Presentation Template

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Problem Statement

Show that the point $\begin{pmatrix} x \\ y \end{pmatrix}$ given by $x = \frac{2at}{1+t^2}$ and $y = \frac{a(1-t^2)}{1+t^2}$ lies on a circle for all real values of t such that $-1 \leq t \leq 1$, where a is any given real number.

Usage of variables

S.No	variables used	description
1	t	a variable which takes the real values in the range $(-1, 1)$
2	a	it is a fixed real number
3	$\mathbf{A}(\mathbf{t})$	it is a transformation matrix of parameter t
4	$\mathbf{v}(\mathbf{t})$	it represent the parameter t and allows to define x and y
5	$\mathbf{p}(\mathbf{t})$	a point with coordinates x and y.

Parametric form

Given x and y in the parametric form,

$$x = \frac{2at}{1+t^2}, \tag{3.1}$$

$$y = \frac{a(1-t^2)}{1+t^2} \tag{3.2}$$

Let $\mathbf{p}(\mathbf{t})$ be equal to,

$$\mathbf{p}(t) = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{2at}{1+t^2} \\ \frac{a(1-t^2)}{1+t^2} \end{pmatrix}. \tag{3.3}$$

Matrix equation

The transformation matrix $\mathbf{A}(\mathbf{t})$ and $\mathbf{v}(\mathbf{t})$ with parameter t are,

$$\implies \mathbf{A}(t) = \begin{pmatrix} \frac{2a}{1+t^2} & 0 \\ 0 & \frac{a(1-t^2)}{1+t^2} \end{pmatrix}, \tag{3.4}$$

$$\implies \mathbf{v}(\mathbf{t}) = \begin{pmatrix} t \\ 1 \end{pmatrix}, \tag{3.5}$$

$$\mathbf{p}(\mathbf{t}) = \mathbf{A}(\mathbf{t})\mathbf{v}(\mathbf{t}), \tag{3.6}$$

$$\implies \mathbf{p}(t) = \begin{pmatrix} \frac{2a}{1+t^2} & 0 \\ 0 & \frac{a(1-t^2)}{1+t^2} \end{pmatrix} \begin{pmatrix} t \\ 1 \end{pmatrix}, \tag{3.7}$$

$$\tag{3.8}$$

Verification

Now, if we check the value of ,

$$\mathbf{p}(t)\top \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{p}(t) \tag{3.9}$$

We get,

$$\mathbf{p}(t)\top \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{p}(t) = a^2 \tag{3.10}$$

$\implies$ We proved that the given points lie on a circle $x^2 + y^2 = a^2$.
Since we have the values of t in $(-1, 1)$, the y-coordinate of the points is always positive.
We get a semi-circle with those points.
The codes for verification:

https://github.com/DESABOINASRISATHWIK/EE1030/blob/main/presentation/codes/plot.py
https://github.com/DESABOINASRISATHWIK/EE1030/blob/main/presentation/codes/code.c

Plot

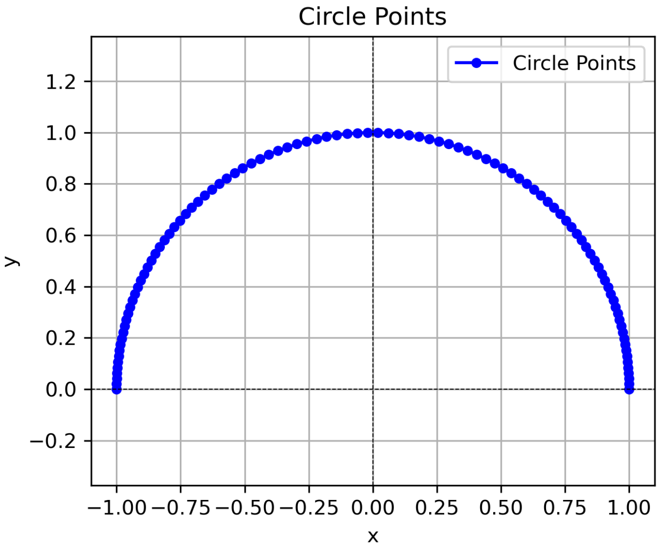

Figure: Circle Points