

# CS & IT ENGINEERING

Algorithms

Analysis of Algorithms

**DPP** (Discussion Notes)



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# TOPICS TO BE COVERED

01 Question

02 Discussion



Q.1

Sort the functions in ascending order of asymptotic (big-O) complexity.

$f_1(n) = n$ ,  $f_2(n) = 80$ ,  $f_3(n) = n^{\log n}$ ,  $f_4(n) = \log \log^2 n$ ,  $f_5(n) = (\log n)^{\log n}$

[MCQ]

A.  $f_2(n)$ ,  $f_4(n)$ ,  $f_1(n)$ ,  $f_5(n)$ ,  $f_3(n)$

B.  $f_2(n)$ ,  $f_1(n)$ ,  $f_4(n)$ ,  $f_5(n)$ ,  $f_3(n)$

C.  $f_2(n)$ ,  $f_1(n)$ ,  $f_4(n)$ ,  $f_3(n)$ ,  $f_5(n)$

D.  $f_1(n)$ ,  $f_1(n)$ ,  $f_4(n)$ ,  $f_3(n)$ ,  $f_2(n)$

$f_2$

$f_5 > f_4$

$\log \log^2 n = \log(\log n * \log n)$

$\log(m * n) = \log m + \log n$   
 $= \log \log n + \log \log n = 2 \log \log n$

$f_4 < f_1$



$$T_4 = 2 \log \log n$$

$$T_5 = \log n^{\log n}$$

$$T_4 \quad | \quad T_5$$

$$\boxed{\log \log n} < \log n^{\log n}$$

$$T_5 > T_4$$

$$\boxed{T_5 > T_1}$$

$$T_1 \quad | \quad T_5$$

$$n \quad \log n^{\log n}$$

$$\log n = \log \log n^{\log n}$$

$$\log n = \log n \times \log \log n$$

Q.2

Consider two function  $f(n) = 10n + 2\log n$  and  $g(n) = 5n + 2(\log n)^2$ , then which of the following is correct option?



[MSQ]

a, b

A.

$f(n) = \theta(g(n))$

B.

$f(n) = O(g(n))$

C.

$f(n) = \omega(g(n^2))$

D.

None of the above



$$f(n) = 10n + 2 \log n = O(n)$$

$$g(n) = \underline{5n} + 2(\log n)^2 = O(n)$$

$$f(n) = O(g(n))$$

$$g(n) = O(f(n))$$

$$\underline{f(n) = g(n)}$$

if  $\Theta$  possible

then  $\Theta \Omega$

~~$\phi \omega$~~

Q.3

Consider two function  $f(n) = \sqrt{n}$  and  $g(n) = n \log n + n$  then  $f(n)/g(n)$  is equivalent to how many of the following given below?

[NAT]

2

A.  $O(n^{-1/2})$  True

B.  $O(n^{-1/2})$  True

C.  $\Omega(1/\log n)$  False

D.  $\theta(n^{-1/2})$  False



$$f(n) = \sqrt{n}$$

$$g(n) = n + n \log n$$

$$\frac{f(n)}{g(n)} = \frac{\cancel{\sqrt{n}}}{\cancel{\sqrt{n}}(1 + \log n)} = \frac{1}{\sqrt{n}(1 + \log n)}$$

$$= \frac{1}{\sqrt{n} + \sqrt{n} \log n}$$

(a)

$$\begin{aligned} f(n) &< c \cdot g(n) \quad 0 \\ f(n) &\leq c \cdot g(n) \quad ① \end{aligned}$$

$$O(n^{-1/2}) = \frac{1}{\sqrt{n}}$$



$$f(n) \quad g(n)$$

$$\frac{1}{\sqrt{n} + \sqrt{n} \log n} = O\left(\frac{1}{\sqrt{n}}\right) \quad \text{True}$$

( < )

( < )

if 0 possible then 0 also possible

if w 11      " R " 11

Q.4

[MCQ]



Consider the following C-code

```
void foo (int n)
{
    int a = 1;
    if (n == 1)
        return;
    for (; a ≤ n; a++)
    {
        printf("GATEWALLAH");
        break;
    }
}
```

$O(1)$

only time for loop run

Print

$TC = O(1)$

What is the worst time complexity of above program?

☒ A.  $O(1)$

☐ B.  $O(n)$

☐ C.  $O(\log n)$

☐ D.  $O(\sqrt{n})$



Q.5



Consider the following asymptotic functions :

$$f_1 = 2^n$$

$$\Rightarrow 2^n$$

$$\Rightarrow 2^n$$

$$f_2 = 1.001^n$$

$$\Rightarrow$$

$$(1.001)^n$$

$$\Rightarrow$$

$$(1.001)^n$$

$$f_3 = e^n$$

$$\Rightarrow e^n$$

$$\Rightarrow (2.71)^n$$

$$f_4 = n!$$

$$\Rightarrow n!$$

[MCQ]

Which of the following is correct increasing order of above functions?

$$f_4 > f_3 > f_1 > f_2$$

A.

$f_3, f_4, f_1, f_2$

B.

$f_2, f_4, f_1, f_3$

C.

$f_3, f_2, f_1, f_4$

D.

$f_2, f_1, f_3, f_4$

$$n! = n^n$$

Q.6

How many of the following expressions correctly describes  $T(n) = n \log(n^2)$ ? \_\_\_\_\_



[NAT]

2

A.  $\theta(n)$

B.  $O(n)$

☒ C.  $\Omega(n)$

☒ D.  $O(n^2)$

$$T(n) = n \log n^2$$

$$= n^2 \log n \Rightarrow \underline{2n \log n} \nearrow$$



$$O$$

$$f(n) \leq C \cdot g(n)$$

$$\Omega$$

$$f(n) \geq C \cdot g(n)$$

$$(b) \quad \underline{n \log n} \neq O(n)$$

$$(a) \quad n \log n \neq \underline{\Theta(n)}$$

$$(c) \quad \underline{n \log n} \underset{w}{\neq} \Omega(n)$$

$$(d) \Rightarrow \underline{n \log n} = \underline{O(n^2)}$$

Q.7

Consider two function  $f_1(n) = n^{2^n}$  and  $f_2(n) = n^{n^2}$  then which of the following is true. [MCQ]



- A.  $f_1(n) = O(f_2(n))$  ✗
- B.  $f_1(n) = \theta(f_2(n))$  ✗
- C.  $f_1(n) = \omega(f_2(n))$  ✓
- D. None of these

(a)  $f_1(n) > f_2(n)$

$2^n = O(n^2)$

(b)  $2^n \neq \theta(n^2)$

(c)  $2^n = \omega(n^2)$



$$f_1(n) = n^{2^n}$$

$$f_2(n) = n^{n^2}$$

$$f_1(n) \quad | \quad f_2(n)$$

$$n^{2^n} = n^{n^2}$$

$$2^n \log n = n^2 \log n$$

$$\boxed{2^n \gg n^2}$$

$$\boxed{f_1(n) > f_2(n)}$$

