

Partial Persistent Data-structure

Pointer Machine Model

PROJECT MEMBERS: ANANNYO DEY, SOUMYAJIT RUDRA SARMA, DEBASMIT ROY, KANKO GHOSH AND KUSHAL DAS

Ephemeral Linked List

Operations:

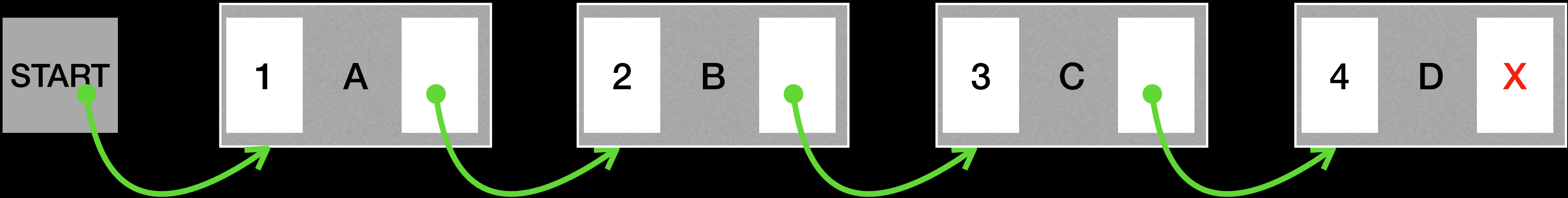
- `start = init()` : To initiate linked list and “start” pointer holds the starting position
- `add(x, y)` : Add new node x after y
- `remove(x)` : Remove node x
- `iterate_over_LL()` : Iterate over the whole linked list
- `update(f_i,x,val)` : Update the i-th field in node x to new value ‘val’

Ephemeral Linkedlist

Node structure



LINKED LIST

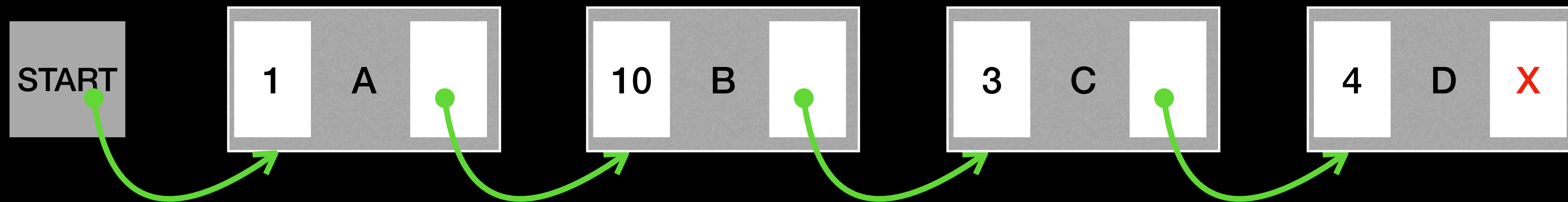


HERE IN-DEGREE OF A NODE IS ≤ 1 AND OUT-DEGREE OF A NODE IS ≤ 1

SOME OPERATIONS

update(f1,B,10)

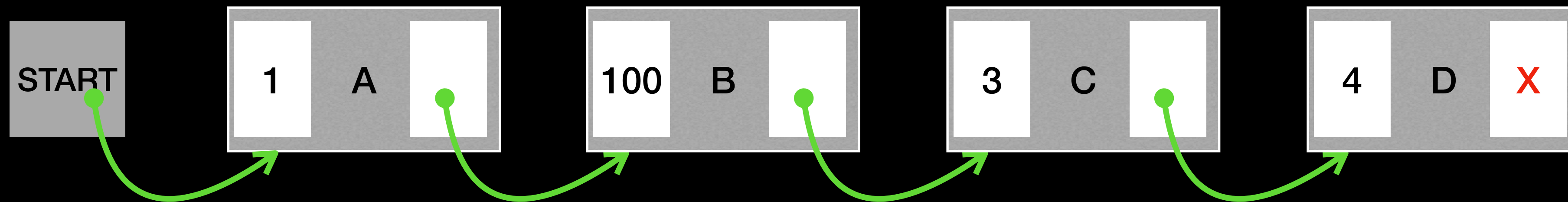
LINKED LIST



SOME OPERATIONS

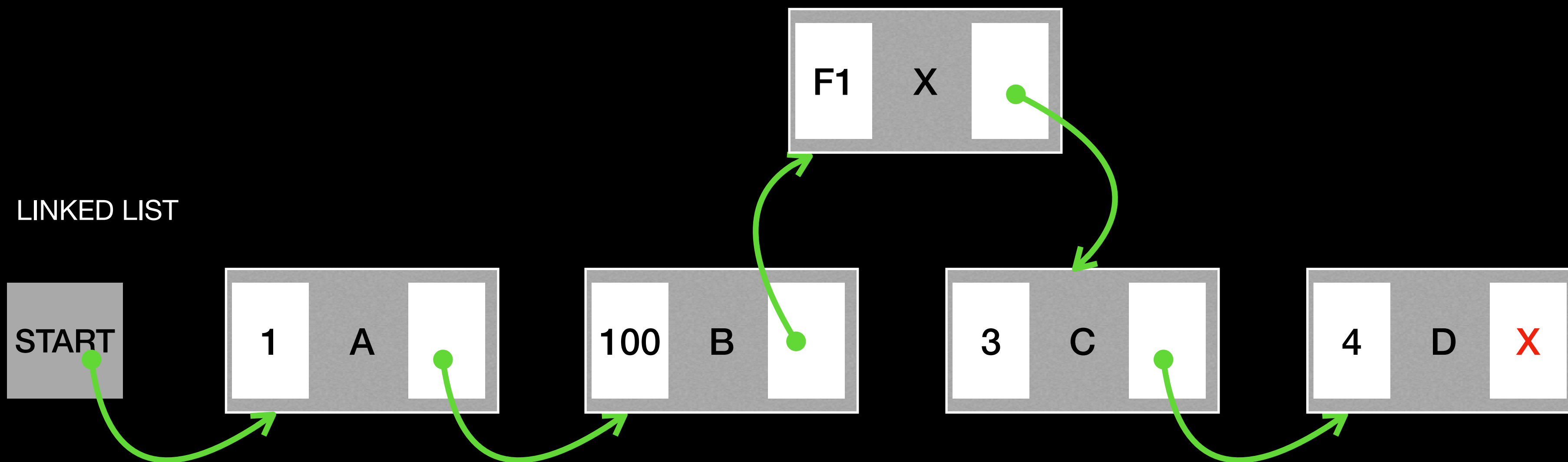
`update(f1,B,100)`

LINKED LIST



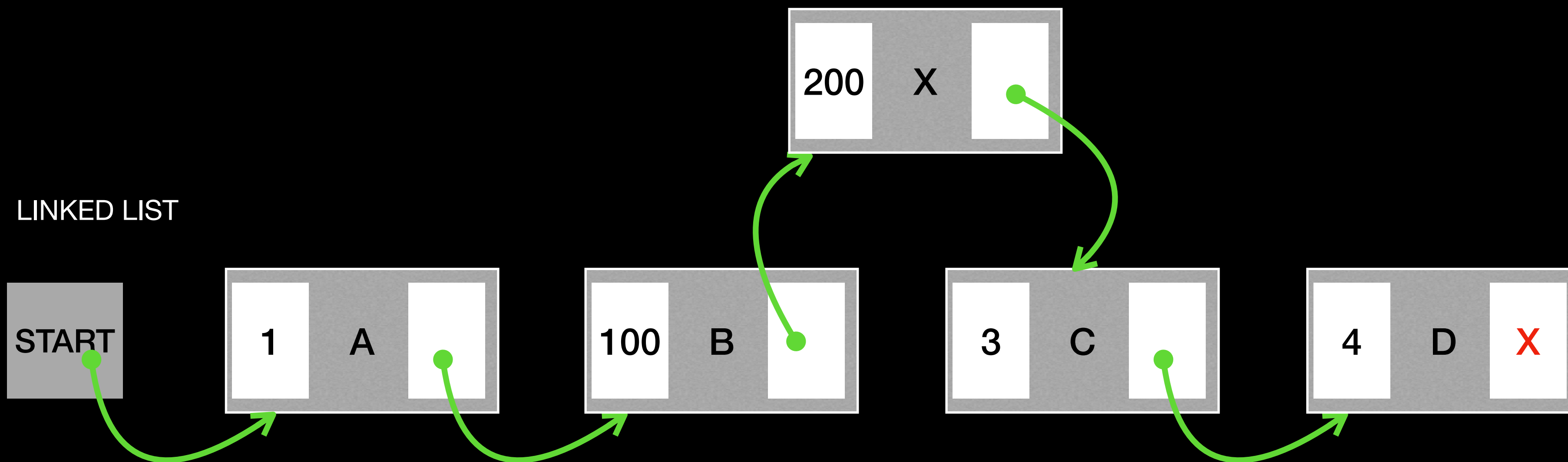
SOME OPERATIONS

add(X,B)



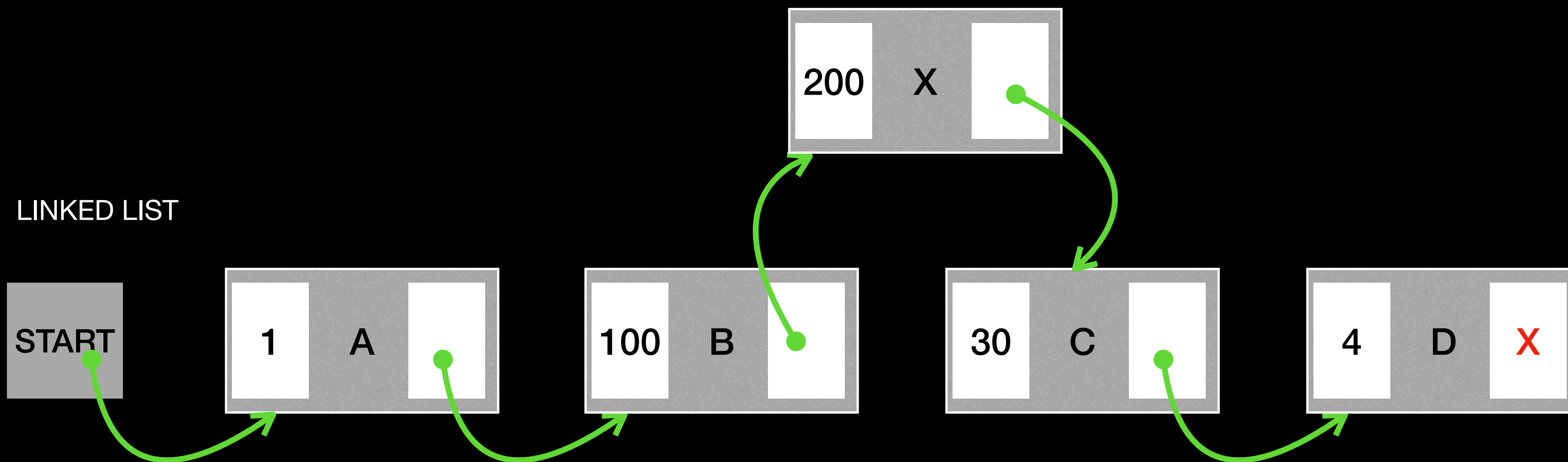
SOME OPERATIONS

update(f1,X,200)



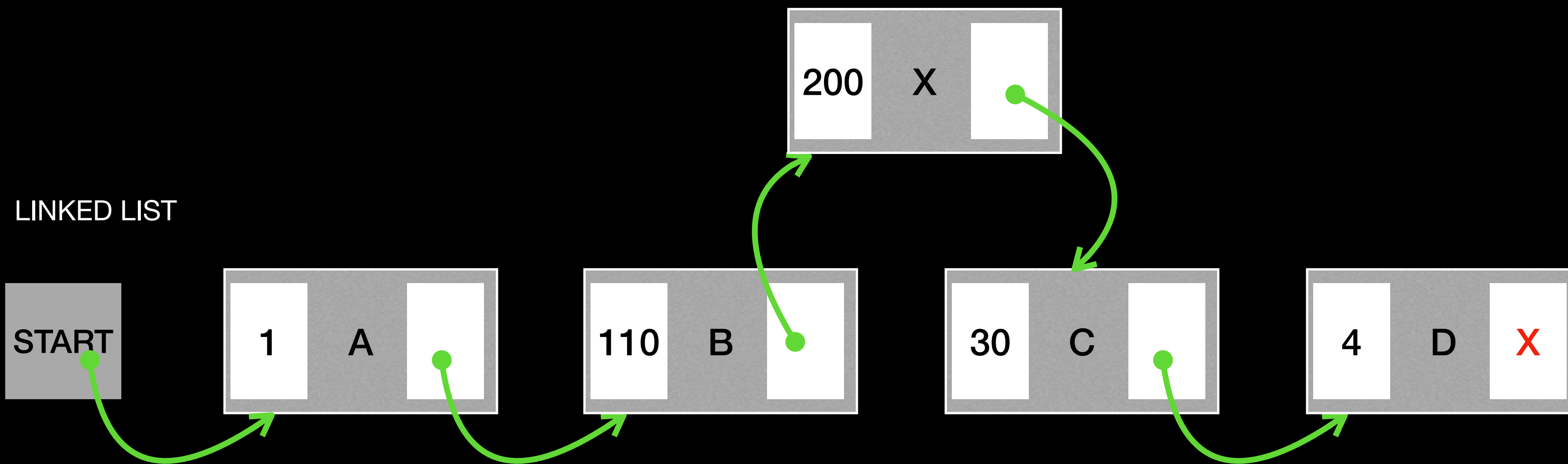
SOME OPERATIONS

update(f1,C,30)



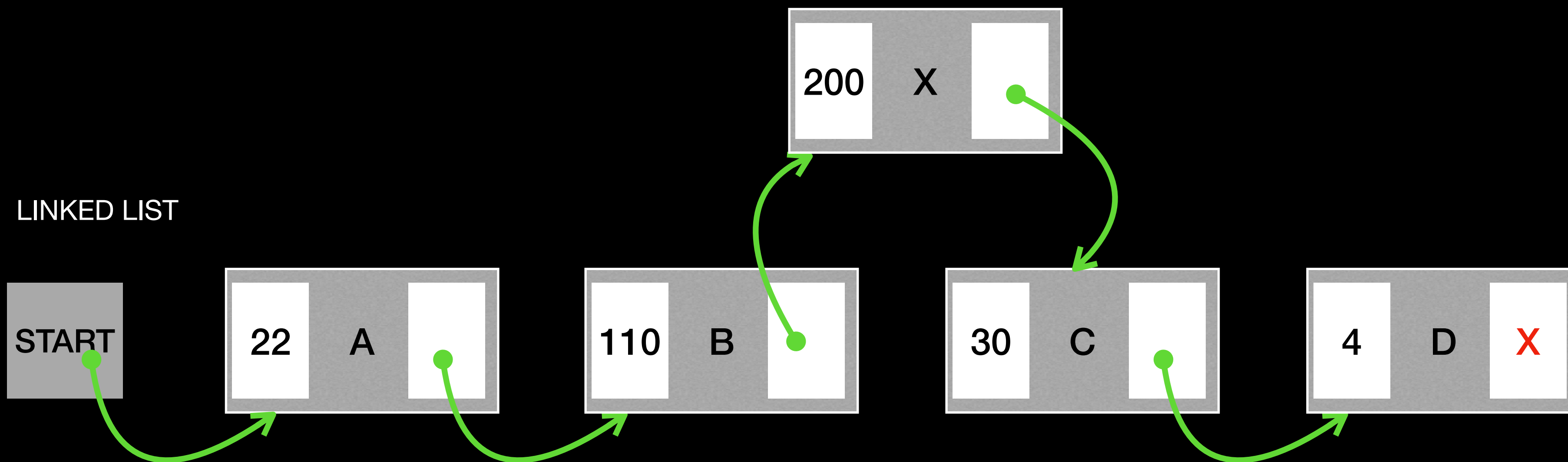
SOME OPERATIONS

update(f1,B,110)



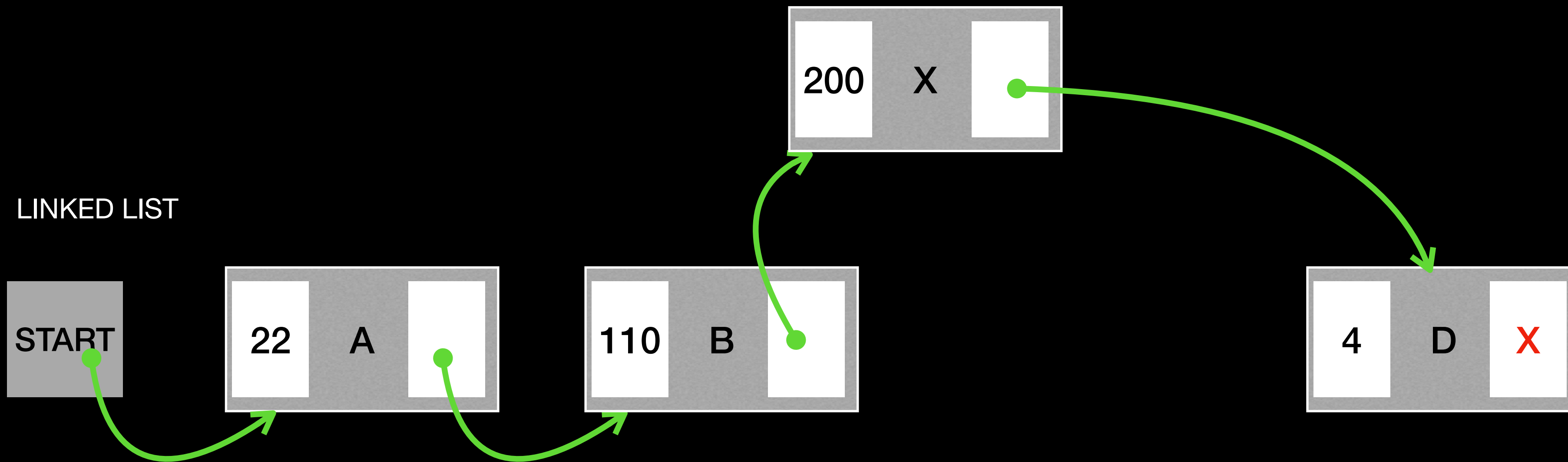
SOME OPERATIONS

update(f1,A,22)



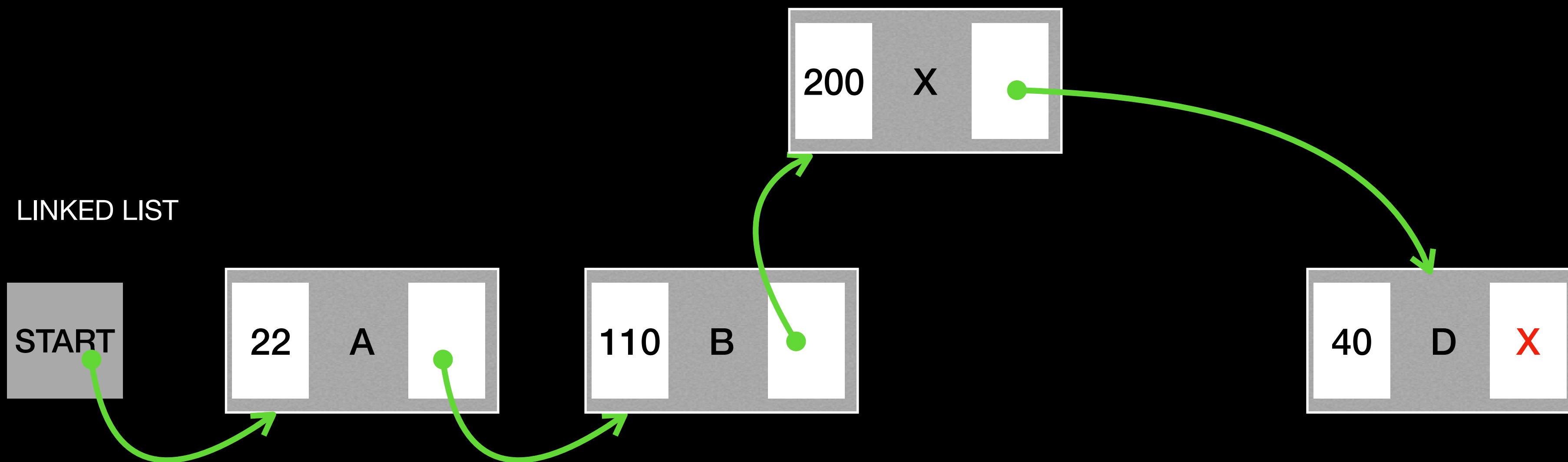
SOME OPERATIONS

remove(C)



SOME OPERATIONS

update(f1,D,40)



Partial Persistent Linked List

Operations:

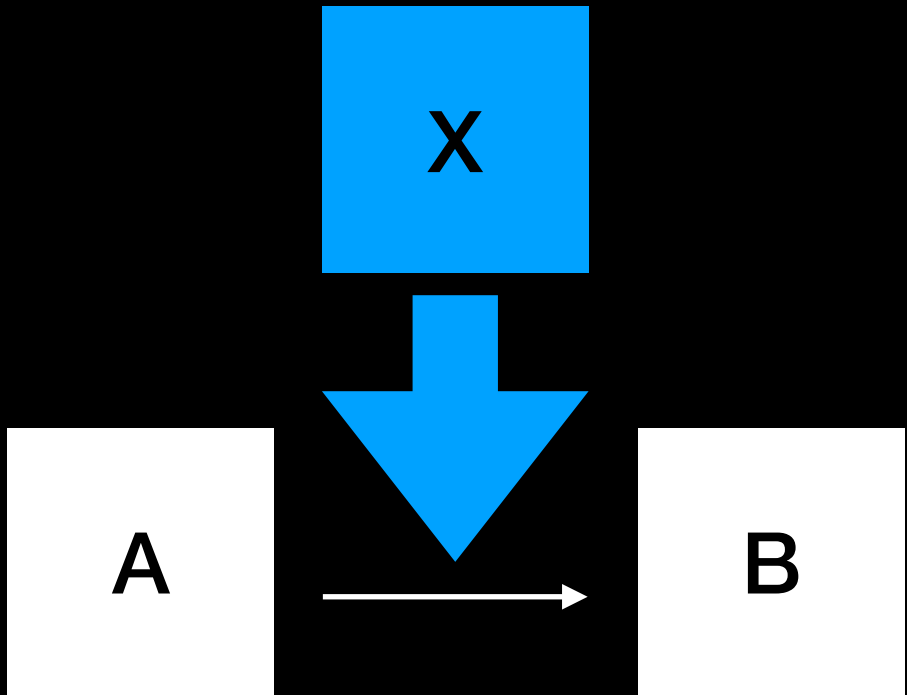
- `start = init()` : To initiate linked list and “start” pointer holds the starting position in `v0`
- `create_node(x, a)`: Allocate a new node `x` with `f1 = a` and `f2 = NULL`, and set its default version to current version
- `add(x, y, a)` : Add new node `x` with value = `a` after `y` and update the version
- `remove(x)` : Remove node `x` and update the version
- `iterate_over_LL(v)` : Iterate over the whole linked list in version `v`
- `update(f_i, x, val)` : Update the `i`-th field in node `x` to new value ‘`val`’ and update the version

Interesting thing!

`add(x,y)` and `remove(x)` are not Elementary operations

`add(X,C,123)` consists of

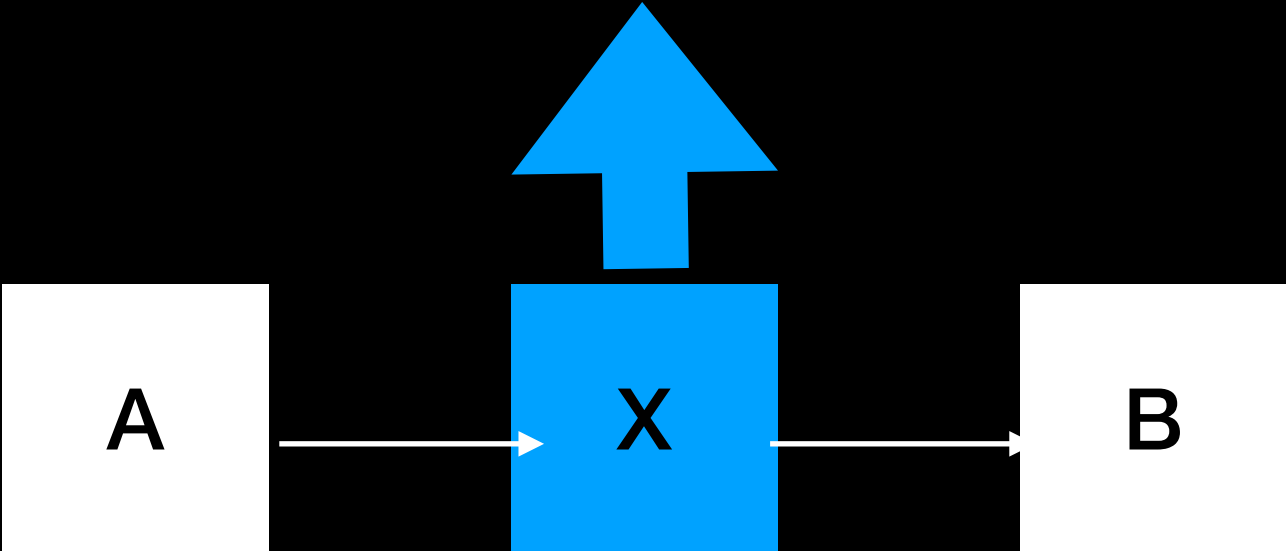
Create node X with value 123
Modify f2 of A to X : <code>update(f2, A, X)</code>
Modify f2 of X to C : <code>update(f2, X, C)</code>
Set BP of X to A: <code>X.bp = A</code>
Set BP of B to X: <code>B.bp = X</code>
Set f1 of X(optional)



`remove(x)` consists of

Modify F2 of Parent C (i.e., X) -> F2 of C (successor of C)

<code>update(f2, A, B)</code>
If all shared reference of X is removed Then free up the memory associated with X



Elementary Operations:

- `start = init()`

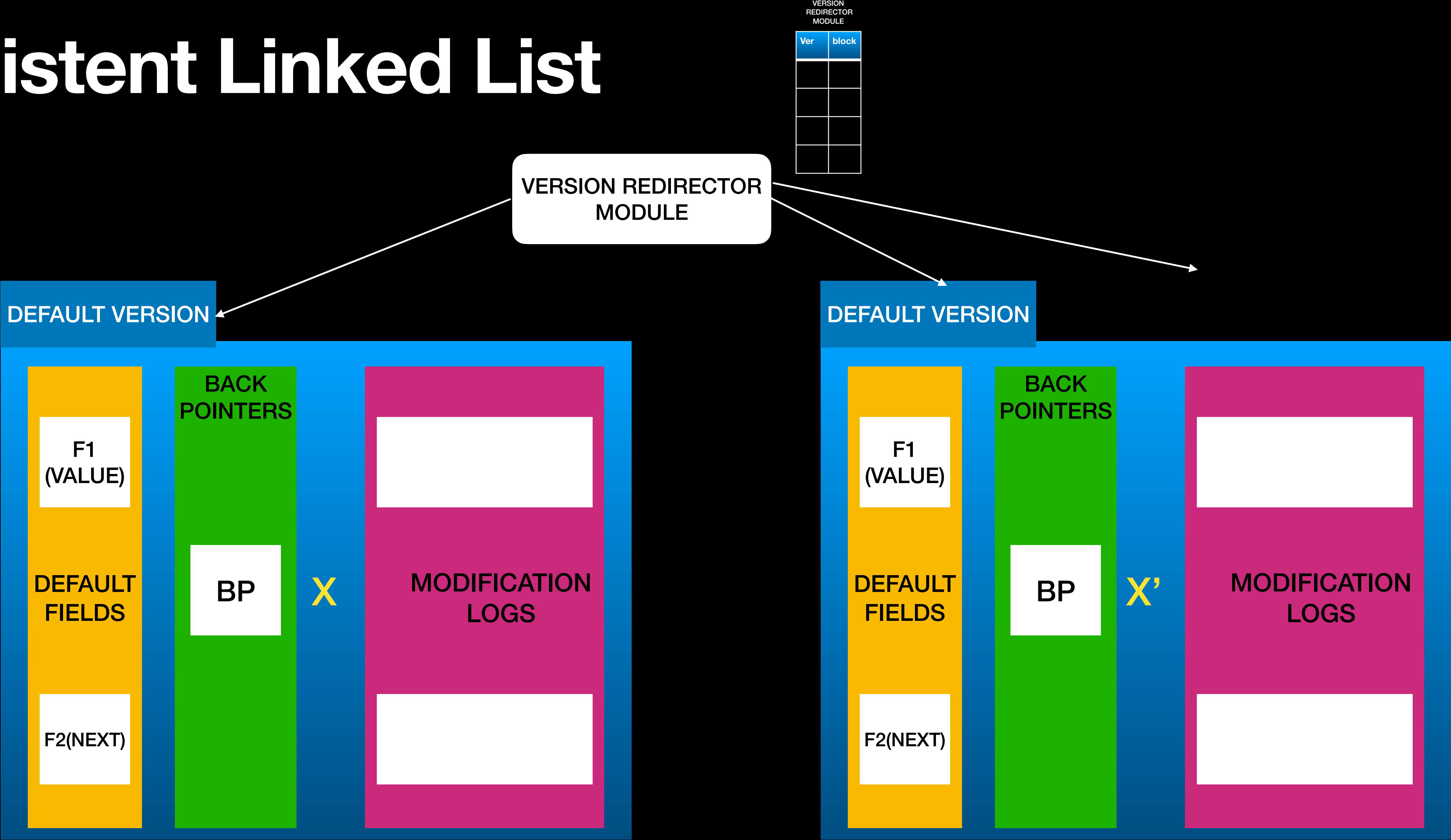
- `create_node(x, a)`

- `iterate_over_LL(v)`

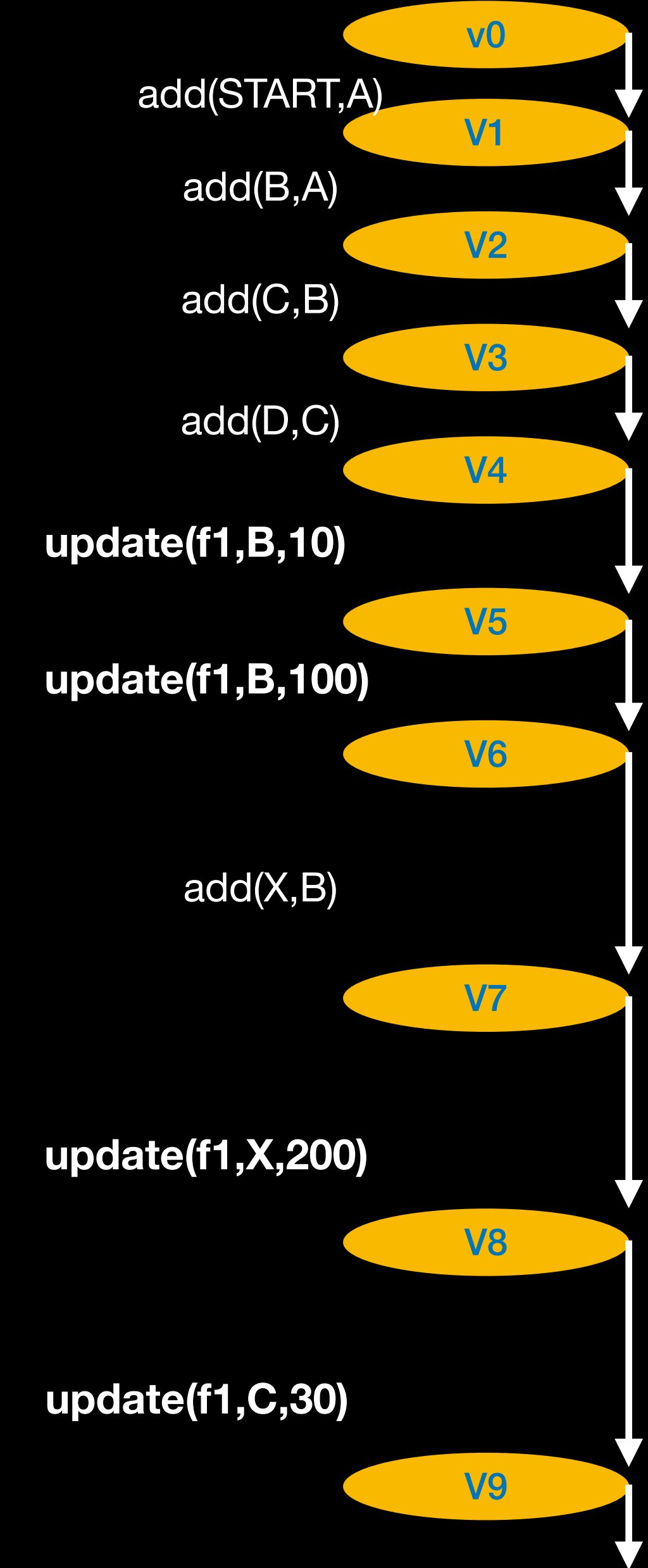
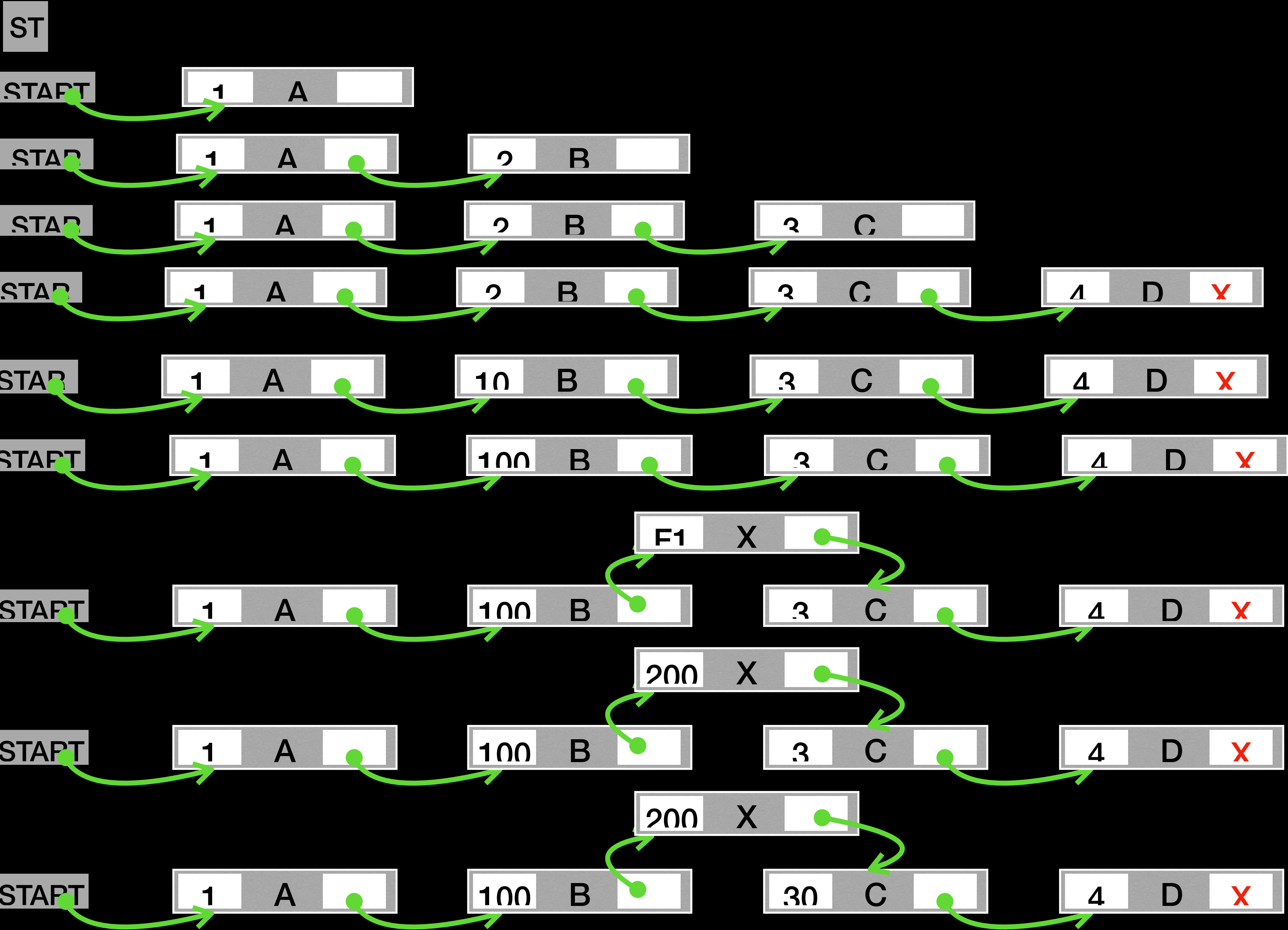
- `update(f_i,x,val)`

Persistent Linked List

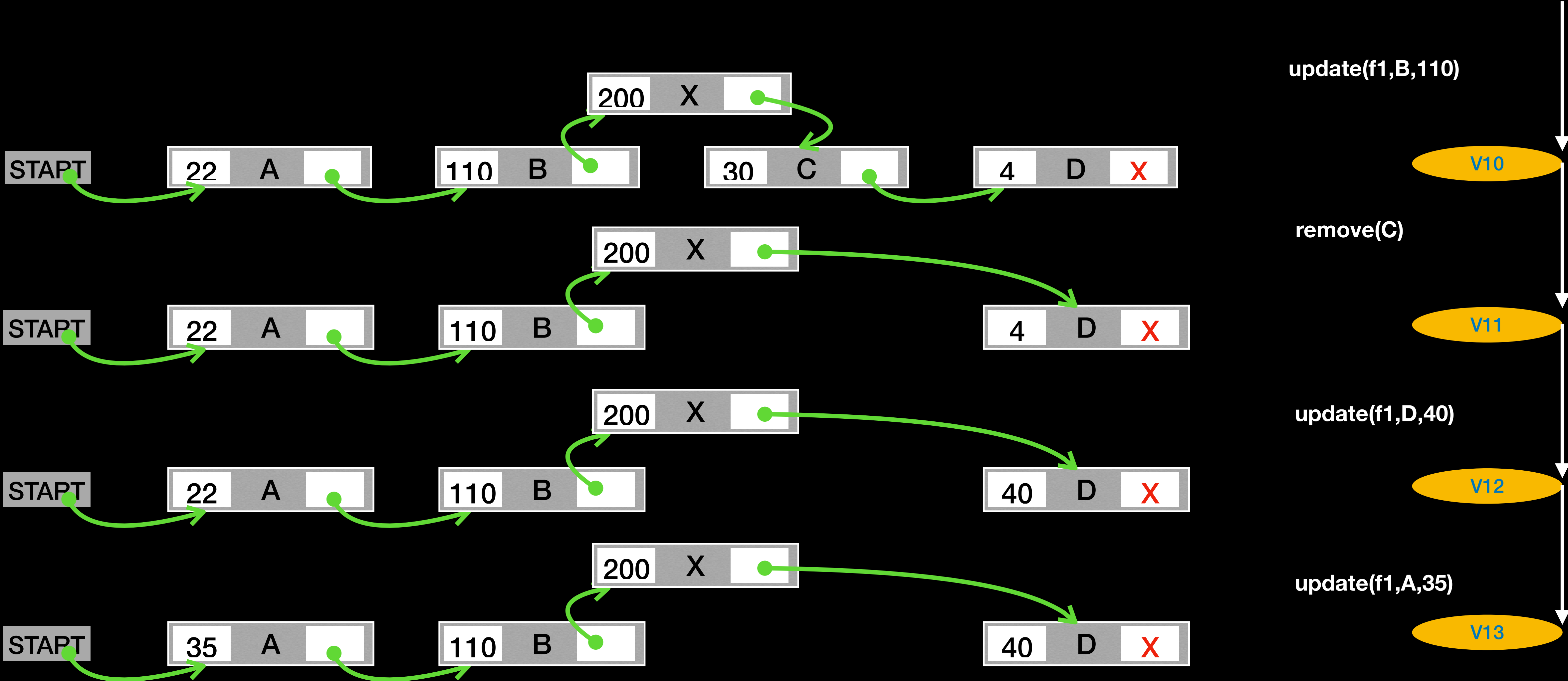
Node



Ephemeral List and Corresponding Versions



Ephemeral List and Corresponding Versions (contd.)



IMPLEMENTATION IN PERSISTENT POINTER MACHINE MODEL

START MODULE

v0

Current Version: v1

START MODULE

V0

add(A,START,1)

V1

1
DEFAULT
FIELDS

X

BACK
POINTER
S

X

A

MODIFICATION
LOGS

VERSION
REDIRECTOR
MODULE

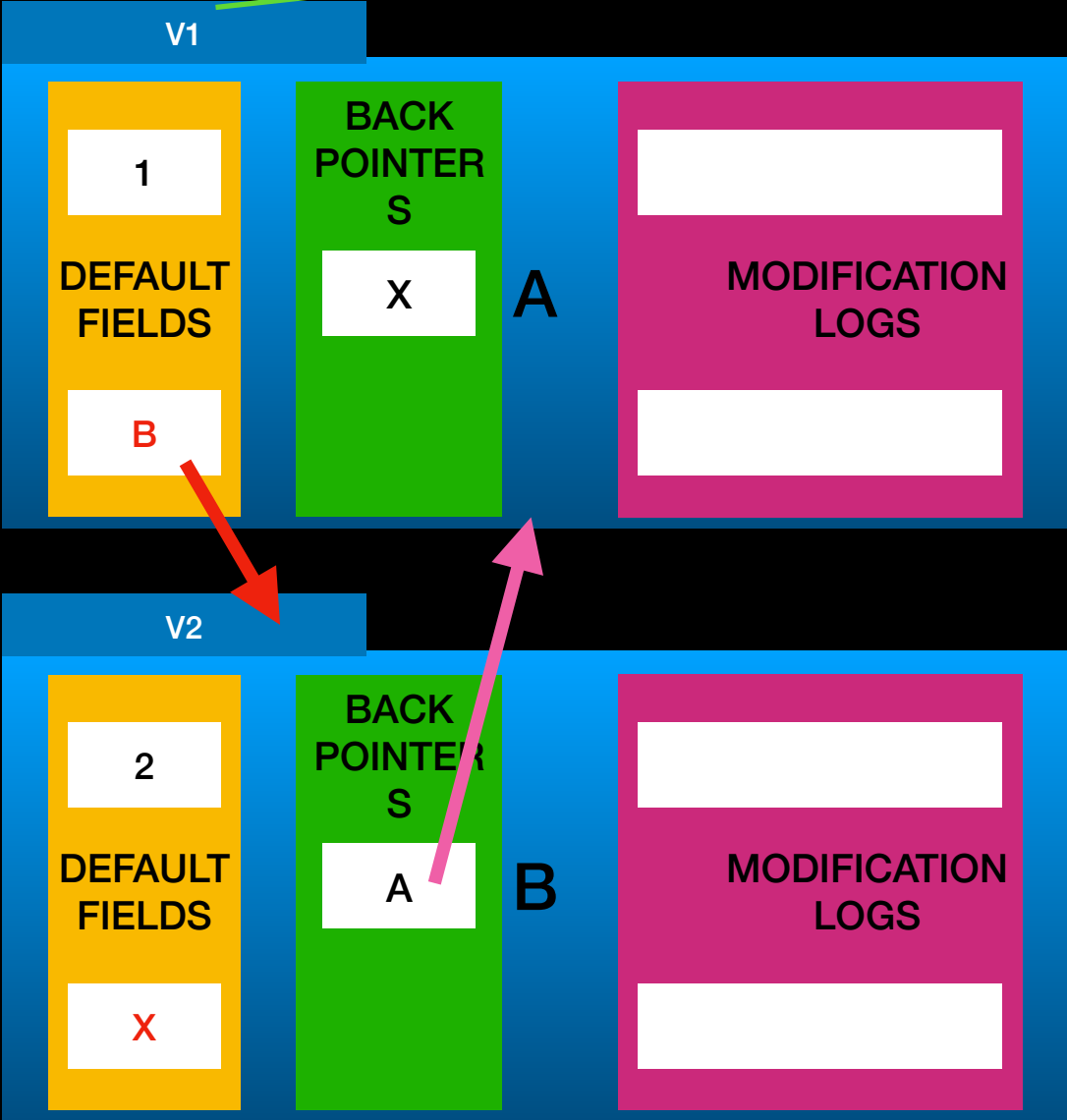
Ver	block
1	A

Current Version: v2

START MODULE

V0

add(B,A,2)



VERSION
REDIRECTOR
MODULE

Ver	block
1	A

VERSION
REDIRECTOR
MODULE

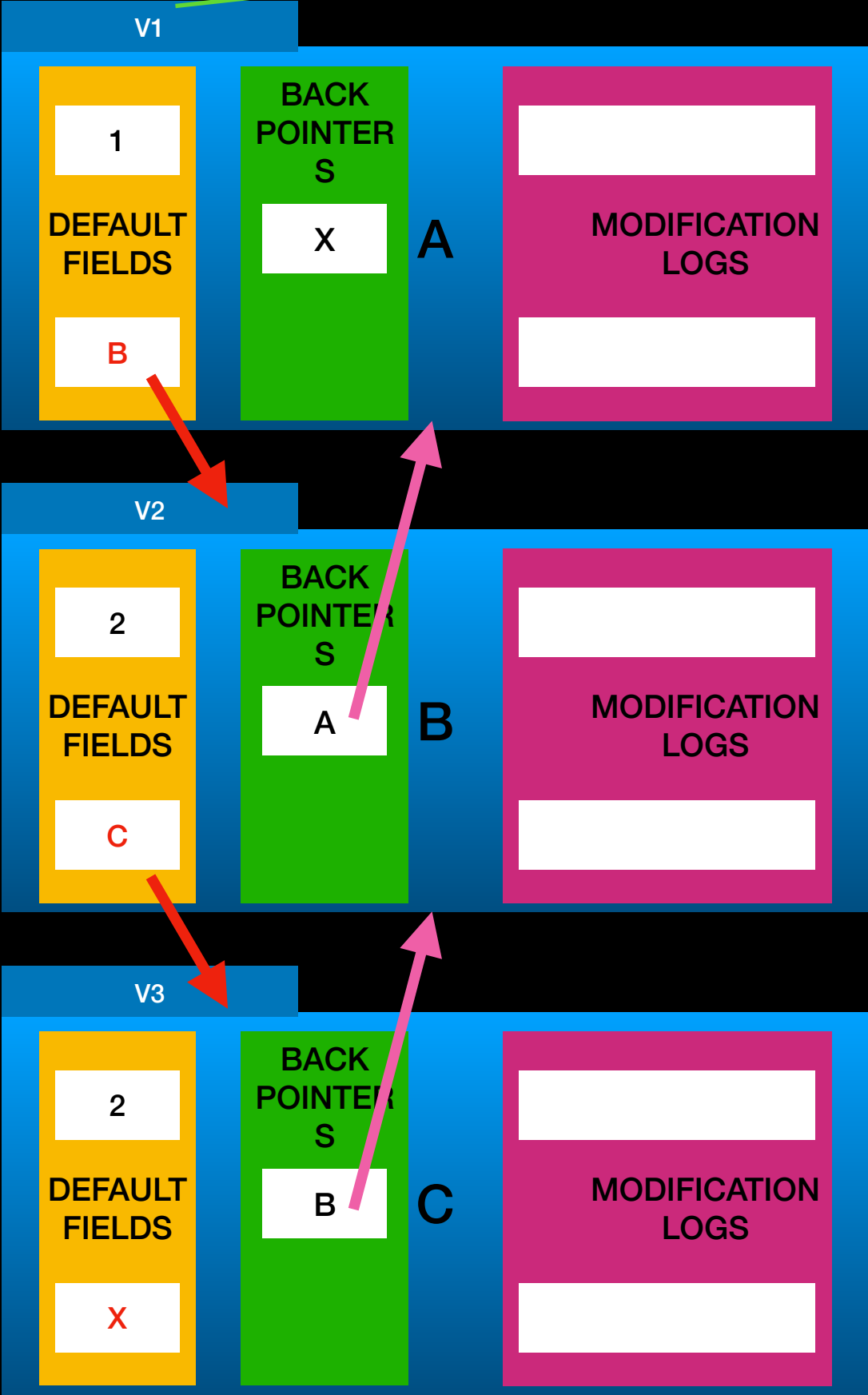
Ver	block
2	B

Current Version: v3

START MODULE

V0

add(C,B,3)



VERSION
REDIRECTOR
MODULE

Ver	block
1	A

VERSION
REDIRECTOR
MODULE

Ver	block
2	B

VERSION
REDIRECTOR
MODULE

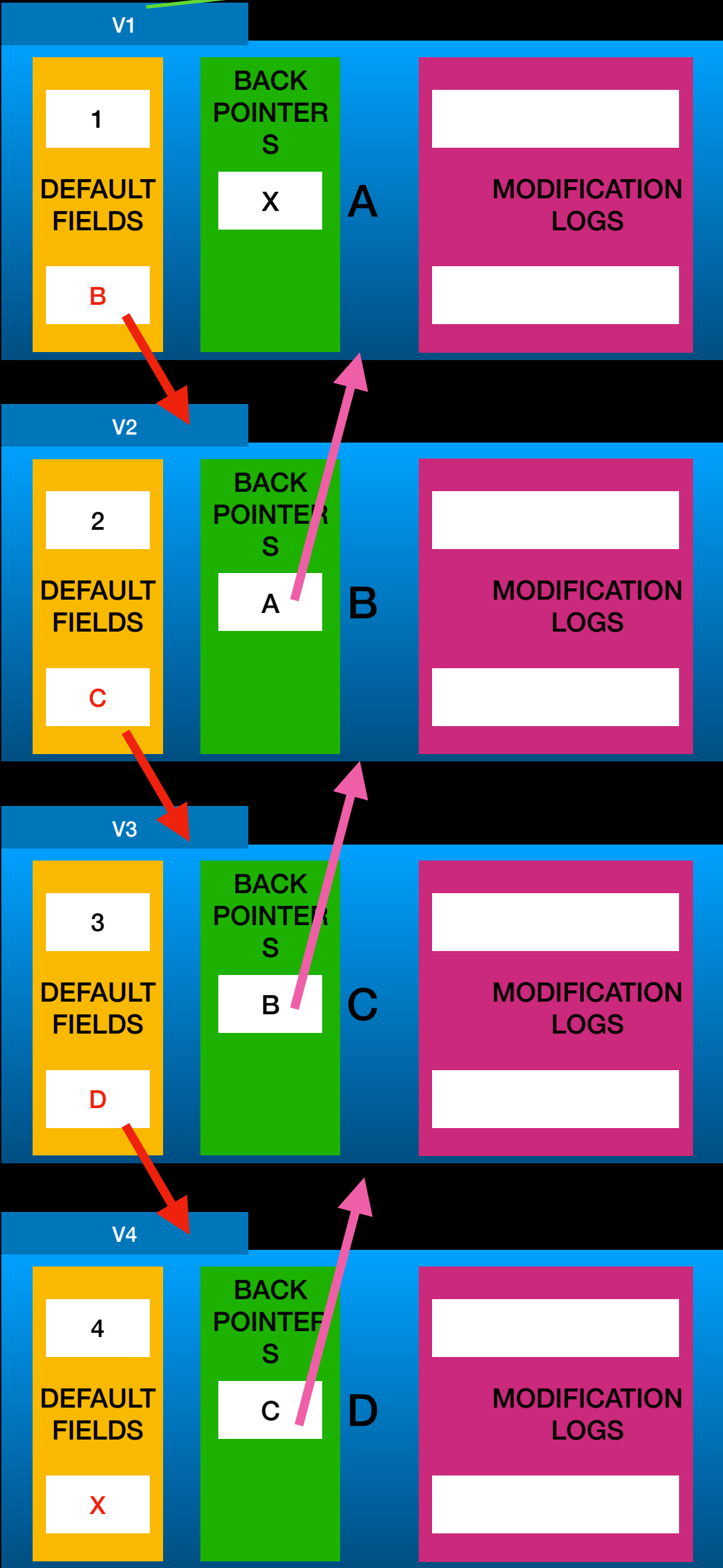
Ver	block
3	C

Current Version: v4

START MODULE

V0

add(D,C,4)



VERSION REDIRECTOR MODULE

Ver	block
1	A

VERSION REDIRECTOR MODULE

Ver	block
2	B

VERSION REDIRECTOR MODULE

Ver	block
3	C

VERSION REDIRECTOR MODULE

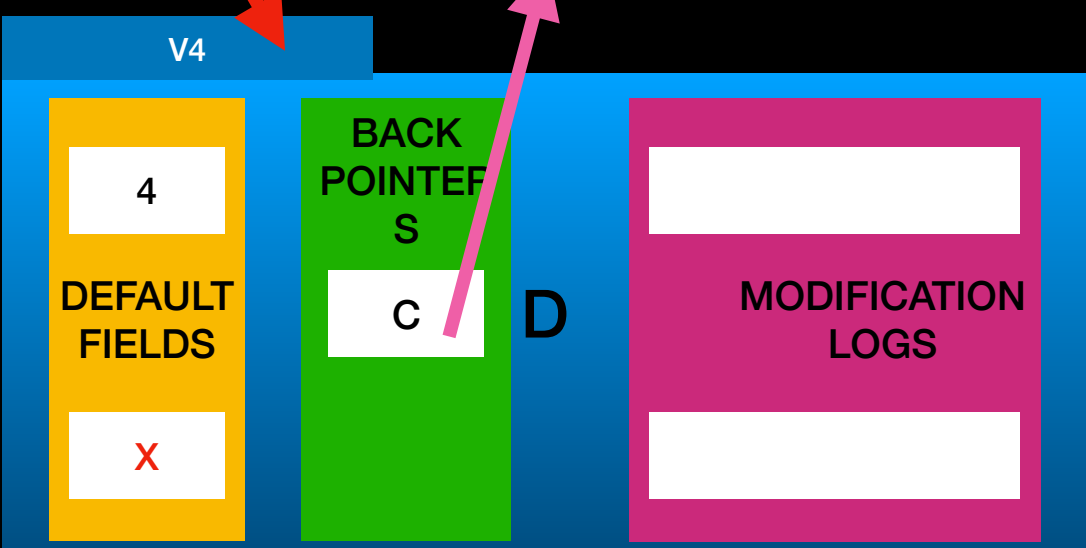
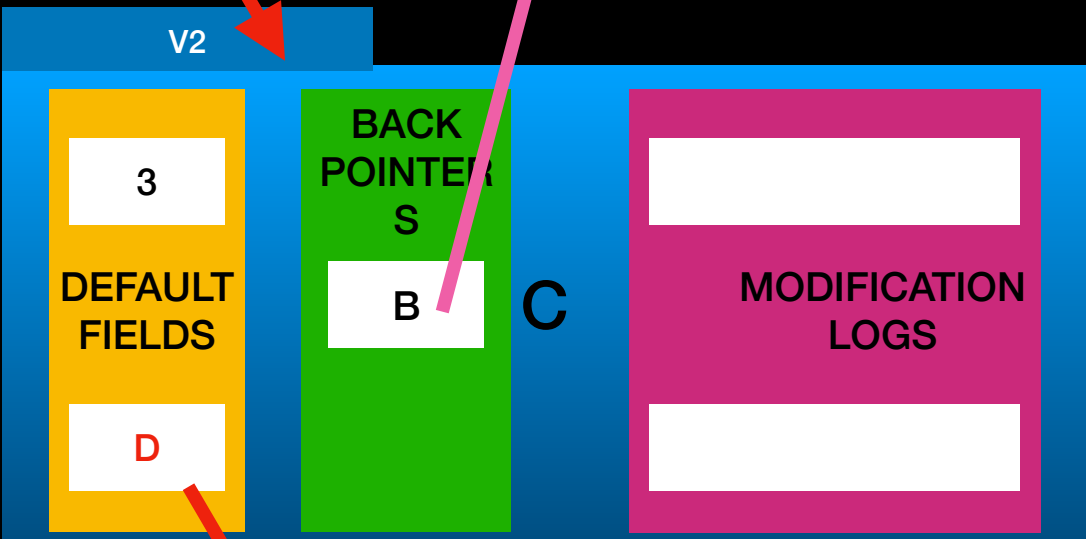
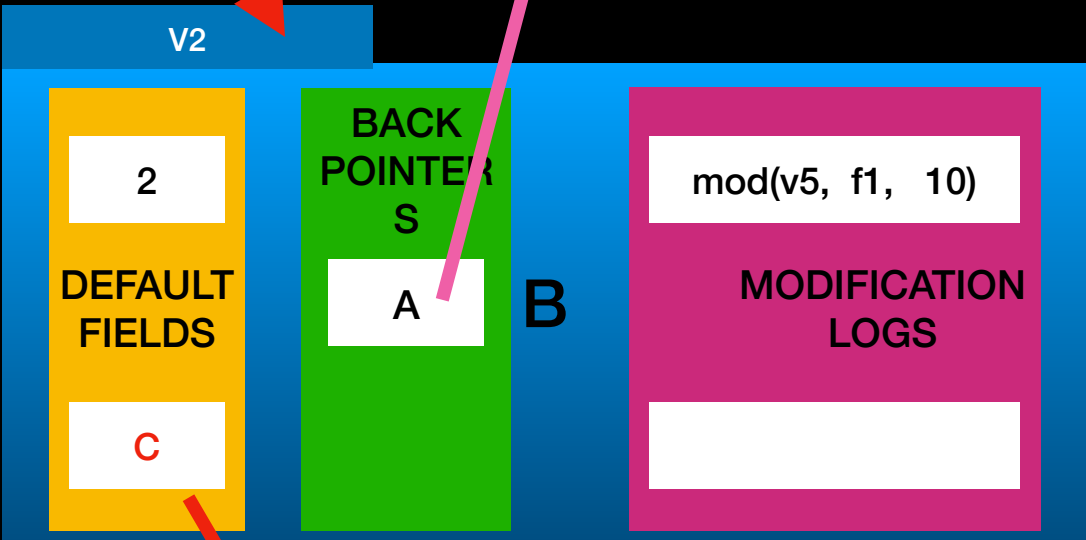
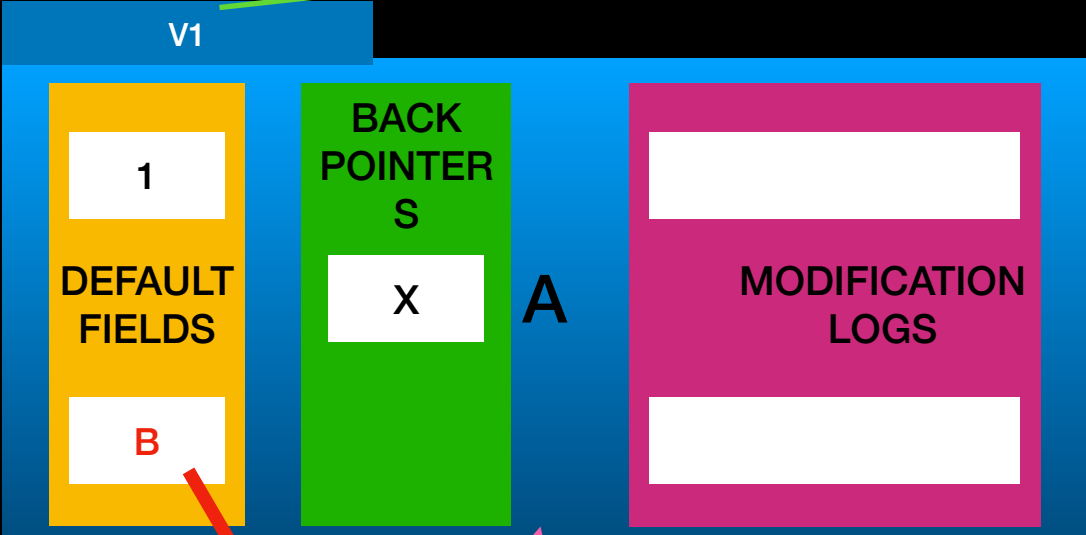
Ver	block

Current Version: v5

START MODULE

V0

update(f1,B,10)



VERSION
REDIRECTOR
MODULE

Ver	block
1	A

VERSION
REDIRECTOR
MODULE

Ver	block
2	B

VERSION
REDIRECTOR
MODULE

Ver	block
3	C

VERSION
REDIRECTOR
MODULE

Ver	block
4	D

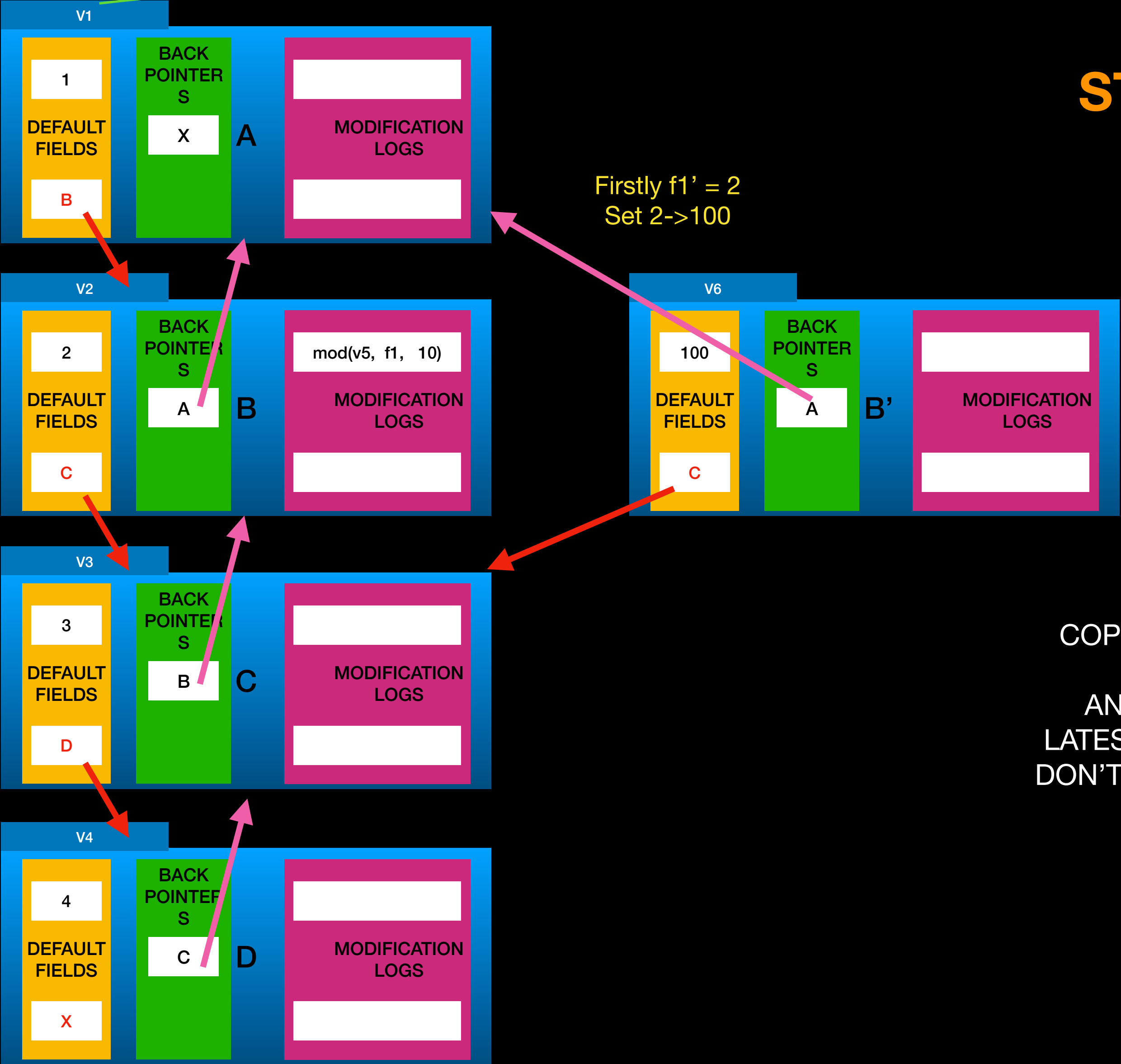
START MODULE

V0

STEP 1

Firstly f1' = 2
Set 2->100

COPY THE WHOLE
NODE
AND KEEP THE
LATEST VALUES AND
DON'T COPY MODLOG



VERSION REDIRECTOR MODULE

Ver	block
1	A

VERSION REDIRECTOR MODULE

Ver	block
2	B
6	B'

VERSION REDIRECTOR MODULE

Ver	block
3	C

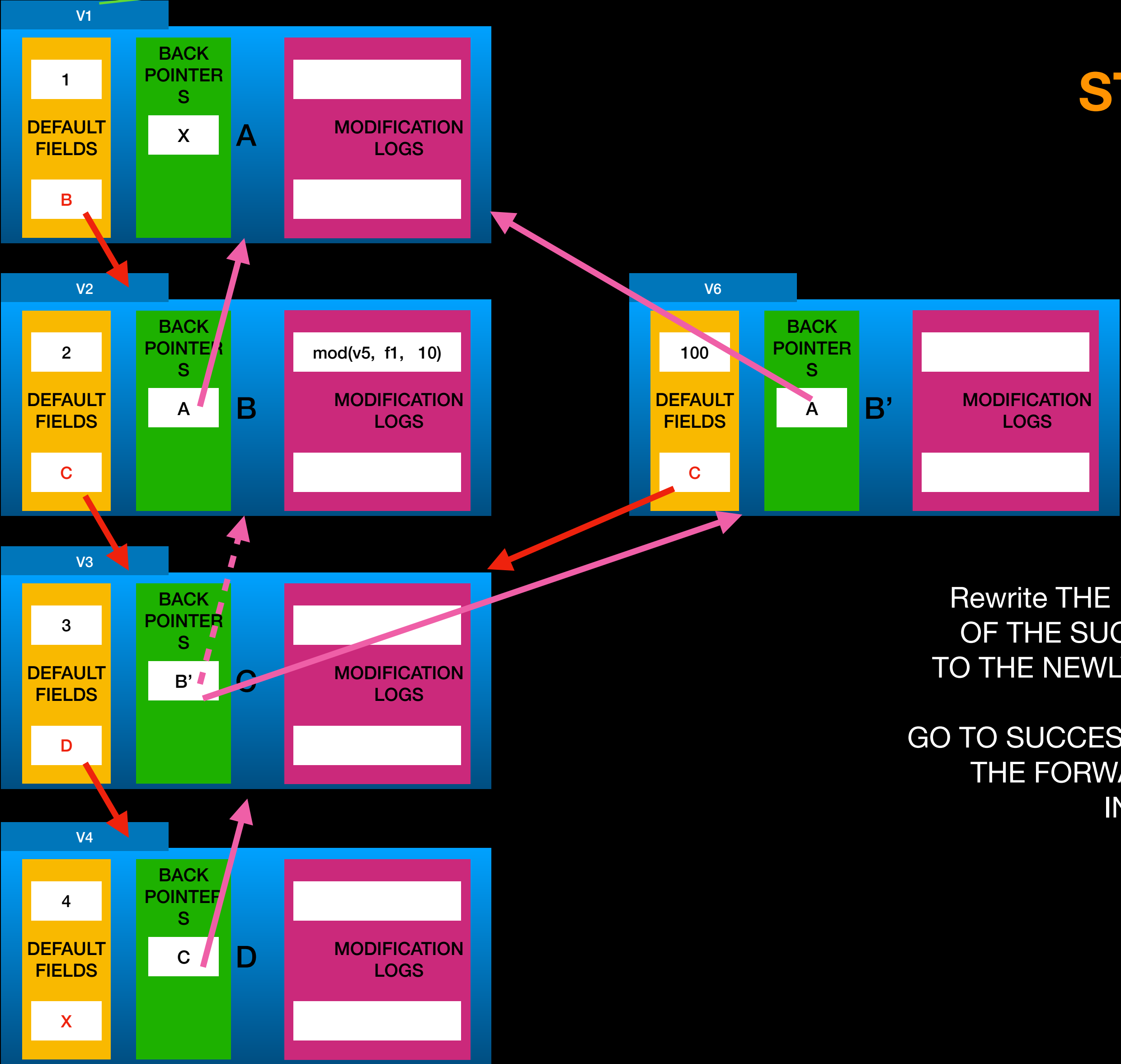
VERSION REDIRECTOR MODULE

Ver	block
4	D

START MODULE

V0

STEP 2



VERSION REDIRECTOR MODULE	
Ver	block
1	A

VERSION REDIRECTOR MODULE	
Ver	block
2	B
6	B'

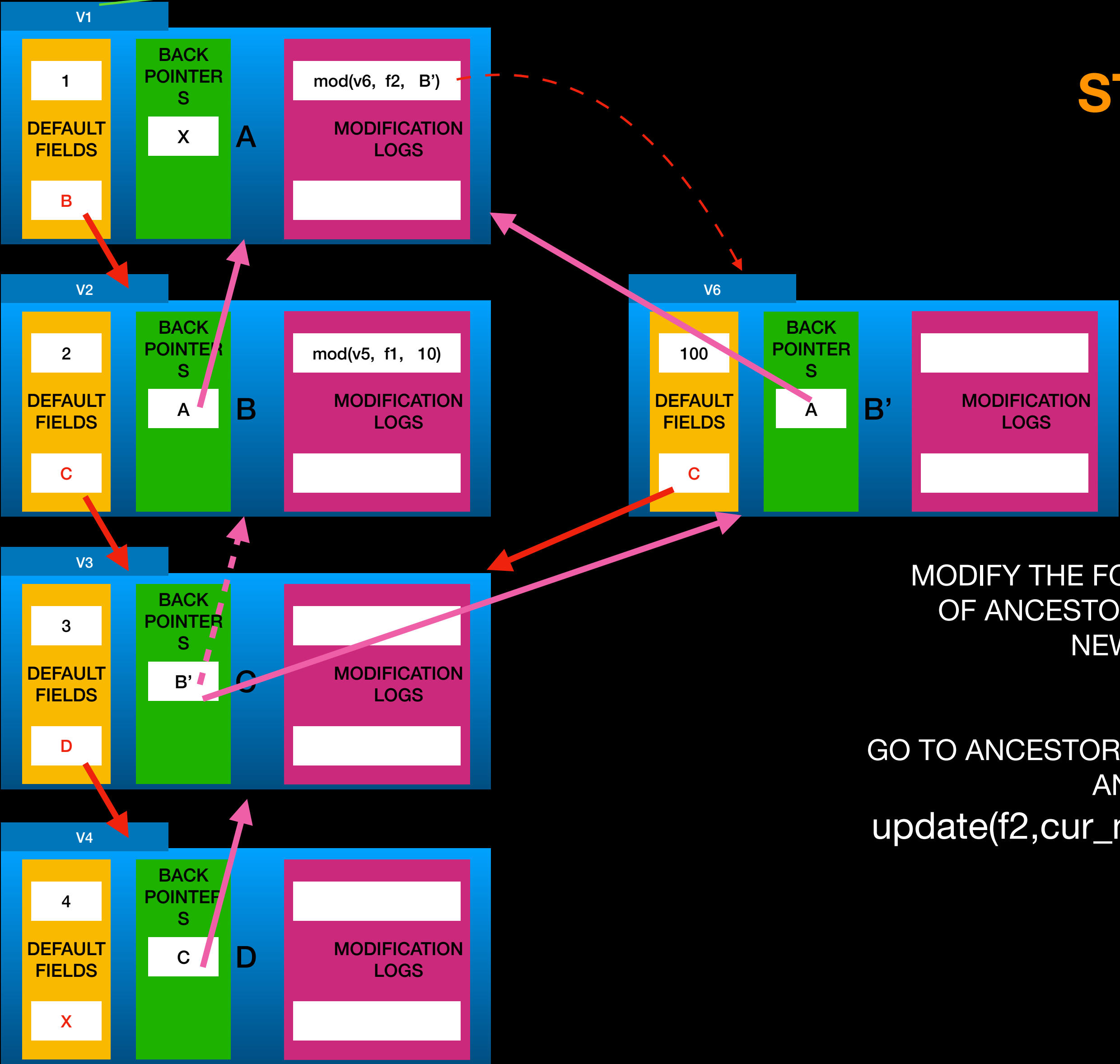
VERSION REDIRECTOR MODULE	
Ver	block
3	C

VERSION REDIRECTOR MODULE	
Ver	block
4	D

START MODULE

V0

STEP 3



MODIFY THE FORWARD POINTERS
OF ANCESTOR NODES TO THE
NEW NODE

GO TO ANCESTOR NODES RECURSIVELY
AND DO
update(f2,cur_node,NEW_NODE)

VERSION REDIRECTOR MODULE	
Ver	block
1	A

VERSION REDIRECTOR MODULE	
Ver	block
2	B
6	B'

VERSION REDIRECTOR MODULE	
Ver	block
3	C

VERSION REDIRECTOR MODULE	
Ver	block
4	D

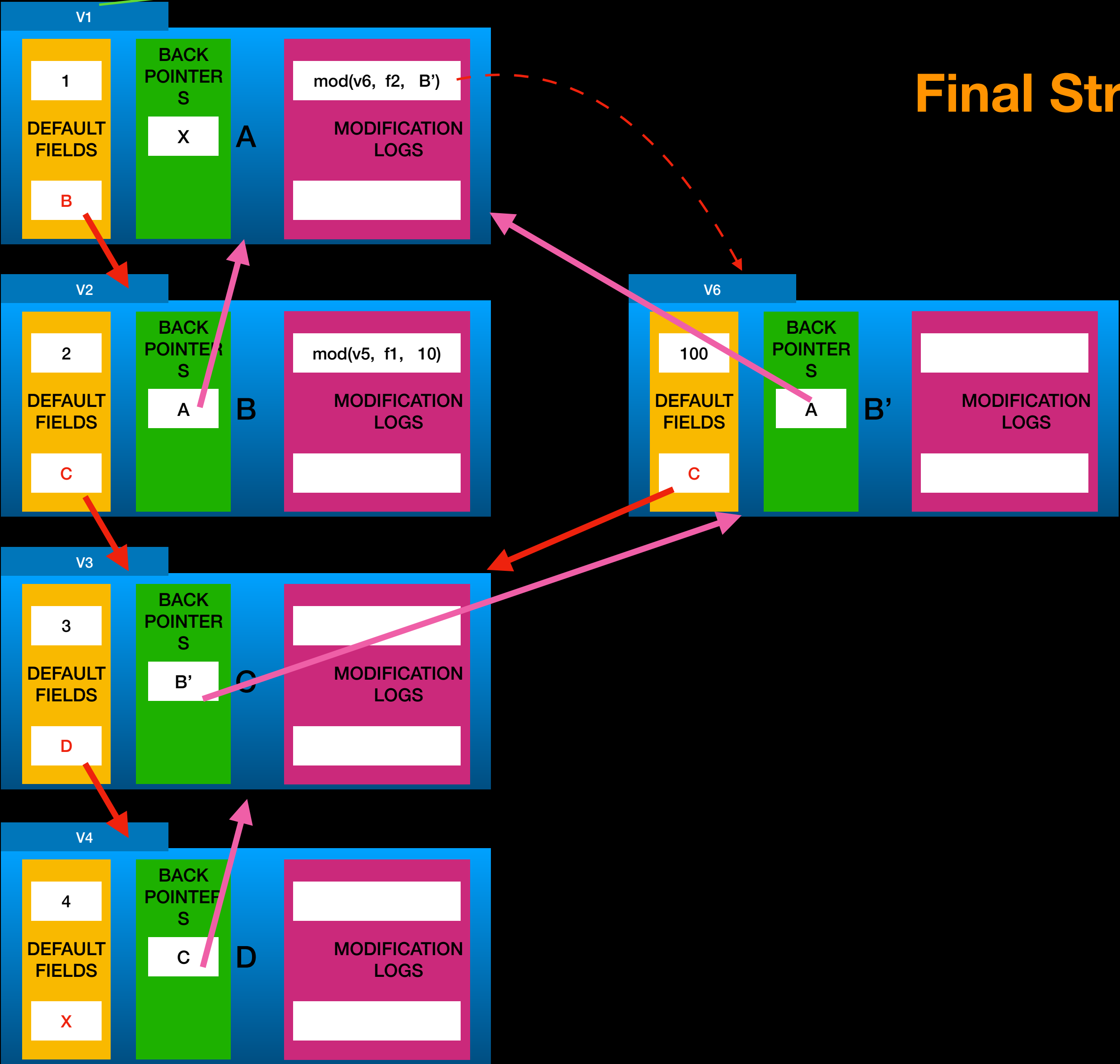
Current Version: v6

START MODULE

V0

update(f1,B,100)

Final Structure of v6



VERSION REDIRECTOR MODULE	
Ver	block
1	A

VERSION REDIRECTOR MODULE	
Ver	block
2	B
6	B'

VERSION REDIRECTOR MODULE	
Ver	block
3	C

VERSION REDIRECTOR MODULE	
Ver	block
4	D

Current Version: v7

START MODULE

V0

add(X,C,_) consists of

add(X,C,_) consists of

Create node X

Modify f2 of B' to X

Modify f2 of X to C + Set BP of X to B'

Update BP of C to X

Set f1 of X

VERSION REDIRECTOR MODULE

Ver	block
1	A

VERSION REDIRECTOR MODULE

Ver	block
2	B
6	B'

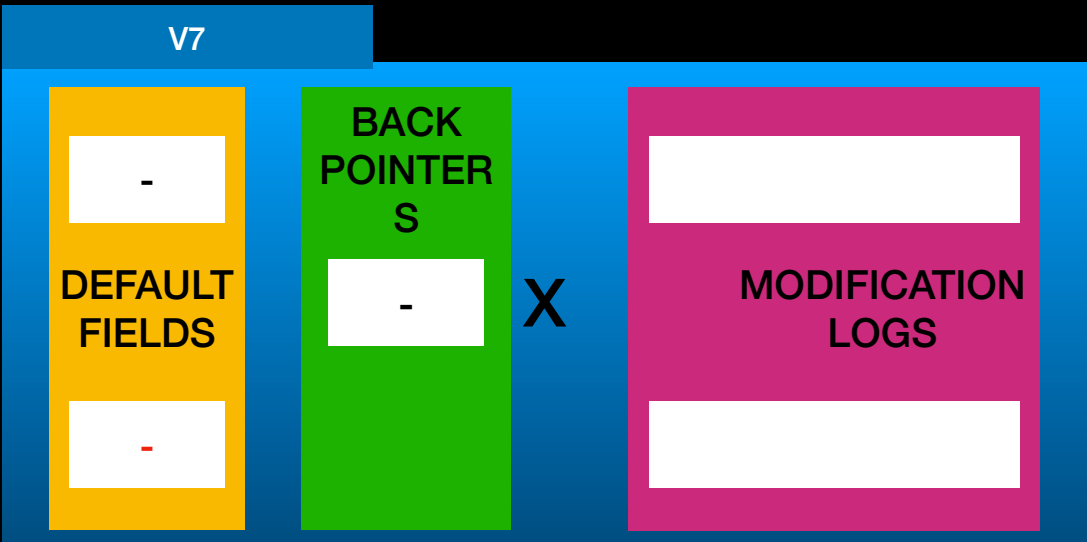
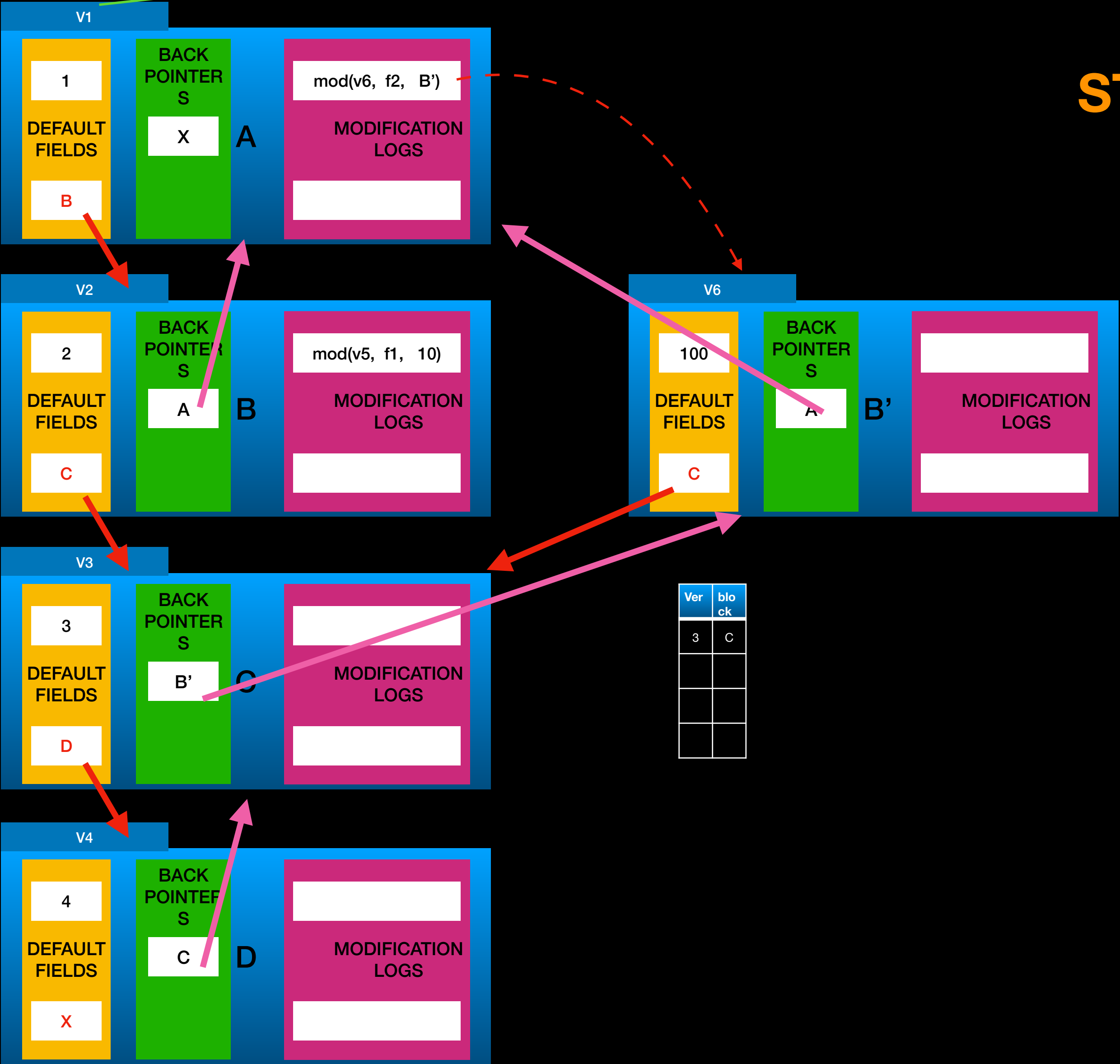
VERSION REDIRECTOR MODULE

Ver	block
7	X

VERSION REDIRECTOR MODULE

Ver	block
4	D

STEP 1



Current Version: v7

START MODULE

V0

add(X,C,_) consists of

add(X,C,_) consists of

Create node X

Modify f2 of B' to X

Modify f2 of X to C + Set BP of X to B'

Update BP of C to X

Set f1 of X

VERSION
REDIRECTOR
MODULE

Ver	block
1	A

VERSION
REDIRECTOR
MODULE

Ver	block
2	B
6	B'

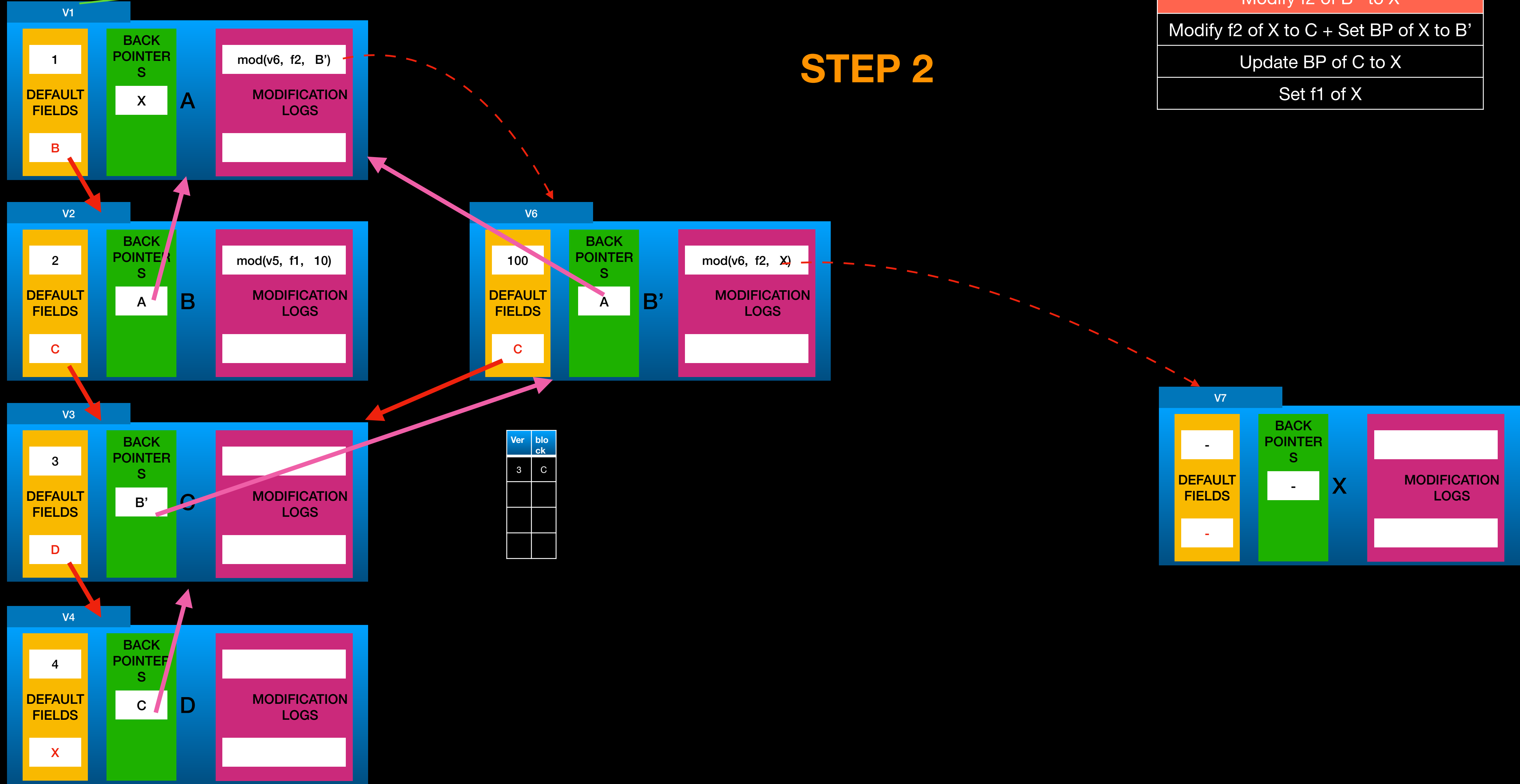
VERSION
REDIRECTOR
MODULE

Ver	block
7	X

VERSION
REDIRECTOR
MODULE

Ver	block
4	D

STEP 2



Current Version: v7

START MODULE

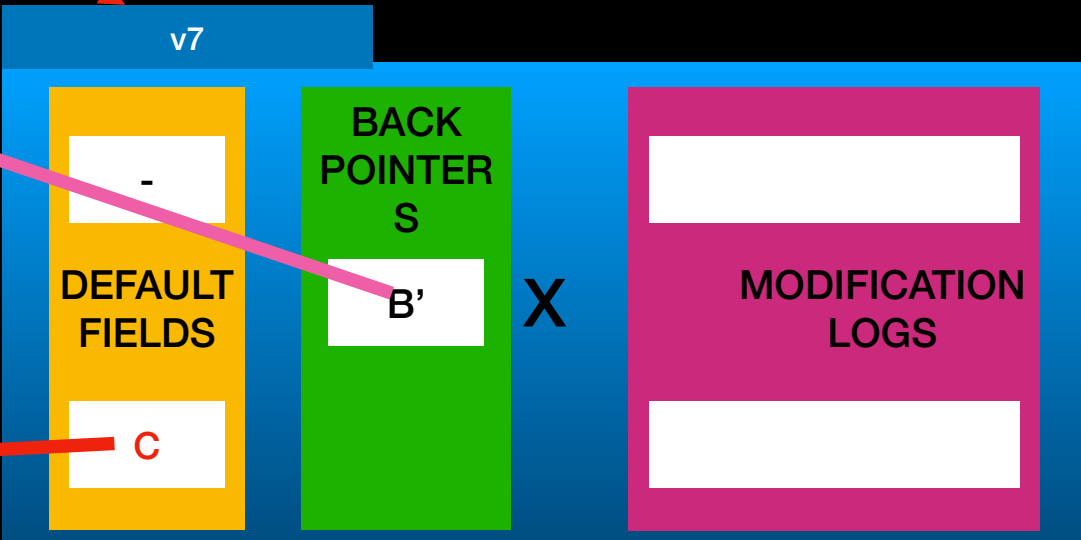
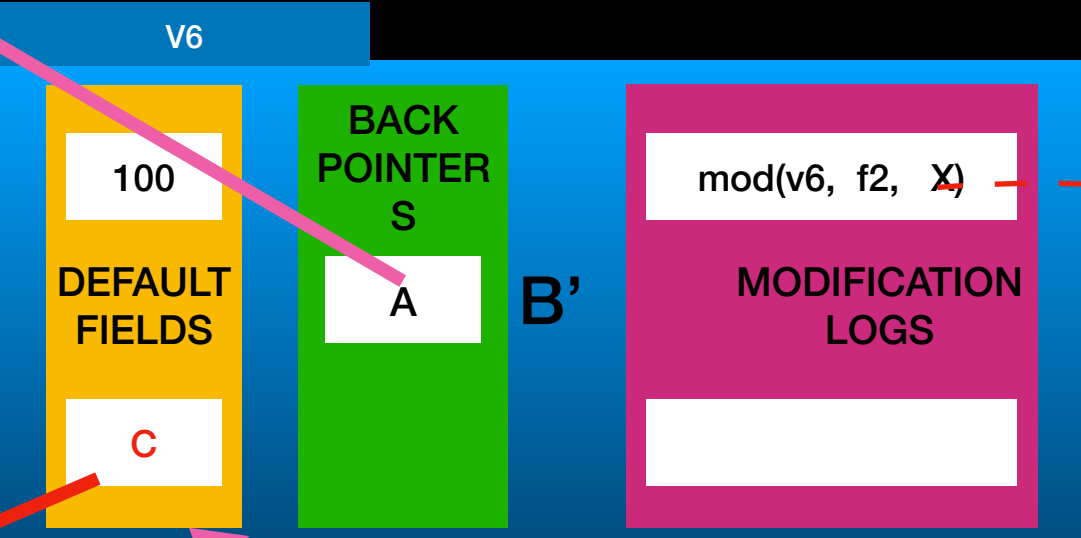
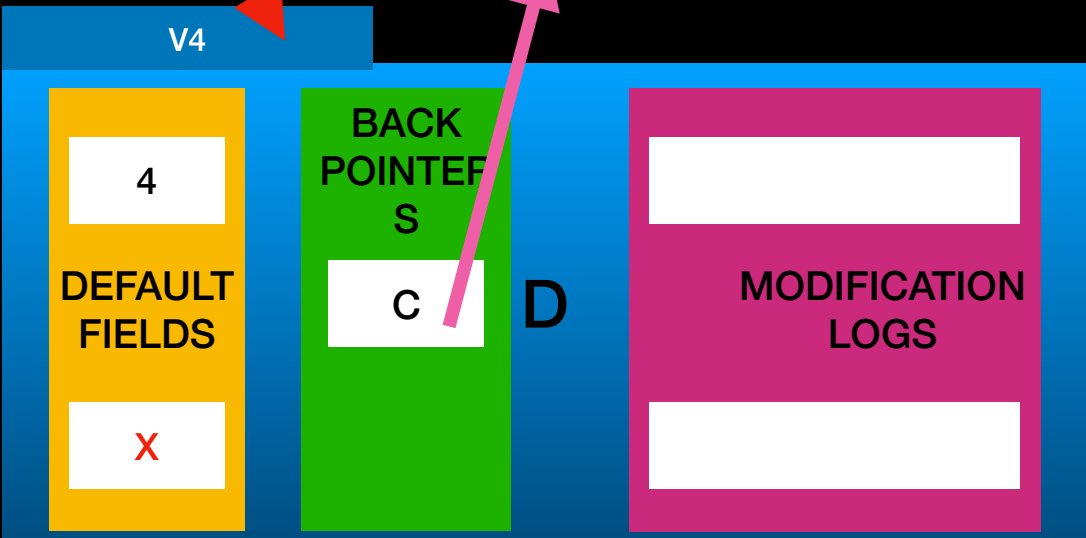
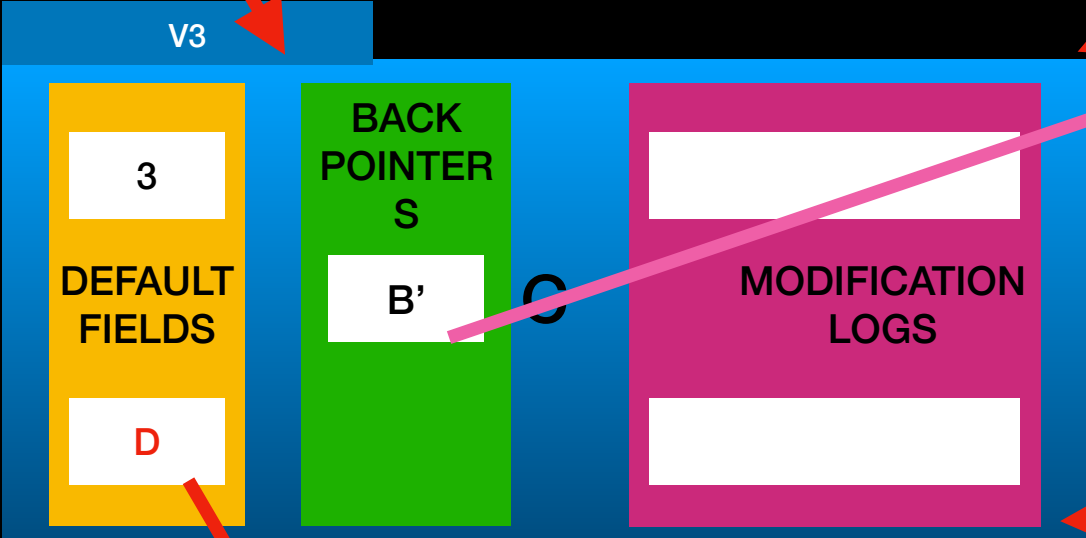
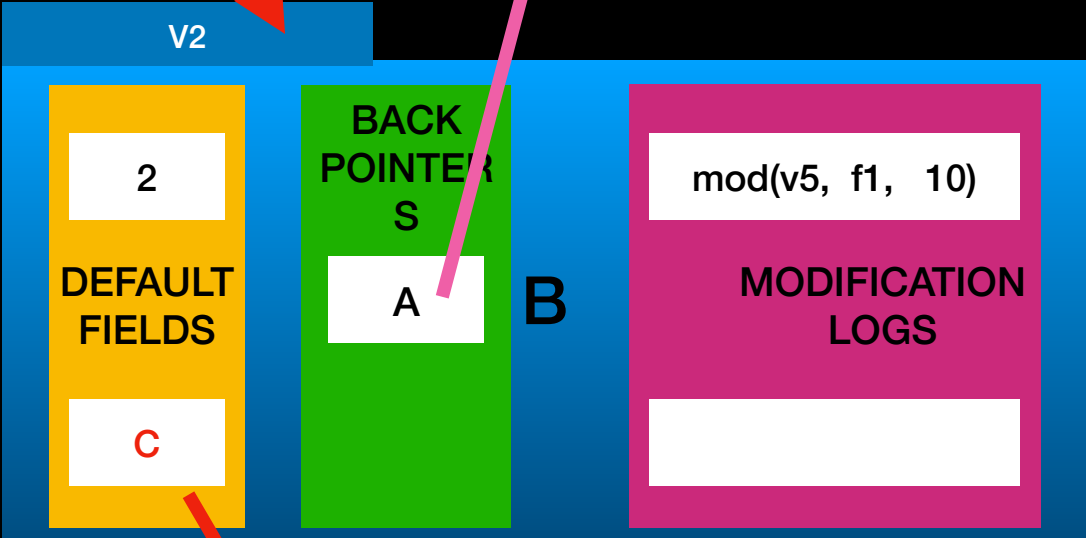
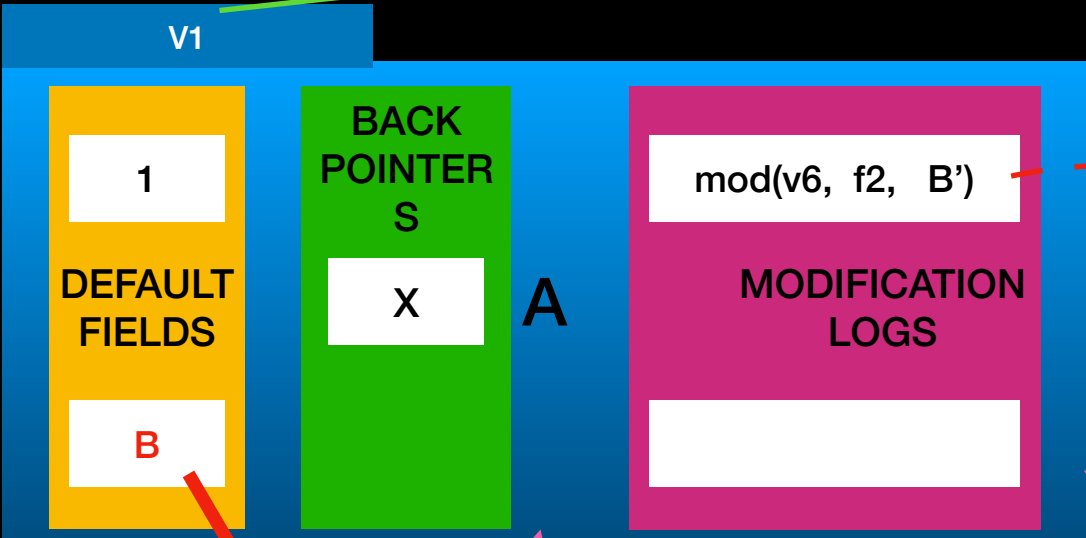
V0

add(X,C,_) consists of

Create node X
Modify f2 of B' to X
Modify f2 of X to C + Set BP of X to B'
Update BP of C to X
Set f1 of X

add(X,B,_)

STEP 3



Ver	block
3	C

VERSION REDIRECTOR MODULE

Ver	block
1	A

VERSION REDIRECTOR MODULE

Ver	block
2	B
6	B'

VERSION REDIRECTOR MODULE

Ver	block
7	X

VERSION REDIRECTOR MODULE

Ver	block
4	D

Current Version: v7

START MODULE

V0

add(X,C,_) consists of

Create node X

Modify f2 of B' to X

Modify f2 of X to C + Set BP of X to B'

Update BP of C to X

Set f1 of X

VERSION REDIRECTOR MODULE

Ver	block
1	A

VERSION REDIRECTOR MODULE

Ver	block
2	B
6	B'

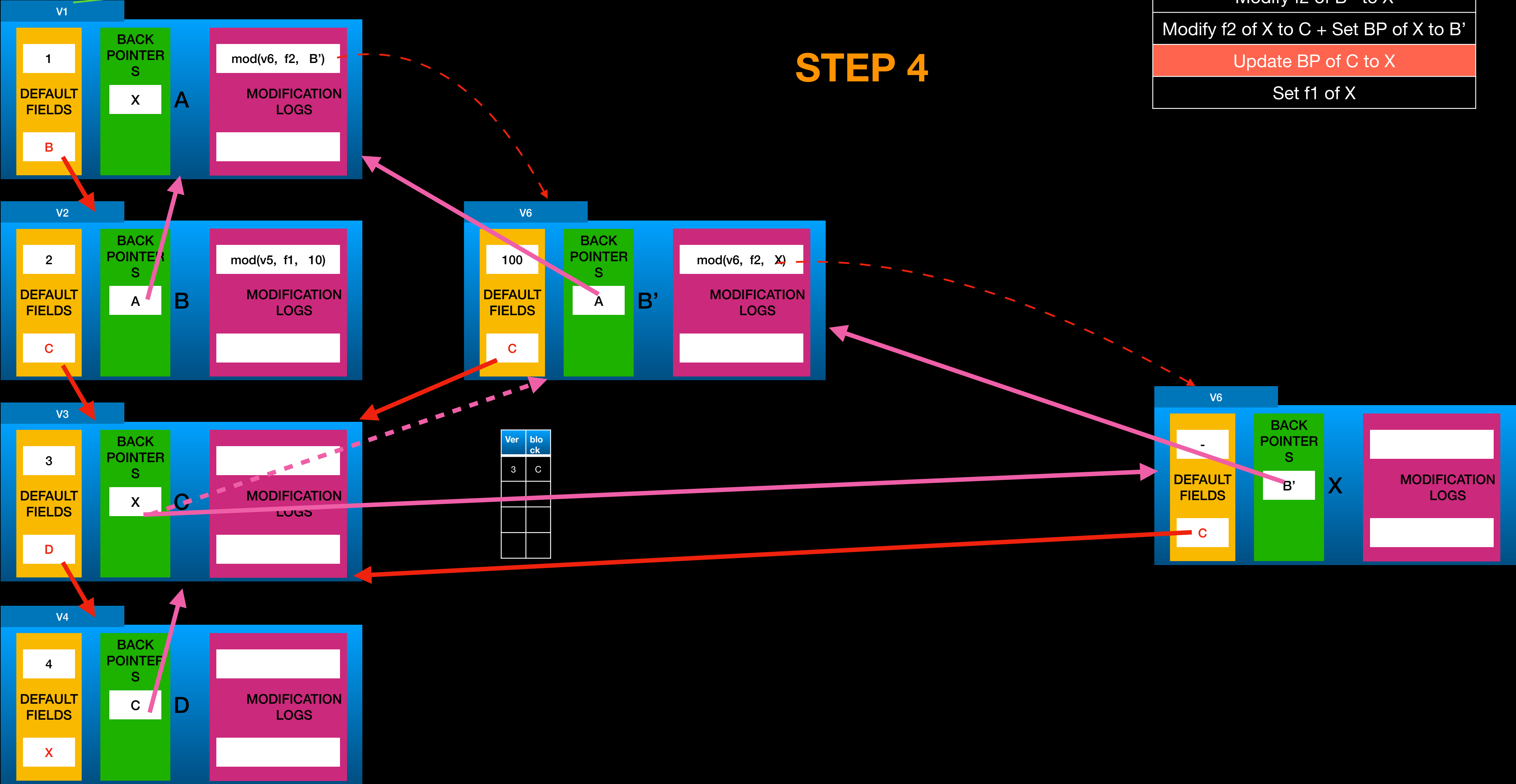
VERSION REDIRECTOR MODULE

Ver	block
7	X

VERSION REDIRECTOR MODULE

Ver	block
4	D

STEP 4



Current Version: v7

START MODULE

V0

add(X,C,_) consists of

Create node X
Modify f2 of B' to X
Modify f2 of X to C + Set BP of X to B'
Update BP of C to X
Set f1 of X

add(X,C,_) consists of

VERSION REDIRECTOR MODULE

Ver	block
1	A

VERSION REDIRECTOR MODULE

Ver	block
2	B
6	B'

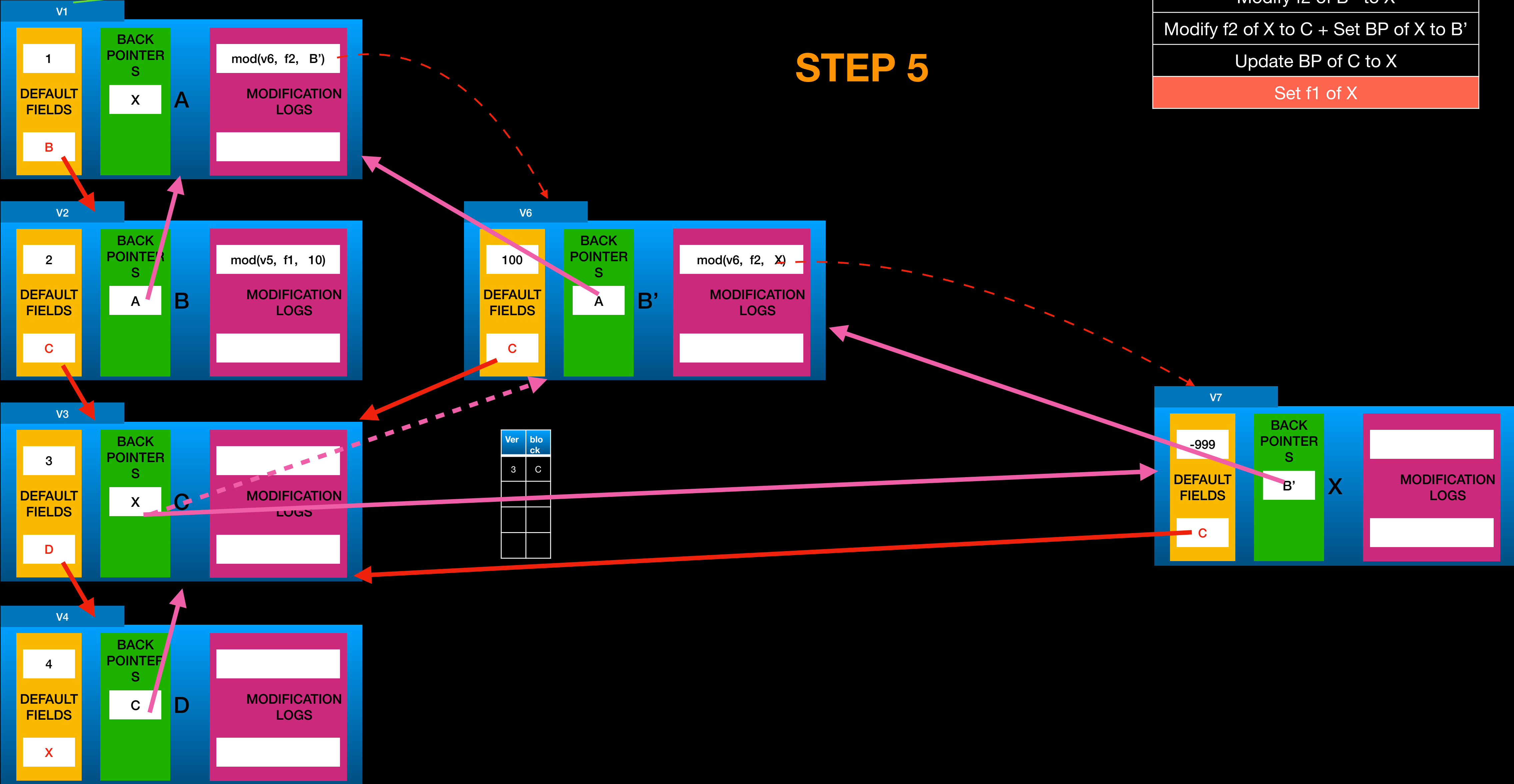
VERSION REDIRECTOR MODULE

Ver	block
7	X

VERSION REDIRECTOR MODULE

Ver	block
4	D

STEP 5



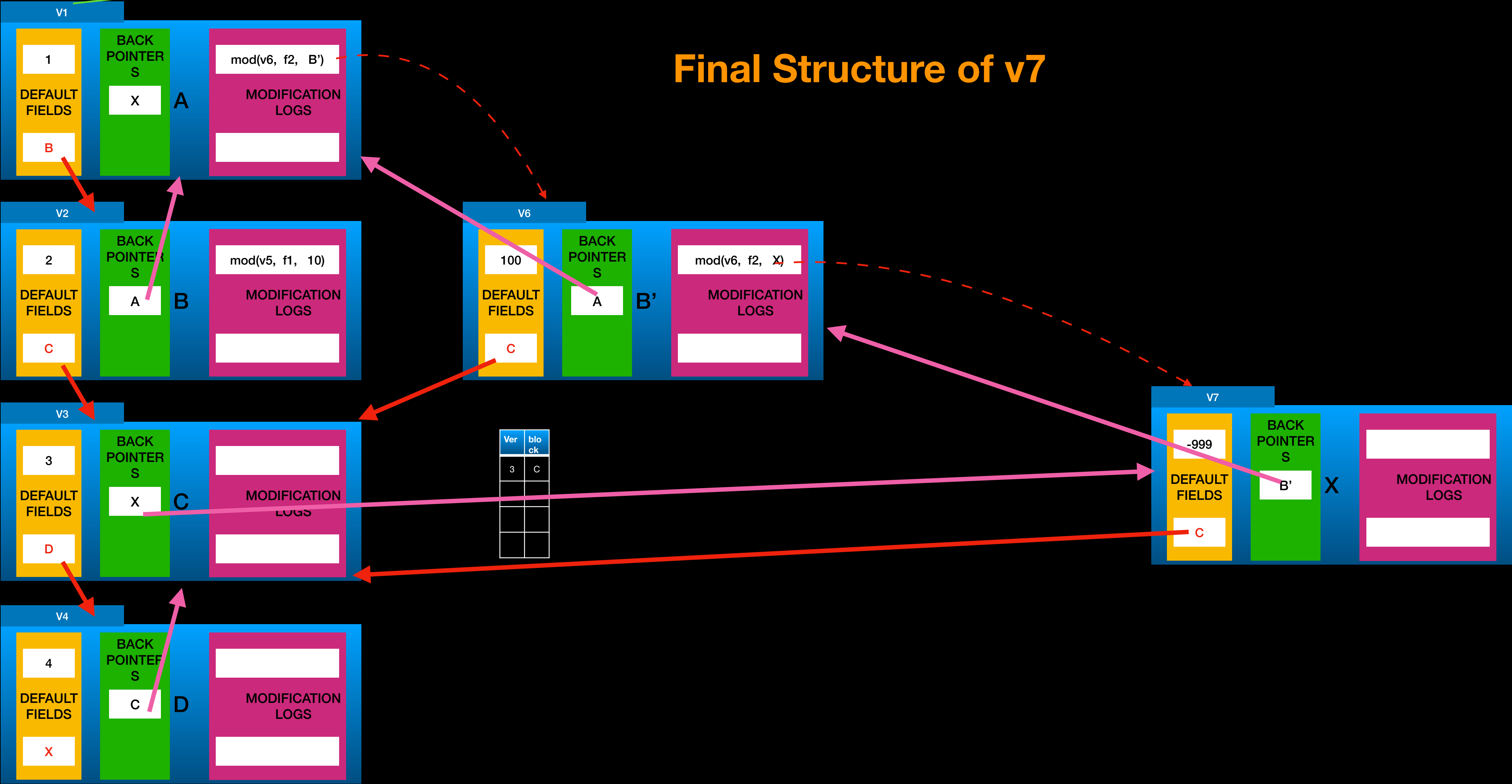
Current Version: v7

START MODULE

vo

add(X,C,_)

Final Structure of v7



VERSION
REDIRECTOR
MODULE

Ver	bloc
1	A

VERSION
REDIRECTOR
MODULE

Ver	blocc
2	B
6	B'

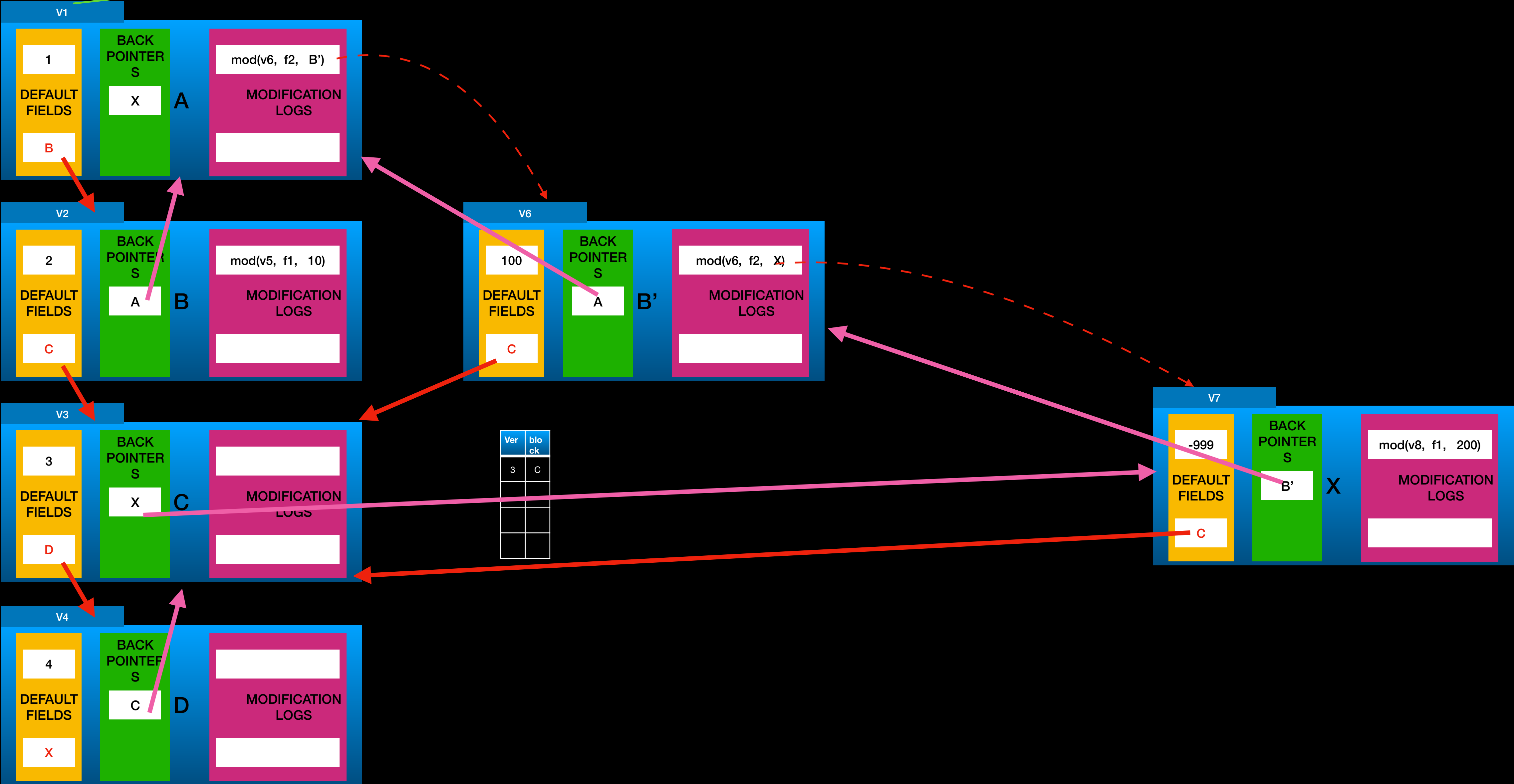
VERSION
REDIRECTOR
MODULE

Ver	block
7	X

VERSION
REDIRECTOR
MODULE

Ver	blocc
4	D

V0



VERSION REDIRECTOR MODULE

Ver	block
1	A

VERSION REDIRECTOR MODULE

Ver	block
2	B
6	B'

VERSION REDIRECTOR MODULE

Ver	block
7	X

VERSION REDIRECTOR MODULE

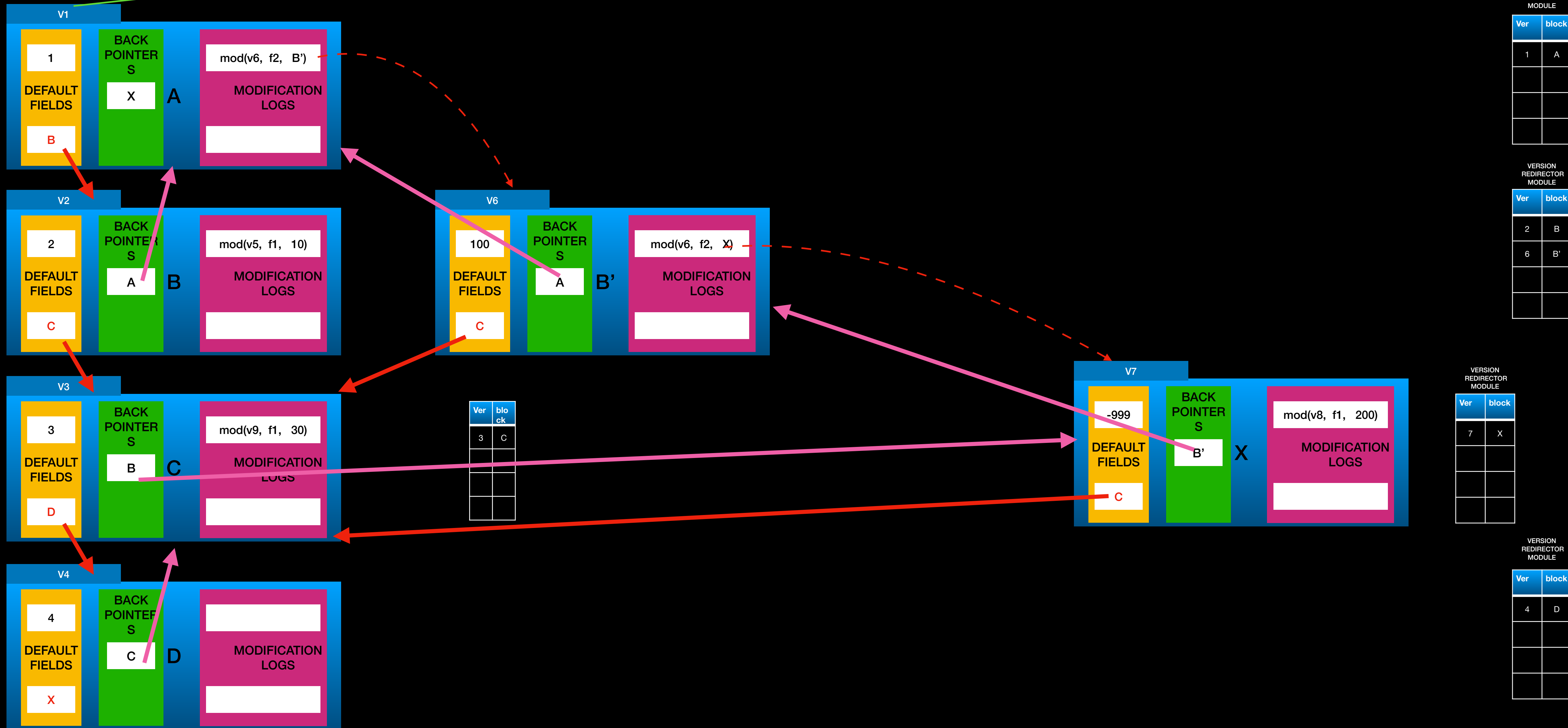
Ver	block
4	D

Current Version: v9

START MODULE

VO

```
update(f1, C,30)
```



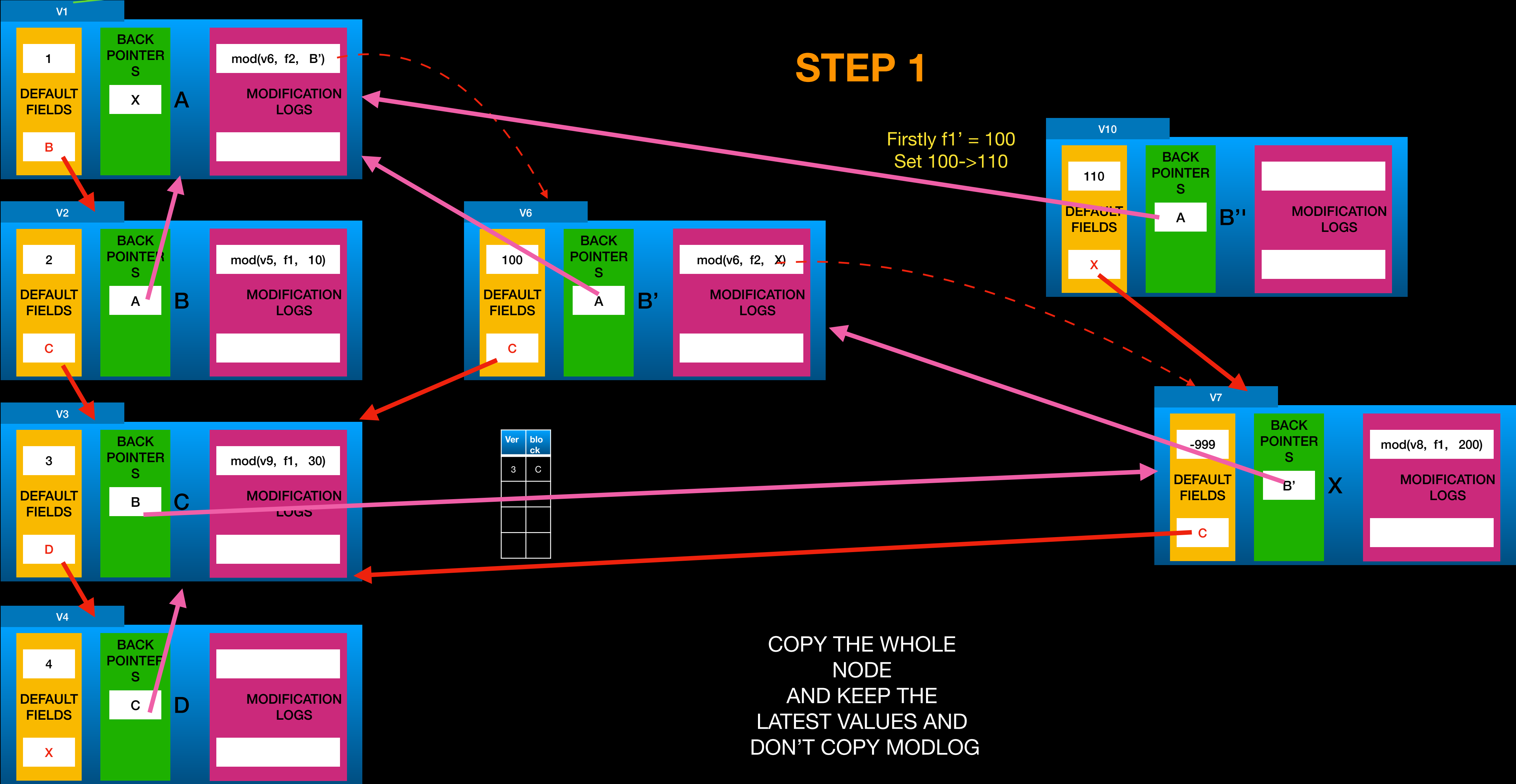
START MODULE

V0

STEP 1

Firstly f1' = 100
Set 100->110

COPY THE WHOLE
NODE
AND KEEP THE
LATEST VALUES AND
DON'T COPY MODLOG



VERSION REDIRECTOR MODULE

Ver	block
1	A

VERSION REDIRECTOR MODULE

Ver	block
2	B
6	B'
10	B''

VERSION REDIRECTOR MODULE

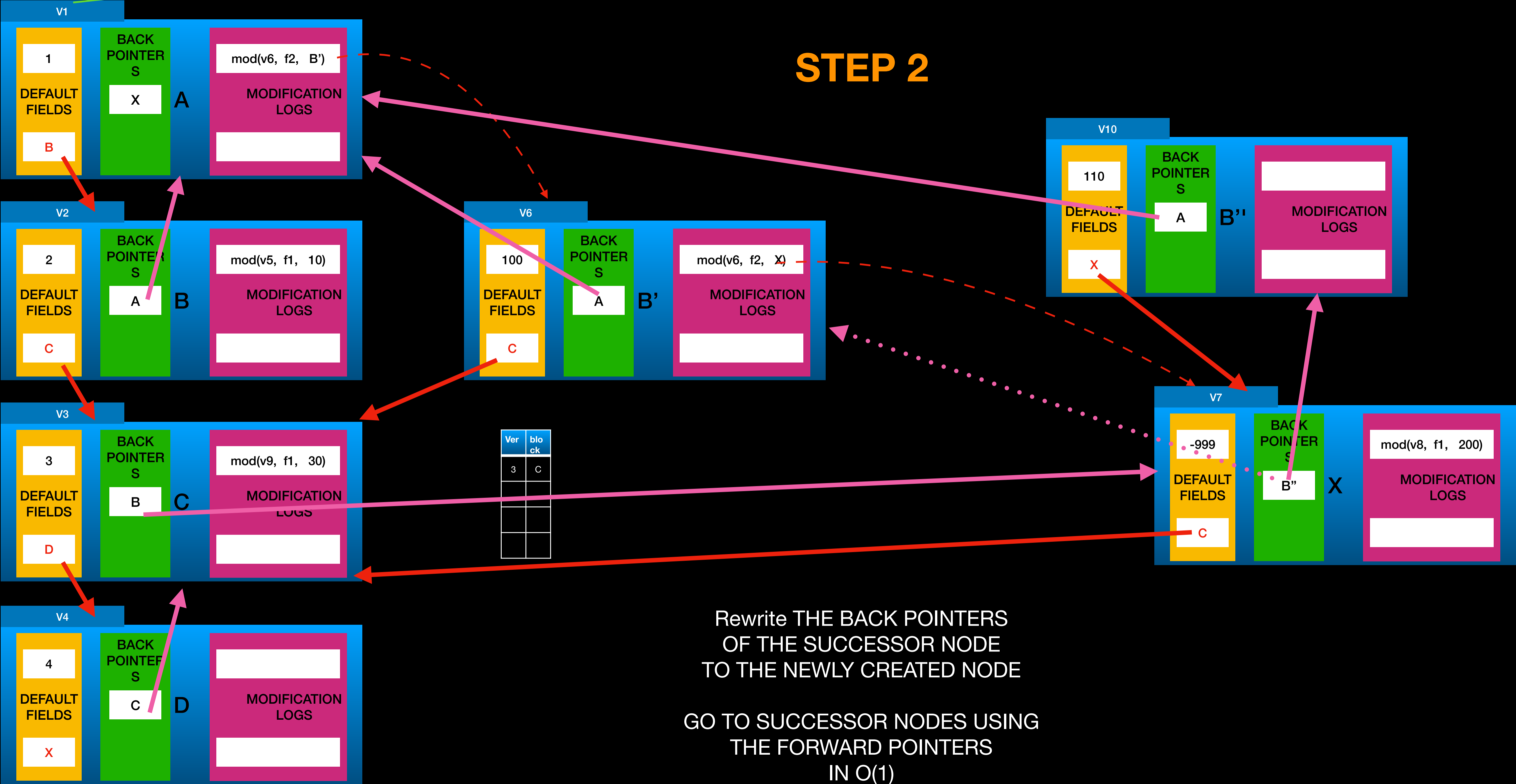
Ver	block
7	X

VERSION REDIRECTOR MODULE

Ver	block
4	D

V0

STEP 2



VERSION REDIRECTOR MODULE

Ver	block
1	A

VERSION REDIRECTOR MODULE

Ver	block
2	B
6	B'
10	B''

VERSION REDIRECTOR MODULE

Ver	block
7	X

VERSION REDIRECTOR MODULE

Ver	block
4	D

Rewrite THE BACK POINTERS
OF THE SUCCESSOR NODE
TO THE NEWLY CREATED NODE

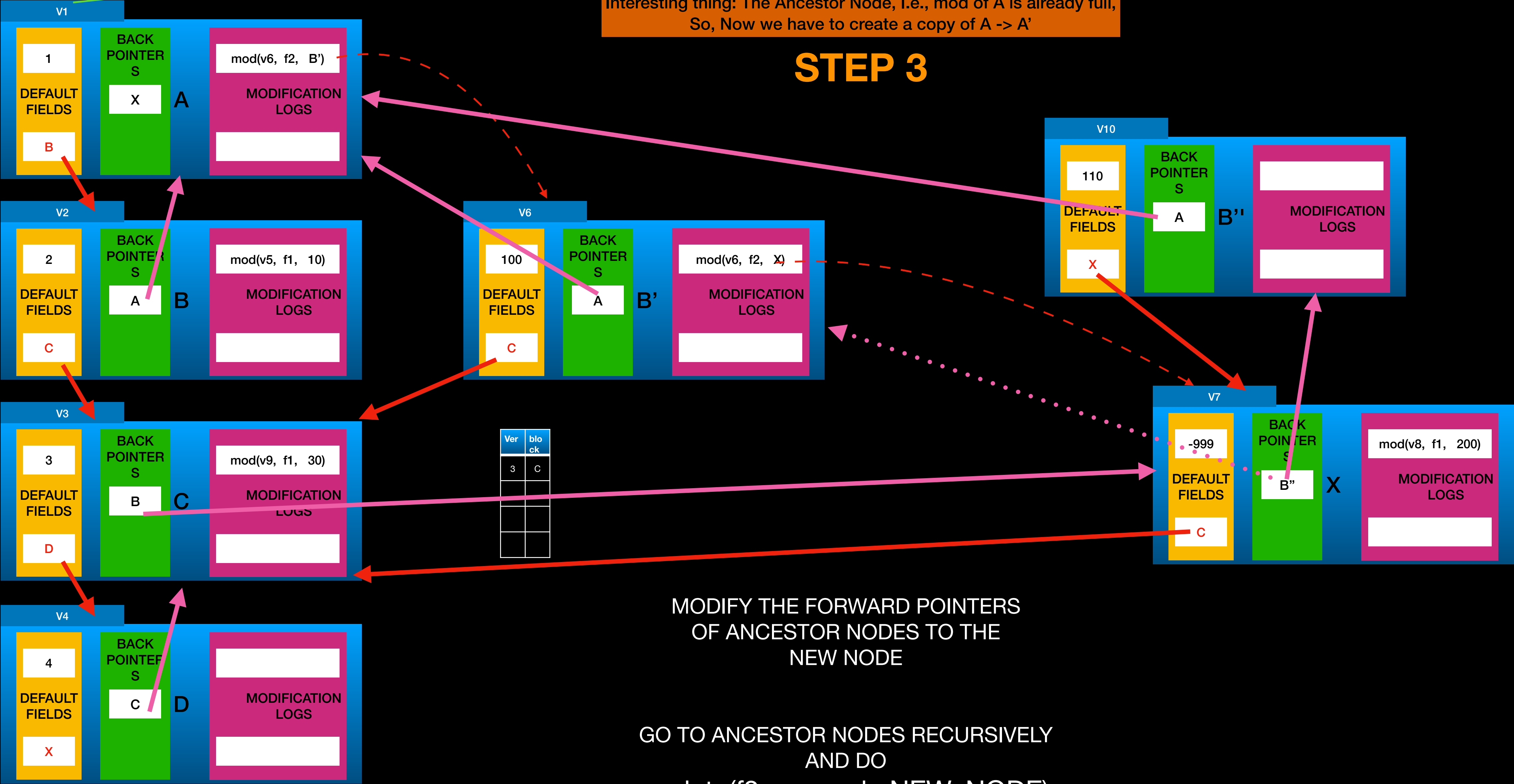
GO TO SUCCESSOR NODES USING
THE FORWARD POINTERS
IN O(1)

START MODULE

V0

Interesting thing: The Ancestor Node, I.e., mod of A is already full,
So, Now we have to create a copy of A -> A'

STEP 3



VERSION REDIRECTOR MODULE

Ver	block
1	A

VERSION REDIRECTOR MODULE

Ver	block
2	B
6	B'
10	B''

VERSION REDIRECTOR MODULE

Ver	block
7	X

VERSION REDIRECTOR MODULE

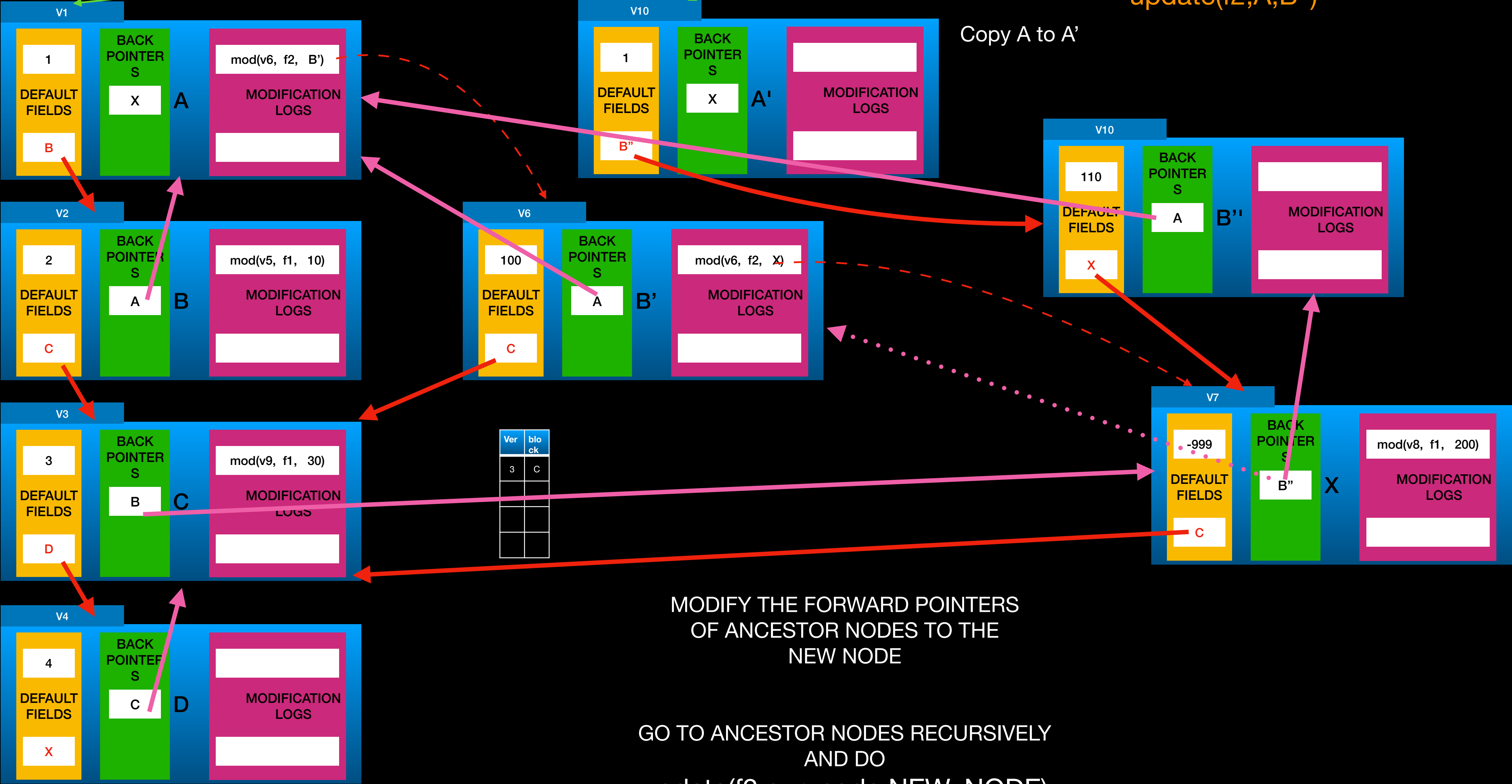
Ver	block
4	D

START MODULE

V0V10

STEP 3
SUBSTEP 1
update(f2,A,B'')

update(f1, B,110)



START MODULE

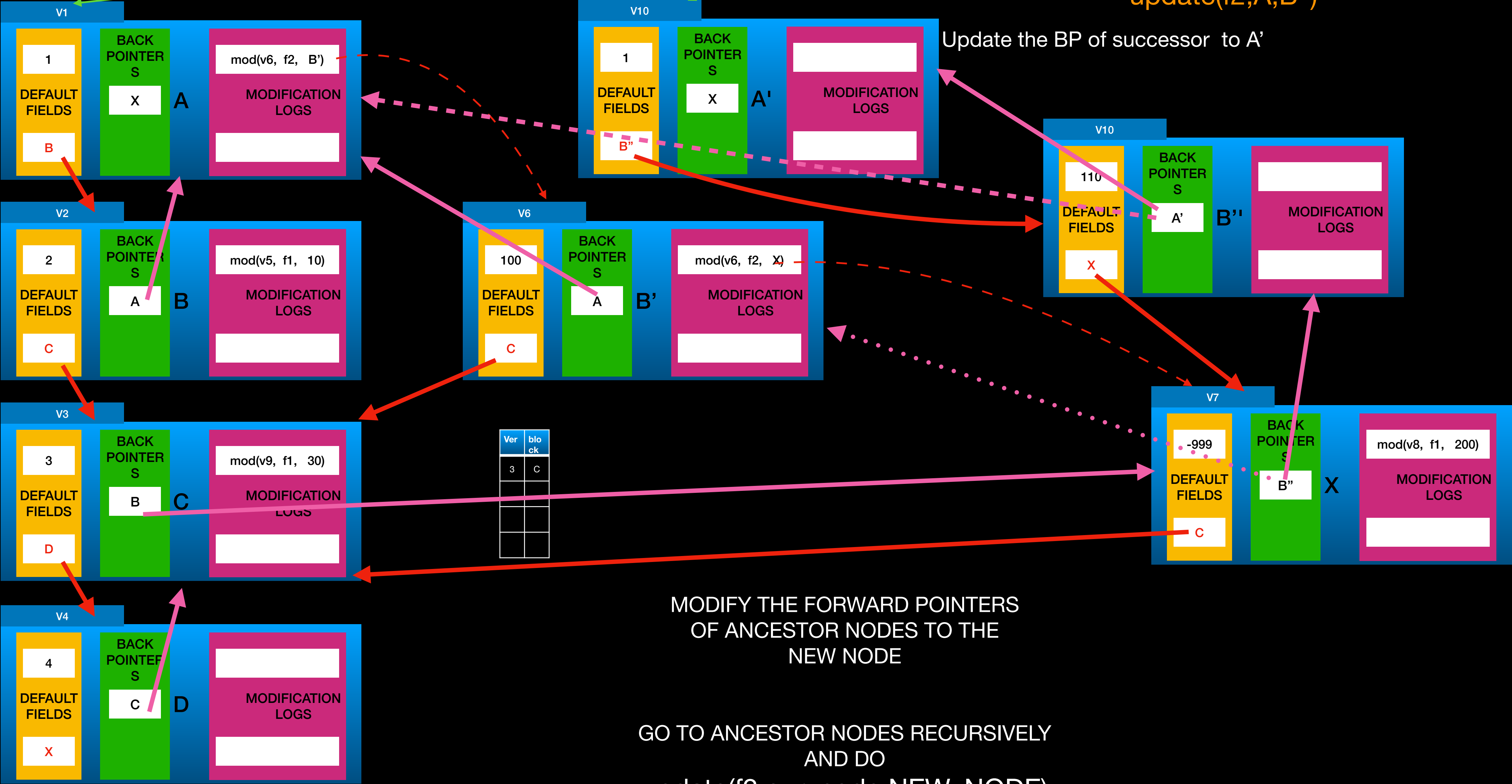
V0V10

STEP 3

SUBSTEP 2

update(f2,A,B'')

update(f1, B,110)



VERSION REDIRECTOR MODULE

Ver	block
1	A
10	A''

VERSION REDIRECTOR MODULE

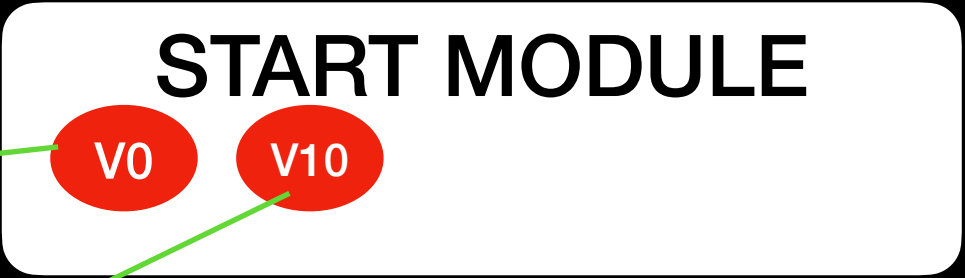
Ver	block
2	B
6	B'
10	B''

VERSION REDIRECTOR MODULE

Ver	block
7	X

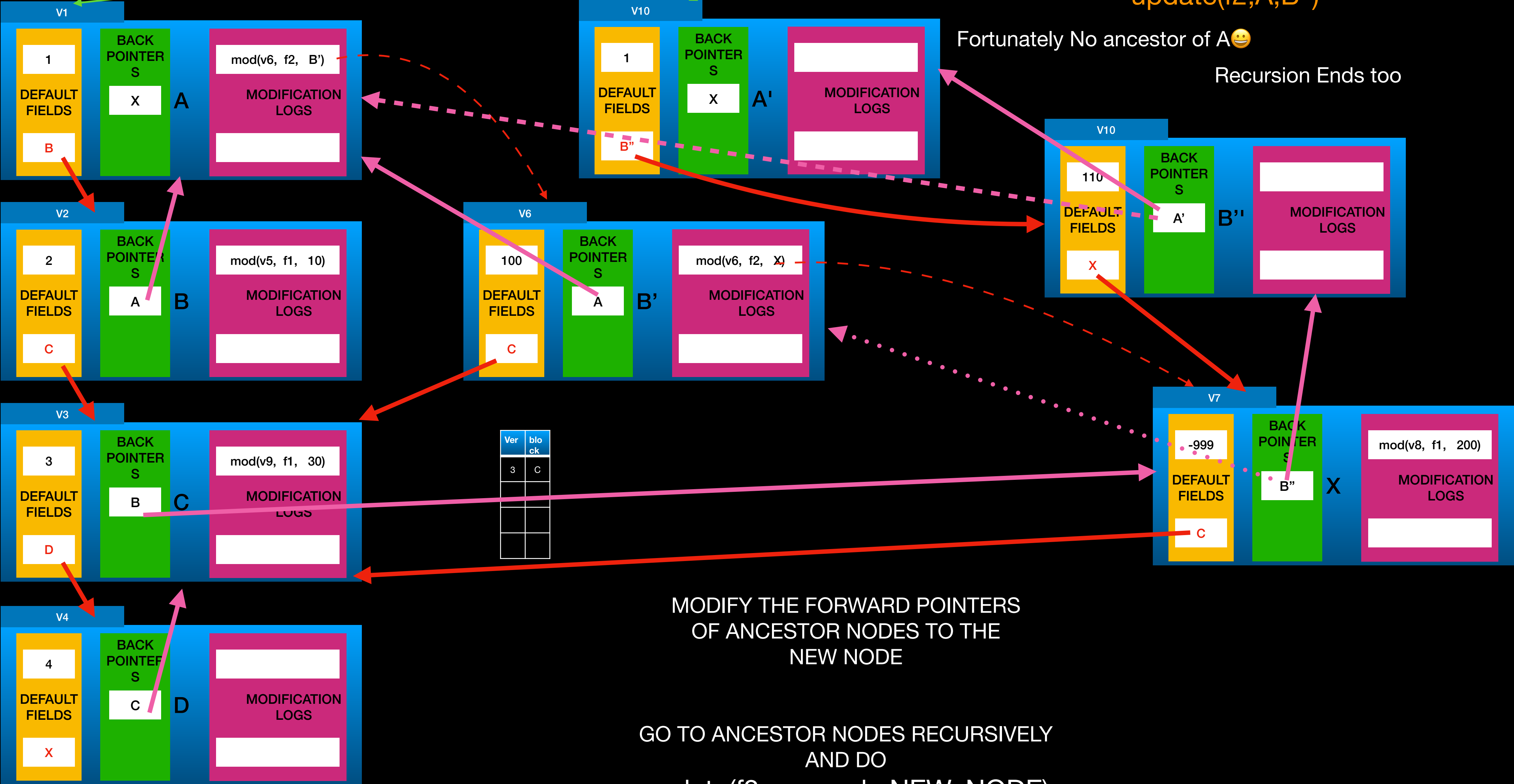
VERSION REDIRECTOR MODULE

Ver	block
4	D



STEP 3
SUBSTEP 3
update(f2,A,B'')

update(f1, B,110)



VERSION REDIRECTOR MODULE

Ver	block
1	A
10	A''

VERSION REDIRECTOR MODULE

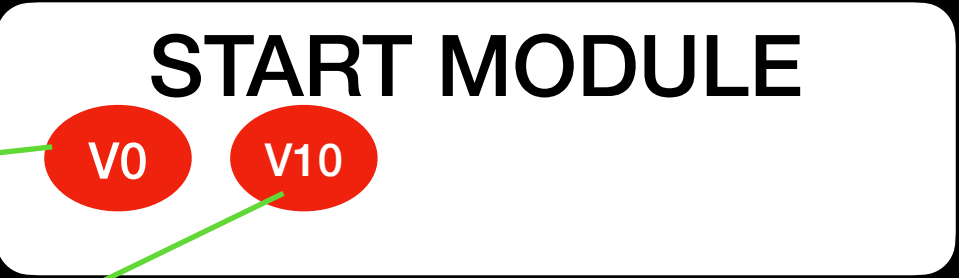
Ver	block
2	B
6	B'
10	B''

VERSION REDIRECTOR MODULE

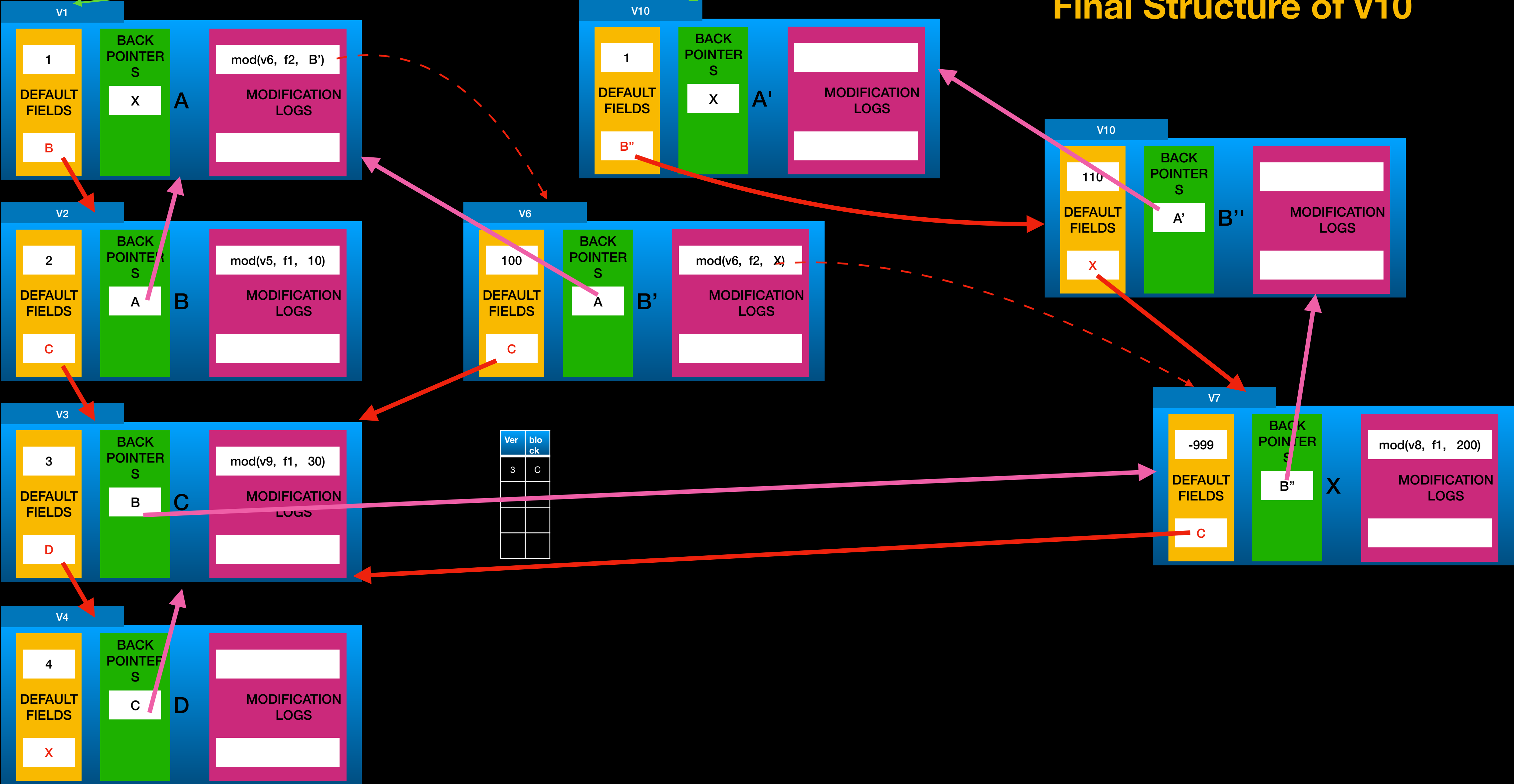
Ver	block
7	X

VERSION REDIRECTOR MODULE

Ver	block
4	D



Final Structure of v10



VERSION REDIRECTOR MODULE

Ver	block
1	A
10	A''

VERSION REDIRECTOR MODULE

Ver	block
2	B
6	B'
10	B''

VERSION REDIRECTOR MODULE

Ver	block
7	X

VERSION REDIRECTOR MODULE

Ver	block
4	D

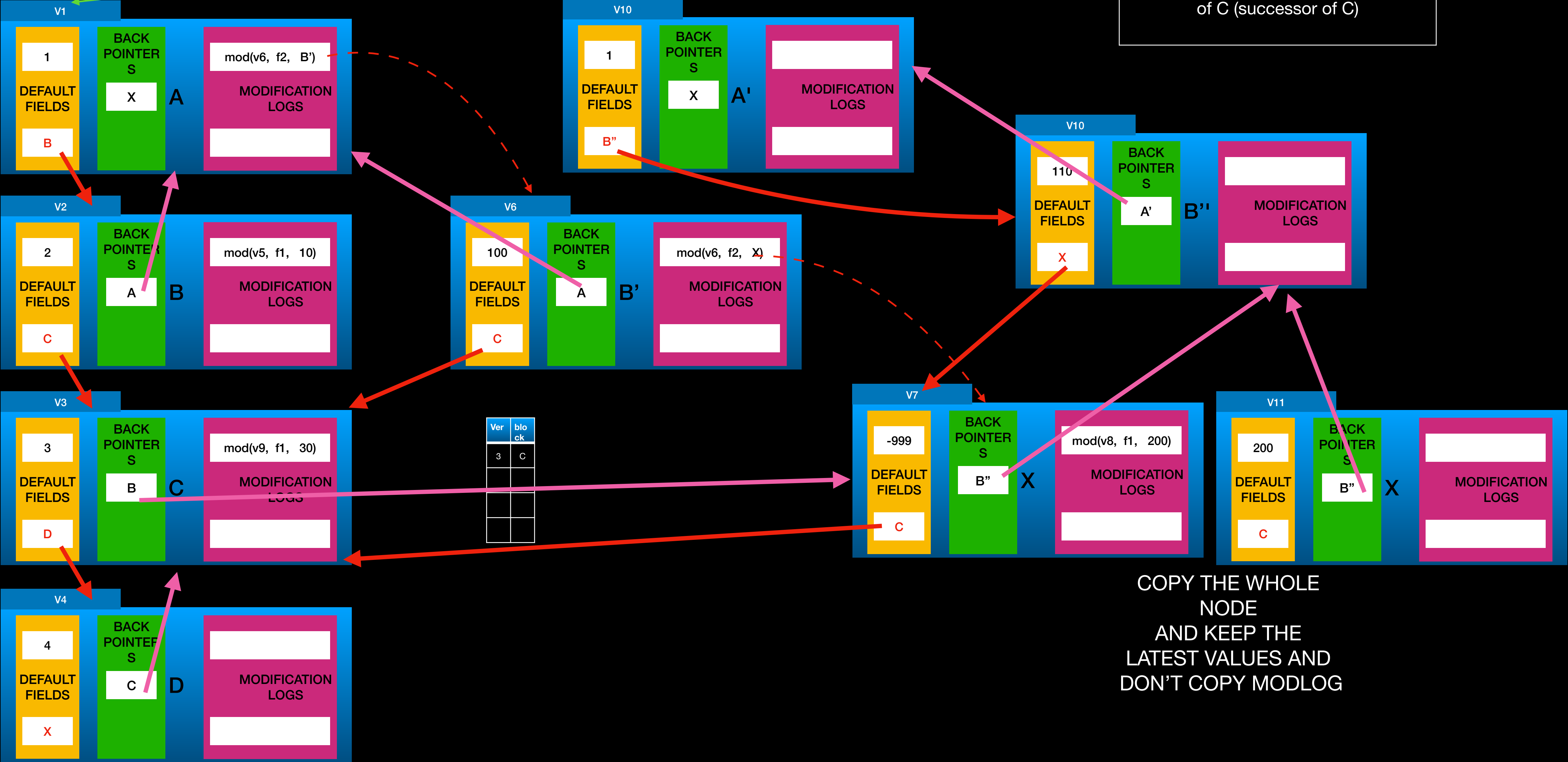
START MODULE

V0V10

Remove C consists of

Modify F2 of Parent of C (i.e., X) -> F2 of C (successor of C)

remove(C)



VERSION REDIRECTOR MODULE	
Ver	block
1	A
10	A''

VERSION REDIRECTOR MODULE	
Ver	block
2	B
6	B'
10	B''

VERSION REDIRECTOR MODULE	
Ver	block
7	X
11	X'

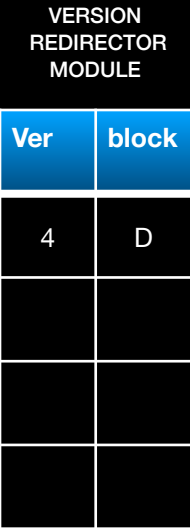
VERSION REDIRECTOR MODULE	
Ver	block
4	D

START MODULE

```
graph TD; subgraph START_MODULE [START MODULE]; V0((V0)); V10((V10)); end; START_MODULE -- V0 --> START_MODULE_UPPER[START MODULE]; START_MODULE -- V10 --> START_MODULE_LOWER[START MODULE];
```

Remove C consists of

Modify F2 of Parent C (i.e., X) \rightarrow F2 of C
(successor of C)

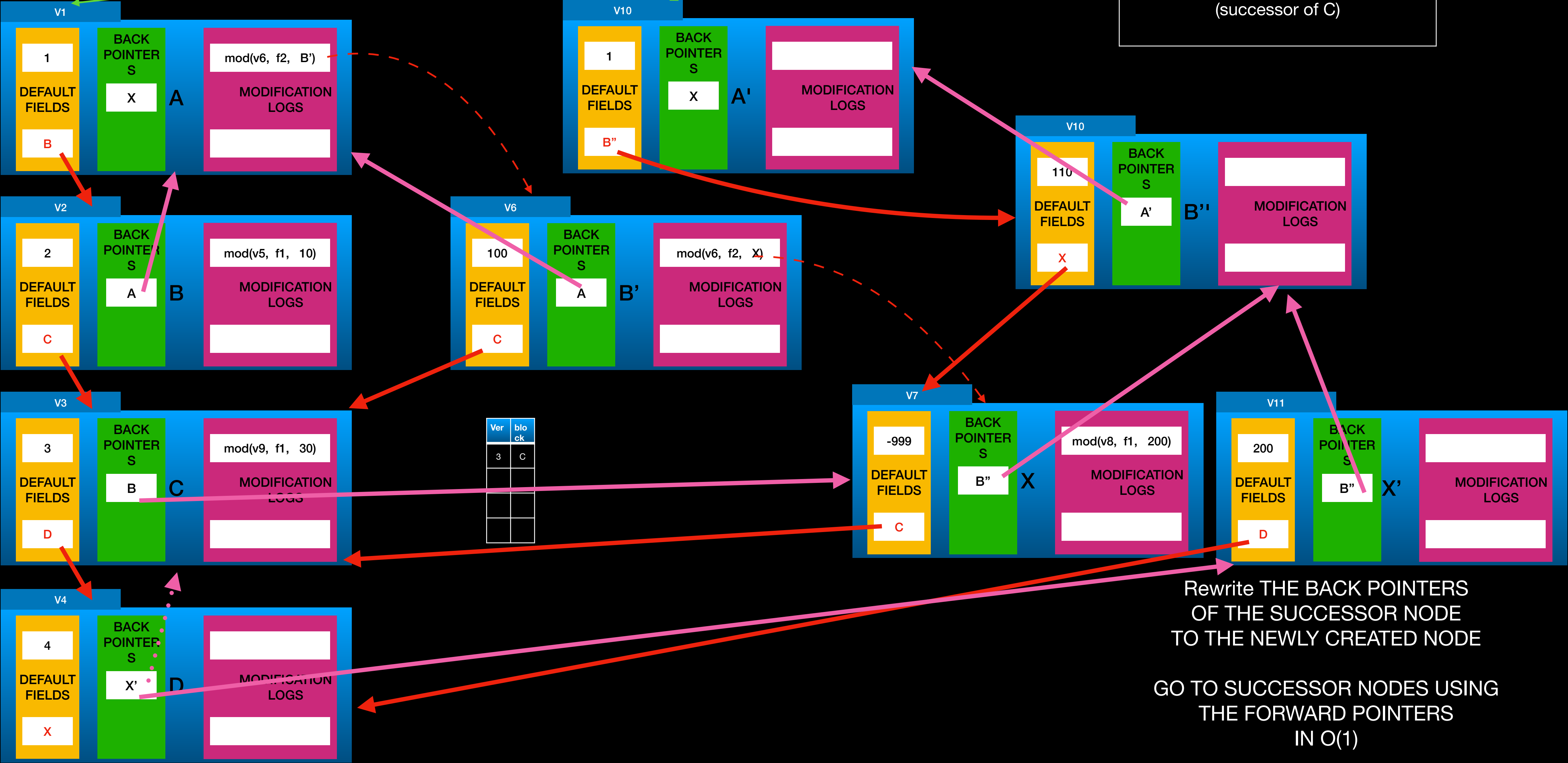


START MODULE

V0V10

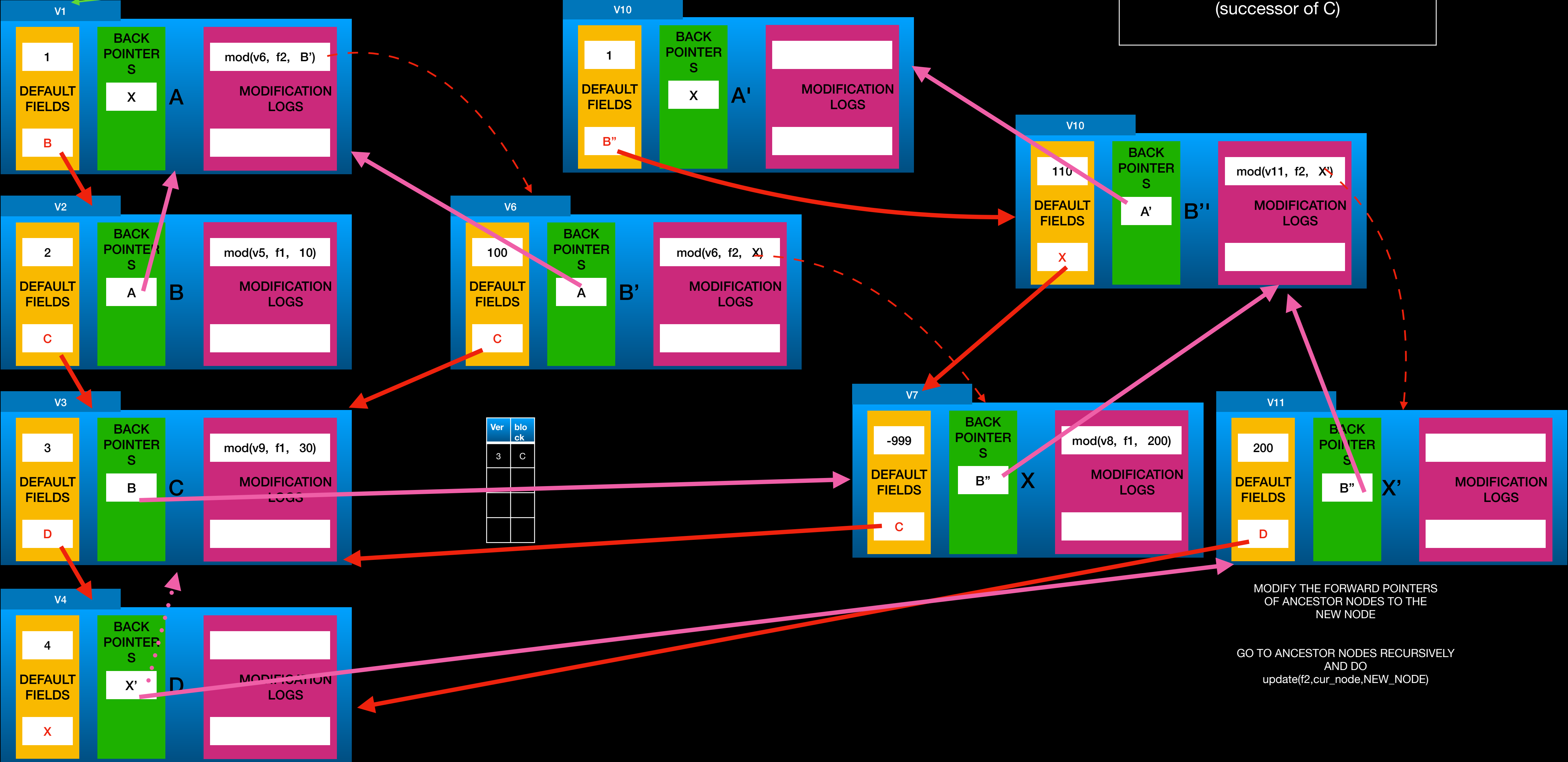
Remove C consists of

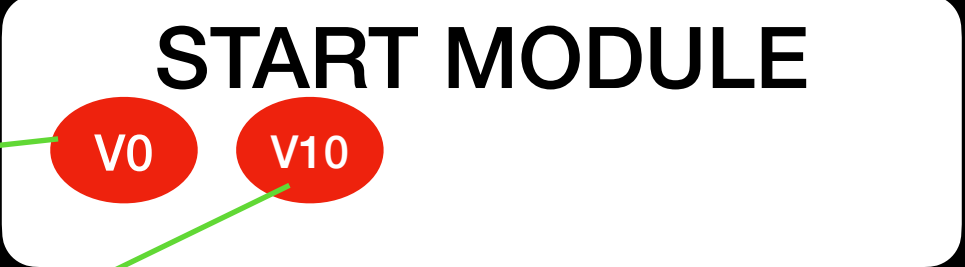
Modify F2 of Parent C (i.e., X) -> F2 of C (successor of C)



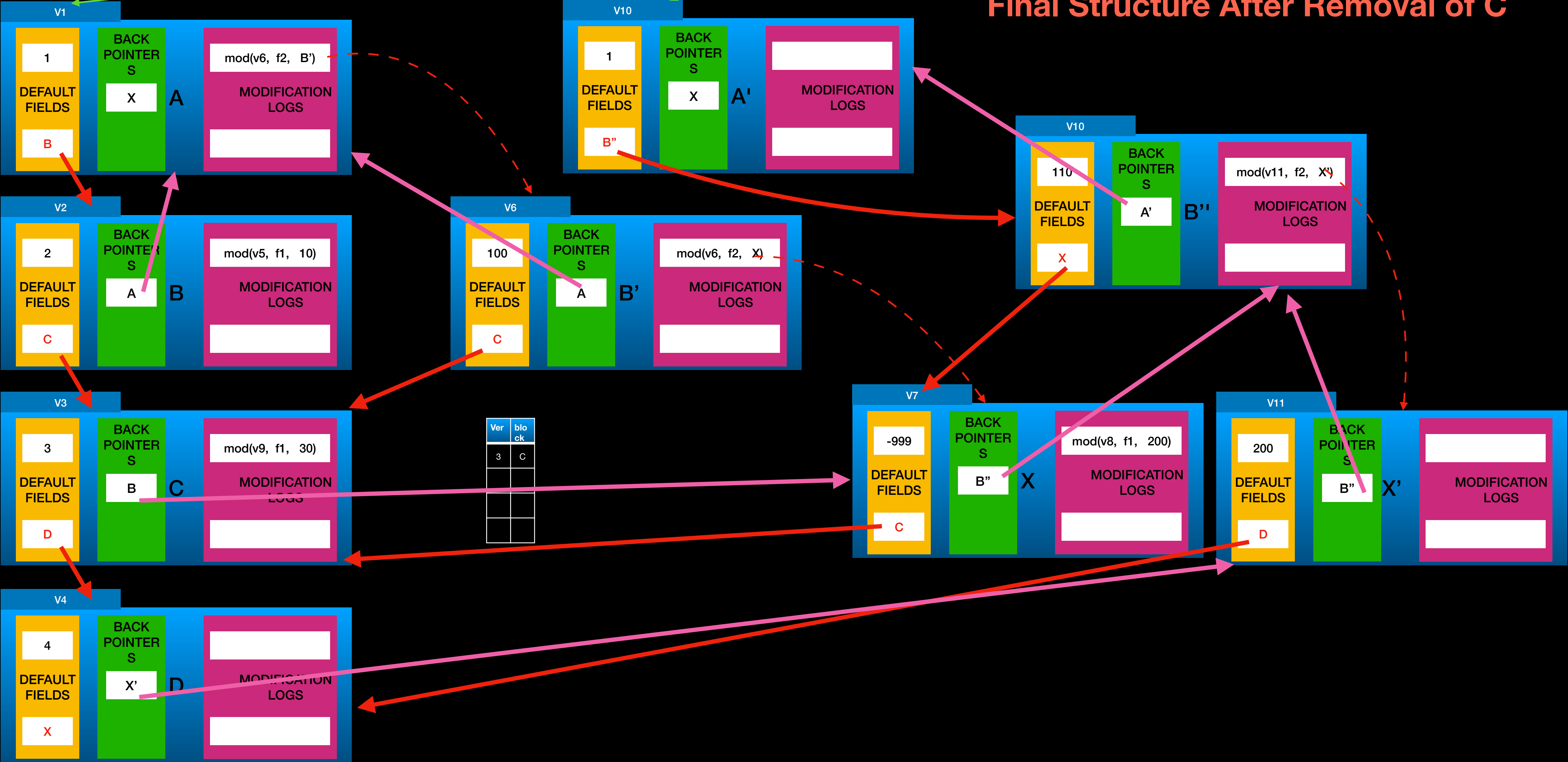
Remove C consists of

Modify F2 of Parent C (i.e., X) -> F2 of C (successor of C)





Final Structure After Removal of C



VERSION REDIRECTOR MODULE

Ver	block
1	A
10	A''

VERSION REDIRECTOR MODULE

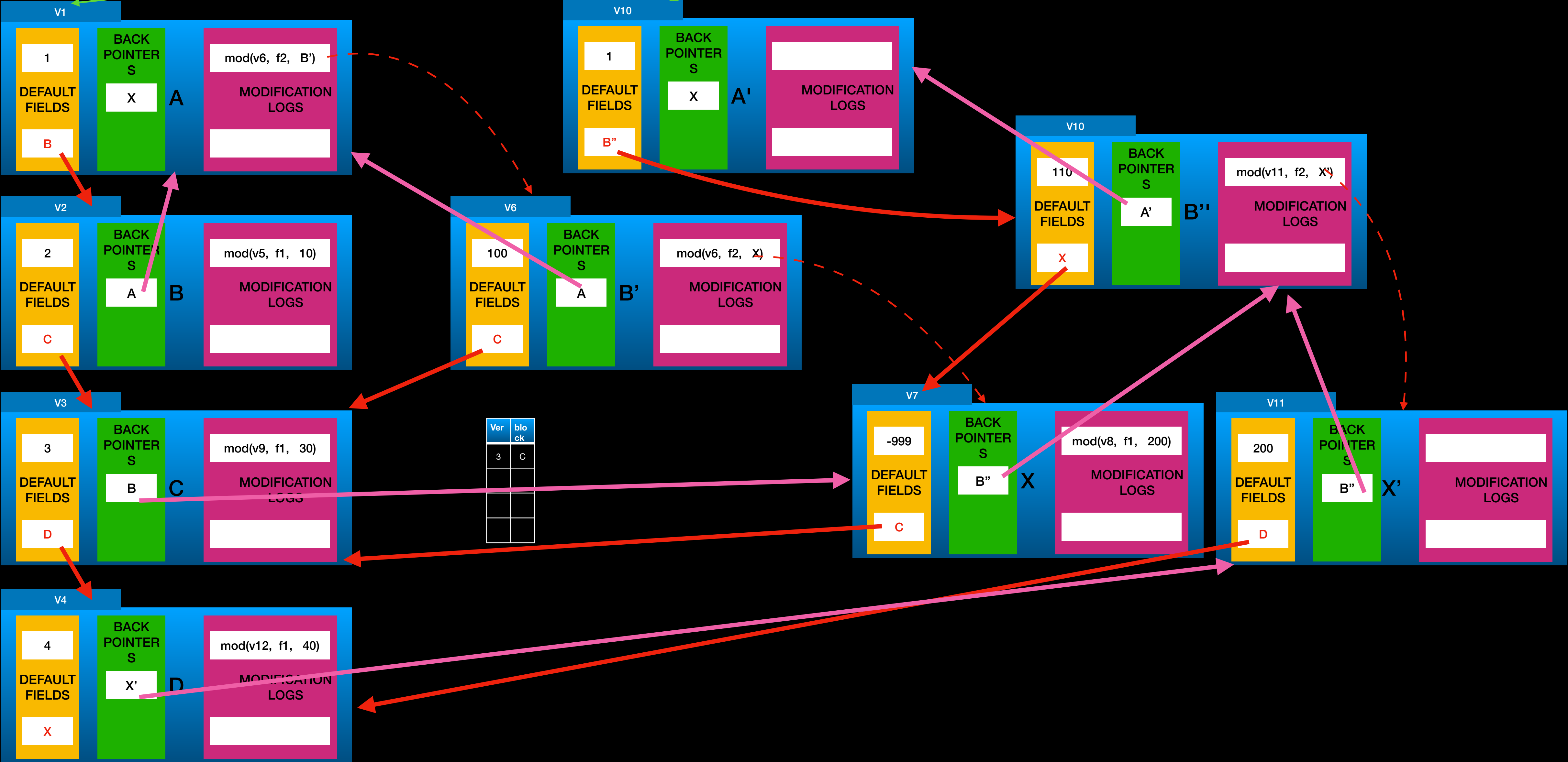
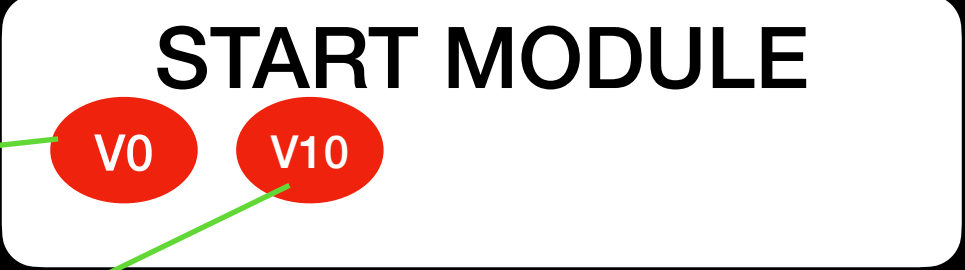
Ver	block
2	B
6	B'
10	B''

VERSION REDIRECTOR MODULE

Ver	block
7	X
11	X'

VERSION REDIRECTOR MODULE

Ver	block
4	D



VERSION REDIRECTOR MODULE

Ver	block
1	A
10	A''

VERSION REDIRECTOR MODULE

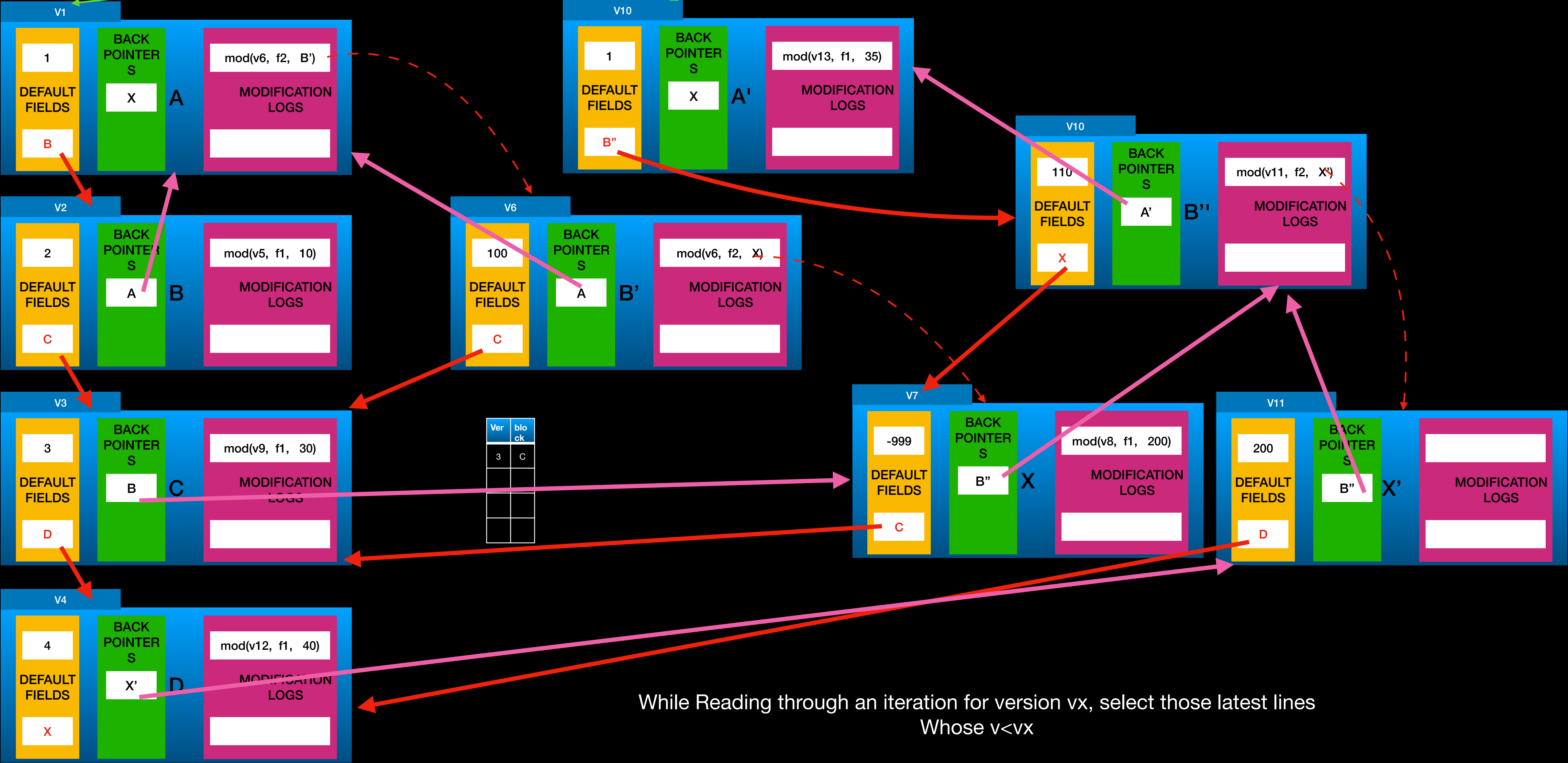
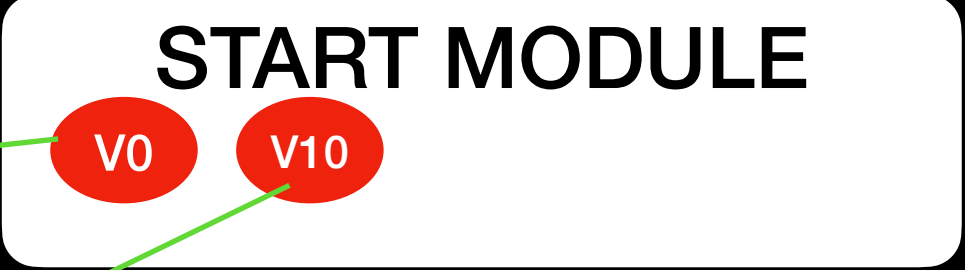
Ver	block
2	B
6	B'
10	B''

VERSION REDIRECTOR MODULE

Ver	block
7	X
11	X'

VERSION REDIRECTOR MODULE

Ver	block
4	D



While Reading through an iteration for version vx, select those latest lines
Whose v<vx

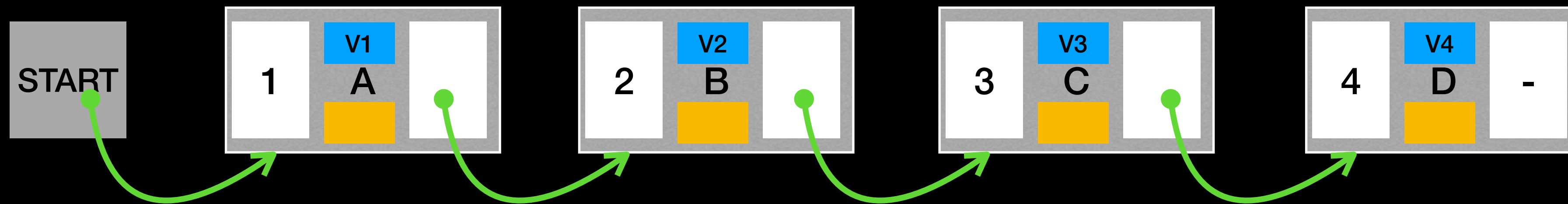
iterate_LL_at_v(vx)

To print the list at v_x

- Start from start module
- Choose those lines whose version **IS JUST LESS THAN OR EQUAL TO** v_x (as, suppose if we are traversing for v5 , either v0,v1,v2,v3, v4 or v5 can be on that path, lines>v5 can't be on that).
- The word “JUST LESS” is written because if a NODE has two version lines in that, e.g. v2 and v4 and we are searching for v5, then we should prefer v4 line over v2.

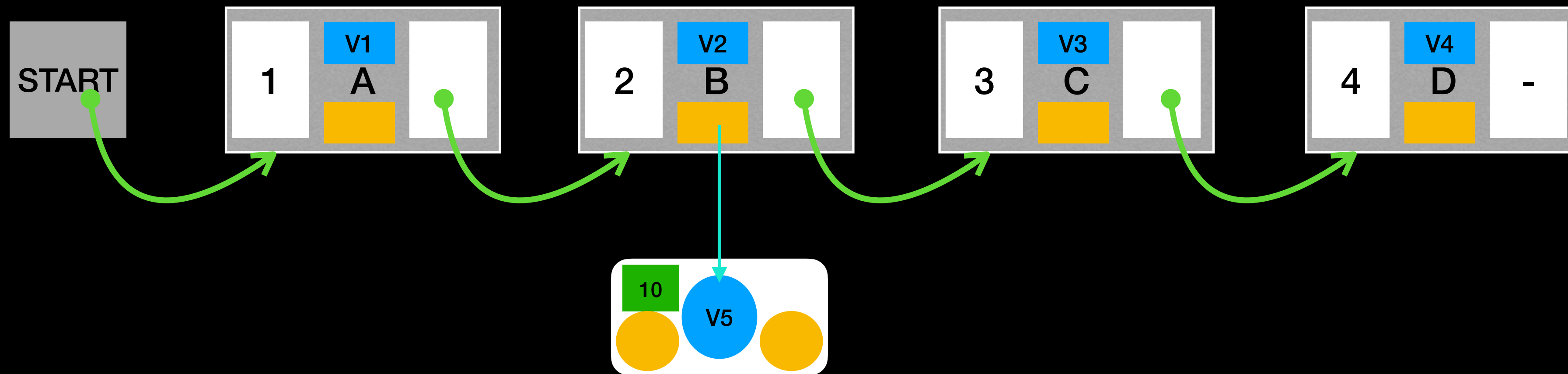
Why such approach is better ? 🥲

Normal Implementation (using balanced BST [may use just LL])



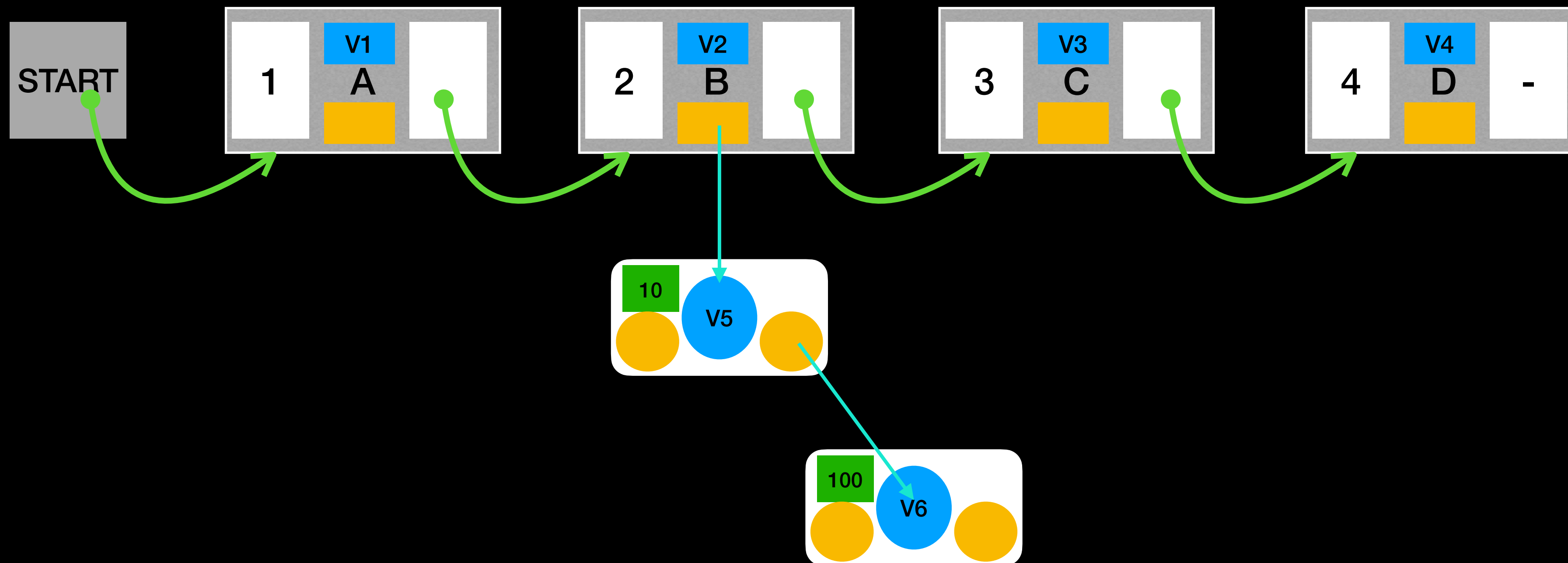
Why such approach is better ? 🥲

Normal Implementation (using balanced BST)



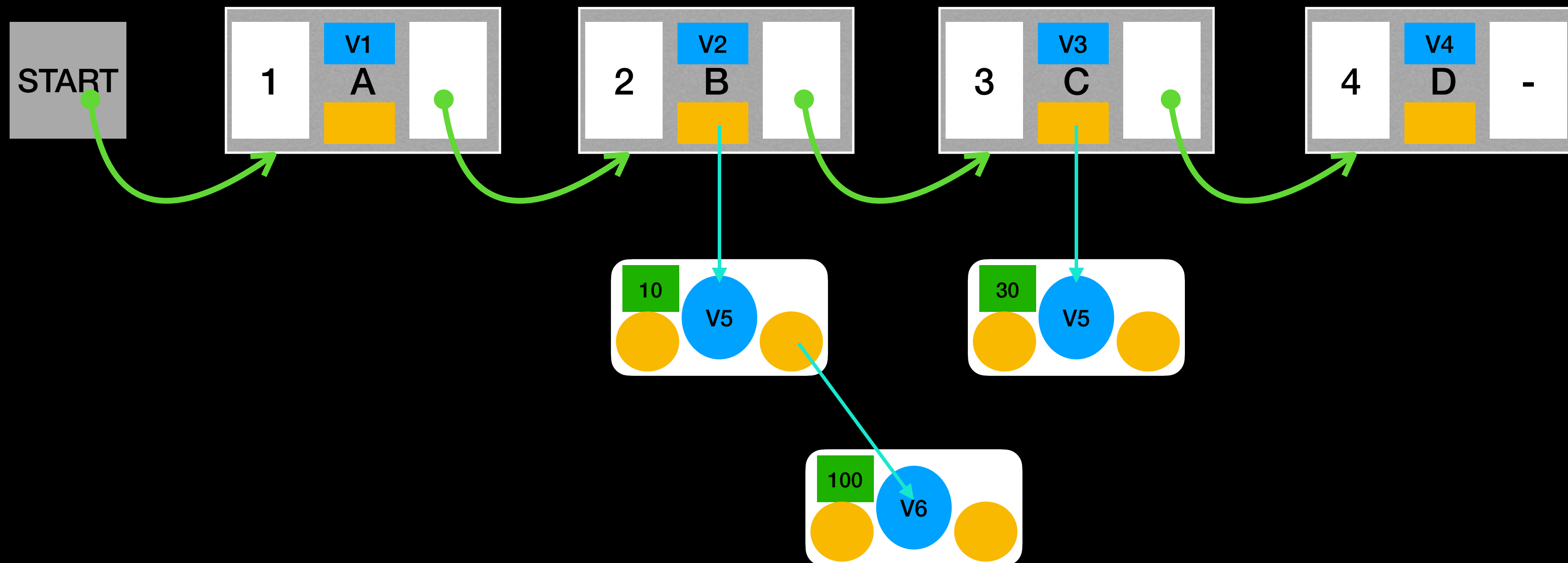
Why such approach is better ? 🥲

Normal Implementation (using balanced BST)



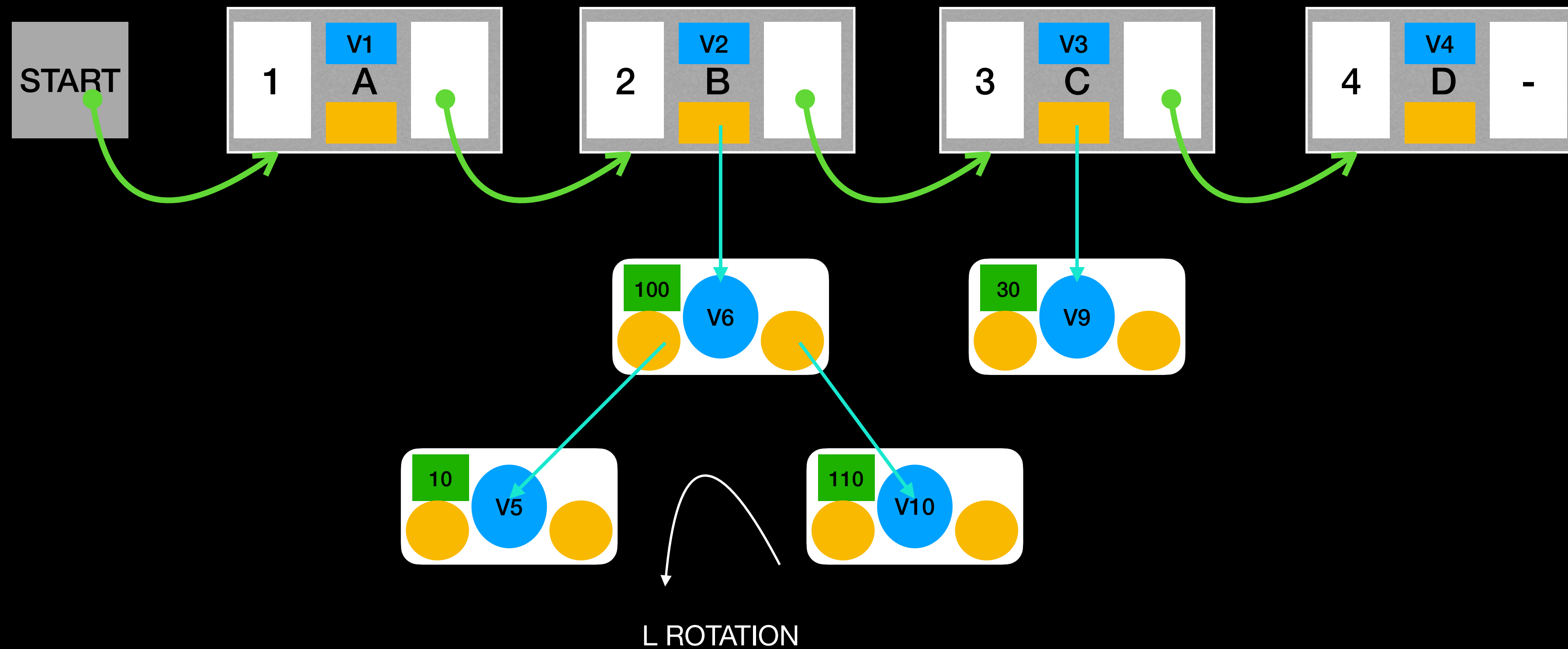
Why such approach is better ? 🥲

Normal Implementation (using balanced BST)



Why such approach is better ? 🥲

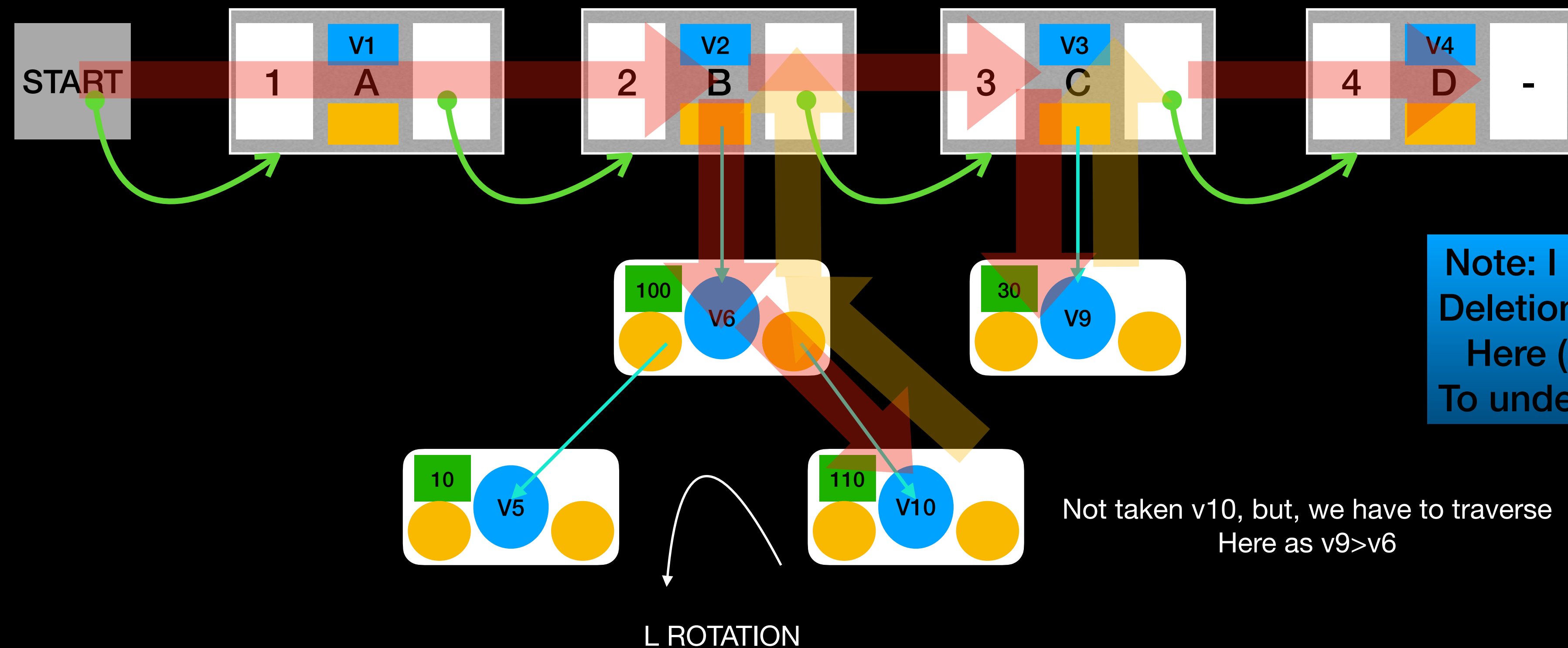
Normal Implementation (using balanced BST)



Why such approach is better ? 🥲

Normal Implementation (using balanced BST) (Reading v9)

Output: 1 -> 100 -> 30 -> 4



Note: I have not considered the Deletion and Insertion operation Here (But, this is not relevant To understand the performance)

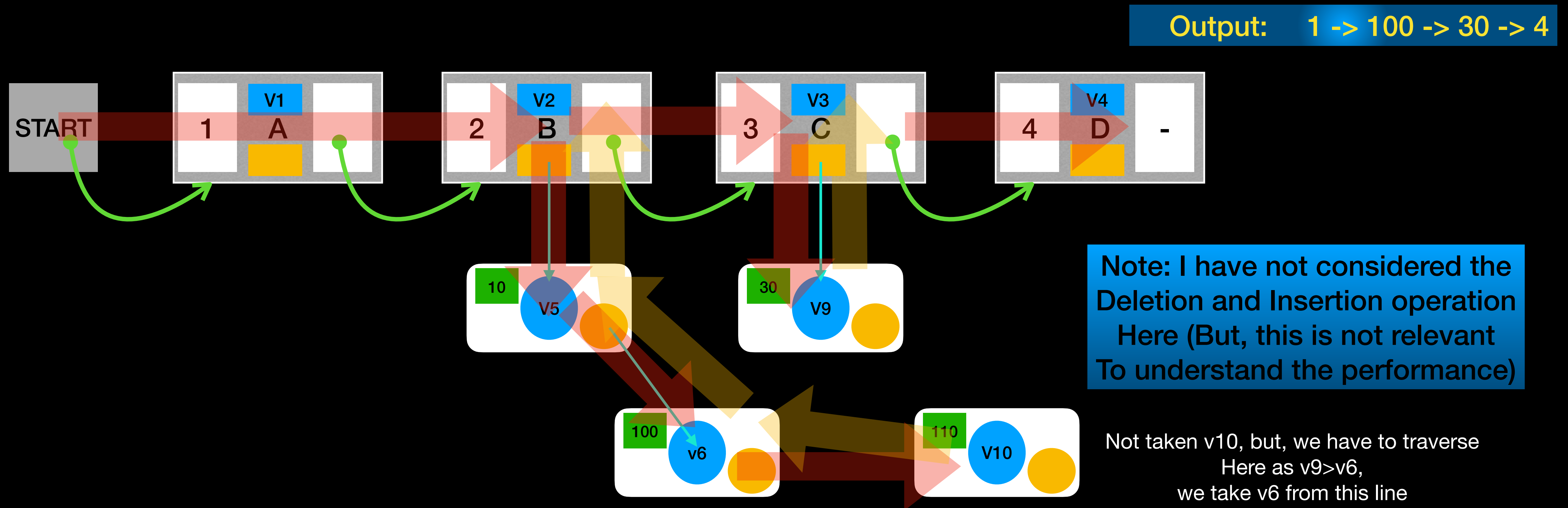
Not taken v10, but, we have to traverse Here as $v9 > v6$

So, the time complexity of reading list v9 ... $n * O(\log m)$
n is length of LL and m is is #mods per node

Also the creation of the AVL of version to node
Takes $O(\log m)$ each time

Why such approach is better ? 🥲

Normal Implementation (Using List) (Reading v9)



So, the time complexity of reading list v9 ... $n * O(m)$
n is length of LL and m is is #mods per node

The creation of the List of version to node
Takes $O(1)$ each time

Why such approach is better ? 🥲

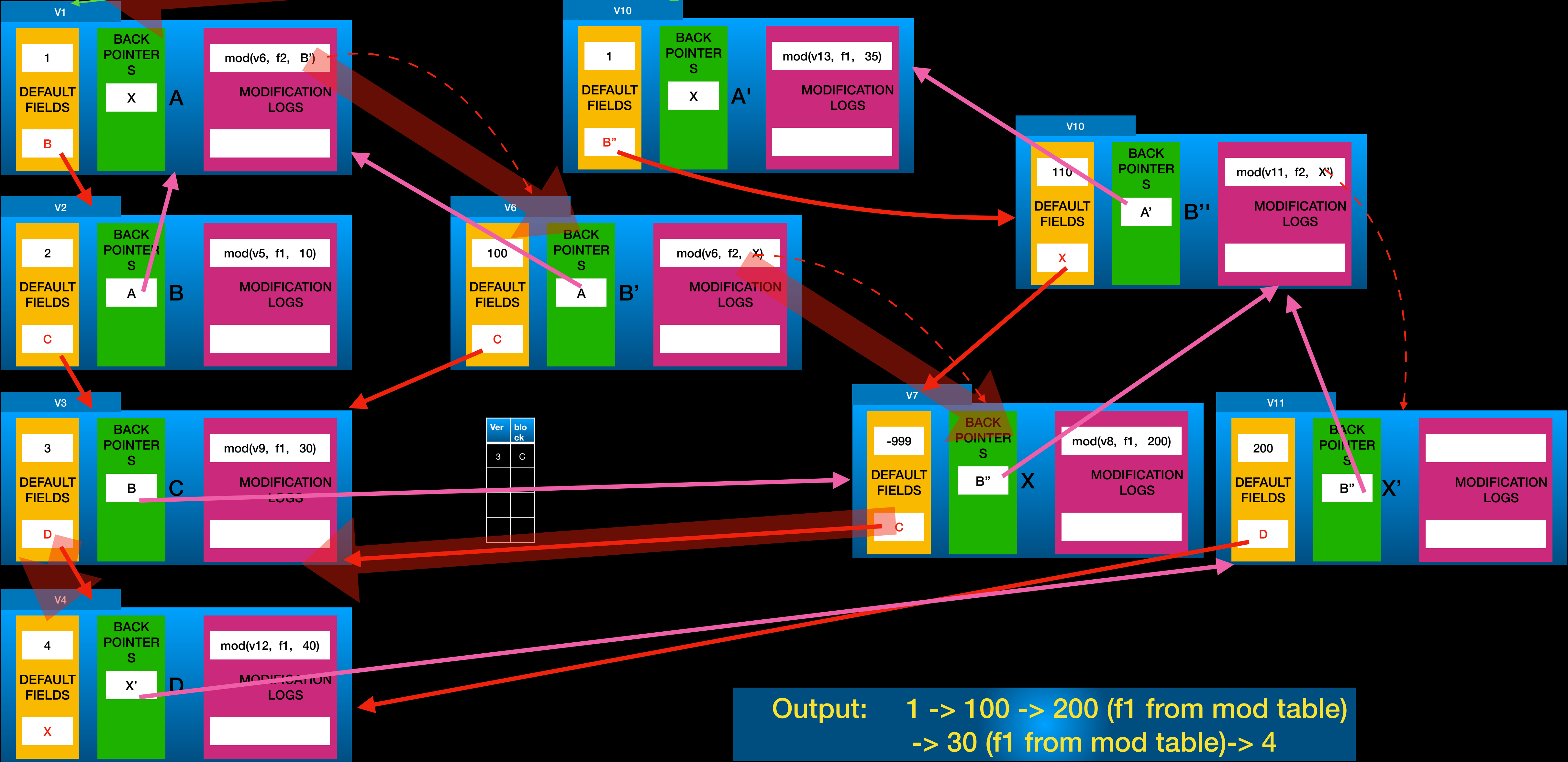
Pointer Machine (Reading v9)

We are taking $O(2k.n) \rightarrow O(n)$ time for iterating over the Linked List
Here k is the in-degree (Here 1) of the DS

We are taking $O(2)$ amortised time for modifying/adding each node to the Linked List
Here k is the in-degree (Here 1) of the DS

START MODULE

V0 V10



VERSION REDIRECTOR MODULE

Ver	block
1	A
10	A''

VERSION REDIRECTOR MODULE

Ver	block
2	B
6	B'
10	B''

VERSION REDIRECTOR MODULE

Ver	block
7	X
11	X'

VERSION REDIRECTOR MODULE

Ver	block
4	D

Output: 1 -> 100 -> 200 (f1 from mod table)
-> 30 (f1 from mod table)-> 4