Parul® University

Information Theory & Coding

Prof. Pankaj Chaudhary, Assistant Professor
Information Technology

CHAPTER-3

Probability Theory

Review of Probability Concept

- Probability means possibility or how likely an event is to occur, or how likely it is that a proposition is true or how likely something is to happen.
- The concept of probability is used to predict the occurrences of yes or no from an event.
- For example the possibility of getting head or tail from the tossing the coin. We say that the probability of the coin landing H is ½, and the probability of the coin landing T is ½.

Review of Probability Concept

- When a single die is thrown, there are six possible outcomes: 1, 2, 3, 4, 5, 6.The probability of any one of them is 1/6.

- In general the formula of probability is define as,

$$Probability\ of\ an\ event\ (P) = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$$

- There are 5 bottles in a bag: 4 are blue, and 1 is red. What is the probability that a blue bottle gets picked? [P=4/5=0.8]

Concept of random variable

- “A random variable is a variable that can take on different values randomly”.
- Measuring a possible values from random experiment. For example tossing a coins, predicting a rain for tomorrow.
- Random variable is a different from simple algebra variable, for an example of algebra variable is $x+2=9$.

Concept of random variable

- Put the value of x as 5 and you will get the result.
- But the random variable is different from Algebra variable. A Random Variable has a whole set of value and it could take on any of those values, randomly.
- X = {0, 1, 2}. X could be 0, 1, or 2 randomly. And they might each have a different probability.

Concept of random variable

Suppose that a coin is tossed twice so that the sample space is S= {HH, HT, TH, TT}. Let X represent the number of heads that can come up. The value of X could be 0, 1, or 2.

Table 3.1

Sample	HH	HT	TH	TT
X	2	1	1	0

Concept of random variable

A random variable can be classify into two types, a random variable that takes on a finite or countable infinite number of values is called a discrete random variable while one which takes on a non-countable infinite number of values is called continuous random variables.

Function of random variable

If X is a random variable and Y=g(X), then Y itself is a random variable.

- **Distribution and density function Moments**

- **Discrete Probability Distributions:** Suppose a random variable X may take k different values, with the probability that X = xi defined to be P(X = xi) = pi. The probabilities pi must satisfy the following:

- 0 < pi < 1 for each i
- p1 + p2 + ... + pk = 1.
- A probability distribution over discrete variables may be described using a probability mass function (PMF).

Function of random variable

- **Mean of discrete random variable:**

$$\mu_X = x_1 p_1 + x_2 p_2 + \cdots + x_k p_k$$
$$= \sum x_i p_i$$

If X is a random variable with mean E(X) = μ, then the variance of X is defined by

Var (X) = E ((X – μ)2).

Var (X) = E(X2) – E(X)2

Function of random variable

- Suppose that a coin is tossed twice so that the sample space is S= {HH, HT, TH, TT}. Find the probability function corresponding to the random variable X.

$$(X = 2) = P(HH) = \frac{1}{4}$$

$$P(X = 1) = P(HT) = \frac{1}{4}$$

$$P(X = 1) = P(TH) = \frac{1}{4}$$

$$P(X = 0) = P(TT) = \frac{1}{4}$$

$$P(X = 1) = P(HT) = \frac{1}{4} + P(X = 1) = P(TH) = \frac{1}{4} = \frac{1}{2}$$

Function of random variable

❖ **Continuous Random Variables:**

When working with continuous random variables, we describe probability distributions using a probability density function (PDF).

$$\Pr[a \le X \le b] = \int_a^b f_X(x)\,dx.$$

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1.$$

Function of random variable

Let X be a continuous random variable with range [a, b] and probability density function f(x). The expected value of X is defined by,

$$E(X) = \int_a^b x f(x)\, dx.$$

$$E(X) = \sum_{i=1}^{n} x_i p(x_i).$$

Function of random variable

❖ **Uniform Distribution:**

X is uniform on [a, b] if X is equally likely to take any value in the range from a to b. The value of area under the curve is 1.0

$$f(x) = \begin{cases} \dfrac{1}{b-a} & for \quad a \le x \le b \\ 0 & for \quad \text{all other values} \end{cases}$$

Characteristic function and conditional statistics

- A characteristic function is simply the Fourier transform, in probabilistic language. The characteristic function always exists when treated as a function of a real-valued.
- Function is called real-valued if it corresponds to a random variable that is symmetric around the origin.

$$\phi_X(t) = \mathrm{E} \exp(it^\top X)$$

This function is called characteristic function (cf) of X

Characteristic function and conditional statistics

- Probability of an event occurring given that another event has already occurred is called a conditional probability.

$$P(B|A) = P(A \text{ and } B) / P(A)$$

The probability that event B occurs, given that event A has already occurred.

Sequence of Random Variables

- A random sequence or a discrete-time random process is a sequence of random variables X1,⋯,Xn. For sample space S,

$$S = \{s_1, s_2, \cdots, s_k\}.$$

Then, a random variable X is a mapping that assigns a real number to any of the possible outcomes si, i=1,2,⋯,k. Thus, we may write

$$X(s_i) = x_i, \qquad \text{for } i = 1, 2, \cdots, k.$$

Rayleigh distribution

- It is a continuous probability distribution for nonnegative-valued random variables. The probability density function and cumulative distribution are represented as:

$$p(x) = \frac{x}{q^2} \exp\left(-\frac{x^2}{2q^2}\right)$$

$$F(x) = 1 - \exp\left(-\frac{x^2}{2q^2}\right)$$

Rice Distribution

- A Rice distribution is one way to model the paths scattered signals take to a receiver. Specifically, this distribution models line-of-sight scatter (FM, radio waves, microwaves) — transmissions between two stations in view of each other that have an unobstructed path between them.

Lognormal Distribution

- It is a continuous probability distribution of a random variable whose logarithm is normally distributed. If the random variable X is log-normally distributed, then Y = ln(X) has a normal distribution.
- Amounts of rainfall, Size distributions of rainfall droplets, the volume of gas in a petroleum reserve are the some natural phenomenon where the used of lognormal distribution can be seen..

Poisson distribution

- It is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time.
- For an example is the number of diners in a certain restaurant every day. If the average number of diners for seven days is 500, you can predict the probability of a certain day having more customers.
- The number of cases of a disease in different towns.
- The Poisson distribution can be applied to systems with a large number of possible events, each of which is rare.

Poisson distribution

$$P(X = x) = e^{-\lambda}\frac{\lambda^x}{x!} \qquad x = 0, 1, 2, 3, 4, \ldots$$

Where X=Random variable

Λ= mean or rare event

e=exponential constant=2.7

Poisson distribution

Using the Poisson distribution $\mathrm{P}(X = r) = \mathrm{e}^{-\lambda}\frac{\lambda^r}{r!}$ write down the formulae for $\mathrm{P}(X = 0)$, $\mathrm{P}(X = 1)$, $\mathrm{P}(X = 2)$ and $\mathrm{P}(X = 6)$, noting that $0! = 1$.

$$\mathrm{P}(X=0) = e^{-\lambda} \times \frac{\lambda^0}{0!} = e^{-\lambda} \times \frac{1}{1} \equiv e^{-\lambda}$$

$$\mathrm{P}(X=1) = e^{-\lambda} \times \frac{\lambda}{1!} = \lambda e^{-\lambda}$$

$$\mathrm{P}(X=2) = e^{-\lambda} \times \frac{\lambda^2}{2!} = \frac{\lambda^2}{2} e^{-\lambda}$$

$$\mathrm{P}(X=6) = e^{-\lambda} \times \frac{\lambda^6}{6!} = \frac{\lambda^6}{720} e^{-\lambda}$$

Poisson distribution

Mean (Expectation) and Variance of Poisson Distribution is given that:

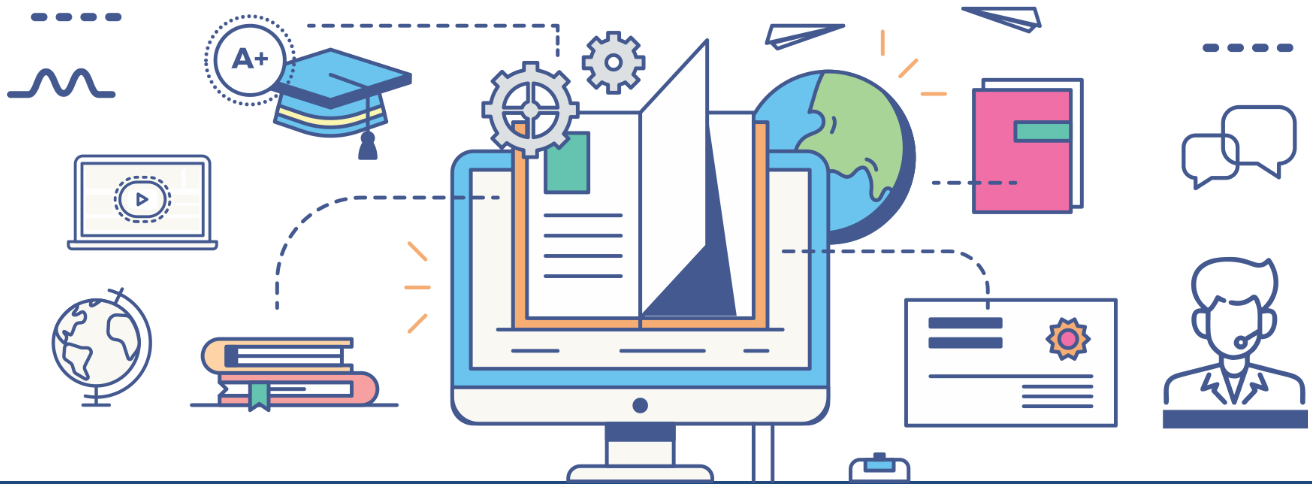
$$\mu = \lambda$$
$$\sigma = \sqrt{\lambda}$$

$$E(X) = np \quad \text{and} \quad V(X) = npq \approx np$$

References

1. https://nptel.ac.in/content/storage2/courses/108106083/lecture11_Part_Two_Types_Of_Random_Variables.pdf
2. https://link.springer.com/chapter/10.1007/978-1-4613-8658-2_8
3. https://www.stat.pitt.edu/stoffer/tsa4/intro_prob.pdf
4. http://www.stat.cmu.edu/~larry/=stat705/Lecture4.pdf.
5. https://internal.ncl.ac.uk/ask/numeracy-maths-statistics/images/Poisson_dist.pdf

DIGITAL LEARNING CONTENT



Parul® University