Branch : CSE/IT   Batch : English

Discrete Mathematics
Graph Theory

DPP-08

[MCQ]
1. Consider the following statements:
   S_1: If a connected graph G has a cut vertex, then G has a cut edge.
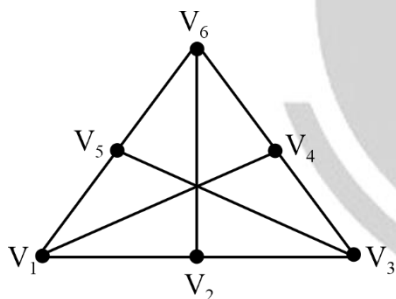 S_2: If a connected graph G has a cut edge then G has a cut vertex.
   Which of the following is true?

   (a) S_1 only
   (b) S_2 only
   (c) Both S_1 and S_2
   (d) Neither S_1 nor S_2

[NAT]
2. For the graph shown below, the chromatic number is _____ .

[NAT]
3. If G is a connected graph with 10 vertices and vertex connectivity is 3, then minimum number of edges necessary in G is_____.

[MSQ]
4. which of the following options is/are correct?
   (a) The chromatic number of a graph with at least 1 edge is at least 2.

   (b) A graph is null graph if and only if its chromatic number is 2.
   (c) For any graph, $K_G \leq 1+\Delta(G) \leq n$, where $\Delta(G)$ is maximum degree and K_G is chromatic number.
   (d) The chromatic number of a multi graph is equal to its equivalent simple graph chromatic number.

[MCQ]
5. Consider the following statements:

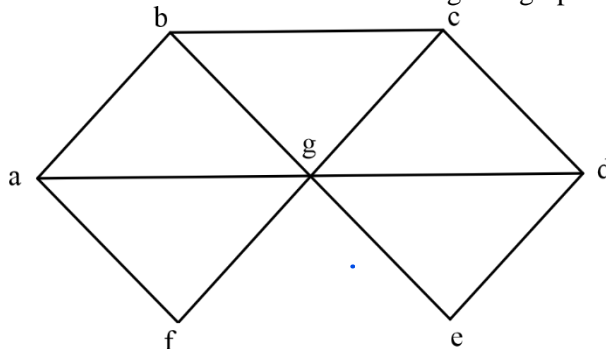 S_1: A graph is bipartite graph if and only if its chromatic number is 2.
   S_2: The chromatic number of a tree is 2. Thus, every tree is bipartite graph.
   Which of the following statement is False?
   (a) S1 only
   (b) S2 only
   (c) Both S_1 and S_2
   (d) Neither S_1 nor S_2

[NAT]
6. What is the chromatic number of the given graph?

Answer Key

1. (d)
2. (2)
3. (15)
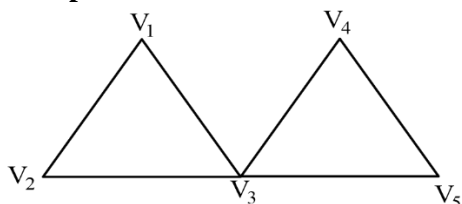4. (a, c, d)

5. (a)
6. (3)

Hints and Solutions
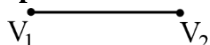
1. **(d)**
Statement S1 : False
Example:

In the above graph G has cut vertex (V_3) but it does not has cut edge.
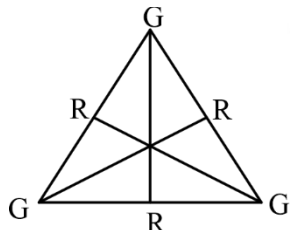Hence, the statement S_1 is false.
Statement S$_2$: False
Example:

The complete graph with 2 vertices have only cut edge (V_1, V_2). It does not have cut vertex.
Hence, statement S2 is also false.

2. **(2)**

Hence, we need only 2 color to completely cover the vertices. So, the chromatic number is 2.

3. **(15)**
In the problem, we have a connected graph G with 10 vertices and VC = 3.
As we know that the relation between VC, EC and minimum degree is:
  $VC \leq EC \leq \delta(G)$

$\therefore$ $3 \leq EC \leq \delta G$

From the above equation, we can conclude that the minimum degree of vertex would be 3.

$\therefore$ $\delta(G) = 3$

Now, by using handshaking lemma:
Sum of degree = $2 * |E|$

$\therefore$ $10 * 3 = 2 * |E|$
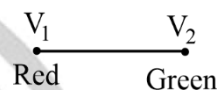
$\therefore$ $|E| = \dfrac{30}{2} = 15$ edges

Hence, 15 edges necessary in graph G to have vertex connectivity = 3.

4. **(a, c, d)**
Option A : Correct
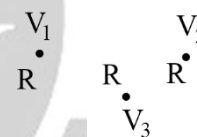The chromatic number of a graph with at least 1 edge is at least 2.
Example,

Option B: Incorrect
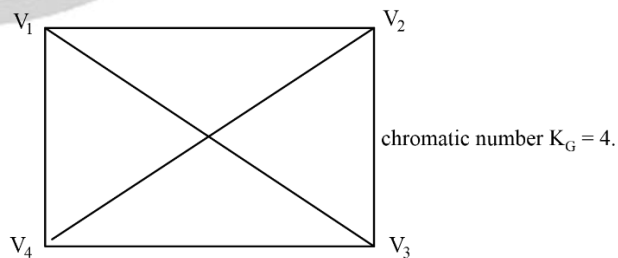A graph is null graph if and only if its chromatic number is 1.
Example:

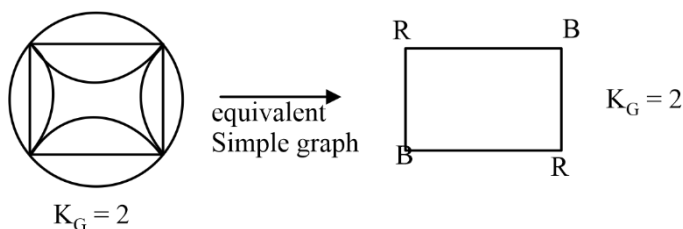**Option C** : Correct
The chromatic number will be less than or equal to "$1 + \Delta(G)$".
Example: The complete graph (K_4) have $\Delta(G) = 3$.

chromatic number $K_G = 4$.

Option d : Correct
Example

5. **(a)**

Statement S_1 : False

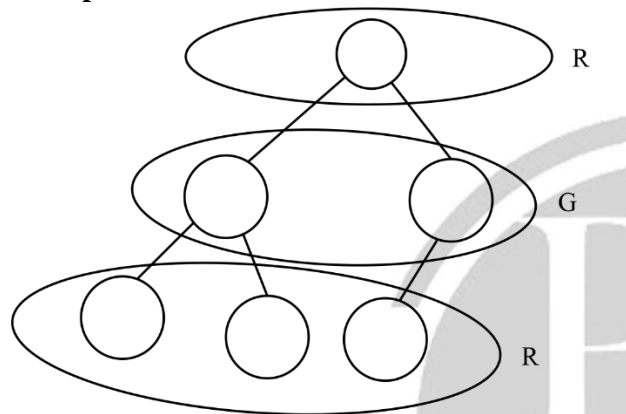Null graph is a graph with no edges and 1 or n vertices and every null graph is bipartite graph.

As we know that a null graph is 1 colorable.

Note: A non – null graph is bipartite graph iff its chromatic number is 2.

Statement S2 : True

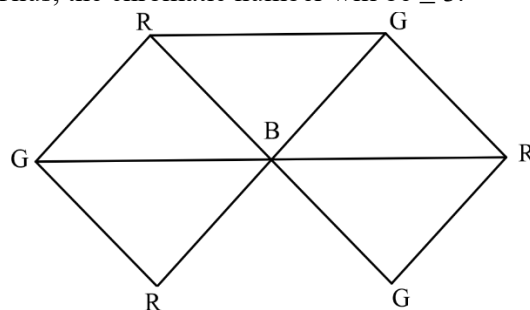Every tree is a bipartite graph. So, the chromatic number is 2.

Example :

6. **(3)**

The given graph have complete graph with 3 vertices (K_3).

Thus, the chromatic number will be ≥ 3.

Hence, the chromatic number is 3.