Problem 1: Evaluate the integral $\int_0^\infty \frac{x^2}{(x^2+a^2)(x^2+b^2)}dx$.

```
clear all
clc
syms x s a b pi real;
assume(a > 0 & b > 0);

% Define functions
f = exp(-a*x);
g = exp(-b*x);

% Fourier sine transform of f(x)
Fs_f = sqrt(2/pi) * int(f * sin(s*x), x, 0, inf);
% Fourier sine transform of g(x)
Fs_g = sqrt(2/pi) * int(g * sin(s*x), x, 0, inf);

lhs = Fs_f * Fs_g;
rhs = int(f * g, x, 0, inf);

% Display results
disp('Fourier sine Transform of f(x):');
disp(Fs_f);
disp('Fourier sine Transform of g(x):');
disp(Fs_g);

lhs=subs(lhs,s,x);
fprintf('%s %s %s %s dx = %s', char(8747), char(8320), char(8734), char(lhs*pi/2),
char(rhs*pi/2));
```

Output:

```
Fourier sine Transform of f(x):
(2^(1/2)*s*(1/pi)^(1/2))/(a^2 + s^2)


Fourier sine Transform of g(x):
(2^(1/2)*s*(1/pi)^(1/2))/(b^2 + s^2)


∫₀∞ x^2/((a^2 + x^2)*(b^2 + x^2)) dx = pi/(2*(a + b))
```

Problem 2: Evaluate the integral $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$.

```
clear all
clc
syms x s a b pi real;
assume(a > 0 & b > 0);

% Define functions
f = exp(-a*x);
g = exp(-b*x);

% Fourier sine transform of f(x)
Fc_f = sqrt(2/pi) * int(f * cos(s*x), x, 0, inf);
% Fourier sine transform of g(x)
Fc_g = sqrt(2/pi) * int(g * cos(s*x), x, 0, inf);

lhs = Fc_f * Fc_g;
rhs = int(f * g, x, 0, inf);

% Display results
disp('Fourier cosine Transform of f(x):');
disp(Fc_f);
disp('Fourier cosine Transform of g(x):');
disp(Fc_g);

lhs=subs(lhs,s,x);
fprintf('%s %s %s %s dx = %s', char(8747), char(8320), char(8734), char(lhs*pi/(2*a*b)),
char(rhs*pi/(2*a*b)));
```

Output:

```
Fourier cosine Transform of f(x):

(2^(1/2)*a*(1/pi)^(1/2))/(a^2 + s^2)


Fourier cosine Transform of g(x):

(2^(1/2)*b*(1/pi)^(1/2))/(b^2 + s^2)


∫ 0 ∞ 1/((a^2 + x^2)*(b^2 + x^2)) dx = pi/(2*a*b*(a + b))
```

Problem 3: Evaluate the integral $\int_0^\infty \frac{x^2}{(x^2+a^2)^2} dx$.

```
clear all
clc
syms x s a b pi real;
assume(a > 0 & b > 0);

% Define functions
f = exp(-a*x);

% Fourier sine transform of f(x)
Fs_f = sqrt(2/pi) * int(f * sin(s*x), x, 0, inf);

lhs = Fs_f ^2;
rhs = int(f ^2, x, 0, inf);

% Display results
disp('Fourier sine Transform of f(x):');
disp(Fs_f);

lhs=subs(lhs,s,x);
fprintf('%s %s %s %s dx = %s', char(8747), char(8320), char(8734), char(lhs*pi/2),
char(rhs*pi/2));
```

Output:

```
Fourier sine Transform of f(x):
(2^(1/2)*s*(1/pi)^(1/2))/(a^2 + s^2)


∫ 0 ∞ x^2/(a^2 + x^2)^2 dx = pi/(4*a)
```

Problem 4: Evaluate the integral $\int_0^\infty \frac{1}{(x^2+a^2)^2}\,dx$.

```
clear all
clc
syms x s a b pi real;
assume(a > 0 & b > 0);

% Define functions
f = exp(-a*x);

% Fourier sine transform of f(x)
Fc_f = sqrt(2/pi) * int(f * cos(s*x), x, 0, inf);

lhs = Fc_f ^2;
rhs = int(f ^2, x, 0, inf);

% Display results
disp('Fourier cosine Transform of f(x):');
disp(Fc_f);

lhs=subs(lhs,s,x);
fprintf('%s %s %s %s dx = %s', char(8747), char(8320), char(8734), char(lhs*pi/(2*a^2)),
char(rhs*pi/(2*a^2)));
```

Output:

```
Fourier cosine Transform of f(x):
(2^(1/2)*a*(1/pi)^(1/2))/(a^2 + s^2)


∫ 0 ∞ 1/(a^2 + x^2)^2 dx = pi/(4*a^3)
```