Digital Image Processing
2nd Edition

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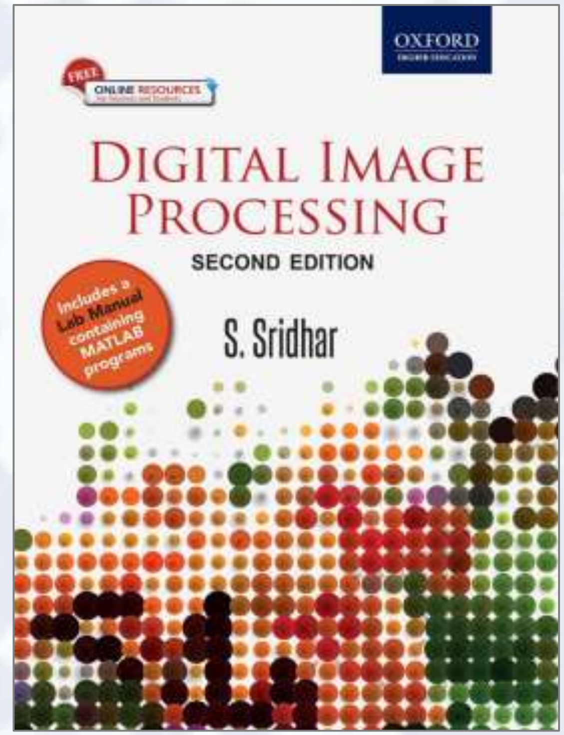


Chapter 11

Image Morphology

Need for morphological processing

- Mathematical morphology is a very powerful tool for analysing the shapes of the objects present in images.
- Morphological operators take a binary image and a mask known as a structuring element as inputs. Then the set operators such as intersection, union, inclusion, and complement can be applied to the images.

Structuring Elements

- Structuring elements are small images that are used to probe the original image.

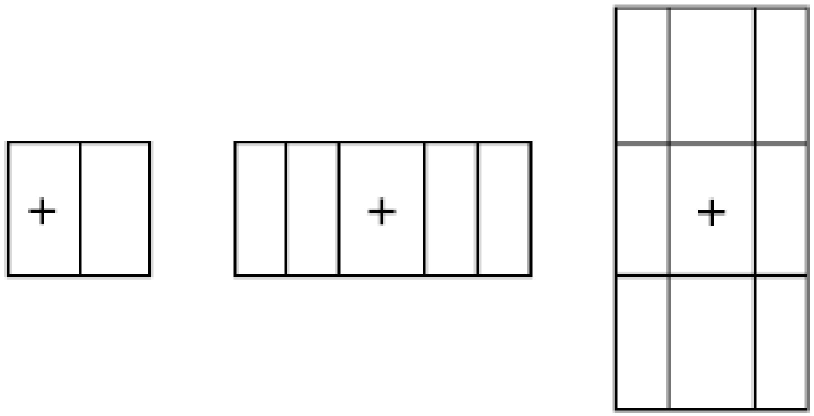


Fig. 11.1 Some general structuring elements (origin marked with +)

MORPHOLOGICAL OPERATORS

- The basic morphological operators are *dilation* and *erosion*.

Let us assume that A and B are two sets of pixels. Then the dilation of A by B can be denoted as

$$A \oplus B = \bigcup_{x \in B} A_x$$

This means that A is translated by every point of the set B. An equivalent definition is

$$A \oplus B = \{(x, y) + (u, v) : (x, y) \in A, (u, v) \in B\}$$

Dilation

Dilation can be considered as a union operation of all the translations of the image A caused by the elements specified in the structuring element B.

$$A \oplus B = \bigcup_{b \in B}^{n} (A)_b$$

Erosion

Erosion is another important operation of mathematical morphology. The objective of this operator is to make an object smaller by removing its outer layer of pixels. If a black pixel has a white neighbour, then all the pixels are made white. This can be expressed mathematically as

$$A \ominus B = \cap \ A_B$$

Algorithms for Dilation and Erosion

Let the number of pixels in the structuring element be k. Let the number of pixels of value 1 in the input image be z. Let the pixel coordinates beneath the origin of the structuring element be (x_0, y_0).

For dilation, the output is given by

$$g(x_0, y_0) = \begin{cases} 1 & \text{for } z > 0 \\ g(x_0, y_0) & \text{otherwise} \end{cases}$$

The output of the erosion operation is

$$g(x_0, y_0) = \begin{cases} 1 \text{ for } z = k \\ 0 \text{ for } z < k \end{cases}$$

Another Approach

Fit A structuring element is said to fit when for each of its elements with value 1, the corresponding element in the set also has the value 1. Set elements whose values are 0, define points where the corresponding elements of the image are irrelevant. For example, if the structuring element is {1 1 1; 1 1 1; 1 1 1} and the image is also {1 1 1; 1 1 1; 1 1 1}, then this represents the fit condition.

Hit This condition is satisfied when there is an exact match between the image and the structuring element for the values of 1 only.

Miss This is a condition where not all the elements of the structuring element match with the elements of the image. For example for $f = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$, the structuring element

$s_1 = \begin{pmatrix} & 1 & \\ 1 & 1 & 1 \\ & 1 & \end{pmatrix}$ is a hit, whereas the structuring element $s_2 = \begin{pmatrix} & 0 & \\ 1 & 1 & 1 \\ & 1 & \end{pmatrix}$ is a miss.

Another Approach

The dilation of an image *f* using a structuring element *s* is written as

$$g(x, y) = \begin{cases} 1 \text{ if } s \text{ hits } f \\ 0 \text{ otherwise} \end{cases}$$

This operation produces a new binary image showing the locations where the structuring element hits the input image.

The erosion of the image *f* by the structural element *s* is defined as a situation where the following rule is applicable:

$$g(x, y) = \begin{cases} 1 \text{ if } s \text{ fits } f \\ 0 \text{ otherwise} \end{cases}$$

Another Approach - Dilation

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} X & X & X \\ 1 & 1 & 1 \\ X & X & X \end{bmatrix}, \begin{bmatrix} X & 1 & X \\ X & 1 & X \\ X & 1 & X \end{bmatrix}, \begin{bmatrix} X & X & 1 \\ X & 1 & X \\ 1 & X & X \end{bmatrix}$$

Fig. 11.4 Masks for the erosion operator

Table 11.1 Dilation as a convolution operation

Input pixel value	Neighbour's value	Output value
1	All neighbours have value 1	1 (Hit)
1	Some are 1 and some are 0	1 (Hit)
0	Some are 1 and some are 0	1 (Hit)
0	All neighbours have value 0	0 (Miss)

Erosion

$$\begin{bmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{bmatrix}, \begin{bmatrix}X & X & X\\1 & 1 & 1\\X & X & X\end{bmatrix}, \begin{bmatrix}X & 1 & X\\X & 1 & X\\X & 1 & X\end{bmatrix}, \begin{bmatrix}X & X & 1\\X & 1 & X\\1 & X & X\end{bmatrix}$$

Fig. 11.4 Masks for the erosion operator

Table 11.2 Erosion operator

Input pixel value	Neighbour's value	Output value
1	All neighbours have value 1	1 (Hit)
1	Some are 1 and some are 0	0 (Miss)
0	Some are 1 and some are 0	0 (Miss)
0	All neighbours have value 0	0 (Miss)

Commutative property This is stated as

$$A \oplus B = B \oplus A$$

Associative property This is stated as

$$A \oplus (B \oplus C) = (A \oplus B) \oplus C$$

$$A \ominus (B \ominus C) = (A \ominus B) \ominus C$$

Distributive property This is stated as

$$A \oplus (B \oplus C) = (A \oplus B) \cup (A \oplus C)$$

$$A \ominus (B \ominus C) = (A \ominus B) \cup (A \ominus C)$$

Duality property Erosion and dilation are complementary.

$$(A \ominus B)^C = A^C \oplus \hat{B}$$

$$(A \ominus B)^c = A^c \ominus \hat{B}$$

Translation property This property implies that erosions and dilations are invariant to translation and preserve the order relationship between the images.

$$(A+h) \oplus B = (A \oplus B) + h$$

$$(A+h) \ominus B = (A \ominus B) + h$$

Decomposition property Suppose $B = B_1 + B_2$, then

$$\begin{aligned}(A \oplus B) &= A \oplus (B_1 \oplus B_2) \\ &= (A \oplus B_1) \oplus B_2\end{aligned}$$

When B is decomposed into B_1 and B_2, the following additional properties hold true:

1. $(A \oplus B_1) \oplus B_2 = A \oplus (B_1 \oplus B_2)$
2. $(A \ominus B_1) \ominus B_2 = A \ominus (B_1 \ominus B_2)$

Opening/Closing

The *opening operation* is an erosion operation followed by a dilation operation. This operation can be defined as

$$A \circ B = (A \ominus B) \oplus B$$

This is equivalent to

$$A \circ B = \cup \ \{B_w; B_w \subseteq A\}$$

Closing is a dilation operation followed by an erosion operation. This can be denoted as

$$A \bullet B = (A \oplus B) \ominus B$$

Properties of Opening/Closing

The properties of the opening and closing operations are as follows:

1. Dual transformation:

$$(A \bullet B)^C = A^C \circ \hat{B}$$

$$(A \circ B)^C = (A^C \bullet \hat{B})$$

2. Ordering relationship:

$$A \circ B \leq A \leq A \bullet B$$

3. Increasing transformations:

$$A_1 \subseteq A_2 \Rightarrow A_1 \circ B \subseteq A_2 \circ B$$

$$A_1 \subseteq A_2 \Rightarrow A_1 \bullet B \subseteq A_2 \bullet B$$

4. Transform invariance:

$$(A + h) \circ B = (A \circ B) + h$$

$$(A + h) \bullet B = (A \bullet B) + h$$

5. Idempotence:

$$(A \circ B) \circ B = A \circ B$$

$$(A \bullet B) \bullet B = A \bullet B$$

Hit or miss Transform

	1	
0	1	1
0	0	

	1	
1	1	0
	0	0

	0	0
1	1	0
	1	

0	0	
0	1	1
	1	

Fig. 11.9 Structuring elements used for detecting corners

Algorithm

1. Translate the centre of the structuring element to all the points of the input image.
2. Compare the structuring element with the image pixels.
3. If there is a complete match, then

   The pixel underneath the structuring element (i.e., the image pixel that coincides with the centre of the structuring element) is set to the foreground colour. This operation is called a hit.

   Else

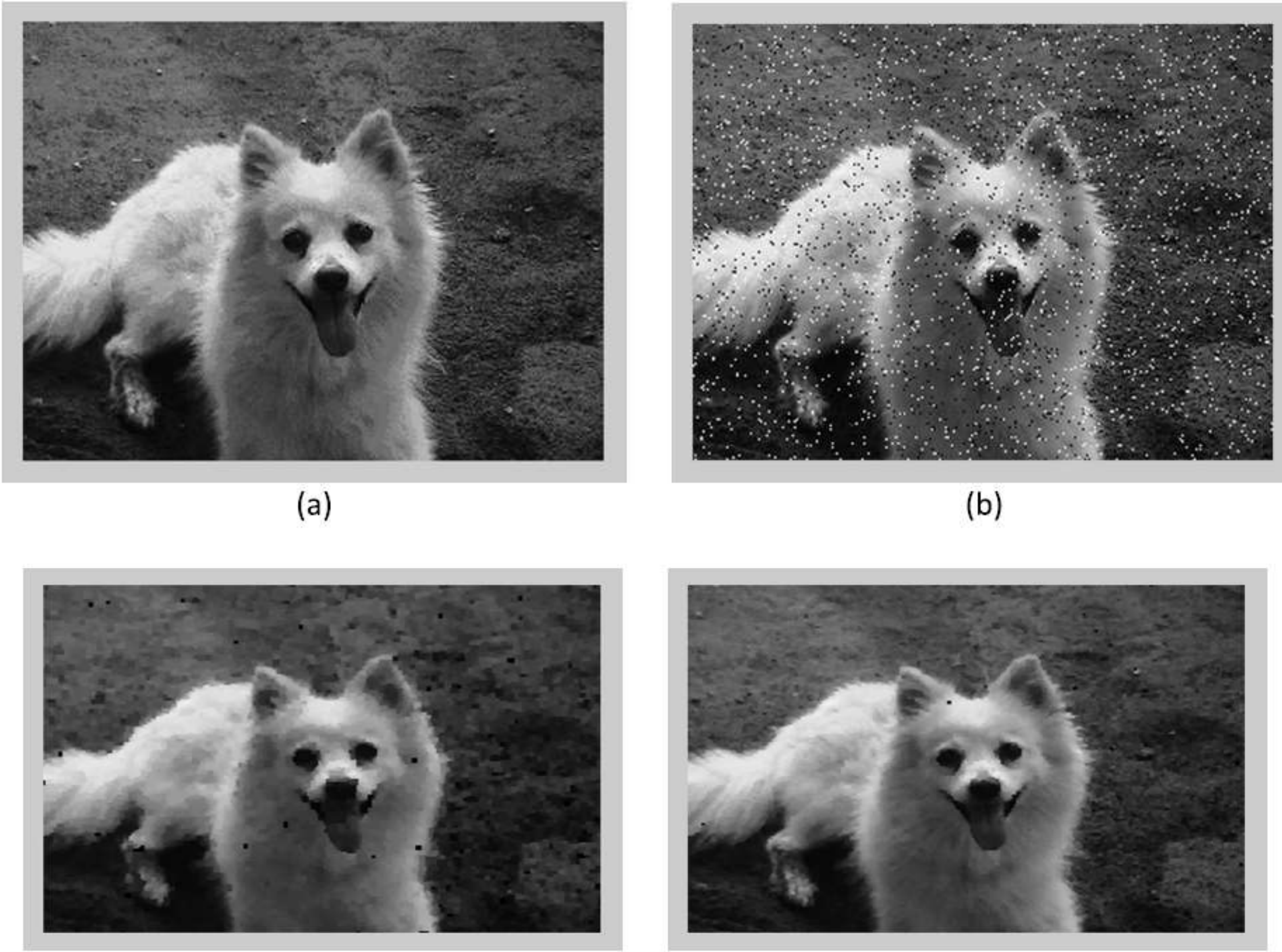
   The pixel underneath the structuring element (i.e., the image pixel that coincides with the centre of the structuring element) is set to the background colour. This operation is called a miss.

Applications – Boundary Extraction

1. Internal boundary: $A - (A \ominus B)$
2. External boundary: $(A \oplus B) - A$

Noise Removal

- The morphological opening followed by a closing operation can remove the noise.



(a) (b) (c) (d)

Fig. 11.12 (Contd) (c) Result with 5 × 5 mask (d) Result with 3 × 3 mask

Thinning

The thinning operation can be expressed in terms of the hit-or-miss transform as

$$\text{Thinning}(A, B) = A - \text{Hit-or-miss transform}(A, B)$$

Here A is the image and B is the structuring element. The operation subtraction here is logical subtraction (i.e., $A - B = A \cap NOT\ (B)$), that is, a set difference operation.

Thickening

The relationship between the thickening operation and the hit-or-miss transform can be expressed as

$$\text{Thickening}(A, B) = A \cup \text{Hit-or-miss transform}(A, B)$$

Here A is the image and B is the structuring element.

Convex Hull

- A region is called convex if a line drawn between any two parts of the region, also lies within the region. *Convex hull* is the smallest polygon that encompasses the region completely like an elastic band or an envelope.

Algorithm

Let B_i be the structuring element where $i = 1, 2, 3$, and 4. The convex hull involves a process

$$X_k^i = (X_{k-1}^i \otimes B^i) \cup A$$

for $k = 1, 2, 3, \ldots, n$.

This process is repeated till there is a convergence. Let D^i be X^i.

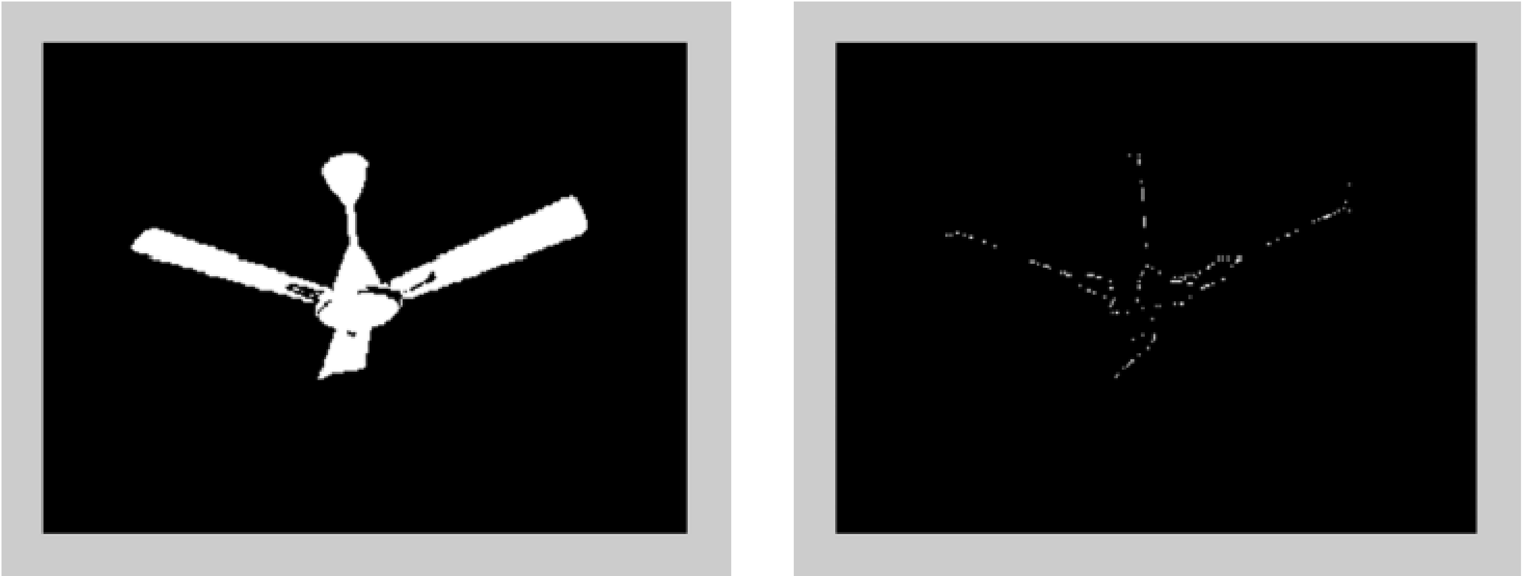
In other words, use the structuring element repeatedly and apply the hit-or-miss transform till there is no change. Then repeat this with the next structuring element which is a rotation of the previous mask by 90°. Perform this for the entire structuring element, producing the intermediate D. Perform union of all four parts of D to form the convex hull, that is, the connected component $C(A) = \bigcup_{i=1}^{4} D^i$.

Skeletonization

- A skeleton (or medial axis) is a geometrical description that describes a shape using lesser number of pixels than the original. Skeletonization is useful in applications where image recognition and compression are used.

$$\text{Skeleton}(S) = \bigcup_{n=0}^{\infty} \text{skel}(S)$$

$$\text{Skeleton}(S) = \bigcup_{n=0}^{\infty} (S \ominus nB) - [(S \ominus nB) \circ B].$$



(a) (b)

Fig. 11.13 Skeletonization (a) Original image (b) Result with 3 × 3 mask

Medial Axis Transform and Distance Transform

- Another way of skeletonizing the image is by using the distance transform. The distance transform takes a binary image as input and generates a grey scale image as output, where each pixel represents the distance between that pixel and the nearest background pixel. The ridge of the distance transform output is the skeleton of the object.

0	0	0	1	1	1	0	0	0	0

(a)

0	0	0	1	2	3	0	0	0	0

(b)

0	0	0	1	2	1	0	0	0	0

(c)

Fig. 11.14 Distance transform (a) Original 1D image (b) Scanning from left to right—stores the minimum distance (c) Scanning from right to left and final result

Distance Transform

1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1

(a)

1	1	1	1	1	1
1	2	2	2	2	1
1	2	2	2	2	1
1	2	2	2	2	1
1	1	1	1	1	1

(b)

1	1	1	1	1	**1**
1	**2**	2	2	**2**	1
1	2	**3**	**3**	2	1
1	**2**	2	2	**2**	1
1	1	1	1	1	**1**

(c)

Fig. 11.15 Distance transform (a) Original 2D image (b) First iteration (c) Distance transform result—ridge highlighted

$$\begin{pmatrix} 4 & 3 & 4 \\ 3 & 0 & \\ & & \end{pmatrix}\begin{pmatrix} & & \\ & 0 & 3 \\ 4 & 3 & 4 \end{pmatrix}$$

(a)

$$\begin{pmatrix} & 11 & & 11 & \\ 11 & 7 & 5 & 7 & 11 \\ & 5 & 0 & & \\ & & & & \\ & & & & \end{pmatrix}\begin{pmatrix} & & & & \\ & & & & \\ & & 0 & 5 & \\ 11 & 7 & 5 & 7 & 11 \\ & 11 & & 11 & \end{pmatrix}$$

(b)

Fig. 11.17 Chamfer distance masks (a) Chamfer distance using 3 × 3 mask shown as two masks (forward and backward) (b) Chamfer distance using 5 × 5 mask shown as two masks (forward and backward)

Region Filling

Region filling is the process of filling up an image. The process that achieves this is as follows:

1. Let p be the point of the region to be filled. Initially $p = X_0$. Let A be subset of elements that represents the region. Let k denote the iteration.
2. Let B be the structuring element.
3. Repeat steps 4–6.
4. Set $k = k + 1$.
5. $X_k = (X_{k-1} \oplus B) \cap A^C$ and store the result.
6. If $X_k = X_{k-1}$, then stop.
7. The union of the final X_k with the original image gives the filled region.
8. Exit.

Extraction of Connected Component

1. Let p be a point in the connected component C that needs to be extracted. Let $p = X_0$.
2. Let B be the structuring element.
3. Repeat the steps 4–6.
4. $k = k + 1$.
5. $X_k = (X_{k-1} \oplus B) \cap A$

   The difference with the previous algorithm is that A is used instead of its complement.
6. If $X_k = X_{k-1}$, then stop.
7. Assign $c = X_k$
8. Exit.

Pruning

- Morphological operations create some tail pixels that affect the topology of the object. These pixels are also called spurs or parasitic components. The process of removing these extra tail pixels is called *pruning*.

Pruning

$$B_1 = \begin{pmatrix} 0 & 0 & 0 \\ X & 1 & X \\ 1 & 1 & 1 \end{pmatrix}, B_2 = \begin{pmatrix} X & 0 & 0 \\ 1 & 1 & 0 \\ X & 1 & X \end{pmatrix}, B_3 = \begin{pmatrix} 1 & X & 0 \\ 1 & 1 & 0 \\ 1 & X & 0 \end{pmatrix}, B_4 = \begin{pmatrix} 1 & 1 & X \\ 1 & 1 & 0 \\ X & 0 & 0 \end{pmatrix}$$

$$B_5 = \begin{pmatrix} 1 & 1 & 1 \\ X & 1 & X \\ 0 & 0 & 0 \end{pmatrix}, B_6 = \begin{pmatrix} X & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & X \end{pmatrix}, B_7 = \begin{pmatrix} 0 & X & 1 \\ 0 & 1 & 1 \\ 0 & X & 1 \end{pmatrix}, B_8 = \begin{pmatrix} 0 & 0 & X \\ 0 & 1 & 1 \\ X & 1 & 1 \end{pmatrix}$$

Fig. 11.20 Structuring element masks

GREY SCALE MORPHOLOGY

$$B = \begin{pmatrix} X_3 & X_2 & X_1 \\ X_4 & X & X_0 \\ X_5 & X_6 & X_7 \end{pmatrix}$$

Fig. 11.21 Erosion mask for grey scale morphology

$$\begin{aligned} O(x, y) = A \ominus B = \text{minimum}(&X + A(x, y), X_0 + A(x+1, y), X_1 + A(x+1, y-1), \\ &X_2 + A(x, y-1), X_3 + A(x-1, y-1), X_4 + A(x-1, y), \\ &X_5 + A(x-1, y+1), X_6 + A(x, y+1), X_7 + A(x+1, y+1)) \end{aligned}$$

Dilation

Grey scale dilation For dilation, the mask value can range from 0 to 255. The output value is the maximum value of all the nine addends. This is given as

$$\begin{aligned} O(x, y) = A \oplus B = \text{maximum}(&X + A(x, y), X_0 + A(x+1, y), X_1 + A(x+1, y-1), \\ &X_2 + A(x, y-1), X_3 + A(x-1, y-1), X_4 + A(x-1, y), \\ &X_5 + A(x-1, y+1), X_6 + A(x, y+1), X_7 + A(x+1, y+1)) \end{aligned}$$

Dilation Vs Erosion

Table 11.3 Comparison of grey scale erosion and dilation operations

Erosion	Dilation
Reduces the size of the objects with respect to the background	Increases the size of the objects
Eliminates noise spikes and ragged edges	Eliminates noise spikes and ragged edges
Darkens the bright objects	Brightens the objects
Increases the size of holes and sharpens corners	Connects objects, bridges gaps, smoothens edges, fills holes, and creates outlines in an image

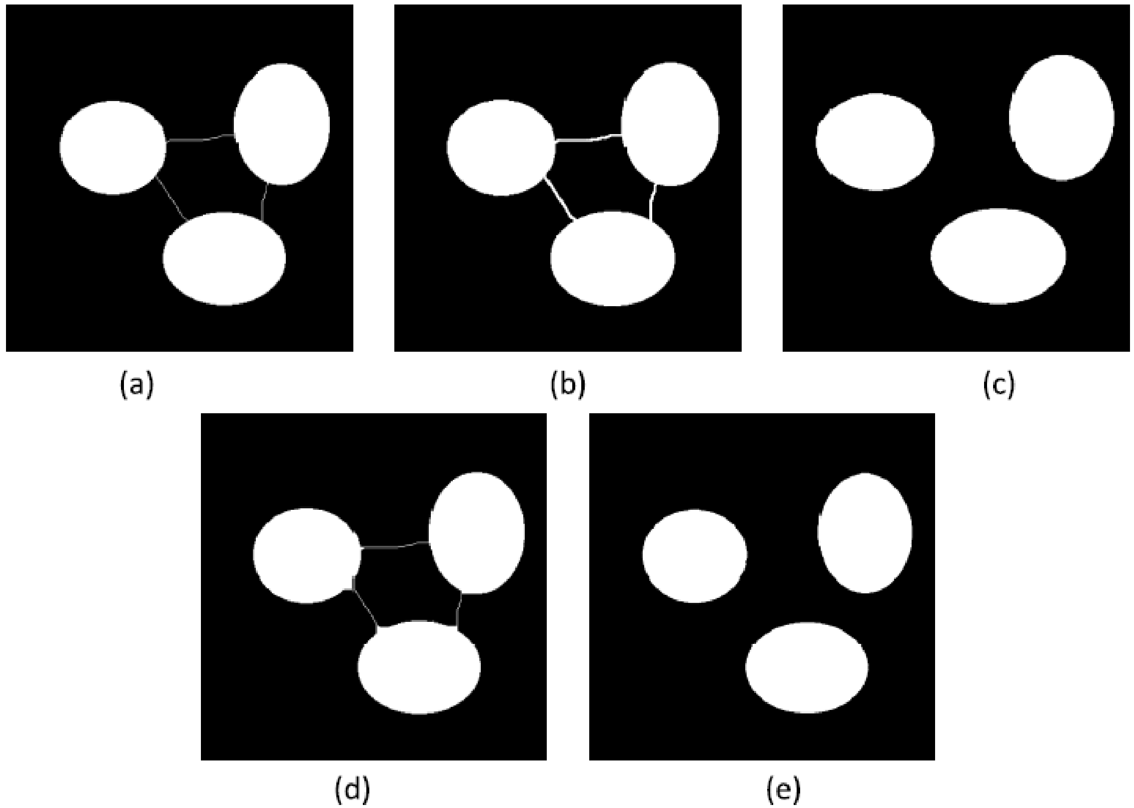


Fig. 11.22 Results of the grey scale morphology operations (a) Original image (b) Dilation (c) Erosion (d) Opening (e) Closing

Morphological Gradient

1. Create a duplicate of the original image. Let it be B.
2. Apply the erosion operation to the original image.
3. Apply the dilation operation to the duplicate image.
4. Edge = dilated image – eroded image, that is,

$$(A \oplus B) - (A \ominus B)$$

5. Display the gradient image.
6. Exit.

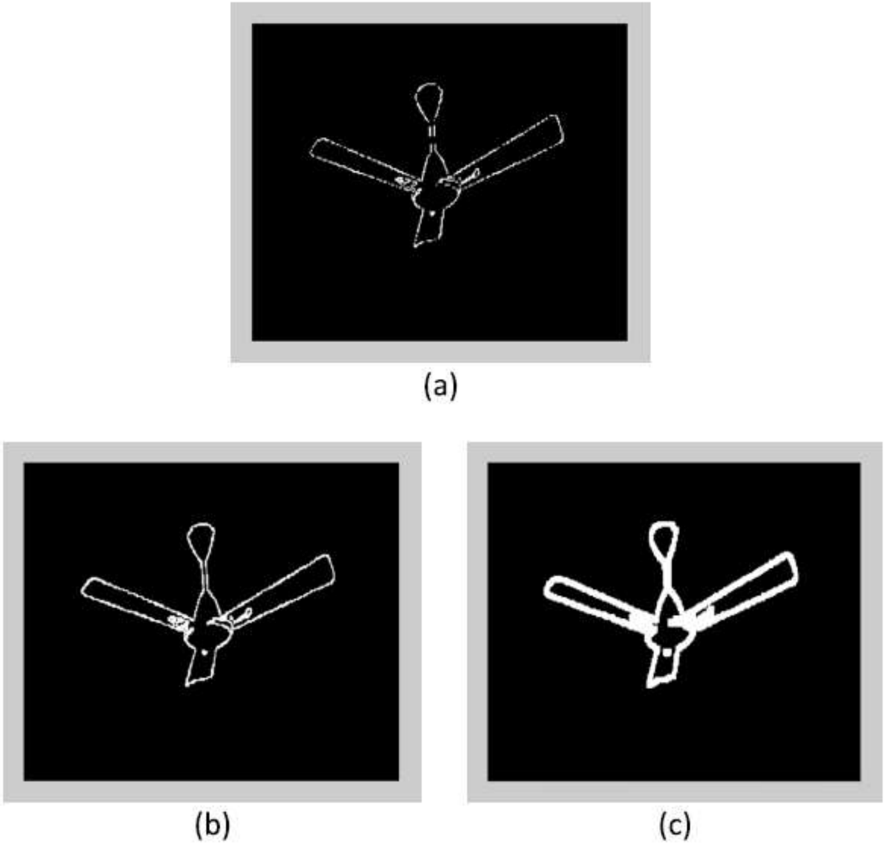


Fig. 11.23 Results of the grey scale morphology gradient operation (a) Gradient with 3×3 mask (b) Gradient with 5×5 mask (c) Gradient with 13×13 mask

Morphological Gradient

Morphological gradient is also used to sharpen an image whose edges are replaced by peaks. It is expressed as a sharpening operation as follows:

$$\text{Morphological gradient} = \frac{1}{2}(\text{Max}(\text{Image}) - \text{Min}(\text{Image}))$$

Top-hat and Well Transformations

1. Apply the grey scale opening operation to an image.
2. Peak = original image – opened image.
3. Display the peak.
4. Exit.

Algorithm

1. Apply the grey scale closing operation to an image.
2. Valley = original image – closed image.
3. Display the valley.
4. Exit.

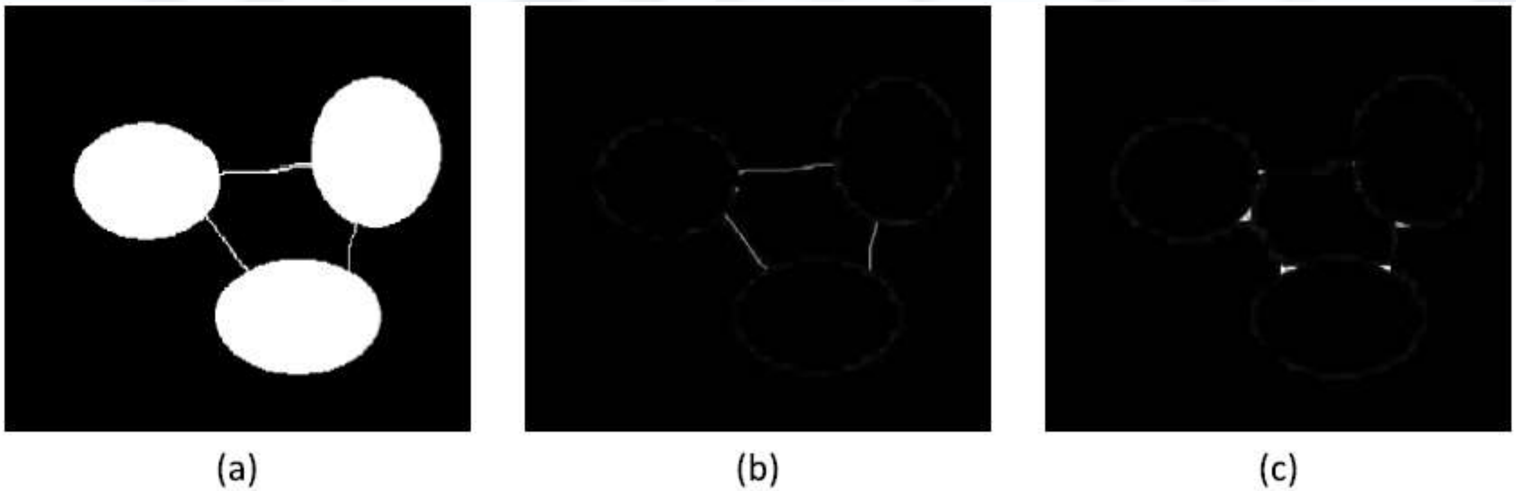


Fig. 11.25 Top-hat and well transformation (a) Original image (b) Top-hat transformation (c) Well transformation

Morphological Reconstruction

- This procedure involves two images. One image is the given original image and the other is called the marker. The marker is the starting point of the transformation. Only elementary operations such as erosion and dilation are used. The transformations force the original image to remain within the limits of the second image.

Watershed Algorithm

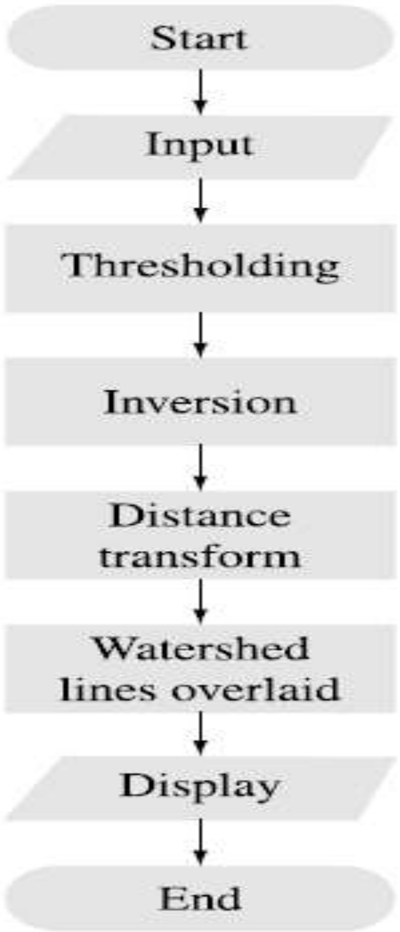
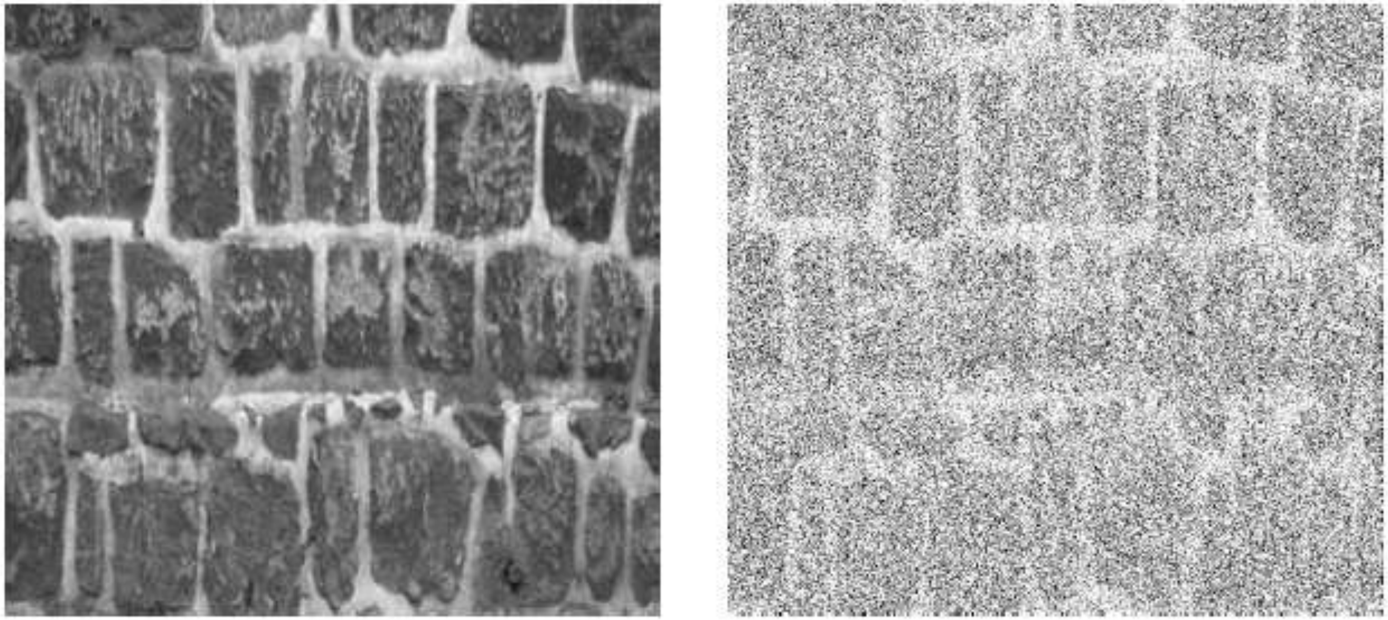


Fig. 11.27 Watershed segmentation process



(a) (b)

Fig. 11.28 Watershed segmentation (a) Original image (b) Segmented image

SUMMARY

- Image morphology is an important tool in the extraction of image features. Image features are necessary for the recognition of objects.
- The theory of mathematical morphology is based on set theory. We can visualize the binary object as a set. Then set theory can be applied to the sample set.
- The structuring element is usually a matrix of size 3×3. It has its origin at the centre of the matrix. It is then shifted over the image and at each pixel of the image its elements are compared with the set of the underlying pixels.
- Dilation is one of the two basic morphological operators. It can be applied to binary as well as grey scale images. The basic effect of the operator on a binary image is that it gradually increases the boundaries of the region while the small holes that are present in the image become smaller.
- One can view the morphological operations as a binary correlation operation involving logical elements.
- The difference between the original image and the eroded image creates a boundary.
- Opening is an erosion operation followed by a dilation operation.
- Closing is a dilation operation followed by an erosion operation.
- The hit-or-miss transform is a general binary morphological operation that can be used to look for particular patterns of foreground and background pixels in an image.
- Thinning is a morphological operator that is used to remove irrelevant foreground pixels that are present in binary images.
- The idea of binary image can be extended to grey scale images. A grey scale image is the input for grey scale morphology operations. Similar to binary morphology operations, the mask moves across the image. The pixel-by-pixel process is performed and the resultant is produced in the output image. The minimum value of the factors is used for the erosion operation and the maximum value is used for the dilation operation.
- The combined operation of grey scale erosion followed by grey scale dilation is called opening operation. It is denoted as $A \circ B$. Similarly,

the combined operation of grey scale dilation followed by the grey scale erosion is called grey scale closing and is denoted as $A \bullet B$.

- Top-hat transformation is a variant of grey scale opening and closing operations that produces only the bright peaks of an image.
- Well transformation is the opposite of the top-hat transformation and produces the dark valleys of an image (i.e., the dark features in the image).