Assignment 4

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30th Oct 2021

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We are going to use the following property of kernels from Lecture slides (Lecture 11):

$$K(x,y) = \sum_{d=1}^{r} \alpha_d (x^T y)^d \text{ where } \alpha_d \geq 0 \text{ is a kernel } (r \text{ can be } \infty)$$

Another property of kernels which we are going to use is:

$K(x,y)$ is a kernel $\implies K'(x,y) = f(x)f(y)K(x,y)$ is also a kernel
Corresponding feature map, $\phi'(x) := f(x)\phi(x)$

An important property of exponential function which we are going to exploit is:

$$\exp(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

Using the above properties, we can conclude that $\exp(\alpha x^T y)$ where $\alpha \geq 0$ is a kernel.
Now, if take $\alpha = \dfrac{1}{\sigma^2}$ and $f(x) = \exp\left(-\dfrac{x^T x}{2\sigma^2}\right)$, we get:
$K_\alpha(x,y) = \exp\left(-\dfrac{||x-y||^2}{2\sigma^2}\right) = \exp\left(-\dfrac{x^T x}{2\sigma^2}\right)\exp\left(-\dfrac{y^T y}{2\sigma^2}\right)\exp\left(\dfrac{x^T y}{\sigma^2}\right)$ is a kernel.
Hence, proved. $\square$

2.2 CS 337: Simple Kernel Design

(i)

(ii)