

DISCRETE MATHEMATICS

Mathematical Logic



DPP-06

[MCQ]

1. Consider a function, $P(x, y, z) = x + y + z = 15$ and domain = z, then which of the following is correct?
- (a) $\forall x \exists y \exists z P(x, y, z)$ (b) $\exists z \forall x \forall y P(x, y, z)$
 (c) $\forall x \exists z \forall y P(x, y, z)$ (d) $\exists z \exists y \forall x P(x, y, z)$

[MCQ]

2. Consider an asymmetric function $P(x, y) = x^2 + y^2 = 10.0$ on domain integer, then which of the following is correct?
- (a) $\exists x \exists y P(x, y)$ (b) $\forall x \exists y P(x, y)$
 (c) $\forall y \exists x P(x, y)$ (d) None of these

[MSQ]

3. Which of the following is/ are negation of $[\forall x \exists y \forall z (P(x, y, z) \oplus Q(x, y, z))]$
- (a) $\exists x \forall y \exists z (\sim P(x, y, z) \oplus \sim Q(x, y, z))$
 (b) $\exists x \forall y \exists z (P(x, y, z) \Rightarrow \sim Q(x, y, z))$
 (c) $\exists x \forall y \exists z (P(x, y, z) \Leftrightarrow Q(x, y, z))$
 (d) $\exists x \forall y \exists z (\sim P(x, y, z) \Leftrightarrow \sim Q(x, y, z))$

[NAT]

4. Consider the following logical expressions
- (a) $\forall x \forall y P(x, y) \leftrightarrow \exists y \forall x P(x, y)$

$$(b) [\forall x P(x)] \vee Q \leftrightarrow \forall x [P(x) \vee Q]$$

$$(c) \forall x [P(x) \wedge Q] \leftrightarrow [\forall x P(x)] \wedge Q$$

$$(d) \exists x [P(x) \vee Q] \leftrightarrow [\exists x P(x)] \wedge Q$$

Total invalid expressions are ____?

[MCQ]

5. Consider the following statements
 S_1 : There is someone who is loved by everyone.
 S_2 : Every real number has its corresponding negative.
 Here $L(x, y)$ denotes "x loves y"
 $P(x, y)$ denotes " $x + y = 0$ "
 Which of the following represent the correct predicate logic of the given statement?
- (a) $S_1: \exists x \forall y L(x, y), S_2: \exists y \forall x p(x, y)$
 (b) $S_1: \forall x \exists y L(x, y), S_2: \forall x \forall y p(x, y)$
 (c) $S_1: \exists y \forall x L(x, y), S_2: \forall x \exists y p(x, y)$
 (d) None of these.

Answer Key

- | | | | |
|----|--------|----|-----|
| 1. | (a) | 4. | (2) |
| 2. | (d) | 5. | (c) |
| 3. | (c, d) | | |



Hints and Solutions

1. (a)

(a) $\forall x \exists y \exists z P(x, y, z)$

$z + y = 15 - x$

$15 - \text{integer} = \text{integer}$ **True**

(b) $z = 15 - x - y$ **False**

z must be independent, here z depends on x and y .

(c) $z = 15 - x - y$ **False**

z should not depend on y .

(d) $y + z = 15 - x$ **False**

the value of $(y + z)$ is depending on x , $(y + z)$ must be independent, so this expression is also **False**.

2. (d)

(a) $\forall x \exists y P(x, y)$ **False**

$x^2 + y^2 = 10.0$

$F(1, 3) = 1 + 9 = 10$

Here, 10 is integer but output must be 10.0, it will never come because 10.0 is not an integer.

(b) $\forall x \exists y P(x, y)$ **False**

10.0 will never come.

(c) $\forall y \exists x P(x, y)$ **False**

Hence, option (d) is correct

3. (c, d)

Negation of XOR operator is biconditional.

p	q	$p \oplus q$	$p \leftrightarrow q$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

(c) $\sim [\forall x \exists y \forall z (P(x, y, z) \oplus Q(x, y, z))]$

$[\exists x \forall y \exists z \sim (P(x, y, z) \oplus Q(x, y, z))]$

$[\exists x \forall y \exists z (P(x, y, z) \leftrightarrow Q(x, y, z))]$ True

(d) Property:

$P \leftrightarrow Q \equiv \sim P \leftrightarrow \sim Q$

$P'Q' + PQ \equiv P'Q' + PQ$

$\sim [\forall x \exists y \forall z (P(x, y, z) \oplus Q(x, y, z))]$

$[\exists x \forall y \exists z (\sim P(x, y, z) \leftrightarrow \sim Q(x, y, z))]$ True

Hence, option (c, d) are correct.

4. (2)

(a): Invalid

$\forall x \forall y P(x, y) \rightarrow \exists y \forall x P(x, y)$ (One way true)

$\forall y \forall x P(x, y) \rightarrow \exists y \forall x P(x, y)$

(b): $[\forall x P(x)] \vee Q \leftrightarrow [\forall x P(x) \vee Q]$

$(P_1 \wedge P_2) + Q \equiv (P_1 \vee Q) \wedge (P_2 \vee Q)$

$P_1 P_2 + Q \equiv P_1 P_2 + P_1 Q + P_2 Q + Q$

$P_1 P_2 + Q \equiv P_1 P_2 + Q$ (valid)

(c): $\forall x [P(x) \wedge Q] \leftrightarrow [\forall x P(x)] \wedge Q$

$(P_1 \wedge Q) \wedge (P_2 \wedge Q) \equiv (P_1 \wedge P_2) \wedge Q$

$P_1 Q P_2 Q \equiv P_1 P_2 Q$

$P_1 P_2 Q \equiv P_1 P_2 Q$

Valid

(d): $\exists x (P(x) \vee Q) \leftrightarrow [\exists x P(x)] \wedge Q$

$(P_1 \vee Q) \vee (P_2 \vee Q) \equiv (P_1 \vee P_2) \wedge Q$

$P_1 + Q + P_2 + Q \equiv (P_1 + P_2) Q$

$P_1 + P_2 + Q \not\equiv (P_1 + P_2) Q$ Invalid

Total 2 expressions are invalid

5. (c)

Statement S₁: There is someone who is loved by everyone.

• Assume, variables x and y denote people

• A predicate $L(x, y)$: denotes “ x loves y ”

$\therefore \exists y \forall x L(x, y)$ there is someone who is loved by everyone.

Statement S₂: Every real number has its corresponding negative.

• Assume, a real number is denoted as x and its negative as y .

• A predicate $p(x, y)$ denotes “ $x + y = 0$ ”

$\therefore \forall x \exists y p(x, y)$

Hence, option c is correct answer.



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