# Assignment 4

## Devansh Jain, 190100044

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## Contents

1 Clustering																												
	1.1	CS 335	K	Me	ans	s Ir	np	len	ner	nta	tic	n					•										•	1
<b>2</b>	Ker	nel desi	igı	n a	$\mathbf{nd}$	K	er	ne	liz	ed	l c	lu	$\mathbf{st}$	er	in	g												2
	2.1	CS 337:	: I	$^{\circ}$ ro	vin	g ŀ	(er	nel	l V	ali	di	ty																2
	2.2	CS 337:	· S	Sim	ple	K	ern	el	De	sig	ζn																	2
		(i)																										2
		(ii)																										

190100044 Assignment 4

## 1 Clustering

## 1.1 CS 335 KMeans Implementation

190100044 Assignment 4

### 2 Kernel design and Kernelized clustering

#### 2.1 CS 337: Proving Kernel Validity

We are going to use the following property of kernels from Lecture slides (Lecture 11):

$$K(x,y) = \sum_{d=1}^{r} \alpha_d(x^T y)^d$$
 where  $\alpha_d \ge 0$  is a kernel  $(r \text{ can be } \infty)$ 

Another property of kernels which we are going to use is:

$$K(x,y)$$
 is a kernel  $\implies K'(x,y) = f(x)f(y)K(x,y)$  is also a kernel Corresponding feature map,  $\phi'(x) := f(x)\phi(x)$ 

An important property of exponential function which we are going to exploit is:

$$\exp(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

Using the above properties, we can conclude that  $\exp(\alpha x^T y)$  where  $\alpha \geq 0$  is a kernel.

Now, if take 
$$\alpha = \frac{1}{\sigma^2}$$
 and  $f(x) = \exp\left(-\frac{x^Tx}{2\sigma^2}\right)$ , we get: 
$$K_{\alpha}(x,y) = \exp\left(-\frac{||x-y||^2}{2\sigma^2}\right) = \exp\left(-\frac{x^Tx}{2\sigma^2}\right) \exp\left(-\frac{y^Ty}{2\sigma^2}\right) \exp\left(\frac{x^Ty}{\sigma^2}\right)$$
 is a kernel. Hence, proved.

2.2 CS 337: Simple Kernel Design

- (i)
- (ii)