Presentation Template

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Problem Statement

Show that the point $\begin{pmatrix} x \\ y \end{pmatrix}$ given by $x = \frac{2at}{1+t^2}$ and $y = \frac{a(1-t^2)}{1+t^2}$ lies on a circle for all real values of t such that $-1 \leq t \leq 1$, where a is any given real number.

Linear Equation

Let the balanced version of (2.1) be

$$x_1 Fe + x_2 H_2 O \rightarrow x_3 Fe_3 O_4 + x_4 H_2 \tag{3.1}$$

which results in the following equations

$$\begin{aligned} (x_1 - 3x_3)\, Fe &= 0 \\ (2x_2 - 2x_4)\, H &= 0 \\ (x_2 - 4x_3)\, H &= 0 \end{aligned} \tag{3.2}$$

Matrix Equation

The linear equations can be expxressed as

$$
\begin{aligned}
x_1 + 0.x_2 - 3x_3 + 0.x_4 &= 0 \\
0.x_1 + 2x_2 + 0.x_3 - 2x_4 &= 0 \\
0.x_1 + x_2 - 4x_3 + 0.x_4 &= 0
\end{aligned}
\tag{3.3}
$$

resulting in the matrix equation

$$
\begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 2 & 0 & -2 \\ 0 & 1 & -4 & 0 \end{pmatrix} \mathbf{x} = \mathbf{0} \tag{3.4}
$$

where

$$
\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \tag{3.5}
$$

Row Reduction

(3.4) can be row reduced as follows

$$\begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 2 & 0 & -2 \\ 0 & 1 & -4 & 0 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow \frac{R_2}{2}} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -4 & 0 \end{pmatrix} \tag{3.6}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - R_2} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -4 & 1 \end{pmatrix} \tag{3.7}$$

$$\xleftrightarrow{R_1 \leftarrow 4R_1 - 3R_3} \begin{pmatrix} 4 & 0 & 0 & -3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -4 & 1 \end{pmatrix} \tag{3.8}$$

$$\xleftrightarrow[R_3 \leftarrow -\frac{1}{4} R_3]{R_1 \leftarrow \frac{1}{4}} \begin{pmatrix} 1 & 0 & 0 & -\frac{3}{4} \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -\frac{1}{4} \end{pmatrix} \tag{3.9}$$

Balanced Equation

Thus,

$$x_1 = \frac{3}{4}x_4, x_2 = x_4, x_3 = \frac{1}{4}x_4 \tag{3.10}$$

$$\implies \mathbf{x} = x_4 \begin{pmatrix} \frac{3}{4} \\ 1 \\ \frac{1}{4} \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 1 \\ 4 \end{pmatrix} \tag{3.11}$$

upon substituting $x_4 = 4$. (3.1) then becomes

$$3Fe + 4H_2O \rightarrow Fe_3O_4 + 4H_2 \tag{3.12}$$

The code in

https://github.com/gadepall/school/blob/master/training/chemistry/codes/chembal.py

verifies (3.10).