Department of
Computer Science and Engineering

Title: Implement 0-1 Knapsack Problem Using Dynamic Programming (DP)

Algorithms Lab

CSE 206

Submitted by

Shamim Ahmed

ID:201902067

Green University of Bangladesh

Objective(s)

- Understand the basic of dynamic programming
- Apply dynamic programming to solve real-life optimal decision making

Problem Analysis

Suppose a thief is going to steal a store. He has a knapsack to carry goods and maximal weight of W is possible to carry. There are n items available in the store and weight of i-th item is w_i and its profit is p_i. What items should the thief take? Problem is he have to take the item entirely or left it behind which is denoted by $x_i = 0, 1$. Therefore, the items should be selected in such a way that the thief will carry those items for which he will gain maximum profit. Hence, the objective of the thief is to maximize the profit –

$$\max \sum_{i=1}^{N} x_i p_i \quad (1)$$

In addition, the constraint is

$$\sum_{i=1}^{N} x_i w_i \leq W \quad (2)$$

Solution Steps

- Take input of list of items, and weights using array
- Construct a DP table $P[n][W]$, where $P[i][w]$ indicates the maximum profit that can be obtained from items 1 to i, if the knapsack has size w
- Case 1: taking the item i, in that case
  $P[i][w] = v_i + P[i - 1][w - w_i]$
- Case 2: not taking the item i, in that case
  $P[i][w] = P[i - 1][w]$
- The final recurrence relation is
  $P[i][w] = max\{v_i + P[i - 1][w - w_i], P[i - 1][w]\}$

We can understand the problem more clearly by the following example

Item	Weight	Value
1	2	12
2	1	10
3	3	20
4	2	15

Figure 1: Weight and profit of each items

i \ w	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	12	12	12	12	12
2	0					
3	0					
4	0					

P[1][1] = P[0][1] = 0
P[1][2] = max{ 12+0, 0} = 12
P[1][3] = max{ 12+0, 0} = 12
P[1][4] = max{ 12+0, 0} = 12
P[1][5] = max{ 12+0, 0} = 12

Figure 2: Iteration 1

i \ w	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	12	12	12	12	12
2	0	10	12	22	22	22
3	0					
4	0					

P[2][1] = max{ 10+0, 0} = 10
P[2][2] = max{ 10+0, 12} = 12
P[2][3] = max{ 10+12, 12} = 22
P[2][4] = max{ 10+12, 12} = 22
P[2][5] = max{ 10+12, 12} = 22

Figure 3: Iteration 2

Calculate the Iteration 3 by yourself

i \ w	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	12	12	12	12	12
2	0	10	12	22	22	22
3	0	10	12	22	30	32
4	0	10	15	25	30	37

P[4][1] = P[3][1] = 10
P[4][2] = max{ 15+0, 12} = 15
P[4][3] = max{ 15+10, 22} = 25
P[4][4] = max{ 15+12, 30} = 30
P[4][5] = max{ 15+22, 32} = 37

Figure 4: Iteration 4

Time Complexity

Time complexity of 0 − 1*Knapsack* problem is $O(nW)$ where, n is the number of items and W is the capacity of knapsack.

Algorithm

Algorithm 1: Dynamic 0-1 Knapsack
Input: Weights, Values
Output: $P[n, W]$

```
for w = 0 to W do
P[0, w] = 0
end
for i = 1 to n do
P[i, 0] = 0
for w = 1 to W do
if w_i ≤ w then
P[i, w] = max{v_i + P[i − 1, w − w_i], P[i − 1, w]} end
else
P[i, w] = P[i − 1, w]
end
end
end
```

Implementation in C++

```
#include<bits/stdc++.h>

using namespace std;

void knapsack (int v[], int w[],int n, int weight)
{
    int k[20][20],i,j;
    for(i=0; i<=n ;i++)
    {
        for(j=0; j<=weight ; j++)
        {
            if(i==0 || j==0)
            {
                k[i][j]=0;
            }
            else if (j<w[i])
            {
                k[i][j]=k[i-1][j];
            }
```

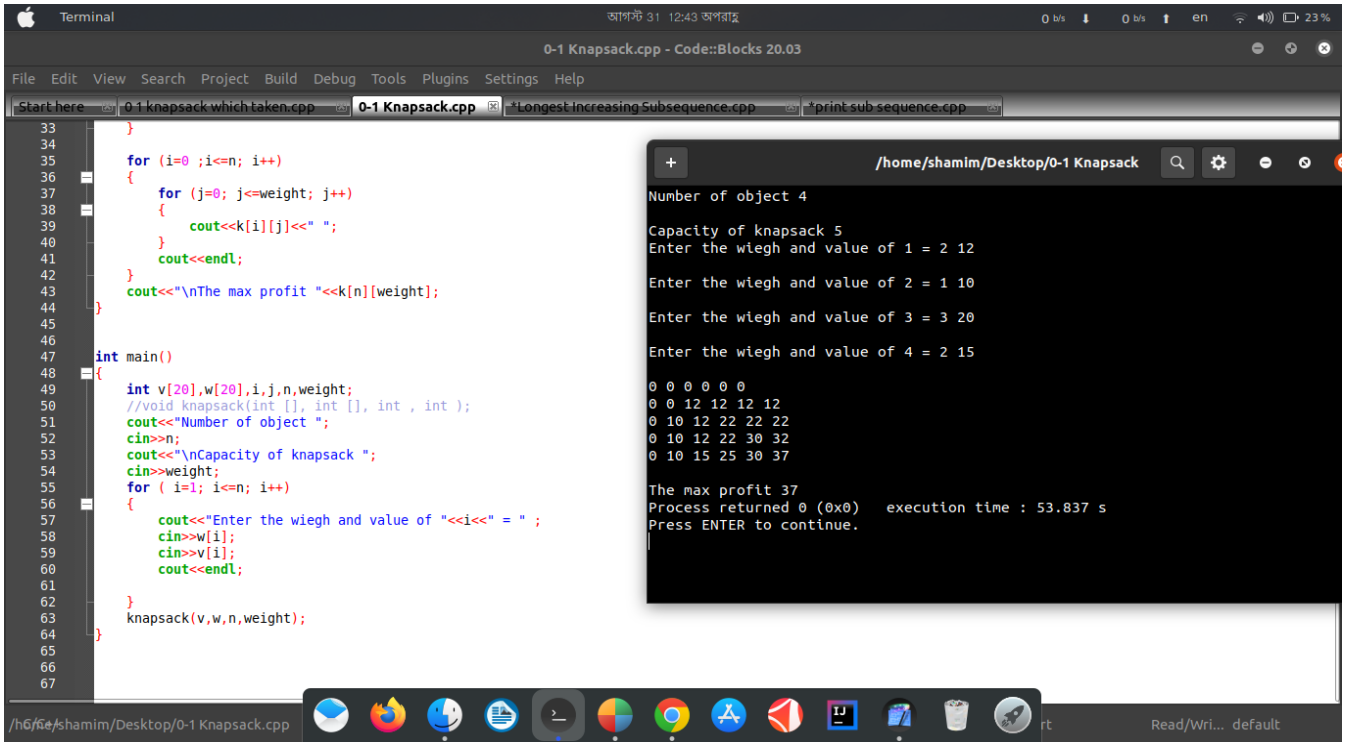
```
        else
        {
            if (k[i-1][j]>k[i-1][j-w[i]]+v[i])
            {
                k[i][j]=k[i-1][j];
            }
            else
            {
                k[i][j]=k[i-1][j-w[i]]+v[i];
            }

        }
    }
}

for (i=0 ;i<=n; i++)
{
    for (j=0; j<=weight; j++)
    {
```

```
            cout<<k[i][j]<<" ";
        }
        cout<<endl;
    }
    cout<<"\nThe max profit "<<k[n][weight];
}

int main()
{
    int v[20],w[20],i,j,n,weight;
    //void knapsack(int [], int [], int , int );
    cout<<"Number of object ";
    cin>>n;
    cout<<"\nCapacity of knapsack ";
    cin>>weight;
    for ( i=1; i<=n; i++)
    {
        cout<<"Enter the wiegh and value of "<<i<<" = " ;
```

```
        cin>>w[i];
        cin>>v[i];
        cout<<endl;


    }
    knapsack(v,w,n,weight);
}
```

Sample Input/Output (Compilation, Debugging & Testing)

```
Output:
Enter No. of Items
4
Enter size of Knapsack
5
Enter the values of items
12 10 20 15
Enter the weights of items
2 1 3 2
Maximum total profit = 37
```



Lab Task

Which one has taken in 0 1 knapsack

#include <iostream>

#define MAX 200

using namespace std;

int weight[] = {0, 2, 1, 3, 2};

int value[] = {0, 12, 10, 20, 15};

int knapsack_weight = 5;

int n = 4;

class KnapsackDP

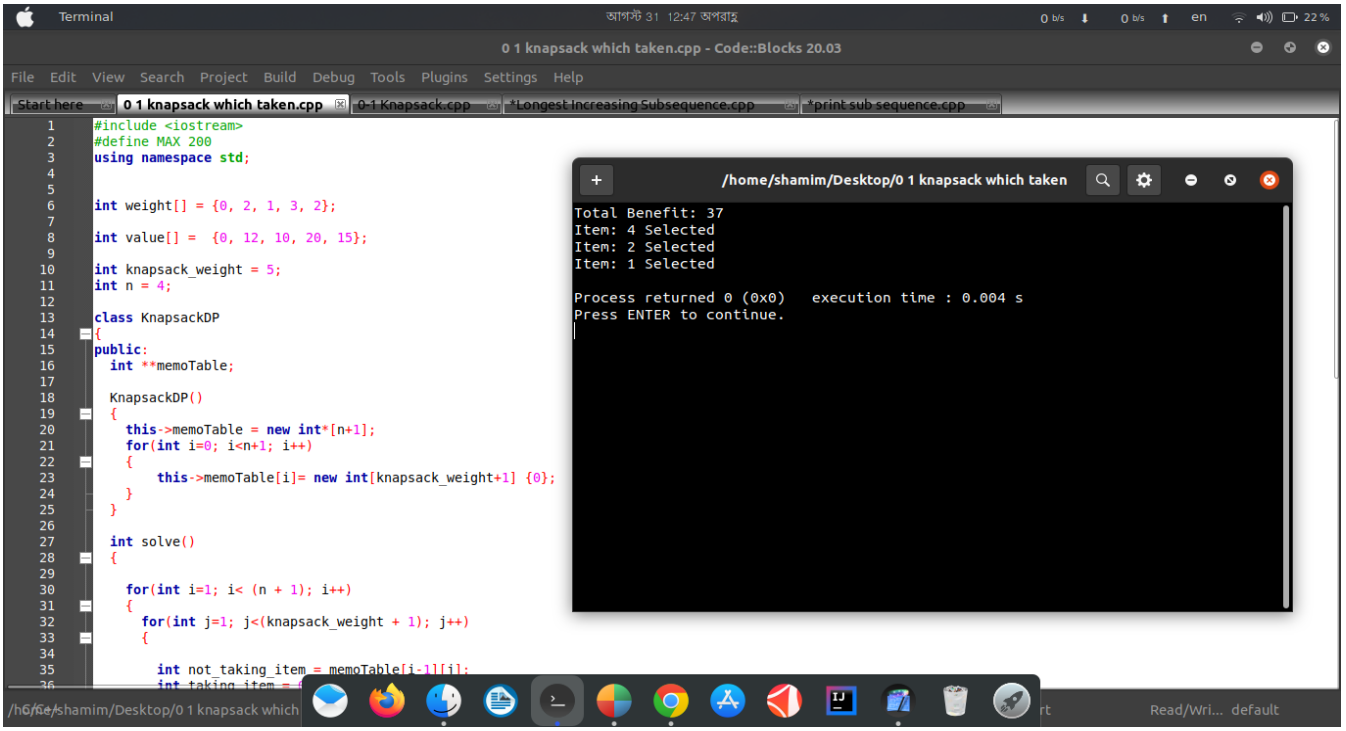
{

public:

```
int **memoTable;

KnapsackDP()
{
  this->memoTable = new int*[n+1];
  for(int i=0; i<n+1; i++)
  {
      this->memoTable[i]= new int[knapsack_weight+1] {0};
  }
}

int solve()
{

  for(int i=1; i< (n + 1); i++)
  {
    for(int j=1; j<(knapsack_weight + 1); j++)
    {

      int not_taking_item = memoTable[i-1][j];
      int taking_item = 0;
      if(weight[i] <= knapsack_weight)
      {
        if(j-weight[i] < 0)
            taking_item = memoTable[i-1][j];
        else
            taking_item = value[i] + memoTable[i-1][j-weight[i]];
      }


      memoTable[i][j] = max(not_taking_item, taking_item);
```

```
      }
    }

    return memoTable[n][knapsack_weight];
  }


  void selected_items()
  {
    for(int i=n, j= knapsack_weight; i>0; i--)
    {
      if(memoTable[i][j] != memoTable[i-1][j])
      {
        cout << "Item: "<<i <<" Selected \n";
        j = j- weight[i];
      }
    }
  }
};

int main()
{
  KnapsackDP kdp;
  cout << "Total Benefit: "<< kdp.solve() <<endl;

  //To print which items are included
  kdp.selected_items();
  return 0;
}
```

Output


Terminal
0 1 knapsack which taken.cpp - Code::Blocks 20.03
File Edit View Search Project Build Debug Tools Plugins Settings Help
Start here
0 1 knapsack which taken.cpp
0-1 Knapsack.cpp
*Longest increasing Subsequence.cpp
*print sub sequence.cpp
#include <iostream>
#define MAX 200
using namespace std;
int weight[] = {0, 2, 1, 3, 2};
int value[] = {0, 12, 10, 20, 15};
int knapsack_weight = 5;
int n = 4;
class KnapsackDP
{
public:
int **memoTable;
KnapsackDP()
{
this->memoTable = new int*[n+1];
for(int i=0; i<n+1; i++)
{
this->memoTable[i]= new int[knapsack_weight+1] {0};
}
}
int solve()
{
for(int i=1; i< (n + 1); i++)
{
for(int j=1; j<(knapsack_weight + 1); j++)
{
int not_taking_item = memoTable[i-1][j];
/home/shamim/Desktop/0 1 knapsack which taken
Total Benefit: 37
Item: 4 Selected
Item: 2 Selected
Item: 1 Selected
Process returned 0 (0x0) execution time : 0.004 s
Press ENTER to continue.