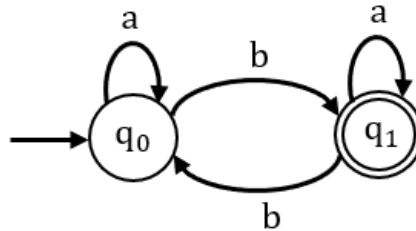


Finite Automata, Regular Expressions and Regular Language

1) Consider the following DFA over $\Sigma = \{a, b\}$



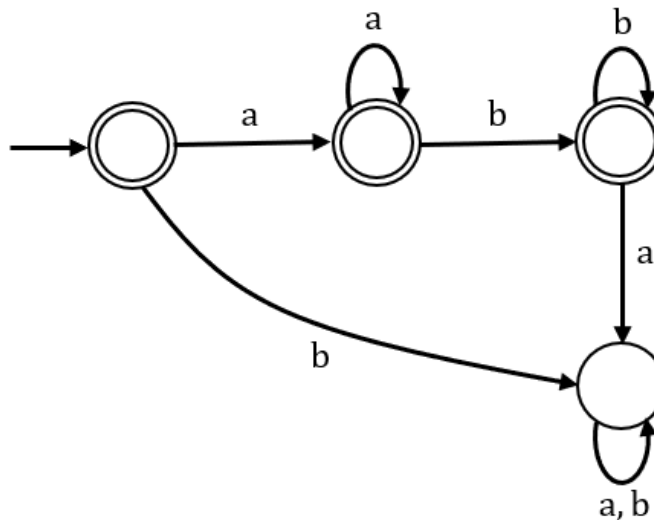
Select the correct option.

- A. The DFA accept all strings which end with “a”.
- B. The DFA accept all strings which has “b” as substring.
- C. The DFA accept all strings which start with “b” and not end with “b”.
- D. None of these

Solution: D

Explanation: DFA accept all strings which end with “a” is false, as String “aa” is not accepted. All strings “b” as substring is false, as sting “abab” is not accepted. All strings which start with “b” and not end with “b” is false, as sting “bbb” is accepted.

2) Consider the following DFA



Select the correct option.

- A. The language accepted by the given DFA is a^*b^* .
- B. The given DFA is a min DFA.
- C. The given DFA accept all strings which don't end with "a".
- D. None of these

Answer: B

Explanation: The given DFA is a min DFA as no further minimization is possible. The language accepted by DFA is not a^*b^* as a^*b^* generate string "b" which is not accepted by DFA. The given DFA don't accept all strings which don't end with "a" for example it don't accept string "bb".

3) Consider a set (S) of finite languages L i.e.,
 $S = \{L \mid L \text{ is a finite language over } \Sigma = \{a, b\}\}$. Select the wrong option with respect to Set S.

- A. S is closed under union.
- B. S is closed under intersection.
- C. S is closed under complement.
- D. S is not closed under Kleene star.

Answer: C

Explanation: Set S contains all the finite languages. Set S is closed under UNION mean that if we take any two languages from S and do UNION then the resulting language must also be in S. In case of UNION and Intersection it is closed, as union and intersection of two finite languages is always a finite language. But complement of a finite language is always infinite, so S is not closed under complement and Kleene star also (as Kleene star on any language of S will result in an infinite language).

4) Consider the following equalities and select the correct equality from the options.

- A. $\{\epsilon, \emptyset\} = \emptyset$
- B. $\emptyset + \epsilon = \epsilon, \emptyset$
- C. $\{\} = \{\epsilon\}$
- D. $\epsilon + R = R$, where R is any regular expression

Answer: A

Explanation:

For any regular expression R , we have the identity $\varnothing.R = \varnothing$, hence $\{\epsilon.\varnothing\} = \varnothing$

$\varnothing + R = R$, so $\varnothing + \epsilon \neq \epsilon$, as $\varnothing + \epsilon = \epsilon$ and $\epsilon.\varnothing = \varnothing$

$\{\} = \varnothing$, so $\{\} \neq \{\epsilon\}$

$\epsilon + R = R + \epsilon$, so $\epsilon + R \neq R$

5) Select the correct option.

- A. $\{\}^* = \{\epsilon\}$
- B. $\{\varnothing\} = \varnothing$
- C. $\varnothing^+ = \varnothing^*$
- D. None of these

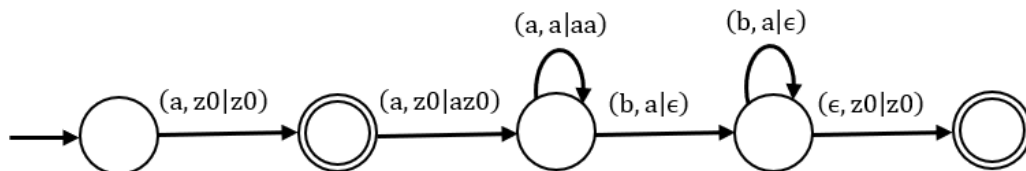
Answer: A

Explanation: $\{\} = \varnothing$ and $\varnothing^* = \{\epsilon\}$, hence $\{\}^* = \varnothing^* = \{\epsilon\}$

$\{\varnothing\} \neq \varnothing$ because the cardinality of \varnothing is zero, as empty set contains zero element, but cardinality of $\{\varnothing\}$ is 1, as this set contains one element (the element is empty set)

$\varnothing^* = \epsilon$ but $\varnothing^+ = \varnothing$, so $\varnothing^+ \neq \varnothing^*$

6) Consider the following PDA and select the language accepted by PDA from the options.



- A. $\{a^n b^n \mid n > 0\} \cup \{a\}$
- B. $\{a^n b^n \mid n \geq 1\} \cup \{a\}$
- C. $\{a^{n-1} b^n \mid n > 0\} \cup \{a\}$
- D. $\{a^n b^{n-1} \mid n > 0\} \cup \{a\}$

Answer: D

Explanation: The PDA accepts one "a" (if there is a single "a"). Also, the PDA accepts string "aaabb", please notice it is ignoring the first "a" and then starts pushing all "a's" and popping

each “a” from stack for every “b” and at the end if stack and input both are over then accepting the string. Hence the PDA accepting $\{a\} \cup \{a^n b^{n-1} \mid n > 0\}$

7) Select the regular language from the given options.

- A. $L1 = \{ww \mid w \in \{a,b\}^*\}$
- B. $L2 = \{ww^R \mid w \in \{a,b\}^*\}$
- C. $L3 = \{wCw^R \mid w \in \{a,b\}^*\}$
- D. None of these

Answer: A

Explanation: Since $w \in \{a,b\}^*$ so, w can be “a” or “b” and hence L1 is a finite language, as it has only two strings $L1 = \{aa, bb\}$

8) Let L is a language over $\Sigma = \{a,b\}$ such that $L = \{ab, bb\}$, select the complement of L from the given options.

- A. $\bar{L} = \{w \mid w \in \{a,b\}^* \text{ \& } |w| > 2\}$
- B. $\bar{L} = \{w \mid w \in \{a,b\}^* \text{ \& } |w| > 2\}$
- C. $\bar{L} = \{w \mid w \in \{a,b\}^* \text{ \& } |w| > 2\} \cup \{a, b, aa, ba\}$
- D. None of these

Answer: D

Explanation:

$$\bar{L} = \{w \mid w \in \{a,b\}^* \text{ \& } |w| > 2\} \cup \{\epsilon, a, b, aa, ba\}$$

9) Let $L1 = a^*b^*$ and $L2 = b^*a^*$, select the correct option with respect to L1 and L2.

- A. $\bar{L1} = L2$
- B. $L1 \cup L2 = \Sigma^*$
- C. $(L1)^R = (L2)^R$
- D. None of these

Answer: D

Explanation:

$L1$ (complement) will have strings such as $\{aba, abab..\}$ which is not generated by $L2$. So $\bar{L1} = L2$ is false.

$L_1 \cup L_2$ will not have strings such as {baba, abaab} hence it is not equal to Σ^* .

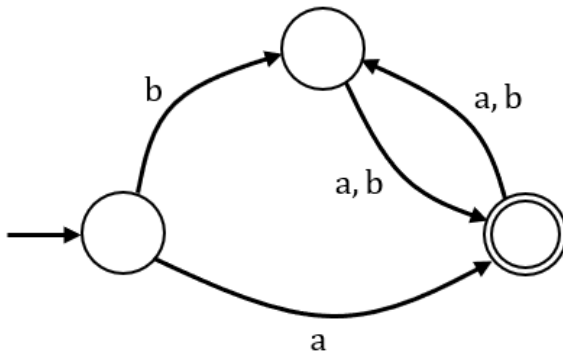
$(L_1)^R = L_2$ so $(L_1)^R = (L_2)^R$ is false statement.

10) Consider the following language L over $\Sigma = \{a, b\}$ such that,
 $L = \{ w \in \{a, b\}^* \mid (|w| \bmod 2 = 0 \text{ \& } w \text{ begins with "a"}) \text{ OR } (|w| \bmod 2 = 1 \text{ \& } w \text{ begins with "b"}) \}$

The number of states in the min DFA for the language L is _____

Answer: 3

Explanation:



11) Consider the following languages L_1 and L_2 , such that
 $L_1 = \{ a^n b^n \mid n > 0 \}$ and $L_2 = \{ a^m b^n \mid m \neq n \text{ \& } m, n \geq 0 \}$

Consider the following statements.

S1: $(L_1 \cup L_2)$ is a regular language.

S2: $(L_1 \cup L_2)$ is equal to Σ^* .

Select the correct option.

- A. Both S1 and S2 are true.
- B. S1 is true while S2 is false.
- C. S1 is false while S2 is true.
- D. Both S1 and S2 are false.

Answer: B

Explanation: S1 is a true statement, but S2 is a false statement.

As, $(L1 \cup L2) = a^*b^*$

12) Consider the following statements.

S1: If L is a regular language then $L_A = \{xy : x \in L \ \& \ |y| < 3\}$ is also a regular language.

S2: If L1 and L2 are two regular languages then $L_B = \{xy : x \in L1 \ \& \ y \in L2\}$ is also a regular language.

Select the correct option.

- A. Both S1 and S2 are true.
- B. S1 is true while S2 is false.
- C. S1 is false while S2 is true.
- D. Both S1 and S2 are false.

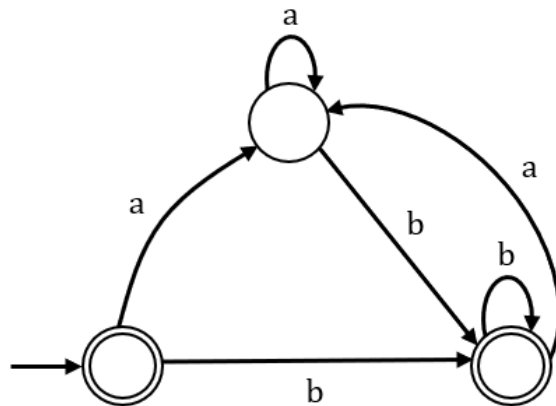
Answer: A

Explanation: If L is a regular language then concatenation three length strings at the end of every string of L will also result in regular language. Assume $L_B = \{y \mid |y| < 3\}$, clearly L_B is a finite language hence it is regular also. So $L_A = L.L_B$

Clearly L_A is a regular language as L and L_B both are regular and concatenation of two regular languages is also a regular language.

Similarly $L_B = L1.L2$ hence it is regular.

13) Consider the following finite automaton

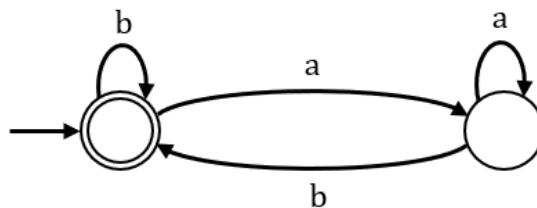


Select the correct option for given FA.

- A. The given FA is a min DFA
- B. The regular expression for given FA is $(a^*b)^*$
- C. The regular expression for given FA is $(aa^*b)^* + b^*$
- D. None of these

Answer: B

Explanation: The given FA is not min DFA, the min DFA is



By comparison we can see that correct regular expression is $(a^*b)^*$. We need to see the equality between string generated by regular expression and the given DFA.

14) Which one of the following doesn't generate the same language as the rest?

- (i) $(a+b)^*a(a+b)^*a(a+b)^*$
- (ii) $b^* a b^* a (a + b)^*$
- (iii) $(a + b)^* a b^* a b^*$
- (iv) $b^* a (a + b)^* a b^*$

- (a) Only (ii)
- (b) Only (iii)
- (c) Only (iv)
- (d) All regular expressions generate same language

Answer: Option (d)

Explanation: Language is having at least 2 a's.

15) Which one of the Regular Expression given defines the same language as defined by $R = (a + b)^* (aa + bb) (a + b)^*$?

- (a) $(a (ba)^* + b (ab)^*) (a + b)^*$
- (b) $(a (ba)^* + b (ab)^*)^* (a + b)^*$
- (c) $(a (ba)^* (a + bb) + b (ab)^* (b + aa)) (a + b)^*$
- (d) $(a (ba)^* (a + bb) + b (ab)^* (b + aa)) (a + b)^+$

Answer: C

Explanation: R generate min string "aa" or "bb".

$(a (ba)^* + b (ab)^*) (a + b)^*$ is not correct as it generate string "a".

$(a (ba)^* + b (ab)^*)^* (a + b)^*$ is not correct as it generate string "ε".

$(a (ba)^* (a + bb) + b (ab)^* (b + aa)) (a + b)^+$ is not correct as it doesn't generate string "aa". It generates the min string "aaa".

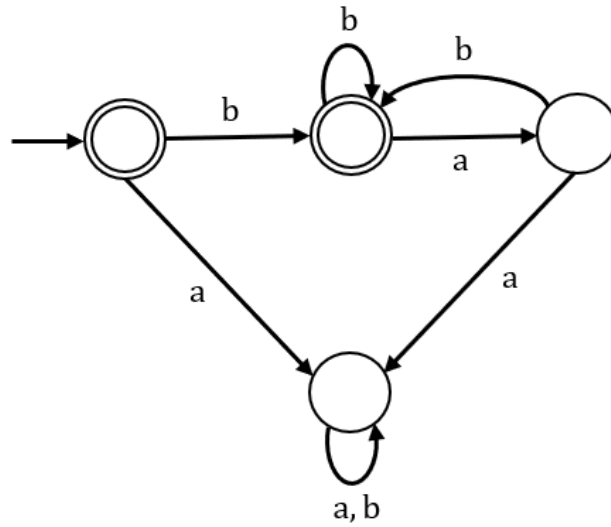
16) Consider a language L such that

$L = \{w \in \{a,b\}^* \text{ \& every "a" in w is immediately preceded and followed by "b" } \}$

The number of states in min DFA which accepts L are _____

Answer: 4

Explanation: The min DFA is



17) The total number of substrings present in “RAVI” is:

A. 7

B. 10

C. 11

D. 8

Ans: C

Explanation: The total substrings include trivial and non – trivial substrings, Trivial substrings are ϵ and “RAVI” itself.

Non – trivial substrings are R, A, V, I, RA, AV, VI, RAV, AVI.

So, the total number of substrings are 11.

Formula to calculate total number of substrings (including trivial) in a string of length “n” is:

$$(n(n+1)/2) + 1$$

In string “RAVI” $n=4$

Total number of substrings is: $(4 \times 5 / 2) + 1 = 10 + 1 = 11$

Please note the formula is applicable only when there is no repetition of any symbol in string, for example we cannot apply formula for the string “RAAVI”.

18) The total number of non-trivial substrings present in “RAVI” is:

A. 7

B. 8

C. 9

D. 11

Ans: C

Explanation:

Non – trivial substrings are R, A, V, I, RA, AV, VI, RAV, AVI.

So, the total number of non trivial substrings are 9.

Formula to calculate total number of substrings (non trivial) in a string of length “n” is:

$$(n(n+1)/2) - 1$$

In string “RAVI” n=4

$$\text{Total number of substrings is: } (4 \times 5 / 2) - 1 = 10 - 1 = 9$$

19) The total number of substrings present in “RAAVI” is:

A. 8

B. 9

C. 15

D. 16

Ans: C

Explanation:

length 1 substrings {R,A,V,I} -> 4

length 2 substrings {RA, AA, AV, VI} -> 4

length 3 substrings {RAA, AAV, AVI} -> 3

length 4 substrings {RAAV, AAVI} -> 2

length 5 substrings {RAAVI} -> 1

length 0 substrings {ε} -> 1

Total substrings are -> 15

Here we cannot apply formula as “A” is repeated two times in string

To apply formula, the process is given below:

First reach to point where all substrings are unique and start with length 1.

1 length substrings {R,A,V,I} -> 4

2 length substrings {RA, AA, AV, VI} -> 4

In two length all are unique (but in length 1 {R, A, A, V, I }, “A” is repeated)

Number of length 2 substrings (assume it as “n”) =4

Apply formula $\rightarrow 4 \times 5 / 2 + 1 = 11$

Now add the number of 1 length substrings in this: $11 + 4 = 15$

20) The total number of substrings present in “RAAAVI” is:

A. 18

B. 19

C. 21

D. 22

Ans: B

Explanation:

First reach to point where all substrings are unique and start with length 1.

1 length substrings {R,A,V,I} $\rightarrow 4$

2 length substrings {RA, AA, AV, VI} $\rightarrow 4$

In length 2 substrings the substrings are like {RA, AA, AA, AV, VI}, but we consider “AA” only one time, as in set repetition is considered as one element only.

3 length substrings {RAA, AAA, AAV, AVI} $\rightarrow 4$

In 3 length substring all are unique

Number of length 3 substrings (assume it as “n”) =4

Apply formula $\rightarrow 4 \times 5 / 2 + 1 = 11$

Now add the number of 1 length and 2 length substrings in this: $11 + 4 + 4 = 19$

21) A Language is said to be regular iff

- There exists a Right Linear Regular Grammar for L
- There exists a Left Linear Regular Grammar for L
- There exists a NFA with a single final state
- There exists a DFA with a single final state
- There exists a NFA without ϵ - move.

Which are true?

A. All are true

B. a, b, c are true

C. a, b, c, e are true

D. a, b, d are true

Solution: Option C

Explanation:

Every regular language can be represented by Right linear grammar, left linear grammar, an NFA with single final state and an NFA without ϵ -moves. But every regular language cannot be represented by a DFA with single final states, as a DFA with multiple final states, cannot be converted to DFA with single final state. Consider a scenario in which an NFA contains two final state, then we can make a single final state and connect it from epsilon move. Hence every NFA which has two or more final states can be converted into single final state (by using epsilon move)

22) Finite automata can be used in

(a) Lexical Analysis

(b) Syntax Analysis

(c) Semantic Analysis

(d) None of these

Solution: Option (a)

Explanation:

Finite Automata is used during Lexical Analysis to recognize tokens.

23) Finite automata requires minimum _____ number of stacks.

(a) 1

(b) 0

(c) 2

(d) None of the mentioned

Solution: Option (b)

Explanation:

Finite automata doesn't require any stack operation (any external memory such as stack) to recognize a regular language.

24) A language is regular if and only if

(a) accepted by DFA

(b) accepted by PDA

(c) accepted by LBA

(d) accepted by Turing machine

Solution: Option (a)

Explanation:

All of above machine can accept regular language but all string accepted by machine is regular only for DFA. In other words, we can use a PDA to recognize a regular language (as PDA is more powerful than DFA, hence can be used), but if a language which is recognized by PDA need not be regular always (since it can be CFL also). Hence a language is regular **iff** it is recognized by a DFA.

25) Consider this regular expression:

$$\text{r.e.} = (a^*a)b + b$$

What is the language?

- (a) All the strings ending with b
- (b) Any string of 1 or more a's followed by single b
- (c) Any string of 0 or more a's followed by single b
- (d) None of above

Solution: Option (c)

Explanation:

All the strings ending with b is false as regular expression doesn't generate strings such as {abb, bab,...}

Any string of 1 or more a's followed by single b is false, as regular expression generate string "b" which has zero a's.

Any string of 0 or more a's followed by single b, since all string generated by regular expression satisfy this condition.

26) Which one of the following is False?

- I. A DFA can contain one initial state and one final state
- II. A NFA can contain many initial states and many final states
- III. A DFA can contain many initial states and many final states
- IV. A NFA can contain one initial state and many final states

- (a) I, II
- (b) II, III
- (c) I, IV
- (d) III, IV

Solution: Option (b)

Explanation:

The definition of finite automata is:

A **deterministic finite acceptor** or **dfa** is defined by the quintuple

$$M = (Q, \Sigma, \delta, q_0, F),$$

where

Q is a finite set of **internal states**,

Σ is a finite set of symbols called the **input alphabet**,

$\delta : Q \times \Sigma \rightarrow Q$ is a total function called the **transition function**,

$q_0 \in Q$ is the **initial state**,

$F \subseteq Q$ is a set of **final states**.

Please note q_0 (only a single state) is initial state and F is set **final states** (more than one is possible and F can be empty i.e., number of final states can be zero also)

Hence a Finite automata (DFA or NFA) always has a single initial state and can contain many final states. Hence I and IV is true and II and III are false.

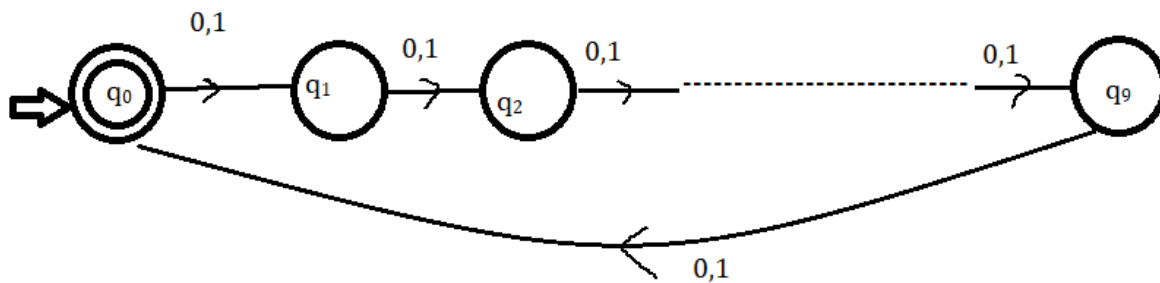
27) Find the number of states in minimal finite automata which accepts a language of strings whose length divisible by both '5' and '2' over $\Sigma = \{0, 1\}$

Solution: 10

Explanation:

Any number which is divisible by both 5 and 2 is (L.C.M. (2, 5)) multiple of 10.

\therefore If the length of the string is divisible by '10' it will produce residue (remainders) 0 to 9 and to represent each residue we require one state. Hence total number of states in min FA contains 10 states.



28) Consider this R.E. = $(0 + 1)^* (00 + 11)$

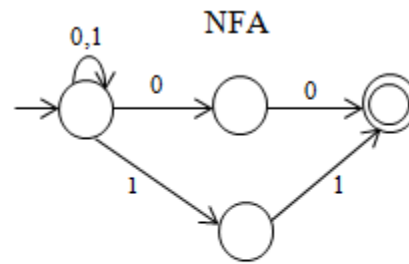
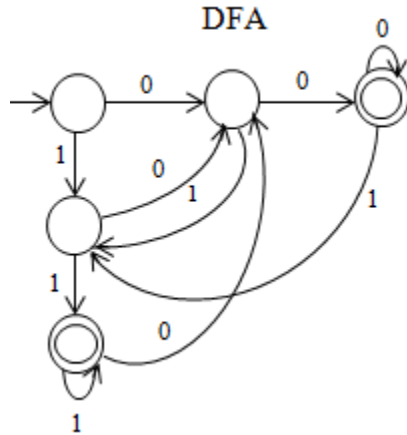
What will be the number of states in minimal DFA and NFA?

- (a) DFA – 5, NFA – 5
(c) DFA – 4, NFA – 4

- (b) DFA – 5, NFA – 4
(d) None

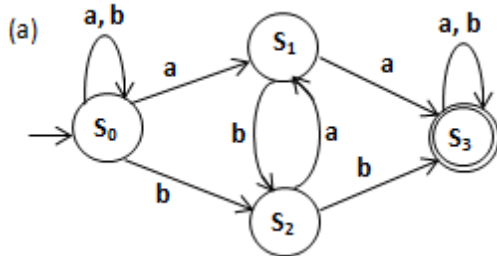
Solution: Option (b)

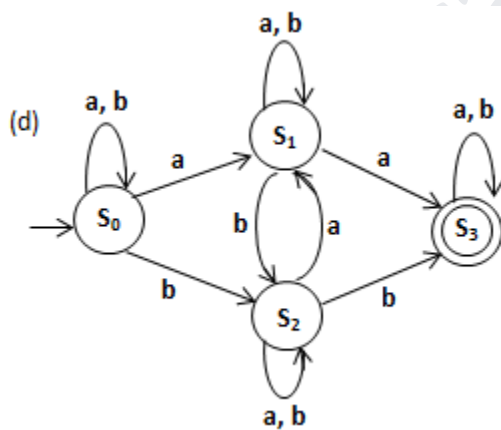
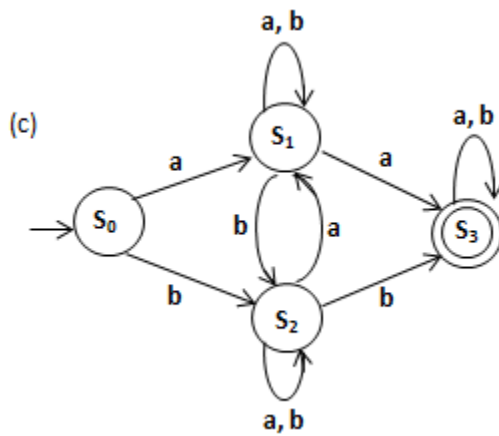
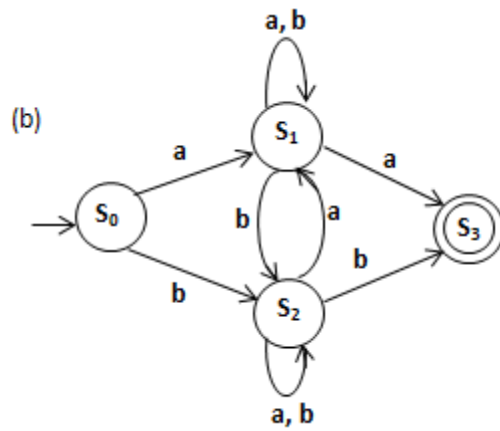
Explanation:



29) Consider $R = (a + b)^* (aa + bb) (a + b)^*$

Which of the following NFA recognizes the language defined by R?

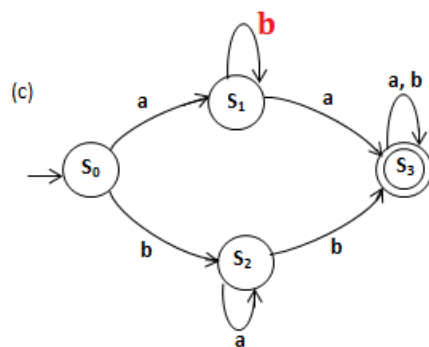
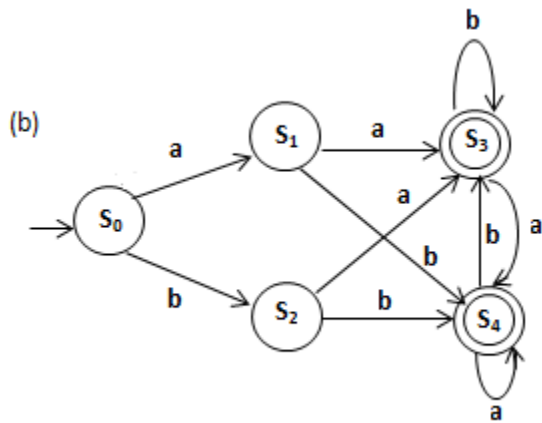
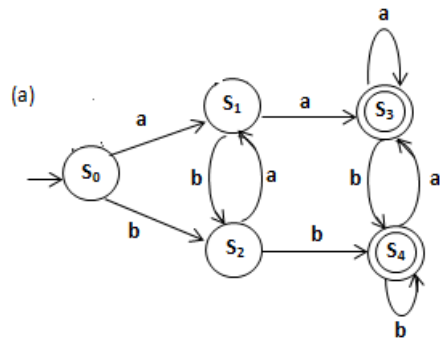




Solution: Option (a)

Explanation: The FA in option b,c and d accept string “aba” which is not generated by regular expression.

30) Which of the following DFA accepts same language accepted by $R = (a + b)^* (aa + bb) (a + b)^*$?

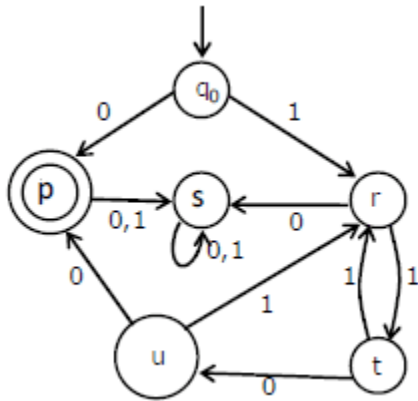


(d) None of above

Solution: Option (a)

Explanation: DFA in option B accept string “ab” and option C accept string “aba” hence B and C are not correct. Moreover option DFA in option A accept all strings generated by regular expression.

31) Which of the following language is accepted by the following finite automata?



(a) $(110)^*01$

(b) $0 + (1(11)^*10)^+0$

(c) $0 + (1(11)^*101)^+0$

(d) $(11 + 10)^*01$

Solution: Option (b)

Explanation:

The FA accept string “0” which is not generated by regular expression $(110)^*01$ and $(11 + 10)^*01$, hence both are wrong.

The regular expression $0 + (1(11)^*101)^+0$ generate string “11010” which is not accepted by given FA, hence it is also wrong.

32) Number of trivial substrings in “GATE2013” are:

- (a) 37 (b) 35
(c) 2 (d) 36

Solution: Option (c)

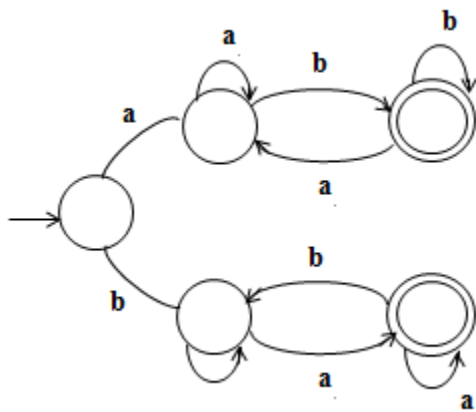
Explanation:

For any string, there will always be only 2 trivial substrings, ϵ and the given string itself.

33) Let the string be defined over symbols a and b then what will be the number of states in minimal DFA, if every string starts and ends with different symbols?

- (a) 5 (b) 4
(c) 3 (d) None

Solution: Option (a)



- 34) The total number of substrings present in "GATE" is: (a) 7 (b) 10
(c) 11 (d) 8

Solution: Option (c)

Explanation:

$L = \{\epsilon, G, A, T, E, GA, AT, TE, GAT, ATE, GATE\}$

Total number of substrings in a string of length n is $\frac{n(n+1)}{2} + 1$

35) Let $\Sigma = \{a, b\}$, what are the number of states in minimal DFA, length of every string congruent to mod 5.

- (a) 2 (b) 3

(c) 5

(d) None

Solution: Option (c)

36) A minimal DFA that is equivalent to a NDFA has:

A. Always more states

(b) Always less no. of states

C. Exactly 2^n states

(d) Sometimes more states

Solution: Option (d)

37) Consider following Regular Expression:

(i) $a^*b^*b(a+(ab)^*)^*b^*$

(ii) $a^*(ab+ba)^*b^*$

What is length of shortest string which is in both (i) & (ii)?

(a) 2

(b) 3

(c) 4

(d) None

Solution: Option (d)

38) What are the number of states needed in minimal DFA, that accepts $(1+1111)^*$, with 1 as alphabet.

(a) 5

(b) 4

(c) 1

(d) None

Solution: Option (c)

Explanation:

The language is 1^* .

39) Let $\Sigma = \{a\}$, assume language, $L = \{a^{2012.K} / K > 0\}$, what is minimum number of states needed in a DFA to recognize L?

(a) $2^{2012} + 1$

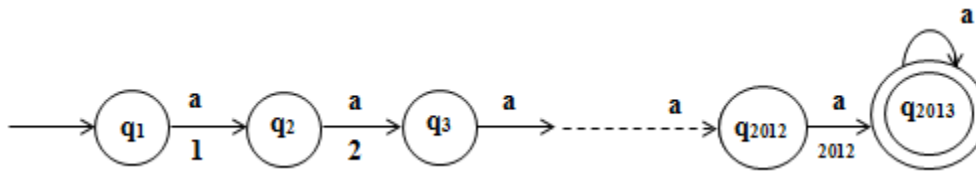
(b) 2013

(c) 2^{2013}

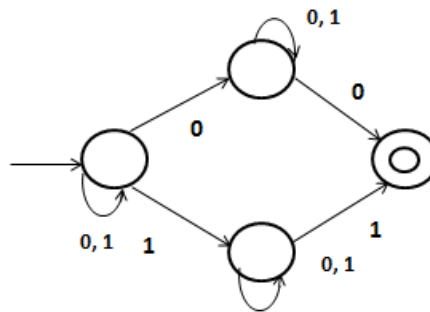
(d) None

Solution: Option (b)

Explanation:



40) Consider the following NFA M over the alphabet $\{0,1\}$.



Let M_1 be the NFA obtained by interchanging final and non-final states of M . Let the language accepted by M be L and that accepted by M_1 be L_1 . Choose correct statement:

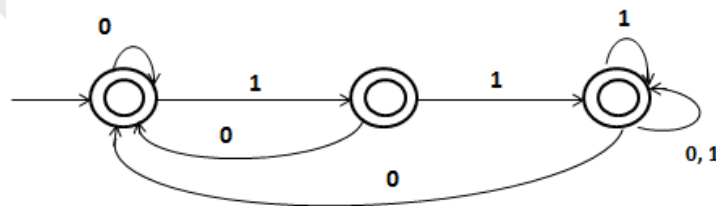
- (a) $L_1 = L$
- (b) $L_1 \cap L_2 = \Phi$
- (c) $L_1 \subseteq L_2$
- (d) $L_1 = (0+1)^*$

Solution: Option (d)

Explanation:

By interchanging final and non-final states, we get $L_1 = (0 + 1)^*$.

41)



The DFA above accepts:

- (a) The set of all strings containing two consecutive 1's
- (b) $(0+1)^*$
- (c) Set of all strings not containing two consecutive 1's
- (d) Set of all strings containing two consecutive 0's

Solution: Option (b)

42) The minimal DFA of the above machine has:

- (a) 1 State (b) 5 States
(c) 3 States
(d) 2 States

Solution: Option (a)

43) Let $r_1 = 1(0 + 1)^*$ and $r_2 = 1(1 + 0)^+$ $r_3 = 11^*0$

Which of the following is true?

- (a) $L(r_1) \subseteq L(r_2)$ and $L(r_1) \subseteq L(r_3)$ (b) $L(r_1) \supseteq L(r_2)$ and $L(r_2) \supseteq L(r_3)$
(c) $L(r_1) \supseteq L(r_2)$ and $L(r_2) \subseteq L(r_3)$ (d) $L(r_1) \subseteq L(r_3)$ and $L(r_2) \subseteq L(r_1)$

Solution: Option (b)

44) Let n_1 and n_2 be the number of states in NFA and minimal DFA of a regular language. Then,

- (a) $n_1 \geq n_2$ (b) $n_1 \leq n_2$
(c) $n_1 < n_2$ (d) $n_2 > n_1$

Solution: Option (b)

45) S_1 : L is regular. Infinite union of L will also be regular i.e. $(L^0 \cup L^1 \cup L^2 \dots)$ S_2 : L is regular. It's subset will also be regular.

- (a) Both are true (b) Both are false
(c) $S_1 \rightarrow T, S_2 \rightarrow F$ (d) $S_1 \rightarrow F, S_2 \rightarrow T$

Solution:

Option (b)

Explanation:

For Statement 1:

Regular languages are closed under finite union. They are not closed under infinite union of regular languages.

Consider

$$L_1 = \{01\}$$

$$L_2 = \{0^2 1^2\}$$

$$L_3 = \{0^3 1^3\}$$

.

.

.

So, infinite union of $L_1 \cup L_2 \cup L_3 \cup \dots = \{(0^n 1^n)/(n > 0)\}$ is CFL but not regular.

For Statement 2:

Subset of regular language may not be regular.

Consider $L = 0^* 1^*$ and $L_1 = \{0^n 1^n / n > 0\}$. L_1 is a subset of L but not regular.

46) Consider 2 scenarios:

C_1 : For DFA $(\phi, \Sigma, \delta, q_0, F)$,
if $F = \phi$, then $L = \Sigma^*$

C_2 : For NFA $(\phi, \Sigma, \delta, q_0, F)$,
if $F = \phi$, then $L = \Sigma^*$

Where F = Final
states set ϕ
= Total
states set

(a) Both are true

(b) Both are False

(c) C_1 is true, C_2 is false

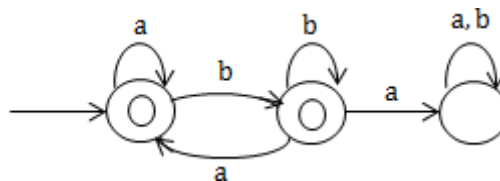
(d) C_1 is false, C_2 is true

Solution: Option (c)

Explanation:

As in case of NFA even if $F = \phi$, dead state rejection is there so $L \neq \Sigma^*$.

47) Consider this FA:



How many strings will be there in the complement of the language accepted by this Finite Automata?

- (a) Infinite (b) 2
(c) 3 (d) 0

Solution: Option (d)

Explanation:

As L is $(a + b)^*$

$\bar{L} = \{ \}$

48) In Programming language, an identifier has to be a letter followed by any number of letters or digits. If L and D denotes the sets of letter and digits respectively, examine the correct expressions?

- (a) $(L \cup D)^*$ (b) $(L \cdot D)^*$
(c) $L \cdot (L \cup D)^*$ (d) $L \cdot (L \cdot D)^*$

Solution: Option (c)

49) Total number of DFA possible with 2 states $q_0 \rightarrow$ start and non-final, $q_1 \rightarrow$ final over $\Sigma = \{a, b\}$ is

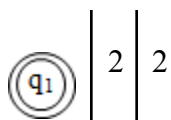
- (a) 16 (b) 32
(c) 48 (d) 64

Solution: Option (a)

Explanation:

16 DFA's are possible.

	0	1
$\rightarrow q_0$	2	2



$$\text{Total} = 2 \times 2 \times 2 \times 2 = 16$$

50) Which one of the following doesn't generate same language as rest?

- (i) $(a+b)^*a(a+b)^*a(a+b)^*$
- (ii) $b^*a b^*a(a+b)^*$
- (iii) $(a+b)^*a b^*a b^*$
- (iv) $b^*a(a+b)^*a b^*$
- (a) Only (ii)
- (b) Only (iii)
- (c) Only (iv)
- (d) All regular expressions generate same language

Solution: Option (d)

Explanation:

Language is having at least 2 a's.

51) $L_1 = \{a^m b^n \mid m+n = \text{Even}\}$ $L_2 = \{a^m b^n \mid m-n = 4\}$

- (a) L_1 is Regular, L_2 is Not Regular
- (b) Both are Regular
- (c) Both are Non- Regular
- (d) L_2 is Regular, L_1 is Not Regular

Solution: Option (a)

Explanation:

$$R = (aa)^* (bb)^* + a(aa)^* b(bb)^* = L_1$$

L_2 involves infinite counting and comparison. So it is not regular.

52) Let r be any Regular Expression:

$$\begin{aligned} S_1 \rightarrow r + \phi &= r = \phi \\ + r S_2 \rightarrow r + \epsilon &= r = \\ \epsilon + r \end{aligned}$$

- (a) Both are true
- (b) Both are False
- (c) $S_1 \rightarrow T, S_2 \rightarrow F$
- (d) $S_1 \rightarrow F, S_2 \rightarrow T$

Solution: Option (c)

Explanation:

r may not contain ϵ , so $r + \epsilon \neq r$

53) Which one of the Regular Expression given defines the same language as defined by $R = (a + b)^* (aa + bb) (a + b)^*$?

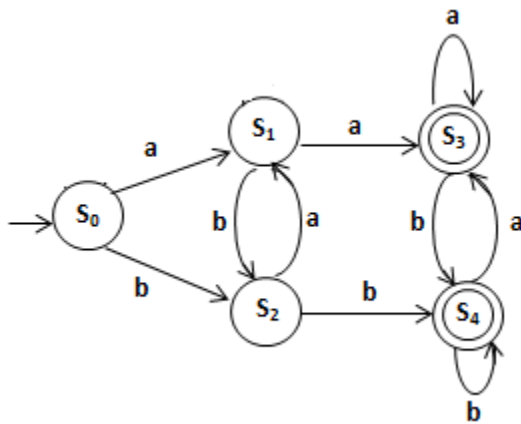
- (a) $(a (ba)^* + b (ab)^*) (a + b)^*$
- (b) $(a (ba)^* + b (ab)^*)^* (a + b)^*$
- (c) $(a (ba)^* (a + bb) + b (ab)^* (b + aa)) (a + b)^*$
- (d) $(a (ba)^* (a + bb) + b (ab)^* (b + aa)) (a + b)^+$

Solution:

Option (c)

Explanation:

This is the DFA for the regular expression R , use state – elimination method to transform the DFA to regular expression.



54) How many two state DFA's can be constructed over the alphabet $\Sigma = \{a, b\}$, with designated initial state?

- (a) 4
- (b) 16
- (c) 64
- (d) 128

Solution: Option (c)

Explanation:

$\Sigma = \{a, b\}$

$Q = \{x, y\}$

Let x be the initial state.

There are 4 types of Finite automata in this case.



In each of these types, there is a transition either from x to x or x to y and y to x or y to y for each terminal in Σ .

δ	a	b
x	*2	*2
y	*2	*2

→ 16

i.e., there can be 2 possibilities for each terminal from each state.

\therefore Total No. of DFA's = No. of types \times No. of DFA's of each type
 $= 4 \times 16 = 64$

55) How many 3 state DFA's with designated initial state can be constructed over the alphabet $\Sigma = \{a, b\}$ _____.

Solution: 5832

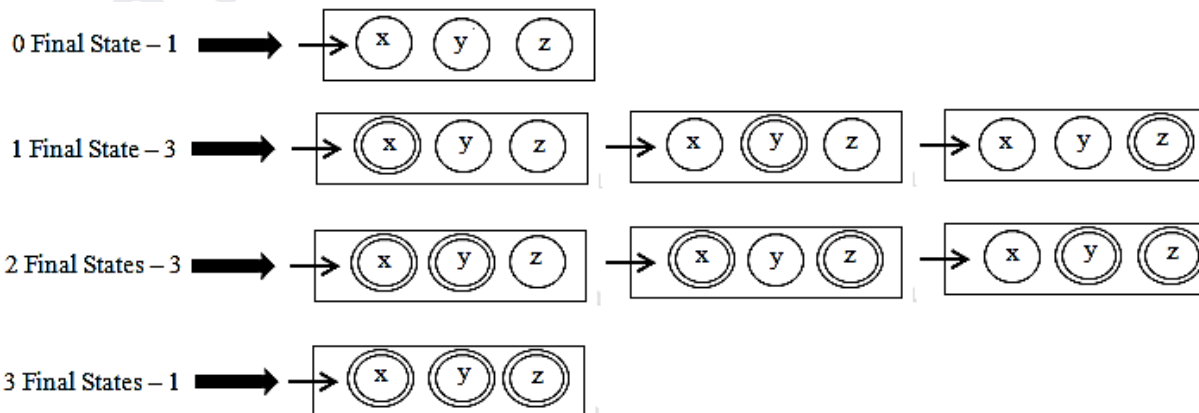
Explanation:

$\Sigma = \{a, b\}$

$Q = \{x, y, z\}$

Let x be the initial state.

There are 8 types of DFA's possible.



In each Finite automata, there can be 3 prefixes transitions from each state over each terminal.

δ	a	b
$\rightarrow x$	*3	*3
y	*3	*3
z	*3	*3

$\longrightarrow 3^6$ DFA's in each type

$$\begin{aligned}\therefore \text{Total No. of DFA's} &= \text{No. of types} \times \text{No. of DFA's in each type} \\ &= 8 \times 3^6 \\ &= 5832\end{aligned}$$

Note: If m = No. of symbols in Σ
 n = No. of states
 Total No. of types = 2^n
 No. of DFA's in each type = $n^{m \cdot n}$
 \therefore Total no. of DFA's = $2^n \times n^{m \cdot n}$

56) Find the no. of DFA's that can be constructed over the alphabet Σ with 5 symbols, and with 10 states.

(a) $2^{50} \times 50^5$

(b) $2^{10} \times 10^{50}$

(c) $2^5 \times 10^{50}$

(d) $2^{50} \times 50^{10}$

Solution: Option (b)

Explanation:

$|\Sigma| = 5$

$m = 5, n = 10$ states

\therefore Total No. of types = 2^{10} (Power set)

No. of DFA's of each type = $10^{5 \cdot 10} = 10^{50}$

\therefore Total No. of DFA's = $2^{10} \times 10^{50}$

57) How many 2 state DFA's with designated initial state can be constructed over the alphabet $\Sigma =$

$\{a, b\}$ that accept empty language ϕ ?

(a) 4

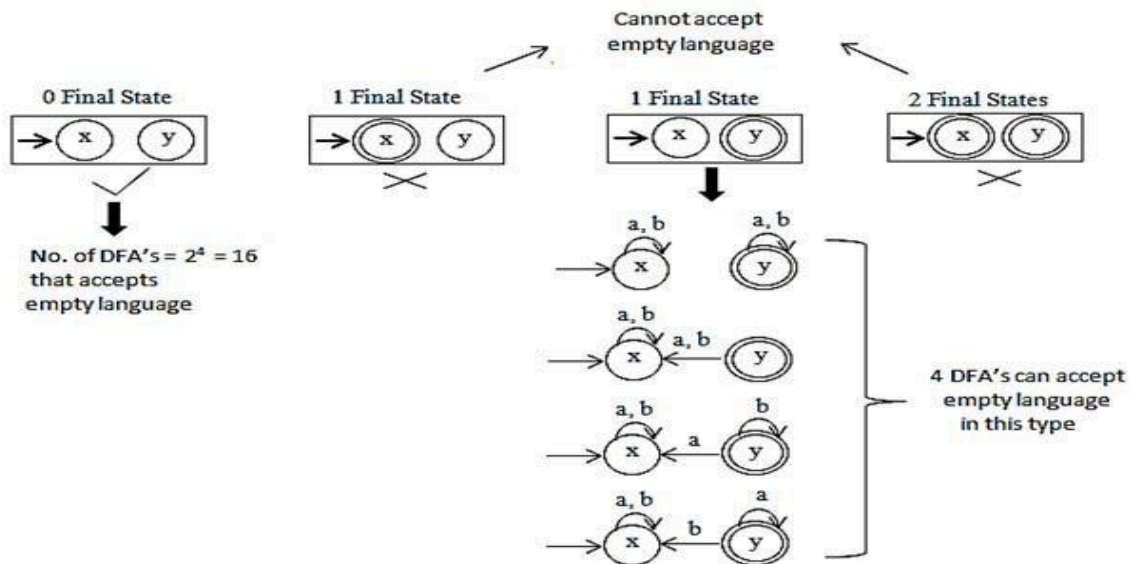
(b) 16

(c) 20

(d) 24

Solution: Option (c)

Explanation:



\therefore Total DFA's with designated initial state that accept empty language = $16 + 4 = 20$

58) How many 2 state DFA's with designated initial state can be constructed over the alphabet over the alphabet $\Sigma = \{a, b\}$ that accept universal language?

(a) 4

(b) 16

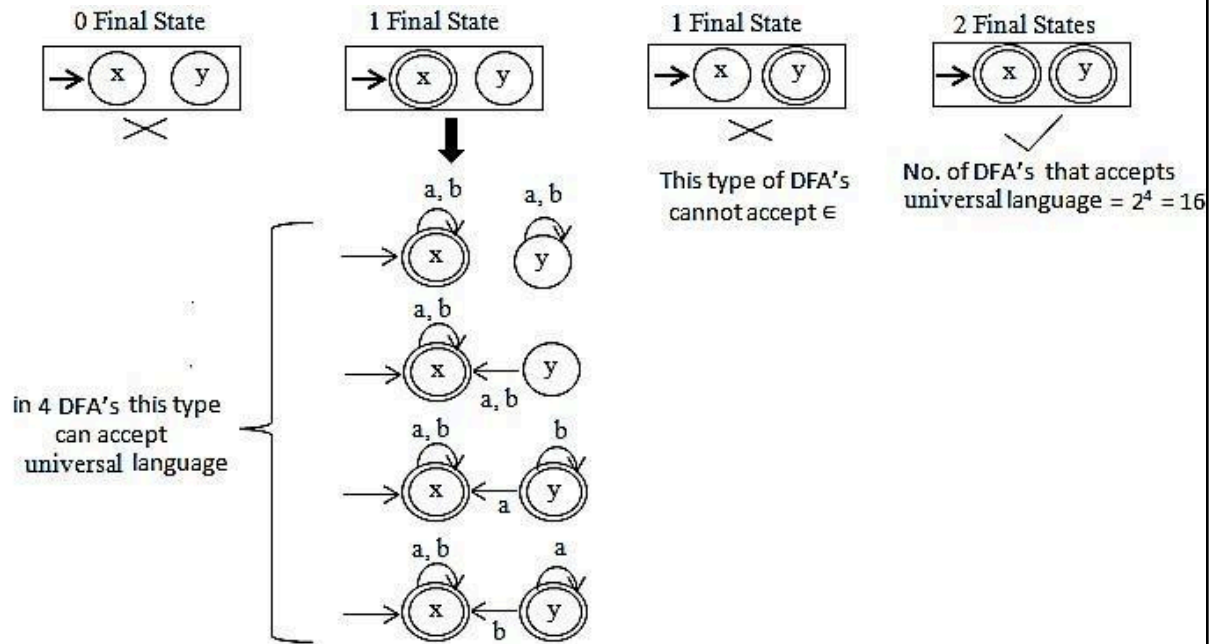
(c) 20

(d) 24

Solution: Option (c)

Explanation:

Universal language in $\Sigma^* = (a+b)^*$



59) Which of the following statement(s) are true about NFA & DFA?

- (i) NFA is more powerful than DFA but DFA is more efficient than NFA.
- (ii) NFA will respond for only valid inputs and no need to respond for invalid inputs.
- (iii) There is no concept of dead states and complement in NFA.
- (iv) NFA is a parallel computing system where we can run multiple threads concurrently.

- (a) only (i) & (ii) are true
- (b) only (ii) & (iii) are true
- (c) only (iii), (iv) & (i) are true
- (d) All statements are true

Solution: Option (d)

60) Which of the following is a Regular language?

- (a) $L_1 = \{wcw^R \mid w \in \{a, b\}^*\}$
- (b) $L_2 = \{wcw^R \mid w, c \in \{a, b\}^*\}$
- (c) $L_3 = \{ww^Rc \mid w \in \{a, b\}^*\}$
- (d) $L_4 = \{cww^R \mid w \in \{a, b\}^*\}$

Solution: Option (b)

Explanation:

(b) is Regular language

$$L_2 = \{wcw^R \mid w, c \in \{a, b\}^*\}$$

$$= \{\epsilon, a, b, ab, ba, aa, bb, aaa, \dots\}$$

$$= \Sigma^* \quad (\text{When } w = \epsilon \text{ and } c \in \{a, b\}^*, \text{ we obtain } L_2 \subset \Sigma^*, \text{ which is regular})$$

$\therefore L_2$ is Regular.

61) Given that a language $L = L_1 \cup L_2$, where L_1 and L_2 are two other languages. If L is known to be a regular language, then which of the following statements is necessarily TRUE?

- (a) If L_1 is regular then L_2 will also be regular
- (b) If L_1 is regular and finite then L_2 will be regular
- (c) If L_1 is regular and finite the L_2 will also be regular and finite
- (d) None of

these **Solution:**

Option (b)

Explanation:

(b) is the answer because we cannot make an irregular set regular by adding a finite number of elements to it. This can be paired as follows:

Let $R \cup F = T$ be regular, where F is a finite set. We remove all elements from F which are in R and let the new set is E . Still $R \cup E = T$ and E is a finite set (But no strings common between E and R).

Now, T be regular means we have a DFA for T . We can find all the states in this DFA where each of the strings in F is being accepted. In each of these accepting states, none of the string in R will be accepted as R and E does not have any string in common.

So, if we make all these states non-accepting, what we get is a DFA for R , meaning that R is regular.

We can prove (a) false by considering $L_1 = \{a\}$ and $L_2 = \{a^n \mid$

$n > 0\}$. Now, $L_1 \cup L_2$ is regular and L_1 is finite but L_2 is not finite

but Regular.

62) Let L be the language represented by the regular expression $\Sigma^*0011\Sigma^*$ where $\Sigma = \{0, 1\}$. What is the minimum number of states in a DFA that recognizes L ?

(a) 4

(b) 5

(c) 6

(d) 8

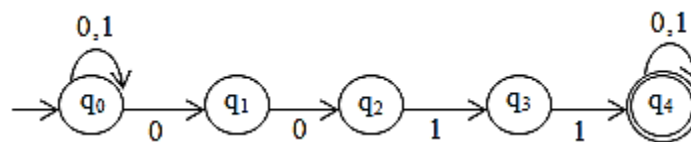
Solution: Option (b)

Explanation:

Given regular expression R.E. is

$(0+1)^*0011(0+1)^*$ NFA for L =

$(0+1)^*0011(0+1)^*$



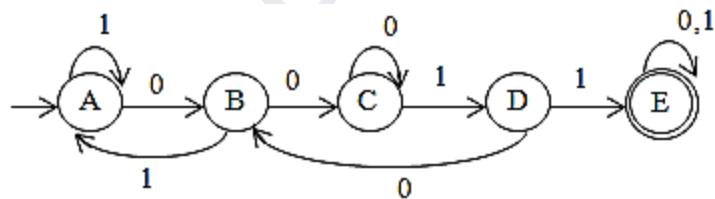
δ	0	1
$\rightarrow q_0$	q_0, q_1	q_0
q_1	q_2	ϕ
q_2	ϕ	q_3
q_3	ϕ	q_4
$*q_4$	q_4	q_4

DFA for the NFA:

δ'	0	1		δ'	0	1
$\rightarrow q_0$	q ₀₁	q ₀		$\rightarrow q_0$	q ₀₁	q ₀
q ₀₁	q ₀₁₂	q ₀		q ₀₁	q ₀₁₂	q ₀
q ₀₁₂	q ₀₁₂	q ₀₃		q ₀₁₂	q ₀₁₂	q ₀₃
q ₀₃	q ₀₁	q ₀₄		q ₀₃	q ₀₁	q ₀₄
*q ₀₄	q ₀₁₄	q ₀₄	Merge	*q ₀₄	q ₀₁₄	q ₀₄
*q ₀₁₄	q ₀₁₂₄	q ₀₄	(q ₀₄ & q ₀₃₄)	*q ₀₁₄	q ₀₁₂₄	q ₀₄
*q ₀₁₂₄	q ₀₁₂₄	q ₀₃₄	(q ₀₄ = q ₀₃₄)	*q ₀₁₂₄	q ₀₁₂₄	q ₀₄
*q ₀₃₄	q ₀₁₄	q ₀₄				

δ'	0	1		δ'	0	1
(A) q ₀	q ₀₁	q ₀		q ₀	q ₀₁	q ₀
(B) q ₀₁	q ₀₁₂	q ₀		q ₀₁	q ₀₁₂	q ₀
(C) q ₀₁₂	q ₀₁₂	q ₀₃		q ₀₁₂	q ₀₁₂	q ₀₃
(D) q ₀₃	q ₀₁	q ₀₄		q ₀₃	q ₀₁	q ₀₄
(E) *q ₀₄	q ₀₁₄	q ₀₄		*q ₀₄	q ₀₁₄	q ₀₄
				*q ₀₁₄	q ₀₁₄	q ₀₄

Minimal DFA

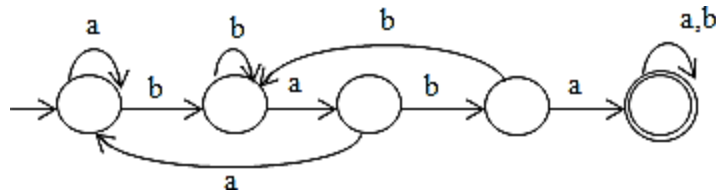


63) Choose the correct statement for the following regular expression over the symbols 0 & 1.
 $0(0+1)^*0 + 1(0+1)^*1$

- (a) To represent all strings over 0's and 1's
- (b) To represent all strings which start with 0's and end with 1's
- (c) To represent all strings which start and end with same symbol
- (d) To represent all strings that starts and ends with 1's

Solution: Option (c)

64) Consider the following DFA:



Which of the following is true for the above DFA?

- (a) It recognizes the strings which contains 'ababa' as substring
- (b) It recognizes the strings which contains 'abbaba' as substring
- (c) It recognizes the strings which contains 'abbabaa' as substring
- (d) It recognizes the strings which contains 'baba' as substring

Solution: Option (d)

65) Which of the following regular expression generates the set of all strings not containing 'baa' as a substring over input alphabet {a, b}?

- (a) $a^*(b^*a)^*$
- (b) a^*b^*ab
- (c) a^*baba^*
- (d) $a^*(ba+b)^*$

Solution: Option (d)

66) Identify the regular expression which denotes all strings of a's and b's where each string contains at least two b's.

- (a) $(a+b)^*ba^*b$
- (b) $(a+b)^*ba^*ba$
- (c) $(a+b)^*ba^*b(a+b)^*$
- (d) None of these

Solution: Option (c)

Explanation:

- (a) $(a+b)^*ba^*b$ □ It will not generate bba
- (b) $(a+b)^*ba^*ba$ □ It will not generate bbaa
- (c) $(a+b)^*ba^*b(a+b)^*$ □ It will generate all the strings containing atleast 2b's.

67) Consider this R.E. = $(0 + 1)^* (00 + 11)$

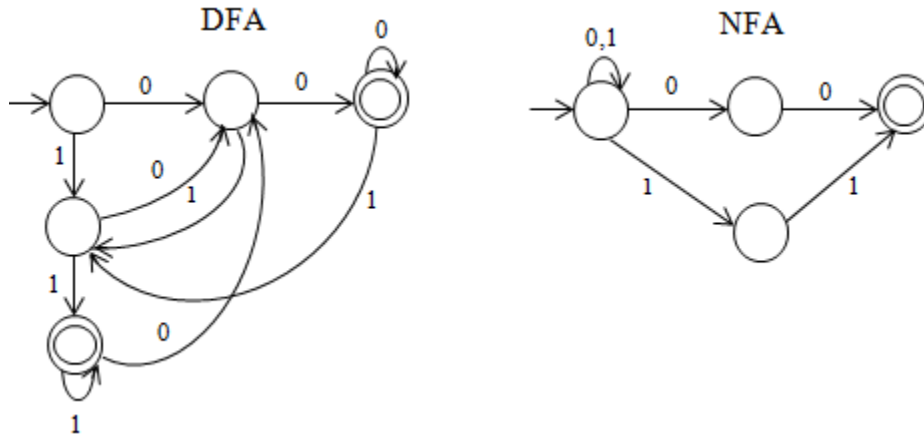
What will be the number of states in minimal DFA and NFA?

- (a) DFA – 5, NFA – 5
(c) DFA – 4, NFA – 4

- (b) DFA – 5, NFA – 4
(d) None

Solution: Option (b)

Explanation:



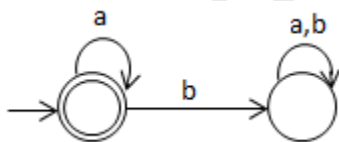
68) Number of states in minimal DFA to accept the language $(a + aaa)^*$ over $\Sigma = \{a, b\}$?

- (a) 1
(c) 3
(b) 2
(d) None

Solution: Option (b)

Explanation:

$(a + aaa)^* \equiv a^*$



69) Consider the following statements:

S_1 : $r_1 = (\epsilon + a + b)^{100}$ represents strings of length strictly less than 100. S_2 : $r_2 = (00 + 11 + 01 + 10)^* (0 + 1)$ represents all odd length strings.

- (a) Both are True
(b) Both are False

(c) $S_1 \rightarrow \text{True}, S_2 \rightarrow \text{False}$

(d) $S_1 \rightarrow \text{False}, S_2 \rightarrow \text{True}$

Solution: Option (d)

Explanation:

S_1

\rightarrow

F

S_2

\rightarrow

T

r_1 represents strings of length at most 100.

70) What will be number of states in DFA to represent the regular expression $r_1 = (01 + 1)^* (\epsilon + 0)$?

(a) 2

(b) 3

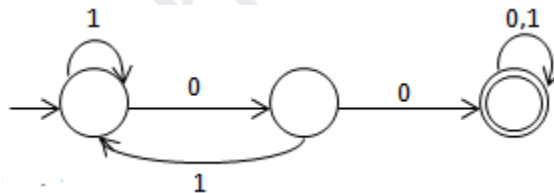
(c) 4

(d) 5

Solution: Option (b)

Explanation:

Let us consider the DFA for the language that accepts all the strings with consecutive 0's.



The complement this DFA is the DFA for r_1 .

71) Let $\text{Prefix}(u) = \{x \mid u = xy\}$

Let u be a string of length n . Total number of Prefixes possible for u will be

(a) n

(c) $n + 1$

(b) $n - 1$

(d) None

Solution: Option (c)

Explanation:

Ex. $u = ab$

Prefix $(u) = \{\epsilon, a, ab\}$

$\epsilon.ab$

$a.b$

$ab.\epsilon$

72) Consider this:

S_1 : Language L and its complement \bar{L} will have same number of states in minimal DFA. S_2 : Language L and its complement \bar{L} will have same number of states in minimal NFA.

(a) Both are True

(b) Both are False

(c) $S_1 \rightarrow T, S_2 \rightarrow F$

(d) $S_1 \rightarrow F, S_2 \rightarrow T$

Solution: Option (c)

Explanation:

Only in DFA, we can say that.

73) Let L be a Finite language in which maximum length of string is n and minimum length is m ($m < n$). Minimum number of states in the DFA will be:

(a) $m + 1$

(b) $n + 1$

(c) $n + 2$

(d) $m + 2$

Solution: Option (c)

Explanation:

In case of finite, maximum n length has to be accepted in $n + 1$ states and 1 dead state to reject strings of length more than n .

74) Let w be any string of length n in $(0, 1)^*$. Let L be set of all sub-strings of w . Minimum number of states in NFA that accepts L ?

(a) n

(b) $n + 1$

(c) $n + 2$

(d) $n - 1$

Solution: Option (b)

Explanation:

Here $|w|_{\max} = n$ and since it's a NFA, no dead state is required. So there should be $n+1$ states in the NFA.

75) Consider these:

S_1 : Kleene closure of a language is always infinite.

S_2 : Concatenation of infinite language and finite language is always infinite.

(a) Both are True

(b) Both are False

(c) $S_1 \rightarrow T, S_2 \rightarrow F$

(d) $S_1 \rightarrow F, S_2 \rightarrow T$

Solution: Option (b)

Explanation:

For S_1 : ϕ^*

$= \epsilon$ For S_2 :

$a^* \cdot \phi = \phi$

76) Consider:

S_1 : Every regular language can be accepted by NFA with only one Final

state S_2 : There is a language for which $L = L^*$

(a) Both are True

(b) Both are False

(c) $S_1 \rightarrow T, S_2 \rightarrow F$

(d) $S_1 \rightarrow F, S_2 \rightarrow T$

Solution: Option (a)

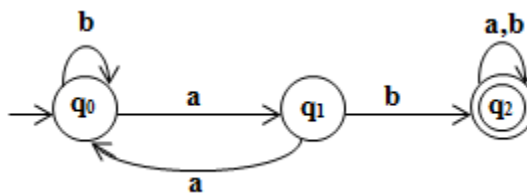
Explanation:

For S_1 : Because of

ϵ -moves For S_2 : $L =$

$\{\epsilon\}$

77) Regular Expression for this DFA:



(a) $(b + aa)^* ab(a + b)^*$

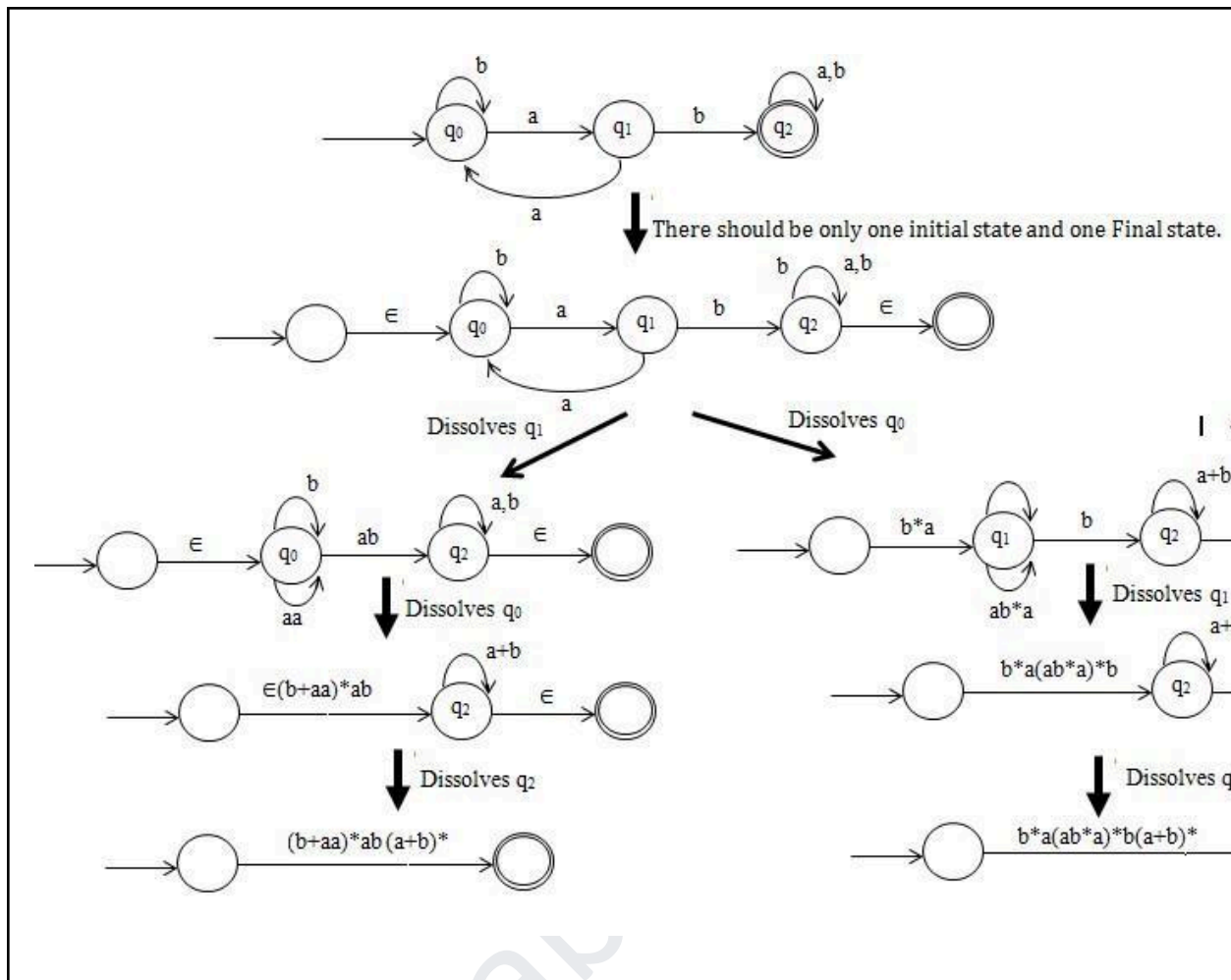
(b) $b^*a(ab^*a)^*b(a + b)^*$

(c) Both (a) and (b)

(d) $b^* ab(a + b)^*$

Solution: Option (c)

Explanation:



78) Consider this regular expression:

$$\text{r.e.} = (a^*a)b + b$$

What is the language?

- (a) All the strings ending with b
- (b) Any string of 1 or more a's followed by single b
- (c) Any string of 0 or more a's followed by single b
- (d) None of above

Solution: Option (c) Explanation:

Option (a) cannot be the answer because the regular expression does not accept strings having more than one b, like 'ababab' which ends with b but not in the given language.

Option (b) is wrong because there can be 0 a's before b.

Option (c) is right as it is exactly the language of given regular expression.

79) Consider 2 regular expression:

- i. $\phi^* + a^+ + b^+ + (a + b)^+ \rightarrow r1$
- ii. $\phi^+ + a^* + b^* + (a + b)^* \rightarrow r2$
- (a) $L(r1) = L(r2)$ (b) $L(r1) \subseteq L(r2)$
- (c) $L(r1) \supseteq L(r2)$ (d) None of above

Solution: Option (a)

Explanation:

Both are $(a + b)^*$ because $\phi^* = \epsilon$

80) Consider this regular expression:

$$r = (a^*b)^* + (b^*a)^*$$

This is equivalent to

- (a) $(a + b)^*$ (b) $(a + b)^* \cdot (ab)^+ + (a + b)^* (ba)^+$
- (c) $(a + b)^*a + (a + b)^*b$ (d) None of above

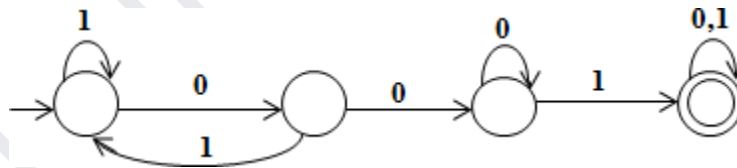
Solution: Option (a)

Explanation:

r = An empty string ϵ and strings ending with either a or b

Option (b) and (c) does not generate ϵ .

81) Consider this DFA:



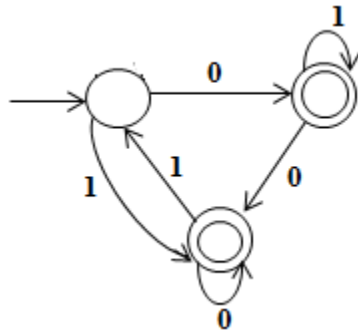
S denotes set of seven bit binary strings in which first, fourth, last bit is 1. Number of strings in L are:

- (a) 5 (b) 6
- (c) 7 (d) 8

Solution: Option (c) Explanation:

$L = \{1001011, 1001101, 1001111, 1001001, 1011001, 1101001, 1111001\}$

82) Which string is not accepted by FSA?



(a) 00111

(b) 01010

(c) 00110

(d) 11010

Solution: Option (a)

83) Can a Deterministic Finite State machine simulate a Non-Deterministic Finite State machine?

(a) No

(b) Yes

(c) Sometimes

(d) Depends on NFA

Solution: Option (b)

Explanation:

Always DFA is possible for an NFA. And DFA is equivalent to NFA for a given language.

84) Consider this:

i. $b^*a^* \cap a^*b^* = a^* \cup b^*$

ii. $a^*b^* \cap c^*d^* = \phi$

(a) Both are True

(b) Both are False

(c) (i) is True and (ii) is False

(d) (i) is False and (ii) is True

Solution: Option (c)

Explanation:

$$a^*b^* \cap c^*d^* = \epsilon$$

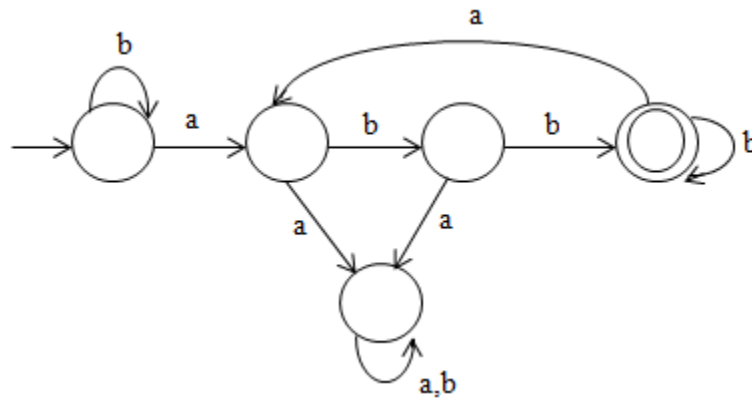
85) Minimum number of states in DFA over $\Sigma = \{0, 1\}$ with each string contains odd number of 0's or odd number of 1's.

- (a) 3
(c) 5

- (b) 4
(d) 6

Solution: Option (b)

86) Consider this FSM 'M' :



Language is

- (a) $\{w \in (a+b)^* \mid \text{every } a \text{ in } w \text{ is followed by exactly 2 b's}\}$
(b) $\{w \in (a+b)^* \mid \text{every } a \text{ in } w \text{ is followed by at least 2 b's}\}$
(c) $\{w \in (a+b)^* \mid w \text{ has substring } abb\}$
(d) $\{w \in (a+b)^* \mid w \text{ does not contain 'aa' as substring}\}$

Solution: Option (b)

87) How many two state DFA's exists over alphabet (0, 1) where X and Y are two states and X is always initial state, Y is always final.

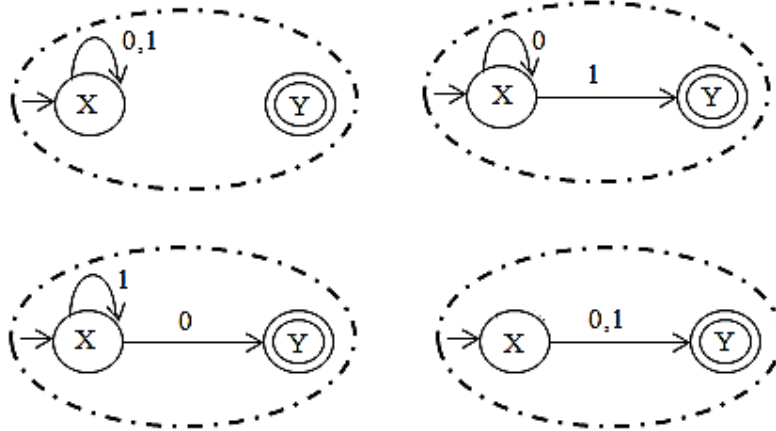
- (a) 16
(c) 32
- (b) 20
(d) 64

Solution: Option (a)

Explanation:

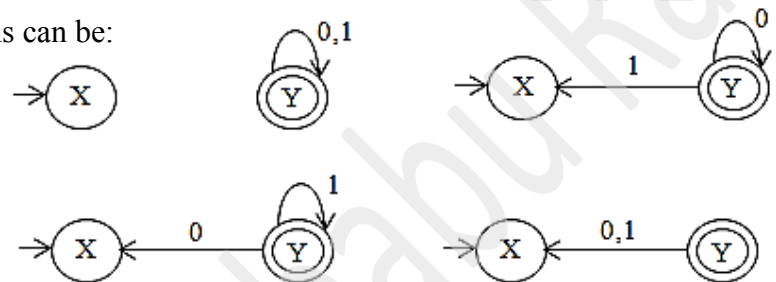
X is initial, Y is always

final. Transition for X:



4 possibilities □ For each of

these Y transactions can be:



So, total combinations $4 \times 4 = 16$

88) How many DFA's exist over alphabet $(0, 1)$ of two states X and Y where both states are always non-final?

- (a) 16
- (c) 32

- (b) 20
- (d) 64

Solution: Option (c)

Explanation:

Both states always Non-final. Same as previous. But here initial state is not mentioned. So, any of X or Y can be initial.

So, Ans = $2 \times 16 = 32$ □ (c)

89) How many total numbers of substrings are possible out of the string abbbccd?

(a) 25 (b) 27

(c) 28

(d) 29

Solution: Option (a)

Explanation:

abbbccd – length = 7

1 bit sub-strings a, b, c, d = 4

2 bit sub-string ab, bb, bc, cc, cd = 5

3 bit sub-string abb, bbb, bbc, bcc, ccd = 5

From 3 bits onwards, no. of substrings possible = $\frac{5 \times 6}{2} + 1$ (for null string) = 16

Total no. of substrings possible = $4 + 5 + 16 = 25$

90) 10. Given $\Sigma = \{A - Z, 0 - 9\}$ be an alphabet where w is a string defined over the alphabet. Let w = GATE2016, then number of substrings and number of trivial substrings is:

(a) 37, 39 (b) 37, 2

(c) 39, 37 (d) 2, 37

Solution: Option (b)

Explanation:

w = GATE2016

$|w| = 8$

∴ If w is a string with distinct symbols and the length is n i.e., $|w| = n$ then

No. of substrings = $(\Sigma n) + 1 = \frac{n(n+1)}{2} + 1$

2

∴ No. of substrings = $\frac{8(9)}{2} + 1 = 37$

2

No. of trivial substrings is 2 $\{\epsilon$ and GATE2016}

91) 11. Given a string $w = \text{"GRAMMAR"}$, the number of prefixes & suffixes respectively are:

- (a) 7, 7 (b) 8, 8
(c) 7, 8 (d) 8, 7

Solution: Option (b)

Explanation:

If $|w| = n$, then No. of prefixes = No. of suffixes = $n + 1$

92) 12. Given $\Sigma = \{a, b\}$, which of the following statement is true?

- (i) $\Sigma^* \cup \Sigma^+ = \Sigma^+$
(ii) $\Sigma^* \cap \Sigma^+ = \Sigma^*$
(iii) $\Sigma^* \cdot \Sigma^+ = \Sigma^+ \cdot \Sigma^* = \Sigma^+$
(iv) $\Sigma^+ \cdot \Sigma^+ = \Sigma^+$

- (a) Only (i), (ii) & (iii) (b) Only (i), (ii) & (iv)
(c) Only (iii) & (iv) (d) All of the above

Solution: Option (c)

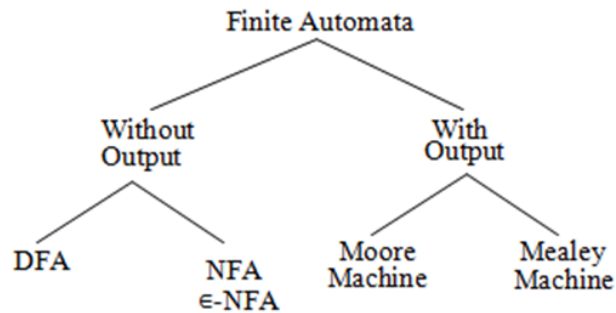
Explanation:

- (i) $\Sigma^* \cup \Sigma^+ = \Sigma^*$
because Σ^* has ϵ whereas Σ^+ does not.
 $\therefore \Sigma^* \cup \Sigma^+$ will contain ϵ in the set.
 \therefore It is Σ^* but not Σ^+ .
(ii) $\Sigma^* \cap \Sigma^+ = \Sigma^+$
since $\Sigma^* = \{\epsilon, a, b, \dots\}$
 $\Sigma^+ = \{a, b, \dots\}$
 $\Sigma^* \cap \Sigma^+ = \{a, b, \dots\} = \Sigma^+$
 \therefore (ii) is false.

93) 15. Which one of the following is/are the Finite Automata with outputs?

- (a) DFA (b) NFA
(c) ϵ -NFA (d) Moore machine
(e) Mealey machine

Solution: Option (d) & (e) Explanation:



94) For a given NFA with n state, the maximum number of state possible in the equivalent DFA is

- (a) 1 (b) n
 (c) $n + 1$ (d) $2n$

Solution: Option (d)

Explanation:

Max. No. of states in DFA is $2n$

& Min. No. of states is 1 for an* NFA with n states.

95) The minimal finite automata that accepts all strings of a's & b's, where the number of a's is atleast 'n' contains number of states.

- (a) $n - 1$ (b) n
 (c) $n + 1$ (d) $n + 2$

Solution: Option (c)

Explanation:

Given $\Sigma = \{a, b\}$

$|w| \geq n$, where $w \in \{a, b\}^*$

The minimal automata accepting this language has $n + 1$ states.

96) Consider the minimal Finite automata that accepts all the strings of a's & b's where each string contains

- (i) exactly 5 a's
 (ii) atleast 5 a's

The No. of states in each case respectively are:

(a) 6, 6

(b) 7, 7

(c) 6, 7

(d) 7, 6

Solution: Option (b)

Explanation:

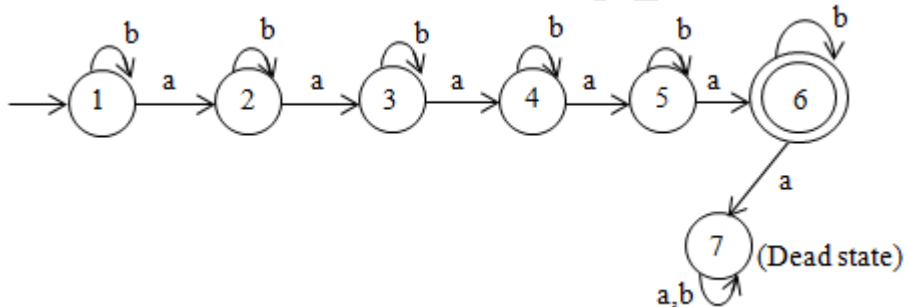
Given $\Sigma = \{a, b\}$

(i) $|w|_a = 5$, where $w \in \{a, b\}^*$

The minimal finite automata accepting exactly 5 a's has 7 states (including a dead state).

(ii) $|w|_a \leq 5$, where $w \in \{a, b\}^*$

The minimal finite automata accepting at most 5 a's has 7 states (including a dead state).



Note: $|w|_a = n \Rightarrow n+2$
states

$|w|_a \leq n \Rightarrow n+2$ states

$|w|_a \geq n \Rightarrow n+1$ states

97) 6. Consider the Minimal Finite Automata that accept all the strings of a's & b's where no. of a's is congruent to 2 mod 3. The no. of states in this automata is

(a) 4 (b) 3

(c) 2 (d) 1

Solution: Option (b)

Explanation:

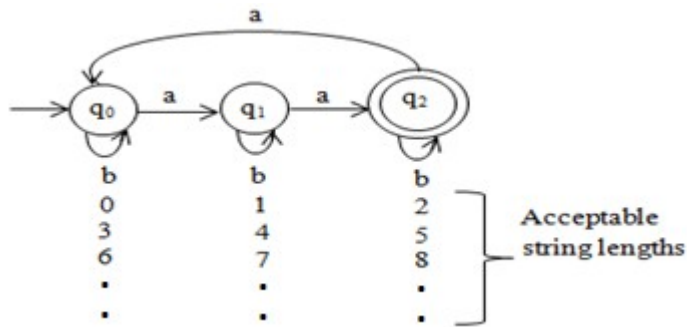
$\Sigma = \{a, b\}$

$w \in \{a, b\}^*, |w|_a \equiv 2 \pmod{3}$

It means, the automata accepts the string having the no. of a's as 2, 5, 8, 11, 14,.....

Clearly, there are 3 states only because of mod 3 and 3rd state will be the final state because of 2.

\therefore MFA is



Note: The no. of states in MFA that accept all the strings of a's and b's where $|w|_a \equiv r \pmod{n}$ or $|w|_b \equiv r \pmod{n}$ is exactly n , and q_r is final state.

98) Construct the minimal finite automata that accept all the strings of 0's & 1's is where the integer equivalent is congruent to 1(mod 4). What is the no. of states in minimal finite automata?

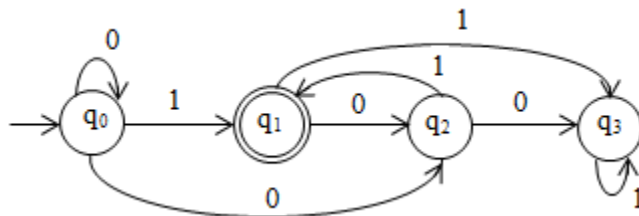
- (a) 4 (b) 3
(c) 2 (d) 1

Solution: Option (b)

Explanation:

Binary number $\equiv 1 \pmod{4}$

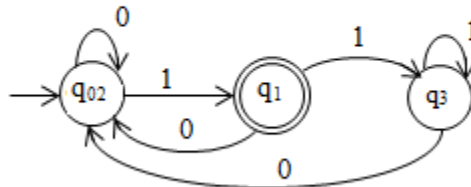
δ	0	1
$\rightarrow q_0$	q_0	q_1
$*q_1$	q_2	q_3
q_2	q_0	q_1
q_3	q_2	q_3



We can further minimize the finite automata:

↓ Merge q_0 & q_2

δ	0	1
q_{02}	q_{02}	q_1
q_1	q_{02}	q_3
q_3	q_{02}	q_3



We can also do this by grouping:

$\{q_0, q_2, q_3\} \{q_1\}$
 $\{q_0, q_2\} \{q_3\} \{q_1\}$

99) Construct the minimal finite automata that accept all the strings of 0's and 1's where the integer equivalent of the binary string is congruent to 3 mod 6. What is the number of states in the minimal finite automata?

- (a) 6 (b) 5
 (c) 4 (d) 3

Solution: Option (c)

Explanation:

Given the integer equivalent of binary string is $\equiv 3 \pmod 6$.

\therefore There are 6 states in DFA.

δ	0	1
$\rightarrow q_0$	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_5
$*q_3$	q_0	q_1
q_4	q_2	q_3
q_5	q_4	q_5

Merge q_1 & q_4 ,

q_2 & q_5

q_1, q_4 are equivalent

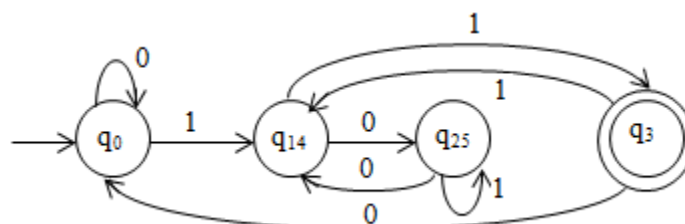
&

q_2, q_5 are equivalent

δ	0	1
$\rightarrow q_0$	q_0	q_{14}
q_{14}	q_{25}	q_3
q_{25}	q_{14}	q_{25}
$*q_3$	q_0	$q_1 \uparrow$

Minimal Finite Automata

Deterministic Finite Automata



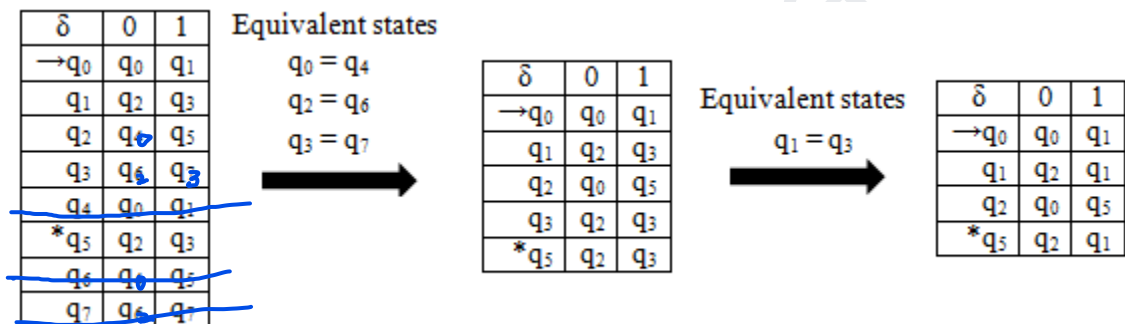
100) Given $\Sigma = \{0, 1\}$, the minimal finite automata that accepts all the strings from the given alphabet where the integer equivalent of the binary string is congruent to 5(mod 8) has _____no. of states.

- (a) 8 (b) 7
(c) 5 (d) 4

Solution: Option (d)

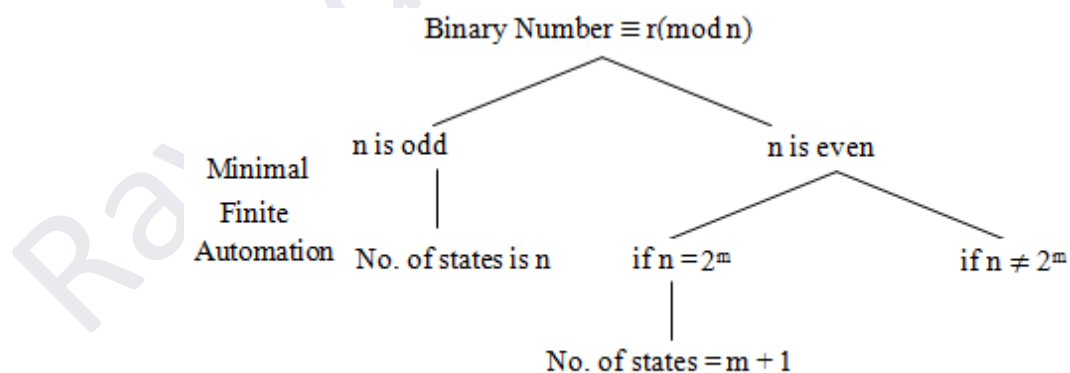
Explanation:

Transition table of DFA:



\therefore The minimal finite automata that accepts the integer equivalent of strings formed by 0's & 1's where $\equiv 5(\text{mod } 8)$ has 4 states.

Note:



Since it is congruent to 5(mod 8) and $8 = 2^3$, number of states in MFA is $3+1 = 4$ states

101) Given language $L = \{a^m b^n c^p \mid m, n, p \geq 0\}$, $\Sigma = \{a, b, c\}$. Find the number of states in

minimal finite automata of the above language.

(a) 3

(b) 4

(c) 5

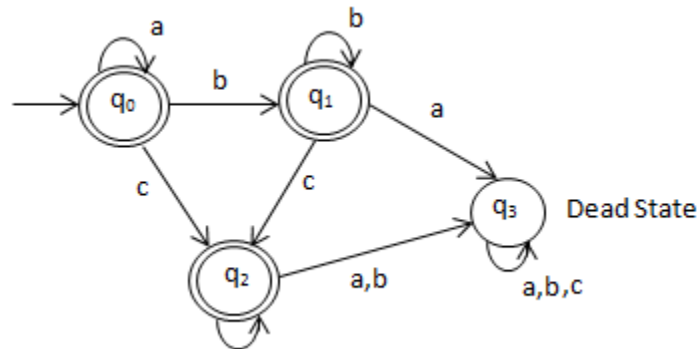
(d) 6

Solution: Option (b)

Explanation:

The given language contains

$\epsilon, a^m, b^n, c^p, a^m b^n b^n c^p, a^m c^p, \dots$



Minimum length of the string = $0+0+0 = 0$

102) Given $L = \{a^m b^n c^p d^q \mid m, n, p, q \geq 0\}$, $\Sigma = \{a, b, c, d\}$, find the number of states in the minimal finite automata of the language.

(a) 3

(b) 4

(c) 5

(d) 6

Solution: Option (c)

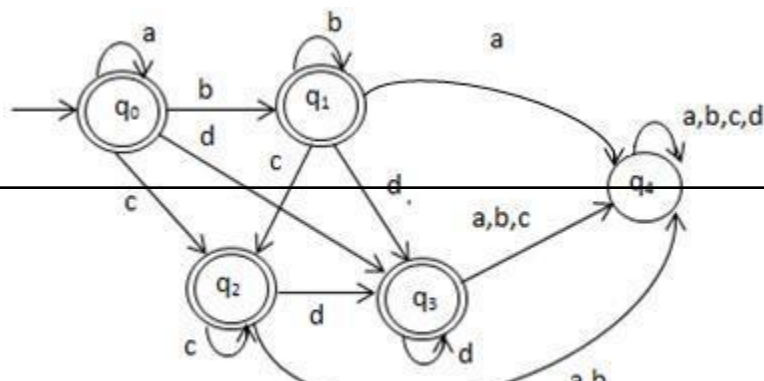
Explanation:

The given language contains

$\epsilon, a^m, b^n, c^p, d^q, a^m b^n, b^n c^p, c^p d^q,$

$a^m b^n c^p, \dots$ Minimum length =

$0+0+0+0 = 0$



Note: If Given language is of the form

$$L = \{a_1^{n_1} a_2^{n_2} \dots a_k^{n_k} \mid n_1, n_2, \dots, n_k \geq 0\} \text{ \& } \Sigma = \{a_1, a_2, \dots, a_k\}$$

Then Number of States in minimal finite automata = $|\Sigma| + 1$ Eg: Let $L = \{a^{n_1} b^{n_2} \dots z^{n_{26}} \mid n_1, n_2, \dots, n_{26} \geq 0\}$

No. of states in MFA is $|\Sigma| + 1 = 26 + 1 = 27$

103) Given a language $L = \{a^m b^n \mid m \geq 0, n \geq 2\}$, $\Sigma = \{a, b\}$. Find the number of states in the minimal finite automata that accepts the above language.

- (a) 3 (b) 4
(c) 5 (d) 6

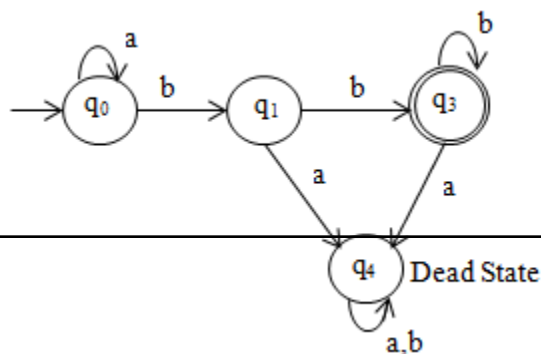
Solution: Option (b)

Explanation:

The language contains b^2 , ab^2 , a^2b^2 , ab^3 ,
.....

Minimum length = 2

\therefore No. states = $2+2 = 4$



MFA

104) $L = \{a^m b^n \mid m \geq 0, n \geq 2016\}$, $\Sigma = \{a, b\}$. Find the no. of states in the minimal finite automata that accepts this language.

(a) 2016

(b) 2017

(c) 2018

(d) 2019

Solution: Option (c)

Explanation:

If the given language is of the form $L = \{a^m b^n \mid m \geq 0, n \geq k\}$

Then no. of states in minimal finite automata that accepts this language is $k+2$ (k is the minimum length of any string).

\therefore Language $L = \{a^m b^n \mid m \geq 0, n \geq 2016\}$ No. of states = 2016 + 2 = 2018

105) The number of states in MFA that accept all the strings of a's & b's where each string contain odd occurrence of the substring ab is

(a) 4

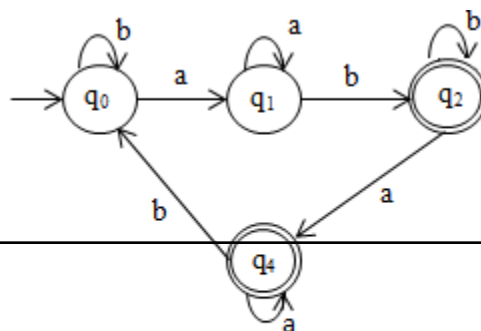
(b) 5

(c) 6

(d) 7

Solution: Option (a)

Explanation:



$L = \{ab, bbaab, babaa, bababab, \dots\}$

106) The number of states in MFA that accepts all the strings of a's & b's where each string contains even occurrences of substring ab

(a) 4

(b) 5

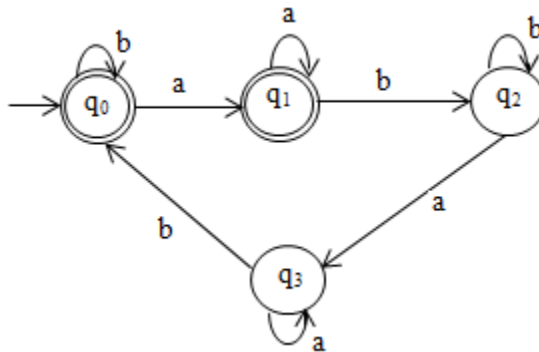
(c) 6

(d) 7

Solution: Option (a)

Explanation:

$L = \{\epsilon, abab, babaaab, \dots\}$



107)

What is the number of final states in the minimal finite automata, where $\Sigma = \{a, b\}$, if every string start with "aa" and length of string is not congruent to $0 \pmod{4}$

(a) 4

(b) 3

(c) 2

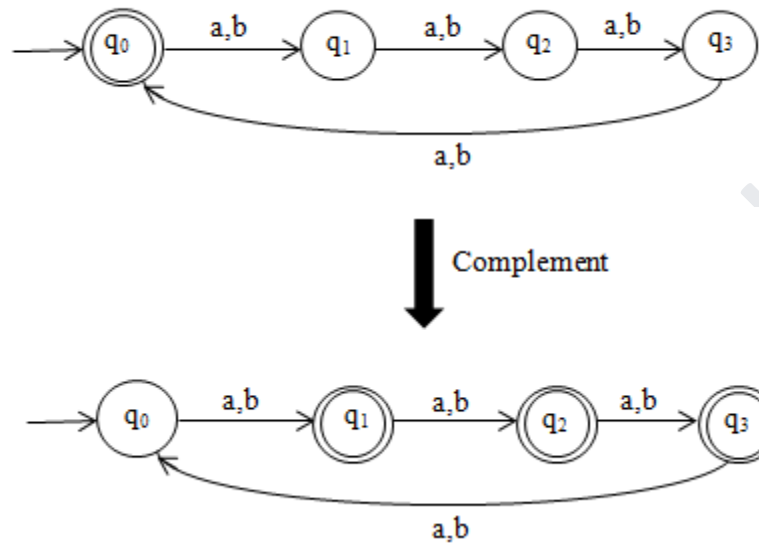
(d) 1

Solution: Option (b)

Explanation:

Given that length of string is not congruent to $0 \pmod{4}$.

Let us construct automata for the language that accepts strings of length congruent to $0 \pmod{4}$ and complement it to get the automata that accepts all the strings of lengths not congruent to $0 \pmod{4}$.



The minimal finite automata that accepts strings that start with "aa" and length of string is not congruent to $0 \pmod{4}$ is:

\therefore Total no. of final state is 3.

