Assignment 1

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MCQs with One Correct Answer

1) The value of the integral $\int_{-\pi/2}^{\pi/2}(x^2 + \ln\frac{\pi+x}{\pi-x})\cos x dx$ is

(2012)

a) 0 b) $\frac{\pi^2}{2} - 4$ c) $\frac{\pi^2}{2} + 4$ d) $\frac{\pi^2}{2}$

2) The area enclosed by the curves $y = \sin x + \cos x$ and $y = |\cos x - \sin x|$ over the interval $[0, \pi/2]$ is

(JEE Adv.2013)

a) $4(\sqrt{2} - 1)$
b) $2\sqrt{2}(\sqrt{2} - 1)$
c) $2(\sqrt{2} + 1)$
d) $2\sqrt{2}(\sqrt{2} + 1)$

3) Let $f : [\frac{1}{2}, 1] \to R$ (the set of all real number) be a positive, non-constant and differentiable function such that $f'(x) < 2f(x)$ and $f(\frac{1}{2}) = 1$. Then the value of $\int_{\frac{1}{2}}^{1} f(x)dx$ lies in the interval

(JEE Adv.2013)

a) $(2e - 1, 2e)$
b) $(e - 1, 2e - 1)$
c) $(\frac{e-1}{2}, e - 1)$
d) $(0, \frac{e-1}{2})$

4) The following integral $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(2\operatorname{cosec} x)^{17}dx$ is equal to

(JEE Adv.2014)

a) $\int_0^{\log(1+\sqrt{2})} 2(e^u + e^{-u})^{16}du$
b) $\int_0^{\log(1+\sqrt{2})} (e^u + e^{-u})^{17}du$
c) $\int_0^{\log(1+\sqrt{2})} (e^u - e^{-u})^{17}du$
d) $\int_0^{\log(1+\sqrt{2})} 2(e^u - e^{-u})^{16}du$

5) The value of $\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2\cos x}{1+e^x}dx$ is equal to

(JEE Adv.2016)

a) $\frac{\pi^2}{4} - 2$
b) $\frac{\pi^2}{4} + 2$
c) $\pi^2 - e^{\frac{\pi}{2}}$
d) $\pi^2 + e^{\frac{\pi}{2}}$

6) Area of the region $\{(x, y) \in R^2 : y \geq \sqrt{|x + 3|}, 5y \leq x + 9 \leq 15\}$ is equal to

(JEE Adv.2016)

a) $\frac{1}{6}$ b) $\frac{4}{3}$ c) $\frac{3}{2}$ d) $\frac{5}{3}$

7) The area of the region$\{(x, y) : xy \leq 8, 1 \leq y \leq x^2\}$ is

(JEE Adv.2018)

a) $8\log_e 2 - \frac{14}{3}$
b) $16\log_e 2 - \frac{14}{3}$
c) $8\log_e 2 - \frac{7}{3}$
d) $16\log_e 2 - 6$

MCQs with One or More than One Correct

1) If $\int_0^x f(t)dt = x + \int_x^1 tf(t)dt$, then the value of $f(1)$ is

(1998-2 Marks)

a) $\frac{1}{2}$ b) 0 c) 1 d) $\frac{-1}{2}$

2) Let $f(x) = x - [x]$,for every real number x, where [x] is the integral part of x. Then $\int_{-1}^{1} f(x)dx$ is

(1998-2 Marks)

a) 1 b) 2 c) 0 d) $\frac{1}{2}$

3) For which of the following values of m, is the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals $\frac{9}{2}$?

(1999-3 Marks)

a) -4 b) -2 c) 2 d) 4

4) Let $f(x)$ be a non-constant twice differentiable function definied on $(-\infty, \infty)$ such that $f(x) = f(1 - x)$ and $f'(\frac{1}{4}) = 0$. Then,

(2008)

a) f"(x) vanishes at least twice on $[0, 1]$
b) $f'(\frac{1}{2}) = 0$
c) $\int_{\frac{-1}{2}}^{\frac{1}{2}} f(x + \frac{1}{2})\sin xdx = 0$
d) $\int_0^{\frac{1}{2}} f(t)e^{\sin\pi t}dt = \int_{\frac{1}{2}}^{1} f(1 - t)e^{\sin\pi t}dt$

5) Area of the region bounded by the curve $y = e^x$ and lines $x = 0$ and $y = e$ is

(2009)

a) $e-1$
b) $\int_1^e \ln(e+1-y)dy$
c) $e-\int_0^1 e^x dx$
d) $\int_1^e \ln y dy$

6) If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x}dx \ n = 0, 1, 2, ...,$ then

(2009)

a) $I_n = I_{n+2}$
b) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$
c) $\sum_{m=1}^{10} I_{2m} = 0$
d) $I_n = I_{n+1}$

7) The value(s) of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$ is(are)

(2010)

a) $\frac{22}{7} - \pi$
b) $\frac{2}{105}$
c) 0
d) $\frac{71}{15} - \frac{3\pi}{2}$

8) Let f be a real-valued function defined on the interval $(0, \infty)$ by $f(x) = \ln x + \int_0^x \sqrt{1 + \sin t}dt$. Then which of the following statement(s) is(are) true?

(2010)

a) $f''(x)$ exists for all $x \in (0, \infty)$
b) $f'(x)$ exists for all $x \in (0, \infty)$ and f' is continuous on $(0, \infty)$, but not differentiable on $(0, \infty)$
c) there exists $\alpha > 1$ such that $|f'(x)| < |f(x)|$ for all $x \in (\alpha, \infty)$
d) there exists $\beta > 0$ such that $|f(x)| + |f'(x)| \leq \beta$ for all $x \in (0, \infty)$