Assignment 5

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1 CS 337: Directed Graphical models

1.1 D-Separation

(a) $C \perp B$ **?**
No - CAB is not blocked.

(b) $C \perp B|A$ **?**
Yes - CAB is now blocked and all other paths have a Head-Head node which isn't A.

(c) $C \perp B|A, J$ **?**
No - $CDGB$ is now not blocked as descendent of Head-Head node G is J.

(d) $C \perp B|A, J, D$**?**
Yes - $CDGB$ is now blocked because of D.

(e) $C \perp G$ **?**
No - $CABG$ and CDG are not blocked.

(f) $C \perp G|B$ **?**
No - CDG is not blocked.

(g) $C \perp G|B, D$ **?**
Yes - $CABG$ and CDG are now blocked and all other paths have a Head-Head node.

(h) $C \perp G|B, D, H$ **?**
No - $CEHFG$ is now not blocked as Head-Head node is H.

(i) $C \perp G|B, D, H, E$ **?**
Yes - $CEHFG$ is now blocked because of E.

(j) $B \perp I|J$ **?**
No - $BGFI$ is not blocked as descendent of Head-Head node G is J.

1.2 Probability distribution

$$\begin{aligned}\mathbb{P}(A, \cdots, J) &= \prod_X \mathbb{P}(X|\text{parents of } X) \\ &= \mathbb{P}(A)\mathbb{P}(B|A)\mathbb{P}(C|A)\mathbb{P}(D|A)\mathbb{P}(E|C)\mathbb{P}(F|D)\mathbb{P}(G|B, D, F)\mathbb{P}(H|E, F)\mathbb{P}(I|F)\mathbb{P}(J|F, G)\end{aligned}$$

1.3 Number of parameters required to learn the probability distribution

Except for H, I, all conditional probability terms would require $2^{\#\text{Parents}}$ parameters.
Conditional probability term for I would require zero parameter as $P(I = 1|F = 1) = 1$ and $P(I = 1|F = 0) = 0$. Similarly, conditional probability term for H would require zero parameter.

Prob. distribution	# Parameters
$\mathbb{P}(A)$	1
$\mathbb{P}(B\|A)$	2
$\mathbb{P}(C\|A)$	2
$\mathbb{P}(D\|A)$	2
$\mathbb{P}(E\|C)$	2
$\mathbb{P}(F\|D)$	2
$\mathbb{P}(G\|B, D, F)$	8
$\mathbb{P}(H\|E, F)$	0
$\mathbb{P}(I\|F)$	0
$\mathbb{P}(J\|F, G)$	4
$\mathbb{P}(A, B, C, D, E, F, G, H, I, J)$	23