Branch : CSE/IT **Batch : English**

Discrete Mathematics

Combinatorics

DPP-04

[MCQ]

1. If ϕ is Euler phi function then $\phi(\phi(1001)$ is

(a) 144 (b) 192

(c) 298 (d) 96

[MCQ]

2. Consider the Euler's phi function given by

$$\phi(n) = n\pi_{p/n}\left(1 - \frac{1}{p}\right)$$

Where p runs over all the primes dividing n. What is the value of $\phi(45)$?

(a) 3 (b) 12

(c) 6 (d) 24

[NAT]

3. How many numbers in $\{1, 2, \ldots, 200\}$ are coprime to 100?

[NAT]

4. Find the number of positive integers $n \leq 168$ such that $gcd(n, 168) = 8$.

[NAT]

5. Let $\phi(n)$ be the Euler's totient function. What is $\frac{\phi(7000000)}{\phi(1000000)}$?

Answer Key

1. **(b)**
2. **(d)**
3. **(80)**
4. **(12)**
5. **(6)**

Hints and Solutions

1. (b)

$\phi(\phi(1001) = ?$

1001 is not a prime number.

$\therefore \quad \phi(1001) = \phi(13 \times 11 \times 7) = \phi(13) \times \phi(11) \times \phi(7)$

$\phi(1001) = 12 \times 10 \times 6 = 720$

Now,

$720 = 2^4 \times 3^2 \times 5$

$$\phi(720) = 720 \times \left(1-\frac{1}{2}\right)\times\left(1-\frac{1}{3}\right)\times\left(1-\frac{1}{5}\right)=192$$

Hence, $\phi(\phi(1001) = 192$.

2. (24)

Euler's Totient function = $\phi(n) = n\pi_{p/n}\left(1-\frac{1}{p}\right)$

Where p = all prime factors of n

Now given $n = 45$

Then prime factors of 45 = 3, 5

$$\phi(45) = 45 \times \left(1-\frac{1}{3}\right)\times\left(1-\frac{1}{5}\right)=24$$

3. (80)

There are $\phi(100) = (4 - 2)(25 - 5) = 40$ coprime numbers to 100 in {1, 2, …, 100}.

Since gcd(a, b) = gcd(a – b, b),

gcd(101, 100) = gcd(1, 100),

gcd(102,100) = gcd(2, 100)

gcd(200, 100) = gcd(100, 100).

So, there are $\phi(100) = 40$ coprime numbers to 100 in {101, 102, …,200}. So the answer is 40 + 40 = 80.

4. (12)

These are the positive integers of the form $8m$, where $m \le 21$ and gcd(m, 21) = 1. So, the answer is $\phi(21) = (3 - 1)(7 - 1) = 12$.

5. (6)

We have to solve for: $\dfrac{\phi(7000000)}{\phi(1000000)}$

Using the number for $\phi(n)$ we can expand it:

$$= \frac{(7000000)\left(1-\frac{1}{2}\right)\left(1-\frac{1}{5}\right)\left(1-\frac{1}{7}\right)}{(1000000)\left(1-\frac{1}{2}\right)\left(1-\frac{1}{5}\right)}$$

After simplifying:

$$= (7)\left(1-\frac{1}{7}\right) = 7\times\frac{6}{7} = 6$$