assignment3:7.3.4

AI24BTECH11007 - Sri Sathwik Desaboina

Question:
Show that the point $\begin{pmatrix} x \\ y \end{pmatrix}$ given by $x = \frac{2at}{1+t^2}$ and $y = \frac{a(1-t^2)}{1+t^2}$ lies on a circle for all real values of t such that $-1 \le t \le 1$, where a is any given real number.
Solution:

S.No	variables used	description
1	t	a variable which takes the real values in the range $(-1, 1)$
2	a	it is a fixed real number
3	$\mathbf{A(t)}$	it is a transformation matrix of parameter t
4	$\mathbf{v(t)}$	it represent the parameter t and allows to define x and y
5	$\mathbf{p(t)}$	a point with coordinates x and y.

TABLE 0
INPUT PARAMETERS

$$\text{Given } x = \frac{2at}{1+t^2}, \quad y = \frac{a(1-t^2)}{1+t^2},$$

$$\text{let } \mathbf{p}(t) = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{2at}{1+t^2} \\ \frac{a(1-t^2)}{1+t^2} \end{pmatrix}.$$

$$\text{Expressing } \mathbf{p}(t) \text{ in matrix form:}$$

$$\implies \mathbf{A}(t) = \begin{pmatrix} \frac{2a}{1+t^2} & 0 \\ 0 & \frac{a(1-t^2)}{1+t^2} \end{pmatrix},$$

$$\implies \mathbf{v}(t) = \begin{pmatrix} t \\ t^2 \end{pmatrix},$$

$$\implies \mathbf{p}(t) = \begin{pmatrix} \frac{2a}{1+t^2} & 0 \\ \frac{a(1-t^2)}{1+t^2} & 0 \end{pmatrix} \begin{pmatrix} t \\ 1 \end{pmatrix}.$$

$$\text{Then } x^2 + y^2 = a^2.$$

$$\text{Thus, } \mathbf{p}(t)^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{p}(t) = a^2.$$

$\therefore$ It is proved that the locus of all these points forms a circle.
Plot:

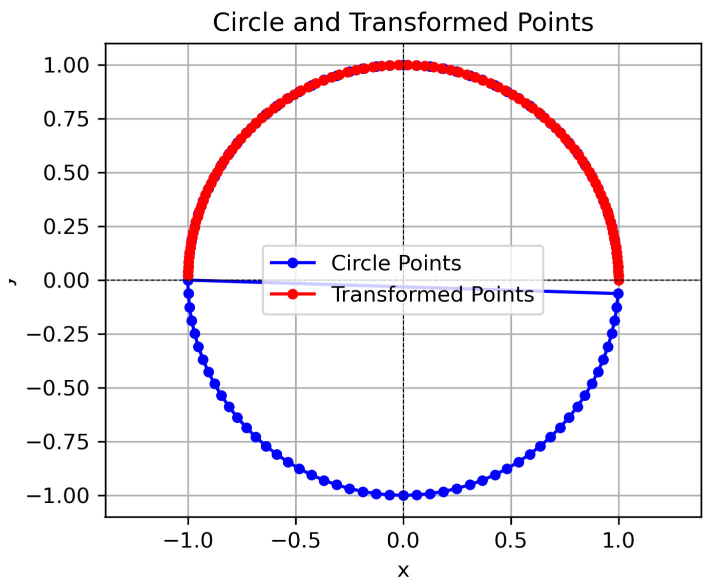

Fig. 0. Circle with center $\mathbf{O}\begin{pmatrix}0\\0\end{pmatrix}$ and with radius $a = 1$ units.