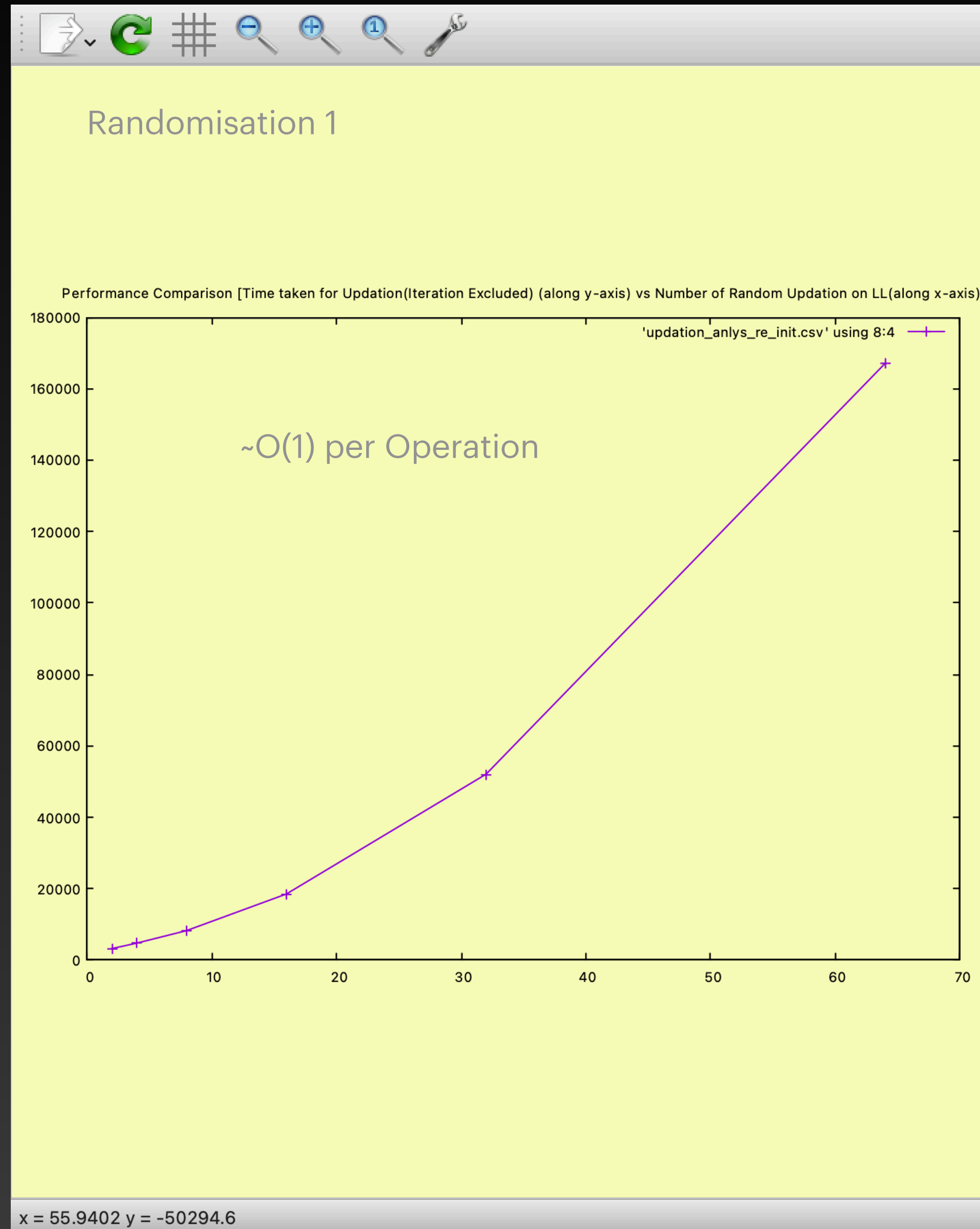
Auxiliary Space

Here, V = Total No. Of Version, N = Average Length Of The Linked List Considering All The Versions

Strategies	Category 1	Category 2
Fat Node	Node Size: # ~ 20byte	$O(N + V)$ to Hold The LinkedList
Path Copying	Node Size: # ~ 12 byte	Best Case: $O(N) + O(V)$ Worst Case: $O(N^2) + O(V)$ to Hold The Tree and Staring Pointers
Pointer Machine	Node Size: # ~ 140 byte	$O(V)$ Amortised to Hold The LinkedList
Ephemeral Linked List	Node Size: # ~ 12 byte	$O(N)$ to Hold The LinkedList

to store 4 byte Integer | 8 Bye pointers

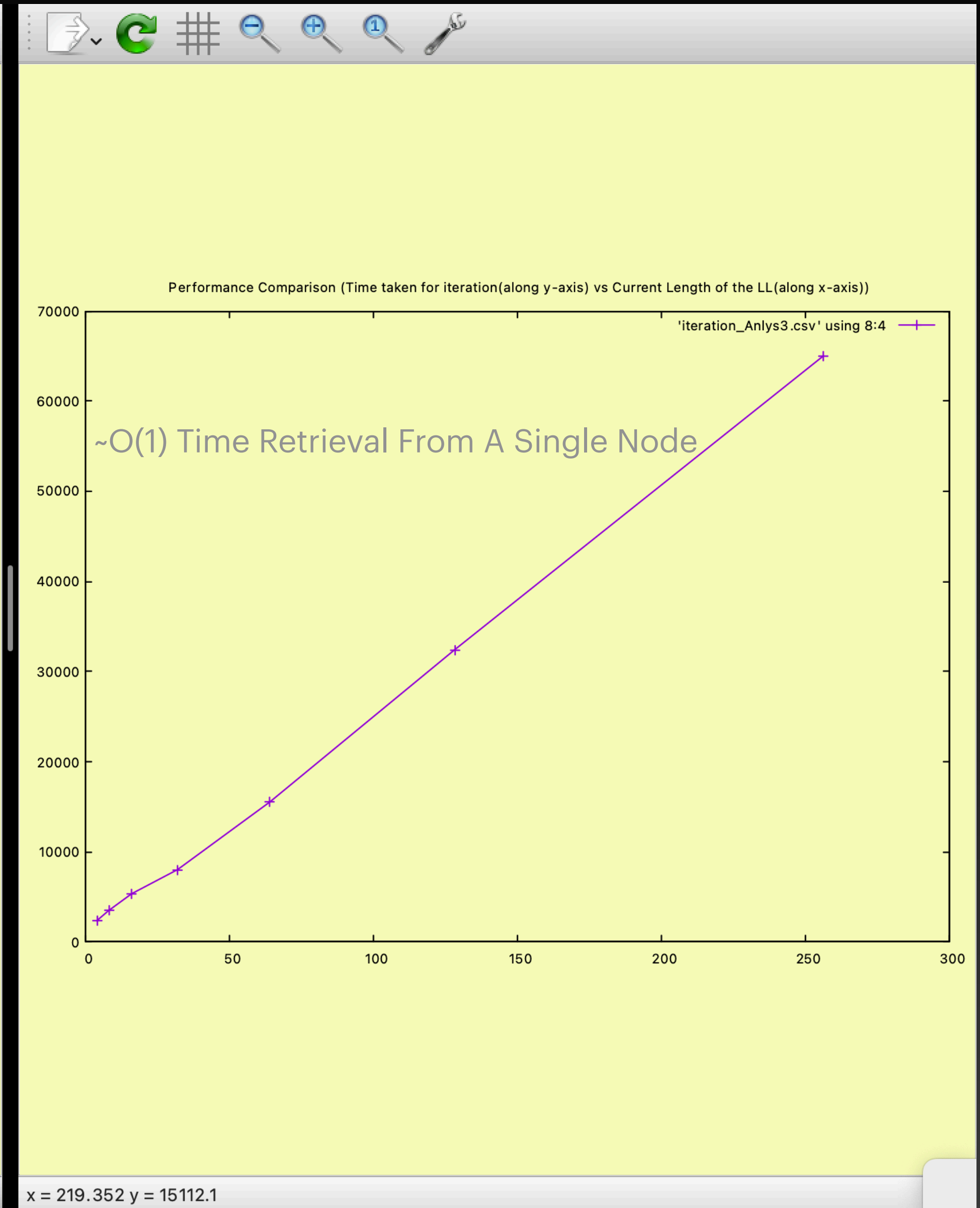
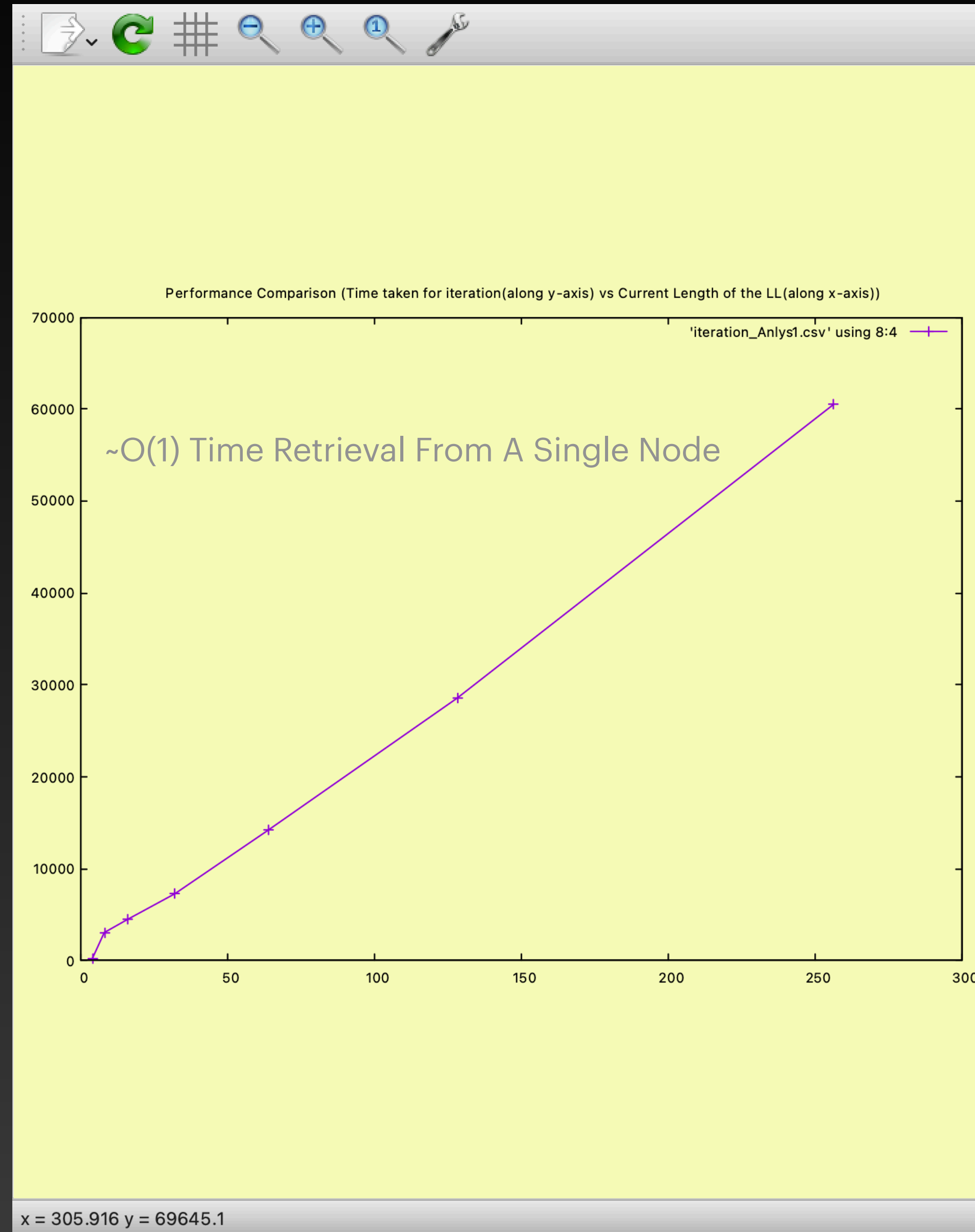
Benchmarking Of Pointer Machine Model



Updation
(Value/
Insertion/ Deletion)
Randomised

Benchmarking Of Pointer Machine Model

Iteration Through
Whole LL
At
Randomised Versions



Full Persistent Linked List

Full Persistent Linked List

Time Complexity

Here, V = Total No. Of Version, N = Average Length Of The Linked List Considering All The Versions

Strategies	InsertAfter(position,ver) #	DeleteAfter(position,ver) #	UpdateData (position,ver) #	RetrieveData (position,version)	traverseWholeLL atVer(version)
Fat Node Pointer Machine [With List Maintenance]	O(1) [Amortized] + O(1) [Amortized] [amortised O(1) due for look up and insertions at version tree]	O(1) [Amortized] + O(1) [Amortized] [amortised O(1) due for look up and insertions at version tree]	O(1) [Amortized] + O(1) [Amortized] [amortised O(1) due for look up and insertions at version tree]	O(N) + O(1) [Amortized] [additional amortised O(1) due for look up at version tree]	O(N) * O(1) [additional amortised O(1) due for look up / insertions at version tree]
Path Copying	O(1) for at front O(N) for at Rear	O(1) for at front O(N) for at Rear	O(1) for at front O(N) for at Rear	O(N)	O(N)
Pointer Machine [With List Maintenance]	O(1) [Amortised]	O(1) [Amortised]	O(1) [Amortised]	O(N)	O(N)
Ephemeral Linked List	O(1) [Only Current Version Supported]	O(1) [Only Current Version Supported]	O(1) [Only Current Version Supported]	O(N) [Only Current Version Supported]	O(N) [Only Current Version Supported]

Full Persistent Linked List

Auxiliary Space

Here, V = Total No. Of Version, N = Average Length Of The Linked List Considering All The Versions

Strategies	Category 1	Category 2	Category 3
Fat Node Pointer Machine [With List Maintenance]	Node Size: # ~ 20byte	$O(N + V)$ to Hold The LinkedList	$O(V)$ to hold the Version Tree Here A ScapeGoat Tree
Path Copying	Node Size: # ~ 12 byte	Best Case: $O(N) + O(V)$ Worst Case: $O(N^2) + O(V)$ to Hold The Tree and Staring Pointers	-
Pointer Machine [With List Maintenance]	Node Size: # ~ 200 byte	$O(V)$ [Amortised] to Hold The LinkedList	$O(V)$ to hold the Version Tree Here A ScapeGoat Tree
Ephemeral Linked List	Node Size: # ~ 12 byte	$O(N)$ to Hold The LinkedList	-

to store 4 byte Integer | 8 Bye pointers

Persistent Stack

Persistent Stack

Model Developed By Our Team

Time Complexity

Here, V = Total No. Of Version, N = Average Length Of The Stack Considering All The Versions

Strategies	push(data, version)	pop(version)	getTop(ver)
Using PPL with PM Model	O(1)	O(1)	O(1)
DAG Model	O(1)	O(1)	O(1)
Ephemeral std::stack (C++)	O(1) [Only Current Version Supported]	O(1) [Only Current Version Supported]	O(1) [Only Current Version Supported]

Persistent Stack

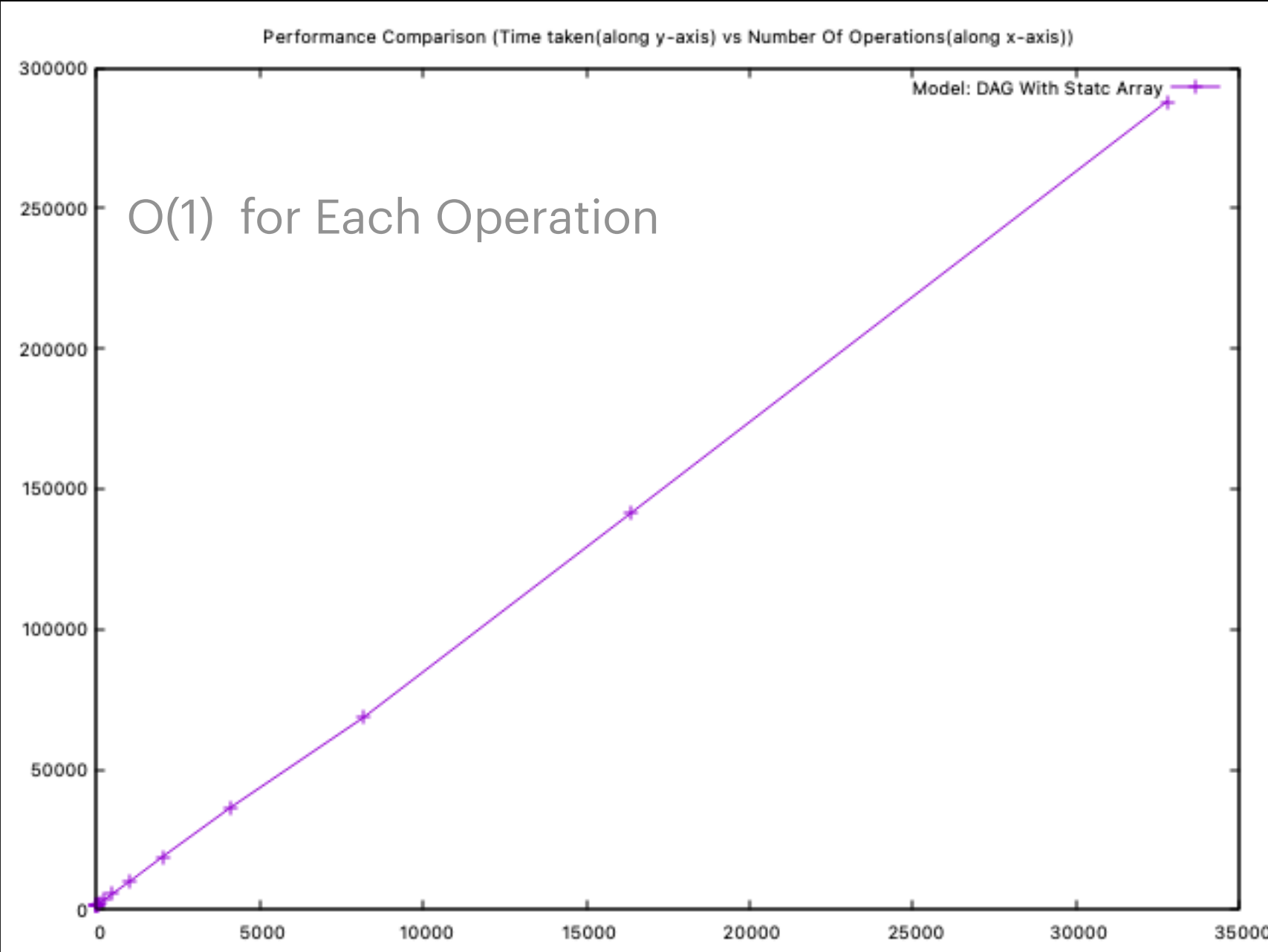
Model Developed By Our Team

Auxiliary Space

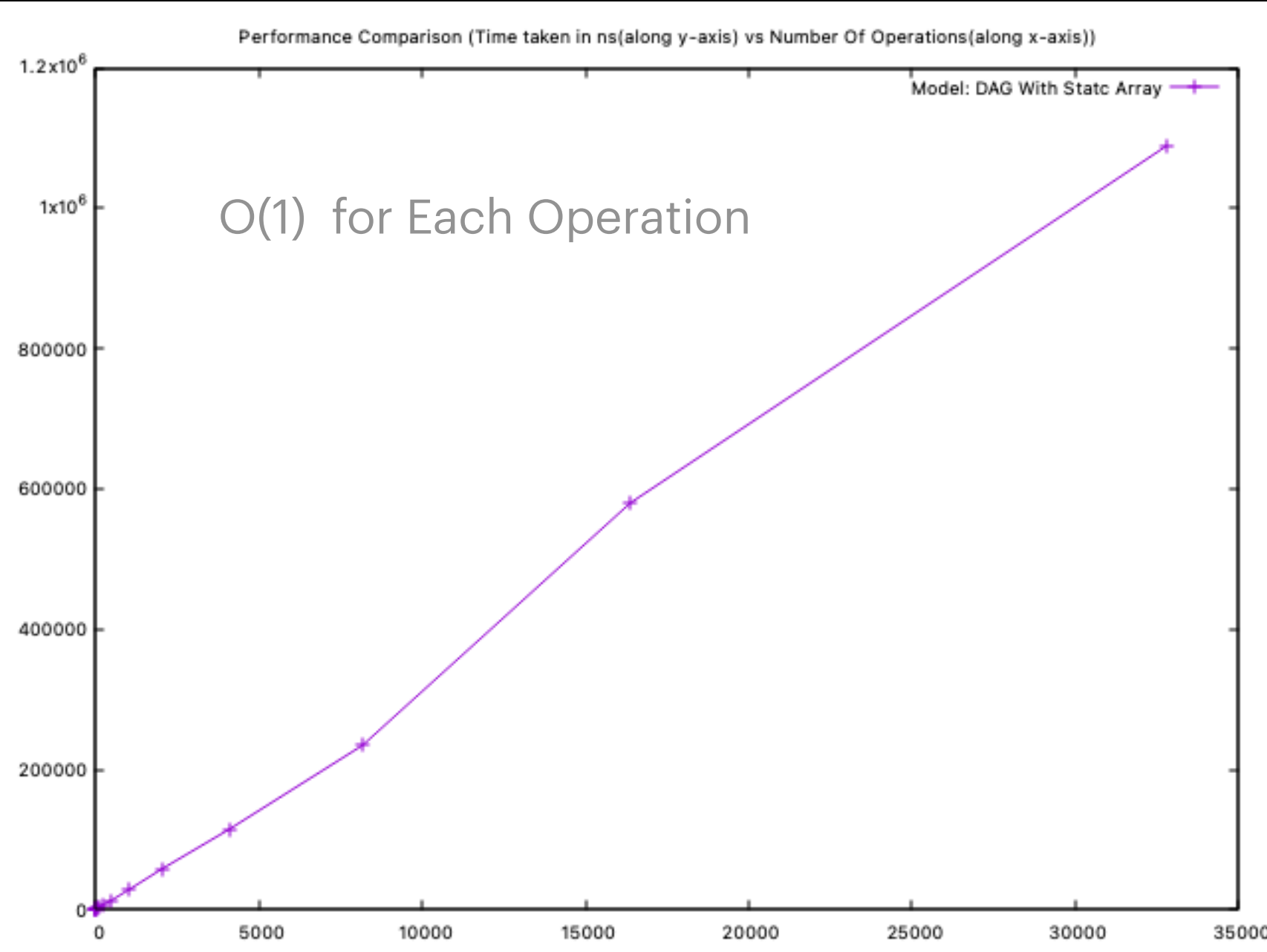
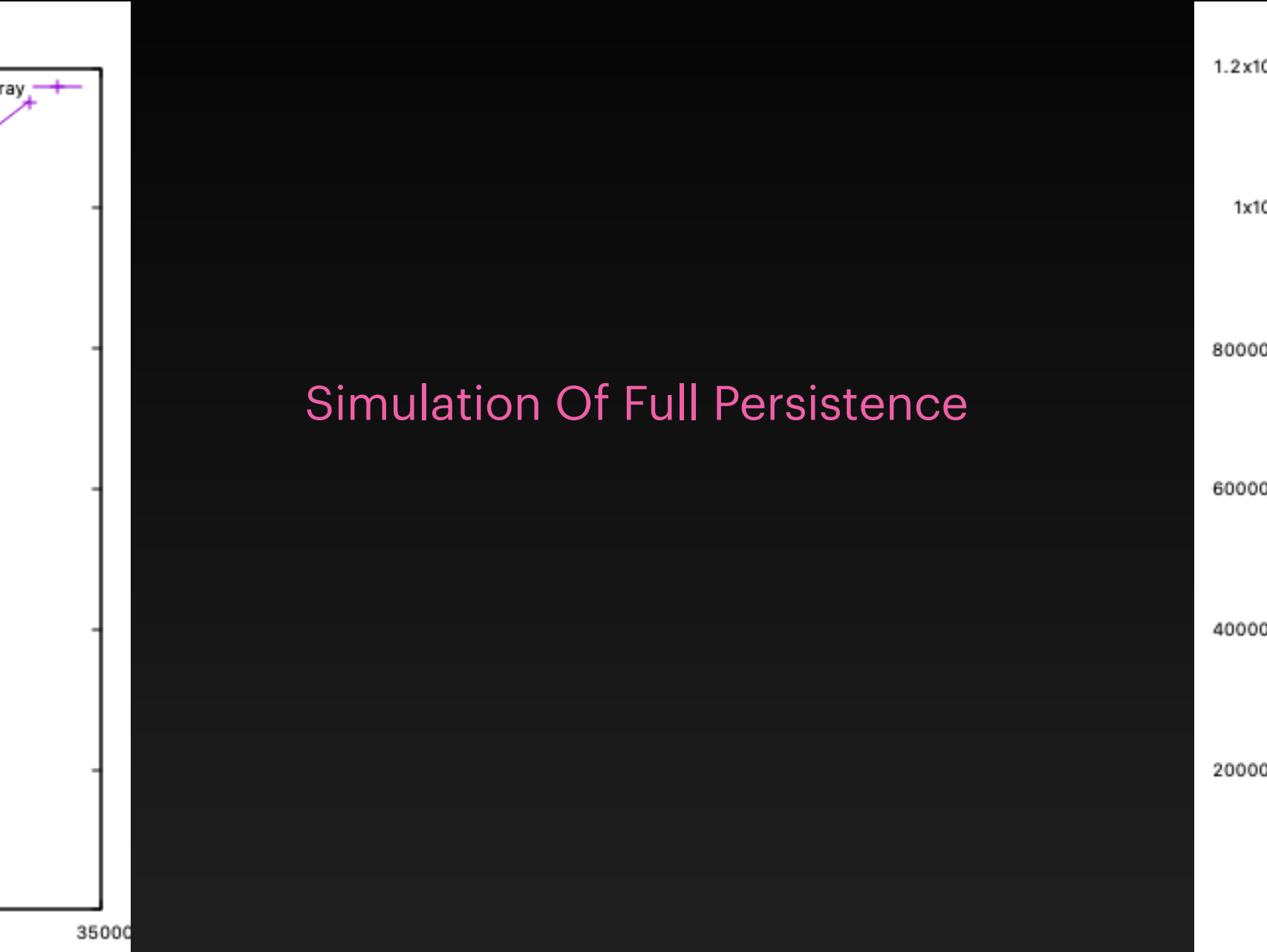
Here, V = Total No. Of Version, N = Average Length Of The Stack Considering All The Versions

Strategies	Category 1	Category 2	Category 3
Using PPL with PM Model	$O(V)$ [Amortized] To Hold The Linked List		
DAG Model	$O(V)$ To Hold The MAP/DAG		
Ephemeral std::stack (C++)	$O(N)$	-	-

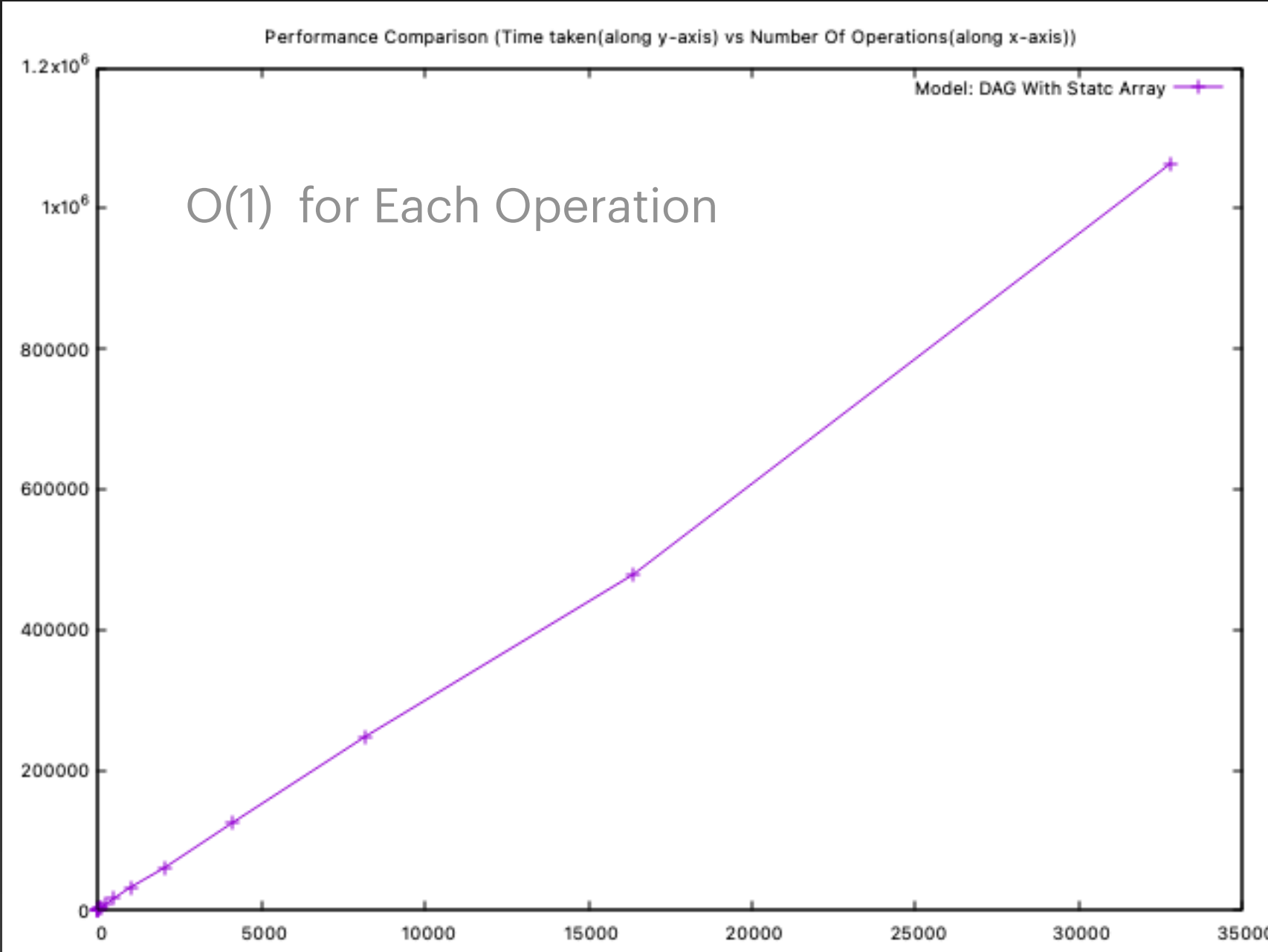
Benchmarking Of DAG Model



OnlyPush In Randomised Versions

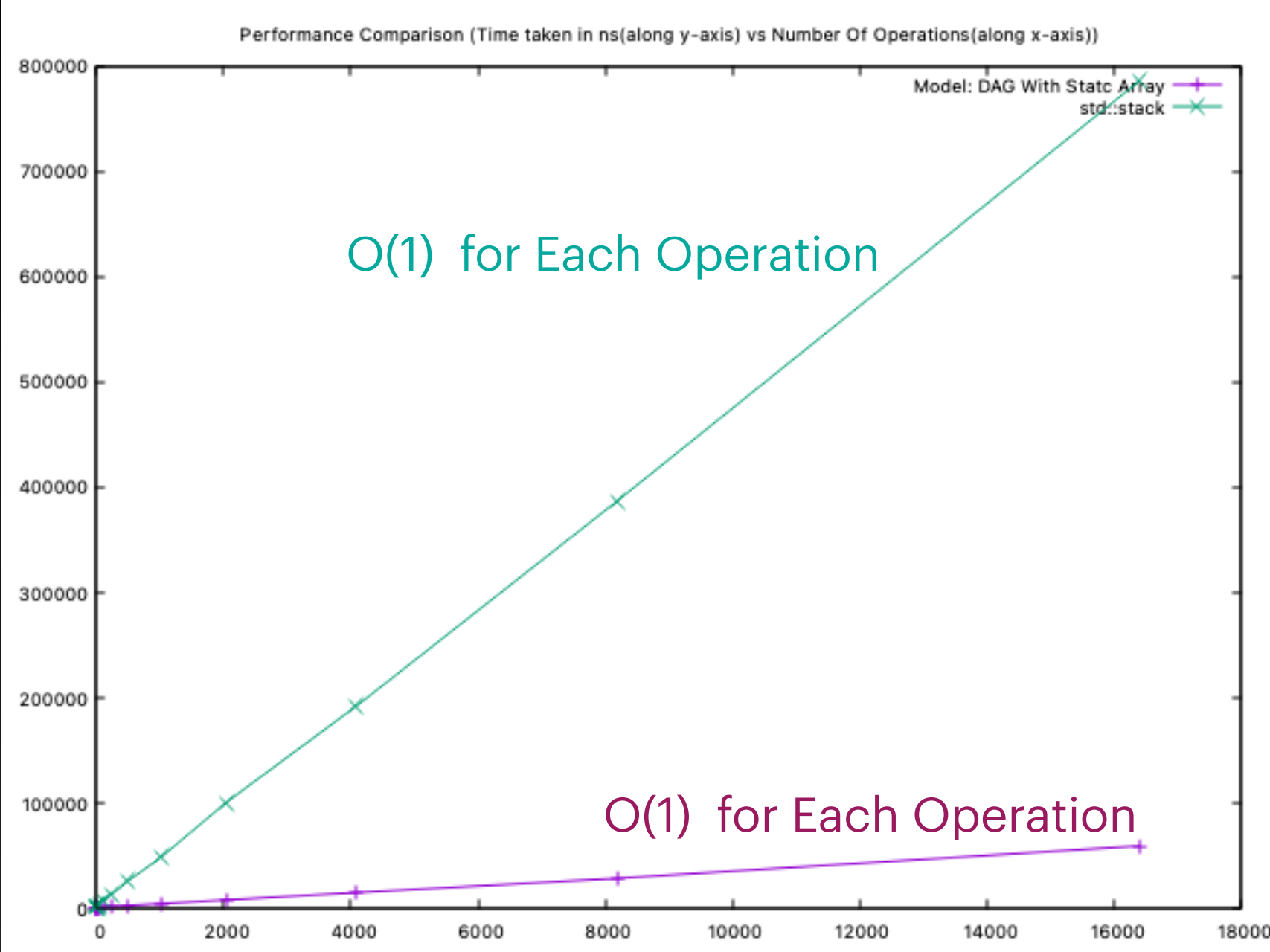


Randomised Push/Pop In Randomised Versions



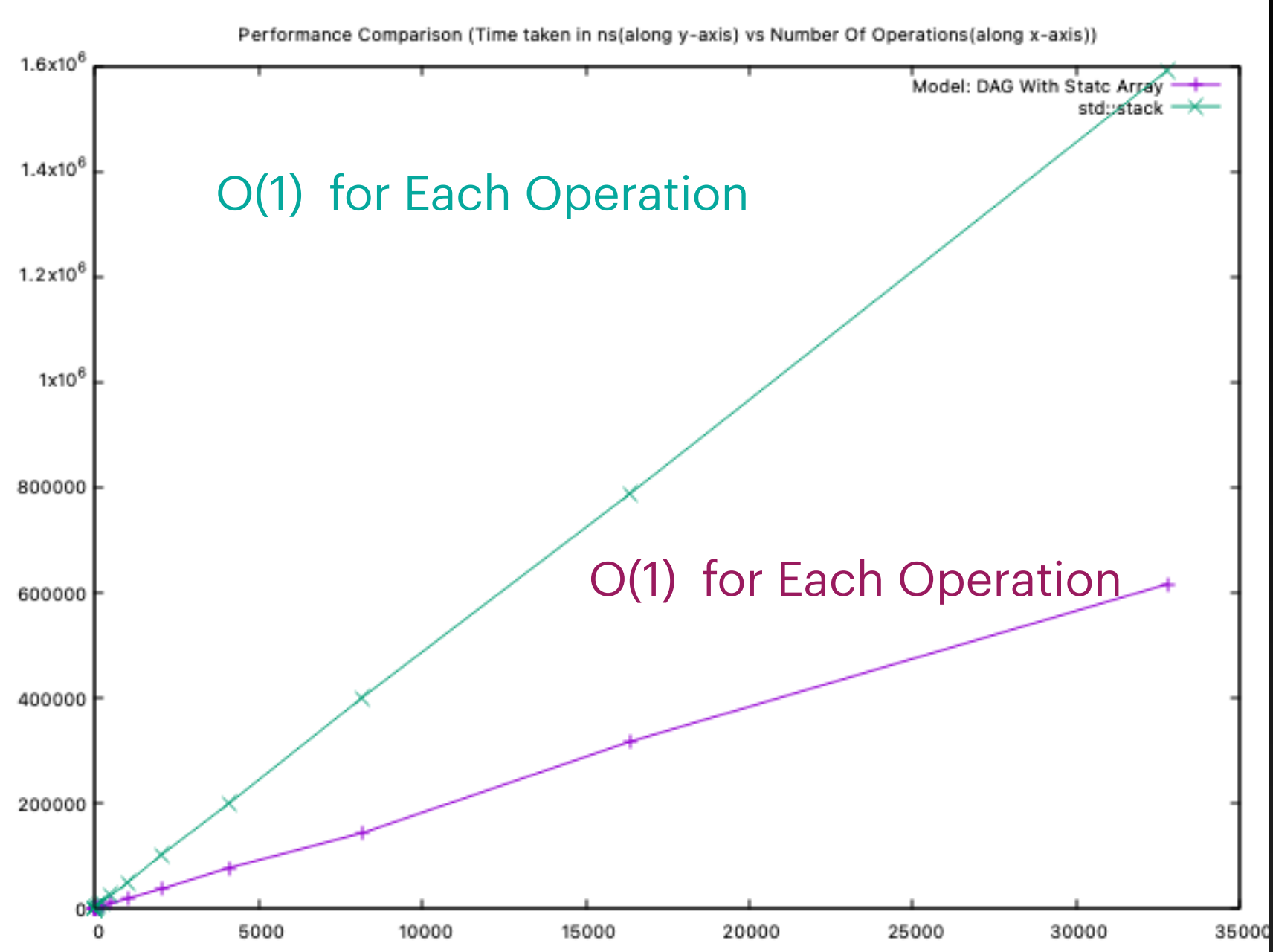
OnlyPop In Randomised Versions
(Pushing Operation is Not Time Profiled)

Benchmarking: DAG Model Vs std::stack

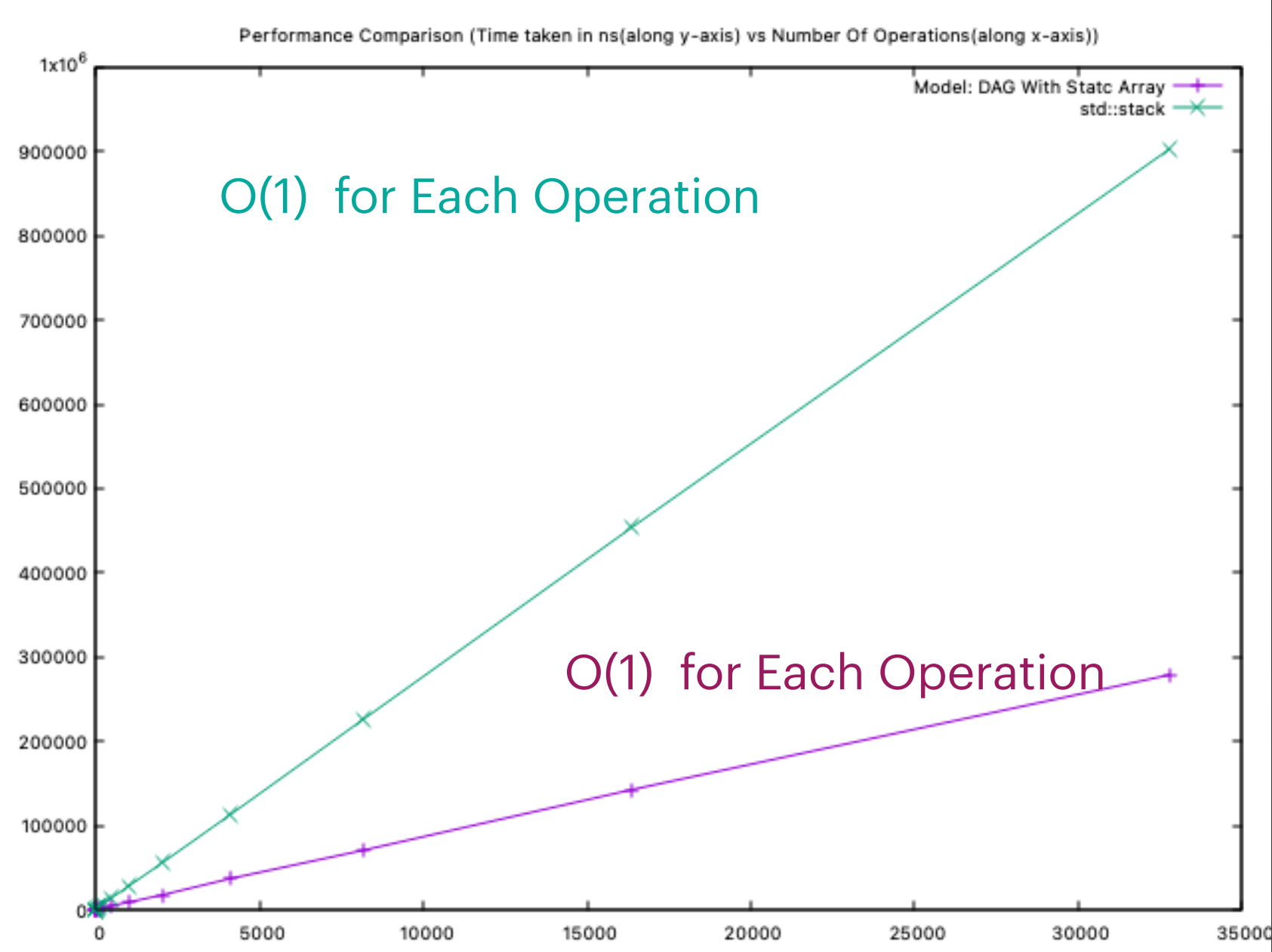


OnlyPush In latest Versions

Simulation Of Partial Persistence



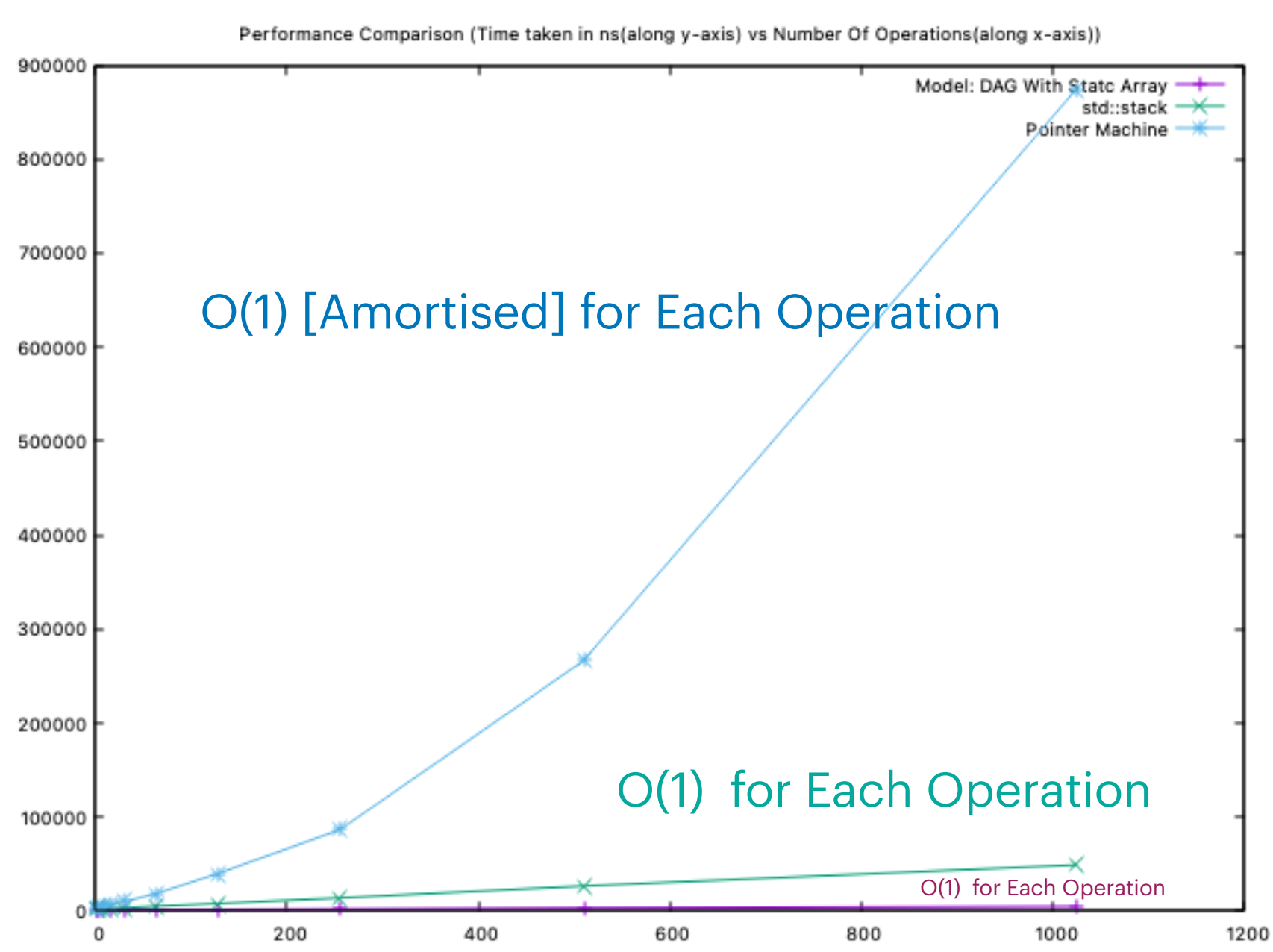
Randomised Push/Pop In Latest Versions



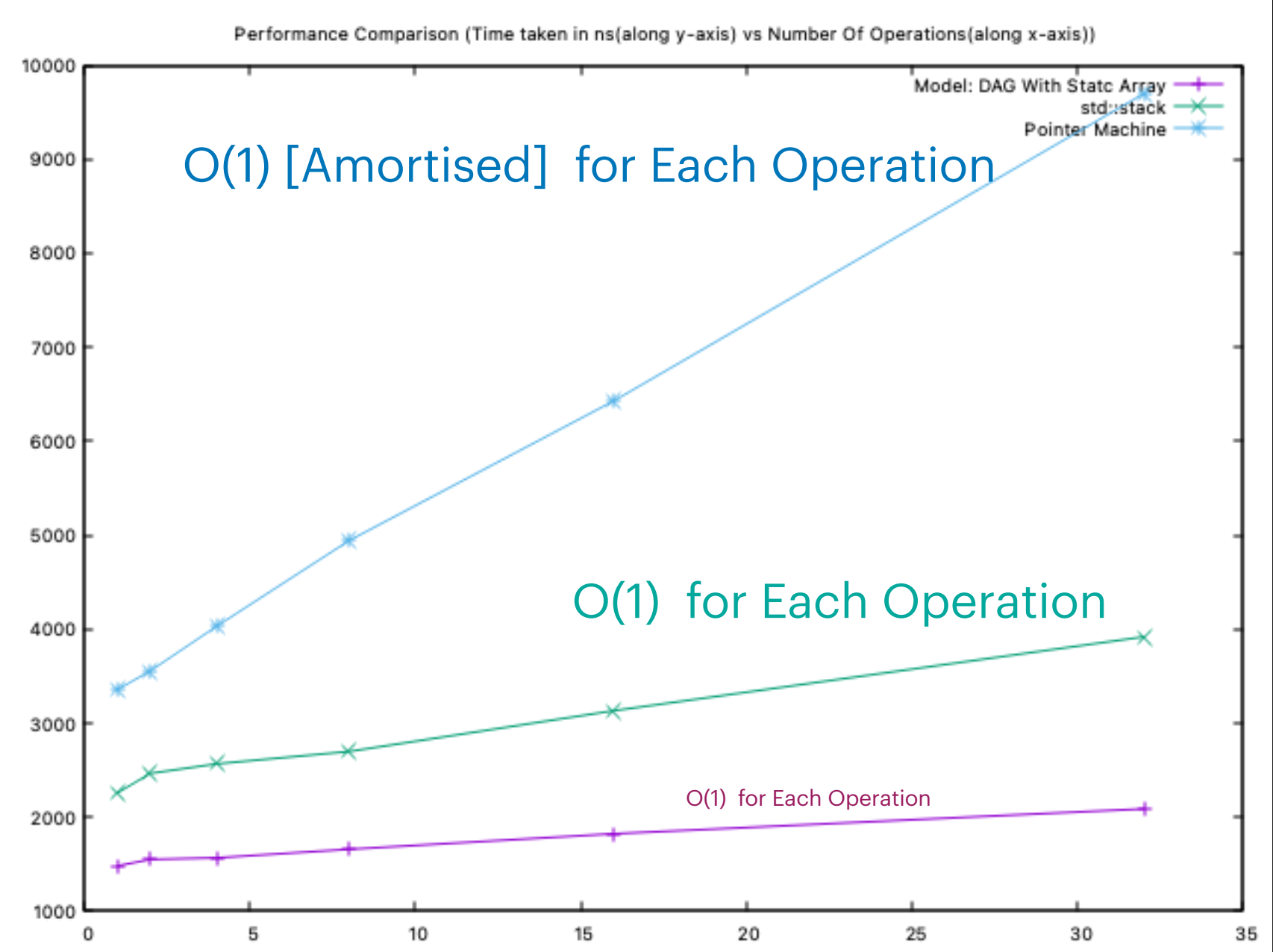
OnlyPop In Latest Versions
(Pushing Operation is Not Time Profiled)

NOTE: We Used Static Array To Handle
In DAG Model, So, It Is Slightly Faster Than
Std::Stack

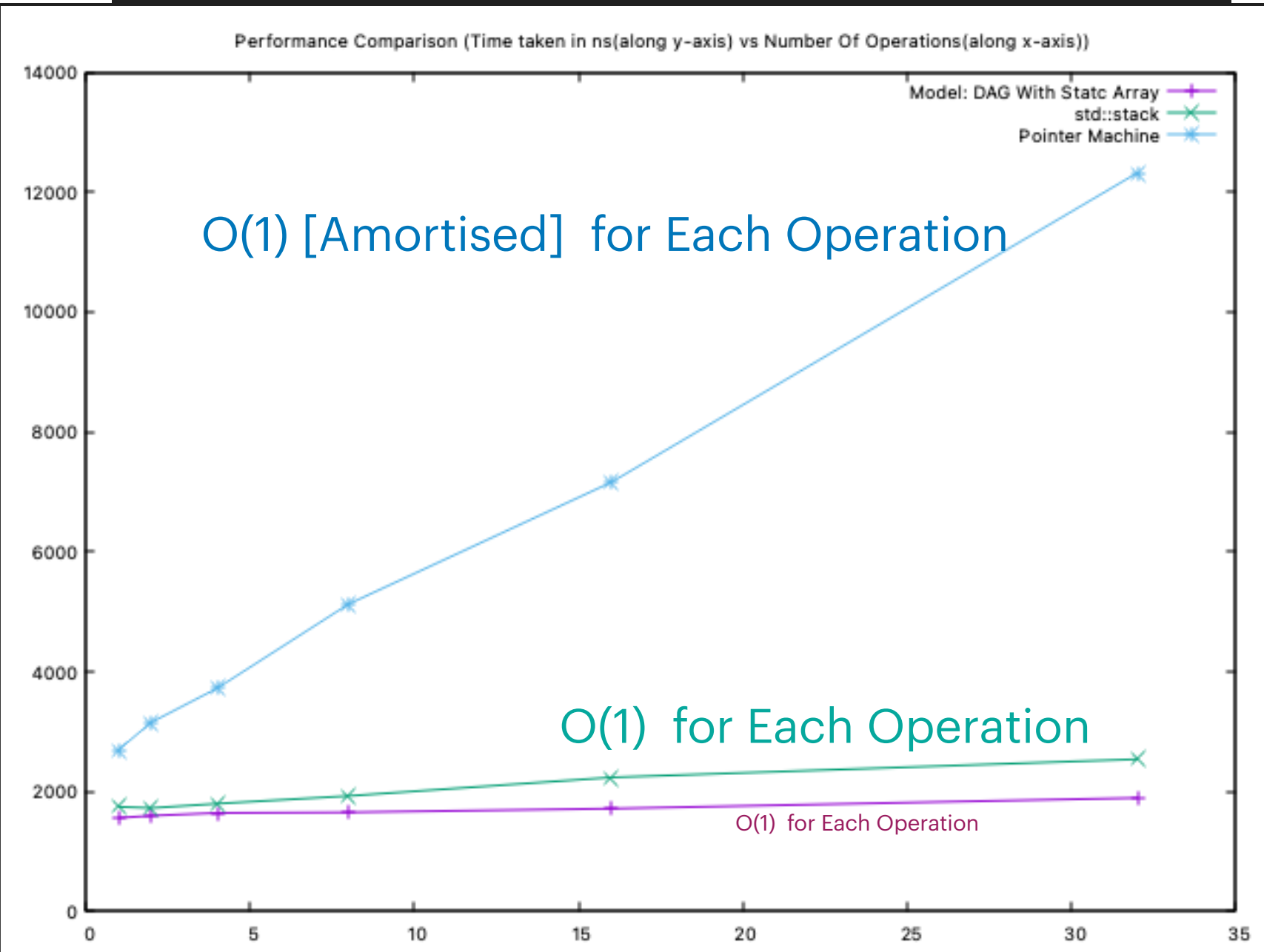
Benchmarking: DAG Model Vs std::stack Vs Stack_Using_PM_PPL



Simulation Of Partial Persistence



OnlyPush In Latest Versions



Randomised Push/Pop In Latest Versions

OnlyPop In Latest Versions
(Pushing Operation is Not Time Profiled)

NOTE: We Used Static Array To Handle
In DAG Model, So, It Is Slightly Faster Than
Std::Stack

Persistent Queue

Persistent Queue

Model Developed By Our Team

Time Complexity

Here, V = Total No. Of Version, N = Average Length Of The Queue Considering All The Versions

Strategies	enqueue(data, version)	dequeue(version)	getFront(ver)	getRear(rear)
Using PPL with PM Model	O(1)	O(1) [Amortized]	O(1)	O(1)
Reduced Sparse Matrix Model	O(log v) [Here v is current version] O((log V!)/V) ~ O(1)	O(log v) [In Average Case] O(v * log v) [In Worst Case]	O(1) [In Average Case] O(log v) [In Worst Case]	O(1)
Threaded_FPQ #	~O(1)	~O(1)	~O(1)	~O(1)
Ephemeral std::queue (C++)	O(1) [Only Current Version Supported]	O(1) [Only Current Version Supported]	O(1) [Only Current Version Supported]	O(1) [Only Current Version

Persistent Queue

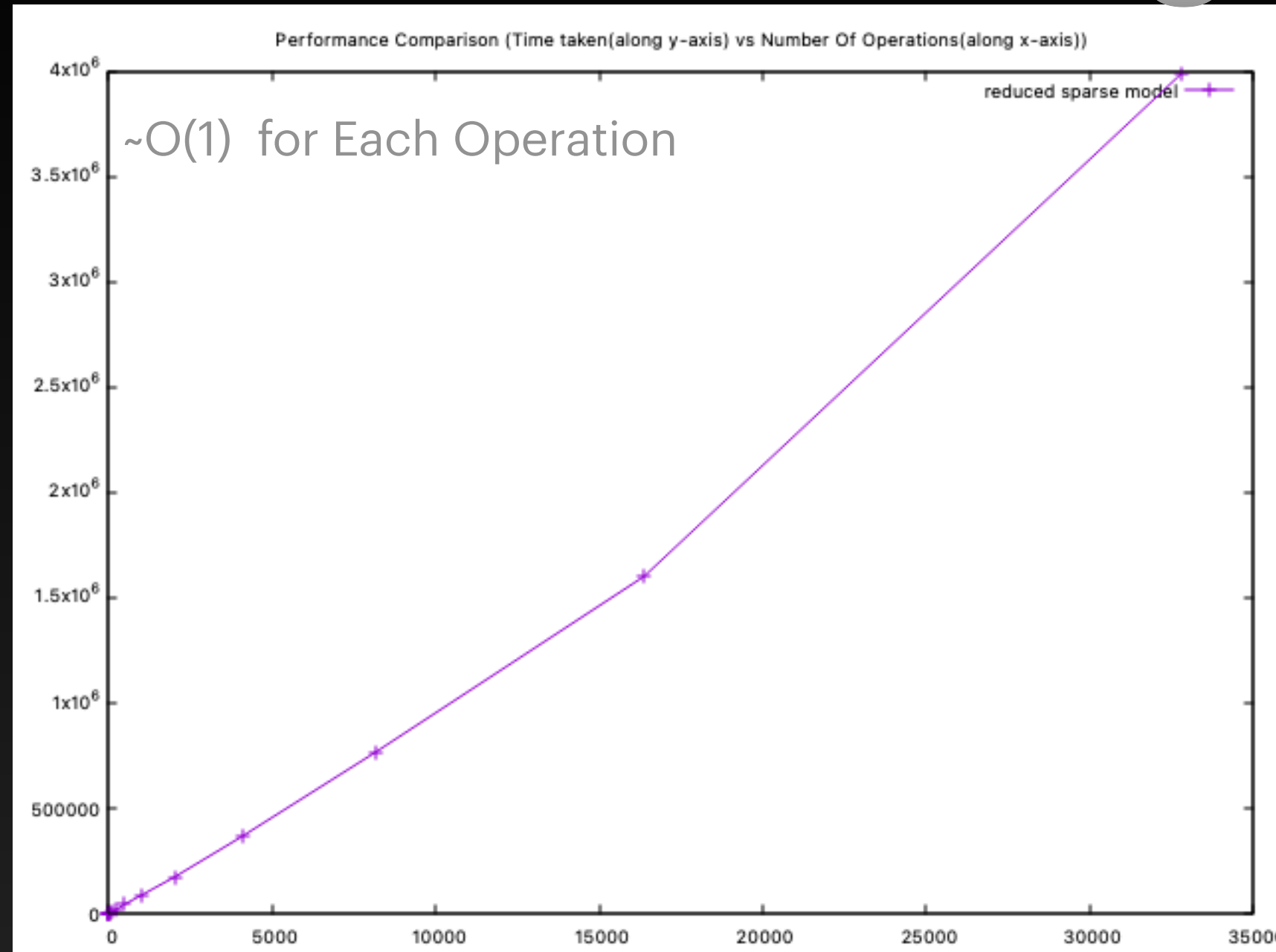
Model Developed By Our Team

Auxiliary Space

Here, V = Total No. Of Version, N = Average Length Of The Queue Considering All The Versions

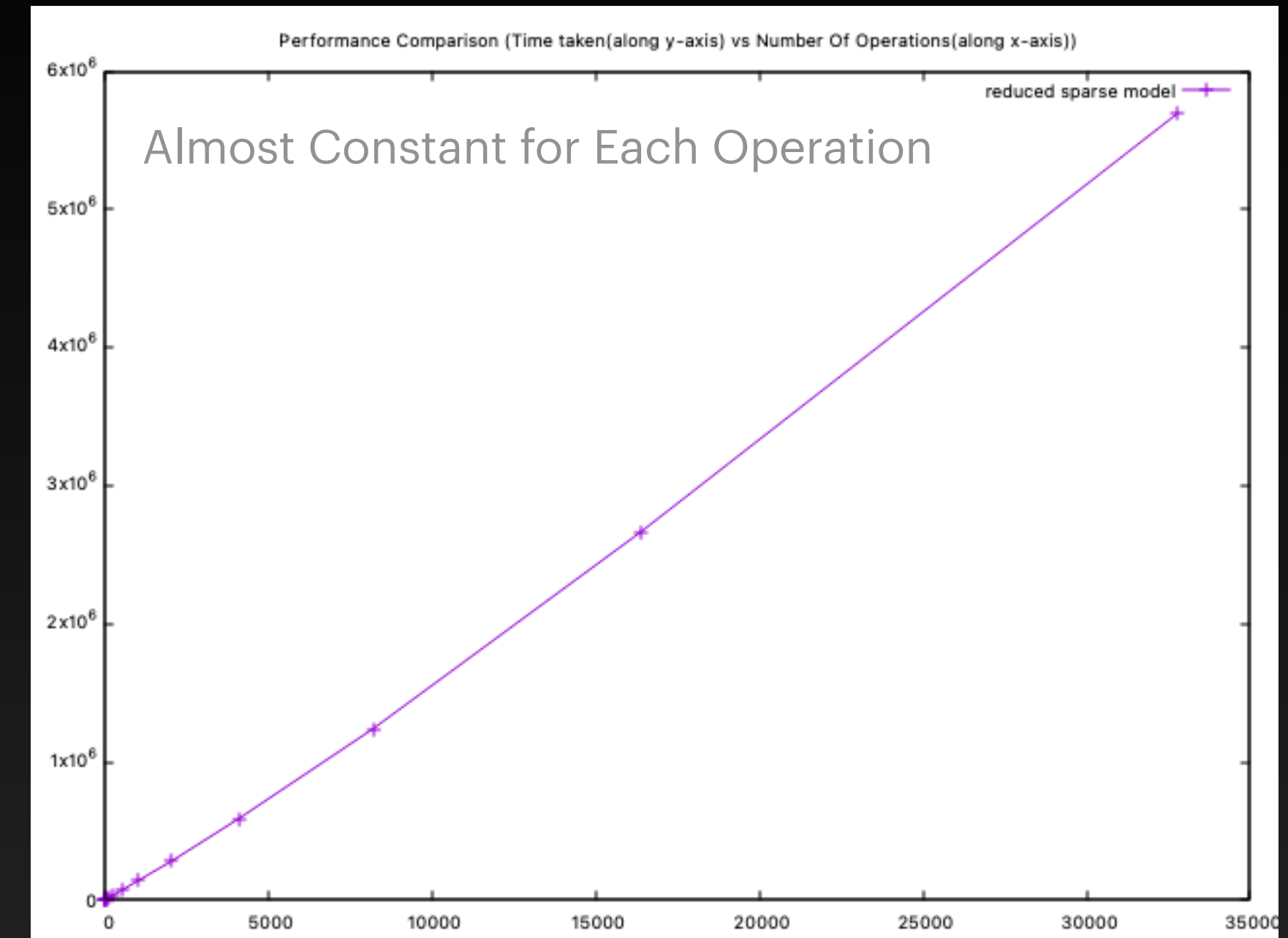
Strategies	Category 1	Category 2		Category 3
Using PPL with PM Model	$O(V)$ [Amortized] To Hold The Linked List	$O(V)$ to Hold The MAP for version->rear		
Reduced Sparse Matrix Model	<ul style="list-style-type: none">$O(V + \log(V!))$ to Hold The UP_TABLE	<ul style="list-style-type: none">$O(V)$ to Hold the MAP	<ul style="list-style-type: none">$O(V)$ to Hold the TYPE_OF_VER	
Threaded FPQ	$O(\text{no_of_enqueues} = n)$ to Hold The Nodes	$O(V)$ to Hold The Versions	$O(1)$ [Best Case] $O(\log V)$ [Balanced Average Case , $O(N*V)$ [Worst Case] To Hold The Thread Directions	Disadvantage Very Low I/O Rate due to Heavy Access To The Heap Memory
Ephemeral std::queue (C++)	$O(N)$			-

Benchmarking Of Reduced Sparse Matrix Model

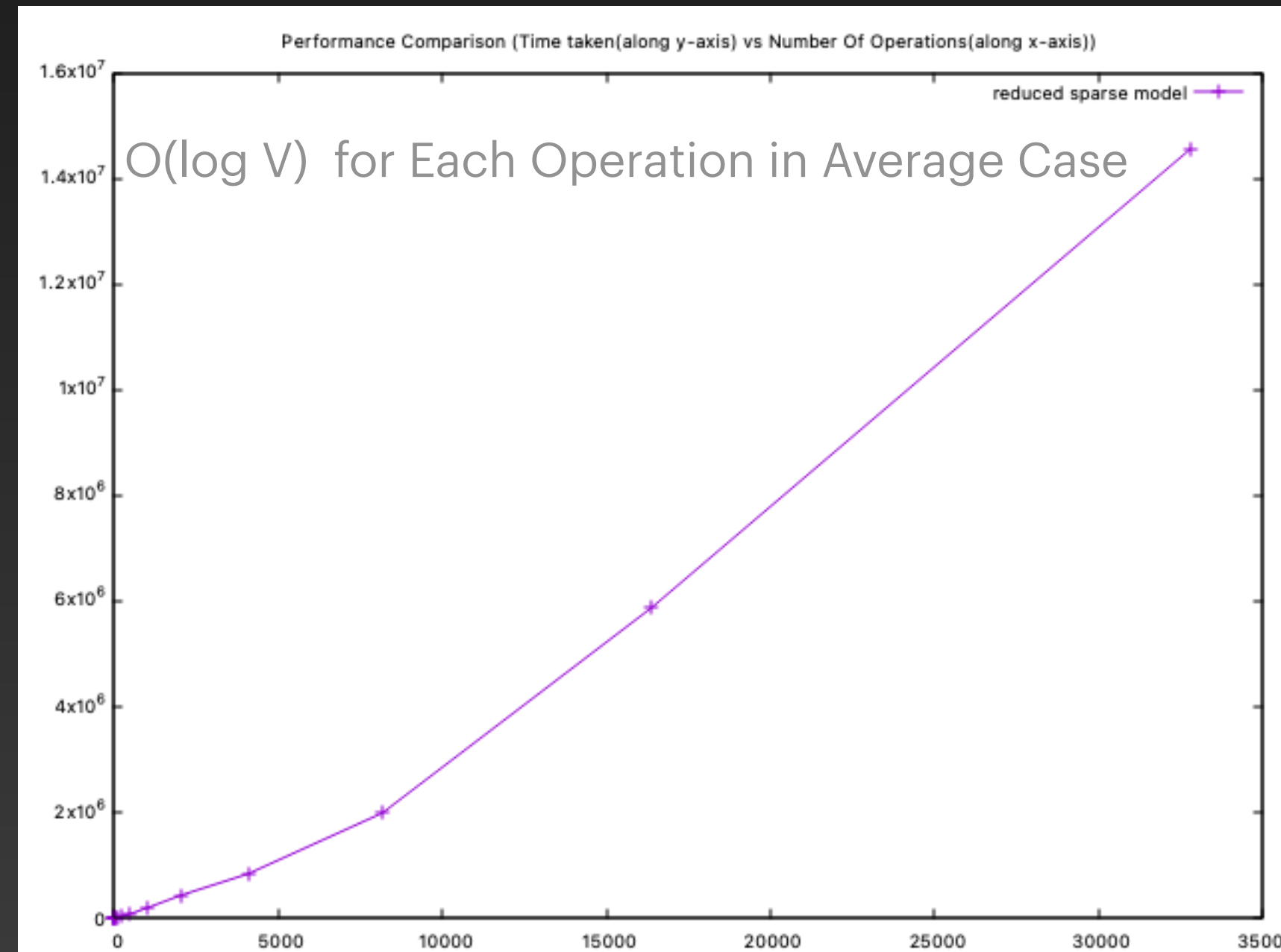


Only Enqueue In Randomised Versions

Simulation Of Full Persistence

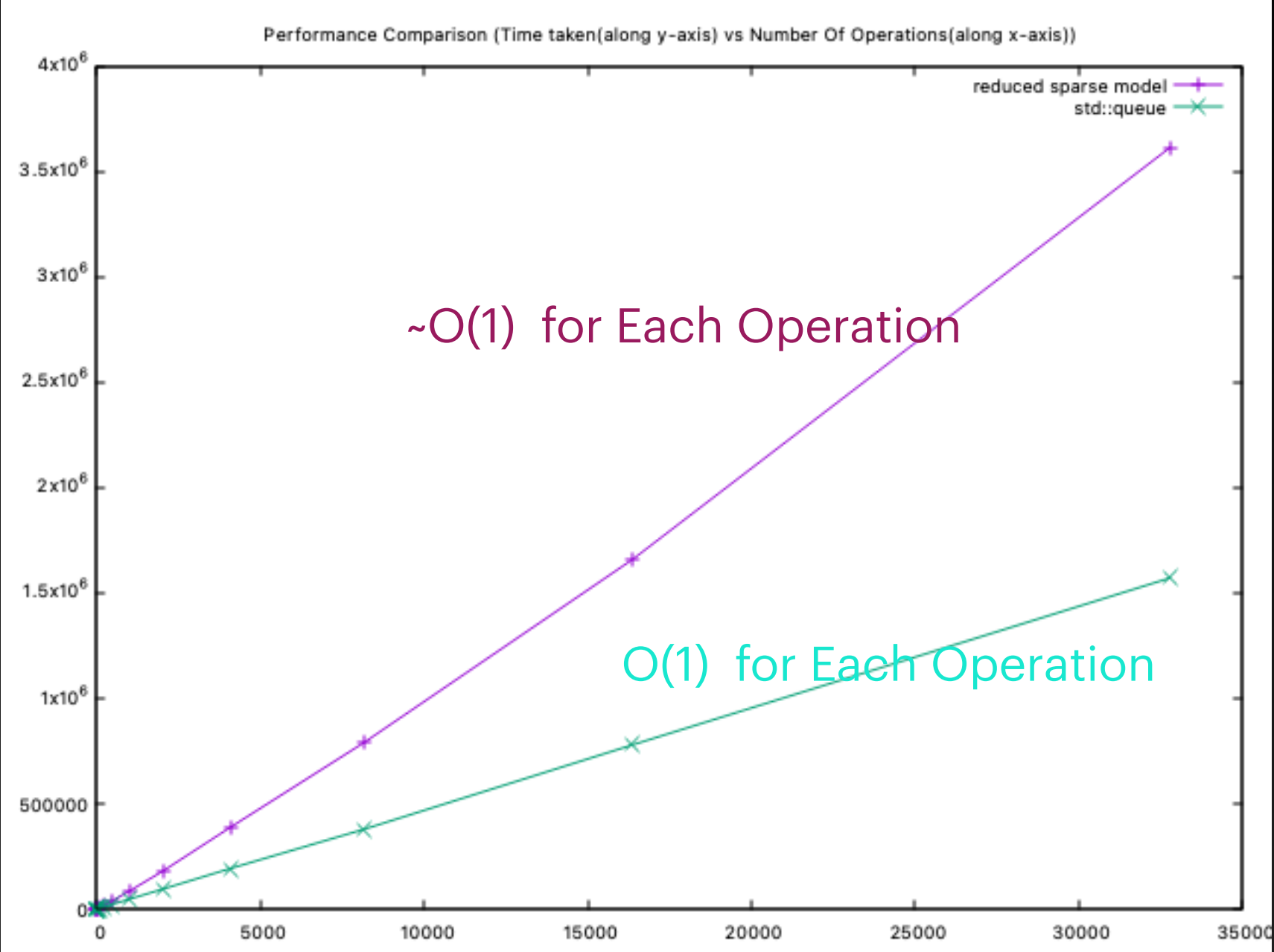


Randomised Enqueue/Deque In Randomised Versions



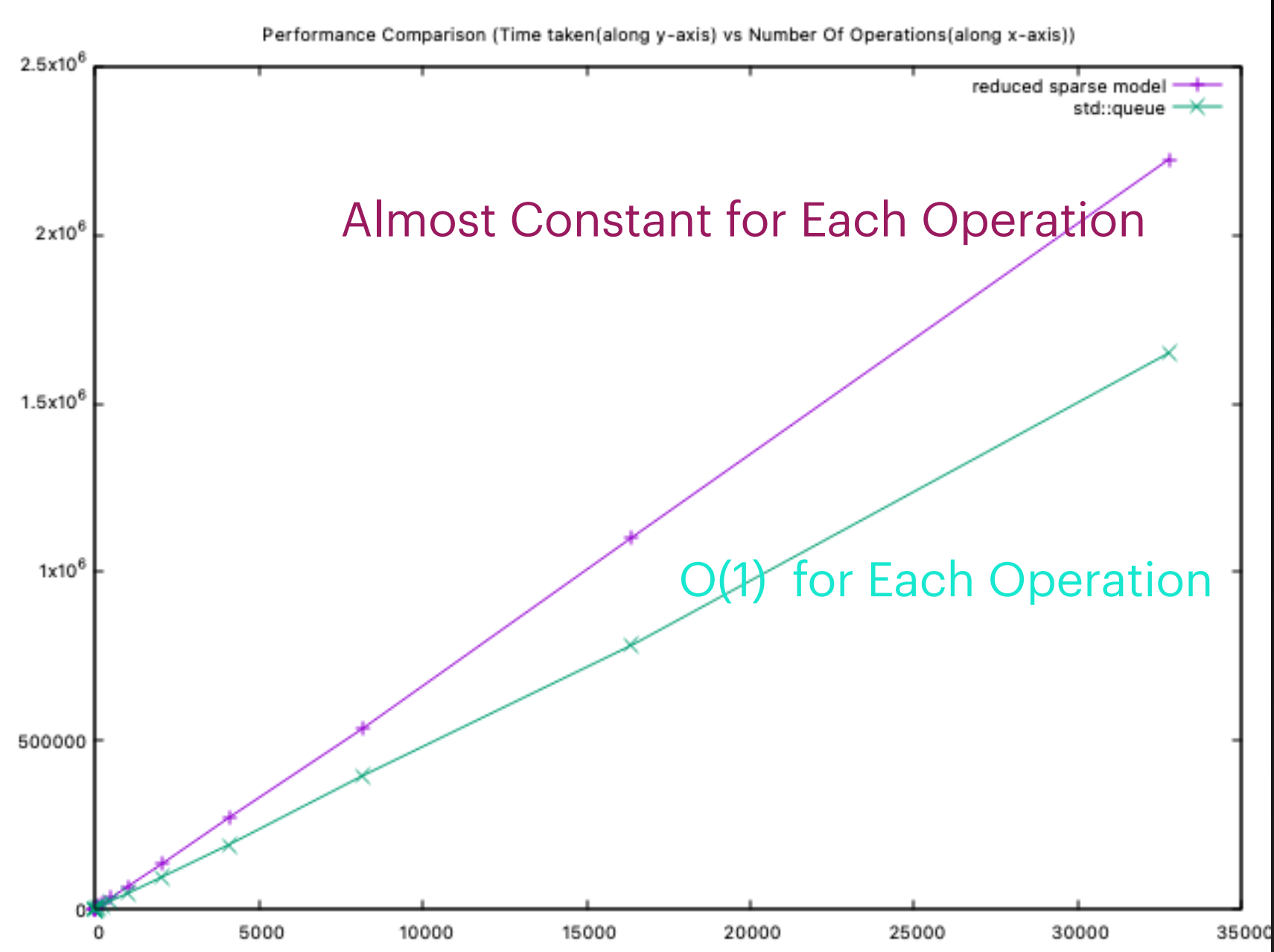
Only Deque In Randomised Versions
(Enqueuing Operation is Not Time Profiled)

Benchmarking: Reduced Sparse Matrix Model vs std::queue

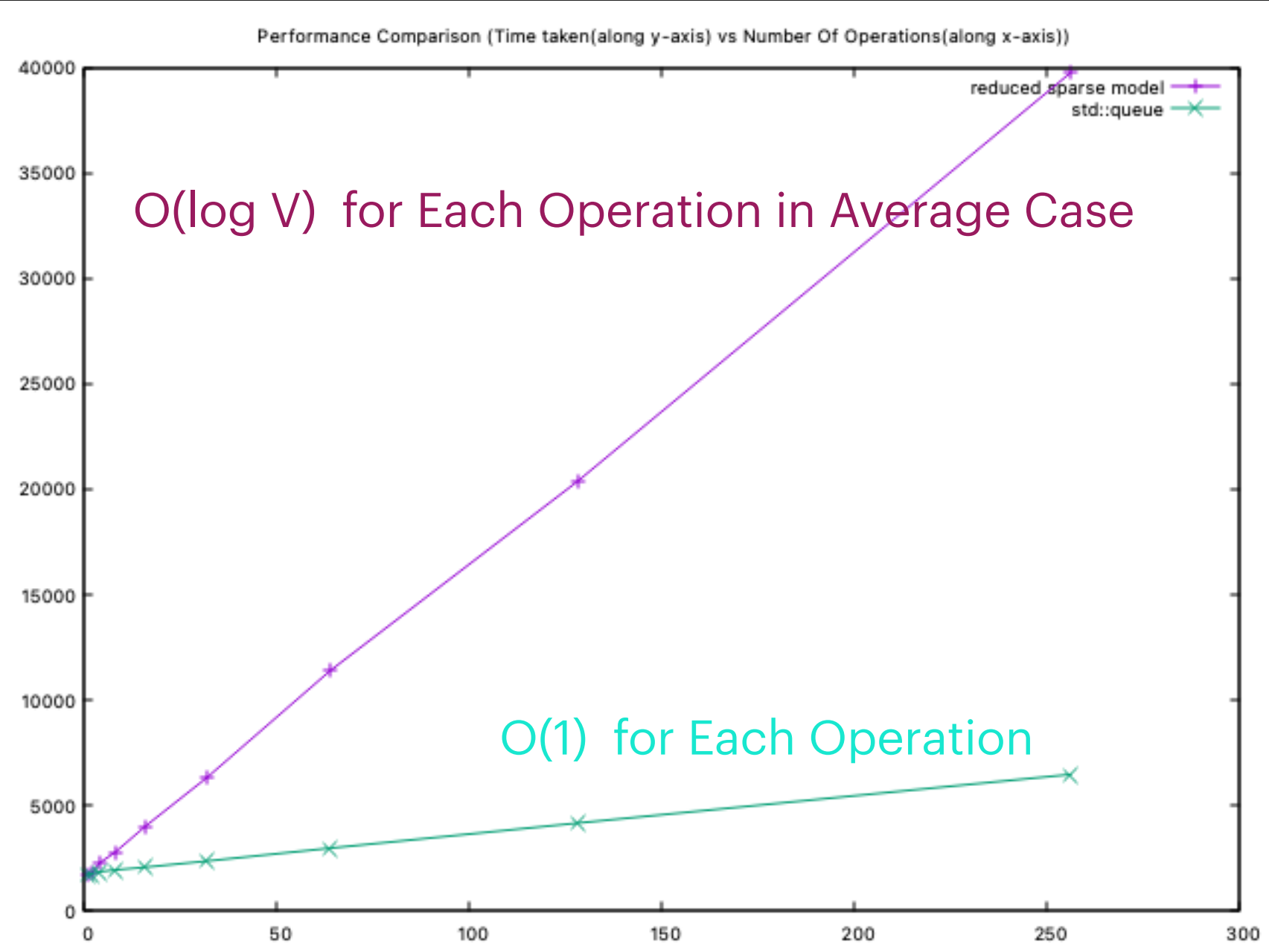


Only Enqueue In Latest Versions

Simulation Of Partial Persistence

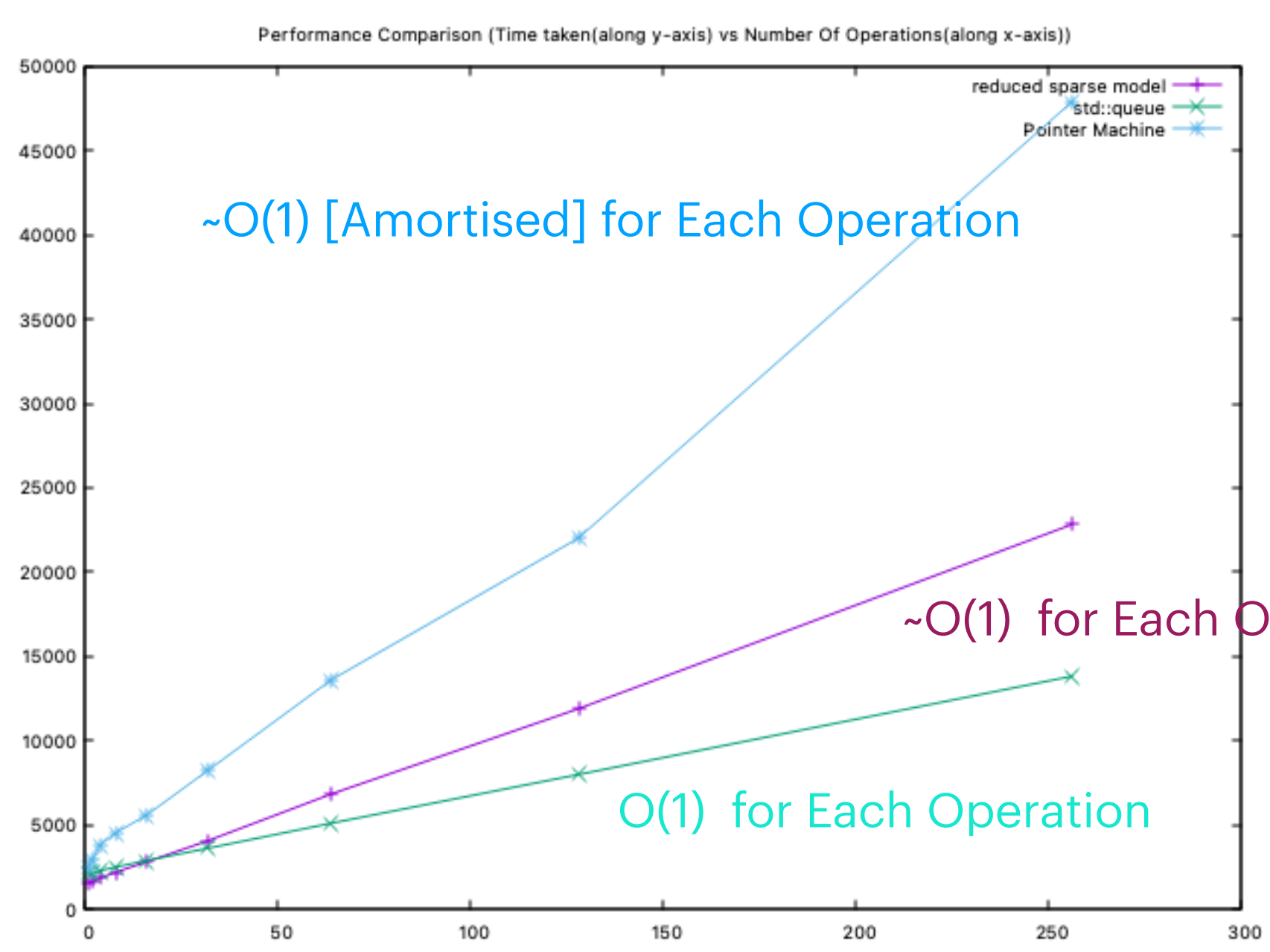


Randomised Enqueue/Deque In Latest Versions

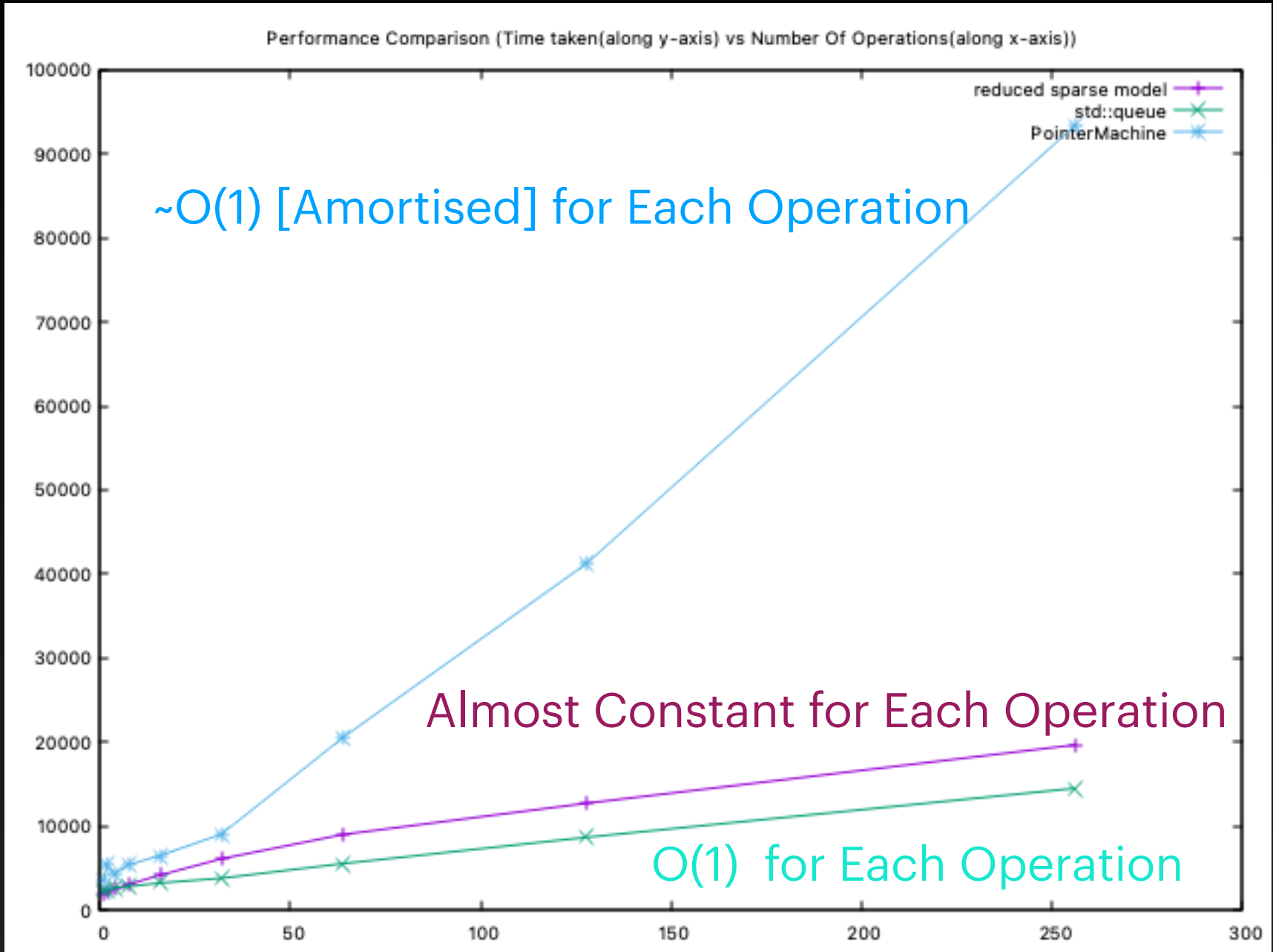
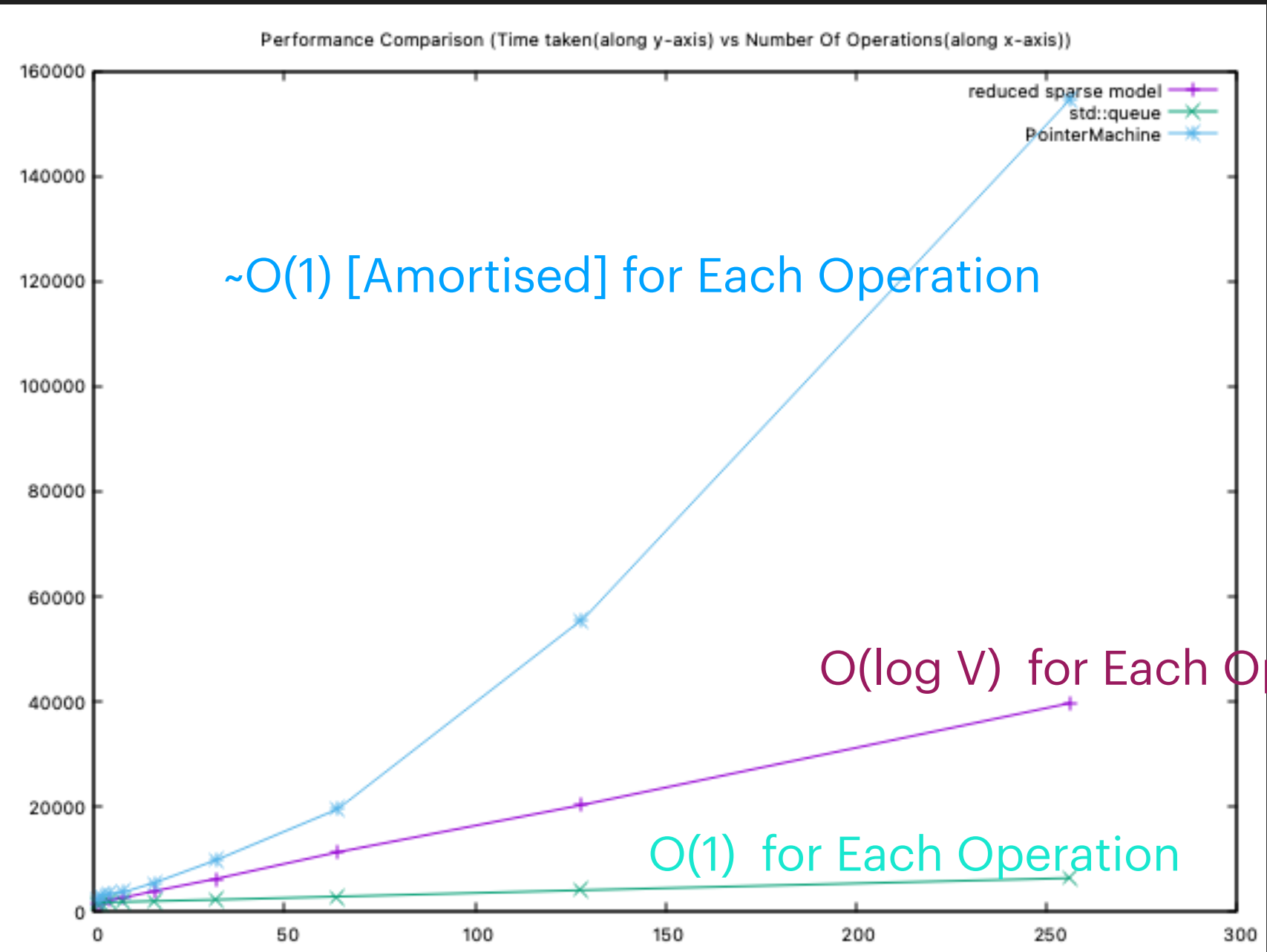


Only Deque In Latest Versions
(Enqueuing Operation is Not Time Profiled)

Benchmarking: Reduced Sparse Matrix Model vs std::queue Vs Queue_Using_PM_PPL



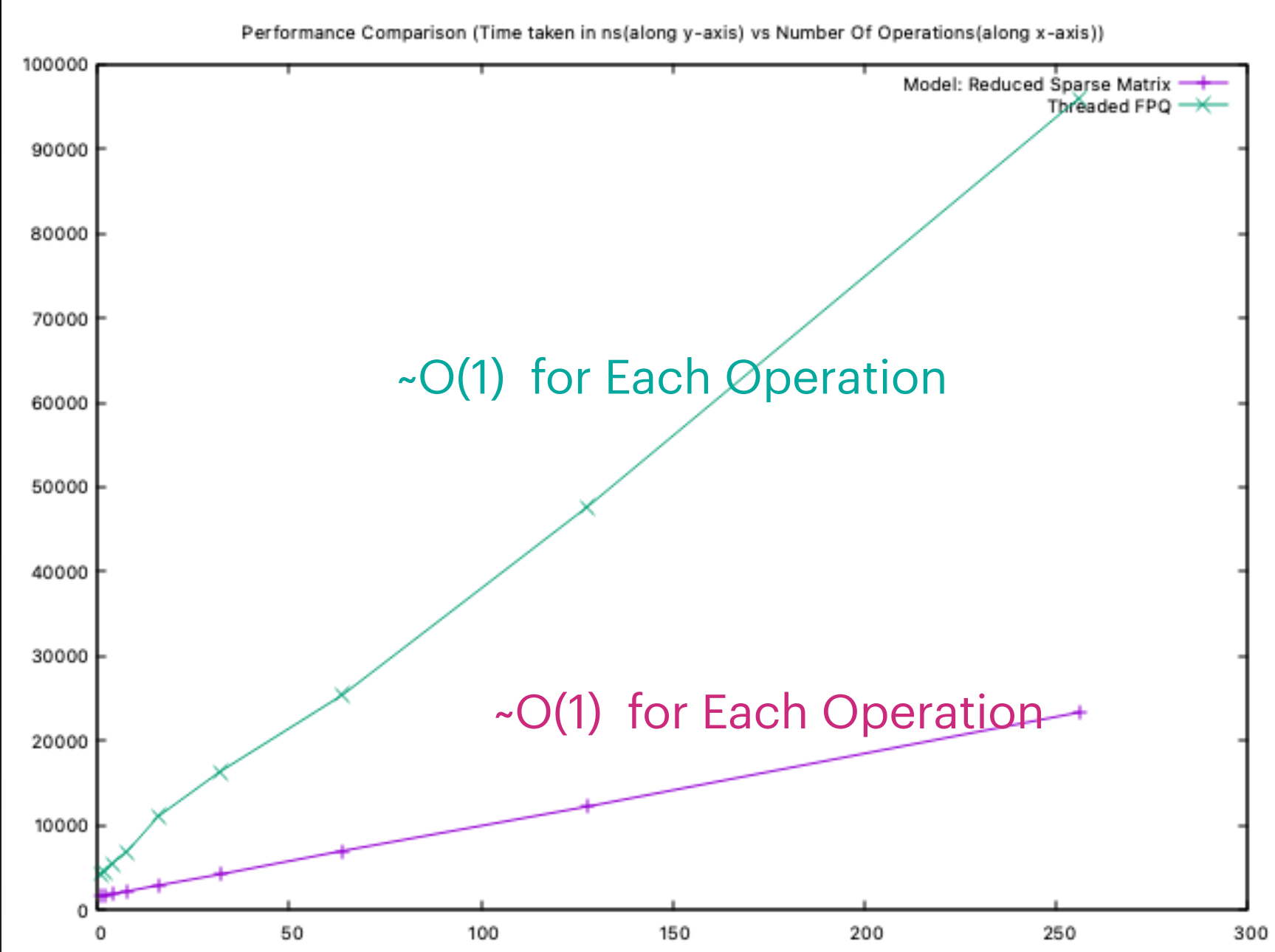
Only Enqueue In Latest Versions



Randomised Enqueue/Dequeue In Latest Versions

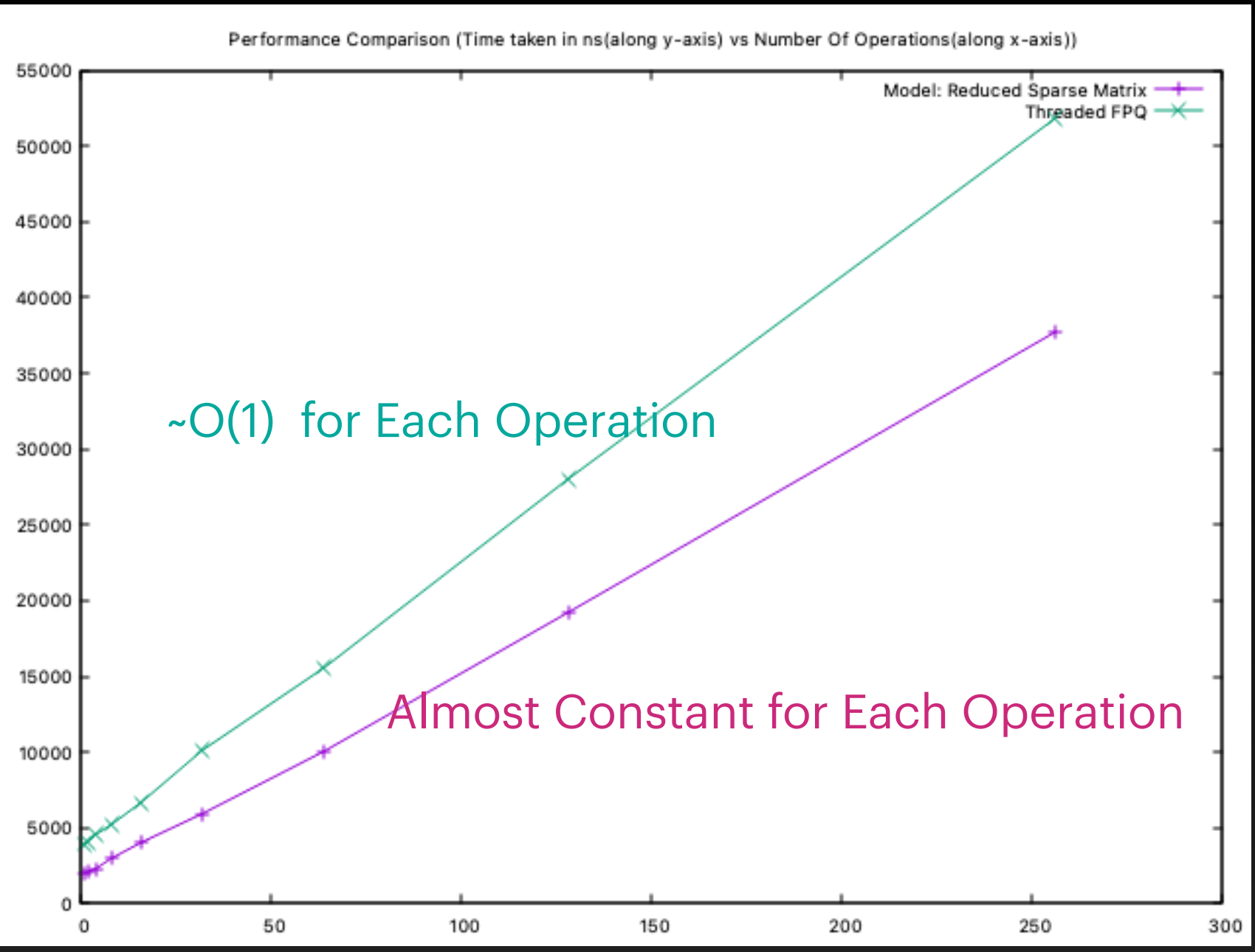
Only Dequeue In Latest Versions
(Enqueuing Operation is Not Time Profiled)

Benchmarking: Reduced Sparse Matrix Model Vs Threaded FPQ

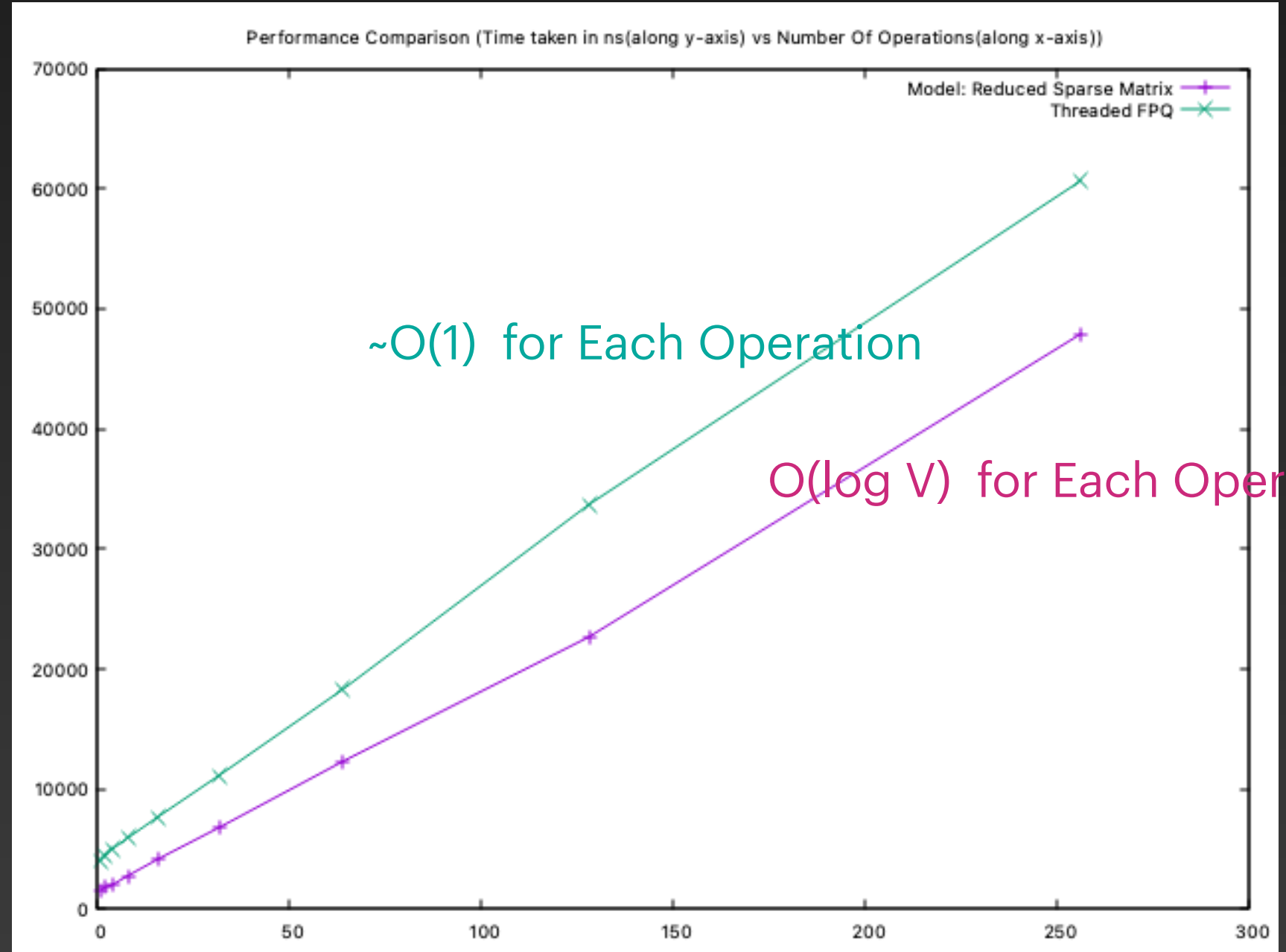


Simulation Of Full Persistence

Only Enqueue In Randomised Versions

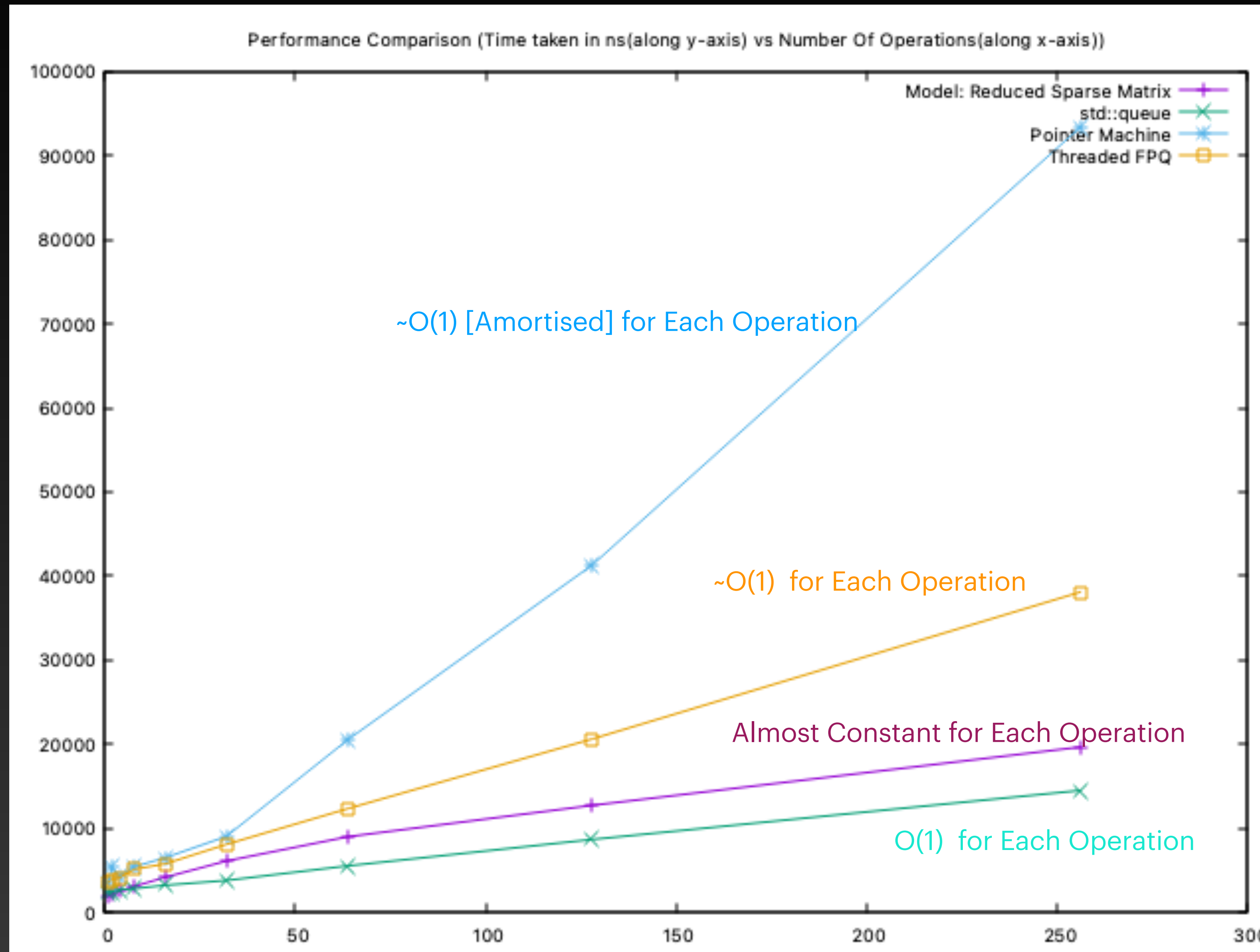


Randomised Enqueue/Deque In Randomised Versions



Only Dequeue In Randomised Versions
(Enqueueing Operation is Not Time Profiled)

Benchmarking: Reduced Sparse Matrix Model vs std::queue Vs Queue_Using_PM_PPL vs Threaded FPQ

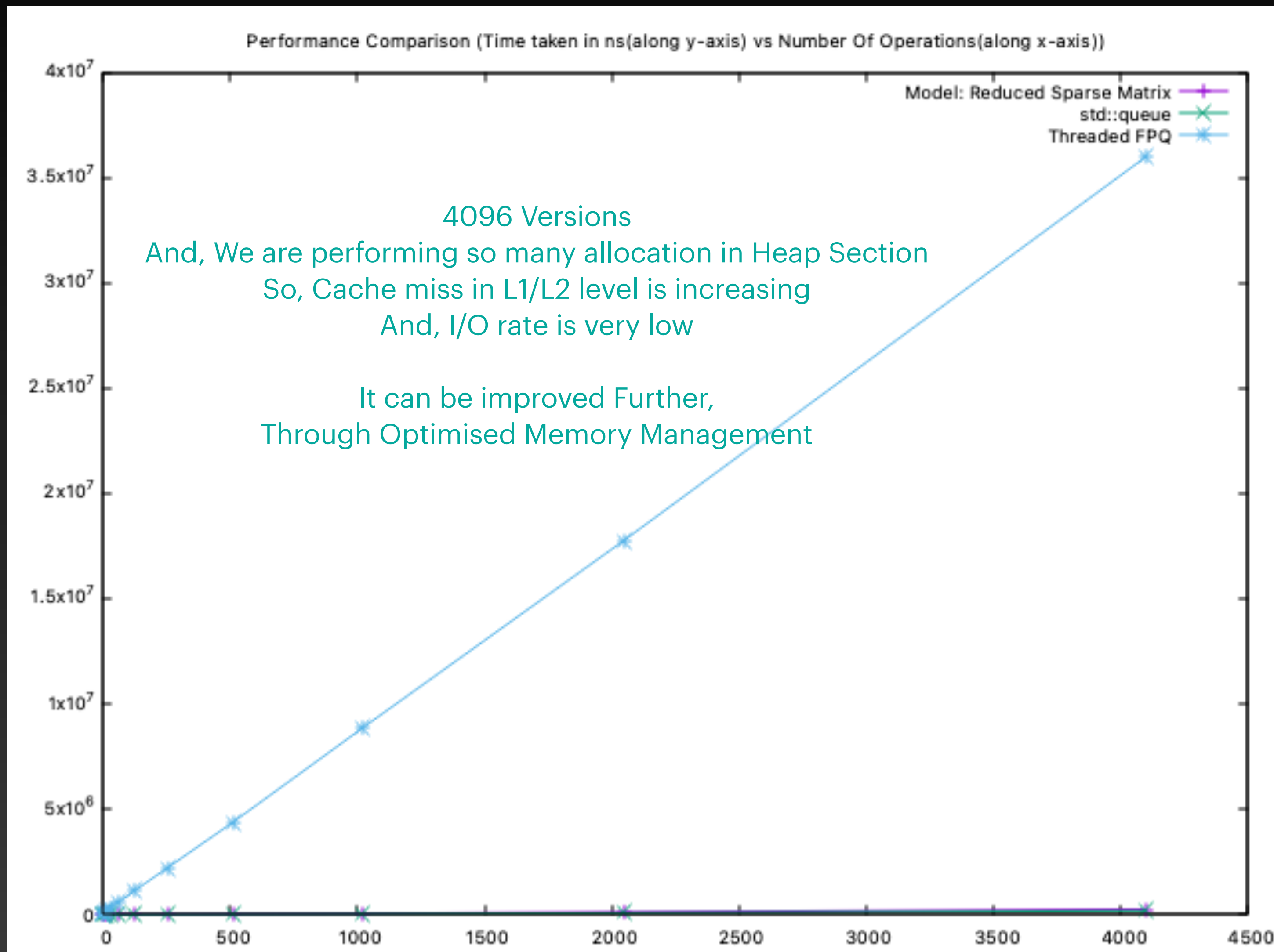


Simulation Of Partial Persistence

Randomised Enqueue/Dequeue In Consecutive Versions

Issue With Threaded FPQ Model ...

Low I/O Rate Due to poor memory management in C



Issue With Threaded FPQ Model ... / BenchMarking Data

```
~/Desktop/programming/XcodePrj/CM_SIR_PROJECT/Annannyo_q2 — -zsh
~/Desktop/persistent/MyCode/BenchMarkingWithGoogleBenchmark/src/main/gpp_run/QUEUE_Anannyo — -zsh +

L1 Data 48 KiB (x4)
L1 Instruction 32 KiB (x4)
L2 Unified 512 KiB (x4)
L3 Unified 6144 KiB (x1)
Load Average: 1.40, 1.98, 2.12

-----
Benchmark              Time              CPU    Iterations UserCounters...
-----
BM_QUEUE_Wrapper/1      19629 ns          19169 ns      46360 items_per_second=52.167k/s 1
BM_QUEUE_Wrapper/2      32168 ns          31566 ns      20668 items_per_second=63.3589k/s 2
BM_QUEUE_Wrapper/4      53112 ns          52226 ns      12222 items_per_second=76.59k/s 4
BM_QUEUE_Wrapper/8      93951 ns          92851 ns       7158 items_per_second=86.1592k/s 8
BM_QUEUE_Wrapper/16     171014 ns         169404 ns      4016 items_per_second=94.4486k/s 16
BM_QUEUE_Wrapper/32     312674 ns         310172 ns      2236 items_per_second=103.169k/s 32
BM_QUEUE_Wrapper/64     602828 ns         599170 ns      1174 items_per_second=106.815k/s 64
BM_QUEUE_Wrapper/128    1180954 ns        1171348 ns       587 items_per_second=109.276k/s 128
BM_QUEUE_Wrapper/256    2344332 ns        2331366 ns       295 items_per_second=109.807k/s 256
BM_QUEUE_Wrapper/512    4598501 ns        4579066 ns       152 items_per_second=111.813k/s 512
BM_QUEUE_Wrapper/1024   9090952 ns        9056254 ns        71 items_per_second=113.071k/s 1024
BM_QUEUE_Wrapper/2048   19536364 ns       19327947 ns        38 items_per_second=105.961k/s 2048
BM_QUEUE_Wrapper/4096   40346193 ns       39580063 ns        16 items_per_second=103.486k/s 4096
BM_QUEUE_Wrapper_BigO   9733.55 N         9571.93 N         1
BM_QUEUE_Wrapper_RMS     5 %              4 %              1

(base) debasmitroy@DEBASMITs-MacBook-Air QUEUE_Anannyo % g++ FP_QUEUE_MAIN_ENQ.cpp -std=c++11 -isystem benchmark/include -Lbenchmark/build/src -lbenchmark -lpthread -o mybenchmark
ld: warning: directory not found for option '-Lbenchmark/build/src'
(base) debasmitroy@DEBASMITs-MacBook-Air QUEUE_Anannyo % ./mybenchmark --benchmark_out="rand_enq_FP_256.csv" --benchmark_out_format=csv
[2022-01-23T12:29:27+05:30
Running ./mybenchmark
Run on (8 X 1100 MHz CPU s)
CPU Caches:
  L1 Data 48 KiB (x4)
  L1 Instruction 32 KiB (x4)
  L2 Unified 512 KiB (x4)
  L3 Unified 6144 KiB (x1)
Load Average: 1.51, 1.93, 2.10

-----
Benchmark              Time              CPU    Iterations UserCounters...
-----
BM_QUEUE_Wrapper/1      4349 ns          4286 ns     198018 items_per_second=233.318k/s 1
BM_QUEUE_Wrapper/2      4823 ns          4690 ns     140919 items_per_second=426.455k/s 2
BM_QUEUE_Wrapper/4      5671 ns          5563 ns     132090 items_per_second=719.071k/s 4
BM_QUEUE_Wrapper/8      7155 ns          7044 ns     80764 items_per_second=1.1357M/s 8
BM_QUEUE_Wrapper/16     11422 ns         11129 ns     73691 items_per_second=1.43769M/s 16
BM_QUEUE_Wrapper/32     16605 ns         16381 ns     42350 items_per_second=1.95348M/s 32
BM_QUEUE_Wrapper/64     25580 ns         25460 ns     26858 items_per_second=2.51378M/s 64
BM_QUEUE_Wrapper/128    48136 ns         47762 ns     14310 items_per_second=2.67998M/s 128
BM_QUEUE_Wrapper/256    97095 ns         95942 ns      7683 items_per_second=2.66827M/s 256
BM_QUEUE_Wrapper_BigO   382.95 N         378.79 N         1
BM_QUEUE_Wrapper_RMS    15 %            14 %            1
```

Very Poor I/O Rate
@ 4096 Versions

Very decent I/O Rate
@ 256 Versions

Persistent Search Tree

Persistent Search Tree

Time Complexity

Here, V = Total No. Of Version, N = Average Number Of Elements in The Tree Considering All The Versions

Strategies	Updation/Insertion/ Deletion of Data	Retrieval Of Data
Path Copying With Normal BST	$O(N)$ in Worst Case $O(1)$ in Best Case	$O(V) + O(N)$ in Worst Case $O(V) + O(1)$ in Best Case
Path Copying With AVL/RB Tree	$O(\log_2 N)$ in Worst Case $O(1)$ in Best Case	$O(V) + O(\log_2 N)$ in Worst Case $O(V) + O(1)$ in Best Case
Path Copying With Hash Array Mapped Trie	$O(\log_{32}(2^{64})) \sim O(12)$	$O(V) + O(\log_{32}(2^{64})) \sim O(12)$
With Pointer Machine	$O(1)$ Amortised	$O(1)$ Amortised

Persistent Search Tree

Auxiliary Space

Here, V = Total No. Of Version, N = Average Number Of Elements in The Tree Considering All The Versions

Strategies	Category 1	Category 2
Path Copying With Normal Threaded BST	Node Size: # ~ 28 byte	$O(N + V)$ to Hold The Tree and Staring Pointers
Path Copying With AVL/RB Tree	Node Size: # ~ 32 byte	$O(N + V)$ to Hold The Tree and Staring Pointers
Path Copying With Bitmapped Hash Array Mapped Trie	Node Size: # ~ (4+4+64*8) byte [Worst Case] ~ (4+4) byte [Best Case]	$O(N + V)$ to Hold The Tree and Staring Pointers
With Pointer Machine	Node Size: # ~ 250 byte	$O(N)$ [Amortised] to Hold The LinkedList

to store 4 byte Integer | 8 Bye pointers

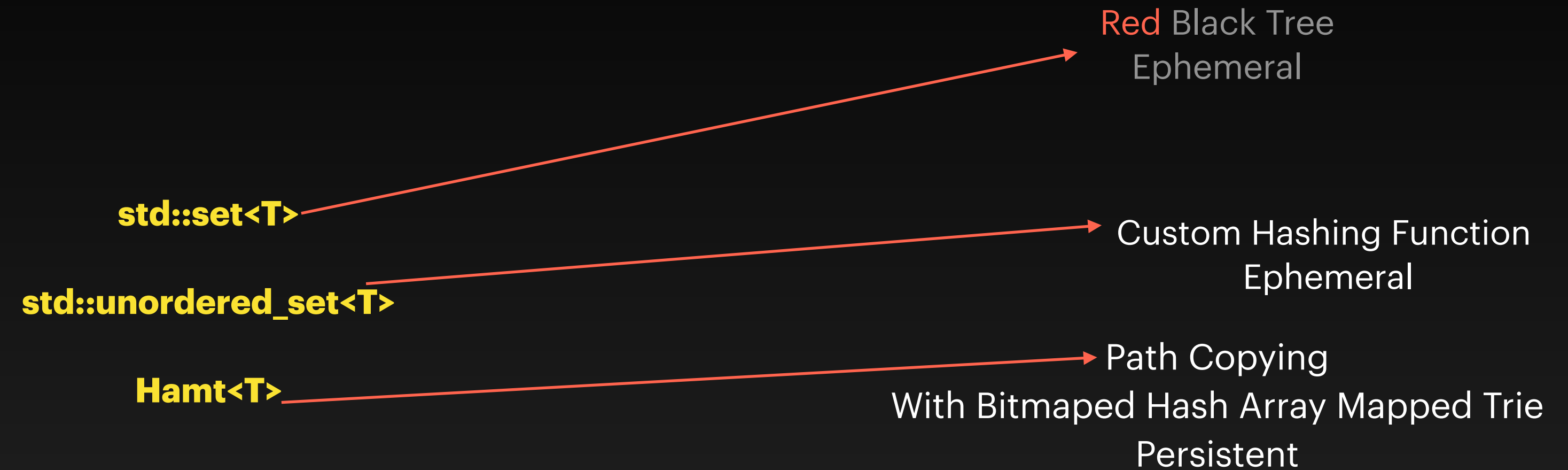
BenchMarking Of Search Tree

BenchMarking Tool:

Nonius

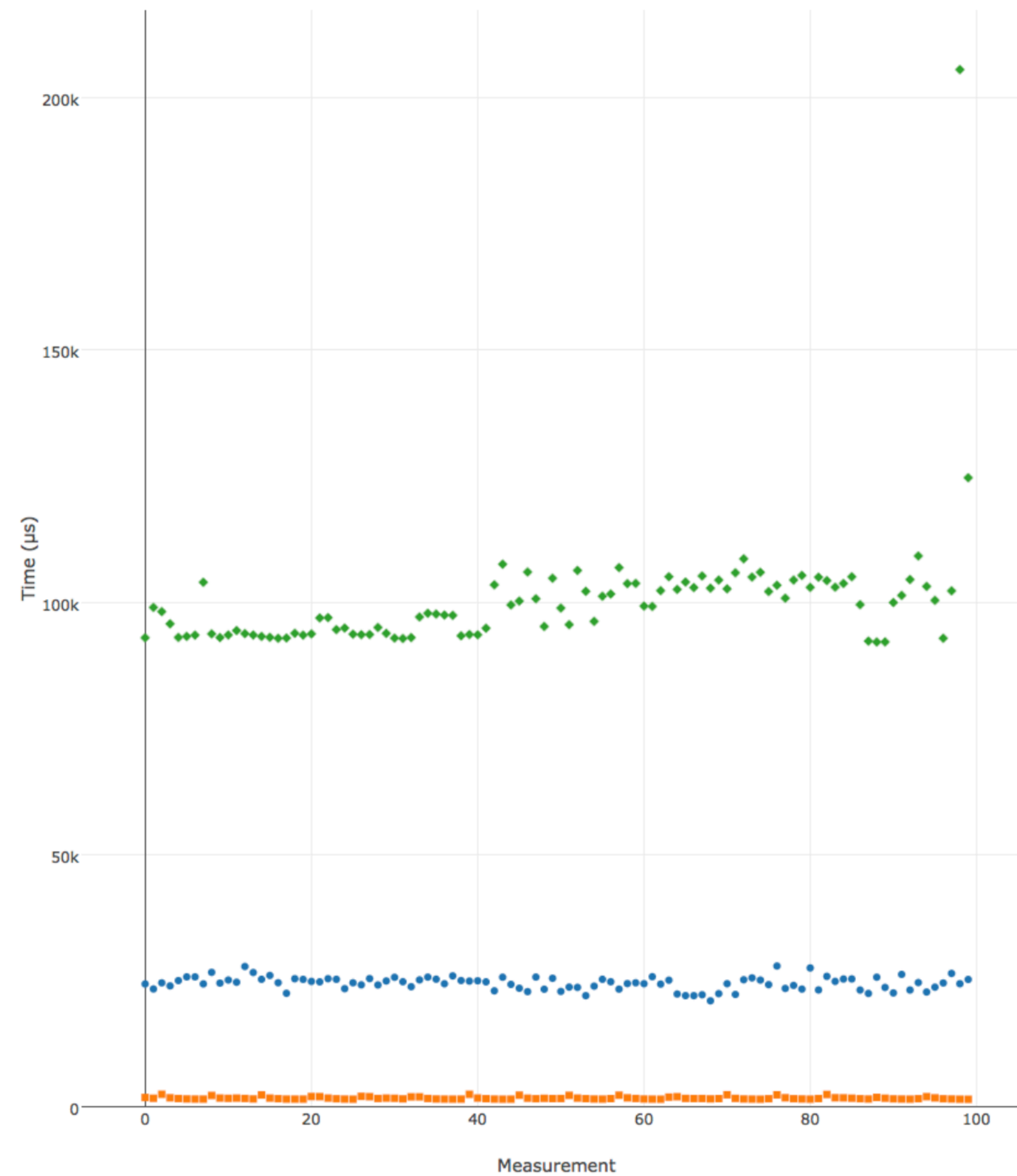
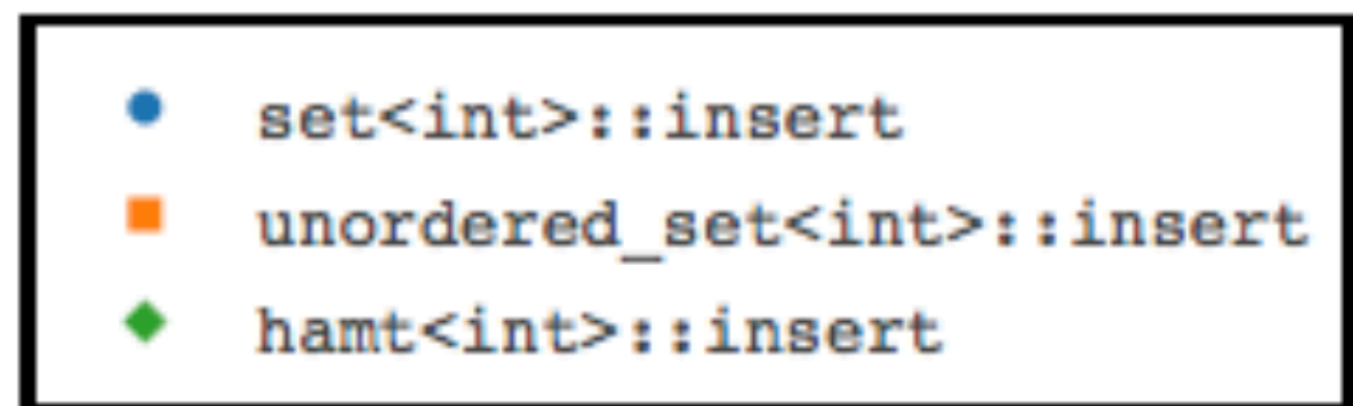
A C++ micro-benchmarking framework

Benchmarking Data Taken From A Talk By
Phil Nash,
Developer Advocate



BenchMarking Of Search Tree

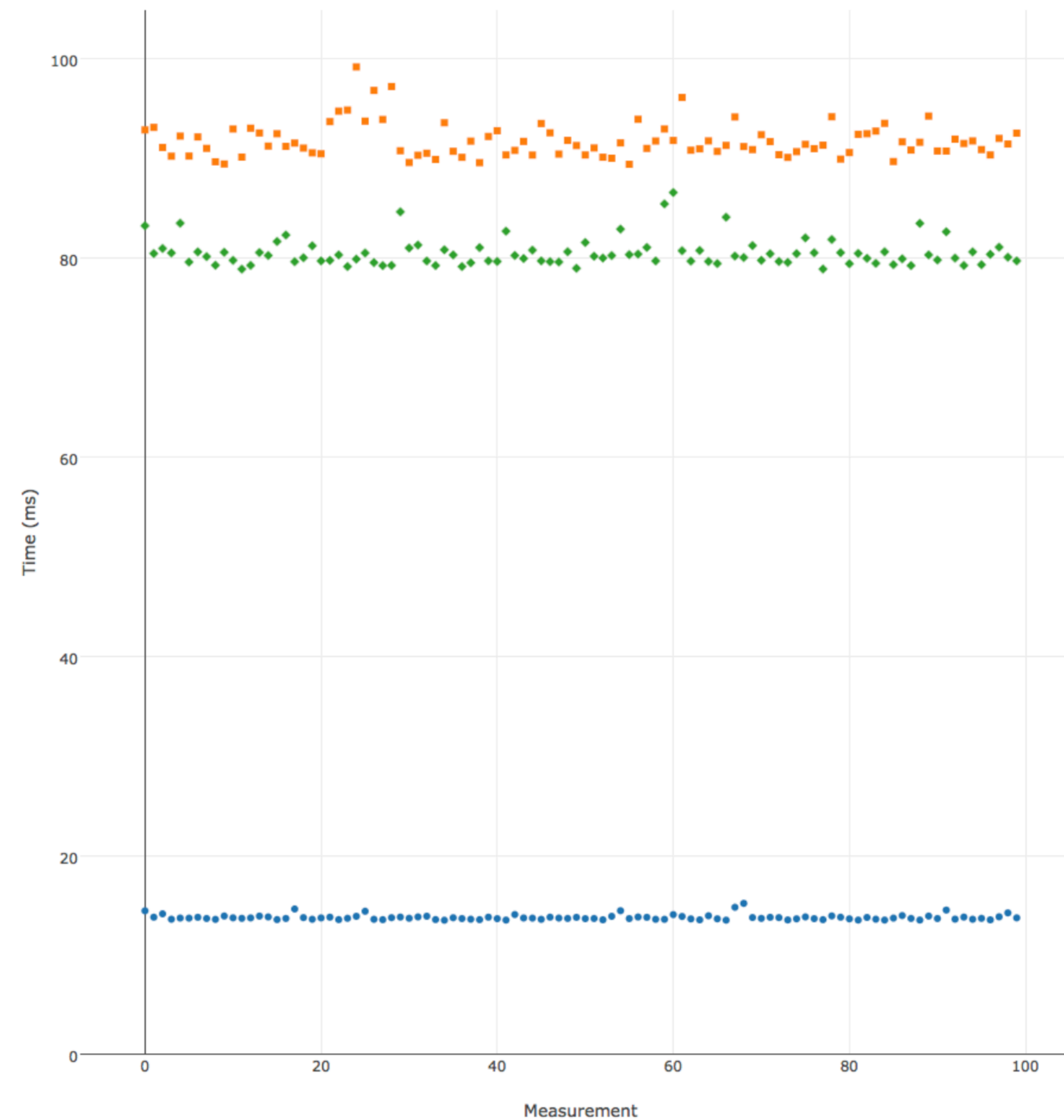
insert
100k ints



BenchMarking Of Search Tree

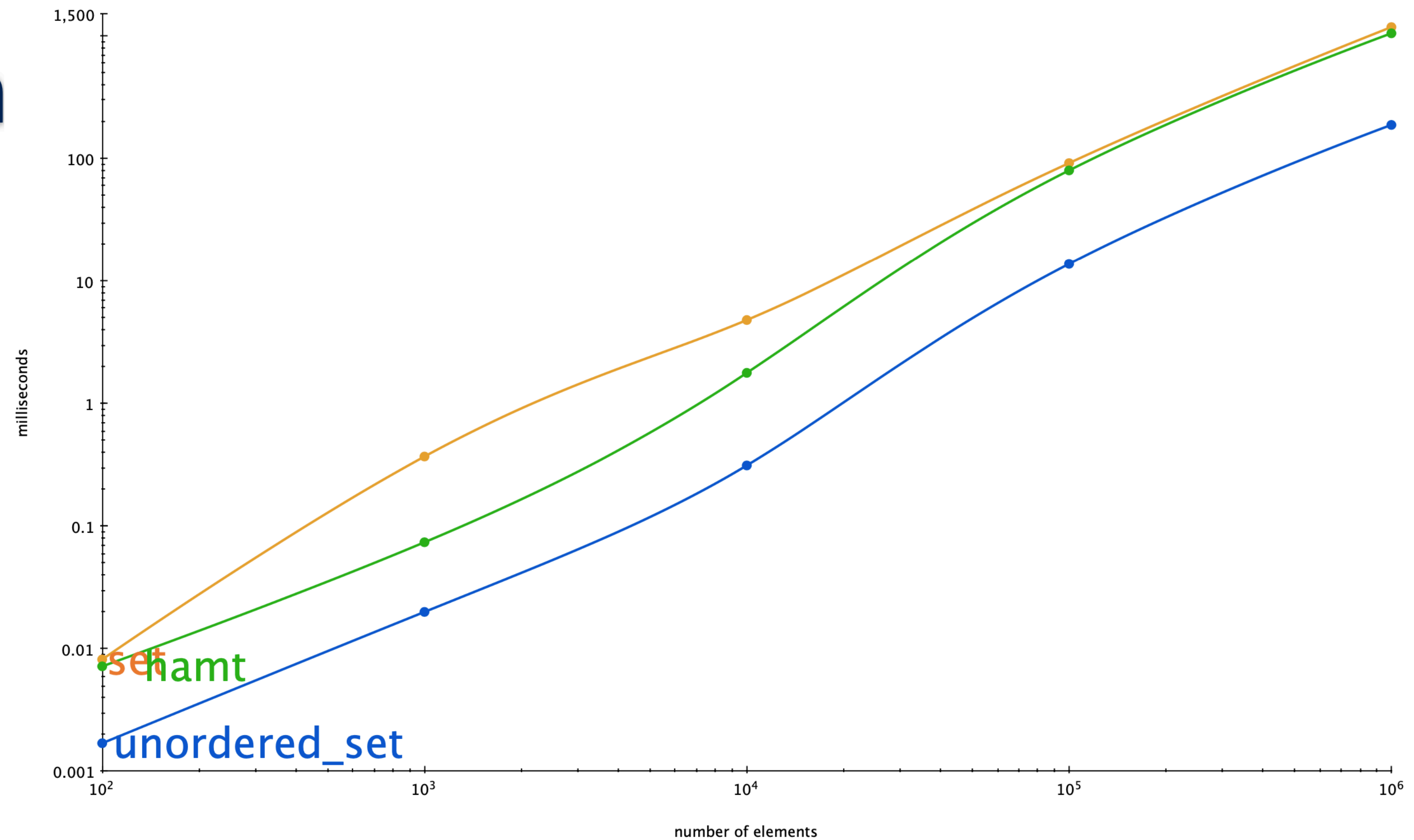
find
100k ints

● `unordered_set<int>::find`
■ `set<int>::find`
◆ `hamt<int>::find`



BenchMarking Of Search Tree

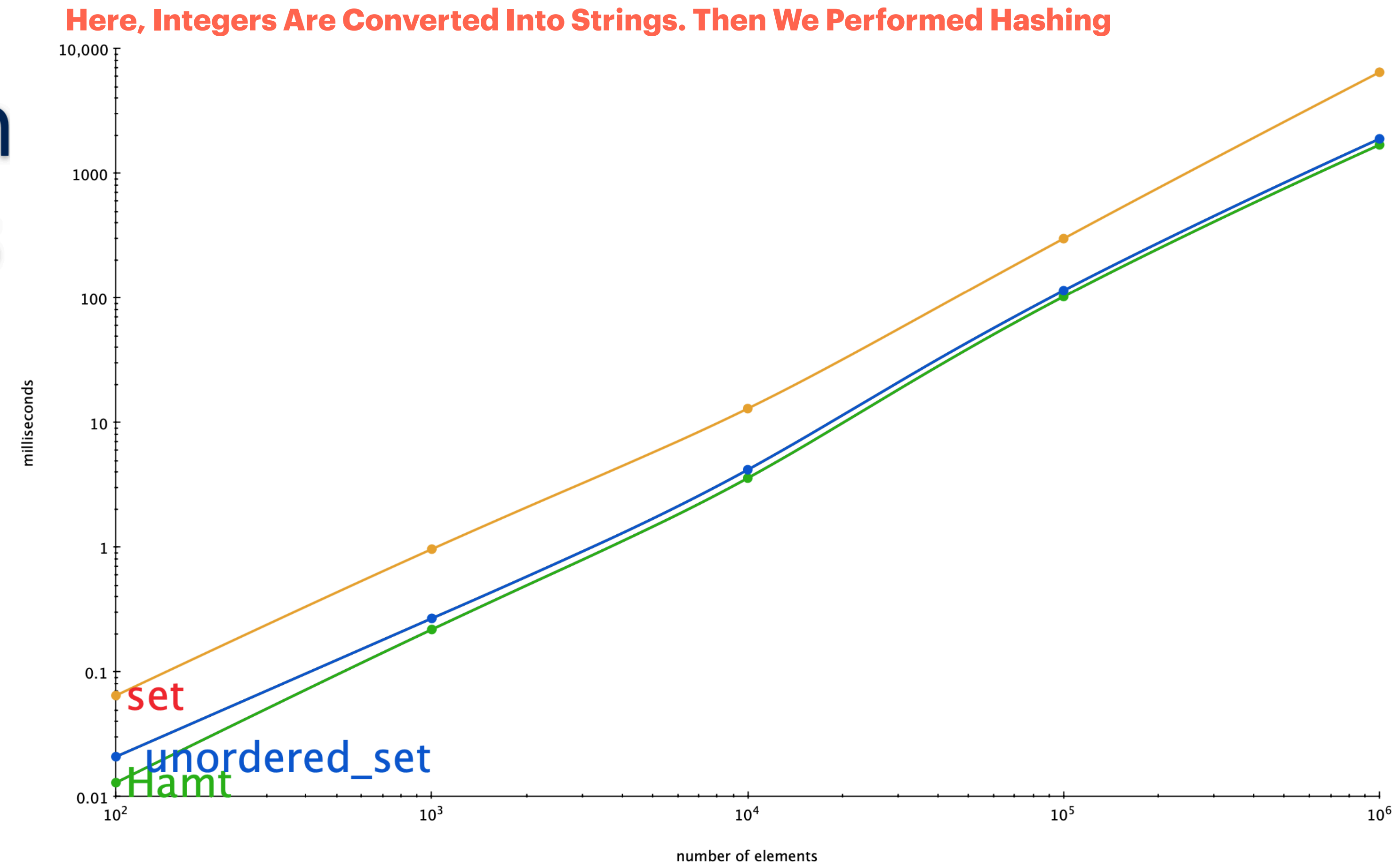
find
100-1m
ints



OmniGraphSketcher

BenchMarking Of Search Tree

find
100-1m
strings

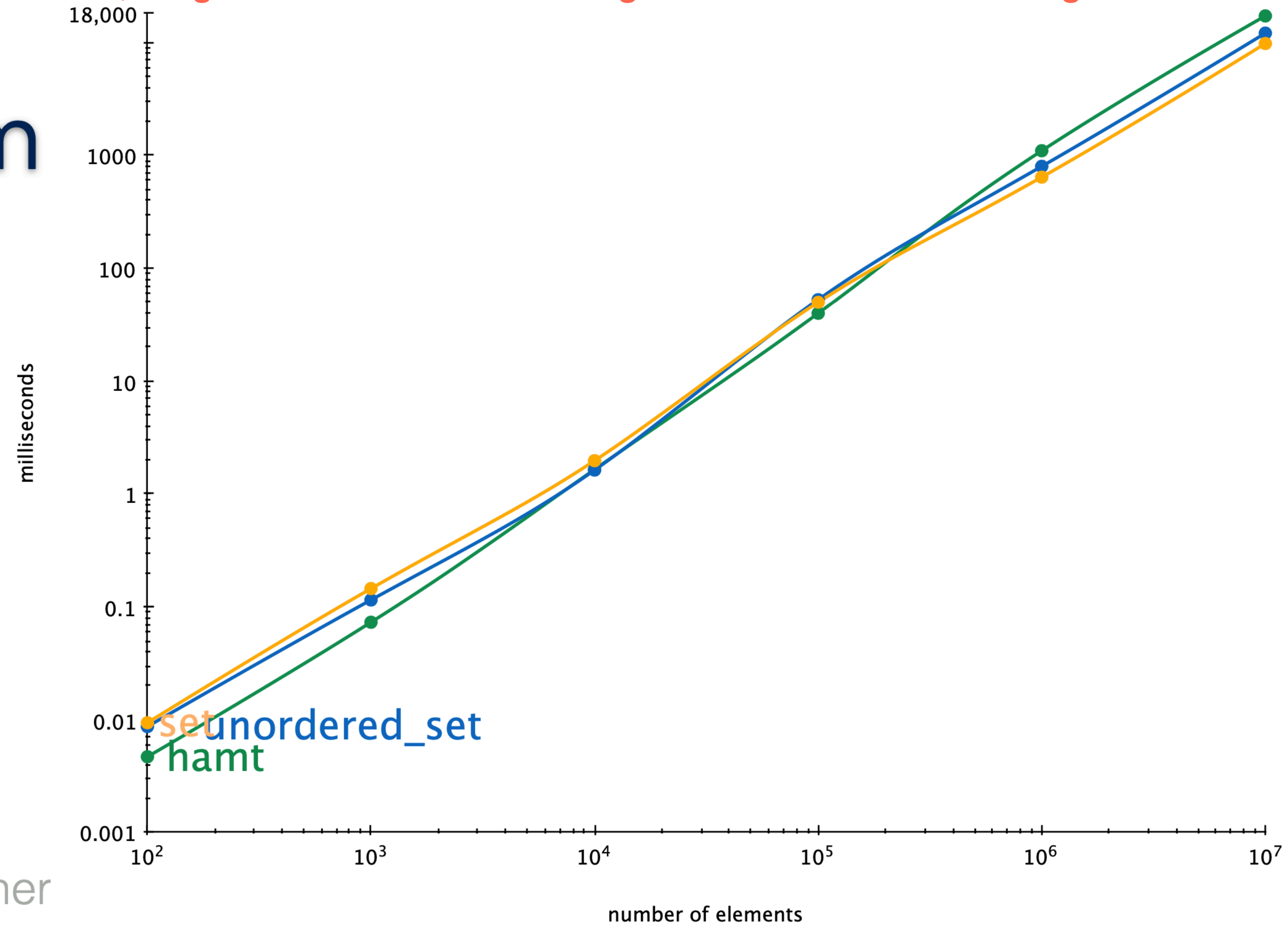


OmniGraphSketcher

BenchMarking Of Search Tree

find
100-10m
hashes

Here, Integers Are Converted Into Strings. Then We Performed Hashing



OmniGraphSketcher