### TRIGONOMETRIC IDENTITIES

When we are dealing with a **right triangle**, we say that

• 
$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{\csc(\theta)}$$

• 
$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{\sec(\theta)}$$

• 
$$tan(\theta) = \frac{opposite}{adjacent} = \frac{1}{cot(\theta)} = \frac{sin(\theta)}{cos(\theta)}$$

And their opposites

• 
$$\csc(\theta) = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{1}{\sin(\theta)}$$

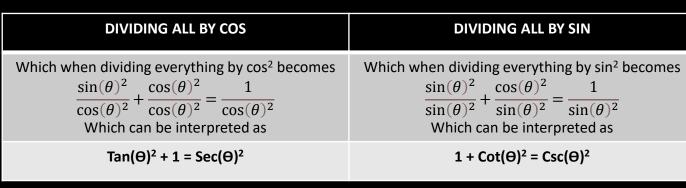
• 
$$sec(\theta) = \frac{hypotenuse}{adjacent} = \frac{1}{cos(\theta)}$$

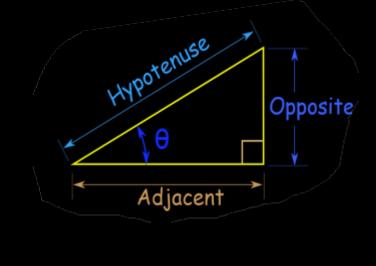
• 
$$cot(\theta) = \frac{adjacent}{opposite} = \frac{1}{tan(\theta)} = \frac{cos(\theta)}{sin(\theta)}$$

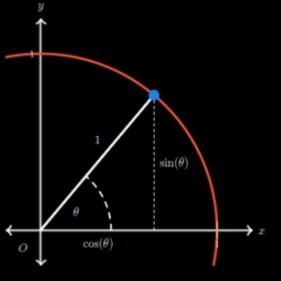


when u are in a unit circle (circle with a radius of 1), you would use this equation

$$Sin(\Theta)^2 + Cos(\Theta)^2 = 1$$







We have otherways to express the pythagorean identities

# Other Variation(s) $\sin^2 \theta = 1 - \cos^2 \theta \qquad \cos^2 \theta = 1 - \sin^2 \theta$ $\tan^2 \theta = \sec^2 \theta - 1$

$$\cot^2 \theta = \csc^2 \theta - 1$$

## **CONFUTATION IDENTITIES**

Let's imagine two angles y and x

Alright so we that we can convert from sin to cos and what not, but let's actually look at what's happening, there is something called the **difference identity for cosine**, it is Cos(x-y) = cos(x) cos(y) + sin(x) sin(y)

Now let us say that

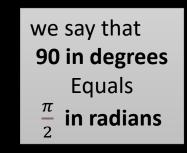
- Cos(90 y) = cos(90) cos(y) + sin(90) sin(y)
- Cos (90 y) = 0 \* cos(y) + 1 \* sin(y)
- Cos(90 y) = sin(y) -> co-function identity for cosine

Now if we replaced y with (90-x)

- Cos(90 (90-x)) = sin(90-x)
- Cos(x) = sin(90-x)
- Sin(90 x) = cos(x) -> co-function identity for sin

we have another thing called the

• Tan(90-x) = Cot(x) -> co-function identity for tangent





عوض you should use these laws to either solve it directly or

# Sum to product

$$\sin A + \sin B = 2 \sin \left(\frac{A+B}{2}\right) \cos \left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2 \cos \left(\frac{A+B}{2}\right) \sin \left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2 \cos \left(\frac{A+B}{2}\right) \cos \left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = -2 \sin \left(\frac{A+B}{2}\right) \sin \left(\frac{A-B}{2}\right)$$

# Sum and difference formulas for sines and cosines

$$\sin A + \sin B = 2 \sin \left(\frac{A+B}{2}\right) \cos \left(\frac{A-B}{2}\right)$$

$$\sin (\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin (\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin (\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin (\alpha + \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos (\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos (\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos (\alpha + \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

# **Product to sum**

$$\cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$$

$$\sin A \sin B = \frac{1}{2} [\cos(A-B) - \cos(A+B)]$$

$$\sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$$

$$\cos A \sin B = \frac{1}{2} [\sin(A+B) - \sin(A-B)]$$

## **DOUBLE ANGLES**

It is called that because we are doubling the angle, so instead of cos(A) it is cos(2A), which is basically

Cos(A+A) = cos(A)cos(A) – Sin(A)Sin(A)

which means that

•  $Cos(2A) = cos(A)^2 - sin(A)^2$ 

Let's take a break and say that

We can take that double angle and put it in a trigonometric identity

• 
$$Cos(A)^2 + Sin(A)^2 = 1$$

Now we can either move the sin or cos

Move cos	Move sin
<ul> <li>Sin(A)² = 1-Cos(A)²</li> <li>Adding that to the equation returns</li> <li>Cos(2A) = cos(A)² - (1 - Cos(A)²)</li> <li>Cos(2A) = cos(A)² - 1 + Cos(A)²</li> </ul>	<ul> <li>Cos(A)<sup>2</sup> = 1-Sin(A)<sup>2</sup></li> <li>Adding that to the equation returns</li> <li>Cos(2A) = 1 - Sin(A)<sup>2</sup> - Sin(A)<sup>2</sup></li> </ul>
Cos(2A) = 2Cos(A) <sup>2</sup> - 1	$Cos(2A) = 1 - 2Sin(A)^2$

Here is the double angle formula

$$Tan(2A) = \frac{2Tan(A)}{1} - Tan(A)^2$$

