Branch : CSE/IT **Batch : English**

Discrete Mathematics
Graph Theory

DPP-03

[MCQ]

1. Consider a graph with order 7. The degree sequence of the graph is 4, 3, 3, 3, 2, 2, 1. Assume x is the number of edges and y is the degree sequence of the complement graph of the given graph. Find x and y?
(a) x = 10 and y = 5, 3, 3, 3, 2, 2, 2
(b) x = 12 and y = 5, 4, 4, 3, 3, 3, 2
(c) x = 14 and y = 5, 5, 4, 4, 4, 4, 2
(d) x = 16 and y = 6, 5, 5, 5, 5, 3, 3

[NAT]

2. What is the maximum number of edges in an complemented graph with 7 vertices?

[MSQ]

3. Which of the following is the number of vertices that form the self-complementary graph?
(a) 13 (b) 12
(c) 15 (d) 16

[NAT]

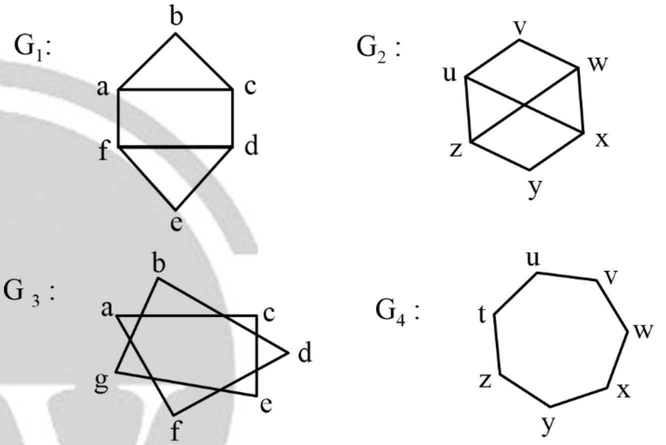
4. In a self-complementary graph G of size 18, then find the number of vertices in the graph G?

[MCQ]

5. Consider a graph with 4 vertices and 3 edges then which of the following statement is True?
S_1 : It may or may not be self-complementary graph.
S_2 : It must be self–complementary graph.
(a) S_1 is True but S_2 is False
(b) S_1 is False but S_2 is True
(c) Neither S1 nor S_2 is True
(d) None of these.

[MSQ]

6. Which of the following options is/are correct for isomorphic graphs?



(a) G_1 and G_2 are isomorphic graph.
(b) G_3 and G_4 are isomorphic graph.
(c) G_1 and G_2 are not isomorphic graph.
(d) G_3 and G_4 are not isomorphic graph.

[MCQ]

7. Consider the following statements:
S_1 : If two graphs are self – complementary graph then they have equal number of vertices and edges.
S_2 : If two graphs G_1 and G_2 have same number of edges, vertices and same degree sequence then they are self-complement graphs.
Which of the following option is true?
(a) S_1 only
(b) S_2 only
(c) Both S_1 and S_2
(d) Neither S_1 nor S_2

Answer Key

1. **(b)**
2. **(21)**
3. **(a, b, d)**
4. **(9)**
5. **(a)**
6. **(b, c)**
7. **(a)**

Hints and solutions

1. **(b)**

I. The degree sequence for the given graph

G is 4, 3, 3, 3, 2, 2, 1.

Now, the degree sequence of the complemented graph $\overline{G}$ will be as follows:

$\therefore K_7 = 6, 6, 6, 6, 6, 6, 6$

$G = 4, 3, 3, 3, 2, 2, 1$

$\overline{G} = 2, 3, 3, 3, 4, 4, 5$

Hence , y = 5, 4, 4, 3, 3, 3,2

II. To find the number of edges apply Handshaking lemma:

Sum of degree = 2 ∗ |E|

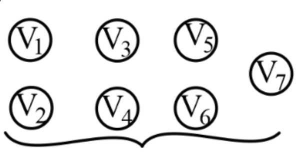
$\therefore\ 5 + 4 + 4 + 3 + 3 + 3 + 2 = 2 * |E|$

$\therefore\ |E| = \frac{24}{2} = 12 \text{ edges}$

Hence, x = 12.

2. **(21)**

I. To get the maximum number of edges in an complement graph, assume we have null graph with 7 vertices.



Null graph G with 7 vertices

II. Now, the complement graph of the above null graph will be complete graph with 7 vertices, so, the maximum number of edges will be:

$\therefore$ Number of edges $= \frac{n(n-1)}{2} = \frac{7 * 6}{2} = 21$ edges.

3. **(a, b, d)**

If graph G is self–Complementary graph then number of vertices must be either 4X or 4X + 1.

Where $X \in Z$

So,

Option A: The number of vertices is 13

$13 = 4 * 3 + 1$ {for x = 3}

Thus, option A can form self – complementary graph.

Option B : 12 = 4 ∗ 3 : can form SC graph

Option C : $15 \neq 4X$ or $4X + 1$ for any $X \in Z$

Hence, can not form SC graph.

Option D : 16 = 4 ∗ 4 : can form SC graph.

4. **(9)**

We know that

If graph G is self –complementary

$\Rightarrow e = \frac{n(n-1)}{4}$

$\therefore$ Number of edges $= \frac{n(n-1)}{4}$

18 edges $= \frac{n(n-1)}{4}$

18 * 4 = n (n – 1)

72 = n (n –1)

So, n = 9

Thus, the number of vertices would be 9.

5. **(a)**

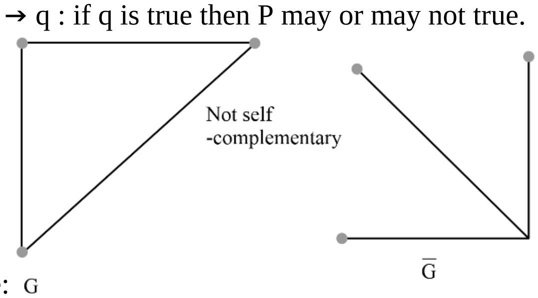
I. If graph G is SC graph ⇒ n = 4x or 4x + 1

$x \in z$

n = No. of vertices.

II. If graph G is SC graph ⇒ $e = \frac{n(n-1)}{4}$

The above points I and II are one way theorem that is p → q : if q is true then P may or may not true.



Example: G

The above graph G and $\overline{G}$ satisfies I and II but not self – Complementary graph.

Hence, S_1 is true only.

6. (b, c)

I. G_1 and G_2 graphs are not isomorphic graphs. G_1 has a circuit of length 3 {a – b – c – a} while G_2 does not. Hence, G_1 and G_2 are not isomorphic.

II. G_3 and G_4 are isomorphic graphs. Pull 'a' and 'c' down. Then move e to the left of g. These graph satisfy the properties for isomorphic.

7. (a)

Statement S_1 : True.

Self–complement graph is a graph which is isomorphic to its own complemented graph.

$\therefore \; G \equiv \overline{G}$

So, if both the graphs are isomorphic mean they have equal number of edges and vertices.

Statement S_2 : False.

To become the self – complement graph, it have to satisfy isomorphic property first.

Now, if two graph have same number of edges, vertices and same degree sequence but it may violate incident property of vertex.

Hence, it is not necessarily self-complement graph