Remember functions?

yk, those F(x) = x

Where F(x) is basically y

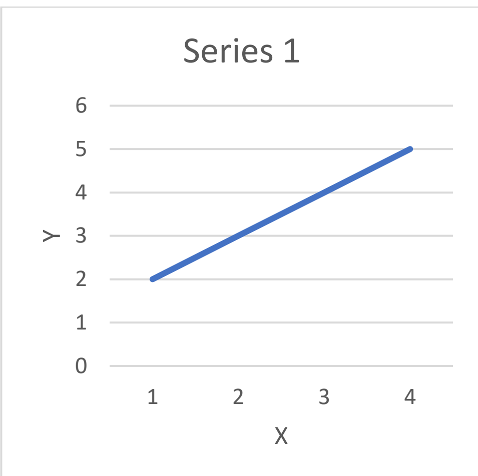
WELL THEY'RE BACK

Linear Functions

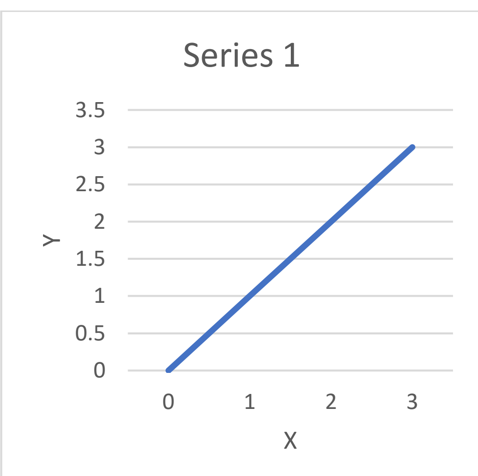
F(x) = mx + b

If m = 1, then they have a direct relationship

If b = 1, then y is Ofsted by b or 1



If m = 1, and b = 0, then f(x)=x, this is what we call an **identity function**



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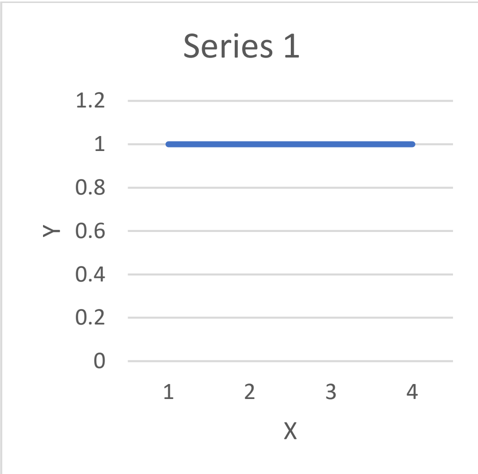
If m = 0, then f(x) = b, which is a constant value



A linear function has 3 forms

Y = mx + b -> slope-intercept form

Ax + By = C -> standard form, its purpose is to check if x & y are on the line.

A, B, C are constants given to you

For example, if you have an equation like $2x + 3y = 6$, it's in standard form. You can pick values for x and y and see if they fit in the equation. If you choose $x = 2$ and $y = 0$, you'll see that $2(2) + 3(0) = 4$, which isn't equal to 6. So, the point $(2, 0)$ is not on this line. But if you try $x = 0$ and $y = 2$, you'll see that $2(0) + 3(2) = 6$, which fits the equation. So, the point $(0, 2)$ is on the line described by $2x + 3y = 6$.

$y-y_1$ = m($x-x_1$) -> point-slope form

this is basically the slope equation but with cross multiplication

$$m = \frac{y - y_1}{x - x_1}$$

The domain of any linear function is all real numbers, no **i**s or what not

If m != 0, the range is also all real numbers

If m = 0, the function is constant and the range is b

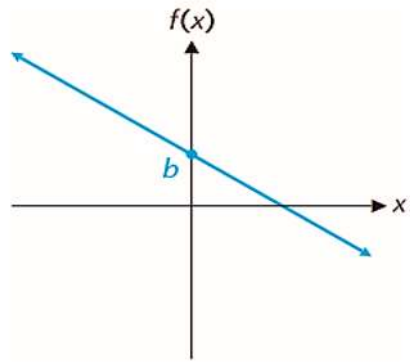
Domain is all the independent values aka (x)

Range is all the dependent values aka (y)

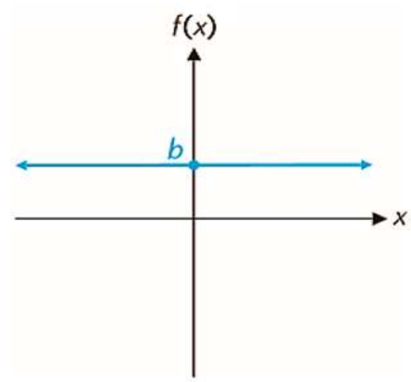
GRAPHING LINEAR FUNCTIONS

The graph of a linear function is a line with slope m and y intercept b.

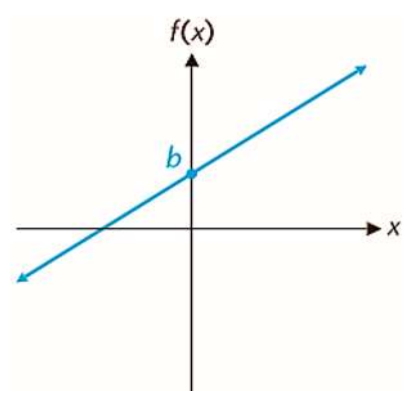
$\boldsymbol{m < 0}$
Decreasing on $(-\infty, \infty)$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$



$\boldsymbol{m = 0}$
Constant on $(-\infty, \infty)$
Domain: $(-\infty, \infty)$
Range: $\{\boldsymbol{b}\}$



$\boldsymbol{m > 0}$
Increasing on $(-\infty, \infty)$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$



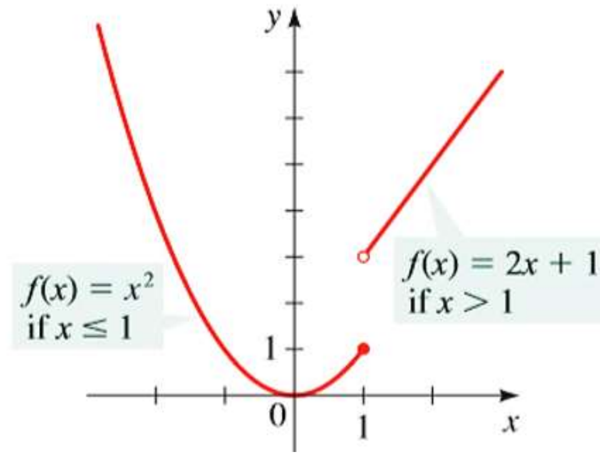
PIECEWISE FUNCTION

A function that combines different equations called **pieces**

A **piece** has a new domain or **set of x values**

Example

$$f(x) = \begin{cases} x^2 & x \leq 1 \\ 2x + 1 & x > 1 \end{cases}$$



You graph a piecewise function by firstly knowing where stuff end and begin

x<0 ->] ∞ , 0 [

x≤0 ->] ∞ , 0]

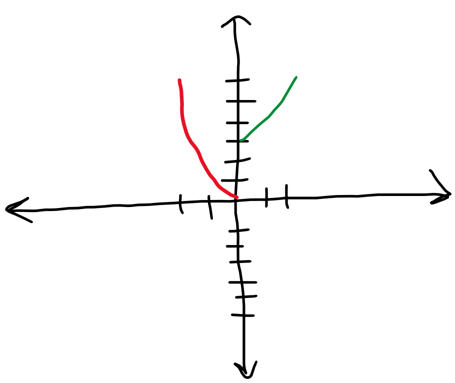
x≥0 ->] 0 , ∞ [

x>0 -> [0 , ∞ [

EXAMPLE

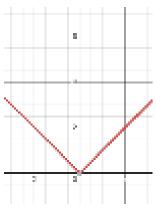
F(x) = { 2x+2   x>0

{ x^2 x≤0

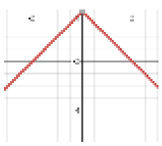


ABSOLUTE VALUE FUNCTION

y=|x| -> It’s a graph that looks like V opening upward and pointing downward



y=-|x| -> it’s a graph that looks like V opening downwards and pointer downwards



STEP FUNCTION

It’s a function whose graph has **breaks** or **discontinuities**

It’s called a **step function** because the graph appears to be a series to be a series of steps

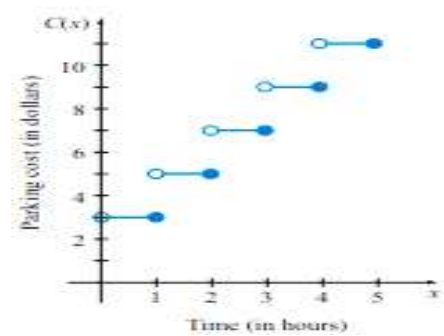
It’s also called the **greatest integer function** or **floor function**

the floor is represented by [[x]], [x], and int(x)

so [1.8] means to get the first larger integer than 1.8

if the number is an integer already, then flooring it gives the same value

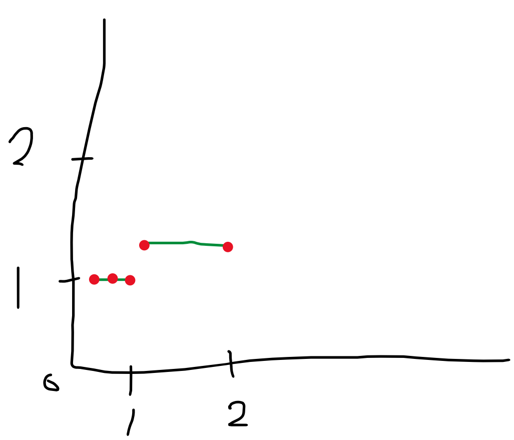
- **The domain (Xs) is the set of real numbers.**
- **The range (Ys) is the set of integers.**



EXAMPLE

F(x) = [x]

X	.5	.7	.D	1.1	2
y	1	1	1	2	2



DIRECT and INVERSE Variation

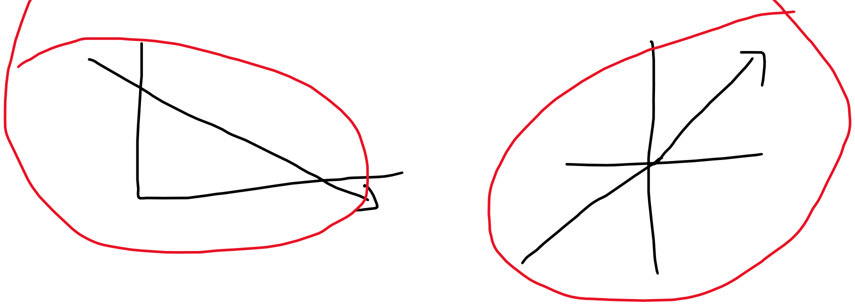
we have something that describes the variation called constant of variation or **k**

if k = 0, then there is no variation, so when there is variation, k != 0

DIRECT VARIATION -> proportional, aka **LINEAR**

$y \propto x$

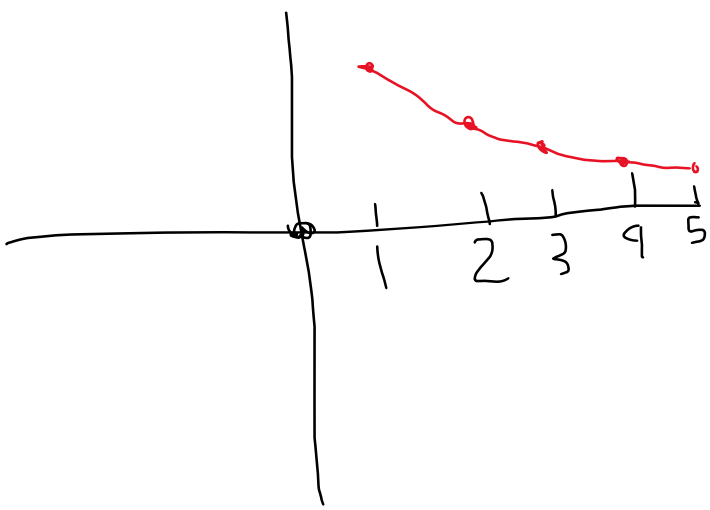
We describe its variation variation as y = kx



INVERSE VARIATION -> inversely proportional, aka **NON LINEAR**

$$y \propto \frac{1}{x}$$

y = k/x



JOINT VARIATION

W is jointly proportional to x and y

$$W = kxy$$

So w will change proportionally with x and y at the same time

NOTES

The three basic types of variation also can be combined.

Certainly! Let's consider a scenario where the total cost of producing a certain product, denoted as C, is directly proportional to the number of units produced (x) and inversely proportional to the number of workers (y) hired to produce those units.

The equation that represents this joint variation could be:

$C = k \times \frac{x}{y}$

Let's assume that for a specific product, when 100 units are produced with 5 workers, the total cost is $5000. We can use this information to solve for the value of the constant k and then use the equation to find the total cost for different scenarios.

Given:

- $x = 100$ (units produced)
- $y = 5$ (number of workers)
- [illegible]

We can plug these values into the equation to find k:

$5000 = k \times \frac{100}{5}$
$5000 = k \times 20$
$k = \frac{5000}{20} = 250$

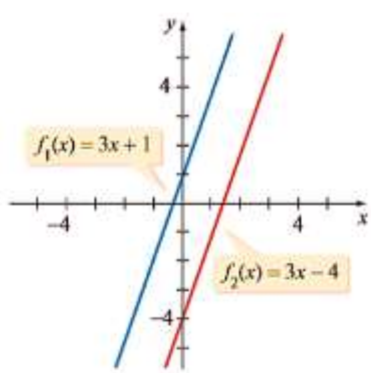
So, the constant k is 250 for this scenario.

Now, let's find the total cost when the company produces 150 units with 8 workers:

$C = 250 \times \frac{150}{8}$
$C = 250 \times 18.75$
[illegible]

Let's say you have two lines

- They are **parallel** when slope1 (m1) = slope2 (m2)



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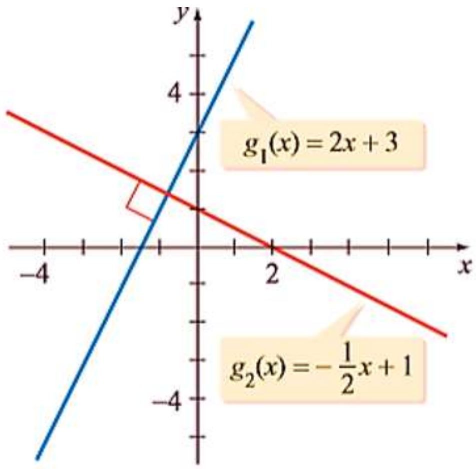
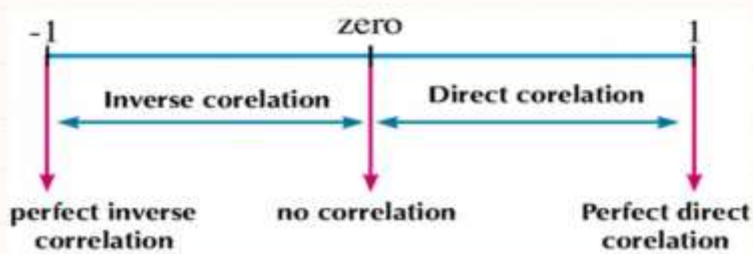
- They are perpendicular if **slope1 = -1 / slope2**, because now slope1 is a negative reciprocal of slope 2



CORRELATION COEFFICIENT

A correlation coefficient (r) is a number between -1 and 1 that tells you the strength and direction of a relationship between variables.



If r = Zero then its no correlation between two variables.
If $0 < r < 0.4$ then its weak correlation.
If $0.4 \le r < 0.6$ then its intermediate correlation.
If $0.6 \le r < 1$ then its strong correlation.
If r = l then its perfect correlation.

How to calculate the correlation coefficient

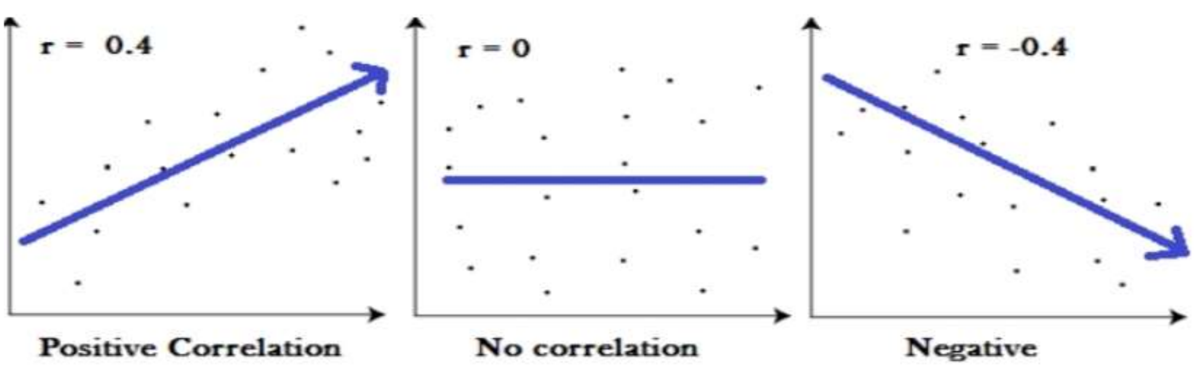
1 – Pearson's correlation

Step 1	Find the mean of x, and the mean of y
Step 2	Subtract every x from the value mean of x (call them "a"), and subtract every y value from the mean of y (call them "b")
Step 3	Calculate ab, a^2, and b^2 for every value
Step 4	Sum up ab, sum up a^2, and sum up b^2
Step 5	Divide the sum of ab by the square root of [(sum of a^2) × (sum of b^2)]

$$r = \frac{n \times \sum(xy) - \sum(x) \times \sum(y)}{\sqrt{n \times \sum(x^2) - \sum(x)^2} \times \sqrt{n \times \sum(y^2) - \sum(y)^2}}$$

Where

- $\sum(x)$ is the sum of x
- $\sum(y)$ is the sum of y
- $\sum(xy)$ is the sum of (x*y)
- $\sum(x^2)$ is the sum of x^2
- $\sum(y^2)$ is the sum of y^2
- N is the number of elements



EXAMPLE 1

1. Calculate the correlation coefficient between the two variables x and y shown below:

X:	1	2	3	4	5	6
Y:	2	4	7	9	12	14

Step 1 : make a table with x, y, xy, x^2, y^2

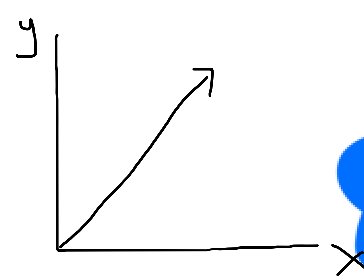
x	y	xy	x^2	y^2
1	2	2	1	4
2	4	8	4	16
3	7	21	9	49
4	9	36	16	81
5	12	60	25	144
6	14	84	36	196
21	**48**	**211**	**91**	**490**

Use this

$$r = \frac{n \times \sum(xy) - \sum(x) \times \sum(y)}{\sqrt{n \times \sum(x^2) - \sum(x)^2} \times \sqrt{n \times \sum(y^2) - \sum(y)^2}}$$

$$R = \frac{6*211-21*48}{\sqrt{6*91-(21)^2}*\sqrt{6*490-(48)^2}} = .998$$

This indicates a very strong direct/linear relationship between x and y



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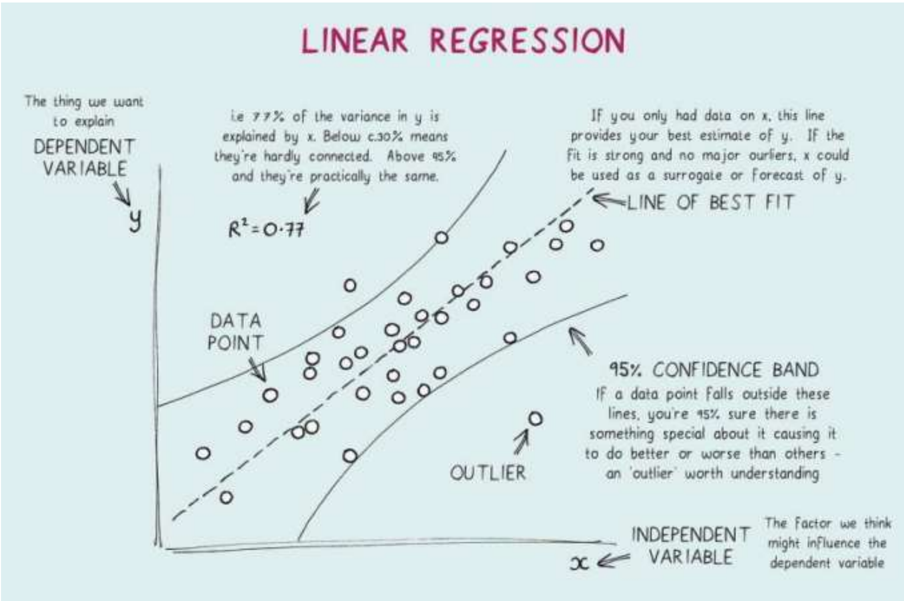
Regression line

displays the connection between

scattered data points in any set



Regression line equation:

$$y = a + bx$$

$$b = \frac{n \times \sum(xy) - \sum(x) \times \Sigma(y)}{n \times \sum(x^2) - (\sum x)^2}$$

$$a = \frac{\sum y - b \sum x}{n}$$

The regression line equation of y on x is used for:

1- predicting the value of Y if the value of X is known.

2- identifying the error which can be identified by the relation:

Error = | Table value - the value satisfying the regression equation |

COMPOSITION OF FUNCTIONS

Is when a function is inside another function

If we have two functions called **f(x)** and **g(x)**

then the composite of **g in f** is (fog)(x) -> f(g(x))

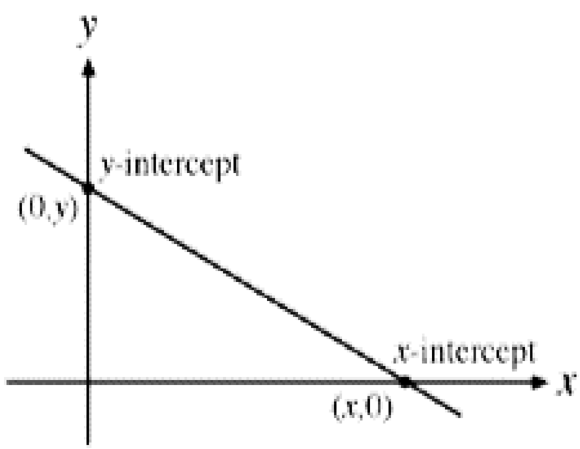
the composite of **f in g** is (gof)(x) -> g(f(x))

> **Example**
>
> $f(x) = 3x - 4$ $\qquad\qquad$ $g(x) = x^2 - 3$
>
> Find: 1) fog $\qquad\qquad$ 2) gof
>
> **Solution**
>
> 1) $fog = f(g(x)) = f(x^2 - 3) = 3(x^2 - 3) - 4 = 3x^2 - 9 - 4$
> $= 3x^2 - 13$
>
> 2) $gof = g(f(x)) = g(3x - 4) = (3x - 4)^2 - 3 = 9x^2 - 24x$
> $+ 16 - 3 = 9x^2 - 24x + 13$

INTERCEPTED PART

You can get the point where the line would intercept the x axis by **setting y to 0** in y=mx+b

You can get the point where the line would intercept the y axis by **setting x to 0** in y=mx+b



TRANSFORMATIONS IN GRAPHS

- If f(x) = x + a -> **x + a** means that the graph or **y-axis** is moved up and down by **a** units
- If f(x+a) is the graph, that means that the graph or the **x-axis** is moved left and right by **a** units
- -f(x) is f(x) but reflected in the x axis
- There is something called **a** that is a multiplier to f(x) and represented as **af(x)**
  - If a is > 1 -> graph stretched vertically
  - If a is in between 0 and 1, the graph will be stretched down
  - If a is negative, the graph is going to be flipped

Exponential Rules

Product Rule

$$a^x \times a^y = a^{x+y}$$
$$a^2 \times a^3 = a^5$$

Quotient Rule

$$a^x \div a^y = a^{x-y}$$
$$a^7 \div a^3 = a^4$$

Power Rule

$$\left(a^x\right)^y = a^{xy}$$
$$\left(a^7\right)^2 = a^{14}$$

Negative Rule

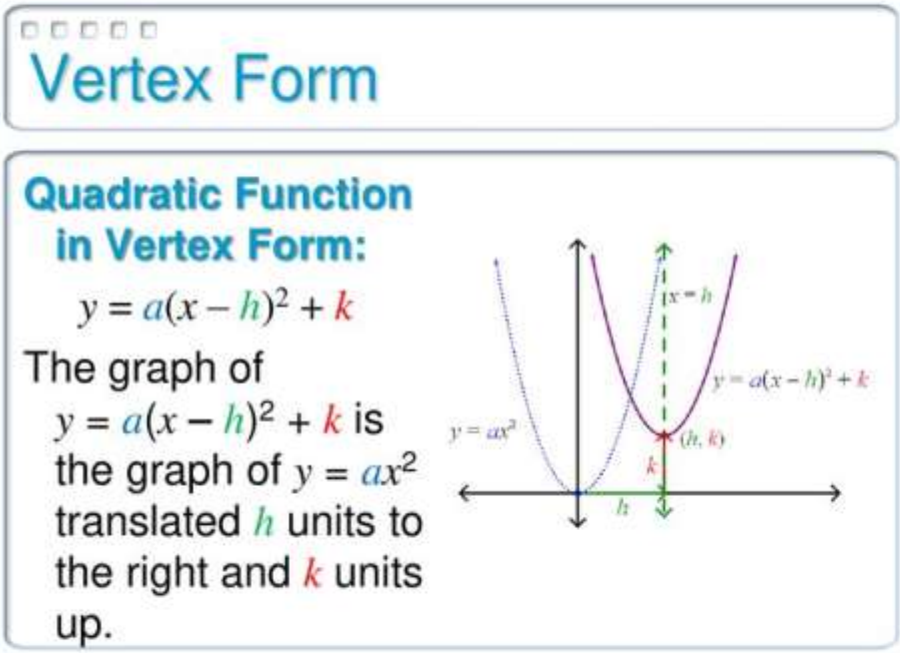
$$a^{-x} = \frac{1}{a^x}$$
$$a^{-4} = \frac{1}{a^4}$$

Zero Rule

$$a^0 = 1$$

Algebra Formula

- $(a+b)^2 = a^2 + b^2 + 2ab$
- $(a+b)^2 = (a-b)^2 + 4ab$
- $(a-b)^2 = a^2 + b^2 - 2ab$
- $(a-b)^2 = (a+b)^2 - 4ab$
- $a^2 - b^2 = (a+b)(a-b)$
- $a^2 + b^2 = (a-b)^2 + 2ab = (a+b)^2 - 2ab$
- $a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$
- $a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$
- $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
- $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
- $a^4 + b^4 = (a+b)(a-b)[(a+b)^2 - 2ab]$
- $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$



K increase -> graph goes up

K decrease -> graph goes down

H increase -> graph goes right

H decrease -> graph goes left

A increase -> quadratic function increase slope

A decrease -> quadratic function decrease slope

H can be replaced with **b**

K can be replaced with **c**

So it can be called

$$\mathbf{y = a(x - b)^2 + c}$$