

Discrete Mathematics

Graph Theory

DPP-02

[NAT]

1. Consider a complete graph with size 2016. Suppose after deletion of 2 vertices from the above graph, the modified graph have x number of edges and y number of vertices. Find the value of $x - y$?

[MSQ]

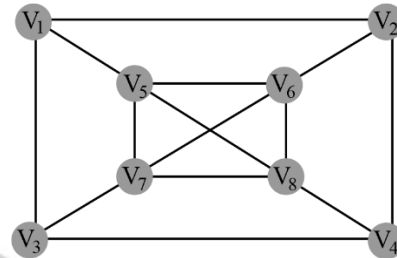
2. Which of the following options is/are True?
- Some k -regular graph can be complete graph.
 - A graph with more than 2 vertices, it must have at least 2 vertices with same degree.
 - The degree sum of odd degree vertices must be even.
 - The degree sum of odd degree vertices must be odd.

[MCQ]

3. Consider a wheel graph (w_n) with $n \geq 4$. Which of the following is minimum number of edges added to the above wheel graph to make it complete graph?
- $n(n-1)$
 - $\frac{(n^2 - 5n + 4)}{2}$
 - $\frac{n(n-1)}{4}$
 - None of these.

[NAT]

4. Consider the given graph $G(V, E)$ with order is 8 $\{V_1, V_2, V_3, \dots, V_8\}$. Find the minimum number of edges to be deleted from the graph, such that the graph become bipartite graph _____?



[MSQ]

5. Which of the following options is/are correct?
- Every NULL graph is always bipartite graph.
 - Some cycle graph is complete graph.
 - A cyclic graph is different from cycle graph.
 - A graph G is bipartite graph if and only if it has even cycle.

[MCQ]

6. Consider a ~~regular~~ graph with order 6 and size 12. Which of the following is the minimum degree (δ) and maximum degree (Δ)?
- $\delta = 3, \Delta = 4$
 - $\delta = 4, \Delta = 3$
 - $\delta = 4, \Delta = 4$
 - None of these

Answer Key

- | | |
|--------------|-----------|
| 1. (1829) | 5. (b, c) |
| 2. (a, b, c) | 6. (c) |
| 3. (b) | |
| 4. (2) | |



Hints and solutions

1. (1829)

As we know that the number of edges in the complete graph is:

$$\text{Number of edges} = \frac{n(n-1)}{2}$$

Where n = Number of vertices

Now,

In the problem size of the complete graph is given that is number of edges is 2016.

$$\therefore 2016 = \frac{n(n-1)}{2}$$

So, the number of vertices (n) = 64.

Now, after deletion of 2 vertices, the modified complete graph would have 62 vertices

$$\therefore \text{Number of edges} = \frac{62 \cdot 61}{2} = 1891$$

So, $x = 1891$ and $y = 62$

Hence, the find value $x - y = 1891 - 62$
 $= 1829$.

2. (a, b, c)

Option a : true

Every complete graph is regular graph.

Thus, a regular graph in which every vertex have degree $n - 1$ is K_n complete graph.

Hence, option A is correct.

Option b : True

In a simple connected undirected graph (with more than 2 vertices), at least 2 vertices must have same degree, since if this is not true, then all vertices would have different degree. A graph with all vertices having different degrees is not possible to construct (By 'Havel-Hakimi theorem').

Option c: True

No graph have odd number of vertices with odd degree.

$$\therefore \sum \deg(v) = 2|E|$$

$$\sum \text{even deg}(v) + \sum \text{odd deg}(v) = \text{Even}$$

$$\therefore \sum \text{odd deg}(v) = \text{Even}.$$

Hence, the sum of degree of odd degree vertices must be even.

3. (b)

I. The total number of edges in the wheel graph (w_n) is $2(n - 1)$ and the number of edges in the complete graph is $\frac{n(n-1)}{2}$

II. Now, the number of edges need to add into the wheel graph is:

$$\begin{aligned} & \frac{n(n-1)}{2} - 2(n-1) \\ \Rightarrow & \frac{(n^2 - n)}{2} - (2n - 2) \\ \Rightarrow & \frac{(n^2 - n) - 2(2n - 2)}{2} \\ \Rightarrow & \frac{n^2 - n - 4n + 4}{2} \\ \Rightarrow & \frac{(n^2 - 5n + 4)}{2} \text{ edges.} \end{aligned}$$

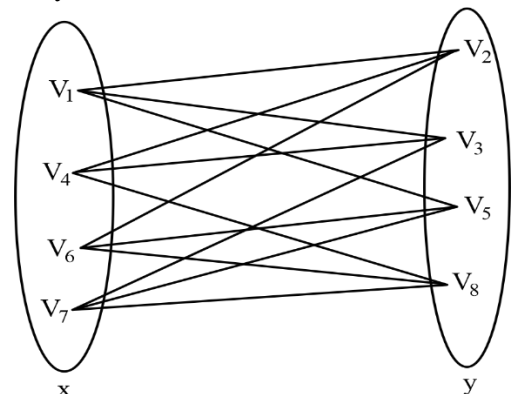
4. (2)

I. We know that bipartite graph consists of two sets of vertices.

If we delete the edge $\{V_6, V_7\}$ and $\{V_5, V_8\}$ then it is possible to divide the modified graph into two sets of vertices

$$x = \{V_1, V_4, V_6, V_7\} \text{ and } y = \{V_2, V_3, V_5, V_8\}$$

II. The vertices of set x are joined only with the vertices of set y and vice - versa.



Hence, we need to delete 2 edges to make it bipartite graph.

5. (b, c)

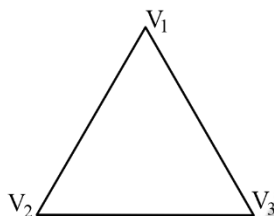
Option a: Incorrect

A null graph with number of vertices ≥ 2 is always bipartite graph but null graph with number of vertex either 0 or 1 is not bipartite graph.

Hence, every null can not be bipartite.

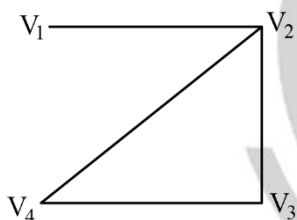
Option b: Correct

A cycle graph with number of vertices $n = 3$ is also a complete graph (K_3)

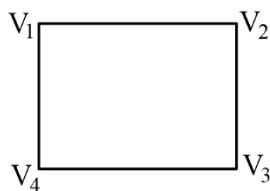


Option c: Correct

I. A cyclic graph is a graph in which a cycle is present.



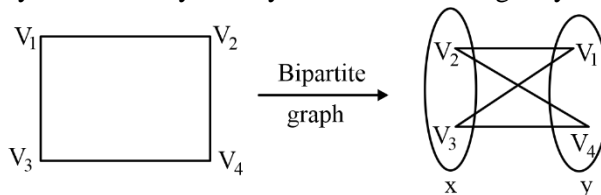
II. A cycle graph itself is one single cycle, where degree of each vertex is 2.



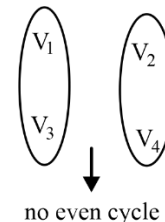
III. A cycle graph is always cyclic but every cyclic graph need not be cycle graph.

Option d: incorrect

A bipartite graph does not consist of odd length cycle but it may or may not have even length cycle.



II. Null graph with $n \geq 2$



Hence, the statement is incorrect.

6. (c)

A graph in which degree of all vertices is same is called as regular graph.

Hence,

$$n \cdot \delta(G) = 2 |E| = n \cdot \Delta(G)$$

$$\text{So, } 6 \cdot \delta(G) = 2 \cdot 12 = 6 \cdot \Delta(G)$$

$$\therefore \delta(G) = 4 = \Delta(G)$$

Hence, the minimum and maximum degree of the graph is 4.



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