Branch : CSE/IT **Batch : English**

Discrete Mathematics

Graph Theory

Planarity Part-2

DPP-11

[MCQ]

1. How many perfect matchings are there in a complete bipartite graph $K_{n, n}$?

(a) 2n (b) n^2

(c) 2^n (d) n!

[MCQ]

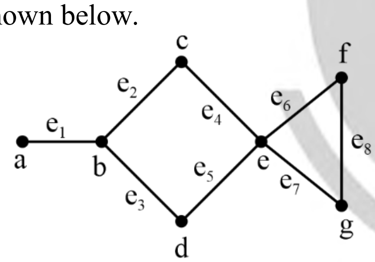
2. Find total number of perfect matchings in K_8?

(a) 100 (b) 102

(c) 105 (d) 107

[MCQ]

3. Which of the following is perfect matching for the graph shown below.



(a) $\{e_1, e_4, e_8\}$

(b) $\{e_1, e_5, e_8\}$

(c) $\{e_1, e_3, e_4, e_8\}$

(d) None of these

[MCQ]

4. Consider a 3-regular graph with number of vertices 10. How many faces in planar embedding for connected planar?

(a) 5 (b) 7

(c) 9 (d) 10

[NAT]

5. Consider a graph with 10 vertices, 15 edges and 3 components then how many closed faces are there____?

Answer Key

1. (d)
2. (c)
3. (d)
4. (b)
5. (8)

Hints and solutions

1. (d)

Number of perfect matchings in $K_{n,n} = \angle n$.

2. (c)

As we known that the perfect matching for complete graph K_{2n} is:

Perfect matching = $\dfrac{(2n)!}{(2!)^n n!}$

So, here we can write K_8 as $K_{(2*4)}$

where n = 4.

∴ Perfect matching $(K_{(2*4)})$

$$= \frac{8!}{(2!)^4 4!} = \frac{8*7*6*5*4!}{2\times2\times2\times2\times4!} = 105$$

3. (d)

I. A graph have perfect matching if it has matching and covering, means set edges in which none of them are adjacent to each other and they cover all the vertices.

II. Perfect matching is possible for a graph, when number of vertices is even.

III. If a graph has even number of vertices then it may or may not have perfect matching.

Now, from the above point we can conclude that the given graph does not have perfect matching because it has 7 vertices.

Option A: False

It do not cover vertex "d"

Option B: False

It do not cover vertex "c"

Option C: False

The edges e_1 and e_3 are adjacent to each other.

4. (b)

I. First find the total number of edges for the given 3-regular graph with 10 vertices.

∴ Number of edges = $\dfrac{nk}{2} = \dfrac{10*3}{2} = 15$ edges

II. Now, we know that for a connected planar graph, number of faces is:

r = e – n + 2 = 15 – 10 + 2 = 7

Hence, we have total 7 faces.

5. (8)

As we know that the Euler formula for the planar graph is:

r = e – n + k + 1

where e = Number of edges

n = Number of vertices

k = Number of components

r = Number of faces

∴ Number of faces = e – n + k + 1

= 15 – 10 + 3 + 1

= 9

Now, the closed faces = 9 – 1 = 8

Hence, we have total 8 closed faces.