Branch : CSE/IT   Batch : English

Discrete Mathematics
Mathematical logic

DPP-01

[MCQ]

1. Which of the following is tautology?

   (a) $(\sim p \wedge (p \to q)) \to \sim q$

   (b) $\sim (p \to q) \to \sim q$

   (c) $[(\sim p \wedge q) \wedge [q \to (p \to q)]] \to \sim r$

   (d) None of these

[MCQ]

2. The statement $[P \vee (p \leftrightarrow Q) \vee Q]$ is equivalent to

   (a) P   (b) Q

   (c) A tautology   (d) $(P \wedge Q)$

[MCQ]

3. Consider the following statement

   $S_1: [(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (r \Rightarrow p)$

   $S_2: [((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)]$

   Which of the following is/are correct?

   (a) S_1 is contingency

   (b) S_2 is tautology

   (c) S_1 and S_2 both contingency

   (d) S_1 and S_2 both Tautology

[MCQ]

4. Which of the following is valid?

   S1: $p \Rightarrow (q \vee r) \equiv (p \Rightarrow q) \vee (p \Rightarrow r)$

   S2: $p \Rightarrow (q \wedge r) \equiv (p \Rightarrow q) \wedge (p \Rightarrow r)$

   (a) S_1 is valid and s_2 is not valid

   (b) S_1 is not valid and s_2 is valid

   (c) Both S_1 and S_2 are valid

   (d) Neither S_1 nor S_2 is valid

[MCQ]

5. Which of the following is not a tautology?

   (a) $((p \to q) \to r) \to (p \to (q \to r))$

   (b) $((p \to (r \vee q)) \to ((p \to r) \vee (p \to q))$

   (c) $(p \to (r \wedge q)) \to ((p \to r) \vee (p \to q))$

   (d) $(p \to (q \to r)) \to ((p \to q) \to r)$

Answer Key

1. (b)
2. (c)
3. (a, b)

4. (c)
5. (d)

Hints and solutions

1. **(b)**

$\sim (p \to q) \to \sim q$

$\sim [\sim (p \to q)] \lor \sim q$

$(p \to q) \lor \sim q$

$\sim p \lor q \lor \sim q$

$\sim p \lor 1 \equiv 1$

Hence, option B is tautology.

2. **(c)**

The statement : $P \lor (P \leftrightarrow Q) \lor Q$

$P \lor P'Q' \lor PQ \lor Q$

$P \lor P'Q' \lor (P \lor 1) Q$

$P \lor P'Q' \lor Q$

Apply absorption LAW

$\underbrace{P \lor \bar{P}\,\bar{Q}} \lor Q$

$P \lor \bar{Q} \lor Q$

$\therefore P \lor 1 \equiv 1$

Hence, the given statement is tautology.

3. **(a, b)**

Statement S1 : Contingency

$[(p \Rightarrow q) \land (q \Rightarrow r)] \Rightarrow (r \Rightarrow p)$

$[(\bar{p} + q) \land (\bar{q} + r)] \Rightarrow (\bar{r} + p)$

$\overline{(\bar{p}+q)} + \overline{(\bar{q}+r)} + (\bar{r}+p)$

$\therefore p\bar{q} + q\bar{r} + \bar{r} + p$

$\therefore p + \bar{r}$ [LAW of Absorption]

Hence, S1 is contingency

Statement S2 : Tautology

$[((p \Rightarrow q) \land (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)]$

$\therefore p\bar{q} + q\bar{r} + \bar{p} + r$

$\therefore \underbrace{p\bar{q} + \bar{p}} + q\bar{r} + r$

$\therefore \bar{p} + \underbrace{\bar{q} + q} + r$

$\therefore \bar{p} + 1 + r \equiv 1$

Hence, S2 is tautology

4. **(c)**

Statement S_1 : valid

$p \Rightarrow (q \lor r) \equiv (p \Rightarrow q) \lor (p \Rightarrow r)$

$\bar{p} + q + r \equiv (\bar{p} + q) + (\bar{p} + r)$

$\bar{p} + q + r = \bar{p} + q + r$

Hence, statement S_1 is valid

Statement S_2 : Valid

$p \Rightarrow (q \land r) \equiv (p \Rightarrow q) \land (p \Rightarrow r)$

$\bar{p} + qr \equiv (\bar{p} + q) . (\bar{p} + r)$

$\bar{p} + qr \equiv \bar{p} + qr$

Hence, statement S_2 is also valid.

5. **(d)**

Statement : $(p \to (q \to r)) \to ((p \to q) \to r)$

$(p \to (\bar{q} + r)) \to ((\bar{p} + q) \to r)$

$(\bar{p} + \bar{q} + r) \to (\overline{(\bar{p}+q)} + r)$

$\overline{(\bar{p} + \bar{q} + r)} + p\bar{q} + r$

$pq\bar{r} + p\bar{q} + r$

$p(q\bar{r} + \bar{q}) + r$

$p(\bar{r} + \bar{q}) + r$

$p\,\underbrace{\bar{r} + p\bar{q} + r}$

$\therefore \underbrace{p + r + p\,\bar{q}}$

$p(1 + \bar{q}) + r$

$\therefore p + r \ne 1$

Hence, option D is not tautology.