Branch: CSE/IT

Batch: English

Discrete Mathematics Mathematical Logic

DPP-02

[MSQ]

- 1. Which of the following is/are logical equivalence?
 - I. $\sim (p \rightarrow q)$
 - II. $\sim (p \rightarrow q) \land (q \rightarrow r)$
 - III. $p \land \sim q$
 - IV. $(p \lor q) \rightarrow r$
 - (a) I and II
- (b) I and III
- (c) II and IV
- (d) II and III

[MSQ]

2. Consider the following statement

$$S_1: (p \to q) \land (p \to r)$$

$$S_2: p \to (q \land r)$$

Which of the following is True?

- (a) S_1 is tautology
- (b) S_1 is contingency
- (c) S_1 is logically equivalence to S_2
- (d) None of these

[MSQ]

- **3.** Which of the following is logically equivalence?
 - (a) $(p \rightarrow r) \lor (q \rightarrow r)$
 - (b) $(p \leftrightarrow q) \lor (q \rightarrow r)$
 - (c) $(p \land q) \land r$
 - (d) $(p \leftrightarrow r) \land (q \leftrightarrow r)$

[MCQ]

4. Consider the following statement

$$S_1$$
: ~ $(p \leftrightarrow q)$

$$S_2: p \leftrightarrow \sim q$$

Which of the following is correct?

- (a) S_1 is tautology
- (b) S_2 is contradiction
- (c) S_1 is equivalence to S_2
- (d) None of these

[MCQ]

5. Consider the following statement

$$S_1$$
: ~ $(p \lor (\sim p \land q))$

$$S_2$$
: ~ $p \land ~ q$

Which of the following is correct?

- (a) S_1 is tautology
- (b) S_2 is contradiction
- (c) S_1 is equivalence to S_2
- (d) S_1 is not equivalence to S_2

Answer Key

- (b, c) 1.
- 2. (b, c)
- (a, b, c)

- 4. (c) 5. (c)



Hints and solutions

1. (b, c,)

Two statements forms are logical equivalent if and only if their resulting truth values are identical for each variation of statement variables.

I.
$$\sim (p \rightarrow q)$$

= $\sim (\sim p \lor q)$
= $p \land \sim q$

Hence, I is logically equivalent to III.

II.
$$(p \to r) \land (q \to r)$$

$$= (\overline{p} + r) \land (\overline{q} + r)$$

$$= \overline{p} \overline{q} + \overline{p} r + \overline{q} r + r$$

$$= \overline{p} \overline{q} \mid \overline{p} r + r$$

$$= \overline{p} \overline{q} + r$$

$$= (\overline{p \lor q}) + r \equiv (p \lor q) \to r$$

Hence, II and IV are logically equivalence.

2. (b, c)

Statement
$$S_1$$
: $(p \rightarrow r) \land (p \rightarrow r)$

$$= (\overline{p} + q) \land (\overline{p} + r)$$

$$= \overline{p} + \overline{p} r + \overline{p} q + qr$$

$$= \overline{p} + \overline{p} q + qr$$

$$= \overline{p} + qr$$

$$= p \rightarrow (q \land r) \neq 41$$

Hence, S_1 is not tautology and S_1 is logically equivalent to S_2 .

Statement S₂:
$$p \rightarrow (q \land r)$$

= $\overline{p} + (q \land r)$
= $\overline{p} + qr \neq 1$ or 0

Hence, statement S_2 is contingency.

(a, b, c)

Option A:
$$(p \to r) \lor (q \to r)$$

$$= (\overline{p} + r) \lor (\overline{q} + r)$$

$$= \overline{p} + r + \overline{q} + r$$

$$= \overline{p} + \overline{q} + r$$

$$= \overline{pq} + r \equiv (\overline{p \land q}) + r$$

$$\equiv (\overline{p \land q}) \to r$$

So, option A is logically equvalence to option C.

Option B:
$$(p \leftrightarrow r) \lor (q \to r)$$

$$= \overline{p} \ \overline{r} + pr + \overline{q} + r$$

$$= \overline{p} \ \overline{r} + \overline{q} + pr + r$$

$$= \overline{p} \ \overline{r} + \overline{q} + r$$

$$= \overline{p} \ \overline{r} + r + \overline{q}$$

$$= \overline{p} + r + \overline{q} + r$$

$$= (\overline{p} \land q) + r = (p \land q) \to r$$

So, option B is also logically equvalence to option A.

4. (c)

Statement
$$S_1$$
: $\sim (p \leftrightarrow q)$
= $\sim (\overline{p} \overline{q} + pq)$
= $(p+q)(\overline{p} + \overline{q})$
= $p \overline{q} + q \overline{p}$

Statement
$$S_2$$
: $p \leftrightarrow \sim q$
= $\overline{p} q + p \overline{q}$

Hence, S_1 and S_2 are equivalence to each other.

5. (c)

Statement
$$S_1$$
: $\sim (p \lor (\sim p \land q))$
 $= \sim p \land [\sim (\sim p \land q)]$
 $= \sim p \land [\sim (\sim p \lor \sim q)]$
 $= \sim p \land [p \lor \sim q)]$
 $= (\sim p \land p) \lor (\sim p \land \sim q)$
 $= F \lor (\sim p \land \sim q)$
 $= (\sim p \land \sim q)$
 $= \sim p \land \sim q)$

Hence, S_1 is equivalence to S_2 .





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