Branch : CSE/IT **Batch : English**

Discrete Mathematics
Graph Theory

DPP-05

[NAT]

1. Consider an undirected graph G, which is connected and have 8 vertices and 13 edges. Find the minimum number edges, whose deletion from graph G is always guarantee that it would be disconnected graph.

[MCQ]

2. The order (Number of vertices) of a complete bipartite graph in which there are 162 edges and one of the partitions has twice the number of vertices as of other ____?

(a) 20
(b) 25
(c) 27
(d) 29

[NAT]

3. Consider a simple graph of 10 vertices. If the graph is disconnected, then the maximum number of edges, it can have is _________ ?

[NAT]

4. Consider a wheel graph (w_n) of 9 vertices. Find the number of edges to be deleted from the above graph, such that the resultant graph must be minimal connected graph ____?

[MCQ]

5. If G is a simple disconnected graph with 16 vertices and 3 components, then maximum number of edges possible in G is _______?

(a) 90
(b) 91
(c) 92
(d) 93

[MCQ]

6. Which of the following statements is/are true?

S_1: A graph $G(V, E)$ is called tree if there is an exactly one path between every 2 vertices.

S_2: A graph $G(V, E)$ is tree iff it is connected, and it does not contain cycle.

(a) S_1 only
(b) S_2 only
(c) Both S_1 and S_2
(d) None of these.

Answer Key

1. 7
2. (c)
3. 36
4. 8
5. (b)
6. (c)

Hints and solutions

1. (7)

Here, we have 8-vertices and 13 edges.

Now, a connected graph with n vertices has atleast (n – 1) edges.

So, (n – 2) edges will lead to disconnected graph

and thus we need to delete:

$$e - (n - 2) = 13 - (8 - 2)$$
$$= 13 - 6$$
$$= 7 \text{ edges}$$

Hence to get the disconnected graph we need to delete 7 edges.

2. (c)

As we know that, the total number of edges in a complete bipartite graph is "m ∗ n" and number of vertices is "m + n".

Now,

$$m * n = 162 \text{ - - - - - (I)}$$

Where

m = Number at vertices in partition 1

n = Number of vertices in partition 2

Also, in the problem it is given that: m = 2n

So, after substitution into eq (I):

$$2n \times n = 162$$
$$2n^2 = 162$$
$$n^2 = 81$$
$$n = 9$$

Now, the order of the graph = m + n

$$= 2n + n$$
$$= 3n$$
$$= 3 \times 9$$
$$= 27$$

Hence, the order of the given graph is 27.

3. (36)

In the problem, we, have 10 vertices in the graph. To make it disconnected divide the 10 vertices into partitions

Partition 1: $\underbrace{V_1, V_2, V_3, \ldots.. V_9}_{\text{connected graph with 9 vertices}}$

Partition 2: $\underbrace{\quad V_{10} \quad}_{\text{Single/Isoloted vertices}}$

So, the maximum number of edges with 9 vertices is possible with complete graph:

$\therefore$ Complete graph (k_9) = n_{C_2}

$$= \frac{9 \times 8}{2}$$
$$= 36 \text{ edges}$$

4. (8)

In the problem, a wheel graph (w_n) of 9 vertices. The number of edges in (w_9); $2(n - 1) = 2\,(9 - 1) = 2 \times 8 = 16$ edges.

Now, the minimal connected graph means the graph is connecte with minimum number of edges.

We know that a graph of n vertices is minimal connected with (n – 1) edges.

Number of edges = 16 – (n –1)

$$= 16 - (9 - 1)$$
$$= 16 - 8 = 8 \text{ edges.}$$

5. (b)

Here, a simple disconnected graph with 16 vertices and 3 component is given.

Now, to get the maximum number of edges, try to distribute the maximum vertices in one component. as follows:

Component 1 :1 vertex (V_1)

Component 2 :1 vertex (V_2)

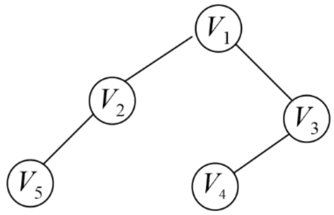
Component 3 :14 vertices ($V_3 \ldots V_{16}$)

$\therefore$ Maximum number of edges

$$= \frac{14 \times 13}{2} = 91 \text{ edges}$$

7. (c)

S_1 : Consider the given graph G:



In the above graph, we have exactly one path between every 2 vertices, that is graph is connected and does not contain cycles.

Hence, S_1 is True.

S_2 : It is the definition of tree. So, statement S_2 is also True.