Branch : CSE/IT **Batch : English**

Discrete Mathematics

Graph Theory

DPP-01

[MCQ]

1. Which of the following is a graphic sequence?

(a) 5, 3, 3, 2, 2, 1
(b) 2, 1, 1, 1, 1, 1
(c) 6, 5, 4, 3, 2, 1
(d) 5, 5, 2, 2, 1, 1

[NAT]

2. Find the number of edges of an undirected graph having degree sequence 2, 4, 4, 3, 4, 1?

[NAT]

3. Let δ denote the minimum degree of any vertices of a given graph and let Δ denote the maximum degree of any vertex in the graph. Suppose a certain graph has 8 vertices and that $\delta = 4$ and $\Delta = 6$, then this graph must contains at least _____edges.

[NAT]

4. There are 24 routers in Physics Wallah. Find the number of cable required to connect them such that each router is connected with exactly 6 others.

[MCQ]

5. What is the maximum value of minimum degree (δ) with a graph of order 10 and size 16?

(a) 4
(b) 3
(c) 2
(d) 1

Answer Key

1. (a)
2. (9)
3. (16)
4. (72)
5. (b)

Hints and solutions

1. (a)

Option a: correct

The degree sequence is : 5, 3, 3, 2, 2, 1

So, by applying "Havel hakimi" theorem,

5, 3, 3, 2, 2, 1 ➔ 2, 2, 1, 1, 0 ➔ 1, 0, 1,0

➔ 1, 1, 0, 0, ➔ it is valid.

The number of 1's is even so, the given graphic sequence is valid.

Option b: Incorrect

Property : After applying 'Havel–hakimi" theorem, the resuit must have evern number of 1's.

2, 1, 1, 1, 1, 1 ➔ 0, 0, 1, 1, 1, ➔ Not valid.

NOTE: In every graph the number of odd degree vertices is always even.

So, the graphical sequence is invalid as it has odd number of odd degree vertices.

Option c: Incorrect.

Any graphical sequence must have atleast one repetition.

2. (9)

Handshaking Lemma:

In any graph G (V,E) the sum of degress of all the vertices is equal to the twice of number of edges in that graph.

$$\sum_{v \in V} \deg(v) = 2\,|E|$$

Now, in the problem degree sequence given as : 2, 4, 4, 3, 4, 1

sum of degree = $\sum_{v \in V} \deg(v) = 2 + 4 + 4 + 3 + 4 + 1 = 18$

$\therefore$ Number of edges = $\dfrac{\sum \deg(v)}{2}$

$= \dfrac{18}{2} = 9$ edges

3. (16)

The given graph have 8 vertices and the minimum degree $\delta = 4$, and the maximum degree $\Delta = 6$.

Now, the relation between the number of edges, minimum degree and the maximum degree is as follows:

$$n \cdot \delta(G) \le 2\,|E| \le n \cdot \Delta(G)$$

$\therefore \quad 8 \cdot 4 \le 2\,|E| \le 8 \cdot 6$

$32 \le 2\,|E| \le 48$

$16 \le |E| \le 24$

Hence, the graph contains at least 16 edges.

4. (72)

The complete arrangement can be viewed as a graph in which routers are represented using vertices and cables using the edges.

Now. We have 24 vertices and the degree of each vertex is 6.

From Handshaking lemma:

Sum of degree of all vertices = 2 * (number of edges)

$\therefore$ 24 * 6 = 2 * number of edges

So, number of edges = $\dfrac{24 * 6}{2}$

= 72 edges

Thus, we need total 72 cables to connect the routers.

5. (b)

The relation between the number of edges, and minimum degree is given as :

$$\delta(G) \le \frac{2\,|E|}{n}$$

$$n \cdot \delta(G) \le 2\,|E|$$

Now, in the problem order is given n = 10 and size (number of edges) is |E| = 16

So, Maximum value of $\delta(G) = \left\lceil \dfrac{2\,|E|}{n} \right\rceil$

$$= \left\lceil \frac{2*16}{10} \right\rceil$$

$$= \left\lceil \frac{32}{10} \right\rceil = [3.2]$$

$$\delta(G) \le 3$$

Hence, the minimum degree can be at most 3.

So, The maximum value of minimum degree δ is 3.

Thus option b is correct .