

# CS & IT ENGINEERING

Discrete Mathematics

Combinatorics

DPP 04 Discussion



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# TOPICS TO BE COVERED

01 Question

02 Discussion



Q.1

If  $\phi$  is Euler phi function then  $\phi(\phi(1001))$  is

[MCQ]



A.

144

B.

192

C.

298

D.

96

$$\phi(\phi(1001))$$

$$\begin{aligned} 720 &= 8 \cdot 9 \cdot 10 \\ &= 2^3 \cdot 3^2 \cdot 2 \cdot 5 \\ &= 2^4 \cdot 3^2 \cdot 5 \end{aligned}$$

$$\phi(720) = \phi(2^4 \cdot 3^2 \cdot 5)$$

$$= 720 \cdot \frac{(2-1)(3-1)(5-1)}{2 \cdot 3 \cdot 5}$$

$$\phi(1001) = \phi(13 \times 7 \times 11)$$

$$= \phi(13) \times \phi(7) \times \phi(11)$$

$$= 12 \times 6 \times 10$$

$$= 720$$

$$= \frac{2^3 \cdot 3^2 \cdot 5}{2 \cdot 3 \cdot 5} \cdot (1)(2)(4)$$

$$= 2^3 \cdot 3 \cdot 2 \cdot 4$$

$$= 8 \cdot 3 \cdot 8$$

$$= 64 \cdot 3$$

$$\begin{array}{r} 64 \\ 3 \cdot 1 \\ \hline 192 \end{array}$$

**Q.2**

Consider the Euler's phi function given by

$$\phi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right)$$

**[MCQ]**

Where p runs over all the primes dividing n. What is the value of  $\phi(45)$ ?

$$45 = 5 \cdot 9 \\ = 3^2 \cdot 5$$

$$\phi(45) = 45 \cdot \frac{(3-1)(5-1)}{3 \cdot 5}$$

$$= \frac{3^2 \cdot 5 \cdot (2)(4)}{\cancel{3} \cdot \cancel{5}} = 3 \cdot 2 \cdot 4 = 24$$

A.

3

B.

12

C.

6

D.

24



**Q.3**How many numbers in  $\{1, 2, \dots, 200\}$  are coprime to 100?**[NAT]****80**

$$\phi(100) = \phi(2^2 \cdot 5^2)$$

$$\begin{aligned} 100 &= 25 \times 4 \\ &= 2^2 \cdot 5^2 \\ &= 100 \frac{(2-1)(5-1)}{2 \cdot 5} \\ &= \frac{2^2 \cdot 5^2 (1)(4)}{2 \cdot 5} \\ &= 10 \cdot 4 = \underline{40} \end{aligned}$$

$$\left\{ \begin{aligned} \gcd(1, 100) &= \gcd(101, 100) \\ \gcd(2, 100) &= \gcd(102, 100) \end{aligned} \right\}$$

40 + 40

**Q.4**

Find the number of positive integers  $n \leq 6000$  such that  $\gcd(n, 6000) = 1$ .

**[NAT]**

$$6000 = 6 \times 1000$$

$$= 2 \cdot 3 \times 25 \cdot 40$$

$$= 2 \cdot 3 \cdot 5^2 \cdot 4 \cdot 10$$

$$= \underline{2} \cdot 3 \cdot 5^2 \cdot \underline{2^2} \cdot \underline{2} \cdot 5$$

$$= 2^4 \cdot 3^1 \cdot 5^3$$

$$\phi(6000) = 6000 \cdot \frac{(2-1)(3-1)(5-1)}{2 \cdot 3 \cdot 5}$$

$$= \frac{\cancel{2^4}^3 \cdot \cancel{3^1}^1 \cdot \cancel{5^3}^2 \cdot (1)(2)(4)}{\cancel{2} \cdot \cancel{3} \cdot \cancel{5}}$$

$$= 2^3 \cdot 5^2 \cdot 2 \cdot 4$$

$$= 16 \cdot 100 = \underline{\underline{1600}}$$



Q.5

Let  $\phi(n)$  be the Euler's totient function. What is

[NAT]



$$\frac{\phi(70000000)}{\phi(1000000)}?$$

$$= \frac{\phi(7 \times 1\text{---})}{\phi(1\text{---})} = \frac{\phi(7) \times \cancel{\phi(1\text{---})}}{\cancel{\phi(1\text{---})}}$$

$$= \phi(7) = 7 - 1 = \underline{\underline{6}}$$

