Multiscale skewed heavy tailed model for texture analysis

Nour-Eddine Lasmar, Youssef Stitou, Yannick Berthoumieu

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Nour-Eddine LASMAR, Youssef STITOU, Yannick BERTHOUMIEU

IMS- Groupe Signal - UMR 5218 CNRS, ENSEIRB – Université de Bordeaux, France
Emails : {nour-eddine.lasmar, youssef.stitou, yannick.berthoumieu}@ims-bordeaux.fr

ABSTRACT

This paper deals with texture analysis based on multiscale stochastic modeling. In contrast to common approaches using symmetric marginal probability density functions of subband coefficients, experimental manipulations show that the symmetric shape assumption is violated for several texture classes. From this fact, we propose in this paper to exploit this shape property to improve texture characterization. We present Asymmetric Generalized Gaussian density as a model to represent detail subbands resulting from multiscale decomposition. A fast estimation method is presented and closed-form of Kullback-Leibler divergence is provided in order to validate the model into a retrieval scheme. The experimental results indicate that this model achieves higher recognition rates than the conventional approach of using the Generalized Gaussian model where asymmetry was not considered.

Index Terms— Image texture analysis, Asymmetric Generalized Gaussian density, Kullback-Leibler Divergence, Dual-Tree Complex Wavelet Transform, Texture Retrieval

1. INTRODUCTION

The accurate characterization of texture is fundamental in various image processing applications, ranging from retrieval in large image databases to segmentation and texture synthesis.

Over the last decade, numerous works have been proposed for texture modeling using multiscale image representations [1] [2] [3]. The conventional scheme of multiscale texture analysis consists of extracting features from subband coefficients and uses these features as a signature for a specific texture class. Portilla and Simoncelli presented a statistical model based on joint statistics of steerable pyramid coefficients [2]. In their work, efficient algorithm of texture synthesis was developed giving increased synthesis quality. However this model is not tractable for classification applications due to the largeness of the signature. Some others conventional approaches consist in representing detail subband coefficients by their probability density functions (PDFs). In [1] [3] Generalized Gaussian (GG) density was used as a parametric model for subband marginal PDFs where orthogonal wavelets are employed as a multiscale decomposition. In a related recent work [4], Weibull distribution was proposed to model the detail subbands magnitudes employing Dual-Tree Complex Wavelet Transform (DT-CWT).

The GG distribution was introduced to model the detail subband coefficients because of the almost heavy tailed aspect of the empirical PDFs of these coefficients. It is adequate for Super Gaussian (leptokurtic), Gaussian (mesokurtic) and sub Gaussian (platykurtic) behavior of the marginal distribution of subband coefficients. This tail aspect is directly linked to the Kurtosis which shouldn't be the only statistic taking into account. However, Skewness is also important for modeling textures [2] and GG density presents limitations for fitting asymmetric subband distribution because its Skewness is null by definition. In order to take into account simultaneously the two statistics (Kurtosis and Skewness) we present a new model using a skewed heavy tailed distribution in the framework of texture analysis: the Asymmetric Generalized Gaussian density (AGG) [5], [6], [7].

In this paper, we work with the Dual-Tree Complex Wavelet Transform (DT-CWT) proposed by Kingsbury [8]. However others multiscale decompositions can be used (e.g. steerable pyramid [2], orthogonal wavelets [1] [3]).

The organization of this paper is as follows: we review in section 2 the AGG density model and introduce a new fast deterministic method to estimate the marginal distribution parameters. In section 3 the Kullback-Leibler divergence between AGG densities is derived. Finally, experimental results indicate the significant improvement in retrieval rates using the new model.

2. ASSYMETRY MODELING

PDFs proposed in previous works rely on symmetric assumption for the wavelet coefficient histogram [1] [3]. In our knowledge, no parametric models explicitly incorporate possible non null texture Skewness. Nevertheless, Van de Wouwer et al [9] defined a multiscale asymmetry signature consisting in an extra texture descriptor defined by:

$$A_{ni} = \int_{0}^{+\infty} \left| h_{ni}(u) - h_{ni}(-u) \right| u \, du \tag{1}$$

where h_{ni} is the histogram of a subband coefficients in scale $n = 1,2\ldots Nsc$ and orientation $i = 1,2\ldots Nor$.

Experimental results [9] confirmed that adding asymmetry signatures resulted in an improved texture characterization. However, this non-parametric model has some limitations; it is bins-dependant and signature estimation uses histogram computing and integral approximation which need more computation.

Overall, Skewness is a universal measure of the asymmetry of probability distribution. We present a model that explicitly incorporates possible texture asymmetry. The AGG model is a generalization of the GG one and can track not only the forth high order statistic (HOS) but also the third HOS: the Skewenss.

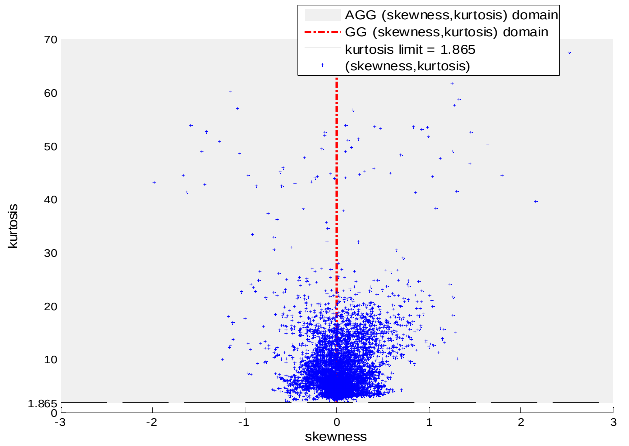


Fig.1. Scatterplot of subbands' (Skewness,Kurtosis) couple

Figure 1 shows how AGG density is more suitable than GG density to respect joint (Skewness,Kurtosis) information. We represent a scatterplot of the two statistics calculated from 1-level DT-CWT coefficients of a textured images set. The GG domain is represented by the vertical dashed straight line. When GG can only fit zero-Skewness subbands, AGG have by definition a huger domain including the GG one and can de facto better model the skewed subbands.

2.1 Skewed GG density: the AGG

The PDF of the AGG density with zero mode is given by [5] [6]:

$$f(x;\alpha,\beta_l,\beta_r) = \begin{cases} \dfrac{\alpha}{(\beta_l+\beta_r)\Gamma(1/\alpha)} \exp - \left(\dfrac{-x}{\beta_l}\right)^{\alpha} & x \prec 0 \\ \dfrac{\alpha}{(\beta_l+\beta_r)\Gamma(1/\alpha)} \exp - \left(\dfrac{x}{\beta_r}\right)^{\alpha} & x \geq 0 \end{cases} \qquad (2)$$

where $\Gamma(x) = \int_0^{+\infty} t^{x-1} e^{-t} dt$ is gamma function; $\alpha \succ 0$ is the shape parameter and $\beta_l \succ 0, \beta_r \succ 0$ are left-scale and right-scale parameters respectively. AGG density extends GG when $\beta_l = \beta_r = \beta$.

The Skewness of this distribution can take any real value (negative, null or positive):

$$s = (\beta_r^4 - \beta_l^4)\Gamma(4/\alpha)\sqrt{\frac{(\beta_r+\beta_l)\Gamma(1/\alpha)}{(\beta_r^3+\beta_l^3)^3\,\Gamma(3/\alpha)^3}} \qquad (3)$$

The AGG distribution was introduced by Tesei and Regazzoni [5] in order to optimize signal detection in non-Gaussian environments. It is able to describe many kinds of wavelet coefficients; symmetric, asymmetric, with variable sharpness of marginal PDFs (Figure 2).

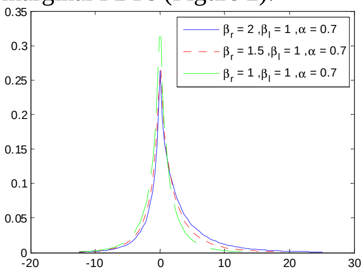


Fig.2. PDFs differentiated by Skewness and with same shape parameter

2.2 Second Order Statistics (SOS) estimation

To estimate AGG density parameters Maximum Likelihood (ML) method was proposed [7]. It uses several steps to find the best value of the shape parameter α. This method provides a better convergence rate for reaching parameter value but does not guarantee convergence for bad initial value. So, ML estimation is inherently complex and needs a lot of computations. We now develop a simple method that allows us to find model parameters given the one-order absolute moment and the second-order moment: SOS estimation. We note that the proposed estimate can be used to initialize the ML approach.

We first calculate the k-order moments μ_k and k-order absolute moments m_k in the case of AGG density with parameters (α,β_l,β_r); $\mu_k = E(X^k)$ and $m_k = E(|X|^k)$.

By using the integral calculation [10]:

$$\int_0^{+\infty} x^{\nu-1} e^{-\mu x^{\gamma}} = \frac{1}{\gamma}\mu^{-\nu/\gamma}\Gamma\left(\nu/\gamma\right) \qquad (4)$$

we establish that

$$\mu_k = \frac{\beta_r^{k+1} + (-1)^k \beta_l^{k+1}}{\beta_r+\beta_l} \times \frac{\Gamma((k+1)/\alpha)}{\Gamma(1/\alpha)} \qquad m_k = \frac{\beta_r^{k+1} + \beta_l^{k+1}}{\beta_r+\beta_l} \times \frac{\Gamma((k+1)/\alpha)}{\Gamma(1/\alpha)} \qquad \text{for } k = 1,2\ldots$$

We use the second order statistic μ_2 with absolute moment m_1 to calculate the ratio $r = \dfrac{m_1^2}{\mu_2}$. We find:

$$r = \frac{(\gamma^2+1)^2}{(\gamma^3+1)(\gamma+1)} \times \rho(\alpha) \qquad (5)$$

where $\gamma = \dfrac{\beta_l}{\beta_r}$ and $\rho(\alpha) = \dfrac{\Gamma(2/\alpha)^2}{\Gamma(1/\alpha)\Gamma(3/\alpha)}$ is the generalized Gaussian ratio function.

The value of γ can be estimated by [6][7]:

$$\hat{\gamma} = \frac{\sqrt{\frac{1}{N_l - 1}\sum_{k=1, x_k \prec 0}^{N_l} x_k^{\ 2}}}{\sqrt{\frac{1}{N_r - 1}\sum_{k=1, x_k \geq 0}^{N_r} x_k^{\ 2}}} \qquad (6)$$

where N_l (N_r) is the number of x_k samples $\prec 0$ (≥ 0).
An unbiased estimate of r is:

$$\hat{r} = \frac{\left(\sum |x_k|\right)^2}{\sum x_k^2} \qquad (7)$$

A very good approximation of $\rho(\alpha)$ and its inverse was proposed in [11], that leads to a fast and deterministic shape parameter estimation. So, we propose to estimate AGG parameters using the following algorithms:

1-we calculate $\hat{R}$ using $\hat{\gamma}$ and $\hat{r}$ estimation above, (6) and (7):

$$\hat{R} = \hat{r}\frac{\left(\hat{\gamma}^3 + 1\right)\left(\hat{\gamma} + 1\right)}{\left(\hat{\gamma}^2 + 1\right)^2} \qquad (8)$$

2-According to $\hat{R}$ value we estimate α using the approximation of the inverse generalized Gaussian ratio[11]:

$$\hat{\alpha} = \hat{\rho}^{-1}\left(\hat{R}\right) \qquad (9)$$

3-finally we estimate left and right scale parameters:

$$\hat{\beta}_l = \sqrt{\frac{1}{N_l - 1}\sum_{k=1, x_k \prec 0}^{N_l} x_k^{\ 2}} \times \sqrt{\frac{\Gamma\left(3/\hat{\alpha}\right)}{\Gamma\left(1/\hat{\alpha}\right)}}$$
$$\hat{\beta}_r = \sqrt{\frac{1}{N_r - 1}\sum_{k=1, x_k \geq 0}^{N_r} x_k^{\ 2}} \times \sqrt{\frac{\Gamma\left(3/\hat{\alpha}\right)}{\Gamma\left(1/\hat{\alpha}\right)}} \qquad (10)$$

3. SIMILARITY MEASURMENTS BETWEEN AGG DISTRIBUTIONS

The conventional scheme of multiscale texture analysis consist in extracting features from subband coefficients and use them as a signature associated with an appropriate distance to distinguish several texture classes. Even that several probabilistic distances can be used, Do and Vetterli justified the use of Kullback-Leibler divergence (KLD) to have relevant results in retrieval application [3].

Considering two AGG PDFs, $agg_1(x;\alpha_1,\beta_1^l;\beta_1^r)$ and $agg_2(x;\alpha_2,\beta_2^l;\beta_2^r)$, KLD is defined by:

$$KLD(agg_1, agg_2) = \int agg_1(x;\alpha_1,\beta_1^l,\beta_1^r)\ln\left(\frac{agg_1(x;\alpha_1,\beta_1^l,\beta_1^r)}{agg_2(x;\alpha_2,\beta_2^l,\beta_2^r)}\right)$$

We calculate a closed-form expression of KLD using the integral expression (4), we find:

$$KLD(agg_1, agg_2) = \ln\left(\frac{\alpha_1(\beta_2^l + \beta_2^r)\Gamma\left(\frac{1}{\alpha_2}\right)}{\alpha_2(\beta_1^l + \beta_1^r)\Gamma\left(\frac{1}{\alpha_1}\right)}\right) + \frac{\beta_2^{r\alpha_2}\beta_1^{l\alpha_2+1} + \beta_2^{l\alpha_2}\beta_1^{r\alpha_2+1}}{\beta_2^{l\alpha_2}\beta_2^{r\alpha_2}(\beta_1^l + \beta_1^r)} \times \frac{\Gamma\left(\frac{\alpha_2+1}{\alpha_1}\right)}{\Gamma\left(\frac{1}{\alpha_1}\right)} - \frac{1}{\alpha_1}$$

4. EXPERIMENTAL RESULTS

The experiments give an evaluation of the proposed model in the framework of texture retrieval. We use almost the same experimental setup presented in [3] [4]. We work with 40 texture classes from Brodatz album [12] (Figure 3). From each of these texture images of size 640x640 pixels, 16 subimages of 160x160 are created. A test database of 640 texture images is thus obtained. A query image is any one of these images in the database. The relevant images for each query are the other 15 images obtained from the same original 640x640 image. We must note that this dataset was not selected in any way to contain textures that showed specific asymmetry. Statistical analysis of all computed asymmetry parameter mentioned above (1) showed that the 90th quantile is equal to 9.68 10^{-5}.

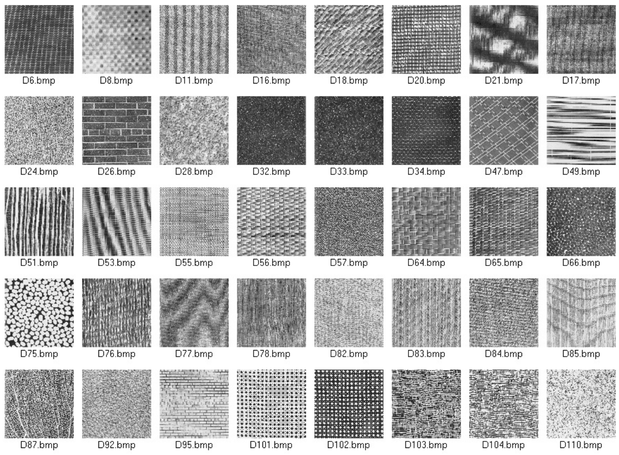
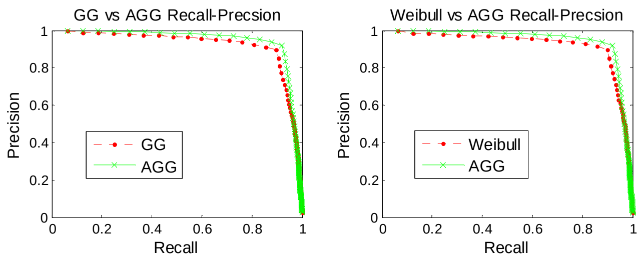


Fig.3. Selection of images from Bordatz album: from top to bottom and left to right: D6, D8, D11, D16, D17, D18, D20, D21, D24, D26, D28, D32, D33, D34, D47, D49, D51, D53, D55, D56, D57, D64, D65, D66, D75, D76, D77, D78, D82, D83, D84, D85, D87, D92, D95, D101, D102, D103, D104, D110.

The Kingsbury's DT-CWT is employed to have a multiscale image representation with six complex detail subbands at each level. Advantages of the DT-CWT can be resumed in its nearly shift invariance and directional selectivity. We use Kingsbury's Q-Shift (14,14)tap filters in combination with (13,19)-tap near-orthogonal filters [13].



(a) (b)

Fig.4. Recall-Precision curves showing the impact improvement of skewed model, with DT-CWT and 1 scale

First, we tested the impact of using a skewed model by comparing the performance of retrieving. The conventional criterion on recall/precision shows the improvement obtained with the skewed model AGG (Figure 4 (a)).

Second, we compare the performances of the proposed skewed model with those obtained using Weibull distribution to model DT-CWT coefficients magnitude [4]. As we can see in Figure 4 (b), AGG model outperforms Weibull model. But we must note that Weibull model has three times less parameters to be estimated.

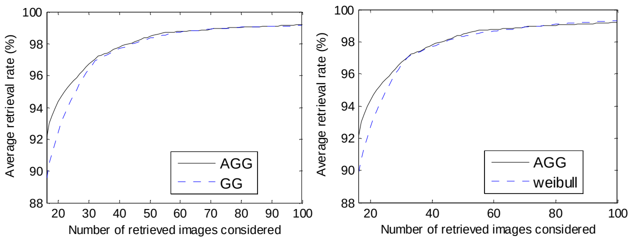


Fig.5. models convergence comparison

It can be seen from the Figure 5 that, compared to the two others models, the AGG converge faster. For example, we retrieve 94% of the relevant images with 20 retrieved images when we require 24 ones to have the same percentage if we employ the two others models (GG and Weibull).

	Retrieval Rate		
Number of scales	AGG	GG	Weibull
1	**87.9980**	87.2656	87.4023
2	**93.7988**	91.8945	92.4316
3	**98.3301**	94.9805	94.5898

Table 1: Average retrieval rate (%) comparison

A second series of experiments was conducted using orthogonal wavelets with Daubechies' D4 filters. The results of average retrieval rates are summarized in Table 1. As expected, the use of AGG model improves the performance of retrieval and in consequence texture characterization.

5. CONCLUSION

In this work we have addressed the issue of modeling textures in multiscale scheme. The probability density functions (PDFs) of detail subband coefficients are skewed and heavy tailed. So, Asymmetric Generalized Gaussian model is proposed in order to take into account third and fourth Higher Order Statistics. The model is validated in the retrieval system and achieves better recognition rates compared to GG-based approach. Future work includes the AGG parameter estimation by Negentropy Matching and the model extension to multivariate joint PDFs.

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