Branch : CSE/IT   Batch : English

Discrete Mathematics
Graph Theory

DPP-07

[NAT]

1. If G is a bipartite graph with 9 vertices and maximum number of edges, then vertex connectivity of G = _____.

[MSQ]

**2.** Which of the following options is/are correct?
(a) A graph G is Euler iff it is connected and $\forall v \in G$ degree (v) = even.
(b) A K – regular graph is Euler iff K is even
(c) A wheel graph (w_n) can have Euler circuit.
(d) A graph will contain an Euler path if it contains at most two vertices of odd degree.

[MCQ]

3. A forest is disconnected graph in which each component is a tree. Let F be a forest on 80 vertices with 21 connected components. Then number of edges in G is _____.
(a) 58   (b) 60
(c) 59   (d) 101

[MCQ]

4. For the graph shown below
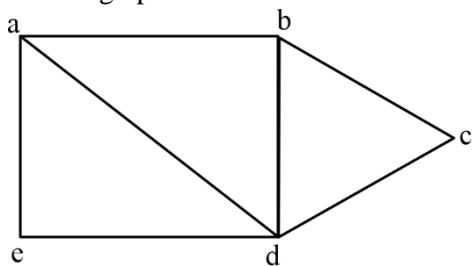
Which of the following statements is/are true?

S_1 : Euler path exists
S_2 : Euler circuit exists
S_3 : Hamiltonian cycle exists
S_4 : Hamiltonian path exists
(a) S_1, S_3 and S_4
(b) S_1, S_2 and S_3
(c) S_1, S_2 and S_4
(d) S_2, S_3 and S_4

[MCQ]

5. Which of the following is Euler Graph?
(a) K_{51}
(b) K_{50}
(c) $\overline{C_{60}}$
(d) 11- regular

[MCQ]

6. If G is not a simple connected graph with n vertices then maximum number of edges possible in G is _____.
(a) $\dfrac{n(n-2)}{2}$

(b) $\dfrac{(n-1)n}{2}$

(c) $\dfrac{(n-1)(n-2)}{2}$

(d) $\dfrac{(n-1)(n-2)}{4}$

Answer Key

1. (4)
2. (a, d)
3. (c)
4. (a)

5. (a)
6. (c)

Hints and Solutions

1. (4)

I. The number of vertices in bipartite graph is given as: m + n

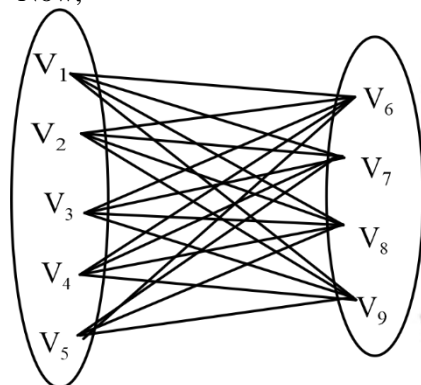where m = No of vertices in partition 1

n = No of vertices in partition 2

Now, we have 9 vertices in the given bipartite graph so, divide the vertices in such a way so, that we have maximum number of edges.

∴ Number of edges = m ∗ n

= 5 ∗ 4 = 20 edges

Here m = 5 is partition 1 vertices and n = 4 is partition 2 vertices.

II. Now,

If we remove partition 2 vertices then graph will disconnect. Hence, vertex connectivity for the given graph is 4.

2. (a, d)

Option a : correct

In the Euler graph every degree must be even because if we enter into a vertex in order to cover the edge then we should exit to complete the cycle or to reach the starting vertex.

Option b : Incorrect

K – regular graph with K is even mean all the degrees are even but the graph may itself not connected.

Hence, the statement is incomplete.

Option c : Incorrect

A wheel graph (W_n) is not Euler graph because the vertices at the outer edge (i.e n –1 vertices) will always have odd degree vertices that is degree 3.

Option d : Correct

To have a Euler path, the graph must have 2 odd degree vertices. Such Euler path will start form one of those odd degree vertex and ends at other odd degree vertex.

3. (c)

We know that a forest on n vertices with K. connected components have (n – K) number of edges.

Now, in the problem,

Number of vertices = 80

Number of Connected Component = 21

∴ Number of edges = 80 – 21

= 59 edges.

4. (a)

I. Euler path exists as in the graph 2 odd vertices are present.

Path : b – c – d – b – a – e – d – a

II. An Euler circuit exists in the graph iff all the degrees are even. In the given we have 2 odd degree vertices so, graph does not have Euler circuit.

III. If a graph have Hamiltonian cycle then it have also Hamiltonian path.

∴ Hamilton cycle : a b c d e a

Hence, option a is correct.

5. (a)

A graph G is Euler if and only if it is connected and all the vertices must have even degree.

Now, the degree of each vertex of K_n complete graph is (n – 1).

Option a : Correct

The degree of each vertex (K_{51})

= (51 – 1)

= 50

So, the degree of all the vertices is even. Hence, K_{51} is Euler graph

Option b : Incorrect

The complete graph K_{50} have odd degree vertices.

Option c : Incorrect

A cycle graph is always Euler graph mean C_{60} is euler graph but its complement $\overline{C_{60}}$ may be disconnect graph. Hence, it is also not a Euler graph.

Option d : Incorrect

It would not be Euler graph if 11 – regular is disconnected graph.

6. **(c)**

Here the graph is disconnected graph with n vertices. So, to get the maximum number of edges, try to make single component with maximum vertices.

So,

$$\underbrace{V_1, \ V_2, \ V_3 \ - - - V_{n-1}}_{\text{Component 1}} \qquad \underbrace{V_n}_{\text{Component 2}}$$

Now, The maximum number of edges with $(n-1)$ vertices is : $\dfrac{(n-1)(n-2)}{2}$.