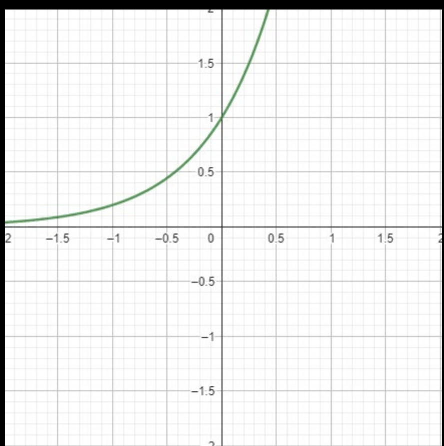
Exponential functions

It’s a function with a base **a** defined like this

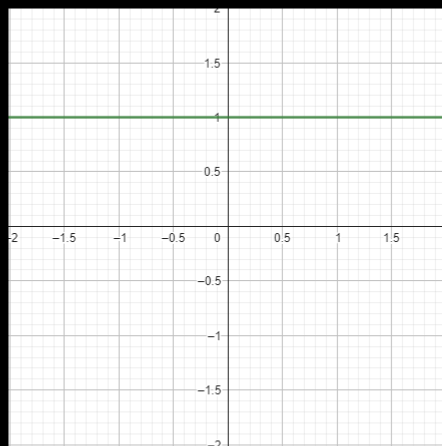
$$f(x) = a^x$$

where a > 0, a ≠ 1, and x is a **real number**

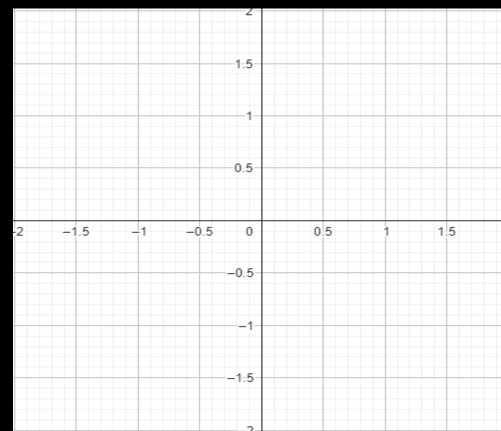
- If the base was a negative number, the value of the function would be a complex number for some values of x
- If a was equal to 1, then f(x) will always be 1 and we will get a constant function



a = 5



a = 1

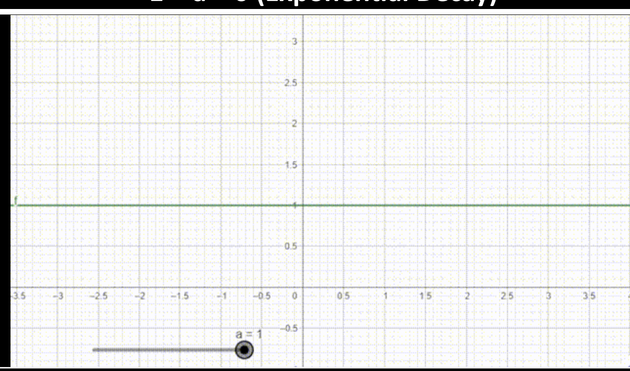


a = -1

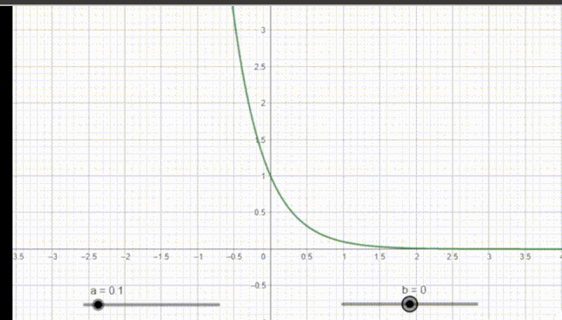
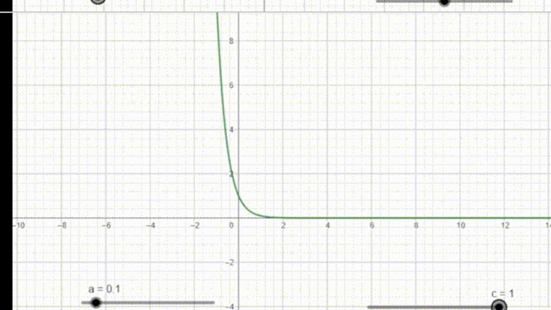
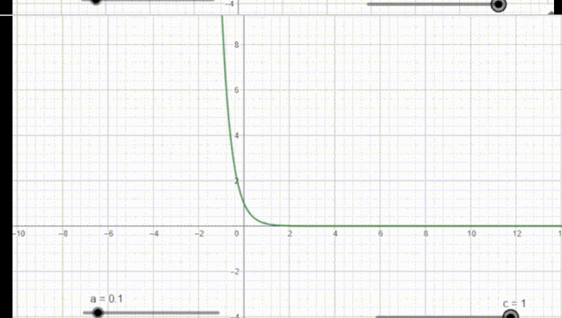
PROPERTIES OF THE EXPONENTIAL FUNCTION

1. The domain (x values) of the function is between (-∞, ∞), and the range (y values) is between (0, ∞)
2. The graph is a smooth continuous curve that has a y-intercept at (0,1), and passes the point (1, a), if it didn’t then it’s not an exponential graph
3. The graph has no x-intercepts no matter the x value, if it did that then it’s not an exponential graph
4. The x-axis is a horizontal asymptote for every exponential function

When **a** is

a > 1 (Exponential growth)	**1 > a > 0 (Exponential Decay)**
	
Because f is an increasing function, the graph rises to the right	f is a decreasing function, the graph rises to the left, until collapsing at 0
When **x becomes ∞**, **y becomes ∞** When **x becomes -∞**, **y becomes 0**	When **x becomes** ∞, **y becomes 0** When **x becomes -**∞, **y becomes** ∞

TRANSFORMATIONS ON EXPONENTIAL FUNCTIONS

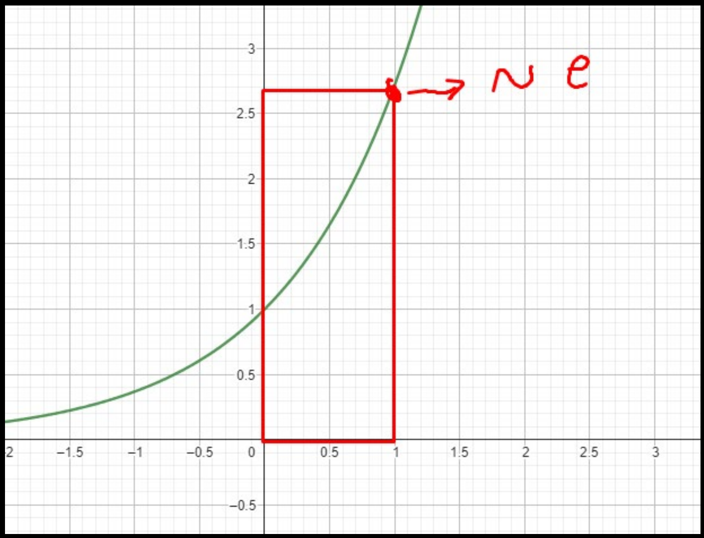
transformation	equation	Effect on graph	Gif for visuals
Horizontal shift	$y = a^{x+b}$	the graph shifts by |b| amount of units -Left when b **increases** -Right when b **decreases**	
Vertical shift	$y = a^{x} + b$	The graph shifts by |b| amount of units -up when b increases -down when b increases	
Stretching or Compressing (vertically)	$y = ca^{x}$	The y coords are multiplied by c, the graph is **->vertically<-** -stretched when c is > 1 -compressed when 1 > c > 0	
Reflection on the x axis	$y = -a^{x}$	Reflects the graph on the x axis	
Reflection on the y axis	$y = a^{-x}$	Reflects the graph on the y axis	

The irrational number **e** is useful in many applications that involve growth/decay

e represents the number that $\left(1+\frac{1}{n}\right)^n$ approaches as n increases without bounds, its value accurate to the eight decimals is 2.71828183

Natural Exponential function

For all real numbers x, the function defined by $f(x) = e^x$



LOGARITHMIC FUNCTIONS

$$f(x) = log_b(x)$$

This equation works when x > 0 and b is a positive constant which isn't equal to 1

The notation is read (the logarithm (or log) base b of x)

Logarithmic functions $\log_b(x)$ are opposite to exponential functions b^x

compositing logarithmic and exponential functions

- When x > 0, b > 0, b ≠ 1, knowing that
  - $g(x) = b^x$
  - $f(x) = \log_b(x)$
- then
  - $g(f(x)) = b^{log_b(x)}$ = x, because b and $\log_b$ cancel each other out
  - $f(g(x)) = log_b(b^x)$ = x

NOTES

- the exponential form of **$y = \log_b(x)$** is **$b^y = x$**
- the logarithmic form of **$b^y = x$** is **$y = \log_b(x)$**

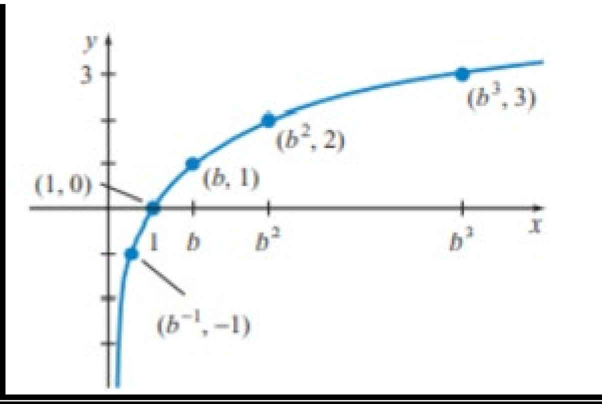
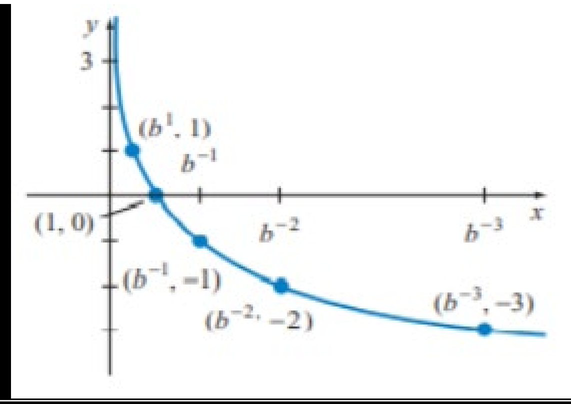
Basic logarithmic properties

1. $\log_b(b) = 1$
2. $\log_b(1) = 0$
3. $\log_b(b^x) = x$
4. $b^{\log_b(x)} = x$

PROPERTIES OF THE LOGARITHMIC FUNCTION

- it has a domain of (0,∞) and a range of (-∞,∞)

- the graph always intercepts the x axis at point (1, 0) and passes through the point (b, 1)
  - When **b** is

<table>
<tr><th>b > 1</th><th>1 > b > 0</th></tr>
<tr><td>
</td><td>
</td></tr>
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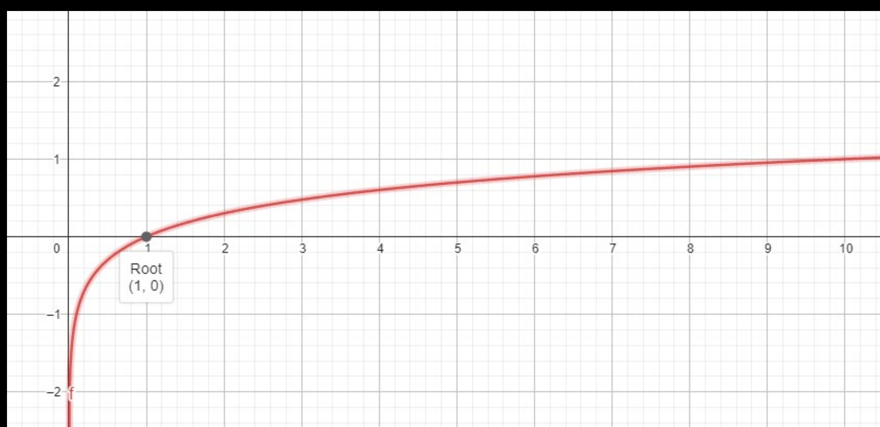
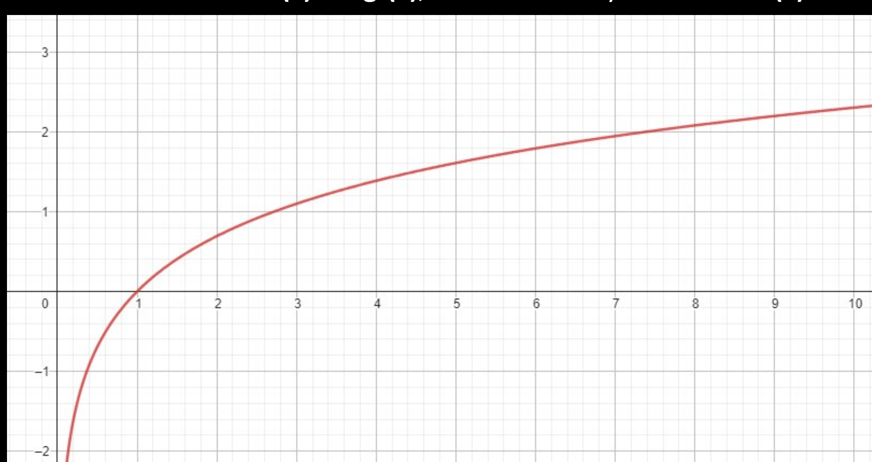
<tr><td>When x becomes ∞, y becomes ∞
When x becomes 0, y becomes -∞</td><td>When x becomes ∞, y becomes -∞
When x becomes 0, y becomes ∞</td></tr>
</table>

Common and Natural Logarithms

The common logarithmic function is defined as **$f(x) = \log_{10}(x)$**, it is customarily written as **$\log(x)$**



the natural logarithmic function is defined as **$f(x) = \log_e(x)$**, it is customarily written as **$\ln(x)$**



Properties of logarithms

in the following properties, b, M, and N are positive real numbers with b ≠ 0

- **Product property**

$$\mathrm{Log}_b(MN) = \log_b(M) + \log_b(N)$$

- **Quotient property**

$$log_b\left(\frac{M}{N}\right) = log_b(M) - log_b(N)$$

- **Power property**

$$\mathrm{Log}_b(M^p) = p \log_b(M)$$

- **Logarithm of each side property**

M = N -> implies that -> **Log_b(M) = Log_b(N**)

- **One to one property**

Log_b(M) = Log_b(N) -> implies that -> **M = N**

LOGARITHMIC EQUATION

$$\mathrm{Log}_a(x^m) = m \log_a(x)$$

EXPONENTIAL GROWTH AND DECAY FUNCTIONS

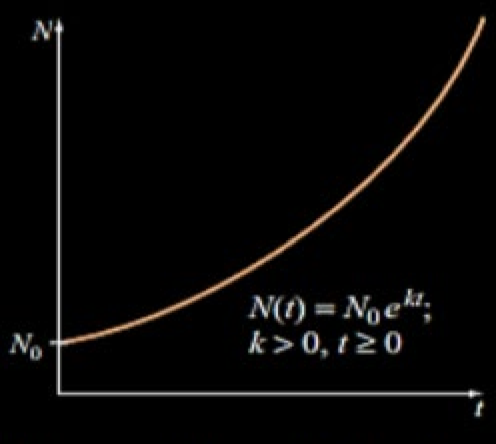
If a quantity N increases/decreases at a rate proportional to the amount present at time

Then the quantity can be modeled as

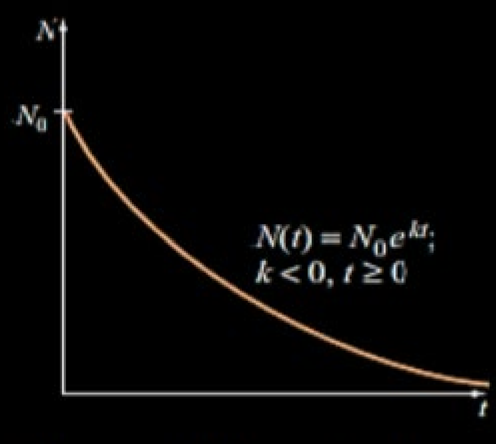
$$\mathbf{N(t) = N_0e^{kt}}$$

- N(t) is the quantity at time t,
- No is the initial quantity,
- e is the base of the natural logarithm (approximately 2.71828),
- k is the growth or decay rate, and
- t is time.

The function shows that the rate of change of the quantity is proportional to the current quantity. This means that the faster the quantity is growing or decaying, the larger the quantity itself.



Exponential growth function



Exponential decay function

COMPOUND INTREST FORMULA

A principal (**p**) is invested at an annual intrest rate (**r**), exprecced as a decimal and compounded (**n**) amount of times per year, for (**t**) years, produces a balance

$$A = p(1 + \frac{r}{n})^{nt}$$

where,,,

A = amount after t years

P = principal

r = annual interest rate (expressed as a decimal)

n = number of times interest is compounded each year

t = number of years

Continuous Compounding intrest formula

If an account with principal (**P**) and annual interest rate (**r**) is compounded continuously for (**t**) amount of years, then the balance is equal to

$$A = pe^{rt}$$

A = amount after t years

P = principal

r = annual rate (expressed as a decimal)

t = number of years

Feature	Compound Interest Formula	Continuous Compounding Formula

Interest application	Discrete intervals	Continuous
Frequency	Defined by compounding period (e.g., annually)	Infinitely small periods
Formula complexity	Relatively simple	More complex, involving natural logarithms (ln)
Practical applicability	Commonly used in finance	Theoretical concept
Impact on final amount	Interest earned grows with more frequent compounding	Highest possible final amount among all compounding frequencies

The difference between the compound interest and continuous compounding interest is that in the first one you have a set number of time of compounding interests in a year, but in the second one, imagine your interest compounding every second of the year, that's the difference in a nutshell