Problem 1:

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clc; clear;

syms l A B C D C_n p x y c n pi

% General solution of wave equation
u(x,y) = (A*cos(p*x) + B*sin(p*x)) * (C*exp(p*y) + D*exp(-p*y))

% BC1: u(0,y)=0
u_0y = u(0,y);
A_sol = solve(u_0y == 0, A);
u = subs(u, A, A_sol)

% BC2: u(l,y)=0
u_ly = u(l,y)
p_sol = solve(sin(p*l)== 0,p,'returnconditions',true);
u = subs(u, p, p_sol.p);
u = subs(u, p_sol.parameters, n)

% BC3: u(x,0)=0
u_x0 = u(x, 0);
D_sol = solve(u_x0 == 0, D,'returnconditions',true);
u = simplify(subs(u, D, D_sol.D))
u=subs(u,2*B*C,C_n)

% BC4: u(x,l)=x(l-x)
u_xl=u(x,l);
f = x*(l- x);

% Fourier series coefficient
Bn= (2/l) * int(f * sin(pi*n*x/l), x, 0, l)
Bn_final=simplify(subs(Bn,[sin(pi*n), sin(pi*n/2)^2],[0, (1 - (-1)^n)/2]))
Cn_new=Bn_final/sinh(n*pi)

% Final solution
u=subs(u,C_n,Cn_new)
disp('The solution is:')
fprintf('u(x,y)= %s n=1 to %s %s \n', char(0x2211), char(8734), char(u));
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Output

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u(x, y) = (C*exp(p*y) + D*exp(-p*y))*(A*cos(p*x) + B*sin(p*x))

u(x, y) = B*sin(p*x)*(C*exp(p*y) + D*exp(-p*y))

u_ly = B*sin(l*p)*(C*exp(p*y) + D*exp(-p*y))

u(x, y) = B*sin((pi*n*x)/l)*(C*exp((pi*n*y)/l) + D*exp(-(pi*n*y)/l))

u(x, y) = 2*B*C*sin((pi*n*x)/l)*sinh((pi*n*y)/l)

u(x, y) = C_n*sin((pi*n*x)/l)*sinh((pi*n*y)/l)

Bn = -(2*((l^3*sin(n*pi))/(n^2*pi^2) - (4*l^3*sin((n*pi)/2)^2)/(n^3*pi^3)))/l

Bn_final = -(4*l^2*((-1)^n - 1))/(n^3*pi^3)

Cn_new = -(4*l^2*((-1)^n - 1))/(n^3*pi^3*sinh(n*pi))

u(x, y) = -(4*l^2*sin((pi*n*x)/l)*sinh((pi*n*y)/l)*((-1)^n -
1))/(n^3*pi^3*sinh(n*pi))
```

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The solution is:
u(x,y)= ∑ n=1 to ∞ -(4*l^2*sin((n*x*pi)/l)*sinh((n*y*pi)/l)*((-1)^n -
1))/(n^3*pi^3*sinh(n*pi))
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Problem 2:

```
clc; clear;

syms A B C D C_n p l x y c n pi k
assume([n,l], 'positive')
% General solution of wave equation
u(x,y) = (A*cos(p*x) + B*sin(p*x)) * (C*exp(p*y) + D*exp(-p*y))

% BC1: u(0,y)=0
u_0y = u(0,y);
A_sol = solve(u_0y == 0, A);
u = subs(u, A, A_sol)

% BC2: u(l,y)=0
u_ly = u(l,y);
p_sol = solve(sin(p*l)== 0,p,'returnconditions',true);
u = subs(u, p, p_sol.p);
u = subs(u, p_sol.parameters, n)

% BC3: u(x,inf)=0
u_x0 = limit(u, y, inf);
C_sol = solve(u_x0 == 0, C,'returnconditions',true);
u = simplify(subs(u, C, C_sol.C))
u=subs(u,B*D,C_n)

% BC4: u(x,0)=kx(l-x)
u_xl=u(x,0);
f=k*x*(l-x);
% Fourier series coefficient
Bn= (2/l) * int(f * sin(pi*n*x/l), x, 0, l)
Bn_final=simplify((subs(Bn,[sin(pi*n), sin(pi*n/2)^2],[0, (1 - (-1)^n)/2])))

% Final solution
u=subs(u,C_n,Bn_final)
disp('The solution is:')
fprintf('u(x,y)= %s n=1 to %s %s \n', char(0x2211), char(8734),char(u));
```

Output

```
u(x, y) = (C*exp(p*y) + D*exp(-p*y))*(A*cos(p*x) + B*sin(p*x))

u(x, y) = B*sin(p*x)*(C*exp(p*y) + D*exp(-p*y))

u(x, y) = B*sin((pi*n*x)/l)*(C*exp((pi*n*y)/l) + D*exp(-(pi*n*y)/l))

u(x, y) = B*D*exp(-(pi*n*y)/l)*sin((pi*n*x)/l)

u(x, y) = C_n*exp(-(pi*n*y)/l)*sin((pi*n*x)/l)

Bn = (2*((4*k*l^3*sin((n*pi)/2)^2)/(n^3*pi^3) - (k*l^3*sin(n*pi))/(n^2*pi^2)))/l

Bn_final = -(4*k*l^2*((-1)^n - 1))/(n^3*pi^3)

u(x, y) = -(4*k*l^2*exp(-(pi*n*y)/l)*sin((pi*n*x)/l)*((-1)^n - 1))/(n^3*pi^3)
```

```
The solution is:
u(x,y)= ∑ n=1 to ∞ -(4*k*l^2*exp(-(n*y*pi)/l)*sin((n*x*pi)/l)*((-1)^n -
1))/(n^3*pi^3)
```