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In this study, a new antibacklash gear mechanism design comprising three pinions and a rack is introduced. This mechanism offers several advantages compared to conventional antibacklash mechanisms, such as lower transmission error as well as lower required preload. Nonlinear dynamic modeling of this mechanism is developed to acquire insight into its dynamic behavior. It is observed that the amount of preload required to diminish the backlash depends on the applied input torque and nature of periodic mesh stiffness. Then, an attempt is made to obtain an approximate relation to find the minimum requiring preload to preserve the system's antibacklash property and reduce friction and wear on the gear teeth. The mesh stiffness of the mated gears, rack, and pinion is achieved via finite element method. Assuming that all teeth are rigid and static transmission error is negligible, dynamic transmission error (DTE) would be zero for every input torque, which is a unique trait, not yet proposed in previous research. [DOI: 10.1115/1.4029582]

Introduction

One of the major problems with gear systems is backlash. Backlash is the rotational clearance between mated gears. The presence of backlash has many adverse effects, such as decline in system accuracy and controllability, as well as noise, vibration, and wear. It is also a major factor contributing to DTE, especially when the rack and pinions are used, since the rotational direction has to change continuously.

Various antibacklash systems have been proposed with different applications like industrial robots, precision machining tools, radar antennas, and precision servo motors. Antibacklash gear systems cause the transmission error to be reduced. As a result, precise and desirable output rotation or displacement is gained. The double electric motor method, double helical gear antibacklash method, adjusting center distance method, and the spring-loaded antibacklash method [1–7] are among the approaches used to eliminate backlash in gear, rack, and pinion systems.

Among the above-mentioned methods utilized to omit backlash, the spring-loaded antibacklash technique is widely employed in numerous fields. In this method, the coefficient of spring needs to be determined so that the mated teeth are always in contact for all input torque. However, high preload in the system causes high contact and bending stress, which may reduce gear lifespan. Other drawbacks of this method are complex manufacturing and installation, low transfer torque and high friction. In addition, if the teeth are assumed to be rigid and static transmission error is neglected, the DTE would always be in the system because of torsional spring deformation triggered by input torque.

In order to gain thorough insight into the dynamic and vibration behavior of gear systems, their modeling has been studied since 1920. Many investigations have been done and accomplishments achieved in the case of dynamic modeling and analysis of gear systems, with summaries compiled by Wang et al. [8].

In this paper, a new antibacklash system with a unique feature is proposed in Ref. [9]. In this new mechanism, assuming that all teeth are rigid and static transmission error is negligible, DTE would be zero for every input torque, a unique trait not proposed in previous research. Nevertheless, dynamic nonlinear system modeling with regard to time-varying stiffness and backlash modeling is investigated in order to thoroughly perceive the system's

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dynamic behavior. Based on a dynamic model, the minimum required preload is also specified to eliminate backlash for different input torques. Eventually, an approximate relation for determining the minimum preload required eliminating backlash is proposed. The developed equations are based on DTE and numerical methods are employed to solve them.

The Concept of a New Antibacklash Gear Mechanism

The configuration of the new antibacklash mechanism comprises three pinions, one rack, two bearing holders, and three adjustment slides, as shown in Fig. 1. In this mechanism, the main pinion is the driver to which input torque is applied. An optimum amount of preload is exerted on the main pinion by an adjusting system to ensure that all gear teeth remain in contact in various operating conditions. It is not essential to use a spring to provide the required preload and it is sufficient to use a screw to supply an accurate amount of preload. Compared with a spring-loaded mechanism, assuming that teeth are rigid and static transmission error is zero, this system would provide zero transmission error.

Nonlinear Dynamic Model

The spur gears, rack, and pinion are modeled using a disk and mass coupled with nonlinear mesh stiffness and damping (Fig. 1). The mesh compliance is considered by periodic variable springs ($k(t)$) and energy loss in the mesh is achieved by the damping elements. In order to obtain equations of motion based on the transmission errors of the relevant gears, first the following definitions are stated:

$$x_{12} = -R_1\theta_1 + R_2\theta_2 \tag{1a}$$

$$x_{13} = R_1\theta_1 - R_3\theta_3 \tag{1b}$$

$$x_{24} = -R_2\theta_2 + x \tag{1c}$$

$$x_{34} = R_3\theta_3 - x \tag{1d}$$

where x_{12}, x_{13}, x_{24}, and x_{34} are relative transmission error between pinion 1 and 2, 1 and 3, pinion 2 and rack, and finally pinion 3 and rack, respectively. R_1, R_2, R_3 are gear radius, $\theta_1, \theta_2, \theta_3$ are the gears' rotational displacements, and x is the linear displacement of the rack. These relations are defined such that compression always gives positive transmission errors.

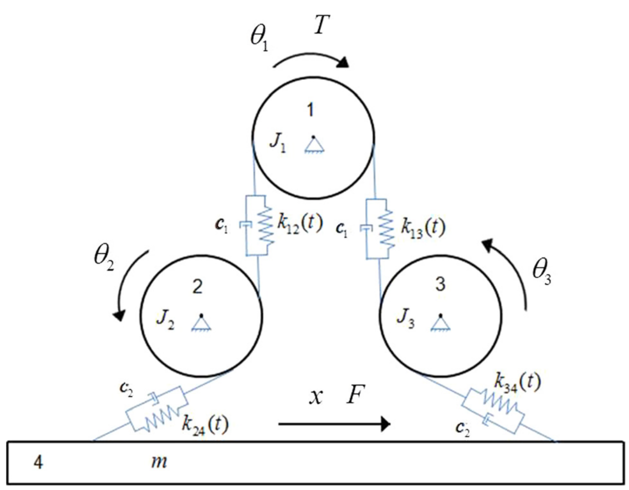

Fig. 1 Model of antibacklash gear mechanism

For simplicity, Eq. (1) can be stated in matrix form as follows:

$$\{x\} = [P]\{\theta\} \tag{2}$$

Matricial form of Newtonian equation of motion leads to following equation:

$$\{\ddot{\theta}\} = [C]\{\dot{x}\} + [K(t)]\{f(\delta + x)\} + \{F\} \tag{3}$$

By multiplying both sides of Eq. (3) by $[P]$ defined in Eq. (2), the following equation that is totally based on relative transmission error vector is obtained:

$$\{\ddot{x}\} = [K'(t)]\{f(\delta + x)\} + [C']\{\dot{x}\} + \{F'\} \tag{4}$$

where

$$[K'(t)] = [P][K(t)]$$

$$[C'] = [P][C]$$

$$\{F'\} = [P]\{F\}$$

where nonlinearity stems from displacement function due to backlash defined as follows:

$$f(\zeta) = \begin{cases} \zeta, & \zeta > 0 \\ 0, & -b < \zeta \le 0 \\ \zeta + b, & \zeta \le -b \end{cases} \tag{5}$$

where b is backlash. Finally, it is possible to obtain the total transmission error of the mechanism using the following relation:

$$T.E. = x - R_1\theta_1 = x_{24} + x_{12} = -(x_{13} + x_{34}) \tag{6}$$

Approximate Preload Requirement for No-Backlash Condition in Constant Rotational Speed

Being aware of the minimum amount of preload needed to preserve tooth contact between mated gears is important to make sure that no excess amount of compression load is presented in the system. First, it is assumed that the system is in normal situation means no separation happened and therefore according to function (5), one can use ζ instead of $f(\zeta)$ and thereafter due to inherent symmetry of mechanism, the effect of compression displacement vanish yields to linear version of equation of motion which will be used in this section to discover limiting case of separation.

The transmission error vector and stiffness matrix is approximated as follows:

$$\{x\} = \{x_0\} + \sum_{n=1}^{m}\{x_{an}\}\cos n\omega t + \sum_{n=1}^{m}\{x_{bn}\}\sin n\omega t \tag{7a}$$

$$[K(t)] = [K_0] + \sum_{n=1}^{m}[K_{an}]\cos n\omega t + \sum_{n=1}^{m}[K_{bn}]\sin n\omega t \tag{7b}$$

Applying Eq. (7) in Eq. (4) and equating sum of similar terms to zero, the general pattern for calculating the $\{x_{an}\}$ and $\{x_{bn}\}$ would be as follows:

$$[K_0]\{x_0\} + \{F\} = 0 \tag{8a}$$

$$([K_0]\{x_{an}\} + [K_{an}]\{x_0\})\cos n\omega t = 0 \tag{8b}$$

$$([K_0]\{x_{bn}\} + [K_{bn}]\{x_0\})\sin n\omega t = 0 \tag{8c}$$

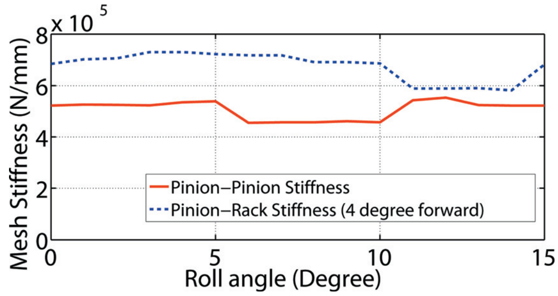

Fig. 2 Mesh stiffness of pinion–pinion and rack–pinion variation relative to roll angle

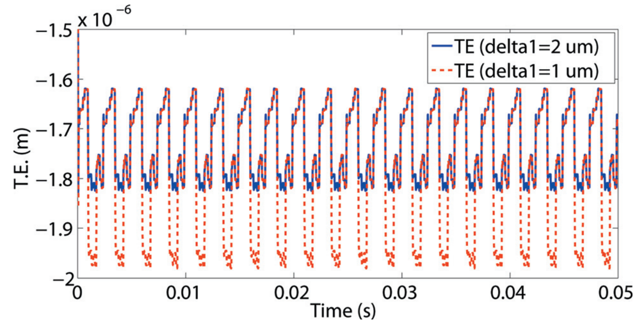

Fig. 3 Time response of total transmission errors

Table 1 Physical parameters of pinions and rack

	Number of teeth Z	Module m, (mm)	Mass (kg)	Face width (mm)	Diameter of pitch circle (mm)	Pressure angle (deg)	Contact ratio
Pinions	24	2	0.222	20	48	14.5	1.97
	Number of teeth Z	Module m, (mm)	Mass (kg)	Face width (mm)	Diameter of pitch circle (mm)	Length (mm)	Height (mm)
Rack	75	2	0.683	20	—	150	32

Case Study

Figure 2 shows the mesh stiffness of pinion–pinion and rack–pinion tooth contact that computed using FINITE ELEMENT software. Table 1 shows the parameters employed in simulation for pinions and rack. The rotational speed, time step, and applied step torque are 1000rpm, 10μs, 24Nm, respectively. The Rung–Kutta method, one of the most important iterative solvers of ordinary differential equations, was employed in MATLAB through the ode45 command. Since the rotational speed is assumed to be constant, therefore, the force applied on rack must be $F = -T/R$.

All simulation were performed for $\delta_1 = 2\mu$m and $\delta_1 = 1\mu$m. The associated δ_2 was calculated using $\delta_2 = \delta_1 k_{24}/k_{12}$ or $\delta_2 = \delta_1 k_{34}/k_{13}$. The initial compression of $\delta_1 = 2\mu$m is associated with no separation condition, while $\delta_1 = 1\mu$m presents system with tooth separation. As Fig. 2 represents the tooth separation increases the transmission error. Due to limitation of paper space, time response of relative transmissions was not presented. However, calculation demonstrated that the limitation for preserving tooth contact is 1.1μm pertained to pinion–pinion teeth.

According to Eq. (7a) for $m = 2$, the approximated absolute amount of transmission error vector could be obtained as follows:

$$\{x\} \approx \begin{Bmatrix} 1.115 \\ 1.115 \\ 0.88 \\ 0.88 \end{Bmatrix} \quad (9)$$

This result is substantially close and slightly higher than the accurate results depicted in Fig. 3.

Conclusion

In this study, a new antibacklash gear mechanism comprising three spur gears and a rack was explained, and a nonlinear dynamic modeling of the system was obtained to gain insight into the system's dynamic behavior. Validity of concept in eliminating backlash is examined by employing Motion Analysis of SOLIDWORKS CAD software. The mesh stiffness of mated gears and rack–-pinion was evaluated with the finite element method and was considered by solving a motion equation. Assuming that the gear and rack teeth are rigid and static transmission error is negligible, the total transmission error of the new mechanism will be zero, which is one of the more significant, considerable features of this system.

The minimum amount of preload at which the system remains antibacklash while operating, has been estimated. This depends on applied torque, external load on rack and varied mesh stiffness behavior. Total system transmission error was obtained by solving the differential equation of motion using the Runge–Kutta method.

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References

[1] Guoming, L., 2001, "Calculation and Analysis of Double Gear's Inherent Frequency," Electro-Mech. Eng., **3**, pp. 11–12.
[2] Imasaki, N., and Tomizuka, M., 1995, "Adaptive Control of Robot Manipulators With Anti-Backlash Gears," The IEEE International Conference on Robotics and Automation, Nagoya, pp. 306–311.
[3] Shim, S. B., Park, Y. J., and Kim, K. U., 2008, "Reduction of PTO Rattle Noise of an Agricultural Tractor Using an Anti-Backlash Gear," Biosyst. Eng., **100**(3), pp. 346–354.
[4] Kwon, Y. S., Hwang, H. Y., Lee, H. R., and Kim, S. H., 2004, "Rate Loop Control Based on Torque Compensation in Anti-Backlash Geared Servo System," American Control Conference, Boston, MA, pp. 3327–3332.
[5] Allan, P. M., and Levy, N. M., 1980, "The Determination of Minimum Pre-Load Torque for Anti-Backlash Gears in a Positional Servomechanism," IEEE Trans. Ind. Electron. Control Instrum., **27**(1), pp. 1232–1239.
[6] Boyuan, M., and Zhiwu, L., 2001, "Drive Chain Return Difference Analysis and Estimate of Spring-Loaded Anti-Backlash Gear," Mech. Des., **6**, pp. 40–42.
[7] Cairnes, C. W., 1953, "Anti-Backlash Gearing," U.S. Patent No. 2663198.
[8] Wang, J., Li, R., and Peng, X., 2003, "Survey of Nonlinear Vibration of Gear Transmission Systems," ASME Appl. Mech. Rev., **56**(3), pp. 309–329.
[9] Besharati, S. R., Sarhan, A. A., and Hamdi, M., 2012, "Gear Backlash Compensation Mechanism (GBCM) in Rack and Pinion System," U.S. Patent Pending, PI No. 2012 701279.