

# Discrete Mathematics

## Graph Theory

DPP-06

[NAT]

1. Let  $G$  be a simple graph with 15 edges and  $\bar{G}$  be a complement graph of  $G$  has 21 edges, then the number of vertices in graph  $G$  is \_\_\_\_.

[MSQ]

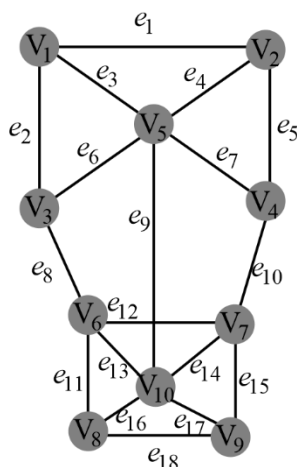
2. Which of the following is true for a graph with vertex connectivity is 3 and edge connectivity is 4?
- Removal of any 3 vertices can disconnect the graph.
  - Removal of any 4 edges can disconnect the graph.
  - Removal of some 3 vertices will increase the number of connected components.
  - Removal of some 4 edges will increase the number of connected components.

[MCQ]

3. What is the maximum value of vertex connectivity and edge connectivity possible with a graph of order 10 and size 16?
- $1 \leq VC, EC \leq 3$
  - $1 \leq VC, EC \leq 4$
  - $0 \leq VC, EC \leq 3$
  - $0 \leq VC, EC \leq 4$

[MSQ]

4. Consider the given connected graph  $G$



Which of the following is not the cut set?

- $\{e_6, e_7, e_9\}$
- $\{e_8, e_9, e_{10}, e_{12}\}$
- $\{e_8, e_9, e_{10}\}$
- $\{e_1, e_2, e_3\}$

[MCQ]

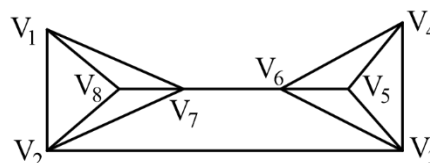
5. Consider the following statements:
- $S_1$ : The vertex connectivity of the graph is 1 if and only if graph has cut vertex.
- $S_2$ : The edge connectivity of the graph is 1 if and only if graph has cut edge.

Which of the following statements is true?

- $S_1$  only
- $S_2$  only
- Both  $S_1$  and  $S_2$
- Neither  $S_1$  nor  $S_2$

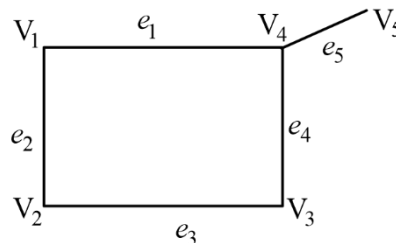
[NAT]

6. For the graph below, vertex. Connectivity is \_\_\_\_ and edge connectivity is \_\_\_\_.



[NAT]

7. Consider the simple undirected graph  $G$ .



Find the number of cut set for the above graph  $G$ ?

## Answer Key

- |           |           |
|-----------|-----------|
| 1. (9)    | 5. (c)    |
| 2. (c, d) | 6. (2, 2) |
| 3. (a)    | 7. (7)    |
| 4. (a, b) |           |



## Hints and Solutions

1. (9)

As we know that the maximum number of edges in

graph with  $n$  vertices is  $n_{c_2} = \frac{n(n-1)}{2}$ .

Now, the graph  $G$  and its complement graph together form a complete graph

So, Complete graph with 9 vertices = 15 + 21

$$\Rightarrow n_{c_2} = 36$$

$$\Rightarrow n_{c_2} = \frac{n(n-1)}{2}$$

$$\Rightarrow n^2 - n = 72$$

$$\Rightarrow n^2 - n - 72 = 0$$

$$\Rightarrow n(n-9) + 8(n-9) = 0$$

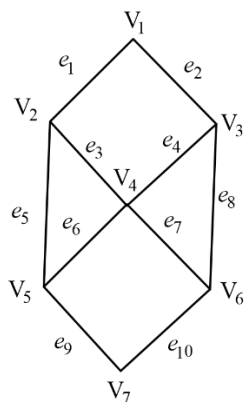
$$\Rightarrow (n+8)(n-9) = 0$$

$$\therefore n = 9$$

Hence, the graph  $G$  have 9 vertices.

2. (c, d)

Consider the given graph below:



Now, the vertex connectivity and edge connectivity for the above graph is 2.

Option a: False

Removal of vertices  $\{V_1, V_2\}$  would not disconnect the graph. Hence, removal of any 2 vertices for the above graph will not disconnect the graph. Thus, option a is False.

Option b: False

Removal of edges  $\{e_6, e_7\}$  would not disconnect the graph.

Option c: True

As removal of some vertices  $\{V_5, V_6\}$  will disconnect the graph. Thus, it increases the number of components.

Option d: True

As removal of some edges  $\{e_9, e_{10}\}$  will disconnect the graph.

3. (a)

Here the number of vertices is given 10 and edges are 16.

Now, first find the minimum degree for the above graph:

$$\delta(G) \leq \left\lfloor \frac{2|E|}{n} \right\rfloor$$

$$\delta(G) \leq \left\lfloor \frac{2 \cdot 16}{10} \right\rfloor$$

$$\delta(G) \leq \lfloor 3.2 \rfloor$$

$\therefore$  Minimum degree  $\delta(G)$  will be 3.

Now, the relation between the VC, EC and minimum degree

$$VC \leq EC \leq \delta(G)$$

$$\therefore VC \leq EC \leq 3.$$

Hence, the possible value of VC and EC will be

$$1 \leq VC, EC \leq 3.$$

4. (a, b)

I. A cut set is a minimal set of edges whose removal disconnect the graph.

II. No proper subset of cut set should be able to disconnect the graph.

Now, let's understand the options:

**Option A:** Correct, as removal of edges  $\{e_6, e_7, e_9\}$  will not disconnect the graph. It is not a cut set.

**Option B:** Correct, as the proper subset of edges  $\{e_8, e_9, e_{10}\}$  will disconnect the graph. Hence, it is also not a cut set. So option c and d are the minimal set of edges whose removal disconnect the graph.

5. (c)

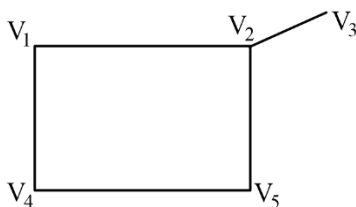
Statement  $S_1$ : True

The vertex connectivity of the graph is 1 mean we have a vertex whose removal disconnect the graph.

Statement  $S_2$ : True

The edge connectivity of the graph is 1 mean a pendant vertex (degree 1) is connect to graph with 1 edge.

Example:



The VC is 1: Removal of  $V_2$  will disconnect the graph.

The EC is also 1: Removal of edge  $(V_2 - V_3)$  will disconnect.

6. (2, 2)

I. First find the minimum degree of the given graph  $\delta(G)$ .

So,  $\delta(G) = 3$ .

II. Now, the relation between VC, EC and  $S(G)$  is as follows:

$$VC \leq EC \leq S(G)$$

$$\therefore VC \leq EC \leq 3$$

From the above relation, we can conclude that the VC and EC would be at most 3.

III. Now, if we delete the vertices either  $\{V_6, V_3\}$  or  $\{V_7, V_2\}$ . It will disconnect the graph as well as if we delete the edges  $\{(V_7 - V_6), (V_2 - V_3)\}$  it will also disconnect the graph.

Hence, the VC and EC for the given graph is 2.

7. (7)

I. The vertex  $V_5$  is pendant vertex. Hence,  $e_5$  will be one of the cut set.

II. Now, in the above given we have a cycle of length '4':  $\{V_1 - V_2 - V_3 - V_4 - V_1\}$

So, if we select any 2 edges from the cycle, it will disconnect the graph.

So, the number of cut set from the cycle

$$= 4C_2 = \frac{4 \times 3}{2} = 6$$

Hence, the total number of cut set is  $6 + 1 = 7$ .

- |                      |                     |
|----------------------|---------------------|
| (I) $\{e_5\}$        | (II) $\{e_1, e_2\}$ |
| (III) $\{e_2, e_3\}$ | (IV) $\{e_3, e_4\}$ |
| (V) $\{e_4, e_1\}$   | (VI) $\{e_2, e_4\}$ |
| (VII) $\{e_1, e_3\}$ |                     |



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