Assignment 4

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JEE MAINS-2021(session-2: 5th shift)

Section-A

1) If the functions are defined as $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$, then what is the common domain of the following functions: $f+g$, $f-g$, f/g?

(Mar-2021)

a) $0 < x \le 1$ b) $0 \le x < 1$ c) $0 \le x \le 1$ d) $0 < x < 1$

2) Let α, β, γ be the roots of the equation $x^3 + ax^2 + bx + c = 0$ (where $a, b, c \in \mathbb{R}$ and $a \neq 0, b \neq 0$). The system of equations in u, v, w given by $\alpha u + \beta v + \gamma w = 0$, $\beta u + \gamma v + \alpha w = 0$, $\gamma u + \alpha v + \beta w = 0$ has non-trivial solutions. Then the value of $\frac{a^2}{b}$ is:

(Mar-2021)

a) 5 b) 1 c) 0 d) 3

3) If the equation $a|Z|^2 + \bar{\alpha}Z + \alpha\bar{Z} + d = 0$ represents a circle where a,d are real constants, then which of the following condition is correct?

(Mar-2021)

a) $|\alpha|^2 - ad \neq 0$
b) $|\alpha|^2 - ad > 0 \quad$ and $\quad a \in \mathbb{R} \setminus \{0\}$
c) $\alpha = 0, \quad a, d \in \mathbb{R}^+$
d) $|\alpha|^2 - ad \geq 0 \quad$ and $\quad a \in \mathbb{R}$

4) $\frac{1}{32-1} + \frac{1}{52-1} + \frac{1}{72-1} + \ldots + \frac{1}{2012-1}$ is equal to:

(Mar-2021)

a) $\frac{101}{404}$ b) $\frac{101}{408}$ c) $\frac{99}{400}$ d) $\frac{25}{101}$

5) The number of integral values of m such that the abscissa of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is:

(Mar-2021)

a) 3 b) 2 c) 1 d) 0

6) The solutions of the equation $\det\begin{bmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin(2x) & 4\sin(2x) & 1+4\sin(2x) \end{bmatrix} = 0$, $(0 < x < \pi)$, are:

(Mar-2021)

a) $\frac{\pi}{6}, \frac{5\pi}{6}$
b) $\frac{7\pi}{12}, \frac{11\pi}{12}$
c) $\frac{5\pi}{12}, \frac{7\pi}{12}$
d) $\frac{\pi}{12}, \frac{\pi}{6}$

7) If $f(x) = \begin{cases} \frac{1}{|x|} & \text{if } x \geq 1 \\ ax^2 + b & \text{if } |x| < 1 \end{cases}$ is differentiable at every point of the domain, then the values of a and b are respectively:

(Mar-2021)

a) $\frac{5}{2}, -\frac{3}{2}$ b) $-\frac{1}{2}, \frac{3}{2}$ c) $\frac{1}{2}, \frac{1}{2}$ d) $\frac{1}{2}, -\frac{3}{2}$

8) A vector a has components $3p$ and 1 with respect to a rectangular Cartesian system. This system is rotated through a certain angle about the origin in the counterclockwise sense. If with respect to the new system, a has components $p+1$ and $\sqrt{10}$, then a value of p is equal to:

(Mar-2021)

a) 1 b) -1 c) $\frac{4}{5}$ d) $-\frac{5}{4}$

9) The sum of all the 4-digit distinct numbers that can be formed with the digits 1,2,2 and 3 is:

(Mar-2021)

a) 26664
b) 122664
c) 122234
d) 22264

10) Choose the correct statement about two circles whose equations are given below:
$x^2 + y^2 - 10x - 10y + 41 = 0$
$x^2 + y^2 - 22x - 10y + 137 = 0$

(Mar-2021)

a) circles have no meeting point
b) circles have two meeting points
c) circles have only one meeting point
d) circles have the same centre

11) If α, β are natural numbers such that $100^{\alpha} - 199\beta = (100)(100) + (99)(101) + (98)(102) + \ldots + (1)(199)$, then the slope of the line passing through (α, β) and the origin is:

(Mar-2021)

a) 510 b) 550 c) 540 d) 530

12) The value of
$3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{3 + \ldots \infty}}}}$
is equal to:

(Mar-2021)

a) $3 + 2\sqrt{3}$
b) $4 + \sqrt{3}$
c) $2 + \sqrt{3}$
d) $1.5 + \sqrt{3}$

13) The integral $\int \frac{(2x-1)\cos\left(\sqrt{(2x-1)^2+5}\right)}{\sqrt{4x^2-4x+6}}\,dx$ is equal to (where c is a constant of integration):

(Mar-2021)

a) $\frac{1}{2}\sin\left(\sqrt{(2x+1)^2+5}\right) + c$
b) $\frac{1}{2}\sin\left(\sqrt{(2x-1)^2+5}\right) + c$
c) $\frac{1}{2}\cos\left(\sqrt{(2x+1)^2+5}\right) + c$
d) $\frac{1}{2}\cos\left(\sqrt{(2x-1)^2+5}\right) + c$

14) The differential equations satisfied by the system of parabolas $y^2 = 4a(x+a)$ is:

(Mar-2021)

a) $y\frac{dy}{dx} + 2x\frac{dy}{dx} - y = 0$
b) $y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y = 0$
c) $y\left(\frac{dy}{dx}\right)^2 - 2x\frac{dy}{dx} - y = 0$
d) $y\left(\frac{dy}{dx}\right)^2 - 2x\frac{dy}{dx} + y = 0$

15) The real-valued function $f(x) = \frac{\text{cosec}^{-1}(x)}{\sqrt{x-[x]}}$ where $[x]$ denotes the greatest integer less than or equal to x, is defined for all x belonging to:

(Mar-2021)

a) all non-integers except the interval $[-1, 1]$
b) all integers except 0, -1, 1
c) all reals except integers
d) all reals except the interval $[-1, 1]$