Optimal Sizing Linear Programme Formulation

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Nomenclature

Constants

i, I Index, of I different storage assets

k, K Index, of K EV fleets

b, B Index, of B different charger types, 0) V2G, 1) Smart

g, G Index, of G different storage assets

t, T Index, of time, t hours within T length simulation

$C_{f,i}$ Fixed Cost of building asset i (£/MWh)

c_i Cost of storage asset i energy throughput (£/(MW that leaves battery)

c_g Marginal cost of energy from generator g (£/MWh).

T_{life} Lifetime of asset (hours)

$P^{\bar{fos}}$ Maximum allowable fraction of passive demand met by fossil fuels during simulation (a number between 0-1).

$\eta_{D,i}$ Discharge efficiency of storage asset i. (if 50% and D = 10MW, 5MW injection onto the grid)

$\eta_{C,i}$ Charge efficiency of storage asset i. (if 50% and C = 10MW, 20MW power from grid)

n_l Hourly leakage rate (between 0 - 1) of storage asset

$\bar{D}$ Maximum battery discharge as a fraction of built capacity (i.e. value between 0-1). This is for the grid side, so the energy into the system (thus more than this is taken out of the battery).

$\bar{C}$ Maximum battery charge as a fraction of built capacity (i.e. value between 0-1). This is grid side limit, so energy put into the battery is less.

$\bar{C_k^{ev}}$ Max Charge rate of charger fleet k (MW). (Grid side so the energy into battery will be lower).

$\bar{D_k^{ev}}$ Max Discharge rate of charger fleet k (MW). (Grid side, so energy out of battery can be higher.)

$\bar{E_k^{ev}}$ Max state of charge for EV in fleet k (MWh).

N_k^{tot} The total number of EVs in fleet k.

$C_{k,b}$ Cost of charger type b for fleet k.

$p_{g,t}$ Normalised generator g power at time t (i.e. if had one MW generator, power at time t).

d_t Passive system demand at time t (MW).

Decision Variables

(all Non Negative Real Continuous Unless Otherwise Stated)

$\boldsymbol{B_i}$ Built capacity of storage asset i (MWh)

$\boldsymbol{B_g}$ Built capacity of generation asset g (MWh)

$\boldsymbol{C_{k,b}}$ Chosen number of type b chargers for fleet k.

$\boldsymbol{C_{i,t}}$ Charge rate of asset i at time t (MW). (energy into battery)

$\boldsymbol{D_{i,t}}$ Discharge rate of asset i at time t (MW). (energy out of battery)

$\boldsymbol{P_t^{fos}}$ Power from Fossil Fuels at time t (MW)

$\boldsymbol{P_t^{shed}}$ Curtailed Renewable Power (MW).

$\boldsymbol{E_{i,t}}$ Storage asset i state of charge at the end of timestep t (MWh)

$\boldsymbol{N_{k,b}}$ Number of chargers of type b built in fleet k.

1 General Constraints

Cost Function:

$$\sum_{i}^{I}\Big(\boldsymbol{B_i}\cdot C_{f,i}\cdot\frac{T}{T_{life}}+c_i\sum_{t}^{T}\boldsymbol{D_{i,t}}\Big)+\sum_{k}^{K}\Big(\sum_{b}^{B}\big(\boldsymbol{N_{k,b}}C_{k,b}\frac{T}{T_{life}}\big)\Big)+\sum_{g}^{G}\Big(\boldsymbol{B_g}C_g\frac{T}{T_{life}}+\sum_{t}^{T}\boldsymbol{B_g}p_tc_g\Big) \tag{1}$$

Limit Total Fossil Fuel Generation, $\bar{P^{fos}}$ is usually some specified fraction of demand:

$$\sum_{t}^{T}\boldsymbol{P_t^{fos}}\leq\bar{P^{fos}}\cdot\sum_{t}^{T}d_t \tag{2}$$

Power Balance (if surplus specified):

$$(\sum_{g}^{G} \boldsymbol{B_g} \cdot p_t - \boldsymbol{P_t^{shed}}) - d_t + \boldsymbol{P_t^{fos}} + \sum_{i}^{I} \Big(\boldsymbol{D_{i,t}} \cdot \eta_{D,i} - \boldsymbol{C_{i,t}} \cdot \frac{1}{\eta_{C,i}}\Big) + \sum_{k}^{K} \Big(\boldsymbol{D_{k,t,0}} \cdot \eta_{D,k} - \sum_{b}^{B} \boldsymbol{C_{k,t,b}} \cdot \frac{1}{\eta_{C,k}}\Big) = 0 \quad \forall t \tag{3}$$

1.0.1 Storage Constraints

The following constraints apply to each storage asset, so subscript i is dropped for clarity. The SOC is dependant on charge decisions and previous charge level:

$$\boldsymbol{E_t} = \boldsymbol{E_{t-1}} \cdot (1 - n_l) + \boldsymbol{C_t} - \boldsymbol{D_t} \tag{4}$$

For conservation of energy:

$$\boldsymbol{E_0} = \boldsymbol{E_T} \tag{5}$$

State of charge (SOC) and power constraints:

$$\boldsymbol{E_t} \leq \boldsymbol{B},\ \boldsymbol{D_t} \cdot \eta_D \leq \bar{D} \cdot \boldsymbol{B},\ \boldsymbol{C_t} \cdot \frac{1}{\eta_C} \leq \bar{C} \cdot \boldsymbol{B} \tag{6}$$

Note: the $\bar{D}, \bar{C}$ in the model are defined from the grid side:

1.0.2 Aggregated EV Constraints

The following constraints apply to each fleet asset, so subscript k is dropped for clarity. Two hourly timeseries of the fraction of total EVs (N^{tot}) that plugin and plugout (N_t^{in}, N_t^{out}) at each hour must be specified for the simulation period. From these the hourly timeseries of the proportion of chargers with an EV attached to it can be deduced (N_t^{ev}).

Each fleet is divided into 2 virtual batteries, corresponding to the EVs with installed V2G or Smart chargers:

$$\boldsymbol{E_{t,b}} = \boldsymbol{E_{t-1,b}} + \boldsymbol{C_{t,b}} - \boldsymbol{D_{t,b}} - E^{out} N_t^{out} + E^{in} N_t^{in} \tag{7}$$

For conservation of energy:

$$\boldsymbol{E_{0,b}} = \boldsymbol{E_{T,b}} \tag{8}$$

SOC and power constraints:

$$\boldsymbol{E_{t,b}} \leq \boldsymbol{N_b} \cdot N_t^{ev} \cdot \bar{E^{ev}},\ \boldsymbol{C_{t,b}} \frac{1}{\eta_C} \leq \boldsymbol{N_b} \cdot N_t^{ev} \cdot \bar{C^{ev}} \tag{9}$$

The first constraint is somewhat problematic if $\bar{E^{ev}} \neq E^{out}$ as it is assumed the EVs plug out with the latter. Thus if they are at a higher SOC than this, there would be some SOC from their batteries (e.g. 10%) erroneously left in the aggregate battery. This could be solved via some algorithm that finds a new SOC plugout amount if the batteries are charged over their minimum plugout

SOC, but I have not implemented that here. Instead I have mandated that $\bar{E^{ev}} == E^{out}$.

The SOC must be kept above the minimum energy requirements, :

$$\boldsymbol{E_{t-1,b}} \geq E^{out} N_t^{out} \tag{10}$$

All EVs need a charger:

$$N^{tot} = \sum_{b}^{B} \boldsymbol{N_b} \tag{11}$$

Finally, for V2G case only:

$$\boldsymbol{D_{t,b=0}} \cdot \eta_D \leq \boldsymbol{N_{b=0}} \cdot N_t^{ev} \cdot \bar{D^{ev}} \tag{12}$$