

# Comparisons

Persistent Array  
Persistent LinkedList  
Persistent Stack  
Persistent Queue  
Persistent Search Tree

We have done BenchMarking using: [GoogleBenchMark Tool](https://github.com/google/benchmark)  
<https://github.com/google/benchmark>

**PROJECT MEMBERS:** ANANNYO DEY, SOUMYAJIT RUDRA SARMA, DEBASMIT ROY, KANKO GHOSH AND KUSHAL DAS

# Persistent Array

# Persistent Array

Time Complexity

Here,  $V$  = Total No. Of Version,  $N$  = Average Length Of The Array

Strategies	Update(index, version, newVal)	RetrieveData(index, version)
Copy_On_Write	$O(\text{length\_of\_array\_at\_version}) \sim O(N)$	$O(1)$
Fat_Node	$O(1)$	Between $O(1)$ and $O(V)$ [Worst Case] $V$ = number of versions $O(1)$ (Average)
Log_log_time method (Deitz, 1989)	$O(\log(\log(\min(n, v))))$ , $n$ = size of array $v$ = number of versions	$O(\log(\log(\min(n, v))))$ , $n$ = size of array $v$ = number of versions

# Persistent Array

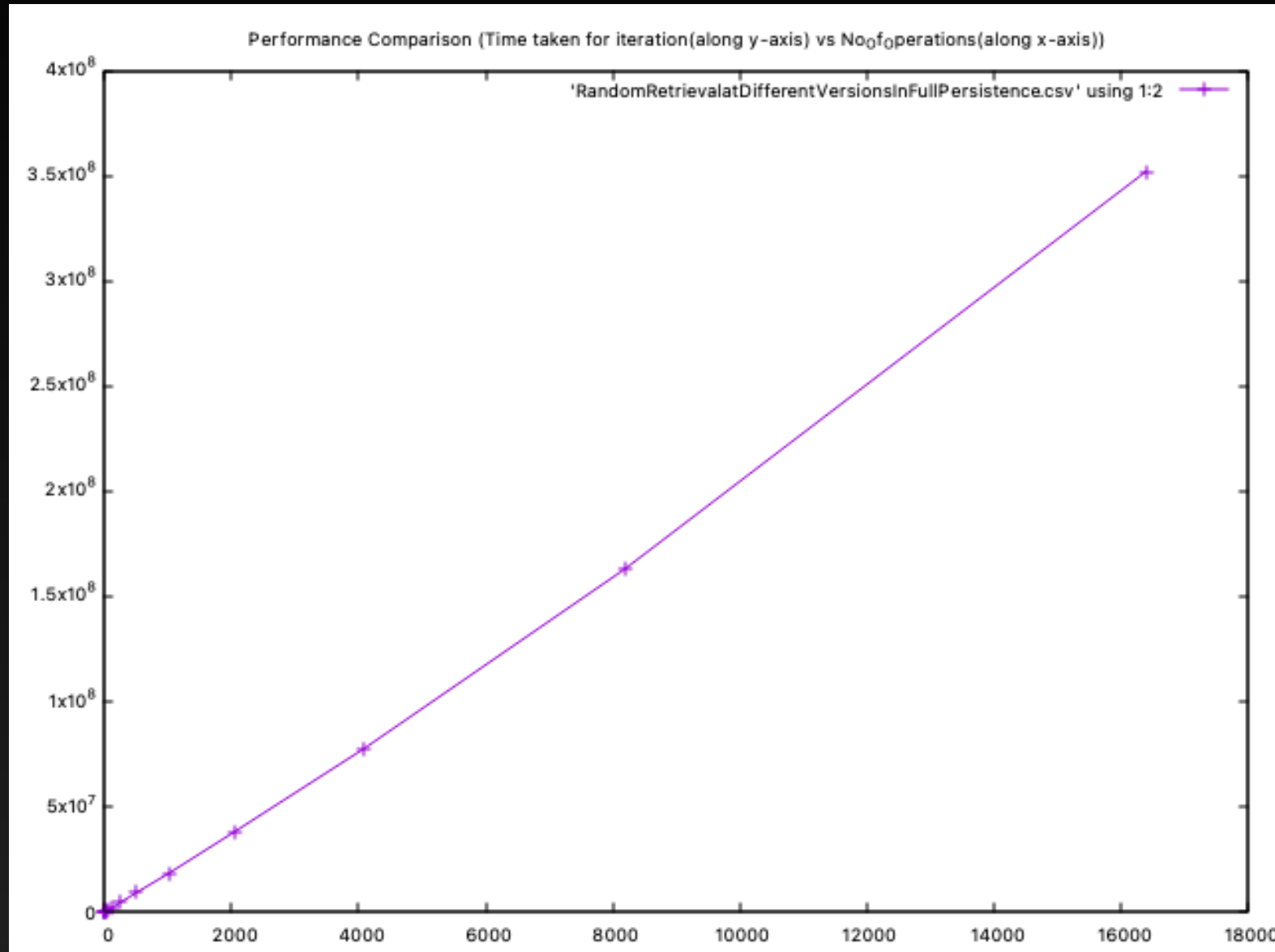
Auxiliary Space

Here,  $V$  = Total No. Of Version,  $N$  = Average Length Of The Array

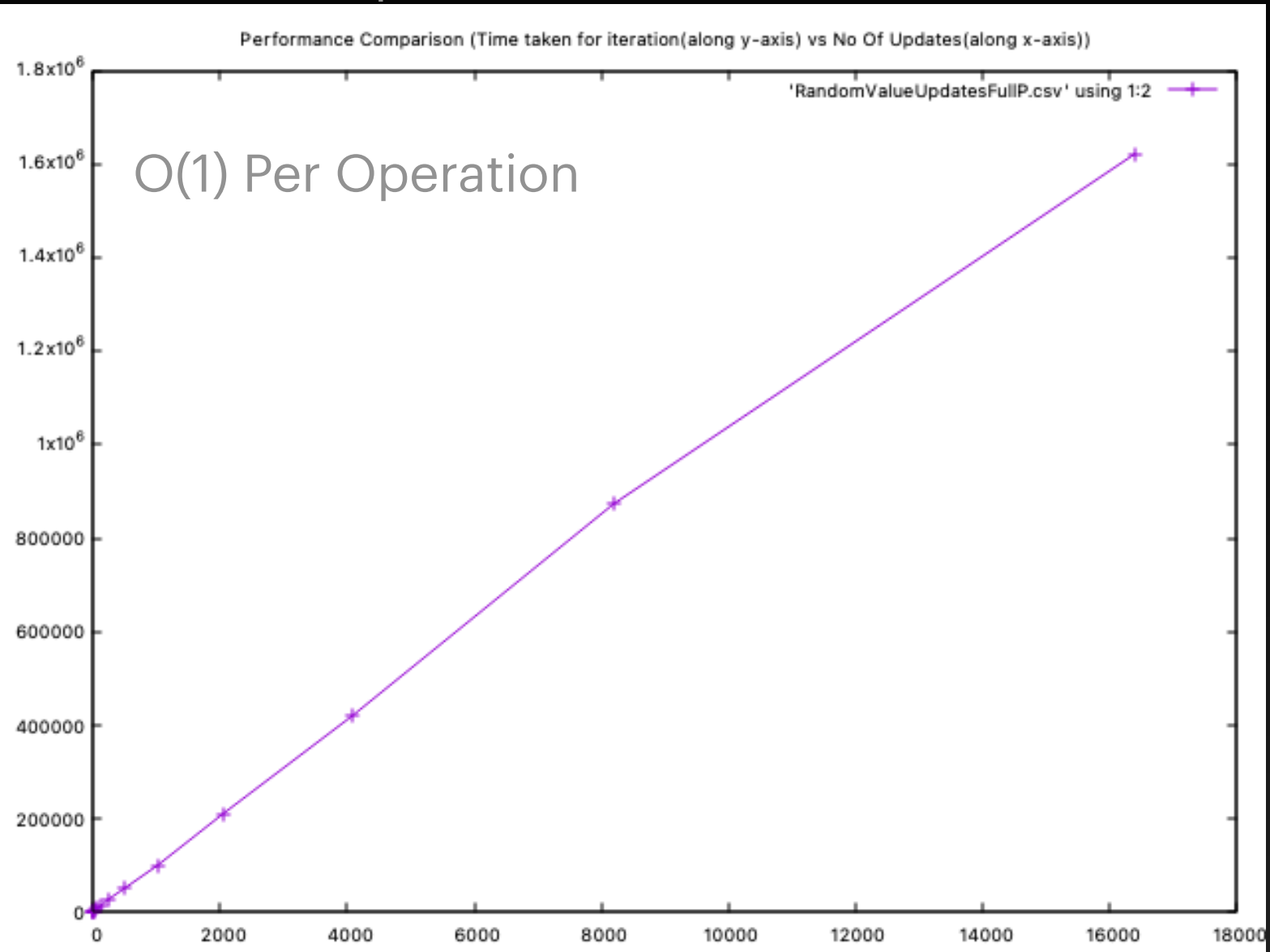
Strategies	Category1	Category2
Copy_On_Write	$\sim O(N^2)$ To Hold All The Copies of Array At Different Versions	$O(V)$ To Hold The Mapping From Version To Array
Fat_Node	$O(N + V)$ $N$ = size of initial array, $V$ = number of version	$O(V)$ To Hold The Mapping From Version To immediate ancestor version
Log_log_time method (Deitz, 1989)	$O(N + V)$ $N$ = size of initial array, $V$ = number of version	$O(V)$ to arrange the versions in form of list order maintenance tree

# Benchmarking Of FatNode Model

RandomRetrievalatDifferentVersionsinFullPersistence

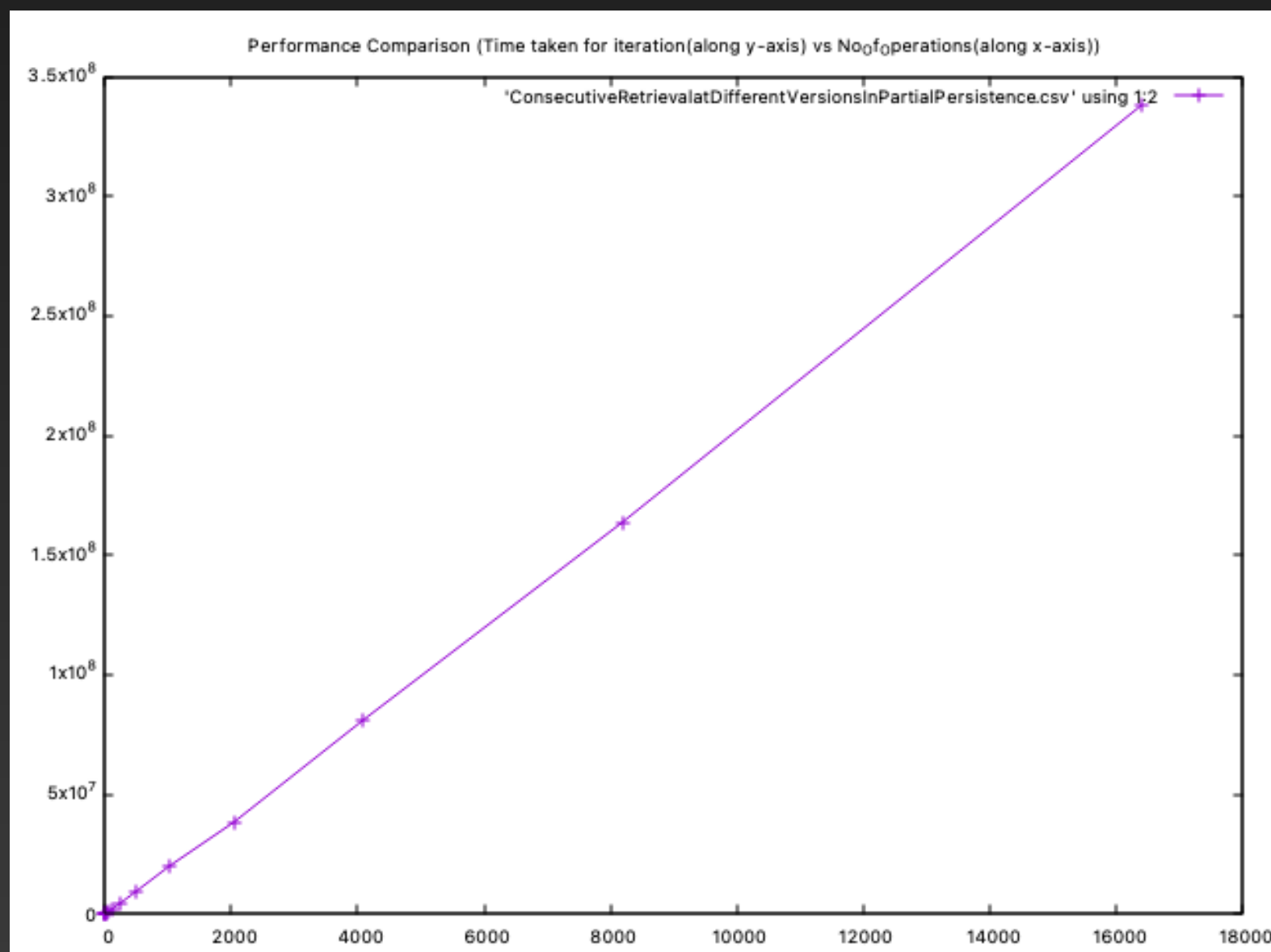


RandomValueUpdatesFullPersistence

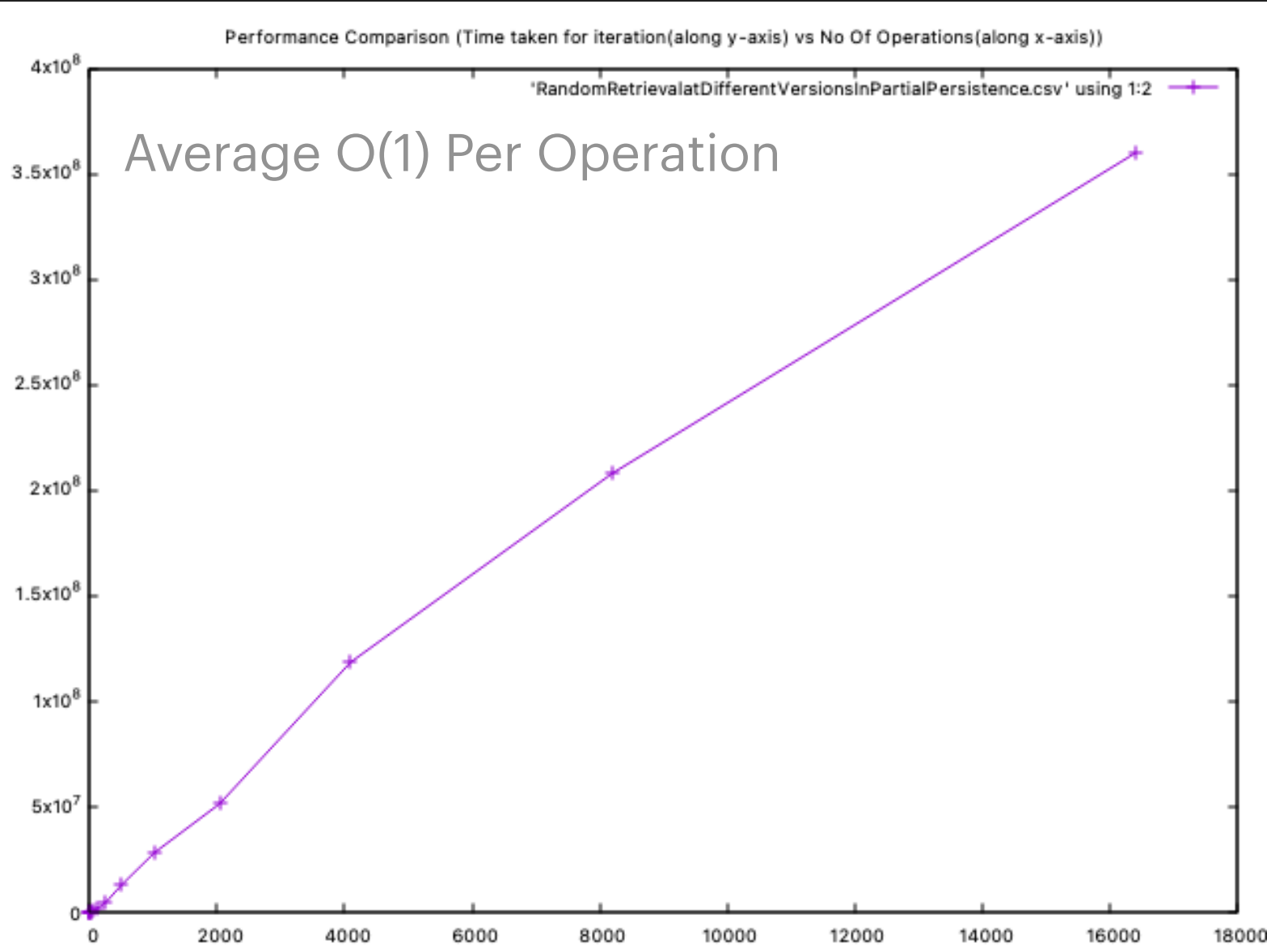


Average O(1) Per Operation

ConsecutiveRetrievalatDifferentVersionsinPartialPersistence



RandomRetrievalatDifferentVersionsinPartialPersistence



# Partial Persistent Linked List

# Partial Persistent Linked List

Time Complexity

Here,  $V$  = Total No. Of Version,  $N$  = Average Length Of The Linked List Considering All The Versions,  
 $m$  = no. of modifications in a particular position

Strategies	InsertAfter(position) #	DeleteAfter(position) #	UpdateData (position) #	RetrieveData (position,version)	traverseWholeLL atVer(version)
Fat Node	$O(1)$	$O(1)$	$O(1)$	$O(\log_x m)$ [If the versions are stored in a Balanced $x\_array$ Tree/ Trie] $O(1)$ [Amortised]	$O(N * \log_x m)$
Path Copying	$O(1)$ for at extreme position $O(N)$ In Average	$O(1)$ for at extreme position $O(N)$ for at Rear	$O(1)$ for at extreme position $O(N)$ for at Rear	$O(N)$	$O(N)$
Pointer Machine	$O(1)$ [Amortised]	$O(1)$ [Amortised]	$O(1)$ [Amortised]	$O(N)$	$O(N)$
Ephemeral Linked List	$O(1)$	$O(1)$	$O(1)$	$O(N)$ [Only Current Version Supported]	$O(N)$ [Only Current Version Supported]

# time to reach the node at that position is not considered

# Partial Persistent Linked List

Auxiliary Space

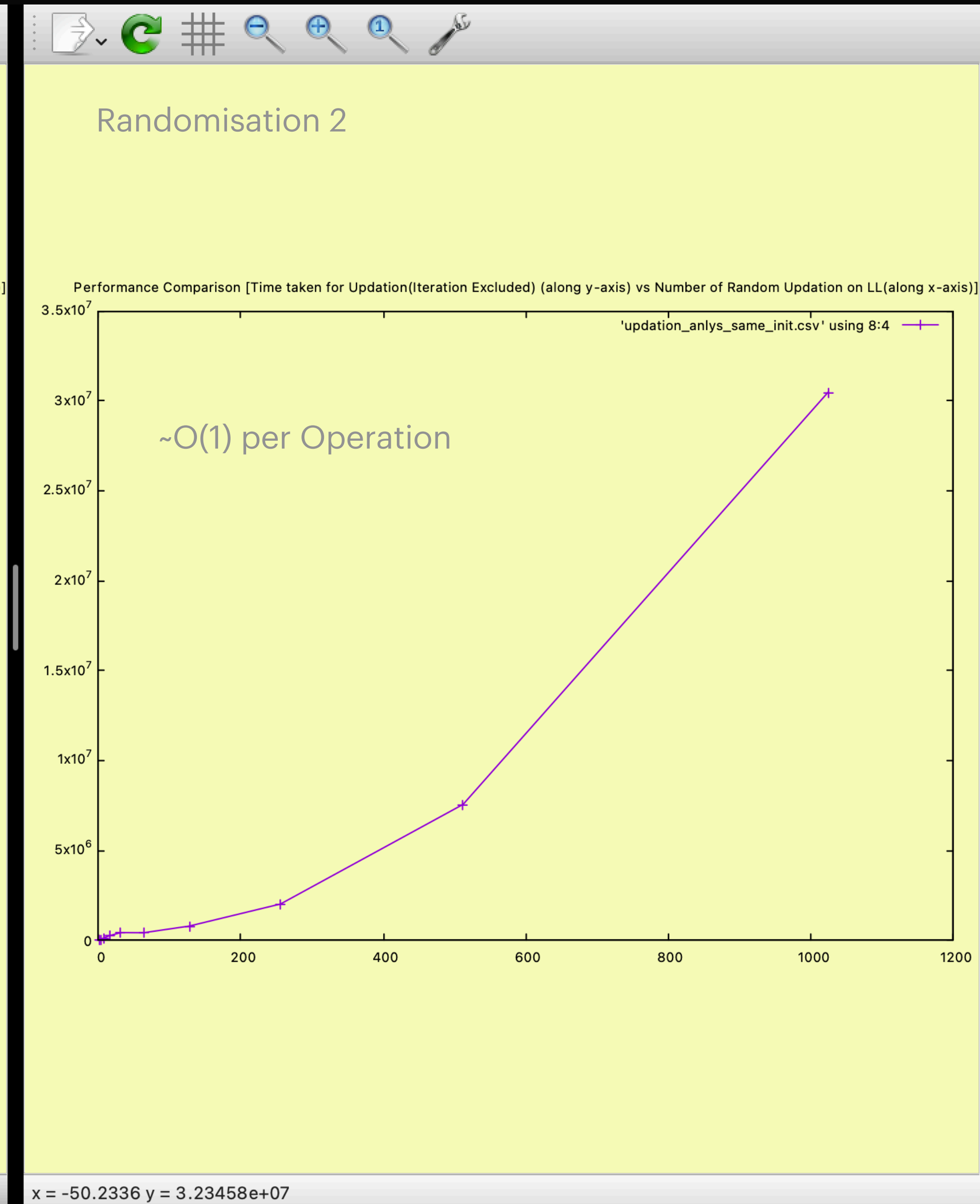
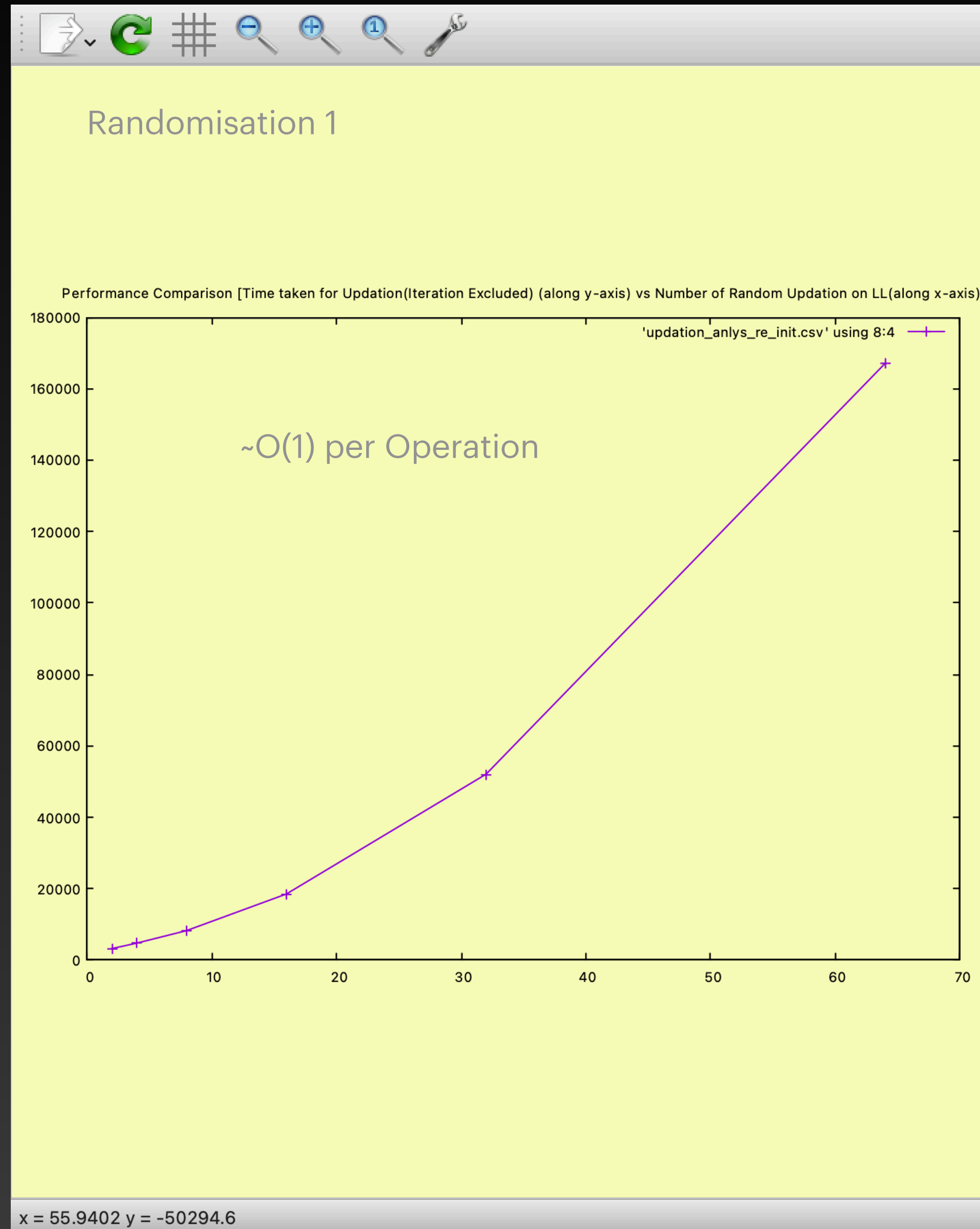
Here,  $V$  = Total No. Of Version,  $N$  = Average Length Of The Linked List Considering All The Versions

Strategies	Category 1	Category 2
Fat Node	Node Size: # ~ 20byte	$O(N + V)$ to Hold The LinkedList
Path Copying	Node Size: # ~ 12 byte	Best Case: $O(N) + O(V)$ Worst Case: $O(N^2) + O(V)$ to Hold The Tree and Staring Pointers
Pointer Machine	Node Size: # ~ 140 byte	$O(V)$ Amortised to Hold The LinkedList
Ephemeral Linked List	Node Size: # ~ 12 byte	$O(N)$ to Hold The LinkedList

# to store 4 byte Integer | 8 Bye pointers



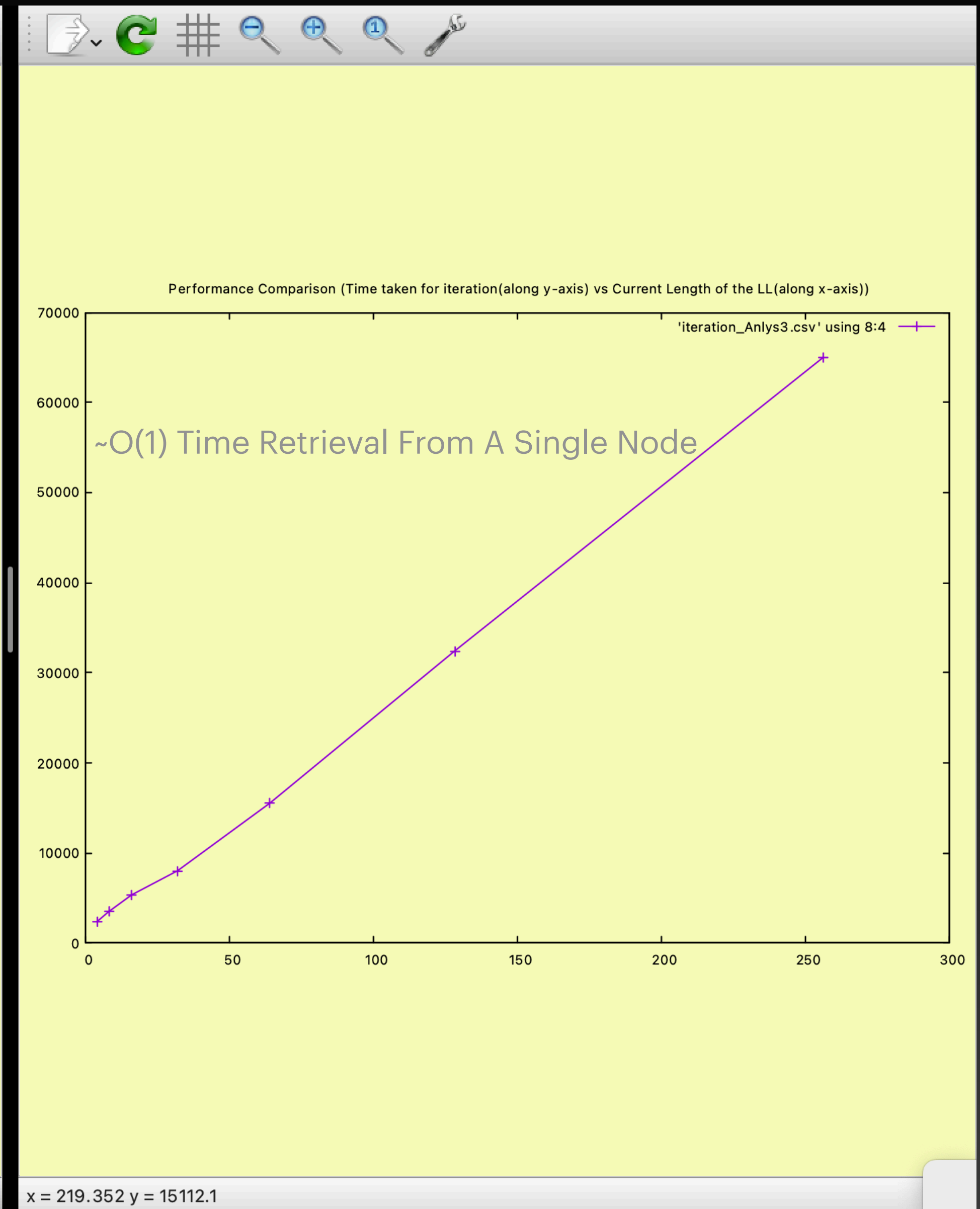
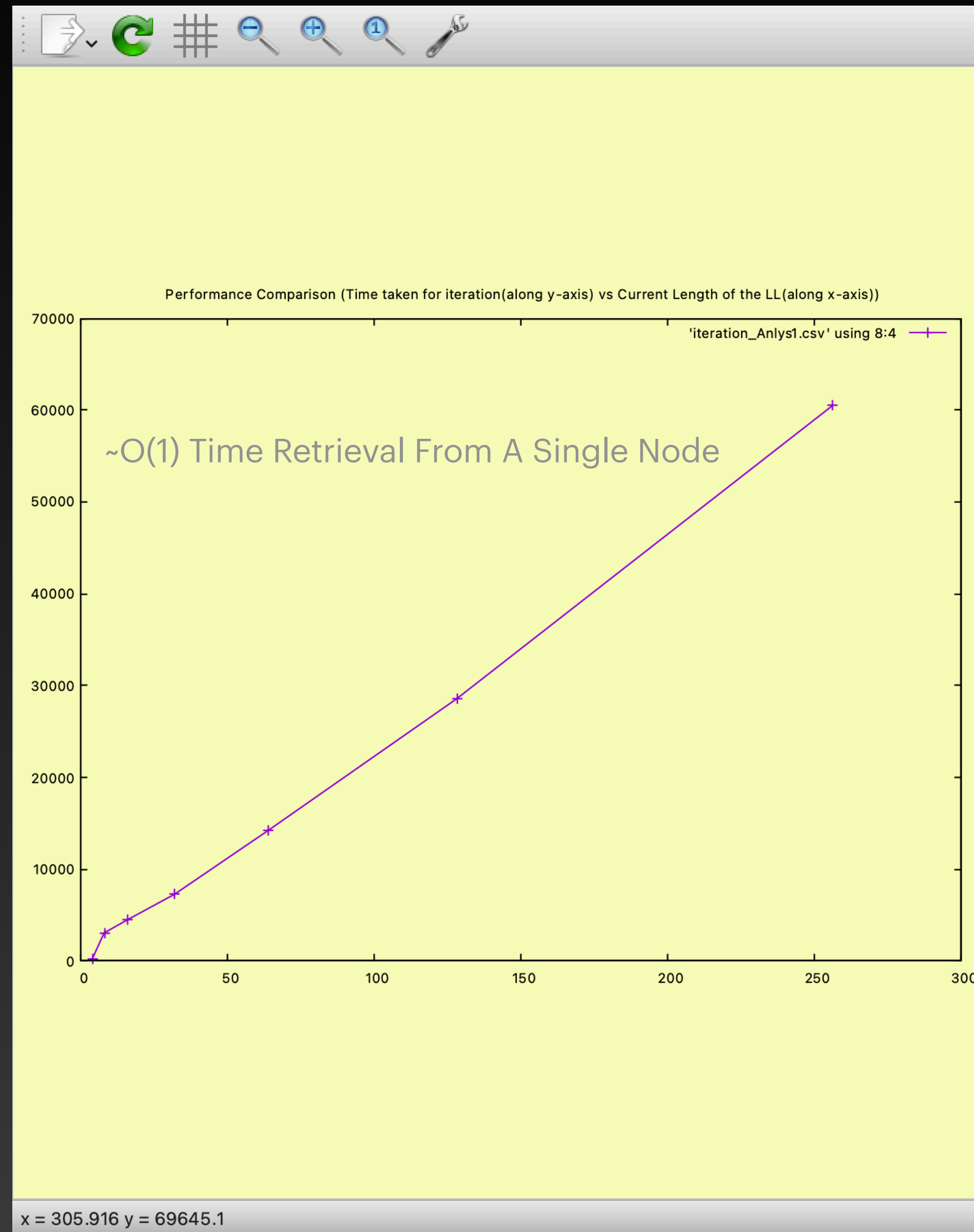
# Benchmarking Of Pointer Machine Model



Updation  
(Value/  
Insertion/ Deletion)  
Randomised

# Benchmarking Of Pointer Machine Model

Iteration Through  
Whole LL  
At  
Randomised Versions



# Full Persistent Linked List

# Full Persistent Linked List

Time Complexity

Here, V = Total No. Of Version, N = Average Length Of The Linked List Considering All The Versions

Strategies	InsertAfter(position,ver) #	DeleteAfter(position,ver) #	UpdateData (position,ver) #	RetrieveData (position,version)	traverseWholeLL atVer(version)
Fat Node Pointer Machine [With List Maintenance]	O(1) [Amortized] + O(1) [Amortized] [amortised O(1) due for look up and insertions at version tree]	O(1) [Amortized] + O(1) [Amortized] [amortised O(1) due for look up and insertions at version tree]	O(1) [Amortized] + O(1) [Amortized] [amortised O(1) due for look up and insertions at version tree]	O(N) + O(1) [Amortized] [additional amortised O(1) due for look up at version tree]	O(N) * O(1) [additional amortised O(1) due for look up / insertions at version tree]
Path Copying	O(1) for at front O(N) for at Rear	O(1) for at front O(N) for at Rear	O(1) for at front O(N) for at Rear	O(N)	O(N)
Pointer Machine [With List Maintenance]	O(1) [Amortised]	O(1) [Amortised]	O(1) [Amortised]	O(N)	O(N)
Ephemeral Linked List	O(1) [Only Current Version Supported]	O(1) [Only Current Version Supported]	O(1) [Only Current Version Supported]	O(N) [Only Current Version Supported]	O(N) [Only Current Version Supported]

# Full Persistent Linked List

Auxiliary Space

Here,  $V$  = Total No. Of Version,  $N$  = Average Length Of The Linked List Considering All The Versions

Strategies	Category 1	Category 2	Category 3
Fat Node Pointer Machine [With List Maintenance]	Node Size: # ~ 20byte	$O(N + V)$ to Hold The LinkedList	$O(V)$ to hold the Version Tree Here A ScapeGoat Tree
Path Copying	Node Size: # ~ 12 byte	Best Case: $O(N) + O(V)$ Worst Case: $O(N^2) + O(V)$ to Hold The Tree and Staring Pointers	-
Pointer Machine [With List Maintenance]	Node Size: # ~ 200 byte	$O(V)$ [Amortised ] to Hold The LinkedList	$O(V)$ to hold the Version Tree Here A ScapeGoat Tree
Ephemeral Linked List	Node Size: # ~ 12 byte	$O(N)$ to Hold The LinkedList	-

# to store 4 byte Integer | 8 Bye pointers

# Persistent Stack

# Persistent Stack

Model Developed By Our Team

Time Complexity

Here, V = Total No. Of Version, N = Average Length Of The Stack Considering All The Versions

Strategies	push(data, version)	pop(version)	getTop(ver)
Using PPL with PM Model	O(1)	O(1)	O(1)
DAG Model	O(1)	O(1)	O(1)
Ephemeral std::stack (C++)	O(1) [Only Current Version Supported]	O(1) [Only Current Version Supported]	O(1) [Only Current Version Supported]

# Persistent Stack

Model Developed By Our Team

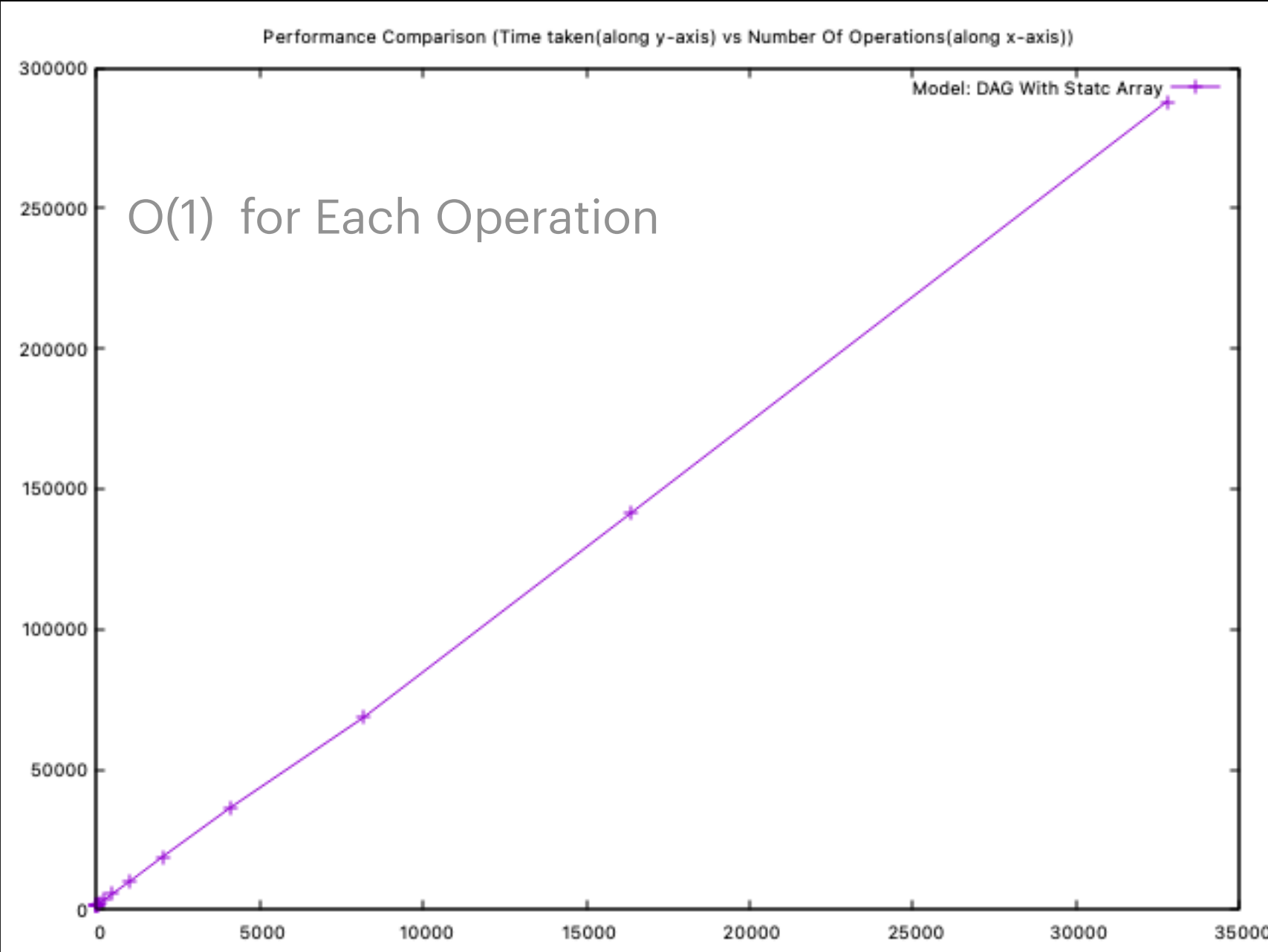
Auxiliary Space

Here, V = Total No. Of Version, N = Average Length Of The Stack Considering All The Versions

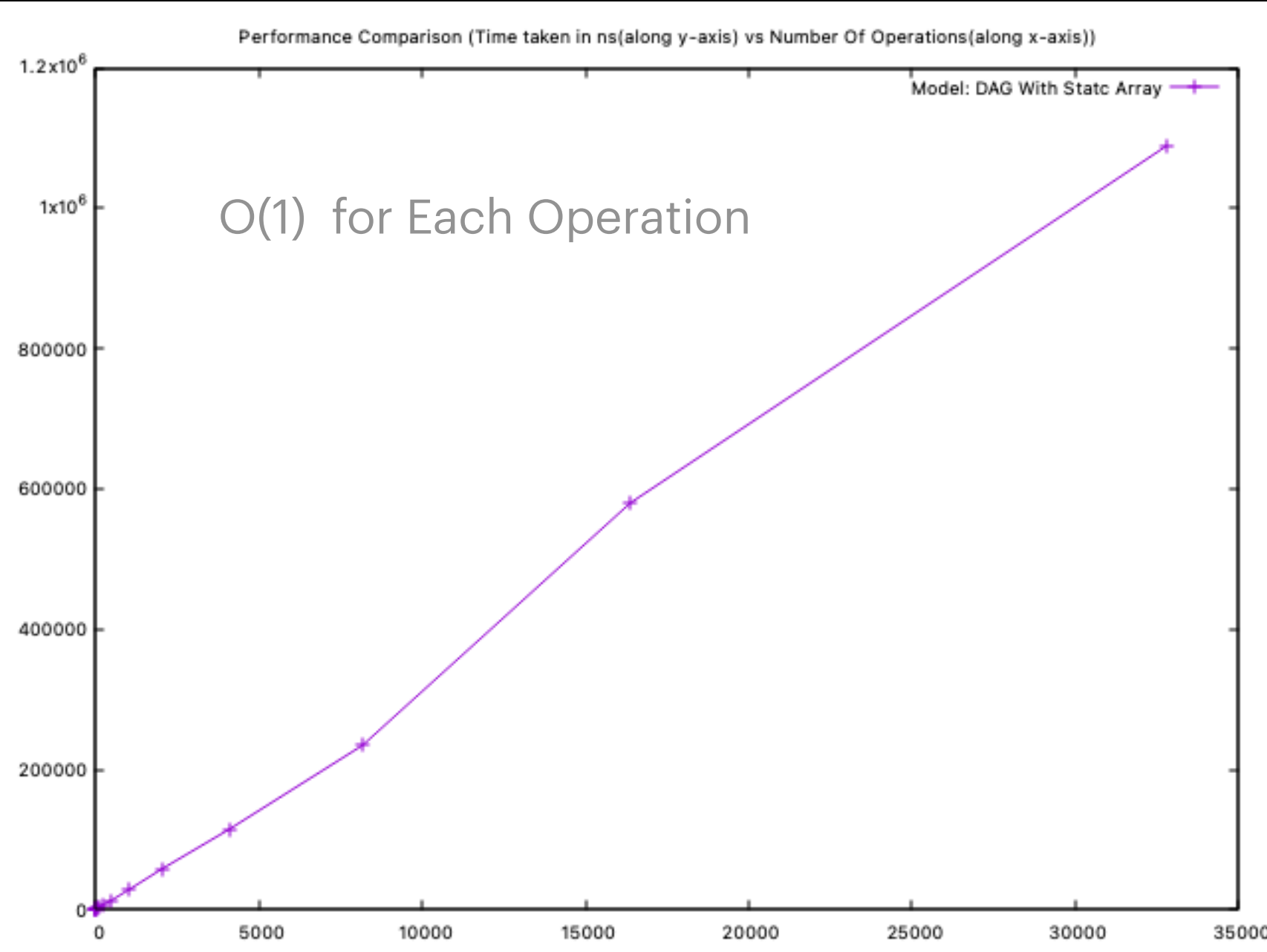
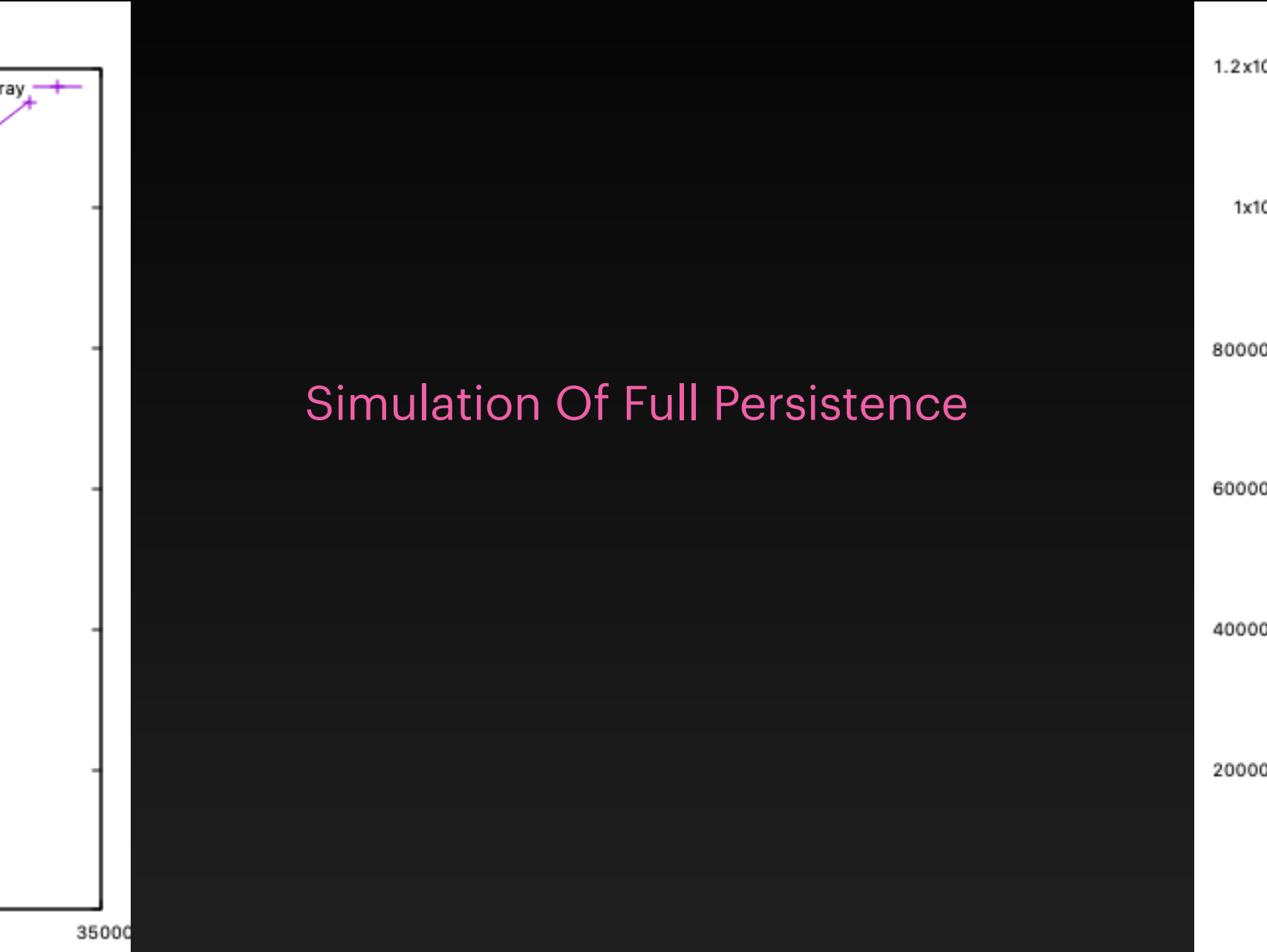
Strategies	Category 1	Category 2	Category 3
Using PPL with PM Model	$O(V)$ [Amortized] To Hold The Linked List		
DAG Model	$O(V)$ To Hold The MAP/DAG		
Ephemeral std::stack (C++)	$O(N)$	-	-



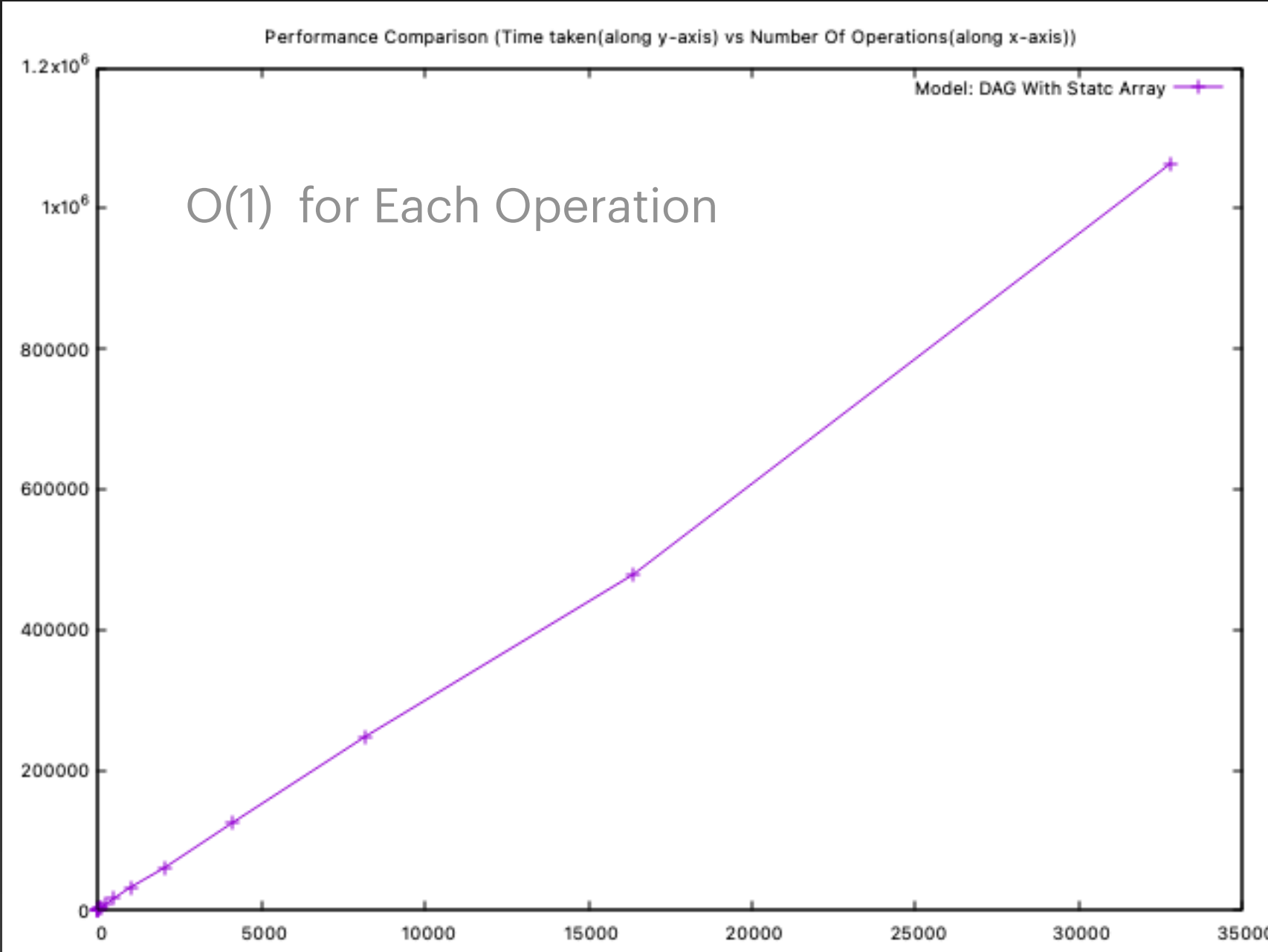
# Benchmarking Of DAG Model



OnlyPush In Randomised Versions

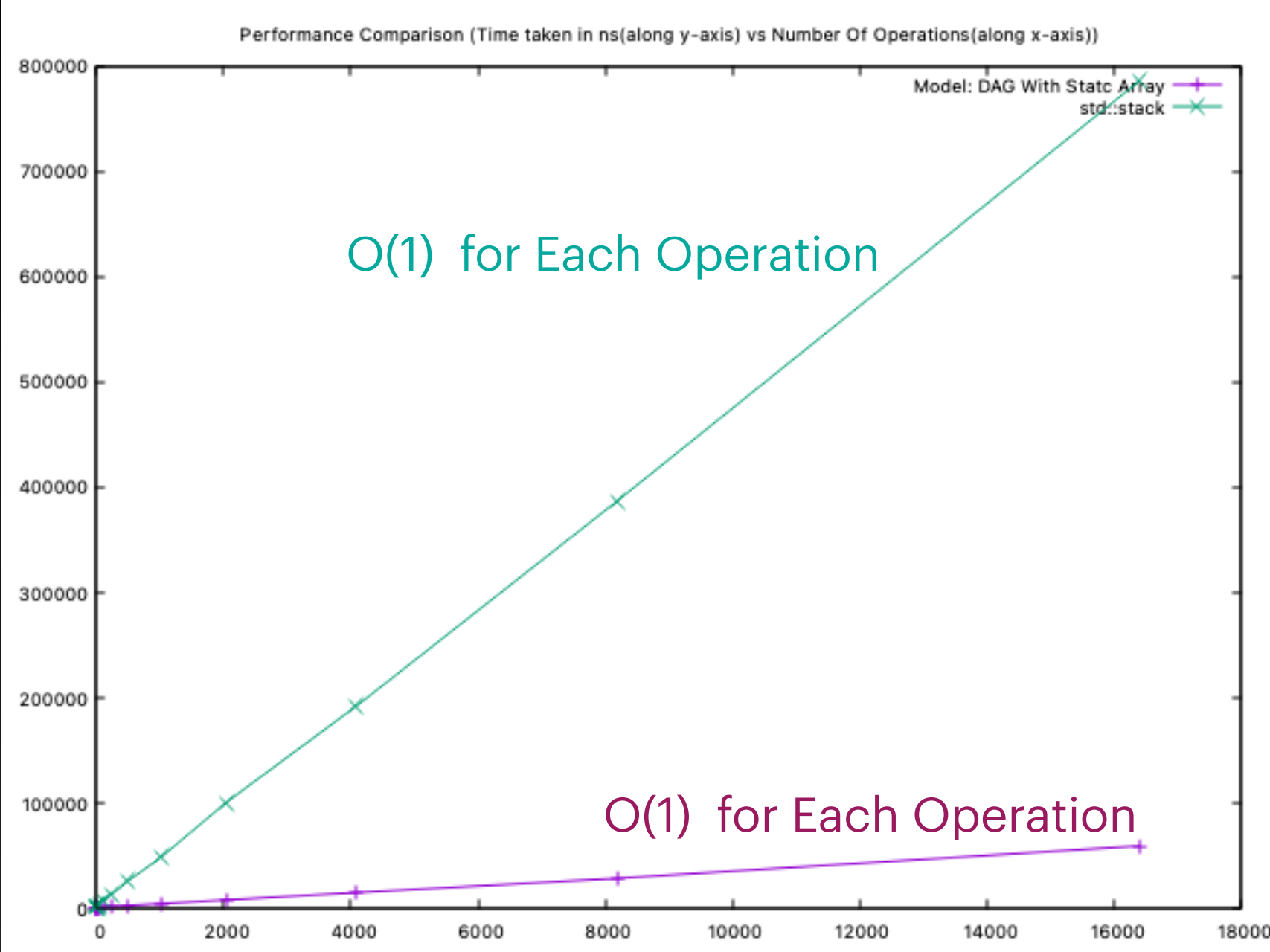


Randomised Push/Pop In Randomised Versions



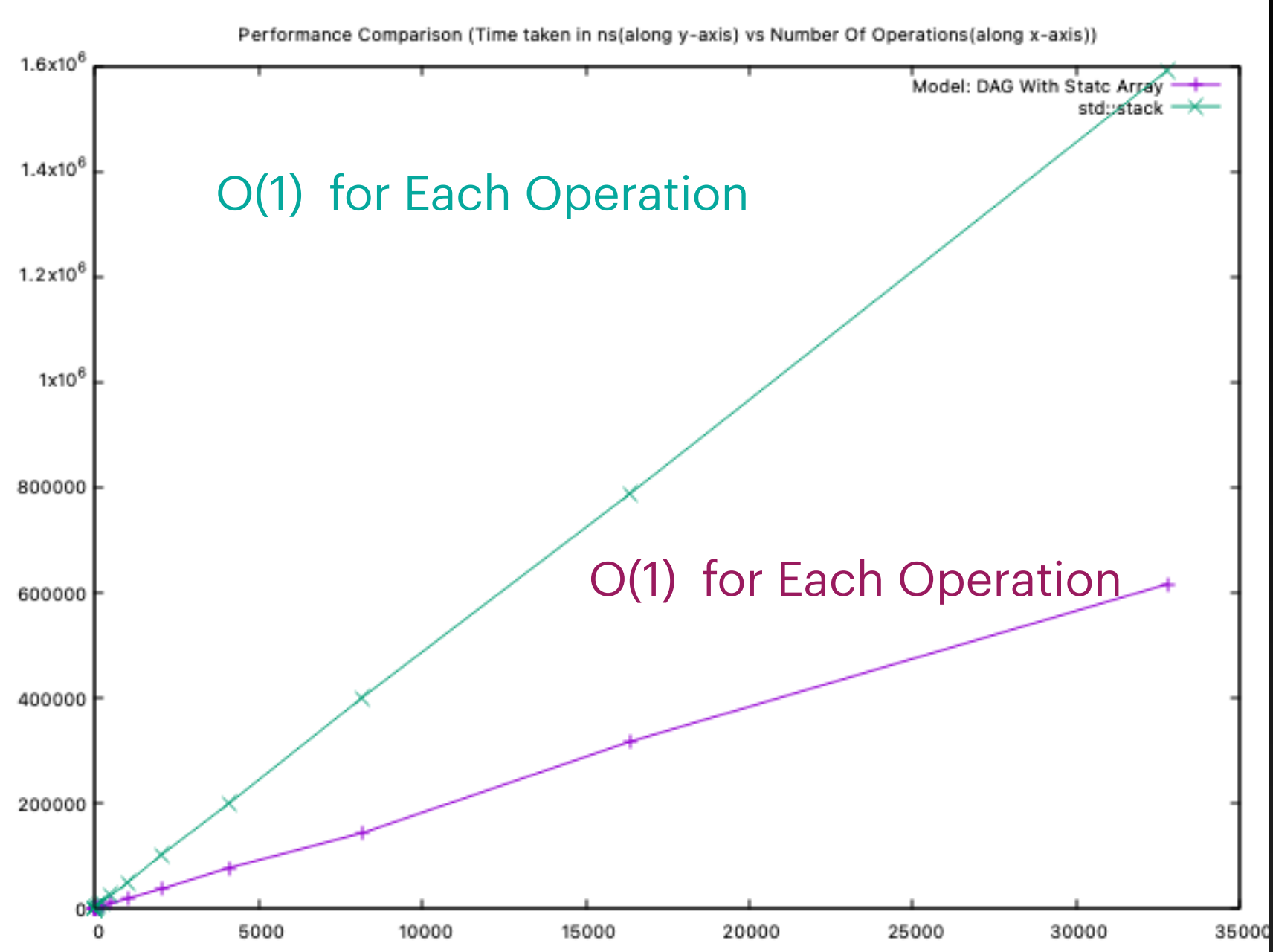
OnlyPop In Randomised Versions  
(Pushing Operation is Not Time Profiled)

# Benchmarking: DAG Model Vs std::stack

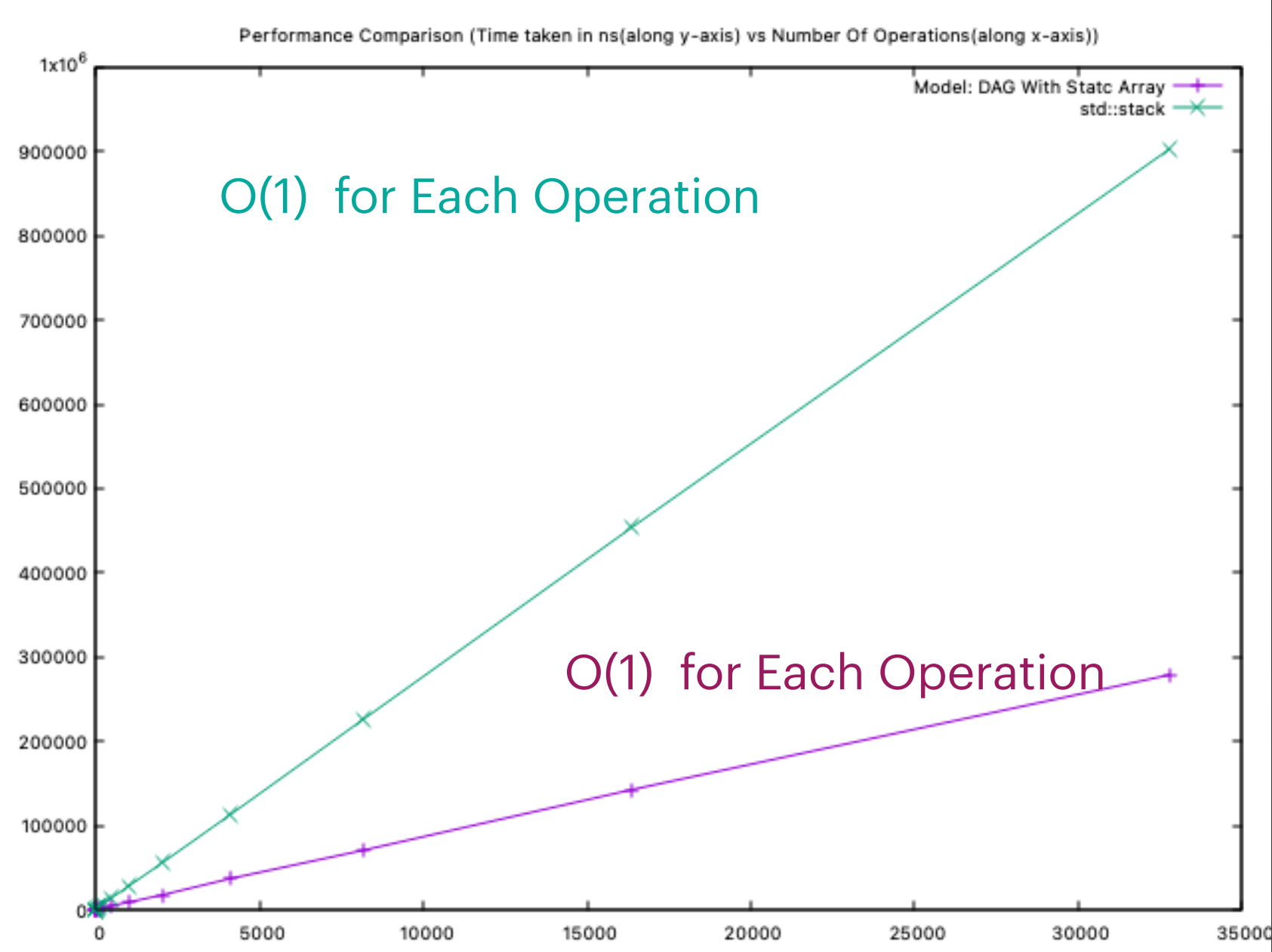


OnlyPush In latest Versions

Simulation Of Partial Persistence



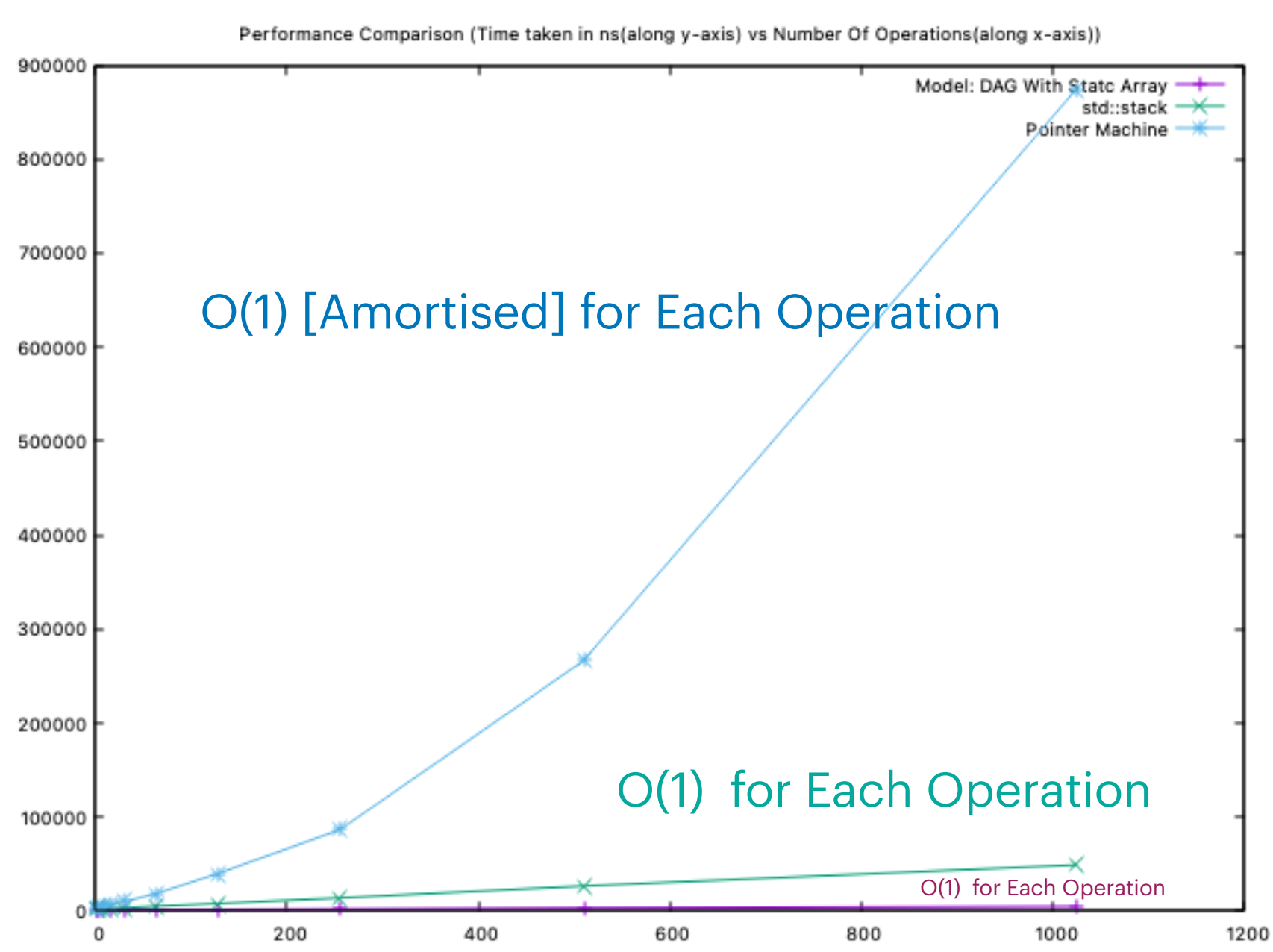
Randomised Push/Pop In Latest Versions



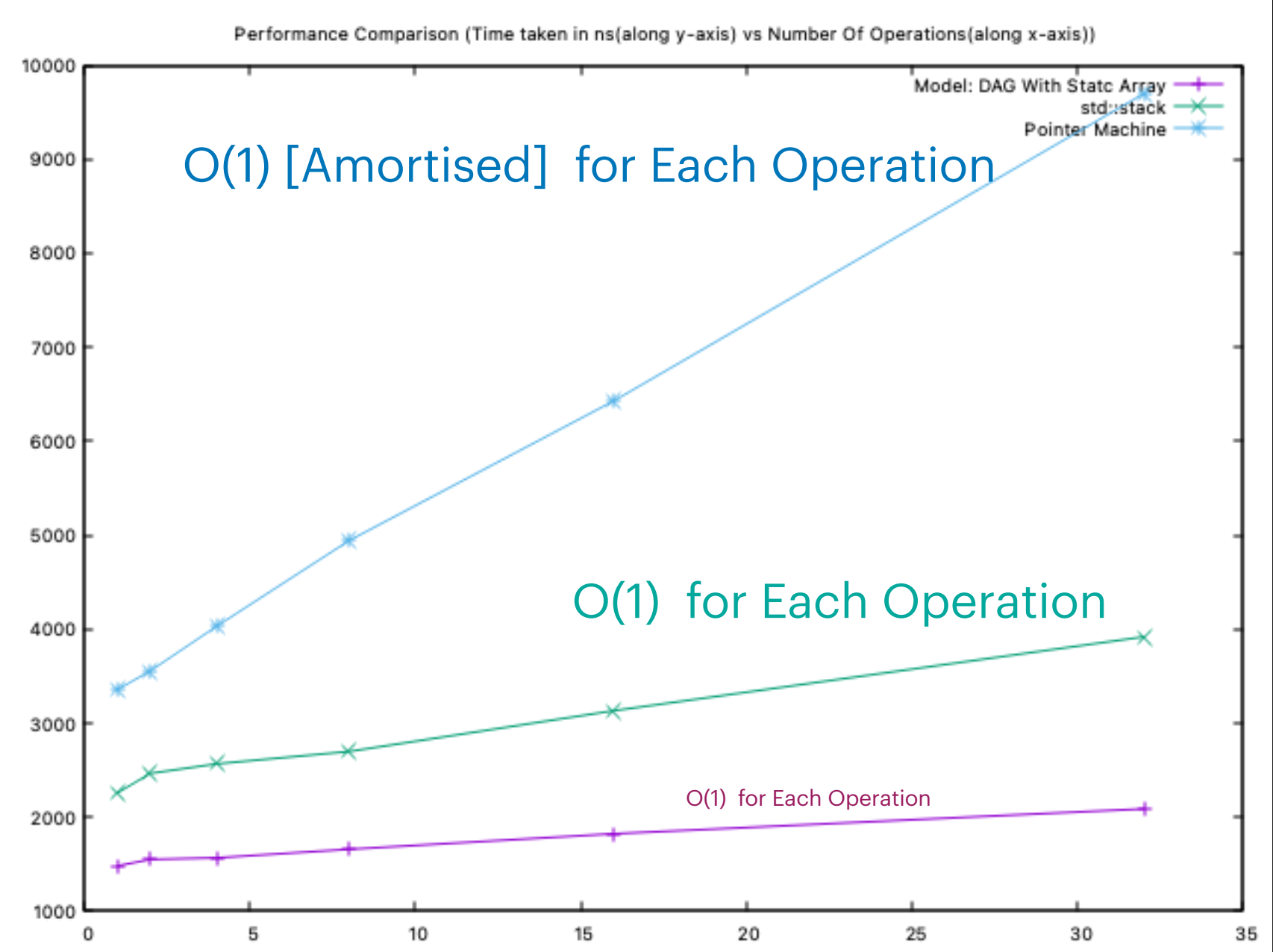
OnlyPop In Latest Versions  
(Pushing Operation is Not Time Profiled)

NOTE: We Used Static Array To Handle  
In DAG Model, So, It Is Slightly Faster Than  
Std::Stack

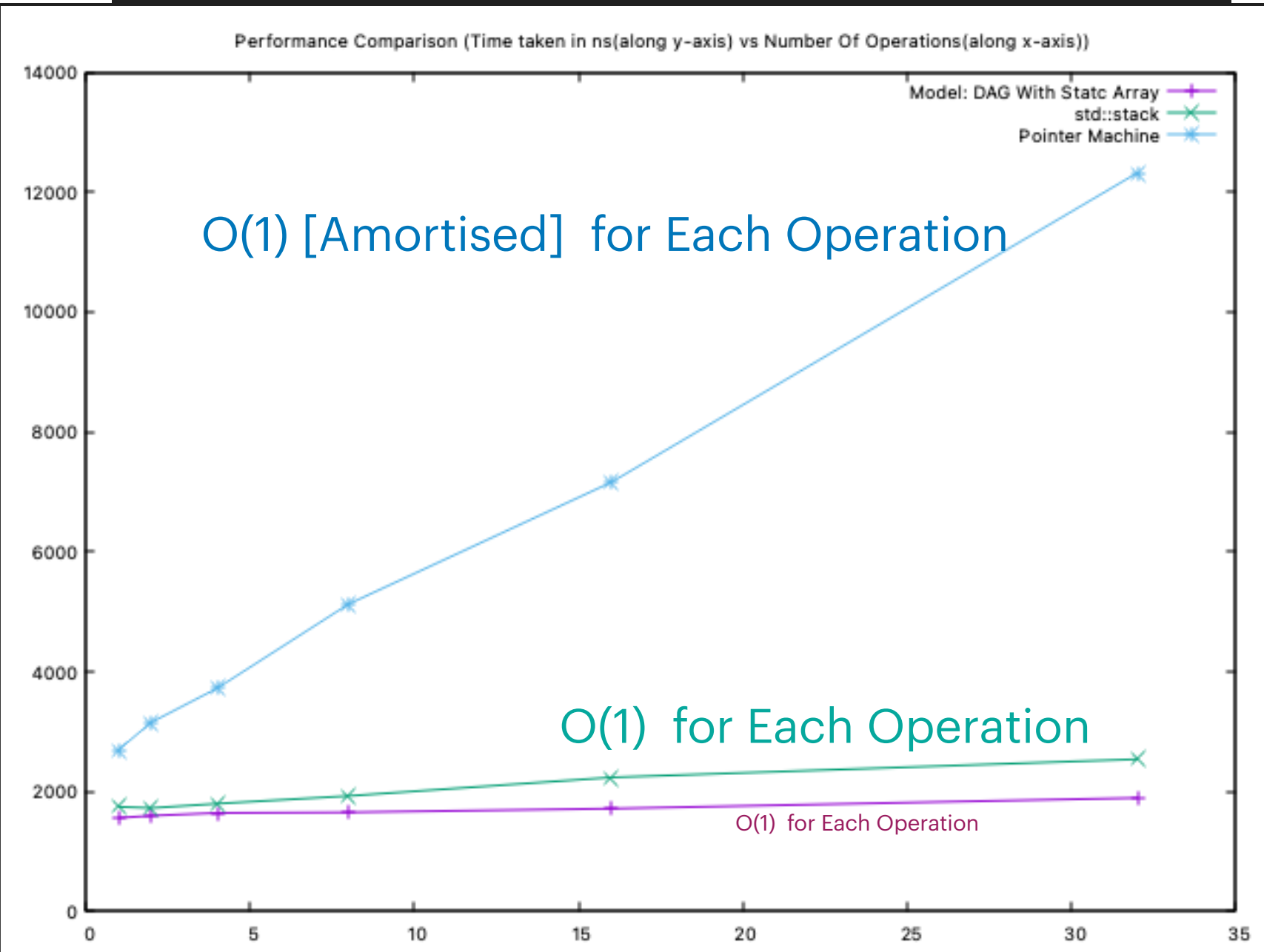
# Benchmarking: DAG Model Vs std::stack Vs Stack\_Using\_PM\_PPL



Simulation Of Partial Persistence



OnlyPush In Latest Versions



Randomised Push/Pop In Latest Versions

OnlyPop In Latest Versions  
(Pushing Operation is Not Time Profiled)

NOTE: We Used Static Array To Handle  
In DAG Model, So, It Is Slightly Faster Than  
Std::Stack

# Persistent Queue

# Persistent Queue

Model Developed By Our Team

Time Complexity

Here, V = Total No. Of Version, N = Average Length Of The Queue Considering All The Versions

Strategies	enqueue(data, version)	dequeue(version)	getFront(ver)	getRear(rear)
Using PPL with PM Model	O(1)	O(1) [Amortized]	O(1)	O(1)
Reduced Sparse Matrix Model	O(log v) [Here v is current version]  O((log V!)/V) ~ O(1) [Amortized]	O(log v) [In Average Case ]  O(v * log v) [In Worst Case ]	O(1) [In Average Case ]  O(log v) [In Worst Case ]	O(1)
Ephemeral std::queue (C++)	O(1) [Only Current Version Supported]	O(1) [Only Current Version Supported]	O(1) [Only Current Version Supported]	O(1) [Only Current Version Supported]

# Persistent Queue

Model Developed By Our Team

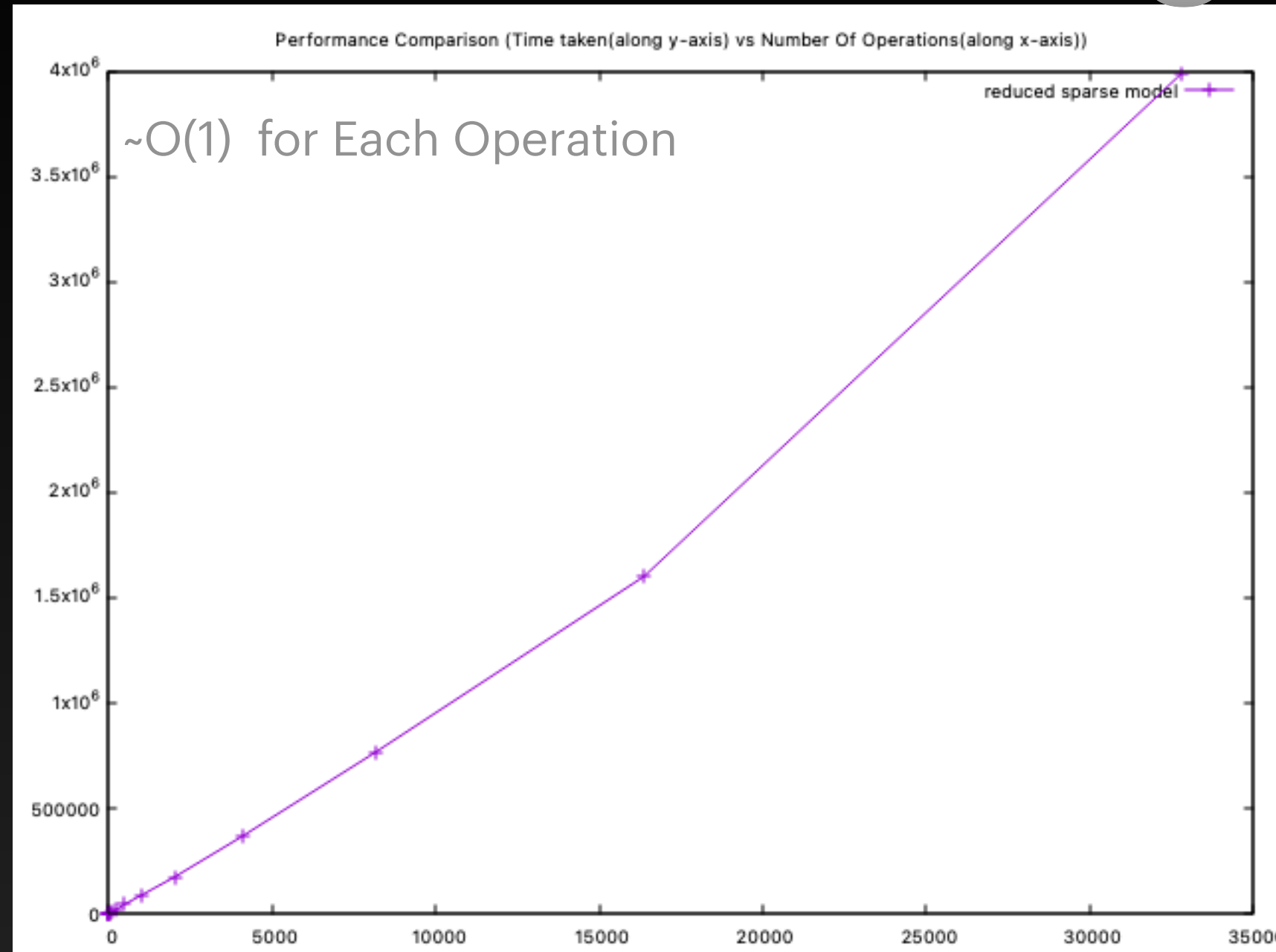
Auxiliary Space

Here, V = Total No. Of Version, N = Average Length Of The Queue Considering All The Versions

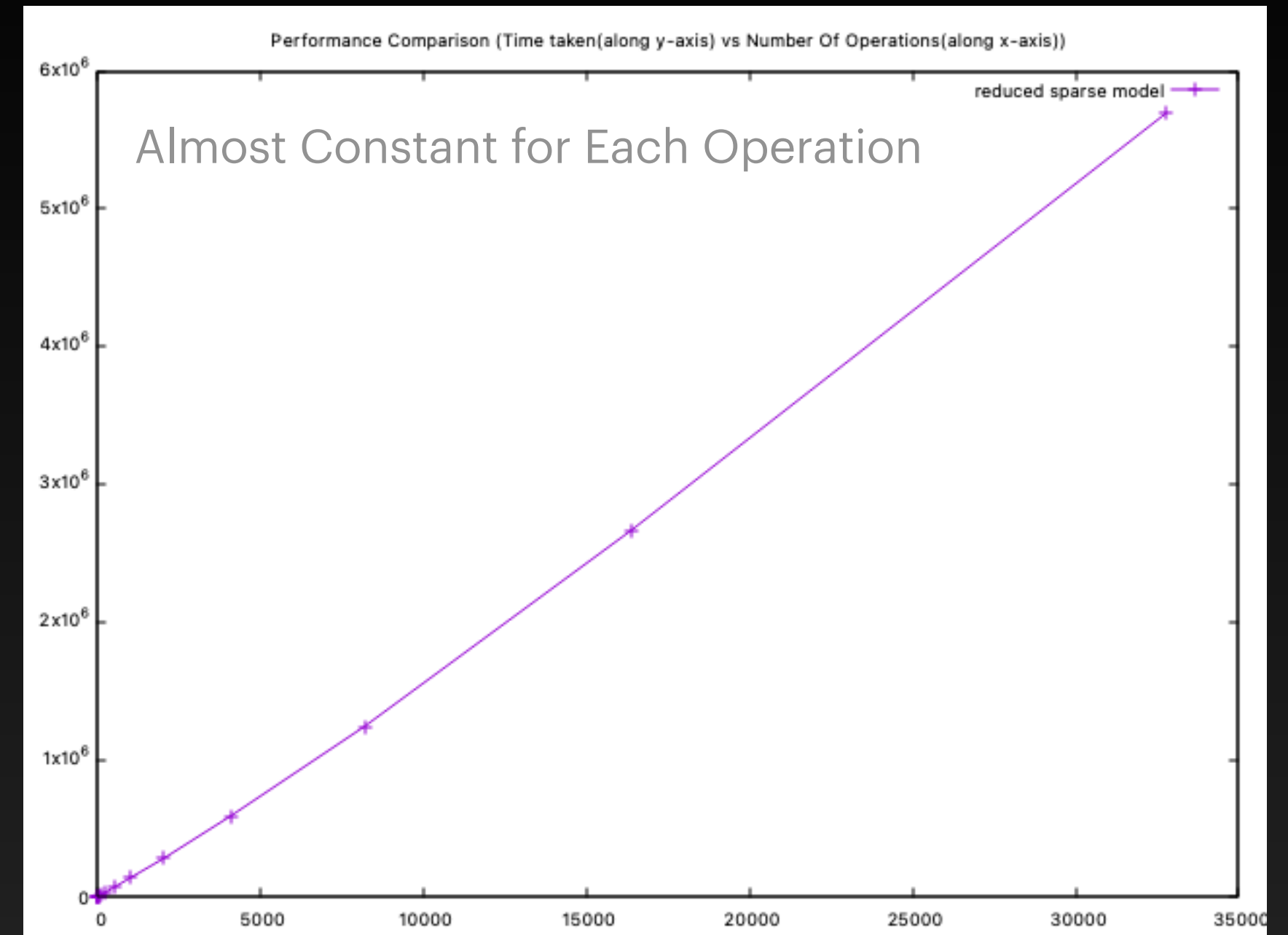
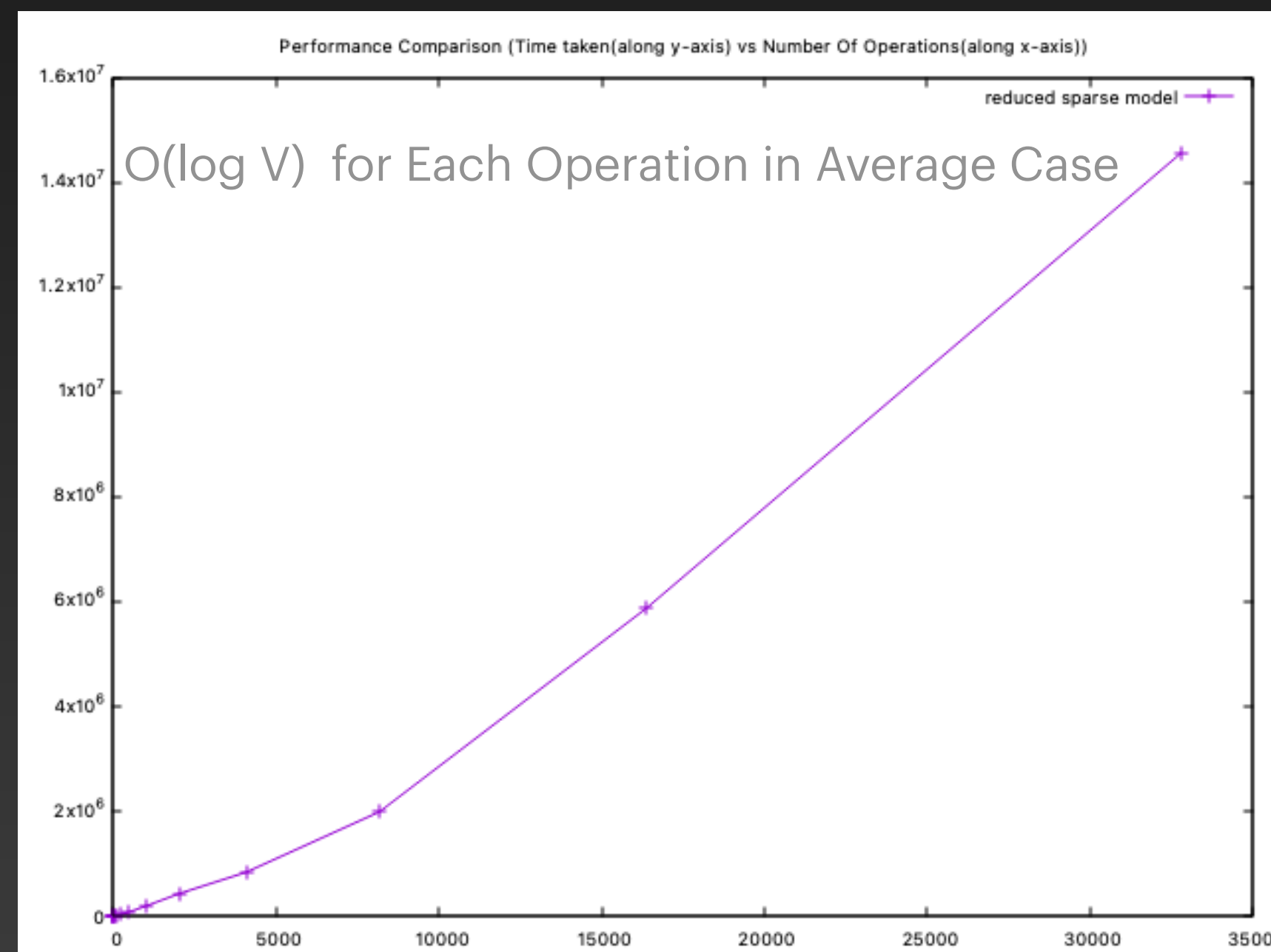
Strategies	Category 1	Category 2	Category 3
Using PPL with PM Model	$O(V)$ [Amortized] To Hold The Linked List	$O(V)$ to Hold The MAP for version->rear	
Reduced Sparse Matrix Model	<ul style="list-style-type: none"><li><math>O(V + \log(V!))</math> to Hold The UP_TABLE</li></ul>	<ul style="list-style-type: none"><li><math>O(V)</math> to Hold the MAP</li></ul>	<ul style="list-style-type: none"><li><math>O(V)</math> to Hold the TYPE_OF_VER</li></ul>
Ephemeral std::queue (C++)	$O(N)$		-



# Benchmarking Of Reduced Sparse Matrix Model



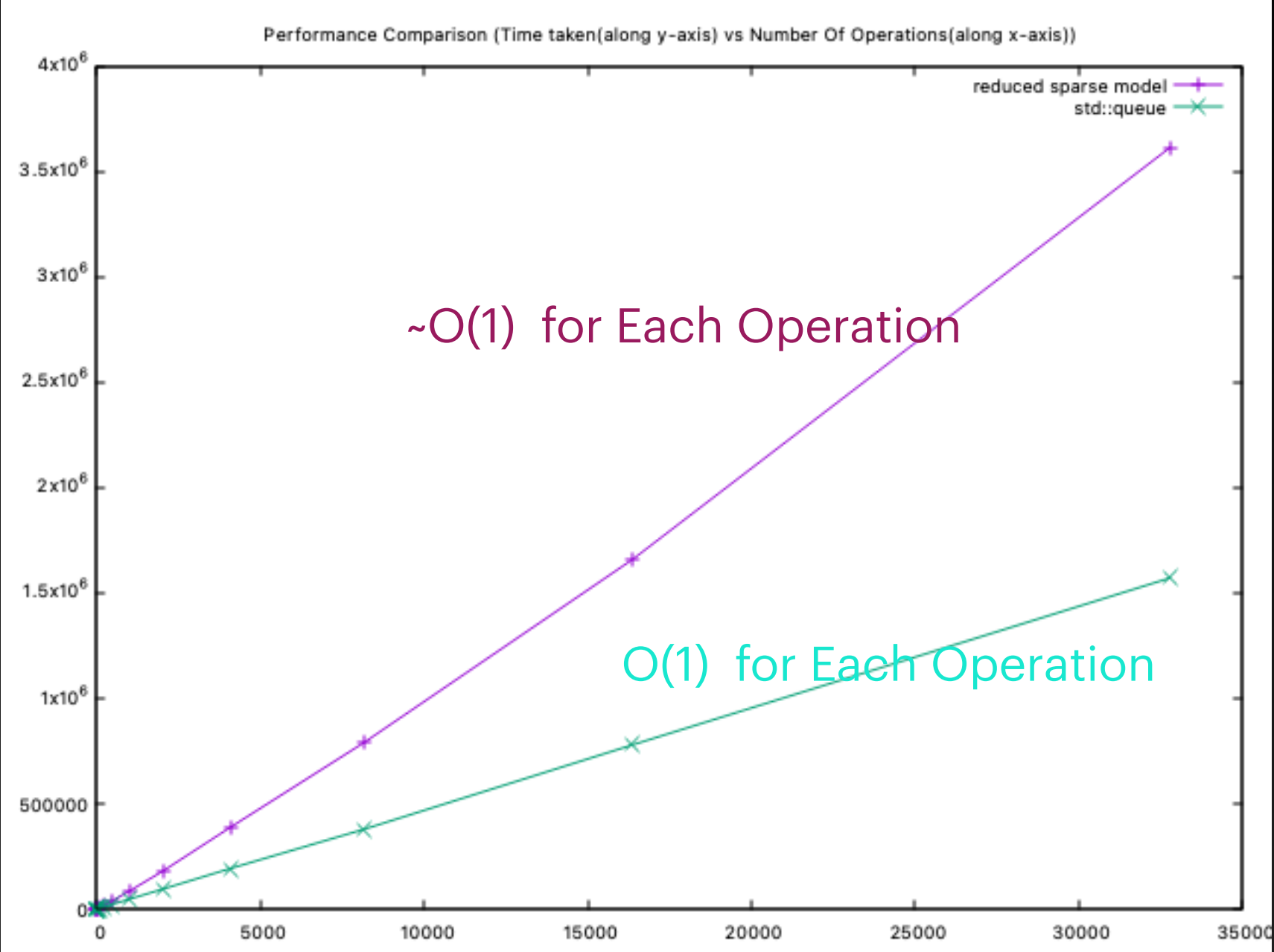
Only Enqueue In Randomised Versions



Randomised Enqueue/Deque In Randomised Versions

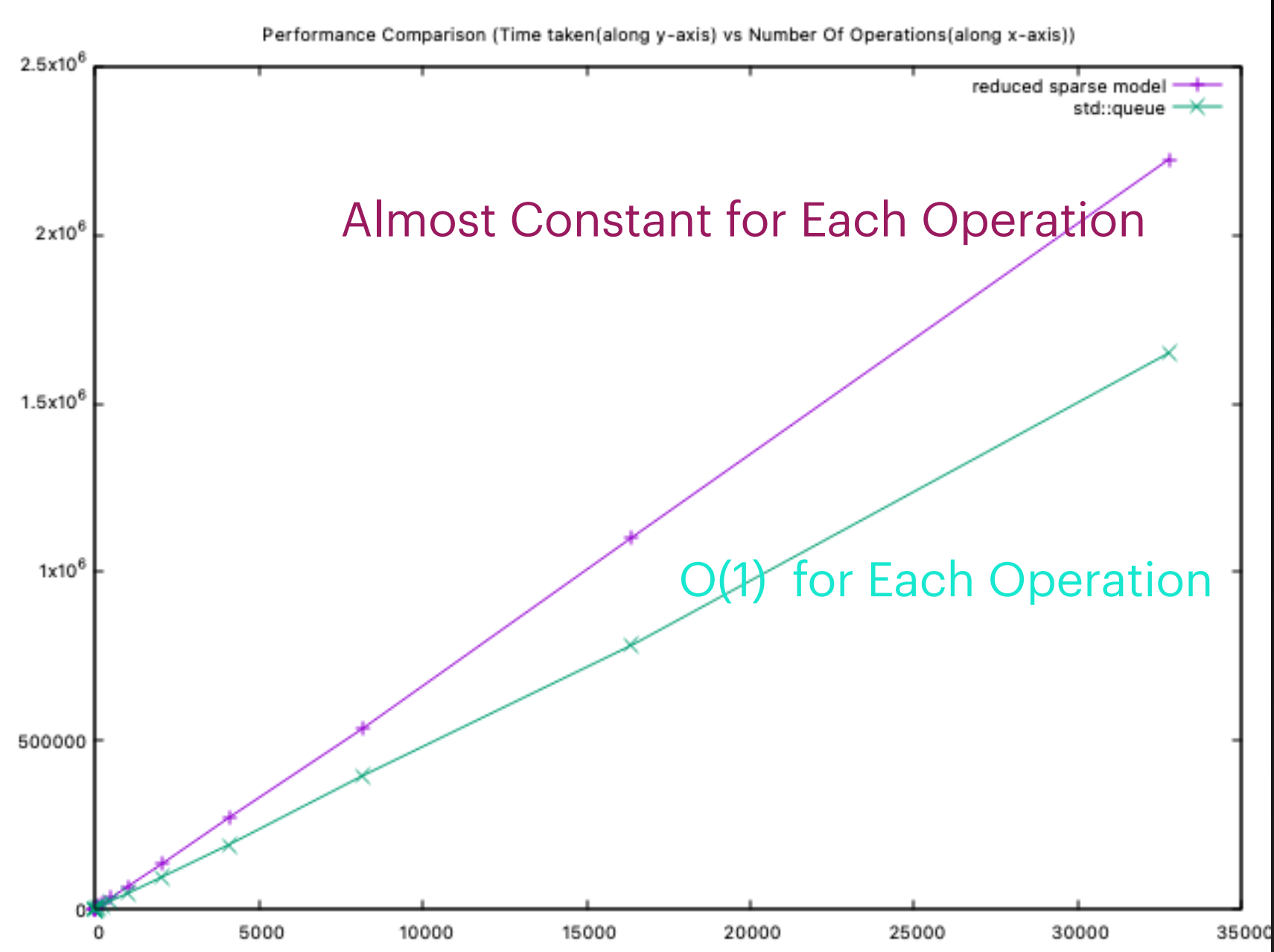
Only Deque In Randomised Versions  
(Enqueuing Operation is Not Time Profiled)

# Benchmarking: Reduced Sparse Matrix Model vs std::queue

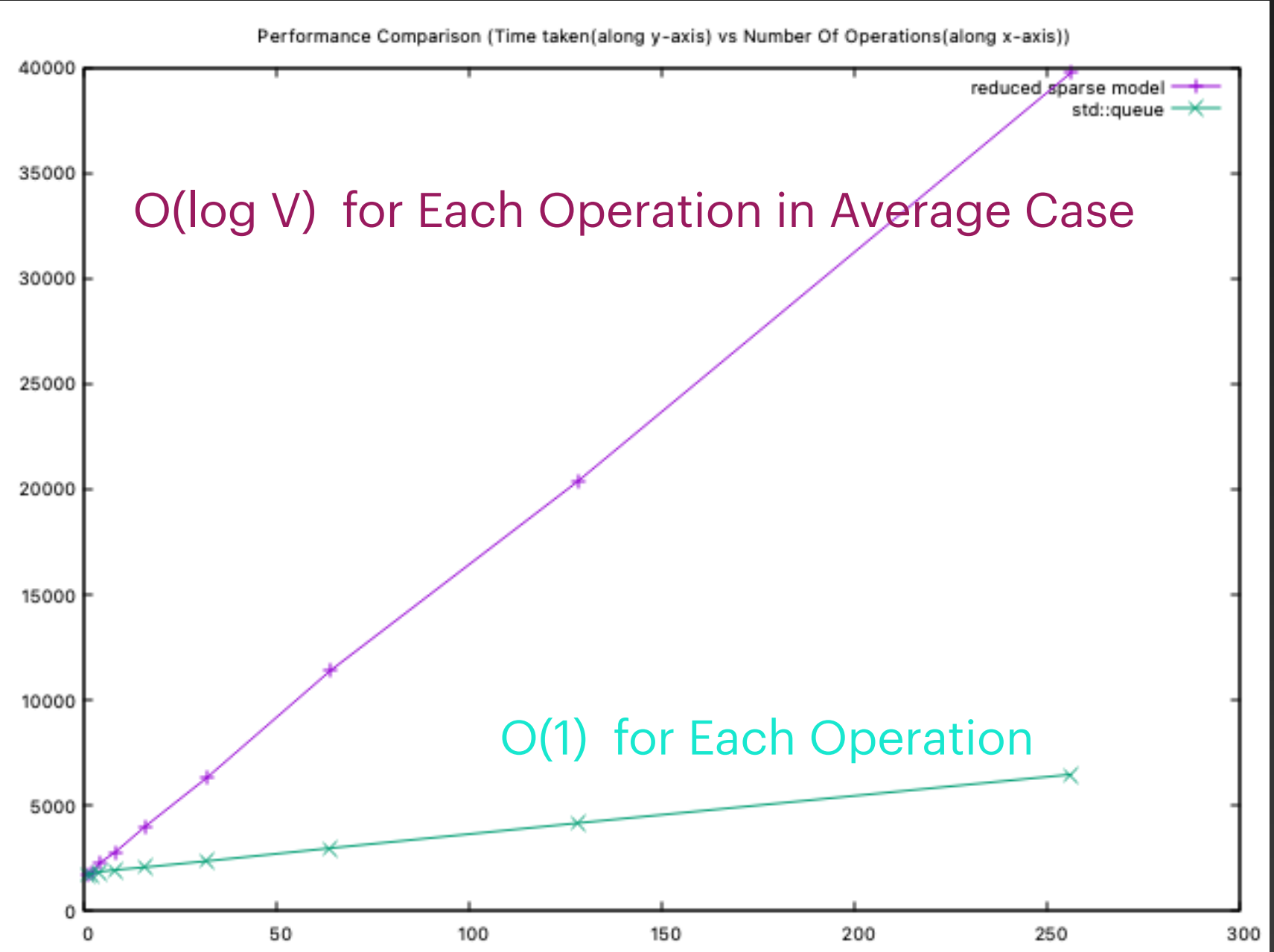


Only Enqueue In Latest Versions

Simulation Of Partial Persistence



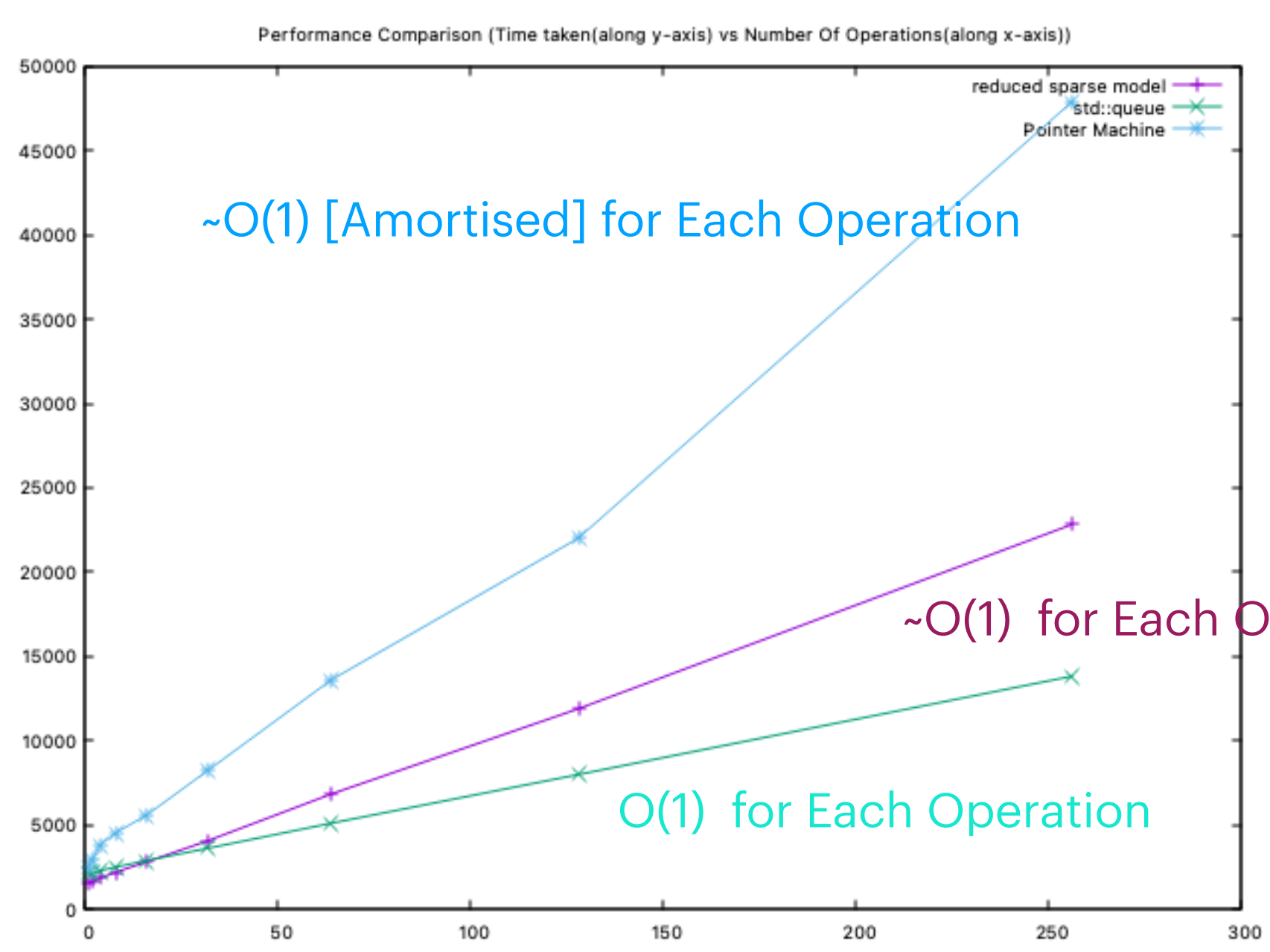
Randomised Enqueue/Deque In Latest Versions



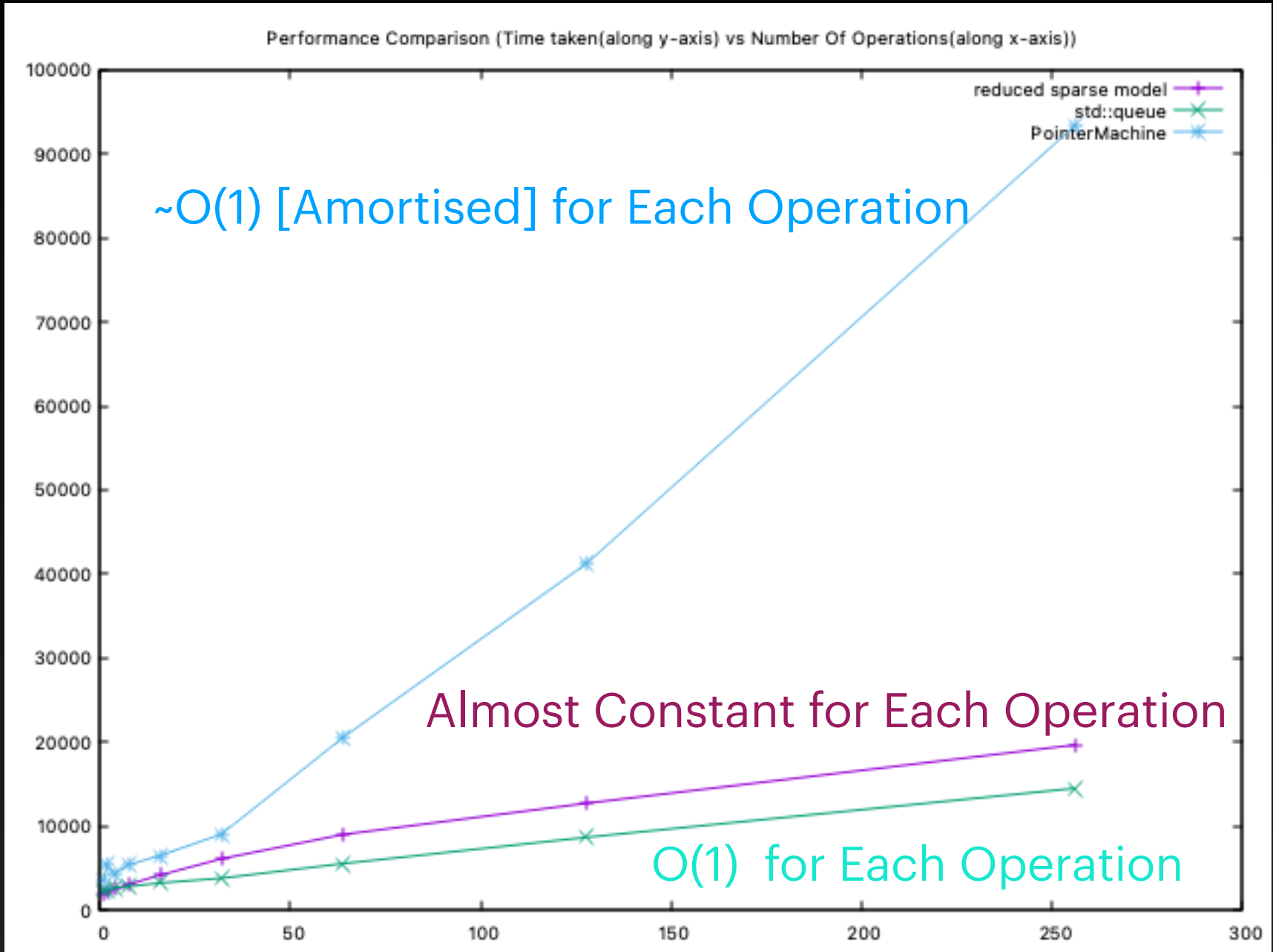
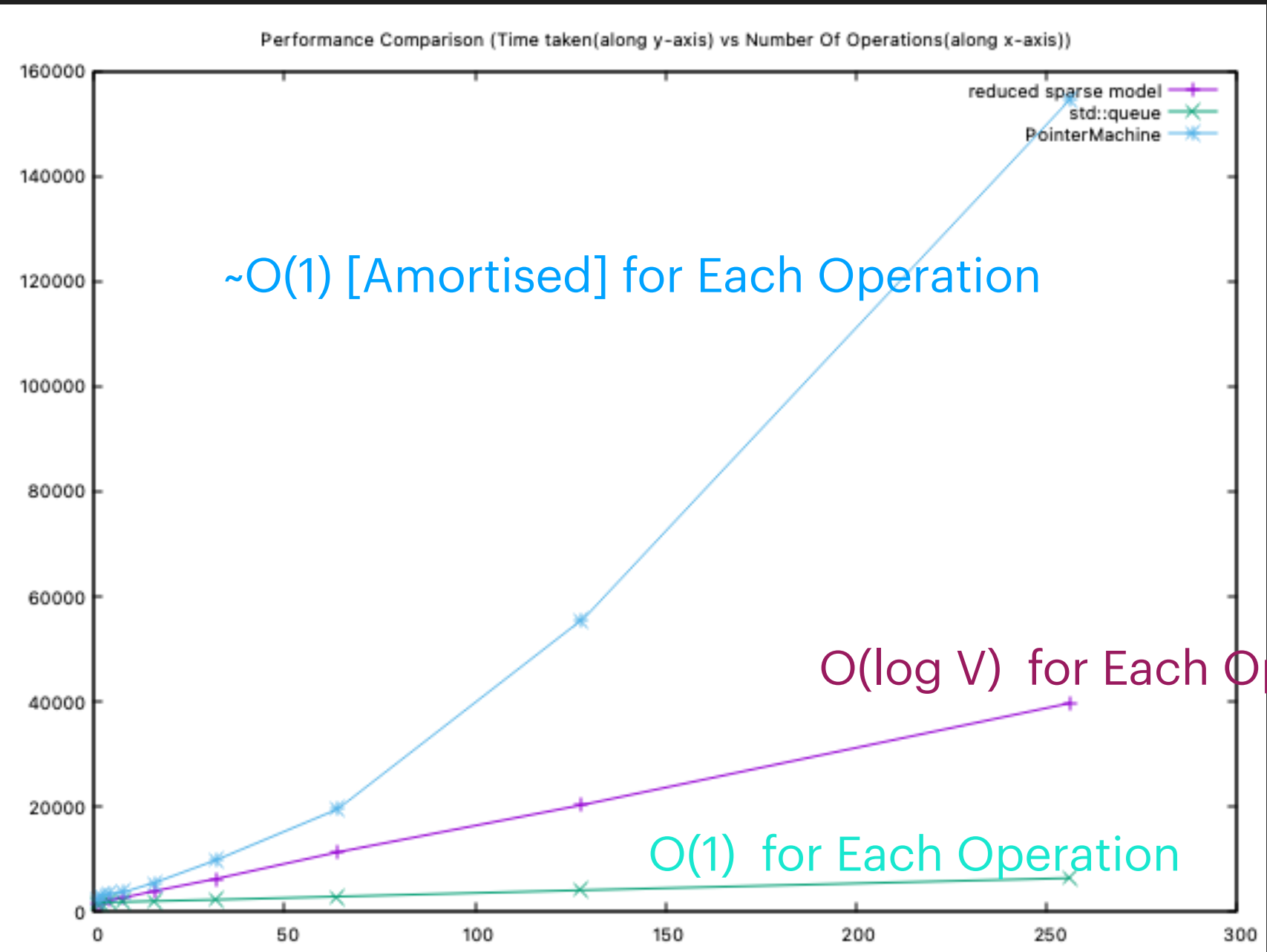
Only Deque In Latest Versions  
(Enqueuing Operation is Not Time Profiled)



# Benchmarking: Reduced Sparse Matrix Model vs std::queue Vs Queue\_Using\_PM\_PPL



Only Enqueue In Latest Versions



Randomised Enqueue/Dequeue In Latest Versions

Only Dequeue In Latest Versions  
(Enqueuing Operation is Not Time Profiled)

# Persistent Search Tree

# Persistent Search Tree

Time Complexity

Here, V = Total No. Of Version, N = Average Number Of Elements in The Tree Considering All The Versions

Strategies	Updation/Insertion/ Deletion of Data	Retrieval Of Data
Path Copying With Normal BST	$O(N)$ in Worst Case $O(1)$ in Best Case	$O(V) + O(N)$ in Worst Case $O(V) + O(1)$ in Best Case
Path Copying With AVL/RB Tree	$O(\log_2 N)$ in Worst Case $O(1)$ in Best Case	$O(V) + O(\log_2 N)$ in Worst Case $O(V) + O(1)$ in Best Case
Path Copying With Hash Array Mapped Trie	$O(\log_{32}(2^{64})) \sim O(12)$	$O(V) + O(\log_{32}(2^{64})) \sim O(12)$
With Pointer Machine	$O(1)$ Amortised	$O(1)$ Amortised

# Persistent Search Tree

Auxiliary Space

Here, V = Total No. Of Version, N = Average Number Of Elements in The Tree Considering All The Versions

Strategies	Category 1	Category 2
Path Copying With Normal Threaded BST	Node Size: # ~ 28 byte	$O(N + V)$ to Hold The Tree and Staring Pointers
Path Copying With AVL/RB Tree	Node Size: # ~ 32 byte	$O(N + V)$ to Hold The Tree and Staring Pointers
Path Copying With Bitmapped Hash Array Mapped Trie	Node Size: # ~ (4+4+64*8) byte [Worst Case] ~ (4+4) byte [Best Case]	$O(N + V)$ to Hold The Tree and Staring Pointers
With Pointer Machine	Node Size: # ~ 250 byte	$O(N)$ [Amortised] to Hold The LinkedList

# to store 4 byte Integer | 8 Bye pointers

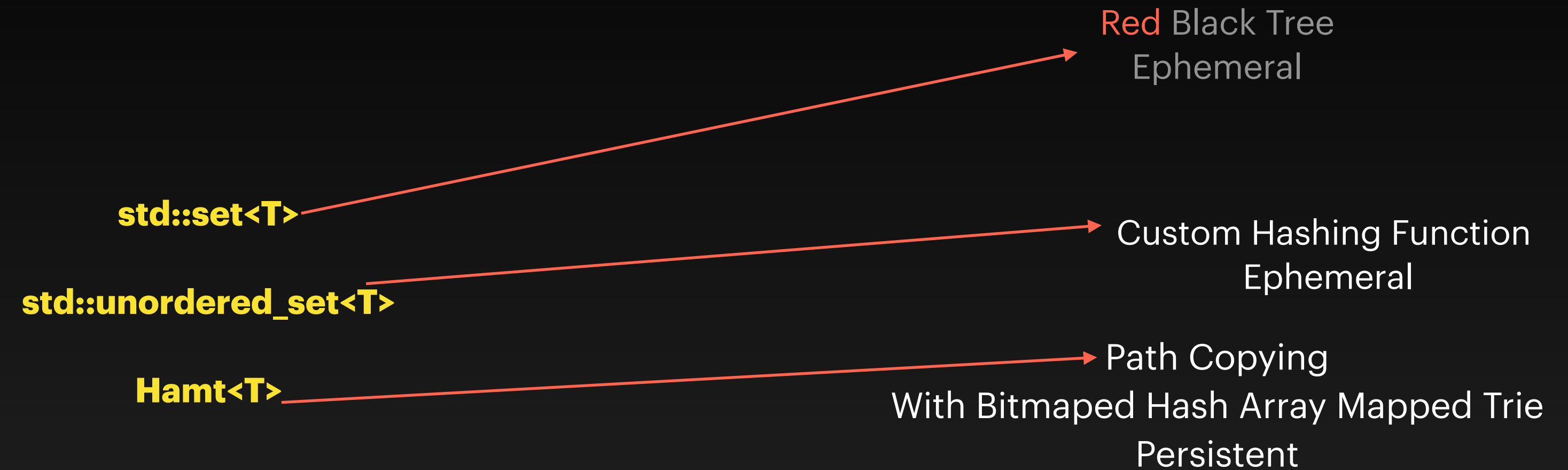
# BenchMarking Of Search Tree

## BenchMarking Tool:

### Nonius

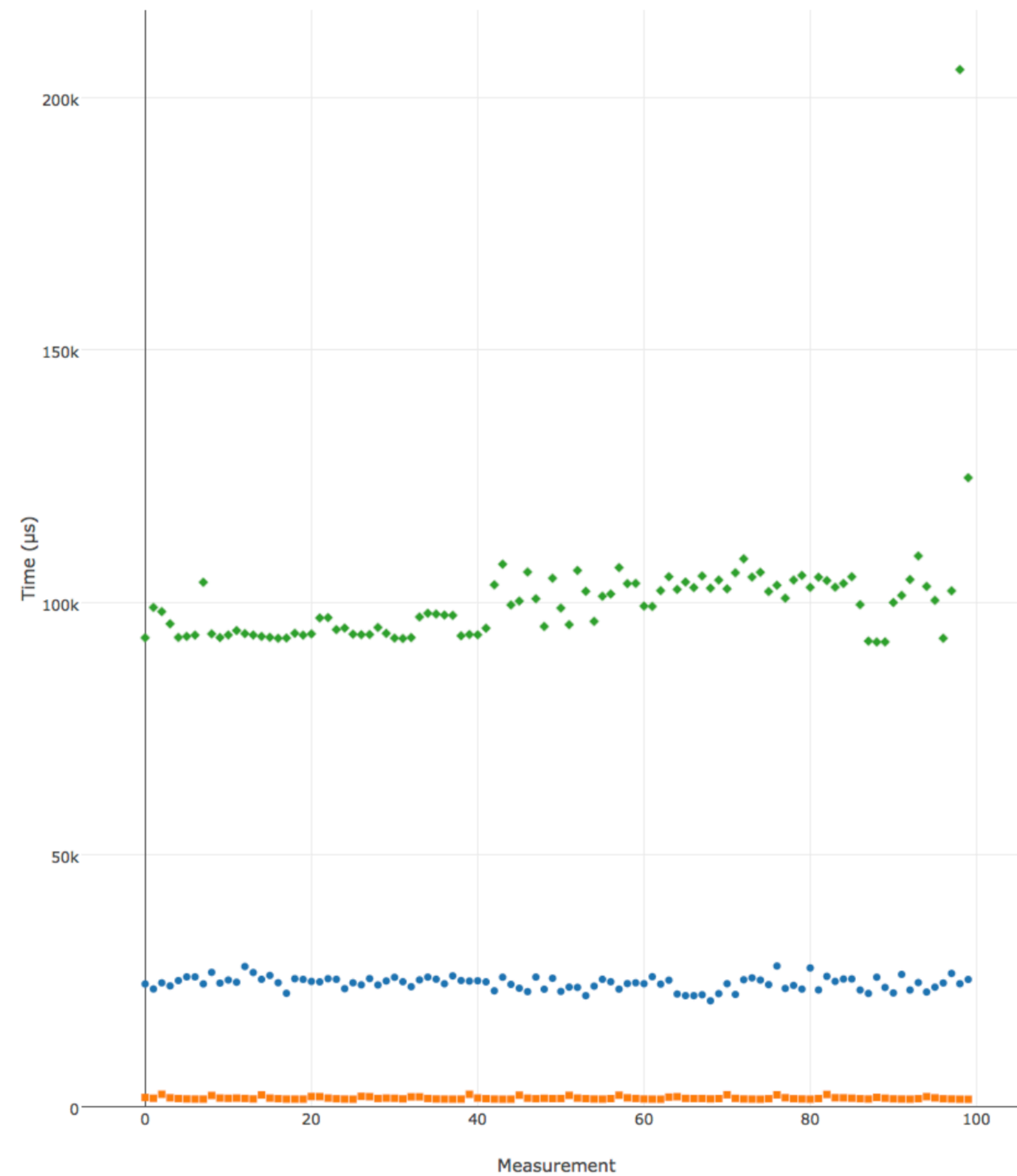
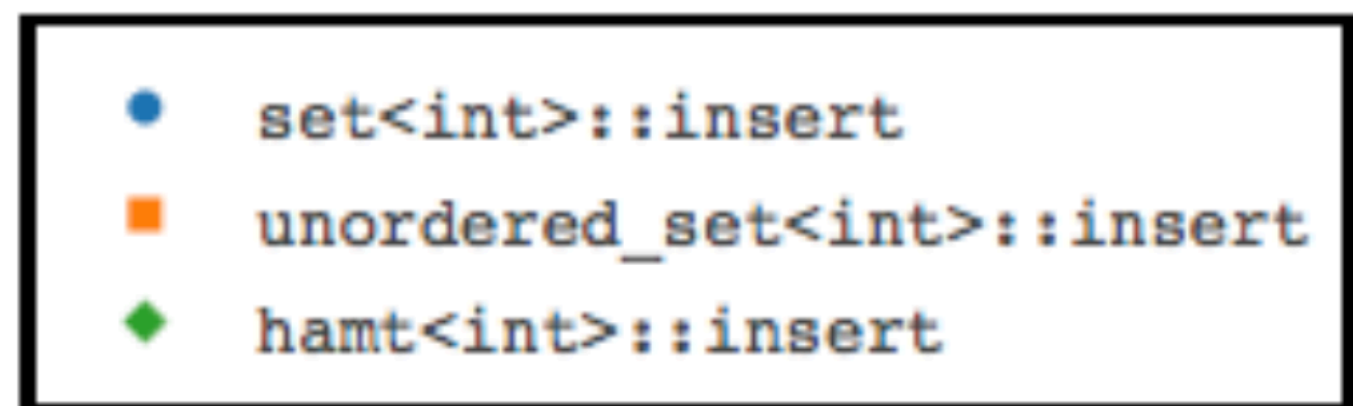
A C++ micro-benchmarking framework

Benchmarking Data Taken From A Talk By  
Phil Nash,  
Developer Advocate



# BenchMarking Of Search Tree

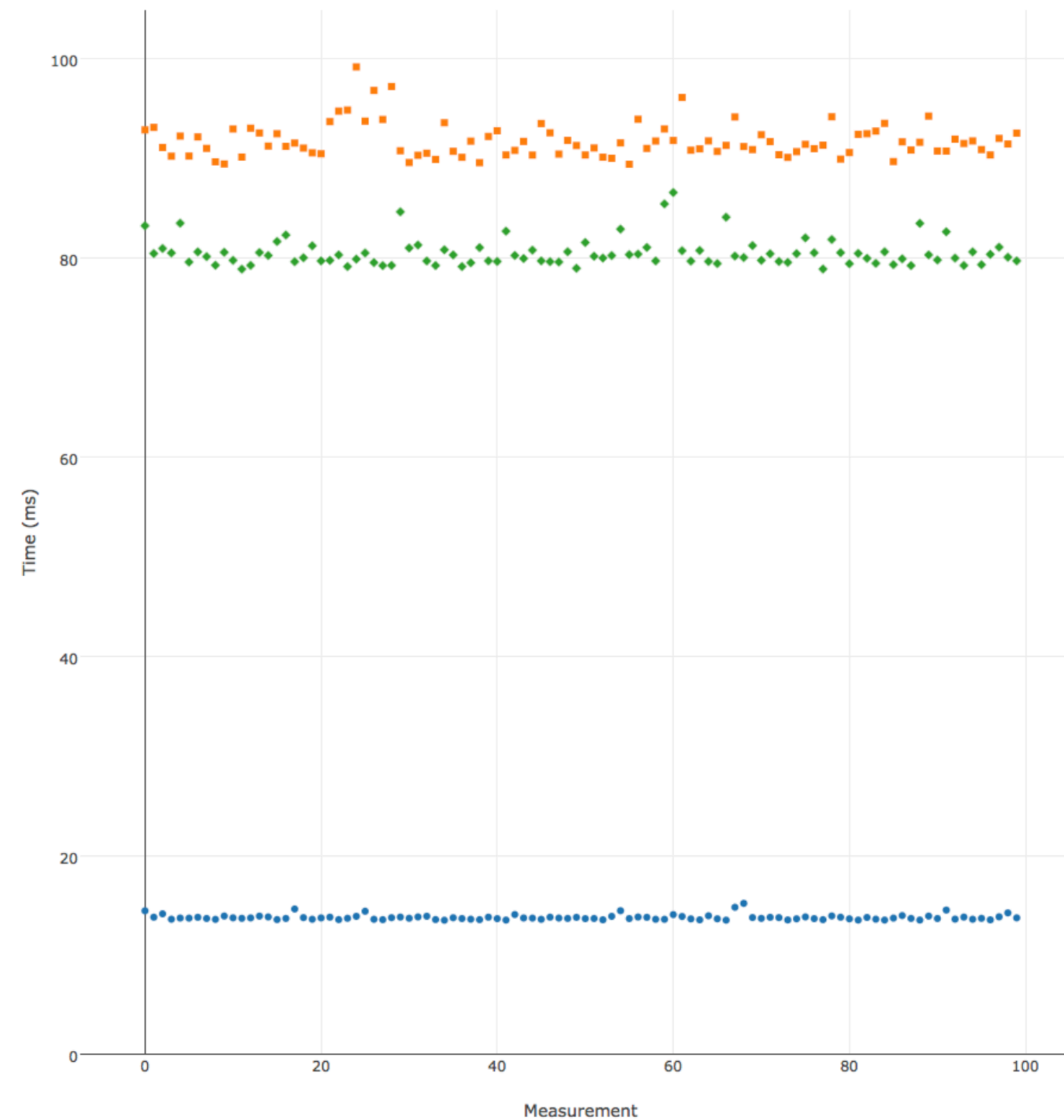
insert  
100k ints



# BenchMarking Of Search Tree

find  
100k ints

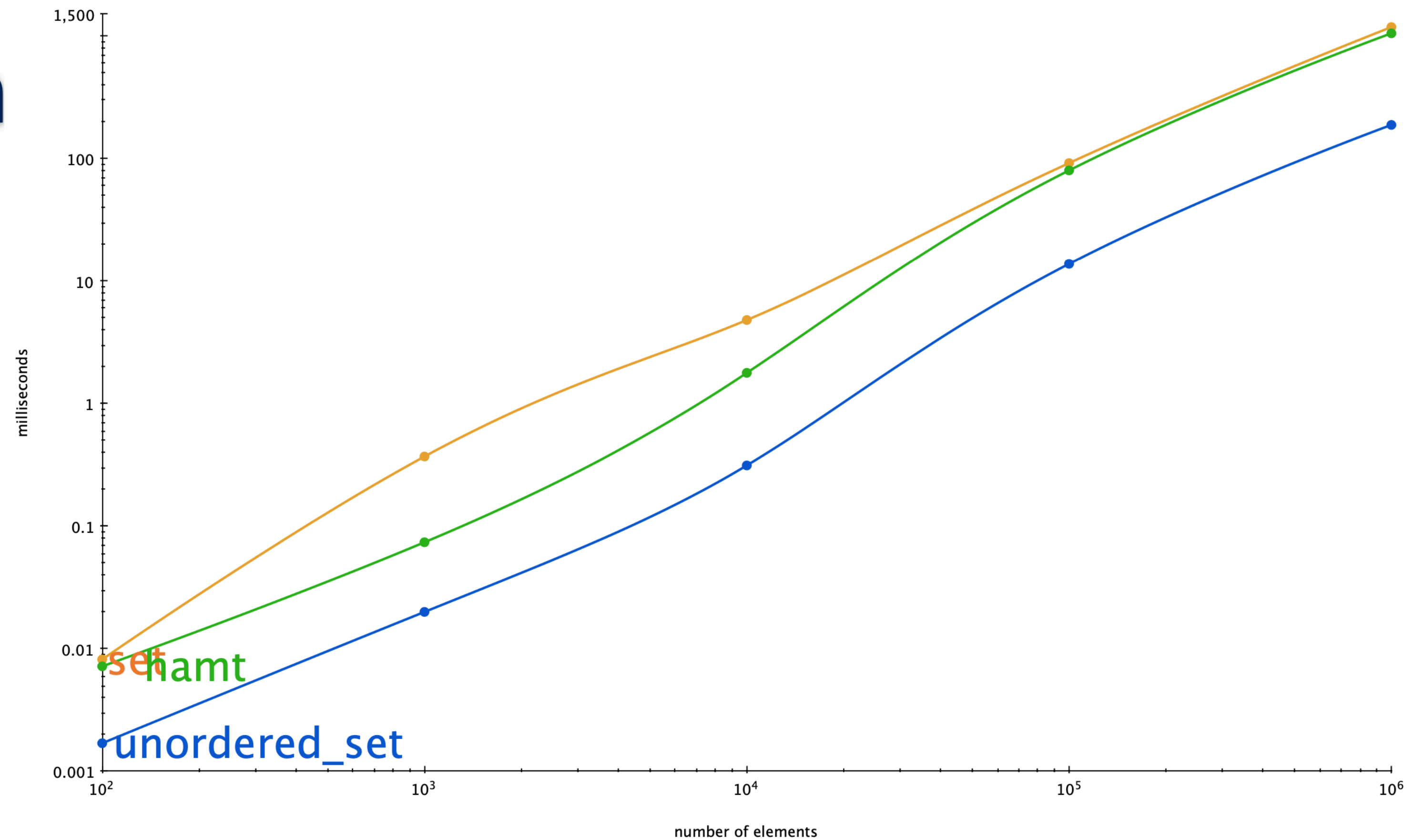
● `unordered_set<int>::find`  
■ `set<int>::find`  
◆ `hamt<int>::find`





# BenchMarking Of Search Tree

find  
100-1m  
ints

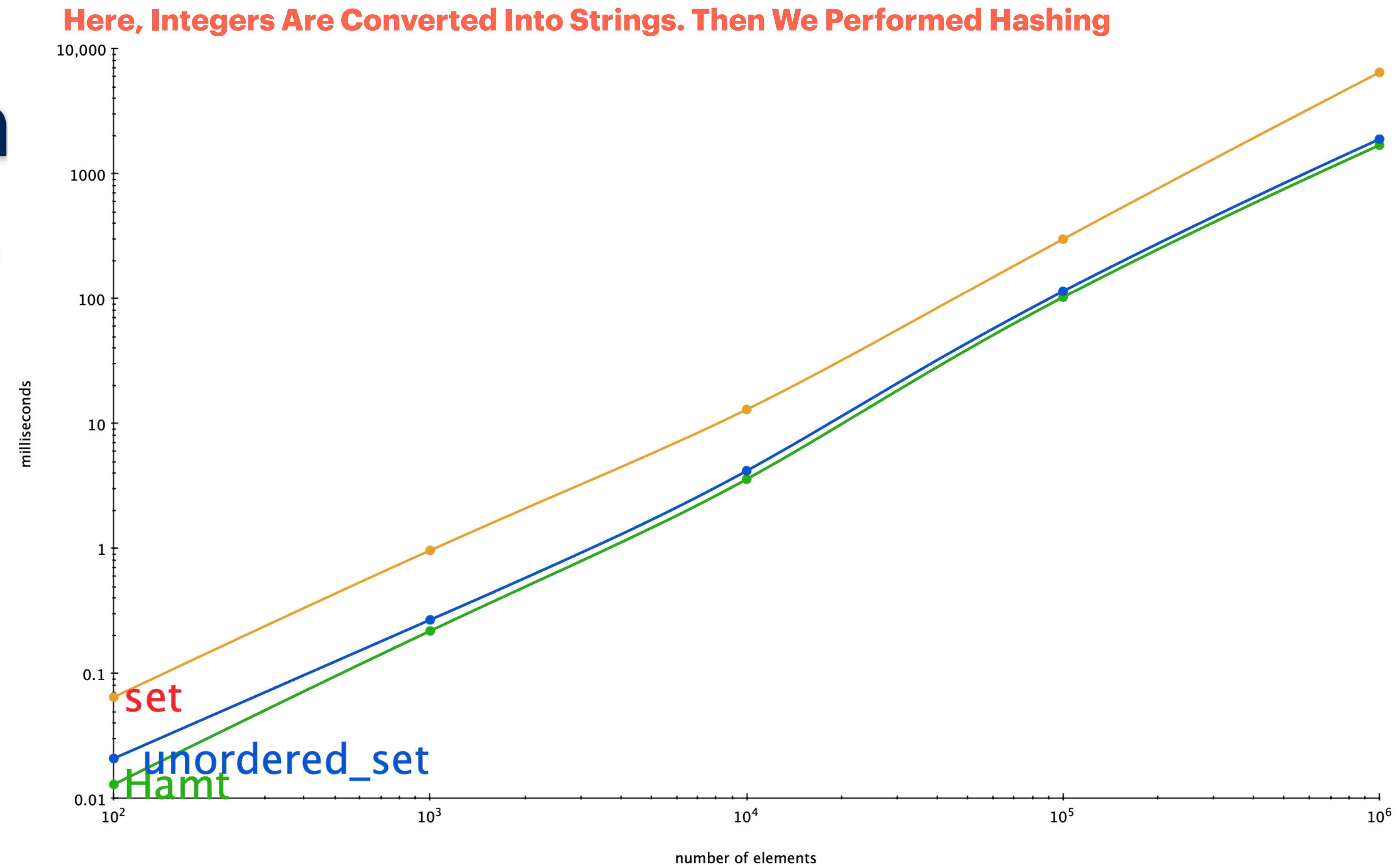


OmniGraphSketcher



# BenchMarking Of Search Tree

find  
100-1m  
strings

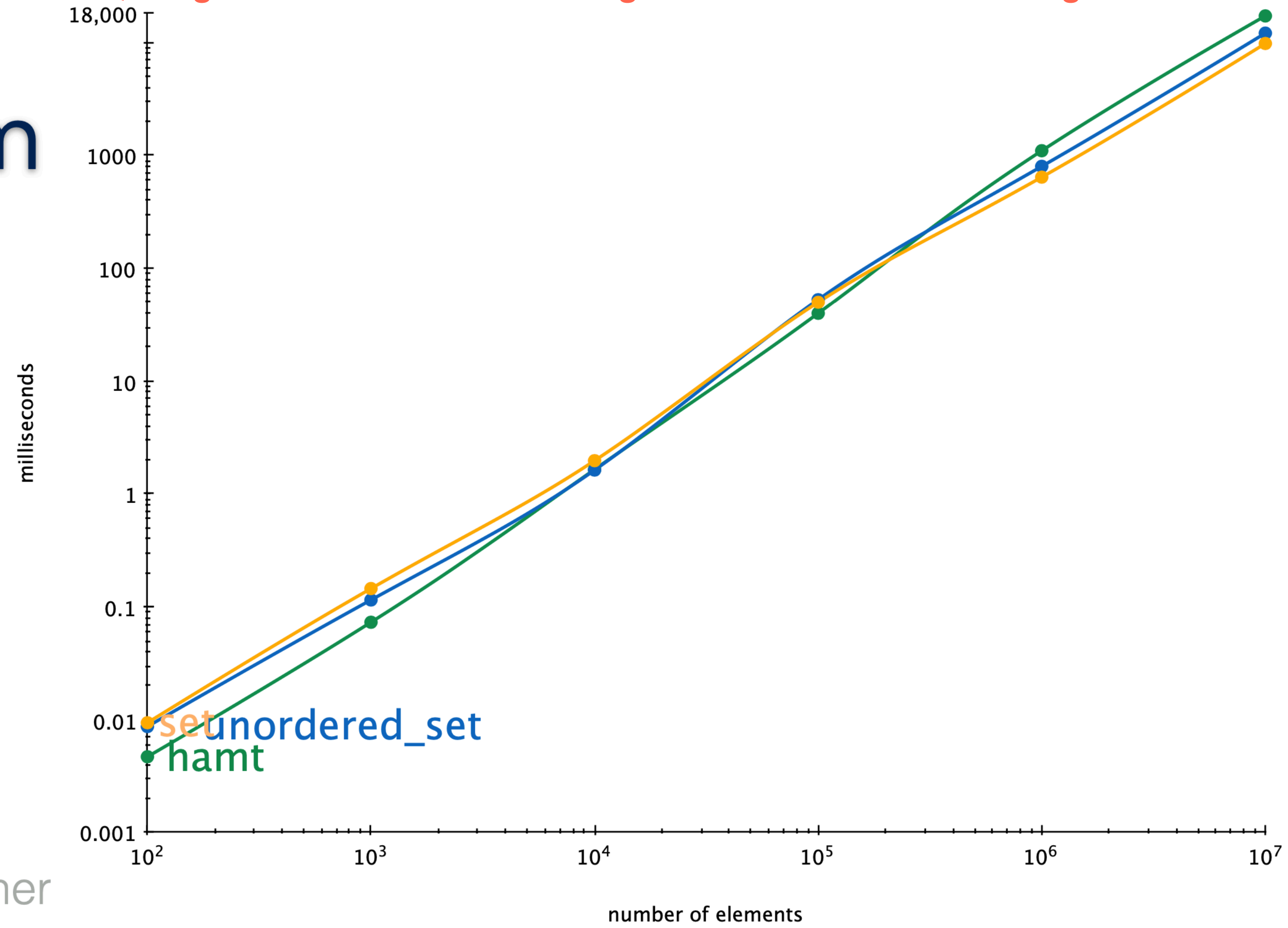


OmniGraphSketcher

# BenchMarking Of Search Tree

find  
100-10m  
hashes

Here, Integers Are Converted Into Strings. Then We Performed Hashing



OmniGraphSketcher