Chapter 4
One-Dimensional Random Variable

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Random Variables

- Let S be the sample space which represents the total outcome of given experiment and let X be real valued function defined over the sample space S, and then X is said to be a **random variable**.
- **A random variable** is a function that assigns a numerical value to each possible outcome of a random experiment. Often random variables are used to summarize the outcome from a random experiment by a simple number.
- A random variable is denoted by an uppercase letter such as X. After an experiment is conducted, the measured value of the random variable is denoted by a lowercase letter.

Types of Random variables

Based on the values of the random variable, random variables can be classified as discrete and continuous.

i. **Discrete random variable**: If the possible values of the random variables are finite or countable infinite.
Examples: number of scratches on a surface, number of defective parts among 1000 tested, number of transmitted bits received in error.

ii. **Continuous random variable**: If the random variable assumes an uncountable or infinite number of possible values or if the random variable assumes any real values between two defined points.
Examples: electrical current, length, pressure, temperature, time, voltage, weight

Types of Random variables...

Depending on the number of variable that determine the output of given experiment random variables can be:

i. **One dimensional random variable**: If the output of that experiment is determined by one variable,
ii. **Two dimensional random variables**: If the output of that experiment is determined by two variables and

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iii. **K dimensional random variable**: If the output of that experiment is determined by k variables. In general the type of a given experiment determines the type of random variable because of the random variable is defined over a given experiment.

Probability distribution

- The probability distribution of a random variable X is a description of the probabilities associated with the possible values of X.
- Consider a simple experiment in which we toss a coin two times.
- Suppose the random variable X is defined as the number of heads that result from two coin tosses.
- then the probability distribution looks like

Number of Heads (X)	Probability
0	0.25
1	0.50
2	025

- Based on the type of a random variable, a probability distribution can be discrete or continuous.

Discrete Probability Distribution

- For a discrete random variable, the distribution is often specified by just a list of the possible values along with the probability of each.
- Probability distribution function for discrete random variable is sometimes called probability mass function.
- **Example**:A Statistics class of 25 students is given a 5 point quiz. 3 students scored 0, 1 scored 1, 4 scored 2, 8 scored 3, 6 scored 4, and 3 students scored 5. If a student is chosen at random, and the random variable S is the student's quiz score then the probability distribution of S is:

S	0	1	2	3	4	5
P(s)	0.12	0.04	0.16	0.32	0.24	0.12

Discrete Probability Distribution...

- For a discrete random variable X with possible values $x_1, x_2, \ldots, x_n$, a probability mass function, $p(x_i)$, is a function such that:
  - i. $0 \leq f(x) \leq 1$
  - ii. $\sum f(x) = 1$
  - iii. $f(x_i) = p(X = X_i)$

  Example

  1. Verify that the following is probability mass function

     a.

X	-2	-1	0	1	2
P(x)	1/8	2/8	2/8	2/8	1/8

     b. $P(x) = (\frac{8}{7})(\frac{1}{2})^x, x = 1, 2, 3$

  2. A family is planning to have 3 children randomly. If X represents the number of girl child they are going to have, construct the probability mass function of X.

Continuous Probability Distribution

- For a continuous random variable, the probability is expressed in terms of a formula/ equation.
- The equation used to describe a continuous probability distribution is called a probability density function (pdf).
- The probability that a random variable assumes a value between a and b is equal to the area under the density function bounded by a and b.
- The probability that X is exactly equal to a i.e. p (X =a) would be zero.
- f(x) is said to be a probability density function for the random variable x if it satisfies the following conditions.
    i. $f(x) \geq 0$ for all the values of X in the defined region
    ii. $\int_{-\infty}^{\infty} f(x)dx = 1$
    iii. $P(a < X < b) = \int_{a}^{b} f(x)dx$, where $-\infty < a < b < \infty$

- If X is a continuous random variable, for any $x_1 and x_2$, then

$$P(x_1 \leq X \leq x_2) = P(x_1 < X \leq x_2) = P(x_1 \leq X < x_2) = P(x_1 < X < x_2)$$

Example

1. Among the following functions identify the one that serve as probability density function
   a. $f(x) = \begin{cases} 3x^2, & 0 \leq x \leq 1; \\ 0, & \text{otherwise} \end{cases}$ is *pdf*
   b. $f(x) = \begin{cases} \frac{x-9}{2}, & 9 \leq x \leq 11; \\ 0, & \text{otherwise} \end{cases}$ is *pdf*
2. Suppose that$f(x) = e^{-x} for 0 < x$. Determine the following probabilities:
   a. $P(1 < X)$
   b. $P(1 < X < 2.5)$
   c. $P(X = 3)$
   d. $P(X < 4)$
   e. $P(3 \geq X)$
   f. determine x such that $P(X < x) = 0.10$
   g. determine x such that $P(X \leq x) = 0.10$

Cumulative Distribution Function

- Let X be a random variable either it can be continuous or discrete,the cumulative distribution of X is defined by $F(x) = P(X \leq x)$.
- The ways we find CDF for the two types of random variable are different.
- If X is a discrete random variable F(x) is given by

$$P(X \leq x) = \sum_{x \leq xi} p(x_i)$$

- If X is a continuous random variable its F(x) is given by

$$P(X \leq x) = \int_{-\infty}^{x} f(t)dt$$

where f(t) here is the function of probability density function of a given random variable.

Example

1. A student is given three true/ false questions, where the student does not know the exact answer of the questions. Let X represents the number of true answers the student answered randomly.Construct a cumulative distribution of X.
2. Assume that the resistance of certain cable is assumed to be a continuous with the following pdf

$$f(x) = \begin{cases} 3x^2, & 0 \leq x \leq 1; \\ 0, & \text{otherwise} \end{cases}$$

Then compute the CDF for the resistance of a cable.

The general properties of CDF

- is non decreasing function
- is right hand continuous
- become decreased when we goes to left