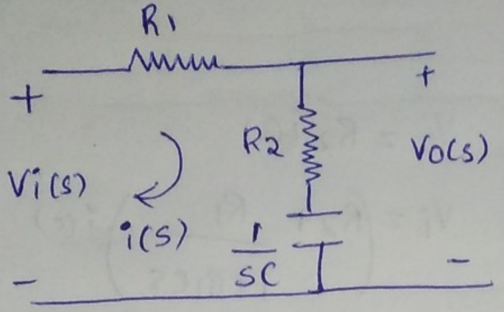
CS LAB (lab-0)

511824

1) Design and study of lag, lead, laglead compensators.

a) Lag compensator :-



$$V_o(s) = \left(R_2 + \frac{1}{sC}\right) i(s)$$

$$V_i(s) = \left(R_1 + R_2 + \frac{1}{sC}\right) i(s)$$

$\Rightarrow$ Transfer function;

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2 + \frac{1}{sC}}{R_1 + R_2 + \frac{1}{sC}}$$

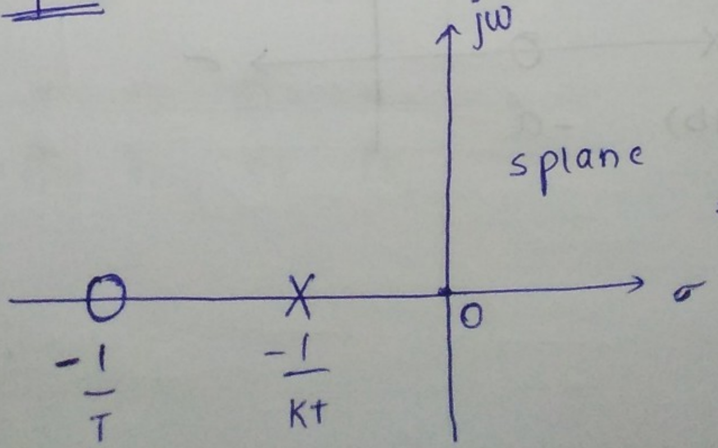
$$= \frac{sR_2C + 1}{s(R_1+R_2)C + 1}$$

$$= \left(\frac{s + \frac{1}{R_2C}}{s + \frac{1}{(R_1+R_2)C}}\right) \frac{R_1+R_2}{R_2}$$

Let $K = \frac{R_1+R_2}{R_2}$; $T = R_2C$

$$\Rightarrow \frac{V_o}{V_i} = \left(\frac{s + \frac{1}{T}}{s + \frac{1}{KT}}\right) \cdot \frac{1}{K}$$

pole zero plot :

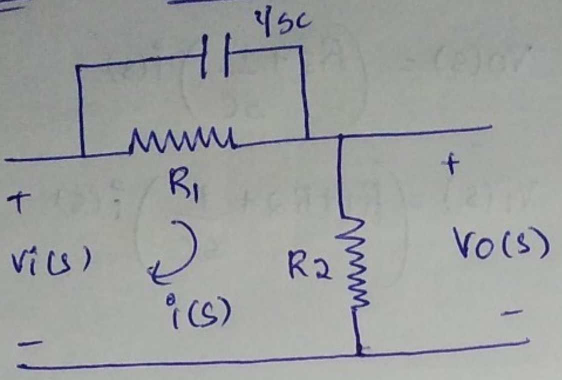


$K = \frac{R_1+R_2}{R_2} = 1 + \frac{R_1}{R_2}$ (>1)

$\Rightarrow \frac{1}{T} > \frac{1}{KT}$

1)

b) **Lead compensator:**



$V_o = R_2\, i(s)$

$V_i = \left(R_2 + \dfrac{R_1}{1+R_1Cs}\right) i(s)$

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_2 + \dfrac{R_1}{1+R_1Cs}}$$

$$= \frac{R_2(1+R_1Cs)}{R_2(1+R_1Cs)+R_1}$$

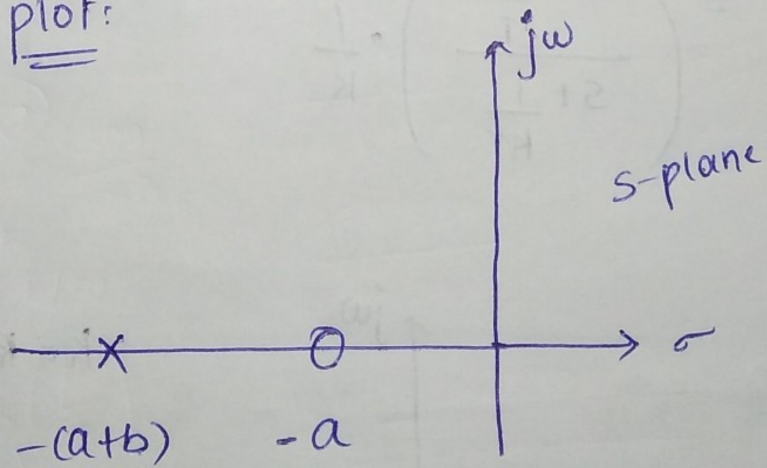
$$= \frac{R_1\cancel{R_2}C\left[s+\dfrac{1}{R_1C}\right]}{R_1\cancel{R_2}C\left[s+\dfrac{1}{R_1C}+\dfrac{1}{R_2C}\right]}$$

Let $a = \dfrac{1}{R_1C}$

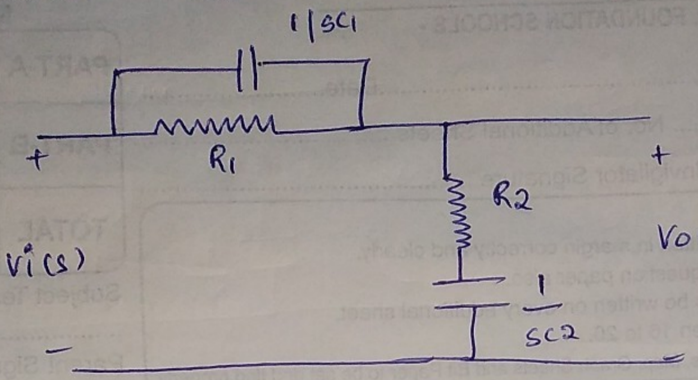
$b = \dfrac{1}{R_2C}$

$$= \frac{s+a}{s+a+b}$$

Pole-Zero plot:



c) *Lag-lead compensator:*



$$\frac{V_o(s)}{V_i(s)} = \frac{R_2 + \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2} + \frac{R_1}{1+sR_1C_1}}$$

$$= \frac{(sR_2C_2+1)\cdot(sR_1C_1+1)}{sR_2C_2 + s^2R_1R_2C_1C_2 + sR_1C_2 + sR_1C_1 + 1}$$

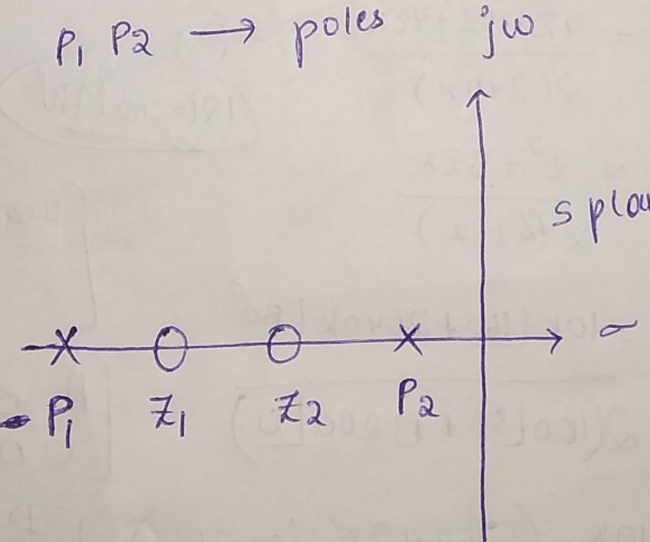
$$= \frac{(sR_2C_2+1)(sR_1C_1+1)}{s^2R_1R_2C_1C_2 + s(R_1C_1 + R_2C_2 + R_1C_2)s + 1}$$

$\Rightarrow$ 2 poles 2 zeroes

combination of both lag and lead compensators

Let $Z_1\, Z_2 \longrightarrow$ zeroes

$P_1\, P_2 \longrightarrow$ poles



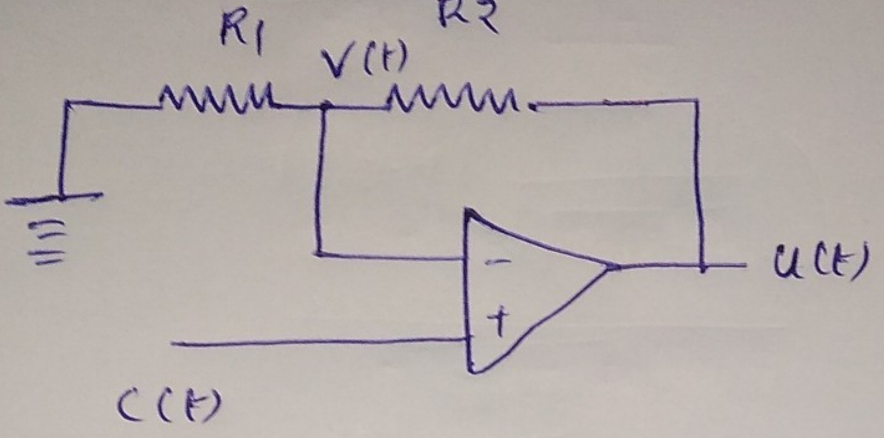
$$Z_1 = -\frac{1}{R_1C_1}$$

$$Z_2 = -\frac{1}{R_2C_2}$$

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}{s^2 + s\left(\frac{R_1C_1+R_2C_2+R_1C_2}{R_1R_2C_1C_2}\right) + \frac{1}{R_1R_2C_1C_2}}$$

1)

d) P- controller:



By virtual groud concept

$$V(t) = C(t) \Rightarrow V(s) = C(s)$$

$$\Rightarrow \frac{C(s)}{R_1} + \frac{C(s) - U(s)}{R_2} = 0$$

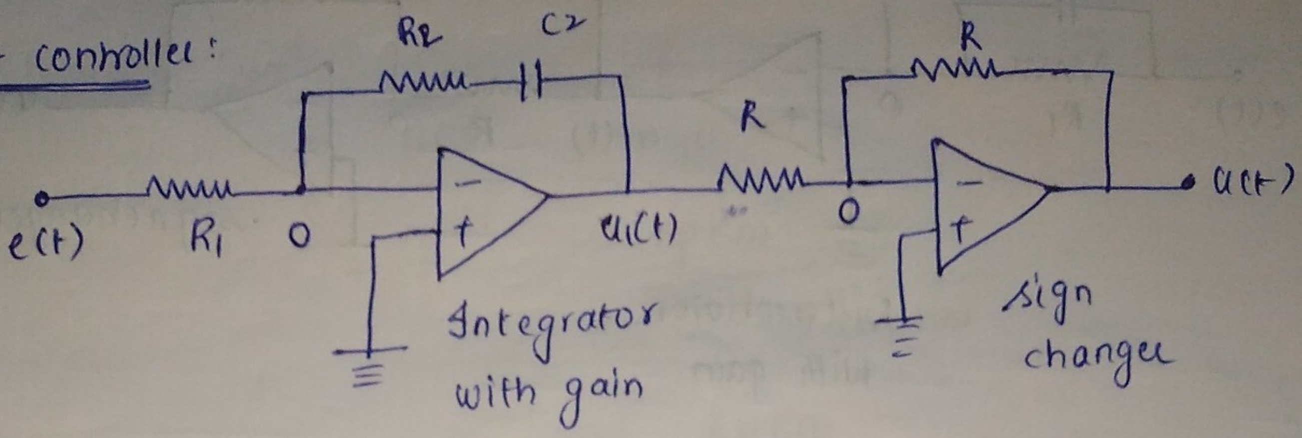
$$\Rightarrow C(s)\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = \frac{U(s)}{R_2}$$

$$\Rightarrow \frac{U(s)}{\cancel{R_2}} = \frac{C(s)\left[R_1 + R_2\right]}{R_1 \cancel{R_2}}$$

$$\frac{U(s)}{C(s)} = \frac{R_1 + R_2}{R_1} = \text{constant value.}$$

1) e)

PI- controller:



$$\frac{-e(s)}{R_1} + \frac{(-u_1(s))}{R_2 + \frac{1}{C_2 s}} = 0 \Rightarrow \frac{e_1(s)}{R_1} = \frac{-u_1(s)\cdot C_2 s}{sR_2C_2 + 1} \longrightarrow 1$$

$$\frac{0 - u_1(s)}{R} + \frac{0 - u(s)}{R} = 0 \Rightarrow u_1(s) = -u(s) \longrightarrow 2$$

$$\Rightarrow \frac{e_1(s)}{R_1} = \frac{-u_1(s)\cdot C_2 s}{sR_2C_2 + 1}$$

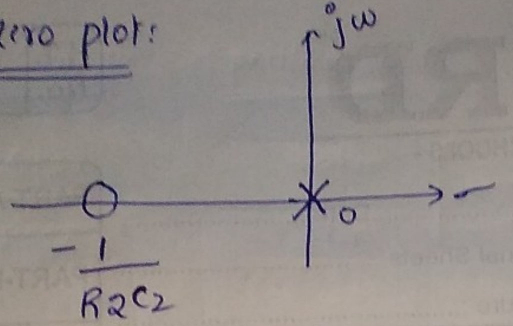
$$= \frac{u(s)\cdot C_2 s}{sR_2C_2 + 1}$$

$$\Rightarrow \frac{e_1(s)}{u(s)} = \frac{sR_1C_2}{sR_2C_2 + 1}$$

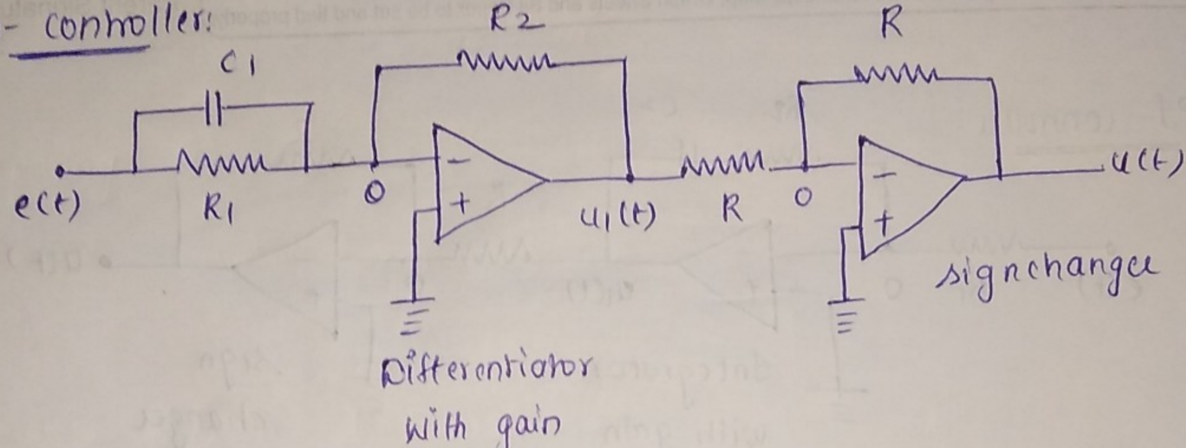
$$\text{Transfer fun}^n = \frac{u(s)}{e(s)} = \frac{sR_2C_2 + 1}{sR_1C_2} = \frac{\left(s + \frac{1}{R_2C_2}\right) R_2 \cancel{C_2}}{sR_1\cancel{C_2}}$$

$$= \frac{R_2}{R_1}\left(\frac{s + \frac{1}{R_2C_2}}{s}\right)$$

Pole-zero plot:



1)
f) **PD- controller:**

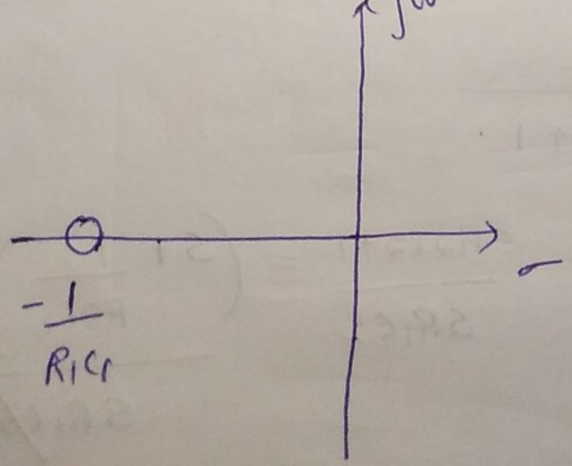


$$\frac{0-u_1(s)}{R}+\frac{0-u(s)}{R}=0 \Rightarrow u_1(s)=-u(s) \rightarrow ①$$

$$\frac{0-e(s)}{\left(\frac{R_1}{1+R_1C_1s}\right)}+\frac{(-u_1(s))}{R_2}=0$$

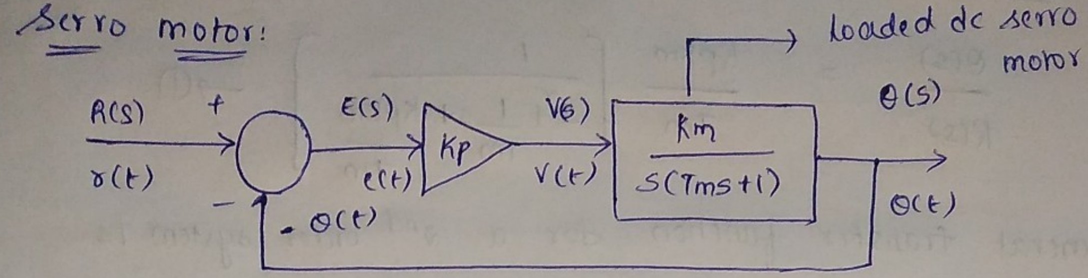
$$\Rightarrow \frac{e(s)(1+R_1C_1s)}{R_1}=-\frac{u_1(s)}{R_2}=\frac{u(s)}{R_2}$$

$$\Rightarrow \frac{u(s)}{e(s)}=\frac{R_2}{R_1}(1+R_1C_1s)=R_2C_1\left[s+\frac{1}{R_1C_1}\right]$$



1)

g) <u>Servo motor:</u>



Gain (openloop) $G = \dfrac{K_p K_m}{S(T_m S+1)}$; $H = 1$

Transfer funⁿ; $\dfrac{\theta(s)}{R(s)} = \dfrac{G}{1+GH}$

$$= \frac{K_p K_m}{S(T_m S+1) + K_p K_m}$$

$$= \frac{K_p K_m}{S^2 T_m + S + K_p K_m}$$

$$= \frac{K_p K_m}{T_m}\left[\frac{1}{S^2 + \frac{1}{T_m} S + \frac{K_p K_m}{T_m}}\right]$$

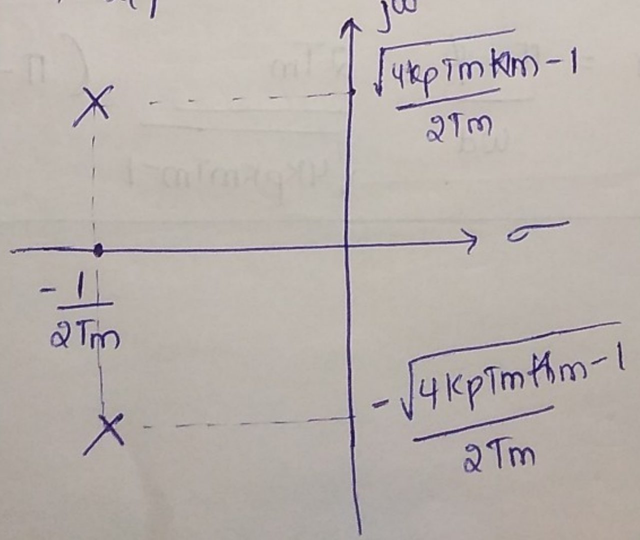
$$S = \frac{-\frac{1}{T_m} \pm \sqrt{\frac{1}{T_m^2} - \frac{4K_p K_m}{T_m}}}{2}$$

Let $\alpha = \dfrac{-1 + \sqrt{1 - 4K_p T_m K_m}}{2T_m}$ $\beta = \dfrac{-1 - \sqrt{1-4K_p T_m K_m}}{2T_m}$

poles → α, β

$$\alpha = \frac{-1}{2T_m} + j\left[\frac{\sqrt{4K_p T_m K_m - 1}}{2T_m}\right]$$

$$\beta = \frac{-1}{2T_m} - j\left[\frac{\sqrt{4K_p T_m K_m-1}}{2T_m}\right]$$



b) Time domain specifications:

$$\frac{\theta(s)}{R(s)} = \frac{K_pK_m}{T_m}\left[\frac{1}{s^2+\frac{1}{T_m}s+\frac{K_pK_m}{T_m}}\right] \rightarrow (1)$$

General transfer function for a 2nd order system is

$$\frac{\theta(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \rightarrow (2)$$

comparing (1) & (2)

$$\omega_n^2 = \frac{K_pK_m}{T_m} \Rightarrow \omega_n = \sqrt{\frac{K_pK_m}{T_m}}$$

$$\xi\omega_n = \frac{1}{2T_m} \leftarrow 2\xi\omega_n = \frac{1}{T_m} \Rightarrow \xi = \frac{\frac{1}{T_m}}{2\sqrt{\frac{K_pK_m}{T_m}}} = \frac{1}{2\sqrt{K_pK_mT_m}}$$

$$\phi^{\circ} = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right) = \cos^{-1}(\xi)$$

$$\phi^{c} = \frac{\pi}{180}\cos^{-1}(\xi)$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = \sqrt{\frac{K_p\cancel{K_m}}{T_m}}\cdot\frac{\sqrt{4K_pK_mT_m-1}}{2\sqrt{T_m\cancel{K_p}\cancel{K_m}}}$$

$$= \frac{1}{2T_m}\sqrt{4K_pK_mT_m-1}$$

i) rise time ; $\boxed{t_r = \frac{\pi-\phi}{\omega_d} = \frac{2T_m\,(\pi-\phi^c)}{\sqrt{4K_pK_mT_m-1}}}$ (seconds)

ii) delay time:

$$T_d = \frac{1 + 0.7\xi}{\omega_n}$$

$$= \frac{1 + 0.7\left(\frac{1}{2\sqrt{K_p K_m T_m}}\right)}{\sqrt{\frac{K_m K_p}{T_m}}}$$

$$= \sqrt{\frac{T_m}{K_m K_p}} + \frac{0.35}{K_m K_p} = \frac{0.35 + \sqrt{K_m K_p T_m}}{K_m K_p} \text{ sec}$$

iii) Peak time:

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$= \frac{2\pi T_m}{\sqrt{4K_p K_m T_m - 1}} \text{ sec}$$

iv) Peak overshoot:

$$M_P = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

$$= e^{-\frac{1}{2\sqrt{K_p K_m T_m}} \times \frac{\pi}{\sqrt{1 - \frac{1}{4K_p K_m T_m}}}}$$

$$= e^{-\frac{\pi}{\sqrt{4K_p K_m T_m - 1}}}$$

v) Settling time = $\frac{4}{\xi\omega_n} \rightarrow 2\%$

$= \frac{3}{\xi\omega_n} \rightarrow 5\%$

$\Rightarrow T_s = 8T_m \rightarrow 2\%$

$= 6T_m \rightarrow 5\%$

2) Basic difference b/n 1st order & second order system.

1st order	2nd order
* Only one independant energy storage element is present.	* 2 independant energy storage elements
Ex: RC circuit	Ex: RLC circuit
* Response doesnot have any ripples irrespective of position of poles.	* Exhibits oscillatory behaviour.