Branch : CSE/IT **Batch : English**

Discrete Mathematics

Graph Theory

Planarity Part-3

DPP-12

[MCQ]

1. Consider a planar graph G with vertices are 20 and number of edges are 25. The graph G have 8 closed faces then find the number of Components?

(a) 1 (b) 2
(c) 3 (d) 4

[MCQ]

2. Consider the following statements

S_1 : k_n is planar if and only if $n \leq 4$.

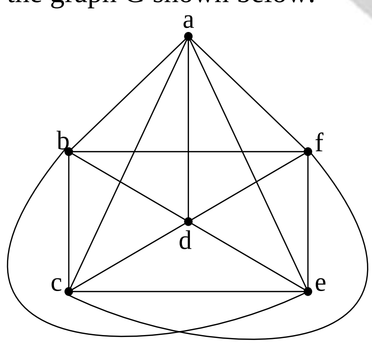
S_2 : $k_{m,n}$ is non-planar if and only if $m \geq 3$ or $n \geq 3$.

Which of the following is False?

(a) Only S_1
(b) Only S_2
(c) Both S_1 and S_2
(d) Neither S_1 nor S_2

[NAT]

3. Consider the graph G shown below.



Find the total number of edges that need to delete to make the above G planar?

[MCQ]

4. Consider a connected simple planar graph with order 10. Find the number of edges such that the minimum degree of graph must be 4?

(a) 25 (b) 20
(c) 30 (d) 40

[NAT]

5. If a 3 – regular graph with number of edges 15. How many closed faces are there in connected planar graph?

Answer Key

1. (c)
2. (b)
3. (3)
4. (b)
5. (6)

Hints and solutions

1. (c)

I. As we know that the Euler formula for planar graph is :
$r = e - n + k + 1$
Where r = number of regions
e and n are edges and vertices
k = number of components.

II. Now, the total number of regions = closed face + open faces
= 8 + 1
= 9 face/ regions

III. So, graph G with 20 vertices, 25 edges and 9 faces.
$\therefore \quad r = e - n + k + 1$
$\therefore \quad 9 = 25 - 20 + k + 1$
$\therefore \quad k = 9 - 6 = 3$
Hence, we have total 3 connected componets.

2. (b)

According to the kuratowski theorem, a graph is planar if and only if it does not contain any subgraph homeomorphic to k_5 or $k_{3,3}$.

Statement s_1: True
k_n is planar iff $n \le 4$ and
k_n is non – planar iff $n \ge 5$.

Statement s_2 : False
$k_{m,n}$ is planar iff $m \le 2$ or $n \le 2$. and
$k_{m,n}$ is non – planar iff $m \ge 3$ and $n \ge 3$.

Hence, option b is correct answer.

3. (3)

If a graph G contains triangle, then number of edges at most "$3n - 6$"
Also in simple graph, every region is bounded by at least 3 edges

$$\therefore \underbrace{\text{I.}\, r \le \frac{2e}{3} \text{ and II } r = e - n + 2}_{e \le 3n-6}$$

Now, we have total 6 vertices and 15 edges.
$\therefore$ No. of edges $(e) \le 3n - 6$
$(e) \le 3n - 6$
$e \le 3 * 6 - 6$
$e \le 18 - 6$
$e \le 12$
So, number of edges need to delete is = 15 – 12 = 3 edges.

4. (b)

I. As we know that the connected simple planer graph with 10 vertices can have at most "3n – 6" edges
$\therefore$ No. of edges $\le 3n - 6$
$\le 3 * 10 - 6$
≤ 24
So, the edges must be less than or equal to 24.

II. Now, the relation between the minimum degree and number of edges is:

$$\delta(G) = \frac{2 * |E|}{n}$$

$$\delta(G) = \frac{2 * |E|}{10}$$

$$\therefore \quad 4 = \frac{2 * |E|}{10}$$

$$\therefore \quad |E| = \frac{40}{2} = 20$$

So, to get the minimum degree 4, the number of edges will be 20.

5. (8)

I. We have 3- regular graph with 15 edges.

So,

$$\text{No. of edges} = \frac{n * K}{2}$$

$$15 = \frac{n * 3}{2}$$

$$\therefore \quad n = \frac{15 * 2}{3} = 10$$

Thus, we have total 10 vertices in the given graph.

II. Now, the number regions can be given by:

$r = e - n + 2$

$r = 15 - 10 + 2$

$r = 7$

Hence, we have total 7 faces, out of which 6 are closed/bounded and 1 is open face.