Branch: CSE/IT

Batch: English

DISCRETE MATHEMATICS Mathematical Logic

DPP-05

[MCQ]

1. Let p(x) and q(x) denote the following open statements.

 $p(x): x^2 > 0$

q(x): x is odd

for the universe of all integers, determine the truth or falsity of each of the statement.

 $S_1: \forall x [p(x) \rightarrow q(x)]$

 S_2 : $\exists x [p(x) \rightarrow q(x)]$

which of the following is true?

(a) S_1 only

(b) S_2 only

(c) Both S_1 and S_2

(d) Neither S₁ nor S₂

[MCQ]

2. Consider following two First Order Logic Statements:

 $S_1 \!\!: \left[\forall x (\sim\! P(x) \!\vee\! Q(x)) \right] \to \left[\forall x \; P(x) \right] \to \left[\forall x \; Q(x) \right]$

 $S_2: [\exists x \ P(x)] \rightarrow [\exists x \ Q(x)] \rightarrow [\exists x \ (P(x) \rightarrow Q(x))]$

Which of the following is valid?

(a) S1 only

(b) S2 only

(c) Both S1 and S2

(d) None of these

[MSQ]

3. $P(y) = \sqrt{y}$ is real in the domain of Z^+ , then which of the following is / are correct?

(a) $\forall y P(y)$

(b) $\exists y P(y)$

(c) $\forall y \sim P(y)$

(d) $\exists y \sim P(y)$

[MCQ]

4. Which of the following is not valid logical expression?

(a)
$$\forall x [P(x) \rightarrow Q(x)] \rightarrow [\forall x P(x)] \rightarrow [\forall x Q(x)]$$

(b)
$$\forall x [P(x) \lor Q(x)] \rightarrow [\forall x P(x)] \lor [\forall x Q(x)]$$

(c)
$$\exists x [P(x) \land Q(x)] \rightarrow [\exists x P(x)] \land [\exists x Q(x)]$$

(d)
$$\forall x [P(x) \leftrightarrow Q(x)] \rightarrow [\forall x P(x)] \leftrightarrow [\forall x Q(x)]$$

[MCQ]

5. Consider following logical expressions:

I:
$$\forall y[P(y) \rightarrow Q] \leftrightarrow [\forall y P(y)] \rightarrow Q$$

II:
$$\exists y[P(y) \rightarrow Q] \rightarrow [\exists y P(y)] \rightarrow Q$$

which of the following logical expression is valid?

(a) I only

(b) II only

(c) Both I and II

(d) None of these

Answer Key

1. **(b)**

2. **(c)**

(a, b) 3.

(b) (d) 5.



Hints and Solutions

1. **(b)**

Statement S₁: $\forall x[p(x)\rightarrow q(x)]$

As we know the $\forall x$ connected through ' \land ' operator.

So, check the statement for x = 3

:.
$$[p(3) \rightarrow q(3)] = [(3^2 > 0) \rightarrow (3 \text{ is odd})]$$

$$\equiv$$
 [True \rightarrow True] \equiv True

Now, check the statement for x = 2

$$\therefore$$
 [p(2) \to q(2)] = [(2²>0) \to (2 is odd)]

$$\equiv$$
 [True \rightarrow False] \equiv False

Here S_1 is false.

Statement S₂: True

If $\exists x$ is true for one value then the overall the truth value of the statement will be true.

So, Check the statement for x = 3

$$[p(3) \rightarrow q(3)] = [(3^2 > 0) \rightarrow (3 \text{ is odd})]$$

$$\equiv$$
 [True \rightarrow True] \equiv True

Hence, S₂ is True.

2. (c)

$$S_1: \left[\forall x \left(P(x) \rightarrow Q(x) \right] \rightarrow \left[\forall x P(x) \right] \rightarrow \left[\forall x Q(x) \right] \right]$$

(Property of Predicate Logic)

$$\left[\forall x \left(\sim P(x) \lor Q(x)\right] \rightarrow \left[\forall x P(x)\right] \rightarrow \left[\forall x Q(x)\right]$$

$$S_2: [\exists x \ P(x)] \rightarrow [\exists x \ Q(x)] \rightarrow [\exists x \ (P(x) \rightarrow Q(x))]$$

Proof:

$$(P_1 \vee P_2) \rightarrow (Q_1 \vee Q_2) \rightarrow [(P_1 \rightarrow Q_1) \vee (P_2 \rightarrow Q_2)]$$

$$\left(P_1'P_2'+Q_1\vee Q_2\right) \rightarrow \left[\left(P_1'+Q_1\right)+\left(P_2'+Q_2\right)\right]$$

$$(P_1 + P_2) \cdot (Q_1' Q_2') + P_1' + Q_1 + P_2' + Q_2$$

$${P_1}{Q_1}^{\prime}{Q_2}^{\prime} + {P_2}{Q_1}^{\prime}{Q_2}^{\prime} \ + {P_1}^{\prime} + {Q_1} + {P_2}^{\prime} + {Q_2}$$

$$A'B + A = A + B$$

$$P_1 + P_2 + P_1' + Q_1 + P_2' + Q_2$$

$$P_1 + P_1' = 1$$
 and $1 +$ anyting $= 1$

$$1 + P_1 + P_2 + Q_1 + Q_2 + P_2'$$

1 True

Hence both are valid.

3. (a, b)

$$P(y) = \sqrt{y}$$
 is real

domain = positive integers (z^+)

For every values of y, \sqrt{y} is real because domain is positive integer

For some values of y, \sqrt{y} is real

(c)
$$\forall y \sim P(y)$$
 False

(d)
$$\exists y \sim P(y)$$
 False

4. (b)

$$(P_1 \vee Q_1) \wedge (P_2 \vee Q_2) \rightarrow (P_1 \wedge P_2) \vee (Q_1 \wedge Q_2)$$

$$(P_1 + Q_1) \wedge (P_2 + Q_2) \rightarrow P_1P_2 + Q_1Q_2$$

$$P_1' Q_1' + P_2' Q_2' + P_1 P_2 + Q_1 Q_2$$

$$P_1P_2 + Q_1Q_2 + P_1' Q_1' + P_2' Q_2'$$
 (Invalid)

Remaining all are valid.

I:
$$\forall y [P(y) \rightarrow Q] \leftrightarrow [\forall y P(y)] \rightarrow Q$$

$$(P_1 \rightarrow O) \land (P_2 \rightarrow O) \equiv (P_1 \land P_2) \rightarrow O$$

$$(P_1' + Q) \wedge (P_2' + Q) \equiv P_1' + P_2' + Q$$

$$P_1' P_2' + P_1' Q + P_2' Q + Q \equiv P_1' + P_2' + Q$$

$$P_1' P_2' + Q \not\equiv P_1' + P_2' + Q \text{ (invalid)}$$

II:
$$\exists y [P(y) \rightarrow Q] \rightarrow [\exists y P(y)] \rightarrow Q$$

$$(P_1{\rightarrow}Q)\vee(P_2{\rightarrow}Q)\rightarrow(P_1{\vee}\ P_2)\rightarrow Q$$

$$P_1' + P_2' + Q \rightarrow P_1' P_2' + Q$$

$$P_1 P_2 Q' + P_1' P_2' + Q$$

$$P_1P_2 + P_1' P_2' + Q$$
 (Invalid)

Hence, option (d) is correct





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