Design Specification for Delegation and Incentives in Shelley

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2 Purpose

This document describes the requirements and design for a delegation- and incentivesmechanism to be used in the Shelley release of Cardano.

Delegation will allow holders of Ada to transfer their rights to participate in the proof of stake (PoS) protocol to stake pools. Stake pools are run by stake pool operators (also called pool leaders), and a person delegating to a stake pool is called delegator, member, or participant of a stake pool.

Introducing delegation is important to increase the stability and performance of the system:

• We cannot expect every holder of Ada to continuously run a node that is well-connected to the rest of the network, in order to write a block on rare occasions. Some users might lack the expertise to do so. Most users will not have enough

stake to warrant running their own node. Delegation allows all holders of Ada to participate in the protocol, regardless of their technical abilities and the amount of stake that they hold. Thus we expect less stake to be offline, making the system faster and more resilient against an adversary.

• Even if every user were to run a node that was online all the time, it would be hard to keep all those nodes well enough in sync to avoid forks and still keep a short slot length. Our delegation design is aimed at keeping the number of nodes that produce a significant amount of blocks reasonably small (about 100 nodes), so that effective communication between them is feasible.

This document covers the design of necessary additions to Cardano in order to support and incentivise delegation.

3 Prerequisites

3.1 HD Wallets

We will use a Hierarchical Deterministic wallet (HD wallet) structure, as described in BIP-32.

4 Assumptions

5 Requirements

The delegation mechanism should meet a number of requirements. They can be grouped into functional requirements that the delegation system should provide, requirements to the security (both of the overall system and the funds of individual users), and existing features that should not be impeded when we add delegation to the system.

5.1 Functional Requirements

5.1.1 Proof of Eligibility

Any slot leader – and in particular stake pool operators, who are elected through stake that is delegated to them – should be able to prove when they are eligible to produce a block in a given slot.

5.1.2 Visibility of Delegation on the Blockchain

We expect stake pools to share their rewards with the delegators. In order to do this, there must be evidence for the delegation happening. Furthermore, we want the sharing of rewards to be enforced by the protocol, so the evidence must be recorded on the blockchain.

5.1.3 Restricting Chain Delegation

We do not want to allow stake to be re-delegated along a chain arbitrarily. We will admit some level of indirection, but not more than necessary to meet the rest of the requirements.

One reason that we do not want arbitrary chain delegation is that it makes it harder for delegators to figure out who is ultimately controlling their stake. Another is that unlimited chain delegation could open up a Denial-of-Service (DoS) attack vector on the system, where the attacker posts long delegation chains in order to slow down processes that depend on delegation, such as leader election or rewards sharing.

We must also have a mechanism to prevent cycles (such as A delegates to B, and B delegates to A) which would introduce ambiguity to the question of who manages stake in the end.

5.1.4 Cheap Re-Delegation

Changing delegation preferences should be as cheap as possible (while still using appropriate fees to prevent a denial of service attack on the blockchain).

5.1.5 Neutral Addresses

We should provide addresses that can hold value, but do not contribute to the PoS protocol. Those might be appropriate for use by exchanges, which will hold large amounts of value, without legally owning it.

5.2 Security Requirements

5.2.1 Sybil Attack Protection at Stake Pool Level

It is conceivable that an an adversary might try to take over the network by registering a large number of stake pools, hoping they accumulate enough stake to mount an attack just by people randomly delegating to them.

This Sybil attack on the level of stake pools should be made infeasible, by requiring stake pool operators to allocate a finite resource to each individual pool they register. In particular, this resource cannot be the cost of operating a node, since it is possible to run multiple pools with one node, so that cost would be constant in the number of pools an adversary is registering.

5.2.2 Address Nonmalleability

The system should provide protection against the following attack:

Changing Delegation through Address Malleability Suppose that Alice makes a payment to Bob. In preparation, Bob transmits an address belonging to his wallet to Alice, and expects Alice to pay to that address. If his wallets later on shows that his balance is increased by the expected amount, he considers that transaction to be successful. An attacker that wants to increase their influence on the PoS protocol changes the address that Bob sends in such a way that funds in that address are delegated to the attacker, but the funds still show up in Bob's wallet.

The attack is considered successful if the staking rights for the transferred money belong to the attacker after the transaction, without Alice and Bob noticing the attack.

5.2.3 Public Spending Keys Should not be Disclosed Prematurely

Delegation of stake should not involve revealing the public spending key. The public spending key should only be revealed once the funds that are controlled by the corresponding private key are actually transferred to another address.

5.2.4 Mitigate Key Exposure

A node run by a stake pool will need to have some key that controls all the delegated stake, in order to sign blocks. In case of an incident where the node is compromised, it should be possible for the stake pool operator to revoke the key, and replace it with a new one. This should not require any action by the delegators.

5.2.5 Handle Inactive Stake Pools

We anticipate that a stake pool operator can cease to operate – whether they lost their keys, lost interest, died, etc. We want to minimise the effect of this to the security and liveness of the system.

5.2.6 Avoid Hard Transition

When we make the switch from Byron (where all stake is delegated to the nodes controlled by the Cardano Foundation, Emurgo, and IOHK) to Shelley (where Ada holders have the freedom to control their stake), we should avoid a scenario where a significant amount of stake is suddenly offline.

This could happen if we automatically revoked the automatic delegation to the core nodes of the Byron network.

5.2.7 Change Delegation Without Spending Key

Users of a cold wallet, such as a paper wallet or a hardware wallet, should be able to delegate the stake corresponding to the funds in the cold wallet without using its spending key.

5.3 Requirements to Preserve Existing Features

5.3.1 Master Recovery Key

The whole wallet should be recoverable from one single key (without any additional information, such as the delegation preferences of the wallet).

The computational complexity of the recovery process should not be worse than logarithmic in the number of addresses appearing on the blockchain, and linear in the number of addresses in the wallet.

5.3.2 Address Recognition

An HD wallet should be able to recognise its addresses in the UTxO, so that it can report balances and transaction histories to the user.

5.3.3 Wallet should be Runnable on Independent Devices

Different user interfaces, running on different devices, should be able to access and control the same wallet, without transferring state between them.

We will accept some degradation of behaviour when running the wallet on different devices:

• Both copies might generate the same fresh addresses

• There can be differences in the reported balance while there are transactions in flight that only one of the two copies has knowledge of. In particular, when one copy sends a transaction, that transaction will only affect the balance reported by the other wallet once it is recorded on the blockchain.

• If the wallets use different delegation preferences, funds sent to the wallet might end up being delegated to different pools.

5.3.4 Maintain Privacy

HD Wallets maintain some level of privacy by using multiple addresses that are not obviously and publicly tied to the same wallet. Delegating stake should not necessarily link the addresses in the wallet of a delegator.

5.3.5 Short Addresses

Adding delegation to the system should not increase the length of addresses more than necessary. Ideally, we should use the necessary changes to the address scheme to come

up with an address length that is even shorter than in Byron.

6 User Stories

6.1 Basic Delegation User Stories

TODO: Add User Stories

6.2 User Stories Related to Incentives

6.2.1 [CDEC-92] Stake Pool Operator Performance Incentives

6.2.2 [CDEC-91] Optimal stake distribution

TODO: Add User Stories

7 Design of Delegation

7.1 Overview of Delegation

Delegation is a separation of the control over the movements of funds and the rights in the Proof of Stake protocol that are associated with those funds. We achieve this

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separation by introducing another type of key: while the rights to move funds are tied to a payment key pair $K^p = (skp, vkp)$, the rights to take part in the PoS are tied to the staking key pair $K^s = (sks, vks)$. Here, skp and sks are the private keys used for signing, and vkp and vks are the public keys used to verify signatures.

An owner of funds (i.e., the owner of the payment key of those funds) can delegate their stake to the owner of a particular staking key K^s by declaring that the stake rights of those funds should be controlled by K^s . There are different mechanisms for that, aimed for different use cases.

7.2 Address Structure

Shelley will introduce three different types of addresses: base addresses, pointer addresses, and enterprise addresses. Each address has the form

$$\mathcal{H}(vkp)||\beta$$

where $\mathcal{H}(vkp)$ is a cryptographic hash of the public spending key, and || denotes string concatenation. The types of addresses differ in the *staking object* β , which carries the staking information.

In addition to those new addresses, the system will continue to support bootstrap addresses and script addresses as introduced in Byron.

7.2.1 Base Address

A base address sets the staking rights directly to a staking key (sks, vks), and sets $\beta = \mathcal{H}(vks)$. The staking rights associated with funds held in this address are controlled by the owner of sks.

7.2.2 Pointer Address

A pointer address indirectly specifies the staking key that should control the stake of the address. It does so by referencing a certificate that has been published to the blockchain.

Concretely, for a pointer address, β is a *certificate pointer*, given by the tuple $(N_{\text{block}}, N_{\text{tx}}, N_{\text{cert}})$, where N_{block} is the number of a blockin the chain, and N_{tx} is the number of a transaction within that block. This transaction should, as its N_{cert} s metadata, contain a stakepool registration certificate¹ (see 7.3.1 below).

¹The research paper also allows pointer addresses to heavyweight certificates, but this is only needed for offline user wallets with cold staking and enhanced security. This is also the same case that requires a more relaxed version of chain delegation, which we decided to drop. So we can also restrict pointer addresses to point to registration certificates only.

7.2.3 Enterprise Address

Enterprise addresses allow completely opting out of participation in the proof of stake protocol. This might be appropriate for exchanges, which control, but not own, large amounts of Ada.

For enterprise addresses, β is set to a fixed constant value, making them easily distinguishable from other types of addresses.

When determining the stake distribution for the follow the Satoshi algorithm, enterprise addresses are completely ignored. Thus, holding funds in an enterprise address will not increase the chances for being elected as slot leader. Note however, that this effectively decreases the total amount of stake, which plays into the hands of the adversary.

7.2.4 Bootstrap Address

Bootstrap addresses were introduced in Byron, and they always delegate their stake to a fixed set of seven staking keys, corresponding to the seven core nodes operated by Cardano Foundation, Emurgo, and IOHK².

Bootstrap addresses will continue to exist in Shelley, but their use will be disincentivised.

7.2.5 Script Address

Another type of addresses present since Byron are script addresses. For those, it is hard to determine whom the funds actually belong to. The stake corresponding to funds in script addresses will be excluded from participation in the proof of stake protocol, just as is done for enterprise addresses.

7.2.6 HD Wallet Structure in Shelley

The wallet will be a hierarchical deterministic wallet, according to BIP-32.

Furthermore, we will require that the tree of addresses has a fixed depth, and that the wallet will only generate a certain number of new addresses before old ones have been used. These requirements allow us to keep wallet restoration from seed cheaper than linear in the total number of addresses in the blockchain. For details, see 7.8.1.

7.3 Certificates

Certificates allow transferring stake rights to specific staking keys. They can either be posted to the blockchain as transaction metadata (stakepool registration or retirement, as

²TODO: insert detailed description of bootstrap addresses.

well as heavyweight delegation certificates), or revealed upon use (lightweight certificates).

7.3.1 Certificates on the Blockchain

Certificates can be publicly announced to all participants by posting them to the blockchain, as transaction metadata. They will remain valid until explicitly overwritten or revoked, as an automatic expiry would likely increase the amount of undelegated, offline stake. The following certificates can be posted to the blockchain:

Stakepool Registration Certificates A person planning to operate a stake pool can anounce this by posting a *stakepool registration certificate* to the blockchain.

The certificate must contain the following information:

- the public staking key, vks_{delegate}
- the parameters that specify the reward sharing function of the stake pool (cost and margin of the pool) 3
- the minimal amount of Ada that the stake pool operator promises to deposit to the stake pool
- an address to which the rewards for the stake pool operator will be sent⁴
- optionally, a stake pool can include an address to which the rewards of the pool that exceed the costs and margin are sent.

If they do, the stake pool members will not get rewards for delegating, and their share will go to the specified address instead. This will allow stakeholders who do not want to get rewards (possibly for regulatory or tax reasons) to delegate to a stake pool that benefits a charity.

Additional, personal, information on the stake pool operator will be hosted separately from the blockchain, see 7.4.1.

Stakepool Retirement Certificate If a stakepool can foresee that it will cease operations, it can announce this intent by posting a *stakepool retirement certificate*. It contains

- the public staking key vks_{pool} of the pool
- the epoch number, starting from which the stakepool will cease to operate

It must be signed by the staking key pks_{pool} of the pool.

After the retirement epoch, any stake that is delegated to this stake pool will be disregarded for the PoS protocol. It will not take part in the leader election process

 $^{^{3}}$ This will be elaborated once this document is merged with the incentives design document.

⁴A priori, the rewards for the stake pool could also be distributed amongst the addresses that use the stake pool's staking key. However, this would allow anybody to piggyback on the stake pool by using addresses that use the stake pool's staking key.

(similarly to how stake in an enterprise address is not considered during the election process).

Stakeholders who delegated to this pool should be notified and asked to redelegate by their wallet the next time they are online.

Heavyweight Delegation Certificates Users can transfer the rights of participation in the PoS protocol from one staking key to another, by posting a *heavyweight delegation certificate* to the blockchain. A heavyweight delegation certificate is a tuple containing

- the public staking key delegating its staking rights, vks_{source}
- the public staking key to which stake is delegated, $vks_{\text{delegate}}^{5}$

It must be signed by pks_{source} .

Delegation Revocation Certificate Users might want to take control of stake that they had previously delegated. They can do that by posting a *delegation revocation certificate*, containing the key for which they want to invalidate previously posted delegation certificates. It must be signed by the corresponding secret key.

7.3.2 Lightweight Delegation Certificates

In addition to certificates posted on the blockchain, the system will also support *lightweight* delegation certificates. They specify that the staking rights are transferred from a source key vks_{source} to a delegate key vks_{delegate} . In contrast to heavyweight certificates, they are not posted to the blockchain, but instead included in the block header when a block is signed with pks_{delegate} (or in a message of the coin-tossing algorithm when pks_{source} is elected as a member of the committe for randomness generation).

The purpose of lightweight certificates is to enable stake pool operators to mitigate key exposure, 5.2.4. The setup is as follows:

- The stake pool operator registers their stake pool, using a key vks_{cold} . This cold key is kept securely and off-line.
- The stake pool operator uses pks_{cold} to sign a lightweight certificate C, transferring the staking rights to a hot key vks_{hot} .
- The stake pool operator keeps pks_{hot} , as well as C, on a node that is on-line, and can sign blocks. A block signed with pks_{hot} will be considered valid, provided that C is included in its header.
- Should the node get hacked, and the hot key compromised, the stake pool operator will create a new lightweight delegation certificate C', delegating the staking rights to a new hot key $vks_{\text{hot}'}$.

⁵It might make sense to use a certificate pointer here instead?

In order to render pks_{hot} useless, it must be established that C' takes precedence over C. For this purpose, the lightweight delegation certificate will have an additional integer field, and certificates with a larger value for this field will take precedence.

7.3.3 Chain Delegation

Chain delegation is the notion of having multiple certificates chained together, so that the source key of one certificate is the delegate key of the previous one.

We will only allow a very simple form of chain delegation, where we have zero or one of each of the following certificates, in that order:

- 1. heavyweight certificate
- 2. stake pool registration certificate
- 3. lightweight certificate

This restricted pattern of chain delegation allows us to satisfy all requirements, but avoids problematic cycles in the graph of delegation certificates, and makes it easier for the nodes to track the delegation patterns.

7.3.4 Certificate Precedence and Validity

The following rules determine precedence and validity of certificates. In particular, they describe what happens when multiple certificates are issued for a given staking key.

The ordering of blocks and transactions induces a canonical ordering amongst certificates. Thus, the terms older/newer certificate are well defined and are used below.

7.3.4.1 Stake Pool Registration and Retirement Certificates

• There can be at most one active stake pool registration certificate for any given staking key. A newer certificate will override an older one.

This will allow stake pool operators to update their costs and margin if they need to. Stake pool members should be notified of such changes by their wallet the next time they are online.

A revocation certificate is only valid if there is an older registration certificate.

7.3.4.2 Heavyweight Delegation and Revocation Certificates

• Newer heavyweight certificates override older heavyweight certificates. This allows delegators to move from one stake pool to another.

• Revocation certificates revoke the effect of older (but not newer) heavyweight certificates. So users can join a staking pool, leave it and control their stake directly, and still have the opportunity to join a staking pool at a later point in time.

7.3.4.3 Lightweight Delegation Certificates

For lightweight certificates, we cannot rely on the ordering induced by the blockchain. But we do have the counter field, which serves the purpose of establishing precedence:

• A lightweight certificate with a higher counter overrides one with a lower counter.

7.3.5 Additional Local Node State

It is not sufficient for certificates to be posted to the blockchain: since nodes will need to validate signatures on new blocks in a timely manner, they need ready access on all valid certificates without resorting to the blockchain itself. Distributing rewards (??) requires further additional state.

Nodes will have to maintain the following local databases as they process blocks:

7.3.5.1 Stake Pools

Pointer addresses (7.2.2) need to reference a specific stake pool registration certificate. Since this is part of the address, the key should be short. A canonical unique index that is reasonably short is the certificate pointer described in 7.2.2.

Access patterns:

- Lookup by certificate index whenever the staking rights for a pointer address have to be resolved
- Lookup by public staking key (to retrieve reward sharing policy for a given pool)
- Bulk listing to display active stake pools to the user

7.3.5.2 Active Heavyweight Certificates

All valid heavyweight certificates need to be kept in a local database.

Access patterns:

• Lookup by source staking key for leader election and rewards sharing

In order to determine which certificate was valid during a given epoch, we will have an additional field that specifies when the certificate came into effect, via their certificate index. Old certificates (revoked or overriden) can be dropped from the database once the rewards for their last active epoch have been distributed.

7.3.5.3 Addresses and Associated Balances per Staking Key

TODO: Verify that we indeed have to do this. It should also be possible to traverse the UTxO directly in the Follow-the-Satoshi algorithm, and then follow any delegation indirections.

At two points in the protocol, we will need to know which addresses belong to a specific staking key, and what their balances are: leader election for the upcoming epoch, and sharing out rewards for a past epoch.

The Follow the Satoshi algorithm for leader election needs a list of staking keys and their associated balances.

For rewards sharing, we need, for each staking pool, a list of all the pointer addresses with their balances that delegated directly to the stake pool. We will also need to have the amount of stake that each heavyweight certificate contributed to the pool.

To achieve both, nodes will maintain a database that contains, for every staking key, the addresses that are directly – i.e., ignoring heavyweight certificates – controlled by it (both pointer and base addresses), as well as their balances. Together with the active heavyweight certificates, this gives us everything we need for leader election and rewards sharing.

Note that directly tracking the stake for each key, including heavyweight certificates, would be problematic in case a heavyweight certificate is overridden or revoked.

7.3.5.4 Updating Local State

TODO: Update this section, it is probably out of date and assumes a more complicated scheme which we started with. Possibly, the additional state that we need is simple enough that we do not flesh out how to keep track of it in the design document at all. Nevertheless, I'm leaving the obsolete contents here as comments, as a basis to rewrite this section, if it is needed.

7.4 Delegation Scenarios

7.4.1 Stakepool Registration

Publicly announcing a stake pool for other people to delegate to requires two steps: posting a stakepool registration certificate to the blockchain, and providing additional verifiable personal information.

The second step is essential to establish trust in a stake pool. However, storing personal information directly on the blockchain would lead to violation of legislation like the GDPR, so instead of including it in the certificate, it will be stored on an external key-value

store, using $\mathcal{H}(vks)$ as key. The integrity of the data can be ensured by requiring it to be signed with pks.

A stake pool operator can change its costs and margin by replacing the registration certificate of the pool with a new one. This allows operators to react, for example, to a change in its costs or the exchange rate of Ada. A wallet that is delegating funds to this stake pool should notify the user of such a change whenever it detects it, and ask whether the delegation should be reconsidered.

The rewards that a stake pool gets depend on a deposit of funds that the stake pool operator themself provides. This adds a cost to creating a competitive stake pool, and protects against Sybil attacks on the stake pool level (5.2.1). All funds in base addresses with vks as the staking key are considered to belong to this deposit.

A stake pool operator will pledge to deposit a certain amount of Ada to the pool when registering a pool. This pledge is important: otherwise, an adversarial stake pool operator could circumvent the Sybil protection of the deposit, by placing a deposit in a pool until it attracted stake, and then simply moving the stake to the next pool. The pledge will be enforced at the point of leader election; stake pools that have a deposit less than what they pledged will be excluded from the election, and as a consequence forfeit their rewards for that epoch⁶.

Note that it will still be possible for a stake pool operator to decrease the amount of stake that they pledge to the pool, but this will require them to post a new certificate, which will notify the stakeholders that delegated to the pool, possibly triggering a re-delegation.

In addition to the above, we will also require pool operators to include a list of IP-adresses and/or domain names in the registration certificate, pointing to publicly reachable relay nodes under their control. (It is necessary to have a sufficient number of such publicly reachable nodes in order to establish a reliable peer-to-peer network.) We will use no technical mechanism to check the validity and availability of these relay nodes, but will rely on social pressure instead: People contemplating joining a pool will check the published data and will put little trust in operators who publish fake or unreliable addresses.

Remark: Due to the nature of our Incentives Mechanism (see below), very large stake-holders are incentivized to split their stake and create several pools. For a future version of Cardano, we plan to facilitate this by allowing such stakeholders to set up all their pools with a single certificate. For the present version, however, these pools will have to be created manually. This seems justified, given that there is only a handful of such very large stakeholders and seeing as such a feature would unnecessarily complicate engineering.

⁶We could also just deny the rewards for this pool, but still let it take part in the protocol.

7.4.2 Display of Stake Pools in the Wallet

The wallet software will keep a list of all the stakepool registration certificates it finds. For each, it will perform a lookup of the contained pks to retrieve the corresponding metadata to display to the user.

In order to prevent relying on a central party to host this key value store, it will be possible to register multiple servers in the wallet, and each of those will be queried. Anybody will have the opportunity to run a stakepool registration server, and announce its existence off band.

In order for stakeholders to be able to delegate their stake to a pool, the wallet will provide a listing of stake pools, in a section of the UI called the *delegation centre*. This listing will be ordered by the rewards that a user should expect if they were to delegate to that pool. Since those expected rewards depend not only on the costs and margin set by the stake pool operator, but also on the performance of the pool and on the amount of stake that it has accumulated, this will prefer pools that are reliable, and have not yet reached saturation. In other words, the users selfish interest to pick a stake pool that is promising large rewards is aligned with the goal of placing the system in the hands of a number of reliable stake pool operators, and of avoiding centralisation. The influence of the stake pool operator's deposit on the rewards provides protection against a Sybil attack on the stake pool level (5.2.1).

For estimating the rewards shared by a pool, the wallet needs to predict the performance of the pool, i.e. the ratio of blocks that the pool added to the chain and the number of slots it was elected as leader. This is done by assuming the performance to be consant, and using the performance during the last epoch, which is visible from the blockchain.

In order to prevent a slight difference in the expected returns to result in people conglomerating to a single stake pool, filling it rapidly, the order of the list of stake pools will be jittered: for each stake pool, the wallet will draw a random number r close to 1, and multiply the expected returns by r. I propose to draw r uniformly from the interval [0.95, 1.05], but this choice is arbitrary and should be re-evaluated during the operation of the testnet.

Since the actual amount of stake that the leader themself uses for the pool might change at any point in time, the ordering of pools will use the amount of stake that the leader pledged when registering the pool, not the amount of stake that the leader currently put into the pool. However, pools where the current deposit is smaller than the amount pledged are expected to give zero rewards, and will end up at the end of the list.

7.4.3 Basic Delegation

When a user has chosen a stake pool P to delegate to, new addresses that the wallet generates will be pointer addresses (7.2.2) pointing to the registration certificate of P.

This will cause all the funds that the wallet will receive to those addresses to be delegated to P.

Additionally, the wallet will provide the option to automatically re-delegate all funds currently in the wallet to P. If this option is chosen, the wallet will create a new address (pointing to P), and transfer the funds it controls to this new address. Note that a single transaction can have multiple inputs, so this will not require a large number of transactions, and incur only moderate costs, as required by 5.1.4.

Using delegation via pointer addresses does not obviously link addresses of the same wallet, as required by 5.3.4, though it does group addresses that delegate to the same pool. Choosing the option of automatically re-delegating *does* link addresses, by using bulk transaction, but that option is not required.

7.4.4 Delegation of Cold Wallets

Using pointer addresses for delegation requires to move funds to a new address in order to re-delegate. For hot wallets, this is fine, but not so for cold wallets: cold wallets are meant to be placed in a vault or buried underground for long-term safe storage, while the owner might still want to re-delegate the funds therein from time to time.

In order to facilitate re-delegation of funds stored in a cold wallet, cold wallets will use base addresses (7.2.1) with one common staking key (sks, vks). In order to (re-) delegate, the owner of the wallet will use a hot wallet to issue a transaction containing a delegation certificate using sks. The second wallet only needs to contain a small amount of funds to pay for the necessary transaction fees, so the requirement of it being a hot wallet is not a significant security risk.

7.4.5 Individual Staking

Stakeholders are not required to delegate their stake to a pool. If they wish to run their own node, they should use base addresses with a common staking key, and use that key to sign blocks with their node.

In addition, they can post a stake pool registration certificate, with a margin of m=1 (for which they are not required to upload any personal information). Usually, this should not be necessary. However, without the registration certificate, it is possible for a third party to piggy-back on such a private node, by using addresses in their wallet that use the same staking key. The rewards distribution mechanism will not be able to discern which addresses truly belong to the stakeholder operating the node, so the third party will get some rewards for this not-asked-for delegation.

Posting a "private" registration certificate with m = 1 will ensure that all rewards are sent to the address specified in the certificate.

TODO: We might want to consider *not* giving rewards to people managing their own stake without a private staking pool, since that could make the implementation simpler (not having two mechanisms for the same thing). It would also give us a mechanism, for free, to exclude the core nodes from getting rewards.

7.4.6 Rewards

For the smooth operation of the system, it is beneficial to have a large portion of the stake delegated to a set of reliable stake pools. Thus, we should incentivise delegating stake to reliable stake pools. One way to do this is to have stake pools share their rewards with their participants.

The reward sharing mechanism should satisfy the following requirements:

- 1. Sharing rewards should be an automatic process that does not require an action, neither by the stake pool operator nor the participants. This requirement is not only meant to ensure that participants get their share reliably. The share of the rewards that are given to a particular participant depends on the amount of stake that that participant delegated in a particular epoch. Thus, any node that verifies a transaction that transfers the rewards for a given epoch needs to access the staking information for that epoch. While this information is archived on the blockchain indefinitely, looking it up for arbitrary past epochs might be too costly. Making the sharing of rewards an automatic process in the following epoch circumvents this problem.
- 2. Sharing rewards should not lead to an excessive growth of the UTxO. In particular, it should avoid creating dust entries.
- 3. Sharing rewards should not lead to a burst of transactions that risks pushing the system to the limits of its predictable region of operation.
- 4. Sharing rewards should not increase the linkability of addresses of a wallet.
- 5. The reward sharing policy of the stake pool should be transparent to potential participants.

In order to satisfy requirements 1-4 above, the rewards for stake pool members will not be dealt out via transactions. Instead, the UTxO set will be adjusted automatically when the system goes from one epoch to the next. Requirement 5 is satisfied, since the rewards for stake pool members are determined by the cost and margin parameters that the stake pool operator includes in the stake pool registration certificate (as well as by the performance of the stake pool).

7.4.6.1 Distributing Rewards

At the end of an epoch, stake pool operators and members will be rewarded, via an update of the UTxO set and a special transaction. Since all the information needed to compute the rewards is publicly available on the blockchain, this update can be calculated and checked by each node independently.

Upon reaching the end of an epoch, each node will calculate the update to the UTxO that distributes the rewards for the preceding epoch, in the following manner⁷:

Let t_0 be the point in time at which the leader election for the epoch took place. For each staking key that had been selected as slot leader for at least one slot during the epoch, calculate the pool rewards \hat{f} , based on the performance during the epoch, and the stake deposit and delegated stake at t_0 . Note that the stake that the leader contributed to the pool can be differentiated from the stake that pool members delegated: the former uses base addresses with the pool's staking key, the latter either pointer addresses or delegation certificates.

Pool Member Rewards If the staking key belongs to a stake pool that did not specify a special address to which all member rewards should be sent, the pool member rewards will be calculated *per UTxO entry that delegated to the pool.* This is necessary, since we do not wish to link together the addresses of individual stake holders, and sharing rewards *per pool member* would require us to do just that. It is possible, since the rewards for each member are linear in the stake that that member delegated (once \hat{f} is fixed).

The value of each UTxO entry that did delegate at t_0 will be updated, adding the reward r_{member} calculated for the stake of this entry.

Note that some transaction outputs which were unspent at t_0 will have been spent during the epoch. Those will not get rewards. The rewards they would have gotten (as well as any fraction of a reward smaller than a Lovelace that got rounded down) will be treated as all rewards that are not distributed: they are partly assigned to the treasury and partly carried over to the next epoch.

This disincentivises stakeholders from re-delegating, and from using or moving their funds. If we want to avoid this, an alternative is described in section 7.4.6.2.

Pool Leader Rewards The rewards for pool leaders can not be distributed via an update of the UTxO, but must be transferred to the address that the operator specified when registering the pool. Instead, pool leader rewards will be shared out in a bulk transaction.

This transaction will have an empty input set, and does not need to be signed. Its validity can be checked by every node, since it can be derived deterministically from the blockchain.

⁷This section contains references to the rewards and incentives design document, which will be merged with this document soon.

Charity Pools The member rewards for charity pools can also not be handled by the UTxO update, and are instead included in the transaction that distributes the pool leader rewards.

Individual Stakeholders If an individual stakeholder posted a stake pool registration certificate (with margin 1), they will be handled just like a stake pool by the rewards sharing mechnism.

Otherwise, the system will assume that all the base addresses using the same staking key belong to the same stakeholder. It will calculate the rewards for the pool operator, and distribute it across the UTxO entries using those addresses, weighted by the coins those entries hold.

Note that the size of special transaction for the rewards of stake pool operators will be rather large, but bounded: it can never have more outputs than twice the number of slots in an epoch. This conservative bound will only be reached if every slot, a different leader was elected, and all of those leaders were "charity" pools (which is where the factor of two comes from). In practice, the number of outputs will be much closer to the number of relevant stake pools.

7.4.6.2 Updating at the Start of an Epoch

Cashing out the pool member rewards as depicted above has the drawback that stakeholders are discouraged from re-delegating, since they would lose their rewards for an epoch. They are also discouraged from using their funds for payments.

This can be avoided if the rewards are not distributed after the epoch, but at t_0 , the time when the leader schedule is determined. At this moment, all the UTxO entries that delegated at t_0 trivially exist, and no rewards will be forfeited.

An obvious problem with this is that there is no way to know in advance how well a given stake pool will perform in the next epoch. Assuming that the performance of a pool is more or less constant, we can however use the performance during the previous epoch (using some fallback, such as the average performance of the whole system for stake pools that were not elected in the previous epoch).

7.4.6.3 Handling of Bootstrap Addresses

Funds in bootstrap addresses will continue to be delegated to the core nodes of the Byron network. As long as there are funds in bootstrap addresses, the core nodes will continue to be elected and produce blocks. However, there will be rewards neither for the operators of the core nodes, nor for stakeholders delegating to them. Thus, the core nodes will contribute to the performance and security of the system, but their use will be disincentivised. This aligns the individual user's short-term interest of receiving rewards with the overall goal of reaching decentralisation of the system.

7.5 Stale Stake

Over time, we expect that an increasing amount of stake will become inactive. Individual stakeholders might lose their keys or interest in the system, and stake pool operators might stop operating in an unorderly fashion without posting a retirement certificate. This poses two problems for the system: the chain growth will decrease, limiting the rate at which transactions can be processed, and increasing the latency. It will also play in the hands of an adversary, since stake which is offline is counted as adversarial.

Luckily, this stale stake can be detected by looking at the blockchain: every time a stale staking key is elected, we will get an empty slot, and a block will be missing in the chain. Of course, a single empty slot does not need to indicate that the elected staking key is indeed stale (there might be network issues, a node might have crashed or been rebooted). But a key that misses multiple slots where it was elected is likely to be inactive.

The system will consider stake keys that satisfy the following two conditions to be inactive:

- The key has failed to sign blocks for the last 10 slots where it was elected as a slot leader⁸.
- It has not been used to sign a single block during the previous epoch

The second criterion is meant to prevent large stake pools or stakeholders from being considered inactive if they experience a temporary outage outage that is shorter than an epoch, but long enough to cover 10 slots for which they were elected.

Inactive stake keys will not be considered during leader election. This ensures that the chain growth is not slowed down by the inactive stake. It also somewhat improves the security problem: instead of the inactive stake becoming adversarial, the overall amount of stake is effectively reduced.

Stakeholders who have delegated to a pool that is considered inactive should be notified by their wallet the next time they come online, and the wallet should advise them to re-delegate.

Owners of a stale staking key – both individual stakeholders and operators of a stake pool – should also be notified when their staking key becomes stale. If they still have the key after it became stale (for isntance, if their node went down temporarily), they should have the possibility to announce to the blockchain that their key should be considered to be active again. They can do that by posting a *heartbeat* message, a message that contains the current slot number, and is signed with the stale key, as transaction metadata. The system shall recognise such messages, and consider the key to be active again if it became stale before the slot number mentioned in the heartbeat. Including the slot number in the heartbeat prevents a malicious third party from re-using a previous heartbeat message.

⁸The number 10 here is arbitrary and subject to discussion.

7.6 Address Recognition

Wallets will recognise addresses that belong to them just as they would without delegation, by looking only at the $\mathcal{H}(vkp)$ part of the address.

After a wallet recognises an address for which it controls the payment key, it will check whether the staking object β is set according to the current delegation preference of the wallet. If there is a discrepancy, it will alert the user, and ask them whether they want to re-delegate according to the current delegation preferences.

This check protects against the malleability attack in 5.2.2.

7.7 Leader election, Block Validity, and Randomness Generation

The process of leader election has to be modified to take delegation into account.

When the schedule for the next epoch has to be constructed, the nodes will compute the stake per staking key, taking into account all pointer addresses and valid heavyweight delegation certificates. The result is passed to the Follow the Satoshi (FtS) algorithm to choose a leader for each slot in the next epoch⁹.

Most stake pool leaders will use lightweight certificates in order to protect the key to which their members delegated. A block for a slot where the key vks_{leader} has been elected as leader will be considered valid by all nodes if either

- The block is signed by vks_{leader}
- The block is signed by vks_{hot} and contains, in its header, a lightweight certificate that transfers the staking rights from vks_{leader} to vks_{hot}

In case there are more than one block for the current slot, each of which are signed using a lightweight certificate, the newest certificate (as per the included counter) takes precedence.

The committee for the randomness generation will be chosen in the same way as the slot leaders, by running FtS algorithm on the stake distribution.

7.8 Related Topics

7.8.1 Wallet Recovery Process

Wallet recovery is the process of reconstructing a wallet from the root key. In order to reconstruct a wallet, all addresses belonging to that wallet which appear on the blockchain need to be identified.

⁹This can be done traversing the stake distribution only once, if we generate a list of *sorted* random numbers, traverse it in lockstep with the stake distribution, and then shuffle the resulting list of leaders).

In the current implementation, this is done by traversing the blockchain, and for each address, checking whether it belongs to the wallet. Unfortunately, this is linear in the size of the blockchain, leading to a very poor user experience.

To speed this up, we will reverse the strategy. Instead of going through the addresses on the blockchain, checking for each whether it belongs to the wallet, we go through the possible addresses of the wallet, and search whether they appeared on the blockchain.

In order for this to be efficient, we need to maintain an index, where we can look up addresses in the blockchain by some key, and we need to have a way of generating the key for an arbitrary range of addresses in the wallet, using only the root key as input.

Recall from 7.2 that the addresses have the form $\mathcal{H}(vkp)||\beta$, where vkp is the spending key, and β depends on the delegation for that address. The $\mathcal{H}(vkp)$ part is derivable from the root key (in particular, it does not depend on the delegation preferences of the wallet), and is a suitable key for the lookup of addresses¹⁰.

Of course, we cannot search for *all* possible addresses of the wallet. Instead, we utilise the tree structure of the HD wallet. We will require that the wallet software populates this tree in a specified way that will allow us to do a kind of exponential search for the addresses of the wallet.

7.8.1.1 Trees of Depth 1

To simplify, let us consider a wallet where the HD wallet tree is of depth 1, so that each address has an index $i \in \mathbb{N}$. We will require that the wallet creates addresses in order, and that there is a maximal address $gap\ \bar{i}$, such that the address α_i will not be generated unless there is an address $\alpha_{i'}$, with $\exists i' \in [i - \bar{i} - 1, i - 1]$ already appearing on the blockchain.

The first step in restoring a wallet is to find an upper bound on the number of addresses of the wallet, i_{up} . This can be done by consecutively looking at the intervals

$$I_n = [2^n + i | i \in [0, \bar{i}]], n \in \mathbb{N}$$

and checking whether any of the addresses in α_i for $i \in I_n$ appears on the blockchain. This check is performed by creating the corresponding spending key, hashing it, and doing a look-up in the index. For some n, this will fail, and we will have found \bar{i} consecutive indices for which there are no addresses of this wallet on the blockchain. Because \bar{i} is the maximal address gap, no address larger than 2^n has been created for the address, and we have $i_{\rm up}=2^n$.

¹⁰Depending on the serialisation format for addresses, it might be possible to not use a separate index at all: if $\mathcal{H}(vkp)$ is a prefix of the serialised address, we can directly do a prefix query in the database.

Afterwards, we can perform a binary search for the maximal address i_{max} , in the interval $[2^{n-1}, 2^n]$. In each step of this binary search, we will probe for \bar{i} consecutive addresses, starting from an offset i. If none of them exist, we know that $i_{\text{max}} < i$, otherwise $i_{\text{max}} \ge i$.

Finally, we will create all spending key hashes in the range $[0, i_{\text{max}}]$, and look up the corresponding addresses.

Two remarks are in order:

Early Finish and Memoisation The above process will perform more lookups than necessary. The binary search can be aborted once the search window gets smaller than \bar{i} . In addition, we should consider memoising the spending keys and/or lookups.

7.8.1.2 Taller Trees

This scheme can be generalised for trees of larger depth. The current wallet in Cardano has a fixed depth of 2. Each address in this wallet has an index $(i, j) \in \mathbb{N} \times \mathbb{N}$. In order to generalise the above wallet restoration procedure for this wallet, we will require that there is no gap in the i, and a maximal gap \bar{j} in j.

Identifying the maximal value i_{max} is straightforward: look at lists of indices

$$[(i,j)|j\in I_0]$$

for increasing values of i, until there is no address found on the chain for a specific value of i. Once i_{max} is found, we can iterate the method for trees of depth 1 over all $i \in [0, i_{\text{max}}]$.

Further generalisations to arbitrary depths are straightforward, provided that

- all the leaves are at the same depth
- at each depth, we can require a certain maximal gap

7.8.1.3 Retrieving Staking Information

After the wallet software has determined the set of addresses that belong to it via the spending keys, it needs to set its delegation preference. In order to do so, it compares the staking objects β of its addresses.

If the wallet consists of pointer addresses to exactly one active staking pool the wallet should set its delegation preference to delegate to that pool via pointer addresses, and show a message to the user to inform them of this.

If the wallet consists of base addresses using the same staking key the wallet should look up this staking key in the list of heavyweight delegation certificates. If there is a certificate for this key, and it points to am active staking pool, the wallet should set its delegation preference to use base addresses with the same staking key, and

inform the user of this choice. If there is none, it should inform the user that the stake is currently undelegated, and that they should consider delegating, or running their own node.

If the wallet consists of addresses with different staking objects the wallet should present the list of stake pools that are delegated to by the wallet, and ask the user to pick one for future addresses, as well as provide an option to re-delegate all funds to that pool.

If there are addresses delegating to an inactive or retired pool the wallet should alert the user and ask them to pick another staking pool to delegate to.

After setting the delegation preferences of the newly restored wallet, the wallet software should encourage the user to visit the delegation centre to make sure that this choice is still competitive.

7.8.2 Transition from Bootstrap Phase

As of the time this document is written, Cardano is in the "bootstrap pahse", where the network is not decentralised, but federated. All stake is automatically delegated to seven stakeholders, by requiring that all the outputs of transactions are to bootstrap addresses (see 7.2.4)¹¹. Those stakeholders have posted one heavyweight delegation certificate to the blockchain, giving control to seven nodes (the *core nodes*) controlled by Cardano Foundation, Emurgo, and IOHK. During the bootstrap phase, rewards are not collected.

The transition from bootstrap phase to delegation is performed by dropping the restriction of using bootstrap addresses as transaction outputs. Moving stake rights away from the core nodes to stake pools or individual users will require user action. This is a deliberate choice: if we, for example, transferred all the stake rights to users at the end of the bootstrap phase, we would risk to have a large portion of the overall stake become offline, since most users will neither be online, nor delegate immediately. This would pose a risk to both the performance and, worse, the integrity of the system.

The obvious drawback of keeping the stake rights with the core nodes until the users intervene is that it will lead to a lesser degree of decentralisation in the transition period between bootstrap phase and full decentralisation. We will probably counter this by incentivising the users to delegate away from the core nodes. This could be done by having the core nodes collect none or fewer rewards (and thus sharing fewer rewards with delegators).

TODO: Needs definitive input from incentives stream regarding how to incentivise users to delegate away from code nodes.

¹¹Check with Erik that this is indeed how it currently works.

8 Design of Incentives

8.1 Overview of Incentives

On a high level, goal of the Incentives mechanism is to incentivize stakeholders to follow the protocol and thereby guaranteeing secure and efficient operation of Cardano.

More specifically, we want a majority of stake (at least 80%) to delegate to a number of k stake pools (where k is a parameter of the system – see below). The pool leaders of those stake pools are supposed to

- provide additional network infrastructure,
- be online for and participate in the election mechanism and
- be online during slots for which they have been elected slot leader and then create a block containing as many transactions as possible.

Stakeholders who do not want to register a pool and become pool leaders can either

- delegate their stake to a registered pool (we hope most will do this) or
- participate in the protocol without registering a pool (at most 20% of stake should belong to such "one-man pools").
- (They can also not do anything, but will not receive any rewards in that case.)

Incentives are provided in the form of *social pressure* (by making pool leader performance and adherence to the protocol public), but mostly by *monetary incentives* in the form of ADA.

Design goal of the mechanism is to align monetary incentives as perfectly as possible with protocol adherence: If every stakeholder follows his own financial interests, the system should settle into a desirable state. If possible, there should never be a conflict of interest between maximizing rewards and "doing the right thing".

Rewards will be paid for each epoch and will be drawn from two sources,

- monetary expansion and
- transaction fees.

Rewards for one epoch will roughly be split proportional to stake. However, there will be several refinements to this general principle:

- Rewards for a stake pool will be capped when the pool gets too large (otherwise, the system would converge towards a state with all stake being delegated to one giant stake pool).
- Rewards will decrease if a pool leader does not create the blocks he is supposed to create.
- Pool leaders will be compensated for their trouble and risk by
 - reimbursing their costs and

- giving them a *margin* before distributing pool rewards proportionally amongst pool leader and pool members. (Pool leaders publicly declare their margin, which they can freely choose.)
- Pool rewards will slightly increase with the stake of their leader. There is no minimal stake required to create a pool anybody can do this. However, pools led by leaders with high stake will get higher rewards. (This will discourage pool leaders from splitting their stake to operate several pools. It will also help preventing Sybil attacks, where an attacker with low stake tries to gain control over a majority of stake by creating a lot of pools with low costs.)

Our game theoretic analysis has shown that if stakeholders try to maximize their rewards in a "short-sighted" (*myopic*) way (pool members joining the pool with the highest rewards at this moment, pool leaders raising their margins to get higher rewards at this moment), chaotic behavior will ensue.

Therefore we will calculate *non-myopic* rewards and make them public, thus guiding stakeholders to behave in a way that will benefit them in the long run. Our analysis shows that if everybody follows this advice, the system will stabilize in a *Nash Equilibrium*, meaning that no stakeholder will have incentive to act differently.

Rewards to both the pool leaders and the pool members will be calculated by the system and will be available to all stakeholders after each epoch. No manual intervention (transfer of funds) will be necessary.

8.2 Parameters

There will be a couple of parameters whose values have to be set in advance:

- The desired number of pools $k \in \mathbb{N}_+$.
- The influence $a_0 \in [0, \infty)$ a pool leader's stake should have on the desirability of the pool. Small values of a_0 indicate little influence.
- The expansion rate $\rho \in [0, 1]$, determining the fraction of still available ADA that will be created per epoch.
- The fraction $\tau \in [0,1]$ of rewards going to the treasury.
- An exponent $\gamma \in (0, \infty)$ for penalty calculation.

We will discuss later how one could approach choosing reasonable values for these.

8.3 Reminder: Stakepool Registration

Recall from above that stakeholders who wish to operate and lead a stake pool have to register their pool on the blockchain. From the point of view of reward-calculation (see below), the following information has to be included in the registration:

• The costs of operating the pool (in ADA/epoch).

- The pool leader margin (in [0,1]), indicating the additional share the pool leader will take from pool rewards before splitting rewards amongst members (see below).
- Proof of *ADA pledged to the pool*. This could be provided as a list of addresses, signed by the corresponding secret spending keys.

There will be no lower bound on the amount of ADA that has to be pledged, but we will see below that pool rewards will increase with this amount. This is necessary to prevent people with low stake from registering many pools, gaining control over a lot of stake and attacking the system (see below).

8.4 Epoch Rewards

There will be three sources of rewards for an epoch: transaction fees, monetary expansion and rewards from the previous epoch.

8.4.1 Transaction Fees

All transaction fees from all transactions from all blocks created during the epoch will be used as rewards.

8.4.2 Monetary Expansion

Let T be the total amount of ADA in existence during a specific epoch, and let T_{∞} be the maximal possible amount of ADA in the future. At this moment, T=31,000,000,000 and $T_{\infty}=45,000,000,000$. Then the amount of $\rho \cdot (T_{\infty}-T)$ ADA will be newly created.

Since T_{∞} is finite, rewards from monetary expansion will decrease over time. This has to be compensated by

- rising transaction fees when more and more people use the system and
- higher exchange rates from ADA to USD when the system become more valuable.

8.4.3 Rewards from the Previous Epoch

As we will see below, not all available rewards from an epoch will actually be distributed during that epoch. The rest will be added to the rewards of the following epoch.

8.4.4 Treasury

A fraction τ of the rewards for one epoch will go to the *treasury*.

8.5 Reward Splitting

In this section we describe how the total rewards R from one epoch are split amongst stakeholders.

These calculations proceed in two steps: First, rewards are split amongst pools. Next, each pool splits its share of R amongst its leader and its members.

8.5.1 Pool Rewards

For a given epoch, the maximal rewards for a pool are

$$f(s,\sigma) := \frac{R}{1+a_0} \cdot \left(\sigma' + s' \cdot a_0 \cdot \frac{\sigma' - s' \frac{z_0 - \sigma'}{z_0}}{z_0}\right).$$

Here

- R are the total available rewards for the epoch (in ADA).
- $a_0 \in [0, \infty)$ is a parameter determining leader-stake influence on pool rewards.
- $z_0 := 1/k$ is the size of a saturated pool.
- $\sigma' := \min(\sigma, z_0)$, where σ is the relative stake of the pool.
- $s' := \min(s, z_0)$, where s is the relative stake of the pool leader (the amount of ADA pledged during pool registration.

The actual rewards for a pool j (with relative stake σ_j and relative leader-stake s_j) which should have created N_j blocks in that epoch and actually created $n_j \leq N_j$ out of those are

$$\hat{f}_j := \left(\frac{n_j}{\max(N_j, 1)}\right)^{\gamma} \cdot f(s_j, \sigma_j).$$

So if the pool leader of pool j faithfully creates all blocks in slots for which one of the pool members was elected slot leader, $n_j = N_j$ and $\hat{f}_j = f(s_j, \sigma_j)$, i.e. the pool gets all available rewards.

If on the other hand the pool leader does not create even a single block, $\hat{f}_j = 0$, and the pool will get no rewards whatsoever for that epoch.

What happens in between these two extremes is controlled by parameter $\gamma \in (0, \infty)$: For $\gamma = 1$, the penalty will be proportional to the number of missed blocks. For $0 < \gamma < 1$, penalties for missing the first few blocks will be relatively light, whereas for $\gamma > 1$, penalties will be over-propertionally harsh in the beginning.

The difference $f(s_j, \sigma_j) - \hat{f}_j$ will be sent to the treasury. In particular, this means that no pool can increase its own rewards by somehow preventing another pool from producing blocks.

Note that $\sum_j f(s_j, \sigma_j) \leq 1$ and that the difference $R - \sum_j f(s_j, \sigma_j)$ will normally be strictly positive. This difference will be added to the following epoch's rewards.

8.5.2 Reward Splitting inside a pool

After the rewards for a pool have been determined according to the previous section, those rewards are then split amongst the *pool leader* and the *pool members*.

Consider

- \hat{f} , the pool rewards,
- c, the pool costs (in ADA),
- $m \in [0,1]$, the margin,
- $\sigma \in [0, 1]$, the relative stake of the pool.

Note that the values c and m for registered pools are available from the pool registration. Stakeholders who have *not* registered a pool and participate in the protocol on their own are treated like *pool leaders of one-man pools with margin 1* (costs are irrelevant in this case, because all pool rewards go to the pool leader anyway).

8.5.2.1 Pool Leader Reward

The pool leader reward r_{leader} (in ADA) is calculated as follows (where $s \in [0, 1]$ is the stake of the pool leader):

$$r_{\text{leader}}(\hat{f},c,m,s,\sigma) := \left\{ \begin{array}{ll} \hat{f} & \text{if } \hat{f} \leq c, \\ c + (\hat{f}-c) \cdot \left(m + (1-m) \cdot \frac{s}{\sigma}\right) & \text{otherwise.} \end{array} \right.$$

8.5.2.2 Pool Member Reward

The pool member reward r_{member} (in ADA) is calculated as follows (where $t \in [0, 1]$ is the stake of the pool member):

$$r_{\text{member}}(\hat{f}, c, m, t, \sigma) := \begin{cases} 0 & \text{if } \hat{f} \leq c, \\ (\hat{f} - c) \cdot (1 - m) \cdot \frac{t}{\sigma} & \text{otherwise.} \end{cases}$$

8.6 Non-Myopic Utility

It would be short-sighted ("myopic") for stakeholders to directly use the formulas from section Reward Splitting. They should instead take the long-term ("non-myopic") view. To this end, the system will calculate and display the "non-myopic" rewards that pool leaders and pool members can expect, thus supporting stakeholders in their decision whether to create a pool and to which pool to delegate their stake.

The idea is to first rank all pools by "desirability", to then assume that the k most desirable pools will eventually be saturated, whereas all other pools will lose all their members, then to finally base all reward calculations on these assumptions.

8.6.1 Pool Desirability and Ranking

First we define the *desirability* of a pool whose leader has stake s, costs c and margin m. Simply put, this number indicates how "desirable" or "attractive" this pool is to (potential) members.

If the pool is *saturated*, the pool rewards are

$$\tilde{f}(s) := f(s, z_0) = \frac{R}{1 + a_0} \cdot (z_0 + \min(s, z_0) \cdot a_0).$$

The desirability is then defined as

$$d(c, m, s) := \begin{cases} 0 & \text{if } \tilde{f}(s) \le c, \\ (\hat{f} - c) \cdot (1 - m) & \text{otherwise.} \end{cases}$$

To determine a pool's *rank*, we order pools by decreasing desirability. The most desirable pool gets rank 1, the second most desirable pool gets rank 2 and so on.

We predict that pools with rank $\leq k$ will eventually be saturated, whereas pools with rank > k will lose all members and only consist of the leader.

8.6.2 Non-Myopic Pool Stake

Consider a pool with leader stake s, total stake σ and rank r. We define its non-myopic stake σ_{nm} as

$$\sigma_{\text{nm}}(s, \sigma, r) := \begin{cases} \max(\sigma, z_0) & \text{if } r \leq k, \\ s & \text{otherwise.} \end{cases}$$

8.6.3 Non-Myopic Pool Leader Rewards

The non-myopic pool leader rewards of a pool with costs c, margin m, leader stake s, stake σ and rank r are

$$r_{\text{leader,nm}}(c,m,s,\sigma,r) := r_{\text{leader}} \Big(f \big(s, \sigma_{\text{nm}}(s,\sigma,r) \big), c,m,s, \sigma_{\text{nm}}(s,\sigma,r) \Big).$$

8.6.4 Non-Myopic Pool Member Rewards

The non-myopic pool member rewards of a pool with costs c, margin m, leader stake s, stake σ , member stake t and rank r are

$$r_{\text{member},\text{nm}}(c,m,s,\sigma,t,r) := r_{\text{member}}\Big(f\big(s,\sigma_{\text{nm}}(s,\sigma,r)\big),c,m,t,\sigma_{\text{nm}}(s,\sigma,r)\Big).$$

8.7 Claiming Rewards

All information necessary to calculate each stakeholder's rewards for each epoch are contained in the blockchain, so there is in principle no need to record any extra information related to the Incentives mechanism.

However, there is the challenge to avoid "bloat" caused by thousands of "micro payments" from rewards after each epoch.

We are considering two solutions to this problem:

- Use a "lottery" which gives everybody the same rewards in expectation, but drastically reduces the number of actual payments to a manageable number.
 - Disadvantage of this idea is the potentially high *variance*, but on the other hand, the element of randomness could also add some additional "thrill" to the process.
- Only pay to UTXO's which haven't changed over the duration of the epoch and then modify those UTXO's instead of creating new ones.

This would imply that people holding on to their ADA instead of spending them would get higher rewards, which may or may not be a problem. It would certainly fit with the general narrative that transaction fees (and incentives in general) flow from people *using* the system (spending ADA) to people *operating* the system (holding ADA).

8.8 System Inputs needed for Calculations

In order to calculate rewards, the following information must be available for each pool (including "one-man pools" of individual protocol participants):

- Cost, margin and pledged ADA of the pool leader. (These will be zero, one and zero for "one-man" pools.)
- Staking addresses of pool leader and pool members.
- Number of times per epoch the owner of an address belonging to the pool was elected slot leader and actually created a block.

8.9 Information in Daedalus

Out game theoretic analysis assumes that every stakeholder has all relevant information available at any time.

This means that pool *costs* and *margins* and pool (leader) *stakes*, as well as the (non-myopic) utilities derived from these figures, have to be easily accessible, so that stake-holders can quickly react to changes and always choose the strategy that maximizes their own rewards.

The *Daedalus* wallet software must therefore make this information readily available.

8.10 Deciding on Good Values for the Parameters

We need to decide on reasonable values for the parameters k, a_0 , ρ and τ (see above).

8.10.1 *k*

The desired number of pools k depends on the level of decentralization we want on the one hand and network efficiency of the Cardano protocol on the other hand. A value of k = 100 seems to be reasonable.

8.10.2 a_0

As explained above, parameter a_0 determines the influence that a pool leader's stake has on pool rewards.

Our game theoretic analysis predicts that the k pools with the highest potential, the highest value of

$$P(\lambda, c) := (z_0 + a_0 \cdot \lambda) \cdot \frac{R}{1 + a_0} - c$$

(where λ is the stake committed by the pool leader and c are the pool costs) will create the saturated pools.

Let us consider an attacker with stake $S < \frac{1}{2}$, who wants to gain control over a majority of stake. This means he has to lead $\frac{k}{2}$ pools, committing $\lambda = \frac{2S}{k}$ stake to each.

In order for his $\frac{k}{2}$ pools to be successful, each of these needs to have higher potential than the honest stakeholder with the $\frac{k}{2}$ -highest potential has. If that honest player has committed stake $\tilde{\lambda} \leq \frac{1}{k}$ and has costs \tilde{c} and if our malicious attacker is willing to lie and claim lower "dumping" costs $c = r \cdot \tilde{c}$ (for $r \in [0,1)$), this means

$$P\left(\frac{2S}{k},c\right) > P(\tilde{\lambda},\tilde{c}) \iff \left(z_0 + a_0 \cdot \frac{2S}{k}\right) \cdot \frac{R}{1 + a_0} - c > \left(z_0 + a_0 \cdot \tilde{\lambda}\right) \cdot \frac{R}{1 + a_0} - \tilde{c}$$

$$\iff a_0 \cdot \frac{2S}{k} \cdot \frac{R}{1 + a_0} - c > a_0 \cdot \tilde{\lambda} \cdot \frac{R}{1 + a_0} - \tilde{c}$$

$$\iff a_0 \cdot \left(\frac{2S}{k} - \tilde{\lambda}\right) \cdot \frac{R}{1 + a_0} > c - \tilde{c} = -(1 - r) \cdot \tilde{c}$$

$$\iff \frac{a_0 > 0}{k} - \tilde{\lambda} > -\frac{\tilde{c} \cdot (1 - r) \cdot (1 + a_0)}{R \cdot a_0} = -\frac{\tilde{c}}{R} \cdot (1 - r) \cdot \left(1 + \frac{1}{a_0}\right)$$

$$\iff S > \frac{k}{2} \cdot \left[\tilde{\lambda} - \frac{\tilde{c}}{R} \cdot (1 - r) \cdot \left(1 + \frac{1}{a_0}\right)\right]$$

In the following tables, we can see how the choice of a_0 influences the minimal stake S needed for a successful attack for various values of $\tilde{\lambda}$, \tilde{c} and r:

ĩ 0.01	~ 0.001 0.0	ĩ 0.01	~ 0.005 0.0	ĩ 0.01	~ 0.01 0.0
	$\tilde{c} = 0.001, r = 0.9$		$, \tilde{c} = 0.005, r = 0.9$		$\tilde{c} = 0.01, r = 0.9$
$\frac{a_0}{a_0}$	<u>S</u>	$\frac{a_0}{0.050}$	<u>S</u>	$\frac{a_0}{a_0}$	<u>S</u>
0.010	0.0000	0.050	0.0000	0.050	0.0000
0.020	0.2450	0.100	0.2250	0.100	0.0000
0.030	0.3283	0.150	0.3083	0.150	0.1167
0.040	0.3700	0.200	0.3500	0.200	0.2000
0.050	0.3950	0.250	0.3750	0.250	0.2500
0.060	0.4117	0.300	0.3917	0.300	0.2833
0.070	0.4236	0.350	0.4036	0.350	0.3071
0.080	0.4325	0.400	0.4125	0.400	0.3250
0.090	0.4394	0.450	0.4194	0.450	0.3389
0.100	0.4450	0.500	0.4250	0.500	0.3500
	05, $\tilde{c} = 0.001$, $r = 0.9$		5, $\tilde{c} = 0.005$, $r = 0.9$		05, $\tilde{c} = 0.01, r = 0.9$
$\frac{a_0}{0.010}$	0.0000	$\frac{a_0}{0.050}$	0.0000	$\frac{a_0}{0.100}$	0.0000
	0.0000				
$0.020 \\ 0.030$	0.0783	$0.100 \\ 0.150$	0.0000 0.0583	$0.200 \\ 0.300$	0.0000
0.030 0.040	0.1200	0.130 0.200	0.1000	0.300 0.400	0.0333
		0.250		0.400	0.0750
0.050	0.1450	0.230 0.300	0.1250		0.1000
$0.060 \\ 0.070$	0.1617 0.1736	0.350	0.1417 0.1536	$0.600 \\ 0.700$	0.1167
0.070		0.330 0.400			0.1286
	0.1825	0.450	0.1625	$0.800 \\ 0.900$	0.1375
$0.090 \\ 0.100$	0.1894 0.1950	0.450 0.500	0.1694	1.000	0.1444 0.1500
0.100	0.1950	0.500	0.1750	1.000	0.1300
$\tilde{\lambda} = 0.00$	$\tilde{c} = 0.001, r = 0.9$	$\tilde{\lambda} = 0.00$	$1, \tilde{c} = 0.005, r = 0.9$	$\tilde{\lambda} = 0.00$	$01, \tilde{c} = 0.01, r = 0.9$
a_0	S	$\underline{}$	S	a_0	S
$\frac{a_0}{0.100}$	$\frac{S}{0.0000}$	$\frac{a_0}{0.500}$	S 0.0000	$\frac{a_0}{0.100}$	$\frac{S}{0.0000}$
$ \begin{array}{r} a_0 \\ \hline 0.100 \\ 0.200 \end{array} $	S 0.0000 0.0200	$ \begin{array}{r} a_0 \\ \hline 0.500 \\ 1.000 \end{array} $	S 0.0000 0.0000	$ \begin{array}{r} a_0 \\ \hline 0.100 \\ 0.200 \end{array} $	S 0.0000 0.0000
$ \begin{array}{r} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \end{array} $	S 0.0000 0.0200 0.0283	$ \begin{array}{r} a_0 \\ \hline 0.500 \\ 1.000 \\ 1.500 \end{array} $	S 0.0000 0.0000 0.0083	$ \begin{array}{r} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \end{array} $	S 0.0000 0.0000 0.0000
$ \begin{array}{r} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \end{array} $	S 0.0000 0.0200 0.0283 0.0325	$ \begin{array}{r} a_0 \\ \hline 0.500 \\ 1.000 \\ 1.500 \\ 2.000 \end{array} $	S 0.0000 0.0000 0.0083 0.0125	$ \begin{array}{r} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \end{array} $	S 0.0000 0.0000 0.0000 0.0000
$ \begin{array}{r} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \end{array} $	S 0.0000 0.0200 0.0283 0.0325 0.0350	$ \begin{array}{r} a_0 \\ \hline 0.500 \\ 1.000 \\ 1.500 \\ 2.000 \\ 2.500 \end{array} $	S 0.0000 0.0000 0.0083 0.0125 0.0150	$\begin{array}{r} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ \end{array}$	S 0.0000 0.0000 0.0000 0.0000 0.0000
$\begin{array}{r} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ \end{array}$	S 0.0000 0.0200 0.0283 0.0325 0.0350 0.0367	$ \begin{array}{r} a_0 \\ \hline 0.500 \\ 1.000 \\ 1.500 \\ 2.000 \\ 2.500 \\ 3.000 \end{array} $	S 0.0000 0.0000 0.0083 0.0125 0.0150 0.0167	$\begin{array}{r} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ \end{array}$	S 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
$\begin{array}{c} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ \end{array}$	S 0.0000 0.0200 0.0283 0.0325 0.0350 0.0367 0.0379	$ \begin{array}{r} a_0 \\ 0.500 \\ 1.000 \\ 1.500 \\ 2.000 \\ 2.500 \\ 3.000 \\ 3.500 \end{array} $	S 0.0000 0.0000 0.0083 0.0125 0.0150 0.0167 0.0179	$\begin{array}{c} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ \end{array}$	S 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
$\begin{array}{c} a_0 \\ 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \end{array}$	S 0.0000 0.0200 0.0283 0.0325 0.0350 0.0367 0.0379 0.0388	$\begin{array}{c} a_0 \\ 0.500 \\ 1.000 \\ 1.500 \\ 2.000 \\ 2.500 \\ 3.000 \\ 3.500 \\ 4.000 \end{array}$	S 0.0000 0.0000 0.0083 0.0125 0.0150 0.0167 0.0179 0.0188	$\begin{array}{c} a_0 \\ 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \end{array}$	S 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
a0 0.100 0.200 0.300 0.400 0.500 0.600 0.700 0.800 0.900	S 0.0000 0.0200 0.0283 0.0325 0.0350 0.0367 0.0379 0.0388 0.0394	$\begin{array}{c} a_0 \\ 0.500 \\ 1.000 \\ 1.500 \\ 2.000 \\ 2.500 \\ 3.000 \\ 3.500 \\ 4.000 \\ 4.500 \end{array}$	S 0.0000 0.0000 0.0083 0.0125 0.0150 0.0167 0.0179 0.0188 0.0194	a0 0.100 0.200 0.300 0.400 0.500 0.600 0.700 0.800 0.900	S 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
$\begin{array}{c} a_0 \\ 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \end{array}$	S 0.0000 0.0200 0.0283 0.0325 0.0350 0.0367 0.0379 0.0388	$\begin{array}{c} a_0 \\ 0.500 \\ 1.000 \\ 1.500 \\ 2.000 \\ 2.500 \\ 3.000 \\ 3.500 \\ 4.000 \end{array}$	S 0.0000 0.0000 0.0083 0.0125 0.0150 0.0167 0.0179 0.0188	$\begin{array}{c} a_0 \\ 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \end{array}$	S 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
$\begin{array}{c} a_0 \\ 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \\ 0.900 \\ 1.000 \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0200 \\ 0.0283 \\ 0.0325 \\ 0.0350 \\ 0.0367 \\ 0.0379 \\ 0.0388 \\ 0.0394 \\ 0.0400 \\ ., \tilde{c} = 0.001, r = 0.5 \\ \end{array}$	$\begin{array}{c} a_0 \\ 0.500 \\ 1.000 \\ 1.500 \\ 2.000 \\ 2.500 \\ 3.000 \\ 3.500 \\ 4.000 \\ 4.500 \\ 5.000 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0000 \\ 0.0083 \\ 0.0125 \\ 0.0150 \\ 0.0167 \\ 0.0179 \\ 0.0188 \\ 0.0194 \\ 0.0200 \\ , \tilde{c} = 0.002, r = 0.5 \end{array}$	$\begin{array}{c} a_0 \\ 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \\ 0.900 \\ 1.000 \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ \end{array}$
$\begin{array}{c} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \\ 0.900 \\ 1.000 \\ \\ \tilde{\lambda} = 0.01 \\ a_0 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0200 \\ 0.0283 \\ 0.0325 \\ 0.0350 \\ 0.0367 \\ 0.0379 \\ 0.0388 \\ 0.0394 \\ 0.0400 \\ ., \tilde{c} = 0.001, r = 0.5 \\ S \\ \end{array}$	$\begin{array}{c} a_0 \\ \hline 0.500 \\ 1.000 \\ 1.500 \\ 2.000 \\ 2.500 \\ 3.000 \\ 3.500 \\ 4.000 \\ 4.500 \\ 5.000 \\ \\ \tilde{\lambda} = 0.01 \\ a_0 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0000 \\ 0.0083 \\ 0.0125 \\ 0.0150 \\ 0.0167 \\ 0.0179 \\ 0.0188 \\ 0.0194 \\ 0.0200 \\ , \tilde{c} = 0.002, r = 0.5 \\ S \\ \end{array}$	$\begin{array}{c} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \\ 0.900 \\ 1.000 \\ \\ \tilde{\lambda} = 0.01 \\ a_0 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ \end{array}$
$\begin{array}{c} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \\ 0.900 \\ 1.000 \\ \hline \\ \tilde{\lambda} = 0.01 \\ \hline \\ a_0 \\ \hline \\ 0.050 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0200 \\ 0.0283 \\ 0.0325 \\ 0.0350 \\ 0.0367 \\ 0.0379 \\ 0.0388 \\ 0.0394 \\ 0.0400 \\ ., \tilde{c} = 0.001, r = 0.5 \\ \hline S \\ \hline 0.0000 \\ \end{array}$	$\begin{array}{c} a_0 \\ \hline 0.500 \\ 1.000 \\ 1.500 \\ 2.000 \\ 2.500 \\ 3.000 \\ 3.500 \\ 4.000 \\ 4.500 \\ 5.000 \\ \\ \tilde{\lambda} = 0.01 \\ \hline a_0 \\ \hline 0.050 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0000 \\ 0.0083 \\ 0.0125 \\ 0.0150 \\ 0.0167 \\ 0.0179 \\ 0.0188 \\ 0.0194 \\ 0.0200 \\ , \tilde{c} = 0.002, r = 0.5 \\ \hline S \\ \hline 0.0000 \\ \end{array}$	$\begin{array}{c} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \\ 0.900 \\ 1.000 \\ \hline \\ \tilde{\lambda} = 0.01 \\ \hline \\ a_0 \\ \hline \\ 0.100 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ \end{array}$
$\begin{array}{c} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \\ 0.900 \\ 1.000 \\ \hline \\ \tilde{\lambda} = 0.01 \\ \hline \\ a_0 \\ \hline \\ 0.050 \\ 0.100 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0200 \\ 0.0283 \\ 0.0325 \\ 0.0350 \\ 0.0367 \\ 0.0379 \\ 0.0388 \\ 0.0394 \\ 0.0400 \\ \vdots, \tilde{c} = 0.001, r = 0.5 \\ \hline S \\ \hline 0.0000 \\ 0.2250 \\ \end{array}$	$\begin{array}{c} a_0 \\ \hline 0.500 \\ 1.000 \\ 1.500 \\ 2.000 \\ 2.500 \\ 3.000 \\ 3.500 \\ 4.000 \\ 4.500 \\ 5.000 \\ \\ \tilde{\lambda} = 0.01 \\ \hline a_0 \\ \hline 0.050 \\ 0.100 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0000 \\ 0.0083 \\ 0.0125 \\ 0.0150 \\ 0.0167 \\ 0.0179 \\ 0.0188 \\ 0.0194 \\ 0.0200 \\ , \tilde{c} = 0.002, r = 0.5 \\ \hline S \\ \hline 0.0000 \\ 0.0000 \\ \end{array}$	$\begin{array}{c} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \\ 0.900 \\ 1.000 \\ \hline \\ \tilde{\lambda} = 0.01 \\ \hline a_0 \\ \hline 0.100 \\ 0.200 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0500 \\ \end{array}$
$\begin{array}{c} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \\ 0.900 \\ 1.000 \\ \hline \\ \tilde{\lambda} = 0.01 \\ \hline a_0 \\ \hline 0.050 \\ 0.100 \\ 0.150 \\ \hline \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0200 \\ 0.0283 \\ 0.0325 \\ 0.0350 \\ 0.0367 \\ 0.0379 \\ 0.0388 \\ 0.0394 \\ 0.0400 \\ \vdots, \tilde{c} = 0.001, r = 0.5 \\ \hline S \\ \hline 0.0000 \\ 0.2250 \\ 0.3083 \\ \end{array}$	$\begin{array}{c} a_0 \\ \hline 0.500 \\ 1.000 \\ 1.500 \\ 2.000 \\ 2.500 \\ 3.000 \\ 3.500 \\ 4.000 \\ 4.500 \\ 5.000 \\ \\ \tilde{\lambda} = 0.01 \\ \hline a_0 \\ \hline 0.050 \\ 0.100 \\ 0.150 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0000 \\ 0.0083 \\ 0.0125 \\ 0.0150 \\ 0.0167 \\ 0.0179 \\ 0.0188 \\ 0.0194 \\ 0.0200 \\ , \tilde{c} = 0.002, r = 0.5 \\ \hline S \\ \hline 0.0000 \\ 0.0000 \\ 0.1167 \\ \end{array}$	$\begin{array}{c} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \\ 0.900 \\ 1.000 \\ \hline \\ \tilde{\lambda} = 0.01 \\ \hline a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.1750 \\ \end{array}$
$\begin{array}{c} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \\ 0.900 \\ 1.000 \\ \hline \\ \tilde{\lambda} = 0.01 \\ \hline a_0 \\ \hline 0.050 \\ 0.100 \\ 0.150 \\ 0.200 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0200 \\ 0.0283 \\ 0.0325 \\ 0.0350 \\ 0.0367 \\ 0.0379 \\ 0.0388 \\ 0.0394 \\ 0.0400 \\ \vdots, \tilde{c} = 0.001, r = 0.5 \\ \hline S \\ \hline 0.0000 \\ 0.2250 \\ 0.3083 \\ 0.3500 \\ \end{array}$	$\begin{array}{c} a_0 \\ \hline 0.500 \\ 1.000 \\ 1.500 \\ 2.000 \\ 2.500 \\ 3.000 \\ 3.500 \\ 4.000 \\ 4.500 \\ 5.000 \\ \hline \\ \tilde{\lambda} = 0.01 \\ \hline a_0 \\ \hline 0.050 \\ 0.100 \\ 0.150 \\ 0.200 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0000 \\ 0.0083 \\ 0.0125 \\ 0.0150 \\ 0.0167 \\ 0.0179 \\ 0.0188 \\ 0.0194 \\ 0.0200 \\ , \tilde{c} = 0.002, r = 0.5 \\ \hline S \\ \hline 0.0000 \\ 0.0000 \\ \end{array}$	$\begin{array}{c} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \\ 0.900 \\ 1.000 \\ \hline \\ \tilde{\lambda} = 0.01 \\ \hline a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0500 \\ \end{array}$
$\begin{array}{c} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \\ 0.900 \\ 1.000 \\ \hline \\ \tilde{\lambda} = 0.01 \\ \hline a_0 \\ \hline 0.050 \\ 0.100 \\ 0.150 \\ 0.200 \\ 0.250 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0200 \\ 0.0283 \\ 0.0325 \\ 0.0350 \\ 0.0367 \\ 0.0379 \\ 0.0388 \\ 0.0394 \\ 0.0400 \\ \vdots, \tilde{c} = 0.001, \ r = 0.5 \\ \hline S \\ \hline 0.0000 \\ 0.2250 \\ 0.3083 \\ 0.3500 \\ 0.3750 \\ \end{array}$	$\begin{array}{c} a_0 \\ \hline 0.500 \\ 1.000 \\ 1.500 \\ 2.000 \\ 2.500 \\ 3.000 \\ 3.500 \\ 4.000 \\ 4.500 \\ 5.000 \\ \hline \\ \tilde{\lambda} = 0.01 \\ \hline a_0 \\ \hline 0.050 \\ 0.100 \\ 0.150 \\ 0.200 \\ 0.250 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0000 \\ 0.0083 \\ 0.0125 \\ 0.0150 \\ 0.0167 \\ 0.0179 \\ 0.0188 \\ 0.0194 \\ 0.0200 \\ , \tilde{c} = 0.002, r = 0.5 \\ \hline S \\ \hline 0.0000 \\ 0.0000 \\ 0.1167 \\ 0.2000 \\ 0.2500 \\ \end{array}$	$\begin{array}{c} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \\ 0.900 \\ 1.000 \\ \hline \\ \lambda = 0.01 \\ \hline a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0500 \\ 0.1750 \\ 0.2375 \\ 0.2750 \\ \end{array}$
$\begin{array}{c} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \\ 0.900 \\ 1.000 \\ \hline \\ \lambda = 0.01 \\ \hline a_0 \\ \hline 0.050 \\ 0.100 \\ 0.150 \\ 0.200 \\ 0.250 \\ 0.300 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0200 \\ 0.0283 \\ 0.0325 \\ 0.0350 \\ 0.0367 \\ 0.0379 \\ 0.0388 \\ 0.0394 \\ 0.0400 \\ \vdots \\ \tilde{c} = 0.001, \ r = 0.5 \\ \hline S \\ \hline 0.0000 \\ 0.2250 \\ 0.3083 \\ 0.3500 \\ 0.3750 \\ 0.3917 \\ \end{array}$	$\begin{array}{c} a_0 \\ \hline 0.500 \\ 1.000 \\ 1.500 \\ 2.000 \\ 2.500 \\ 3.000 \\ 3.500 \\ 4.000 \\ 4.500 \\ 5.000 \\ \hline \\ \tilde{\lambda} = 0.01 \\ \hline a_0 \\ \hline 0.050 \\ 0.100 \\ 0.150 \\ 0.200 \\ 0.250 \\ 0.300 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0000 \\ 0.0003 \\ 0.0125 \\ 0.0150 \\ 0.0167 \\ 0.0179 \\ 0.0188 \\ 0.0194 \\ 0.0200 \\ , \tilde{c} = 0.002, r = 0.5 \\ \hline S \\ \hline 0.0000 \\ 0.0000 \\ 0.1167 \\ 0.2000 \\ 0.2500 \\ 0.2833 \\ \end{array}$	$\begin{array}{c} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \\ 0.900 \\ 1.000 \\ \hline \\ \lambda = 0.01 \\ \hline a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0500 \\ 0.1750 \\ 0.2375 \\ 0.2750 \\ 0.3000 \\ \end{array}$
$\begin{array}{c} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \\ 0.900 \\ 1.000 \\ \hline \\ \lambda = 0.01 \\ \hline a_0 \\ \hline 0.050 \\ 0.100 \\ 0.150 \\ 0.200 \\ 0.250 \\ 0.300 \\ 0.350 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0200 \\ 0.0283 \\ 0.0325 \\ 0.0350 \\ 0.0367 \\ 0.0379 \\ 0.0388 \\ 0.0394 \\ 0.0400 \\ , \tilde{c} = 0.001, r = 0.5 \\ \hline S \\ \hline 0.0000 \\ 0.2250 \\ 0.3083 \\ 0.3500 \\ 0.3750 \\ 0.3917 \\ 0.4036 \\ \end{array}$	$\begin{array}{c} a_0 \\ \hline 0.500 \\ 1.000 \\ 1.500 \\ 2.000 \\ 2.500 \\ 3.000 \\ 3.500 \\ 4.000 \\ 4.500 \\ 5.000 \\ \hline \\ \tilde{\lambda} = 0.01 \\ \hline \\ a_0 \\ \hline 0.050 \\ 0.100 \\ 0.150 \\ 0.200 \\ 0.250 \\ 0.300 \\ 0.350 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0000 \\ 0.0003 \\ 0.0125 \\ 0.0150 \\ 0.0167 \\ 0.0179 \\ 0.0188 \\ 0.0194 \\ 0.0200 \\ , \tilde{c} = 0.002, r = 0.5 \\ \hline S \\ \hline 0.0000 \\ 0.0000 \\ 0.1167 \\ 0.2000 \\ 0.2500 \\ 0.2833 \\ 0.3071 \\ \end{array}$	$\begin{array}{c} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \\ 0.900 \\ 1.000 \\ \hline \\ \lambda = 0.01 \\ \hline a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0500 \\ 0.1750 \\ 0.2375 \\ 0.2750 \\ 0.3000 \\ 0.3179 \\ \end{array}$
$\begin{array}{c} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \\ 0.900 \\ 1.000 \\ \hline \\ \lambda = 0.01 \\ \hline a_0 \\ \hline 0.050 \\ 0.100 \\ 0.150 \\ 0.200 \\ 0.250 \\ 0.300 \\ 0.350 \\ 0.400 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0200 \\ 0.0283 \\ 0.0325 \\ 0.0350 \\ 0.0367 \\ 0.0379 \\ 0.0388 \\ 0.0394 \\ 0.0400 \\ \vdots, \tilde{c} = 0.001, r = 0.5 \\ \hline S \\ \hline 0.0000 \\ 0.2250 \\ 0.3083 \\ 0.3500 \\ 0.3750 \\ 0.3917 \\ 0.4036 \\ 0.4125 \\ \end{array}$	$\begin{array}{c} a_0 \\ \hline 0.500 \\ 1.000 \\ 1.500 \\ 2.000 \\ 2.500 \\ 3.000 \\ 3.500 \\ 4.000 \\ 4.500 \\ 5.000 \\ \\ \hline \lambda = 0.01 \\ \hline a_0 \\ \hline 0.050 \\ 0.100 \\ 0.250 \\ 0.300 \\ 0.250 \\ 0.300 \\ 0.350 \\ 0.400 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0000 \\ 0.0083 \\ 0.0125 \\ 0.0150 \\ 0.0167 \\ 0.0179 \\ 0.0188 \\ 0.0194 \\ 0.0200 \\ , \tilde{c} = 0.002, r = 0.5 \\ \hline \hline 0.0000 \\ 0.0000 \\ 0.1167 \\ 0.2000 \\ 0.2500 \\ 0.2833 \\ 0.3071 \\ 0.3250 \\ \end{array}$	$\begin{array}{c} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \\ 0.900 \\ 1.000 \\ \hline \\ \lambda = 0.01 \\ \hline a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.2000 \\ 0.0000 \\ 0.0000 \\ 0.0500 \\ 0.1750 \\ 0.2375 \\ 0.2750 \\ 0.3000 \\ 0.3179 \\ 0.3313 \\ \end{array}$
$\begin{array}{c} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \\ 0.900 \\ 1.000 \\ \hline \\ \lambda = 0.01 \\ \hline a_0 \\ \hline 0.050 \\ 0.100 \\ 0.150 \\ 0.200 \\ 0.250 \\ 0.300 \\ 0.350 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0200 \\ 0.0283 \\ 0.0325 \\ 0.0350 \\ 0.0367 \\ 0.0379 \\ 0.0388 \\ 0.0394 \\ 0.0400 \\ , \tilde{c} = 0.001, r = 0.5 \\ \hline S \\ \hline 0.0000 \\ 0.2250 \\ 0.3083 \\ 0.3500 \\ 0.3750 \\ 0.3917 \\ 0.4036 \\ \end{array}$	$\begin{array}{c} a_0 \\ \hline 0.500 \\ 1.000 \\ 1.500 \\ 2.000 \\ 2.500 \\ 3.000 \\ 3.500 \\ 4.000 \\ 4.500 \\ 5.000 \\ \hline \\ \tilde{\lambda} = 0.01 \\ \hline \\ a_0 \\ \hline 0.050 \\ 0.100 \\ 0.150 \\ 0.200 \\ 0.250 \\ 0.300 \\ 0.350 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0000 \\ 0.0003 \\ 0.0125 \\ 0.0150 \\ 0.0167 \\ 0.0179 \\ 0.0188 \\ 0.0194 \\ 0.0200 \\ , \tilde{c} = 0.002, r = 0.5 \\ \hline S \\ \hline 0.0000 \\ 0.0000 \\ 0.1167 \\ 0.2000 \\ 0.2500 \\ 0.2833 \\ 0.3071 \\ \end{array}$	$\begin{array}{c} a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ 0.800 \\ 0.900 \\ 1.000 \\ \hline \\ \lambda = 0.01 \\ \hline a_0 \\ \hline 0.100 \\ 0.200 \\ 0.300 \\ 0.400 \\ 0.500 \\ 0.600 \\ 0.700 \\ \end{array}$	$\begin{array}{c} S \\ \hline 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0500 \\ 0.1750 \\ 0.2375 \\ 0.2750 \\ 0.3000 \\ 0.3179 \\ \end{array}$

$\tilde{\lambda} = 0.005, \ \tilde{c} = 0.001, \ r = 0.5$		$\tilde{\lambda} = 0.005, \ \tilde{c} = 0.002, \ r = 0.5$	$\tilde{\lambda} = 0.005, \ \tilde{c} = 0.003, \ r = 0.5$	
a_0	S	a_0 S	$a_0 \hspace{1cm} S$	
0.050	0.0000	0.100 0.0000	0.200 0.0000	
0.100	0.0000	0.200 0.0000	0.400 0.0000	
0.150	0.0583	0.300 0.0333	0.600 0.0500	
0.200	0.1000	0.400 0.0750	0.800 0.0812	
0.250	0.1250	0.500 0.1000	1.000 0.1000	
0.300	0.1417	0.600 0.1167	1.200 0.1125	
0.350	0.1536	0.700 0.1286	1.400 0.1214	
0.400	0.1625	0.800 0.1375	1.600 0.1281	
0.450	0.1694	0.900 0.1444	1.800 0.1333	
0.500	0.1750	1.000 0.1500	2.000 0.1375	
$\tilde{\lambda} = 0.00$	$\tilde{c} = 0.001, r = 0.5$	$\tilde{\lambda} = 0.001, \ \tilde{c} = 0.002, \ r = 0.5$	$\tilde{\lambda} = 0.001, \ \tilde{c} = 0.003, \ r = 0.5$	
$\tilde{\lambda} = 0.00$ a_0	11, $\tilde{c} = 0.001$, $r = 0.5$	$\tilde{\lambda} = 0.001, \ \tilde{c} = 0.002, \ r = 0.5$ a_0 S	$\tilde{\lambda} = 0.001, \ \tilde{c} = 0.003, \ r = 0.5$ a_0 S	
	, , , , , , , , , , , , , , , , , , ,			
a_0	S	a_0 S	$\underline{\hspace{1cm}}$ a_0 S	
$\frac{a_0}{0.500}$	S 0.0000	$\begin{array}{c c} a_0 & S \\ \hline 5.000 & 0.0000 \end{array}$	$ \begin{array}{c cc} a_0 & S \\ \hline 5.000 & 0.0000 \end{array} $	
$ \begin{array}{r} a_0 \\ \hline 0.500 \\ 1.000 \end{array} $	S 0.0000 0.0000	$\begin{array}{c c} a_0 & S \\ \hline 5.000 & 0.0000 \\ 10.000 & 0.0000 \\ \end{array}$	$\begin{array}{c cc} a_0 & S \\ \hline 5.000 & 0.0000 \\ 10.000 & 0.0000 \\ \end{array}$	
$ \begin{array}{r} a_0 \\ \hline 0.500 \\ 1.000 \\ 1.500 \end{array} $	S 0.0000 0.0000 0.0083	$ \begin{array}{c cc} a_0 & S \\ \hline 5.000 & 0.0000 \\ 10.000 & 0.0000 \\ 15.000 & 0.0000 \\ \end{array} $	$ \begin{array}{c cc} a_0 & S \\ \hline 5.000 & 0.0000 \\ 10.000 & 0.0000 \\ 15.000 & 0.0000 \\ \end{array} $	
$ \begin{array}{r} a_0 \\ \hline 0.500 \\ 1.000 \\ 1.500 \\ 2.000 \end{array} $	S 0.0000 0.0000 0.0083 0.0125	$ \begin{array}{c cc} a_0 & S \\ \hline 5.000 & 0.0000 \\ 10.000 & 0.0000 \\ 15.000 & 0.0000 \\ 20.000 & 0.0000 \\ \end{array} $	$ \begin{array}{c cc} a_0 & S \\ \hline 5.000 & 0.0000 \\ 10.000 & 0.0000 \\ 15.000 & 0.0000 \\ 20.000 & 0.0000 \\ \end{array} $	
$ \begin{array}{r} a_0 \\ \hline 0.500 \\ 1.000 \\ 1.500 \\ 2.000 \\ 2.500 \\ \end{array} $	S 0.0000 0.0000 0.0083 0.0125 0.0150	$ \begin{array}{c cccc} a_0 & S \\ \hline 5.000 & 0.0000 \\ 10.000 & 0.0000 \\ 15.000 & 0.0000 \\ 20.000 & 0.0000 \\ 25.000 & 0.0000 \\ \end{array} $	$ \begin{array}{c cccc} a_0 & S \\ \hline 5.000 & 0.0000 \\ 10.000 & 0.0000 \\ 15.000 & 0.0000 \\ 20.000 & 0.0000 \\ 25.000 & 0.0000 \\ \end{array} $	
$ \begin{array}{r} a_0 \\ \hline 0.500 \\ 1.000 \\ 1.500 \\ 2.000 \\ 2.500 \\ 3.000 \end{array} $	S 0.0000 0.0000 0.0083 0.0125 0.0150 0.0167	$\begin{array}{c cc} a_0 & S \\ \hline 5.000 & 0.0000 \\ 10.000 & 0.0000 \\ 15.000 & 0.0000 \\ 20.000 & 0.0000 \\ 25.000 & 0.0000 \\ 30.000 & 0.0000 \\ \end{array}$	$ \begin{array}{c cccc} a_0 & S \\ \hline 5.000 & 0.0000 \\ 10.000 & 0.0000 \\ 15.000 & 0.0000 \\ 20.000 & 0.0000 \\ 25.000 & 0.0000 \\ 30.000 & 0.0000 \\ \end{array} $	
$\begin{array}{c} a_0 \\ \hline 0.500 \\ 1.000 \\ 1.500 \\ 2.000 \\ 2.500 \\ 3.000 \\ 3.500 \\ \end{array}$	S 0.0000 0.0000 0.0083 0.0125 0.0150 0.0167 0.0179	$\begin{array}{c cccc} a_0 & S \\ \hline 5.000 & 0.0000 \\ 10.000 & 0.0000 \\ 15.000 & 0.0000 \\ 20.000 & 0.0000 \\ 25.000 & 0.0000 \\ 30.000 & 0.0000 \\ 35.000 & 0.0000 \\ \end{array}$	$\begin{array}{c cccc} a_0 & S \\ \hline 5.000 & 0.0000 \\ 10.000 & 0.0000 \\ 15.000 & 0.0000 \\ 20.000 & 0.0000 \\ 25.000 & 0.0000 \\ 30.000 & 0.0000 \\ 35.000 & 0.0000 \\ \end{array}$	

See figure 1 below for the effect of various choices for a_0 on pool rewards (for k = 10).

8.10.3 ρ

In order to determin the inflation rate per epoch ρ , we need four more pieces of information:

- The expected exchange rate e from ADA to USD (in USD/ADA).
- The average $costs\ c$ (in USD) to run a pool for one year.
- The average $transaction\ fees\ F$ (in ADA) paid during one epoch.
- The expected ratio r of rewards per year per staked ADA.

The available rewards for one epoch (assuming an equilibrium state with k pools and noticing that there are $\frac{365}{5} = 73$ epochs per year) will be

$$(1-\tau)\cdot (F+\rho\cdot (T\infty-T))-\frac{k\cdot c}{73\cdot e}.$$

On the other hand, expected rewards per epoch are

$$T \cdot \left(\sqrt[73]{1+r} - 1\right)$$
.

Equating the two, we get

$$\rho = \frac{T \cdot \left(\sqrt[73]{1+r} - 1\right) - (1-\tau) \cdot F + \frac{k \cdot c}{73 \cdot e}}{(1-\tau) \cdot (T_{\infty} - T)}.$$

For example, using

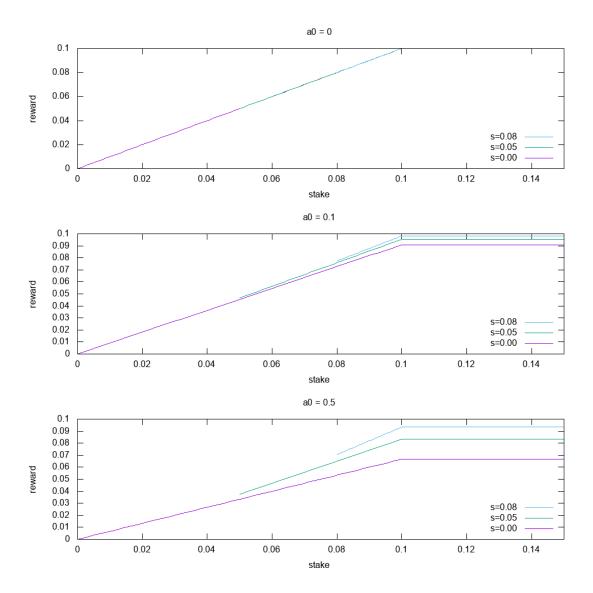


Figure 1: Effect of different choices for a_0

- k = 100,
- T = 31,000,000,000 ADA,
- $T_{\infty} = 45,000,000,000 \, \text{ADA},$
- e = 0.5 USD/ADA,
- c = 1,000 USD,
- $F = 2,000 \, \text{ADA}$ and
- r = 0.05,
- $\tau = 0.2$,

we would get

$$\rho = \frac{31,000,000,000 \cdot \left(\sqrt[73]{1+0.05} - 1\right) - 0.8 \cdot 2000 + \frac{100 \cdot 1000}{73 \cdot 0.5}}{0.8 \cdot \left(45,000,000,000 - 31,000,000,000\right)} \approx 0.0019.$$

This would correspond to reducing the remaining amount of available ADA by $1.0019^{73} - 1 \approx 0.144 = 14.4\%$ per year (which sounds awfully high...).

8.10.4 τ

Setting τ is a policy decision; we will probably use $\tau = 0.2$, i.e. 20% of available epoch rewards will be sent to the treasury.

8.10.5 γ

Setting γ is also a policy decision. Having said this, values of $\gamma < 1$ seem to be preferable, because pool operators occasionally missing one or two slots will not be punished too harshly.

To help with the task of deciding on a reasonable value for γ , we show the effect of different values on a potential pool that was elected to create 100 pools in a given epoch. The table below shows the ratio of rewards paid for varying numbers of missed slots and values of γ :

missed slots	$\gamma = 1$	$\gamma = 0.7$	$\gamma = 0.5$	$\gamma = 0.3$
0	1.0000	1.0000	1.0000	1.0000
1	0.9900	0.9930	0.9950	0.9970
5	0.9500	0.9647	0.9747	0.9847
10	0.9000	0.9289	0.9487	0.9689
50	0.5000	0.6156	0.7071	0.8123
100	0.0000	0.0000	0.0000	0.0000

9 Satisfying the Requirements

In the following, we describe how the requirements listed in 5 are satisfied by the design in this document.

- **5.1.1 Proof of Eligibility** The leader election process takes delegation via pointer addresses and heavyweight certificates into account (7.7), so the leader schedule will contain the key of the party that is expected to sign the block (either a stake pool operator or an individual stakeholder).
 - If a lightweight certificate is used, it will be posted to the block header, which will also prove eligibility.
- **5.1.2 Visibility of Delegation on the Blockchain** Delegation via heavyweight certificates and pointer addresses is visible on the blockchain. Delegation via lightweight certificates should only be used for hot/cold key management. Thus, it is not relevant for the rewards sharing process, and does not need to be visible on the chain.
- **5.1.3 Restricting Chain Delegation** Chain delegation is properly restricted, as described in 7.3.3.
- **5.1.4 Cheap Re-Delegation** Re-delegation can be performed cheaply with multi-input transactions.
- **5.1.5 Neutral Addresses** The design includes enterprise addresses (7.2.3), which are disregarded by the PoS protocol.
- **5.2.1 Sybil Attack Protection at Stake Pool Level** Stake pool operators are expected to pledge a deposit to their pools that has an influence on the rewards that stake pool members will receive, and on the position of the stakepool in the listing displayed to stakeholders (7.4.1, {display-of-stake-pools-in-the-wallet}, **TODO**: add reference to rewards function once document is merged with incentives doc).
 - Since this pledge cannot be shared between multiple pools, creating n viable stake pools will require funds linear in n.
- **5.2.2 Address Nonmalleability** Protection against the malleability attack is described in 5.3.2.
- **5.2.3 Public Spending Keys Should not be Disclosed Prematurely** The introduction of a dedicated staking key (7.2) avoids the need to use the payment key for delegation purposes.
- **5.2.4 Mitigate Key Exposure** Stake pool operators can and should use lightweight certificates for hot/cold key management, as described in 7.3.2.
- **5.2.5 Handle Inactive Stake Pools** We have two mechanisms for dealing with inactive stake pools. Stake pools can be retired via a retirement certificate (7.3.4.1. If a

- stake pool ceises to operate without being properly retired, its key will be detected as a stale staking key as described in 7.5.
- **5.2.6 Avoid Hard Transition** As described in 7.8.2, at the transition from Byron to Shelley, all stake will initially stay delegated with the core nodes of the Byron network, avoiding a hard transition with temporarily undelegated stake.
- **5.2.7 Change Delegation Without Spending Key** Delegation of cold wallets is described in 7.4.4, and does not require having the spending key of the cold wallet online.
- **5.3.1 Master Recovery Key** Wallet recovery is described in 7.8.1, and does not require any information in addition to the master key.
- **5.3.2 Address Recognition** Wallets will recognise addresses belonging to it by looking at the payment key hash part of the address, as described in 7.6.
- **5.3.3 Wallet should be Runnable on Independent Devices** With the caveats listed in that requirement, nothing in this document requires wallets running on different devices to share state.
- **5.3.4 Maintain Privacy** The default delegation mechanism (7.4.3) uses pointer addresses, so addresses of the same wallet are not connected in an obvious way.
- **5.3.5 Short Addresses** The goal of having reasonably short addresses has guided the design of delegation, and we do not see an obvious way of making them even shorter, while still satisfying the rest of the requirements.