CS & IT ENGINEERING

Discussion

Discrete Mathematics

Combinatorics

DPP 05

SATISH YADAV SIR

TOPICS TO BE COVERED

01 Question

02 Discussion

Q.1 In how many ways can 3000 identical envelopes be divided, in packages of 25, among four student groups so that each group gets at least 150, but not more than 1000, of the envelopes?

[MCQ]

A. $\binom{99}{96} - 4\binom{64}{61} + 6\binom{29}{26}$

B. $\binom{99}{96} - 4\binom{62}{61} + 6\binom{29}{26}$

C. $\binom{99}{96} - 4\binom{62}{61} + 5\binom{29}{26}$

D. None

3000

$\frac{3000}{25} = \underline{120}$

$\square$ $\square$ . . . . $\square$

25 25 25

$\frac{150}{25}$

6

$\frac{1000}{6} = 40$

coefficient of x^{120}

$$(x^6 + x^7 \dots x^{40})^4$$

$$(x^6)^4 (1 \dots x^{34})^4$$

$$x^{24}\left(\frac{1-x^{35}}{1-x}\right)^4$$

$$x^{24}(1-x^{35})^4(1-x)^{-4}$$

$$x^{24}(1-4x^{35}+6x^{70}\dots)(1-x)^{-4}$$

$$^{-4}C_{96} - 4 \times {}^{-4}C_{91} + 6\,{}^{-4}C_{26}$$

$$^{4+96-1}C_{96} - 4 \times {}^{4+91-1}C_{91} + 6 \times {}^{4+26-1}C_{26}$$

$$^{99}C_{96} - 4 \times {}^{93}C_{91} + 6 \times {}^{29}C_{26}$$

$$\begin{array}{r} 24 \\ 35 \\ \hline 59 \end{array} \qquad \begin{array}{r} 70 \\ 24 \\ \hline 94 \end{array}$$

Q.2 Determine the coefficient of x^{15} in [NAT]

$$f(x)=\left(x^2+x^3+x^4+\cdots\right)^4$$

$(x^2)^4(1 + x \ldots)^4$ $\quad (-1)^7 \, {}^{-4}C_7 \, (-x)^7 \times x^8$

$x^8\left(\dfrac{1}{1-x}\right)^4$

$4+7-1C_7 = {}^{10}C_7 = {}^{10}C_3$

$x^8(1-x)^{-4}$

Q.3 In how many ways can a police captain distribute 24 rifle shells to four police officers so that each officer gets at least three shells, but not more than eight? [NAT]

x^{24}

$(x^3 \dots x^8)^4$

$(x^3 + x^4 + \dots x^8)^4$

$(x^3)^4 (1 + x \dots x^5)^4$

$x^{12}\left(\frac{1-x^6}{1-x}\right)^4$

$x^{12}(1 - 4x^6 + 6x^{12} \dots)(1-x)^{-4}$

${}^{-4}C_{12} - 4\,{}^{-4}C_6 + 6$

${}^{15}C_{12} - 4 \times {}^{9}C_6 + 6$

Q.4

Determine the sequence generated by the following generating functions:

[MCQ]

$1 - 3x + (3x)^2 - (3x)^3 + (3x)^4.$

$f(x) = 1/(1 + 3x)$

A. $1, 3, 3^2, 3^3, \ldots$

B. $1, 3, 3^2, -3^3, \ldots$

C. $1, -3, 3^2, 3^3, \ldots$

D. $1, -3, 3^2, -3^3, \ldots$

Q.5 In how many ways can two dozen identical robots be assigned to four assembly lines with at least three robots assigned to each line?

[NAT]

x^{24}

$(x^3 - \cdots)^4$

$x^{12}(1+x\cdots)^4$

$-{}^4C_{12} = {}^{4+12-1}C_{12}$

$= \underline{{}^{15}C_{12}}$

Q.6 Find a recurrence relation, with initial condition, that uniquely determines the following geometric progressions: [MCQ]

7, 14/5, 28/25, 56/125

a_0 a_1

$\frac{a_1}{a_0} = \frac{\cancel{14}^{2}}{5} \times \frac{1}{\cancel{7}}$

A. $a_n = 5a_{n-1}, n \geq 1, a_0 = 2$

B. $a_n = -3a_{n-1}, n \geq 1, a_{0=6}$

C. $a_n = (2/5)a_n - 1, n \geq 1, a_0 = 7$

D. None of these

Q.7 If a_n, $n \geq 0$, is the unique solution of the recurrence relation $a_{n+1} - da_n = 0$, and $a_3 = 153/149$, $a_5 = 1377/2401$, What is d?

[MSQ]

$a_{n+1} = d^n a_n$

$a_2 =$

A. $d = 3/7$

B. $d = 4/7$

C. $d = -3/7$

D. $d = -4/7$

$$\frac{d^5 a_0}{d^3 a_0} = \frac{a_5}{a_3} = \frac{1377}{2401} \times \frac{149}{153} = \frac{9}{49} = d^2$$

$a_n = 2a_{n-1}$

$a_1 = 2^1 a_0$

$a_2 = 2^2 a_0$

$d = \pm \frac{3}{7}$

Q.8 If $a_0 = 0$, $a_1 = 1$, $a_2 = 4$, and $a_3 = 37$ satisfy the recurrence relation $a_{n+2} + ba_{n+1} + ca_{n=0}$, where $n \geq 0$ and b, c are constants, determine values for b, c respectively

[MCQ]

A. $b = -4, c = -21$

B. $b = -21, c = -4$

C. $b = 4, c = 21$

D. None of these

$a_{n+2} + ba_{n+1} + ca_n = 0$

$n=0$

$a_2 + ba_1 + ca_0 = 0$

$4 + b \cdot 1 + c \cdot 0 = 0$

$b = -4$

$n=1$

$a_3 + ba_2 + ca_1 = 0$

$37 + (-4)4 + c(1) = 0$

$37 - 16 + c = 0$

$c = -21$

Q.9 Determine the constants b and c if $a_n = c_1 + c_2(7^n)$, $n \geq 0$, is the general solution of the relation $a_{n+2} + ba_{n+1} + ca_{n=0}$, $n \geq 0$.

[MCQ]

A. $b = 7, c = -8$

B. $b = -8, c = 7$

C. $b = -8, c = 8$

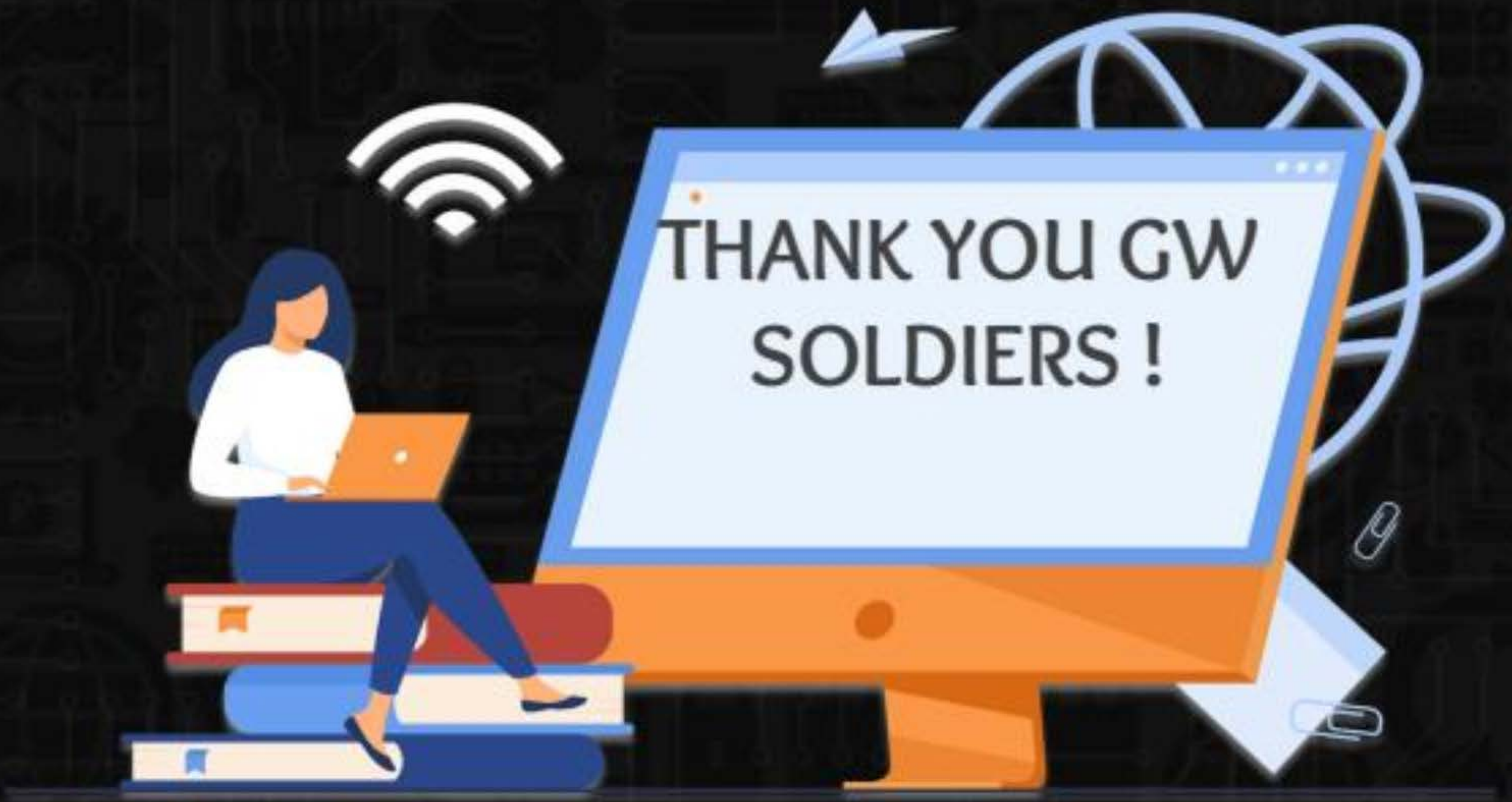
D. None of these

Roots : 1, 7

$(x-1)(x-7)=0$

$x^2 - 8x + 7 = 0.$

$b = -8 \quad c = 7.$


THANK YOU GW
SOLDIERS !