SYSTEM OF EQUATIONS BY THE GRAPHING METHOD AND ALGEBRAIC METHOD

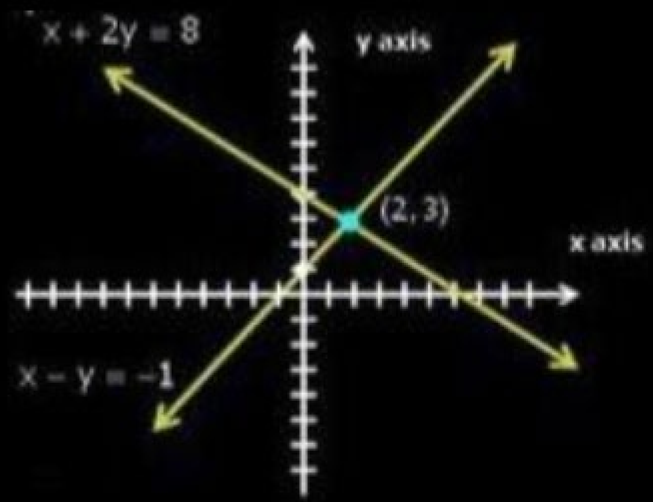
Let's put a system composed of these two equations

1: X + 2Y = 8 **2: X-Y = -1**

This would give us two vectors, to get the solution for both of these equations, we can get the meeting points between them

Which is only (2,3), so the solution is {(2,3)}

Which we can solve with



SUBSTATION

- X+2Y = 8 becomes **X = 8-2Y**
- Add that to the 2nd equation so it becomes **8 – 2Y – Y = -1**
- Which becomes **3Y = 9**
- **Y = 3**
- then in the first equation
- **X = 8 – 2 * 3 = 2**
- **So X = 2, Y = 3**
- **Solution = {(2,3)}**

MULTIPLY & ELIMINATION

- Multiply the 2nd equation by 2 so it becomes **2X – 2Y = -2**
- Add them together **and eliminate the Ys**
- **3X = 6**
- X = 2
- In any equation replace X with 2 and then you'll find that Y is equal to **3**
- **So X = 2, Y = 3**
- **Solution = {(2,3)}**

MATRIX

A system of numbers composed in the shape of **rows (horizontal) & columns (vertical)**

$$\begin{array}{cc} & \text{column} \quad \text{column} \\ \text{row} & a \quad\quad b \\ \text{row} & c \quad\quad D \end{array} \longrightarrow 2 \times 2 \text{ matrix}$$

You get the size of the matrix like this **"row count x column count"** so the one above is a 2x2 matrix

EQUAL OF TWO MATRICES

For two matrices to be equal, they most have the same values in the same order

$$\begin{bmatrix} 1, & 2 \\ 5, & 10 \end{bmatrix} \quad \begin{bmatrix} 1, & 2 \\ 5, & 10 \end{bmatrix} \qquad \Bigg| \qquad \begin{bmatrix} 1, & 5 \\ 2, & 10 \end{bmatrix} \quad \begin{bmatrix} 1, & 2 \\ 5, & 10 \end{bmatrix}$$

EQUAL NOT EQUAL

TRANSPOSE OF MATRIX

converting row to column and column to row by its order

If the matrix **A** Is equal to, then its transpose is equal to A^T

$$A = \begin{matrix} a & d \\ b & e \\ c & f \end{matrix} \longrightarrow A^T = \begin{matrix} a & b & c \\ d & e & f \end{matrix}$$

become row

just like how ^2 cancels out the $\sqrt{\ }$, the transpose of the transpose, returns the normal one

$$(A^T)^T = A$$

$$\begin{matrix} a & b & c \\ d & e & f \end{matrix} \rightarrow \text{column} \qquad \begin{matrix} a & d \\ b & e \\ c & f \end{matrix}$$

become

SYMMETRIC AND SKEW SYMMETRIC MATRIX

Symmetric matrix is a matrix that **is equal** to its transpose

$$A = A^T$$

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 3 & 4 \\ 2 & 4 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 2 \\ 1 & 3 & 4 \\ 2 & 4 & 5 \end{bmatrix}$$

Skew Symmetric matrix is a matrix that **is equal** to its (transpose * -1)

$$B = B^T \times -1$$

$$\begin{bmatrix} 0 & 2 & 4 \\ -2 & 0 & 3 \\ -4 & -3 & 0 \end{bmatrix} = \left(\begin{bmatrix} 0 & -2 & -4 \\ 2 & 0 & -3 \\ 4 & 3 & 0 \end{bmatrix} \times -1 \right)$$

$$\downarrow$$

$$\begin{bmatrix} 0 & 2 & 4 \\ -2 & 0 & 3 \\ -4 & -3 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 4 \\ 4 & 2 \\ 2 & 4 \end{bmatrix}$$

$$\downarrow$$

$$A^T = \begin{bmatrix} 2, 4, 2 \\ 4, 2, 4 \end{bmatrix}$$

$$\downarrow$$

$$A^T \times -1 = \begin{bmatrix} -2, -4, -2 \\ -4, -2, -4 \end{bmatrix}$$

0, [illegible]

OPERATIONS OF MATRICES

ADDITION AND SUBTRACTION

To add/subtract matrices, they need to be on the same order, so you can do it with two 2x2s matrices, but not with smth like a 2x3 and a 3x2

To add/subtract, just add/subtract the parralels

$$\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 5 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 5 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 7 & 8 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 4 \\ 7 & 2 \\ 5 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 1 & 5 \\ 6 & 9 & 7 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 33 & 39 & 43 \\ 33 & 25 & 49 \\ 69 & 86 & 88 \end{bmatrix}$$

MULTIPLICATION

To multiply two matrices, the number of columns in the first matrix **MUST EQUAL** the number of rows in the second matrix

The resultant matrix's size will be

The number of rows in the first matrix x **the number of columns in the second matrix**

Now for each row in the first one, you're gonna multiply it with each column of the 2nd row, and I mean by "multiplying" is taking each item from from the row, multiplying it with its parallel in order from the 2nd column, and then adding them all together, so let's try

$$\underset{1\times 3}{A \begin{bmatrix} 3 & 1 & 4 \end{bmatrix}} \times \overset{3\times 2}{B \begin{bmatrix} 4 & 3 \\ 2 & 5 \\ 6 & 8 \end{bmatrix}}$$

$$AB \begin{bmatrix} 3(4) + 1(2) + 4(6) = 38 & 3(3) + 1(5) + 4(8) = 38 \end{bmatrix}$$

$$1\times 2 = \begin{bmatrix} 38 & 46 \end{bmatrix}$$

NOTE

B x A can't happen because when multiplying B first, the number of columns in the first matrix won't equal the number of rows in the second matrix

Gauss Jordan Elimination

We're going to use it to a system equation with 3 variables

$$x + y - z = 7$$
$$x - y + 2z = 3$$
$$2x + y + z = 9$$

Now, the first thing we need to do, is to convert this into an augmented matrix

We can do this by writing the coefficients/actual numbers, we separate the left and the right side with a line

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 1 & -1 & 2 & 3 \\ 2 & 1 & 1 & 9 \end{array}\right]$$

Now what we need, is to check that these 3 numbers, and these 3, are zeros, and if the numbers in the middle forming a diagonal, are 1s only

This form is what we call a **reduced row echelon form**

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & ? \\ 0 & 1 & 0 & ? \\ 0 & 0 & 1 & ? \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 1 & -1 & 2 & 3 \\ 2 & 1 & 1 & 9 \end{array}\right]$$

When it is in RREF form, the 3 values on the other side are gonna equal to x , y, z

So now, let's convert this matrix into RREF form

Turning the bottom-left side into 0s

- Subtract row 1 from row 2, and replace row 2 with the result "to turn the unwanted 1 to a zero"

$$\begin{array}{cccc} 0 & 2 & -3 & 4 \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 1 & -1 & 2 & 3 \\ 2 & 1 & 1 & 9 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 0 & 2 & -3 & 4 \\ 2 & 1 & 1 & 9 \end{array}\right]$$

it became 0 "what we want"

- Multiply row 1 with -2 and add the result to row 3 "to turn the 2 into a zero"

$$\begin{array}{cccc} 0 & -1 & 3 & -5 \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 0 & 2 & -3 & 4 \\ 2 & 1 & 1 & 9 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 0 & 2 & -3 & 4 \\ 0 & -1 & 3 & -5 \end{array}\right]$$

- Multiply row 3 by 2, and then add it to row 2, and replace row 3 with the result "turn the -1 into 0"

it became 0

$$\begin{array}{cccc} 0 & 0 & 3 & -6 \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 0 & 2 & -3 & 4 \\ 0 & -1 & 3 & -5 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 0 & 2 & -3 & 4 \\ 0 & 0 & 3 & -6 \end{array}\right]$$

"it became 0"

Turning the top-right side into 0s

- Multiply row 1 by 3 and subtract it from row 2 and apply the results to row 1 "turning -1 to 0"

$$\begin{array}{cccc} 3 & 1 & 0 & 17 \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 0 & 2 & -3 & 4 \\ 0 & 0 & 3 & -6 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 3 & 1 & 0 & 17 \\ 0 & 2 & -3 & 4 \\ 0 & 0 & 3 & -6 \end{array}\right]$$

- Add row 2 to row 3 and apply the results to row 2 "turns -3 to 0"

$$\begin{array}{cccc} 0 & 2 & 0 & -2 \end{array}$$
$$\left[\begin{array}{ccc|c} 3 & 1 & 0 & 17 \\ 0 & 2 & -3 & 4 \\ 0 & 0 & 3 & -6 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 3 & 1 & 0 & 17 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 3 & -6 \end{array}\right]$$

- Multiply row 1 by 2, then subtract it from row 2, and apply the results to row 1

$$\begin{array}{cccc} 6 & 0 & 0 & 36 \end{array}$$
$$\left[\begin{array}{ccc|c} 3 & 1 & 0 & 17 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 3 & -6 \end{array}\right] \quad \left[\begin{array}{ccc|c} 6 & 0 & 0 & 36 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 3 & -6 \end{array}\right]$$

Turning the diagonals into 1s

- First row -> multiply with 1/6
- Second row -> multiply with ½
- Third row -> multiply with 1/3

$$\begin{array}{c} \times\frac{1}{6} \\ \times\frac{1}{2} \\ \times\frac{1}{3} \end{array}\left[\begin{array}{ccc|c} 6 & 0 & 0 & 36 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 3 & -6 \end{array}\right] \rightarrow \underbrace{\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{array}\right]}_{RREF} \rightarrow \begin{array}{l} x = 6 \\ y = -1 \\ z = -2 \end{array}$$

So, RREF is basically to make a diagonal line of 1s to cut the left side and everything else is 0 and the right side will be the answer

So to turn a matrix into RREF, just operate on the rows , turning stuff into zeros at first, then make the diagonal line into 1s

LINEAR PROGRAMMING

In it, there are two types of equations (constraint function – objective function)

CONSTRAINTS (2 variables)

- They have 2 variables, they help you solve the objective function, for example

- $x + y \le 20$
- $3x + 4y \le 72$

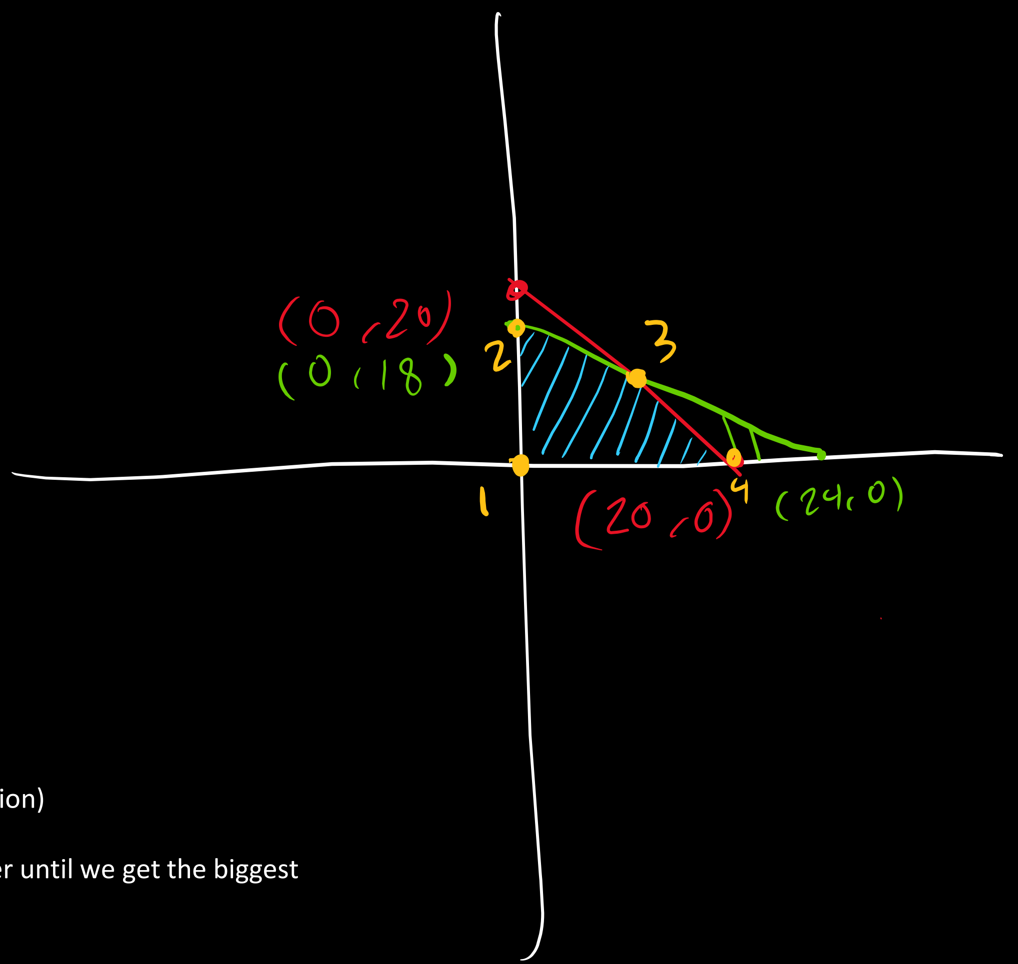
To solve these types of problems, you have to plot them on a graph, you do this by

RED

1. Get x and y intercepts
    1. To get x intercept -> replace y with 0
        1. **$x + 0 \le 20$**
        2. **$x \le 20$**
        3. Because there is an equal, so let's say x is 20
    2. To get y intercept -> replace x with 0
        1. **$0 + y \le 20$**
        2. **$y \le 20$**
        3. Because there is an equal, so let's say y is 20
    3. Just connect them

GREEN

1. Get x and y intercepts
    1. To get x intercept -> replace y with 0
        1. **$3x + 0 \le 72$**
        2. **$x \le 24$**
        3. Because there is an equal, so let's say x is 24
    2. To get y intercept -> replace x with 0
        1. **$0 + 4y \le 72$**
        2. **$y \le 18$**
        3. Because there is an equal, so let's say y is 18
    3. Just connect them



now after you plot the two graphs, identify the corner points (of the overlapping region)
At one of these corner points, we are going to get a maximum of the graph
So in the objective function **Z = 4X + 5Y**, we are going to plug the cords of each corner until we get the biggest one

CORNERS	X	Y	Z
1	0	0	4*0 + 5*0 = **0**
2	0	18	4*0 + 5*18 = **90**
3	8	12	4*8 + 5*12 = **92**
4	20	0	4*20 + 5*0 = **80**

NOTE : how did we get corner 3?
We need to solve the two constraints and get the interception between them (we learnt that in page 1)

$$3x + 4y \le 72$$
$$-4x + -4y \le -4\times 20 \quad +$$
$$-x \le -8$$
$$x \le 8$$
$$8 + y \le 20$$
$$y \le 12$$
$$\{(8, 12)\}$$

So corner 3 has the maximum Z value, where x is 8, and y is 12

EXAMPLE 1 : LINEAR PROGRAMMING

A company receives in sales $20 per book and $18 per calculator. The cost per unit to manufacture each book and calculator are $5 and $4 respectively. The monthly (30 day) cost must not exceed $27,000 per month. If the manufacturing equipment used by the company takes 5 minutes to produce a book and 15 minutes to produce a calculator, how many books and calculators should the company make to maximize profit? Determine the maximum profit the company earns in a 30 day period.

	Book (B)	Calculator (C)
profit	20	18
cost	5	4
time	5	15

Max cost in 30 days = 27000
Max profit in 30 days = ??
(let's say that B is in X, and C is in Y)
Now we need to write 2 constraints, and 1 objective function

OBJECTIVE FUNCTION

So let's say that,
we get 20 dollars per number of books,
and get 18 dollars per number of calculators,
which is equal to the profit **S**

S = 20B + 18C

CONSTRAINTS FUNCTIONS

Let's say that,
We spend 5 dollars per book count, and 4 dollars per calculator counter
and our maximum in costs is 27000, so

5B + 4C ≤ 27000

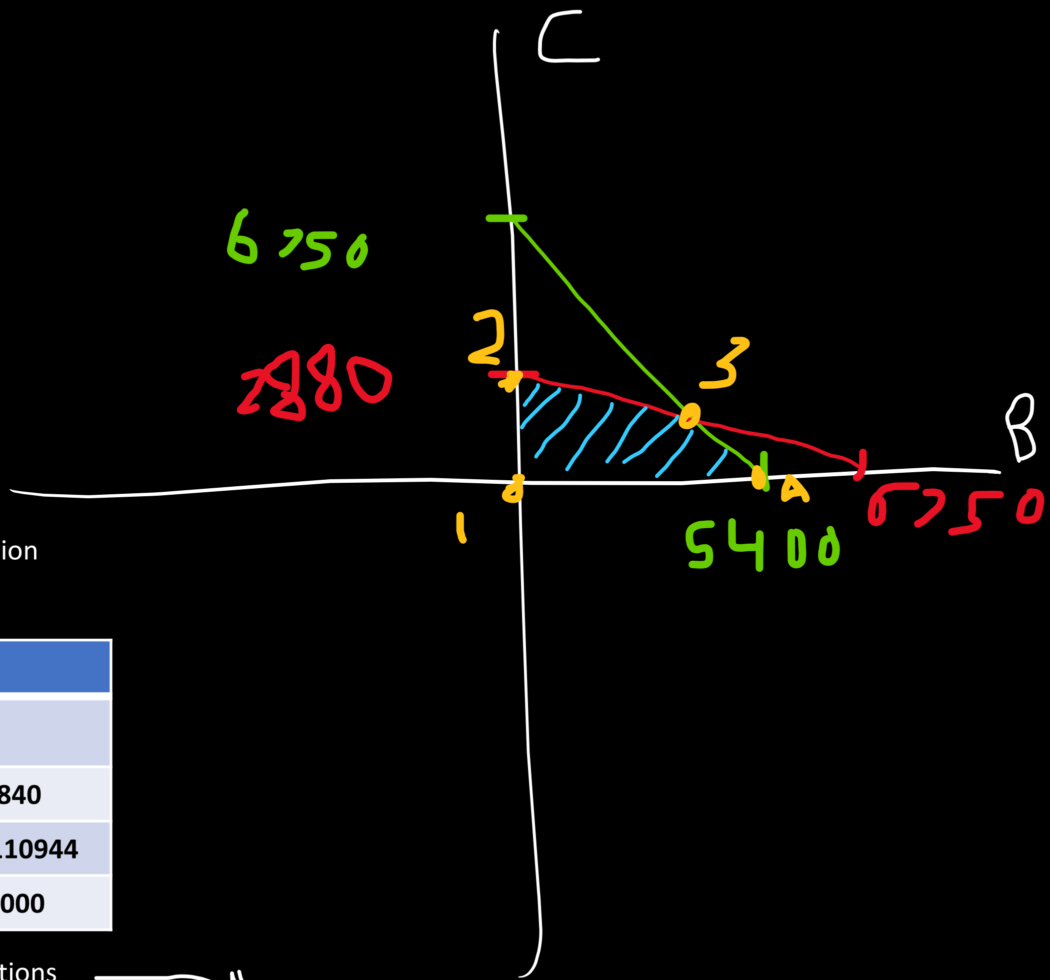
Let's say that,
We spent 5 minutes per book count, and 15 minutes per calculator count,
and our maximum time is (30*24*60) -> 43,200 minutes, so

5B + 15C ≤ 43200

So
Objective function -> S = 20B + 18C
Constraint functions -> 5B + 4C ≤ 27000
5B + 15C ≤ 43200

GREEN
- X (B) intercept
    - 5B ≤ **27000**
    - B ≤ **5400**
- Y (C) intercept
    - 4C ≤ **27000**
    - C ≤ **6750**

RED
- X (B) intercept
    - 5B ≤ **43200**
    - B ≤ **8640**
- Y (C) intercept
    - 15C ≤ **43200**
    - C ≤ **2880**



After plotting, we find the maximum value by replacing B,C in the objective function
By the cords of each corner

CORNERS	B	C	S
1	0	0	20*0 + 18*0 = **0**
2	0	2880	20*0 + 18*2880 = **51,840**
3	4222.4	1472	20*4222.4 + 18*1472 = **110944**
4	5400	0	5400*20 + 18*0 = **108000**

To get corner 3, we need to get the intercept/solution of the two constraint functions

5B + 4C ≤ 27000
-5B – 15C ≤ -43200 (*-1)
-11C ≤ -16200
C ≤ 1472 -> (1)
5B + 4 * 1472 ≤ 27000
B ≤ 4222.4 -> (2)
Intersection {(4222.4, 1472)}

Corner 3 has the maximum profit, so
The amount of books -> **4222.4**
The amount of calculators -> **1472**
The maximum profit -> **110944 $**