INTRO TO TYPES OF NUMBERS.

Real -> include rational numbers like integers, fractions, and irrational numbers like π, $\sqrt{3}$

Imaginary -> is i which is $\sqrt{-1}$

Rational -> are integers and decimals

Irrational -> are numbers that cannot be presented as ratios as they have a lot of numbers after the decimal point, stuff like $\sqrt{2}$ or π

Perfect square -> are numbers that return an integer when put in a square root, like how 25 returns 5 when square rooted

QUADRATIC FUNCTIONS

Are polynomial functions of **degree 2**, so a quadratic function is a function of $f(x) = ax^2 + by + c$

Where $a \neq 0$

The solution for the quadratic equation is given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

If **D > 0**, then the roots are **real and** distinct

If **D = 0**, then the roots are **real and equal**

If **D < 0**, then the roots are **imaginary and distincts**

If **D > 0**, and is a perfect square, then the roots are **rational**

If **D > 0**, and isn't a perfect square, then the roots are **irrational**

If **a = 1** and **c** is a rational number, then the first root is **1** and the second root $\frac{c}{a}$

If **a – b + c = 0**, then the first root is **-1** and the second root is $-\frac{c}{a}$

If the equation $\boldsymbol{ax^2 + by + c = 0}$ has real roots, then these roots will be named $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, and then we can write $\alpha x^2 + bx + c = a(x - \alpha)(x - \beta)$

If α and β are the roots of $\boldsymbol{ax^2 + by + c = 0}$**,** then

- α + β = $-\frac{b}{a}$
- αβ = $\frac{c}{a}$

and using these two results we can turn this $\boldsymbol{ax^2 + by + c = 0}$

into this $x^2 - \left(-\frac{b}{a}\right)x + \frac{c}{a} = 0$

to this $\boldsymbol{x^2 - (\alpha + \beta)x + \alpha\beta}$

Commented [SD1]: If they have a limited decimal count, so numbers like 32.5, 421.54

Commented [SD2]: مختلفين distinct

WAYS TO SOLVE THE QUADRATIC EQUATION

Factorizing

$x^2 + 5x = 24$

$x^2 + 5x - 24 = 0$

$(x + 8)(x - 3) = 0$

$x + 8 = 0$ or $x - 3 = 0$

$x = -8$ or $x = 3$

General formula

$2x^2 + 3x - 2 = 0$

Fill this $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-3 \pm \sqrt{3^2 - 4 * 2 * -2}}{2 * 2}$$

$$x = \frac{-3 \pm \sqrt{25}}{4} = \frac{-3 \pm 5}{4}$$

$x = 2/4 = ½$ **or** $x = -8/4 = -2$

Complete the square

Let's imagine a quadratic equation $ax^2+bx+c=0$

$3x^2 - 12x + 6 = 0$

1. **start with the quadratic expression**
   $3x^2 - 12x + 6 = 0$
2. **move c to the right side**
   $3x^2 - 12x = -6$
3. **take the common factor on the left side**
   $3(x^2-4x) = -6$
4. **add $(b/2)^2$ to the left side, and the right side**
   $3(x^2-4x+4) = -6 + 3(4)$
5. **do the calculation on the right side**
   $3(x^2-4x-4) = 6$
6. **divide both sides by the common factor**
   $x^2 - 4x - 4 = 2$
7. **take the complete square (المربع الكامل)**
   $(x-2)^2 = 2$
8. **take square root**
   $x - 2 = \pm\sqrt{2}$
9. **move 2 to right side**
   $x = 2 \pm \sqrt{2}$

Commented [SD3]: If there is a common factor, multiply the $(b/2)^2$ on the right side with it

First and Second Difference

We can know the degree of the equation without looking at the equation itself
If we just have the x and y values

How to get 1st difference

- arrange the values and make sure the values of x are increasing at a constant rate
- make the first difference between the values of y
- if the first difference is
    - **constant** -> the equation is **linear** and has a degree of 1

x	0	1	2	3	4
y	2	3	4	5	6

1 1 1 1

    - **isn't constant** -> get the 2nd difference, which is between the results of the first difference, if the 2nd difference is constant, then the equation has a degree of 2

x	0	1	2	3	4
y	2	3	6	11	18

First difference: 1 3 5 7

Second difference: 2 2 2

if the 2nd difference is not constant, then you don't care about it, we don't have to study it

TRANSFORMATIONS OF QUADRATIC FUNCTION

You have two ways of translating a quadratic functions (standard form, and vertex form)

STANDARD FORM

$y=ax^2+bx+c$

C increase -> graph goes up

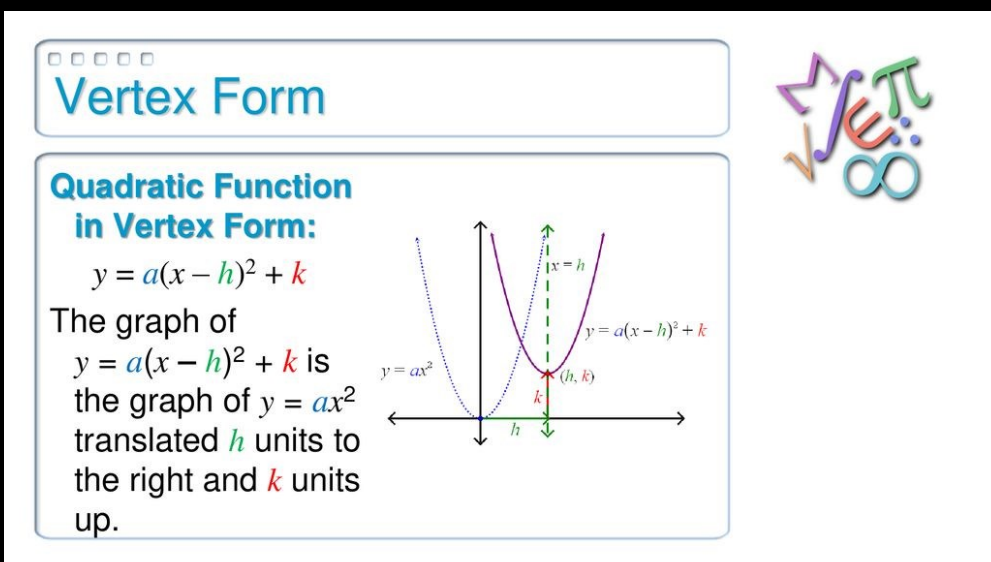
C decrease -> graph goes down

B increase -> graph goes left

B decrease -> graph goes right

A increase -> quadratic function increase slope

A decrease -> quadratic function decrease slope



K increase -> graph goes up

K decrease -> graph goes down

H increase -> graph goes right

H decrease -> graph goes left

A increase -> quadratic function increase slope

A decrease -> quadratic function decrease slope

H can be replaced with **b**

K can be replaced with **c**

So it can be called

$$\mathbf{y = a(x - b)^2 + c}$$

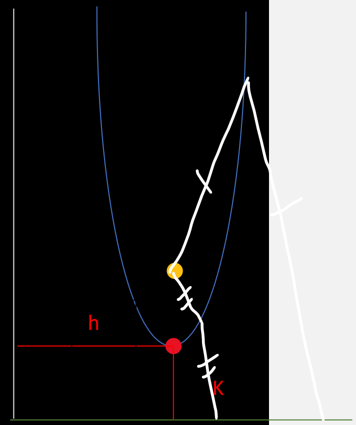
PARABOLA

You can express the head of a parabola with

- **vertex form =>** $(\boldsymbol{h}, \boldsymbol{k})$
- **standard form =>** $\left(-\frac{\boldsymbol{b}}{\boldsymbol{2a}}, \boldsymbol{f}\left(-\frac{\boldsymbol{b}}{\boldsymbol{2a}}\right)\right)$

so you can say that h = $-\frac{\boldsymbol{b}}{\boldsymbol{2a}}$

and k = $a\left(-\frac{b}{2a}\right)^2 + b\left(-\frac{b}{2a}\right) + c$



the focus is a fixed point inside the parabola on its axis of symetr

Commented [SD4]: The x value of the head

the **directrix** is a line where the axis of symmetry is perpendicular to it

a parabola is a set of points with the same distance from them to focus and from them to directrix

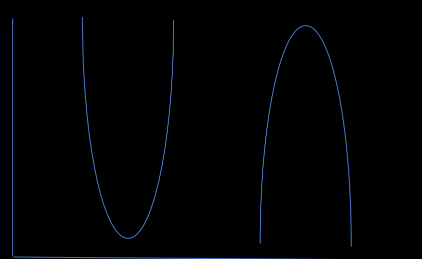
in this equation, **y = a(x-h)² + k**

$$\frac{1}{a} = 4\rho$$

Where ρ = distance from vertex to focus = distance from vertex to directrix

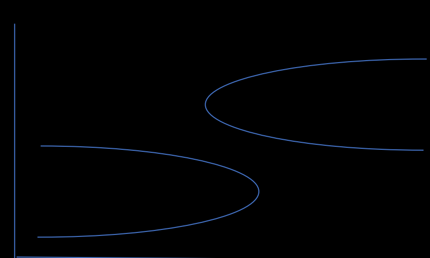
If the parabola is facing upward/downward on the vertical axis

$$y = \frac{1}{4}\rho x^2$$



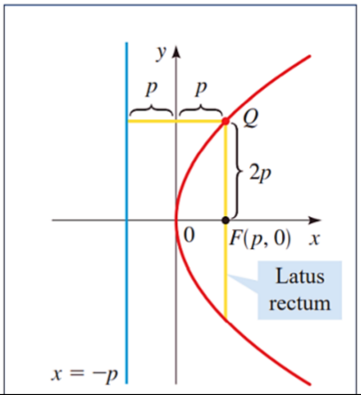
If the parabola is facing upward/downward on the horizontal axis

$$y^2 = 4\rho x$$



Note:

- ✓ The equation $y^2 = 4px$ does not define **y** as a function of x, we must first solve for y. this lead the function to be $y = \sqrt{4px}$, and $y = -\sqrt{4px}$. We need to graph both functions to get the complete graph of the parabola.
- ✓ The line segment that runs through the focus perpendicular to the axis, with endpoints on the parabola, is called the **latus rectum**, and its length is half the length of the **focal diameter** of the parabola.
- ✓ Focal diameter = 4p, latus rectum = 2p

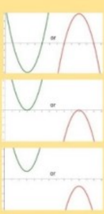


In the general formula x = $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

If b^2 – 4ac (**D**) IS

- > 0 -> then 2 intersects on the x axis
- = 0 -> then 1 intersect on the x axis
- < 0 -> then 0 intersects on the x axis

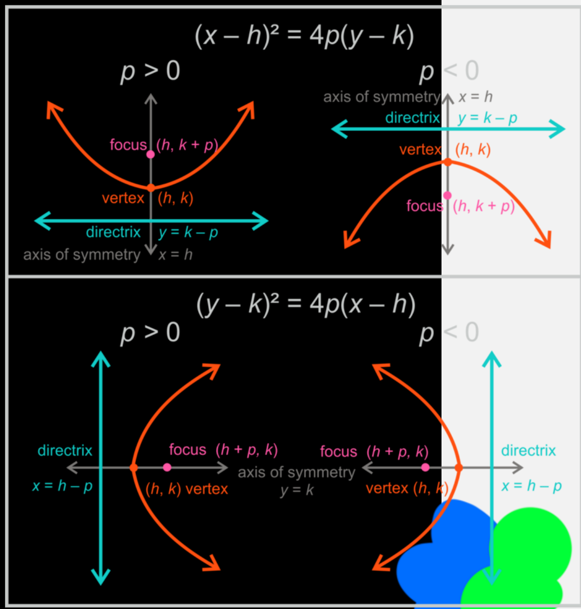
Three Scenarios



$\Leftarrow b^2 - 4ac > 0$

$\Leftarrow b^2 - 4ac = 0$

$\Leftarrow b^2 - 4ac < 0$



An imaginary number like i equals $\sqrt{-1}$

So i^2 = -1

i^3 = -i

i^4 = 1

Subscribe to Sir Devenilla aka: Omar Tarek a complex number has the form of " $a + bi$ " where a & b are real numbers

a complex number is generally denoted as **z**

where a is the real part of z and denoted as **Re(z)**

and b is the imaginary part of z and denoted as **Im(z)**

Notes:

- ✓ Zero is the only real number, which is purely real as well as purely imaginary, but not imaginary number.
- ✓ i is called an imaginary unit, which is introduced by Swedish mathematician **Sir Euler**. Also, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$
- ✓ The period of iota is 4.
- ✓ The sum of the first four ith consecutive powers is zero.
- ✓ $\sqrt{a}\,\sqrt{b} = \sqrt{ab}$ holds good only when at least any one of a or b is non negative.

CALCULATIONS WITH REAL NUMBERS

Addition

you add the imaginary together and the real together

$(a + bi) + (c + di) = (a + c) + i(b + d) =$
$A + Bi$

Subtraction

you subtract the imaginary together and the real together

$(a + bi)(c + di) = (ac - bd) + i(ad + bc) =$
$A + Bi$

You multiply then like this

Step 1: Write the given complex numbers to be multiplied.

$z_1z_2 = (a + ib)(c + id)$

Step 2: Distribute the terms using the FOIL technique to remove the parentheses.

$z_1z_2 = ac + i(ad) + i(bc) + i^2(bd)$

Step 2: Simplify the powers of i and apply the formula $i^2 = -1$.

$z_1z_2 = ac + i(ad) + i(bc) + (-1)(bd)$

Step 3: Combine like terms that mean combine real numbers with real numbers and imaginary numbers with imaginary numbers.

$z_1z_2 = (ac - bd) + i(ad + bc)$

You divide them like this

Let $z_1 = a + bi$ and $z_2 = c + di$

Then $\frac{z_1}{z_2} = \frac{a+bi}{c+di} = \frac{(a+bi)(c-di)}{(c+id)(c-id)} = \frac{(ac+bd)+i(bc-bd)}{(c^2+d^2)} = \left(\frac{(ac+bd)}{(c^2+d^2)}\right) + i\left(\frac{(bc-bd)}{(c^2+d^2)}\right)$

✓ In real numbers, if $a^2 + b^2 = 0$, then $a = 0 = b$, however in complex numbers if $z_1^2 + z_2^2 = 0$ **does not imply** that $z_1 = 0 = z_2$.

We display a complex number (a + bi) on two axes (real , imaginary) called the complex plane (argand)

To graph the complex number a + bi
we plot the pair (a , b)

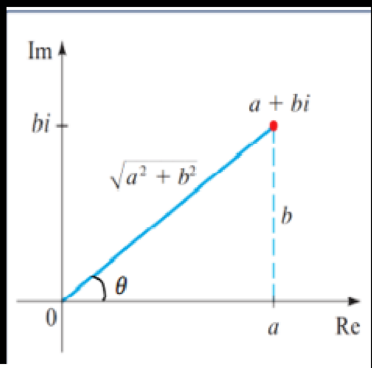
Modulus of a complex number is the absolute value of it, so in a graph

$|z| = \sqrt{a^2 + b^2}$

Argument of a complex number is an angle denoted as Arg(z) = $\Theta = \tan^{-1}(\frac{b}{a})$

The principal argument is denoted as amp(z) which equals Θ

Where Θ lies between $-\pi \le 0 \le \pi$



Representation of a complex number:

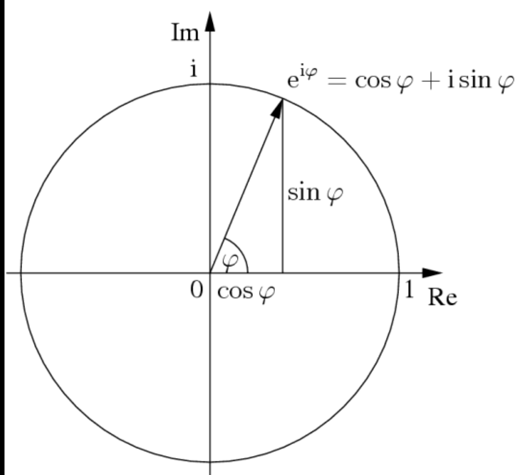
✓ Cartesian form:
Every complex number z = x + yi can be represented by a point on the Cartesian plane, known as complex plane or z-plane or Argand plane and the diagram is called the Argand diagram, by the ordered pair (x, y).

✓ Trigonometric/Polar representation:
$Z = r(\cos\theta + \sin\theta)$

r is the modulus

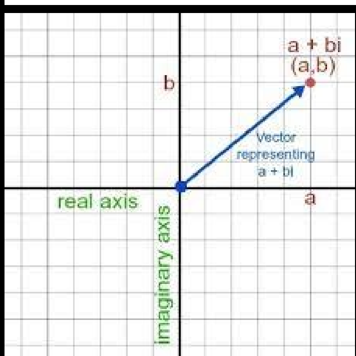
✓ **Euler's Representation:**

$z = re^{i\theta}$, where $|z| = r$, $\arg(z) = \theta$ and $z = re^{i\theta}$, where $|z| = r$, $\arg(z) = -\theta$



✓ **Vector representation:**

Every complex number can be expressed as the position vector of a point, if the point P represents a complex number z such that $\overline{OP} = z$, $|\overline{OP}| = |z|$



Here O is (0,0) and P is (a,b)