

Discrete Mathematics

Mathematical Logic

DPP-04

[MSQ]

1. Let $R(x, y, z)$ denote the statement
 “ $x + y = z$ ”
 Which of the following proposition will evaluate truth value True?
- (a) $R(1, 2, 3)$ (b) $R(0, 0, 1)$
 (c) $R(1, 1, 2)$ (d) $R(2, 3, 4)$

[MCQ]

2. Let $p(x), q(x)$ denote the following open statements.
 $p(x): x \leq 3$ $q(x): x + 1$ is odd
 If the universe consists of all integers, what are the truth values of the following statements?
- $S_1: \sim(p(-4) \vee q(-3))$
 $S_1: \sim(p(-4) \wedge \sim q(-3))$
- (a) S_1 : True, S_2 : False
 (b) S_1 : False, S_2 : True
 (c) S_1 : True, S_2 : True
 (d) S_1 : False, S_2 : False

[NAT]

3. Let $p(x), q(x)$ denote the following open statements.
 $p(x): x + 1 > x$ $q(x): x^2 > 0$
 How many expressions evaluate to True?
- I. $p(3) \vee [q(3) \vee \sim p(3)]$
 II. $p(2) \rightarrow [q(2) \rightarrow p(2)]$
 III. $[p(2) \rightarrow q(2)] \wedge p(-3)$

[MSQ]

4. Consider the english sentence
 “You can not ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old”.
 q : you can ride the roller coaster
 r : you are under 4 feet tall
 s : you are older than 16 years old.
 Which of the following correctly represent the logical expression for the sentence?
- (a) $q \rightarrow \sim(r \wedge \sim s)$
 (b) $(r \vee \sim s) \rightarrow q$
 (c) $(r \wedge \sim s) \rightarrow \sim q$
 (d) None of these

[MCQ]

5. Let $p(x)$ be the statement
 “ $x + 1 > x$ ”
 Now, consider the truth value of quantification, where the domain consists of all real number.

$$L_1 = \forall x p(x)$$

$$L_2 = \exists x p(x)$$

Which of the following evaluate to True?

- (a) L_1 only
 (b) L_2 only
 (c) Both L_1 and L_2 are True
 (d) Neither L_1 nor L_2

Answer Key

1. (a, c)
2. (d)
3. (3)

4. (a, c)
5. (c)



Hints and Solutions

1. (a, c)

I. The proposition $R(1, 2, 3)$ is obtained by setting $x=1, y=2$ and $z=3$ in the statement $R(x, y, z)$

So, $R(1, 2, 3) \equiv 1 + 2 = 3 \equiv \text{True}$

II. $R(1, 1, 2) \equiv 1 + 1 = 2 \equiv \text{True}$

Hence, option a and c is correct.

2. (d)

Statement S_1 :

$$\sim(p(-4) \vee q(-3))$$

$$\begin{array}{ccc} \Downarrow & & \Downarrow \\ -4 \leq 3 & & -3 + 1 = -2 \text{ is not odd} \end{array}$$

$$\therefore \sim(\text{True} \vee \text{False})$$

$$\therefore \sim(\text{True}) \equiv \text{False}$$

Statement S_2 :

$$\sim p(-4) \wedge \sim q(-3)$$

$$\sim(\text{True}) \wedge \sim(\text{False})$$

$$\therefore \text{False} \wedge \text{True} = \text{False}$$

Hence, option d is correct

3. (3)

I.

$$p(3) \vee [q(3) \vee \sim p(3)]$$

$$\begin{array}{ccc} \Downarrow & & \Downarrow \\ 3+1 > 3 & & 3^2 > 0 \end{array} \quad \text{True}$$

$$\begin{array}{cc} \Downarrow & \Downarrow \\ \text{True} & \text{True} \end{array}$$

$$\therefore \text{True} \vee [\text{True} \vee \sim \text{True}]$$

$$\therefore \text{True} \vee \text{True} = \text{True}$$

II.

$$\begin{array}{ccc} p(2) \rightarrow [q(2) \rightarrow p(2)] \\ \Downarrow \quad \quad \quad \Downarrow \quad \quad \quad \Downarrow \\ \text{True} \quad \quad \quad \text{True} \quad \quad \quad \text{True} \end{array}$$

$$\therefore \text{True} \rightarrow [\text{True} \rightarrow \text{True}] \equiv \text{True}$$

III.

$$\begin{array}{ccc} [p(2) \rightarrow q(2)] \wedge p(-3) \\ \Downarrow \quad \quad \quad \Downarrow \quad \quad \quad \Downarrow \\ \text{True} \quad \quad \quad \text{True} \quad \quad \quad \text{True} \end{array}$$

$$\therefore [\text{True} \rightarrow \text{True}] \wedge \text{True} \equiv \text{True}$$

4. (a, c)

I. Let q, r , and s represents:

q : you can ride the roller coaster

r : you are under 4 feet tall

s : you are older than 16 years old.

\therefore The sentence can be translated to

$$(r \wedge \sim s) \rightarrow \sim q$$

II. An implication and its contrapositive always have the same truth value.

So, $q \rightarrow \sim(r \wedge \sim s)$ also represent the sentence.

5. (c)

I. $L_1 = \forall x p(x) : \text{True}$

Here $p(x)$ is true for all real number x , so, the quantification $\forall x p(x)$ is True.

II. $L_2 = \exists x p(x) : \text{True}$

Here $p(x)$ is true for all real number
Thus, it will also true for same.



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