Problem 1:

```
clc; clear;
syms A B C D C_n p x t l c n pi k

% General solution of wave equation
y(x,t) = (A*cos(p*x) + B*sin(p*x)) * (C*cos(p*c*t) + D*sin(p*c*t))

% BC1: y(0,t)=0
y0 = y(0,t);
A_sol = solve(y0 == 0, A);
y(x,t) = subs(y(x,t), A, A_sol)

% BC2: y(l,t)=0
yL = y(l,t);
p_sol = solve(sin(p*l)== 0,p,'returnconditions',true);
y(x,t) = subs(y(x,t), p, p_sol.p);
y(x,t) = subs(y(x,t), p_sol.parameters, n)

% BC3: y'(x,0)=0
y_t = diff(y, t);
y_t0 = y_t(x, 0);
D_sol = solve(y_t0 == 0, D,'returnconditions',true);
y(x,t) = subs(y(x,t), D, D_sol.D);
y(x,t)=subs(y(x,t),B*C,C_n)

% BC4: y(x,0)=kx(l-x)
y_x=y(x,0);
f = k*x*(l- x);

% Fourier series coefficient
Bn= (2/l) * int(f * sin(pi*n*x/l), x, 0, l)
Bn_final=simplify(subs(Bn,[sin(pi*n), sin(pi*n/2)^2],[0, (1 - (-1)^n)/2]))

% Final solution
y(x,t)=subs(y(x,t),C_n,Bn_final)
disp('The solution is:')
fprintf('y(x,t)= %s n=1 to %s %s \n', char(0x2211), char(8734), char(y(x,t)));
```

Output

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y(x, t) = (A*cos(p*x) + B*sin(p*x))*(C*cos(c*p*t) + D*sin(c*p*t))

y(x, t) = B*sin(p*x)*(C*cos(c*p*t) + D*sin(c*p*t))

y(x, t) = B*sin((pi*n*x)/l)*(C*cos((pi*c*n*t)/l) + D*sin((pi*c*n*t)/l))

y(x, t) = C_n*cos((pi*c*n*t)/l)*sin((pi*n*x)/l)


Bn =(2*((4*k*l^3*sin((n*pi)/2)^2)/(n^3*pi^3) - (k*l^3*sin(n*pi))/(n^2*pi^2)))/l

Bn_final = -(4*k*l^2*((-1)^n - 1))/(n^3*pi^3)

y(x, t) = -(4*k*l^2*cos((pi*c*n*t)/l)*sin((pi*n*x)/l)*((-1)^n - 1))/(n^3*pi^3)

The solution is
y(x,t)= ∑ n=1 to ∞ -(4*k*l^2*cos((c*n*t*pi)/l)*sin((n*x*pi)/l)*((-1)^n -
1))/(n^3*pi^3)
```

Problem 2:

```
clc; clear;
syms A B C D C_n p x t l c n pi k

% General solution of wave equation
y(x,t) = (A*cos(p*x) + B*sin(p*x)) * (C*cos(p*c*t) + D*sin(p*c*t))
% BC1: y(0,t)=0
y0 = y(0,t);
A_sol = solve(y0 == 0, A);
y = subs(y, A, A_sol)

% BC2: y(l,t)=0
yL = y(l,t);
p_sol = solve(sin(p*l)== 0,p,'returnconditions',true);
y = subs(y, p, p_sol.p);
y = subs(y, p_sol.parameters, n)

% BC3: y(x,0)=0
y_x = y(x,0);
C_sol = solve(y_x == 0, C,'returnconditions',true);
y = subs(y, C, C_sol.C)
y=subs(y,B*D,C_n)

% BC4: y'(x,0)= kx(l-x)
y_t = diff(y, t);
y_t0 = y_t(x,0)
f = k*(l*x - x^2);

% Fourier series coefficient
Bn= (2/l) * int(f * sin(pi*n*x/l), x, 0, l)
Bn_final=simplify(subs(Bn,[sin(pi*n), sin(pi*n/2)^2],[0, (1 - (-1)^n)/2]))
Cn_new=l/(n*pi*c)*Bn_final

% Final solution:
y=subs(y,C_n,Cn_new)
disp('The solution is:')
fprintf('y(x,t)= %s n=1 to %s %s \n', char(0x2211), char(8734), char(y));
```

Output

```
y(x, t) = (A*cos(p*x) + B*sin(p*x))*(C*cos(c*p*t) + D*sin(c*p*t))

y(x, t) = B*sin(p*x)*(C*cos(c*p*t) + D*sin(c*p*t))

y(x, t) = B*sin((pi*n*x)/l)*(C*cos((pi*c*n*t)/l) + D*sin((pi*c*n*t)/l))

y(x, t) = B*D*sin((pi*c*n*t)/l)*sin((pi*n*x)/l)

y(x, t) = C_n*sin((pi*c*n*t)/l)*sin((pi*n*x)/l)

y_t0 = (C_n*c*n*pi*sin((pi*n*x)/l))/l

Bn = (2*((4*k*l^3*sin((n*pi)/2)^2)/(n^3*pi^3) - (k*l^3*sin(n*pi))/(n^2*pi^2)))/l

Bn_final = -(4*k*l^2*((-1)^n - 1))/(n^3*pi^3)

Cn_new = -(4*k*l^3*((-1)^n - 1))/(c*n^4*pi^4)

y(x, t) = -(4*k*l^3*sin((pi*c*n*t)/l)*sin((pi*n*x)/l)*((-1)^n - 1))/(c*n^4*pi^4)
```

The solution is:
y(x,t)= ∑ n=1 to ∞ -(4*k*l^3*sin((c*n*t*pi)/l)*sin((n*x*pi)/l)*((-1)^n - 1))/(c*n^4*pi^4)