Discrete Mathematics
Mathematical Logic

DPP-03

[MCQ]

1. A logically binary relation $\otimes$ is defined as follows:

A	B	$A \otimes B$
True	True	True
True	False	True
False	True	True
False	False	False

Let $\sim$ be the unary negation (NOT) operator with higher precedence than $\otimes$, which one of the following is equivalent to $A \wedge B$?

(a) $\sim A \otimes \sim B$
(b) $\sim [\sim A \otimes \sim B]$
(c) $\sim [\sim A \otimes B]$
(d) None of these

[MSQ]

2. Consider the following propositional logic statements which of the following is contingency?

(a) $(\sim p \wedge (p \rightarrow q)) \rightarrow \sim p$
(b) $(q \wedge (p \rightarrow q)) \rightarrow \sim p$
(c) $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$
(d) $((p \vee q) \wedge \sim p) \rightarrow q$

[MCQ]

3. Let p be "I will study discrete math".
Let q be "I will study English literature".
Now, consider the logical statement
"I will study discrete math or I will study English literature"
"I will not study discrete math"
from the given premises, which of the following can be conclusion?

(a) Therefore, I will not study English literature
(b) Therefore, I will study English literature.
(c) Both A and B
(d) None of these.

[MCQ]

4. Which of the following can be the conclusion for the given hypothesis?
Hypothesis: $\sim p \wedge q,\ r \rightarrow p,\ \sim r \rightarrow s,\ s \rightarrow t$

(a) $r \wedge p$
(b) t
(c) s
(d) $r \rightarrow s$

[MCQ]

5. P_1: If it rains; the match will not be played
P_2: The match was played which of the following is valid inference?

(a) It rains
(b) It did not rain
(c) It either rain or did not rain
(d) None of these

Answer Key

1. **(b)**
2. **(a, c, d)**
3. **(b)**
4. **(b)**
5. **(b)**

Hints and solutions

1. **(d)**

From the truth table we can conclude that

$A \otimes B \equiv A \vee B$.

Now,

option (a): Incorrect

$\sim A \otimes \sim B \equiv \sim A \vee \sim B$

option (b): Correct

$\sim [\sim A \otimes \sim B] \equiv \sim [\sim A \vee \sim B]$

$= A \wedge B$

Hence, option (b) is the correct answer.

2. **(a, c, d)**

I: we can use the logical properties or truth table to find the truth value of the given logical statement.

II: If we have learned the inference rule then we can identify that

Statement A: modus tollens

Statement C: Hypothetical Syllogism

Statement D: Disjunctive Syllogism

Hence, all the options A, C and D are tautology.

III: Option B: Contingency

$(q \wedge (p \to q)) \to p$

$= \overline{(q + \overline{p} + q)} + \overline{p}$

$= \overline{q}\, p\, \overline{q} + \overline{p}$

$= \overline{q}\, p + \overline{p} = \overline{q} + \overline{p}$

Hence, option B is contingency.

3. **(b)**

By applying Disjunctive syllogism

$p \vee q$

$\underline{\sim p}$

$\underline{\therefore q}$

Therefore, I will study English literature.

4. **(b)**

Step	Reason
1. $\sim p \wedge q$	premise
2. $\sim p$	Simplification using (1)
3. $r \to p$	premise
4. $\sim r$	Modus tollens using (2), (3)
5. $\sim r \to s$	Premise
6. s	Modus ponens using (4) and (5)
7. $s \to t$	Premise
8. t	Modus ponens using (6) and (7)

Hence, ‘t’ will be the conclusion for the given hypothesis.

5. **(b)**

Now for the given problem:

p = It rains

q = the match will not be played

$\therefore ((p \Rightarrow q) \wedge \sim q) \Rightarrow \sim p$

Hence, inference “It did not rain” is valid using modus tollens.