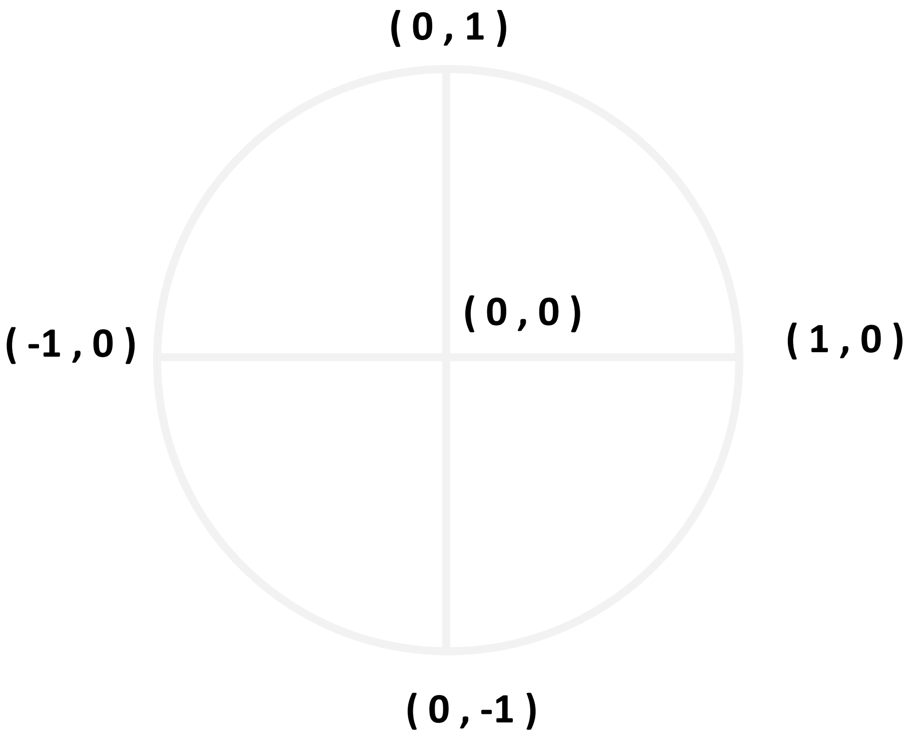
THE UNIT CIRCLE

it’s a circle that has a radius of 1 (a unit), and its center is where the x and y axis cross



OBSERVATION 1:

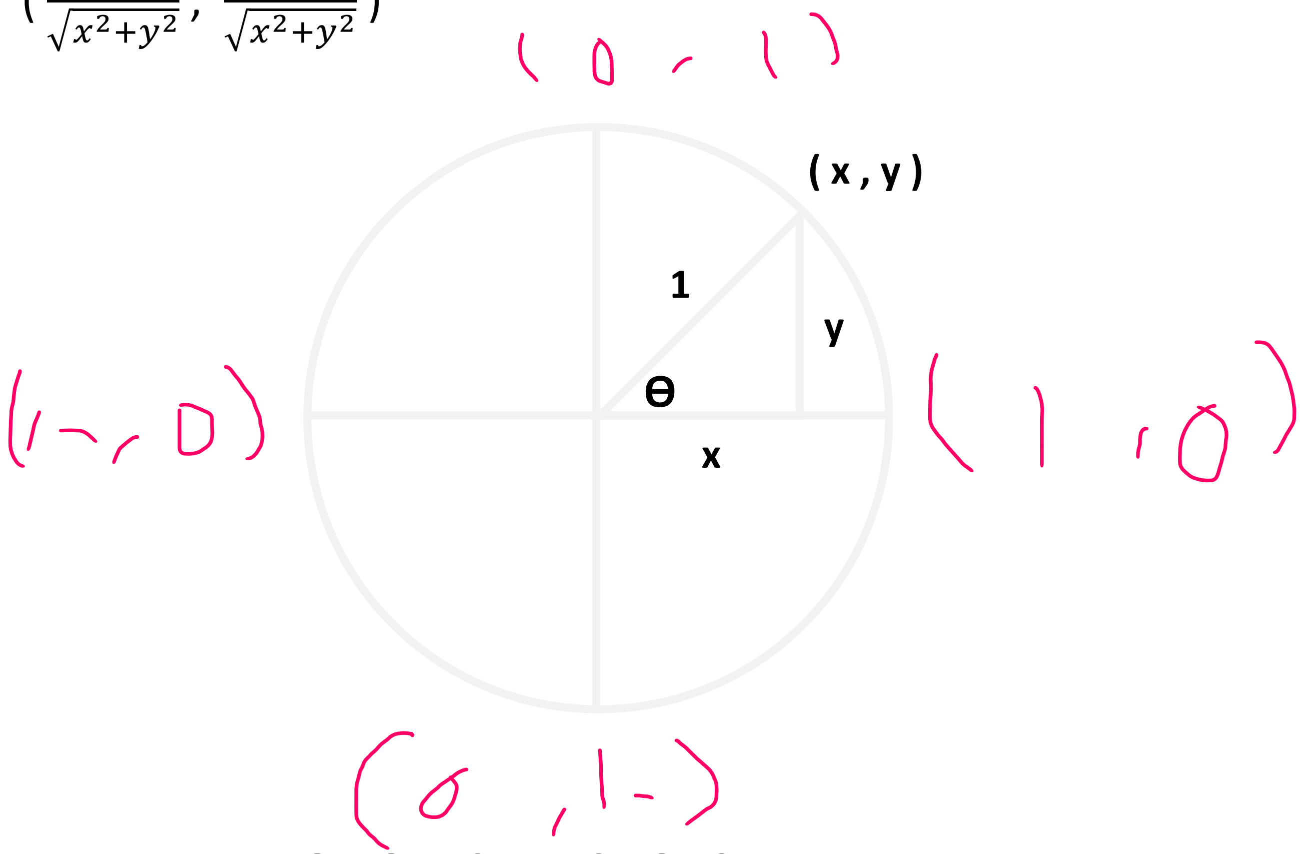
If **X** and **Y** ϵ The unit circle

Then [$x^2 + y^2 = 1$] according to Pythagorean Theorem

X ϵ [-1 , 1] and Y ϵ [-1 , 1]

If x or y are not in that range then we use this formula

$\left(\frac{x}{\sqrt{x^2+y^2}} , \frac{y}{\sqrt{x^2+y^2}} \right)$



THE MAIN TRIGINOMIC FUNCTIONS ARE

- Sin ϴ = $\frac{opposite}{hypotenuse} = \frac{y}{1}$ = y
- cos ϴ = $\frac{adjacent}{hypotenuse} = \frac{x}{1}$ = x
- tan ϴ = $\frac{opposite}{adjacent} = \frac{y}{x}$

OBSERVATION 2:

- you can say (cos ϴ , sin ϴ) instead of (x , y)
- you can say that tan is = $\frac{\sin\theta}{\cos\theta}$
- sinϴ ϵ [-1 , 1]
- cosϴ ϵ [-1 , 1]

The reciprocal trig ratios

Secant (sec) = $\boldsymbol{\frac{1}{cos}}$

cosecant (csc) = $\boldsymbol{\frac{1}{sin}}$

cotan (cot) = $\boldsymbol{\frac{1}{tan}}$ **=** $\boldsymbol{\frac{cos}{sin}}$

Directed angle

What is the difference from saying angle (ABC) to saying angle (CBA)

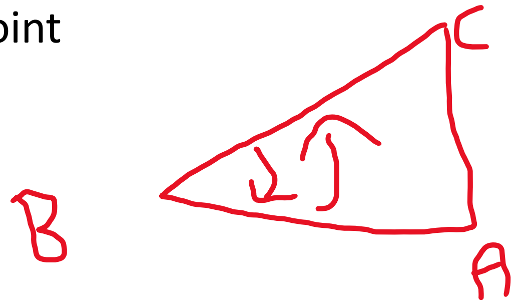
Well, in normal angles, it doesn’t make a difference

But in **Directed Angles**

The type and sign of the angle will change

Firstly, what is a directed angle

It’s an ordered pair of rays that are the sides of the angle where the **x** is called the **beginner ray** and **y** is called the **end ray** and they have a connection point which is the angle’s vertex point



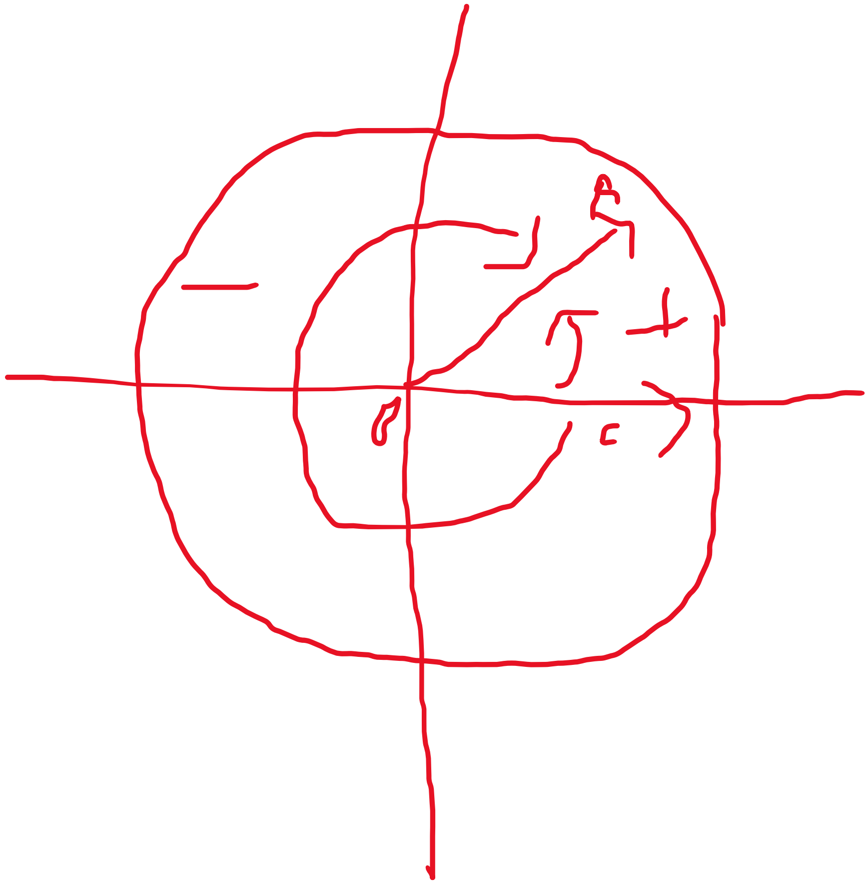
Angle ABC -> (BA , BC)

Angle CBA -> (BC , BA)

If the angle is moving **clockwise**, then we have a **negative** angle

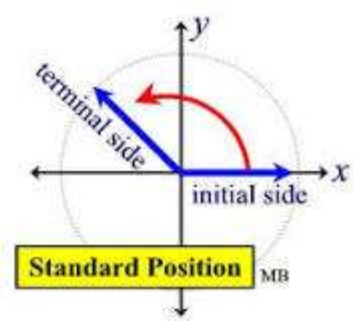
If the angle is moving **counter-clockwise**, then we have a **positive** angle

$\widehat{CBA}$

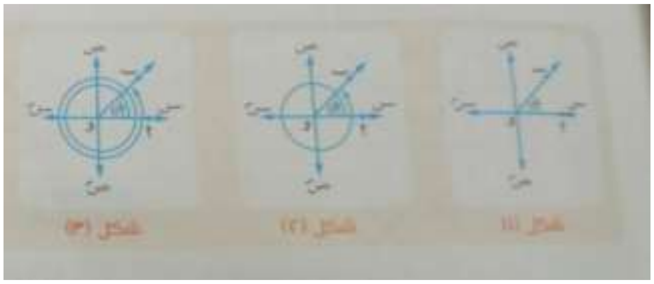


ato have a directed angle, the angle should be in a standard position, AKA

- the vertex of the angle is on the origin point
- and the initial side is on the positive x axis



Co-terminal/equivelant angles



It's a direcred angle that has the same end ray, which means that you can get a coterminal angle by subtracting or adding 360^{o}

So any to get a coterminal angle it has the law of

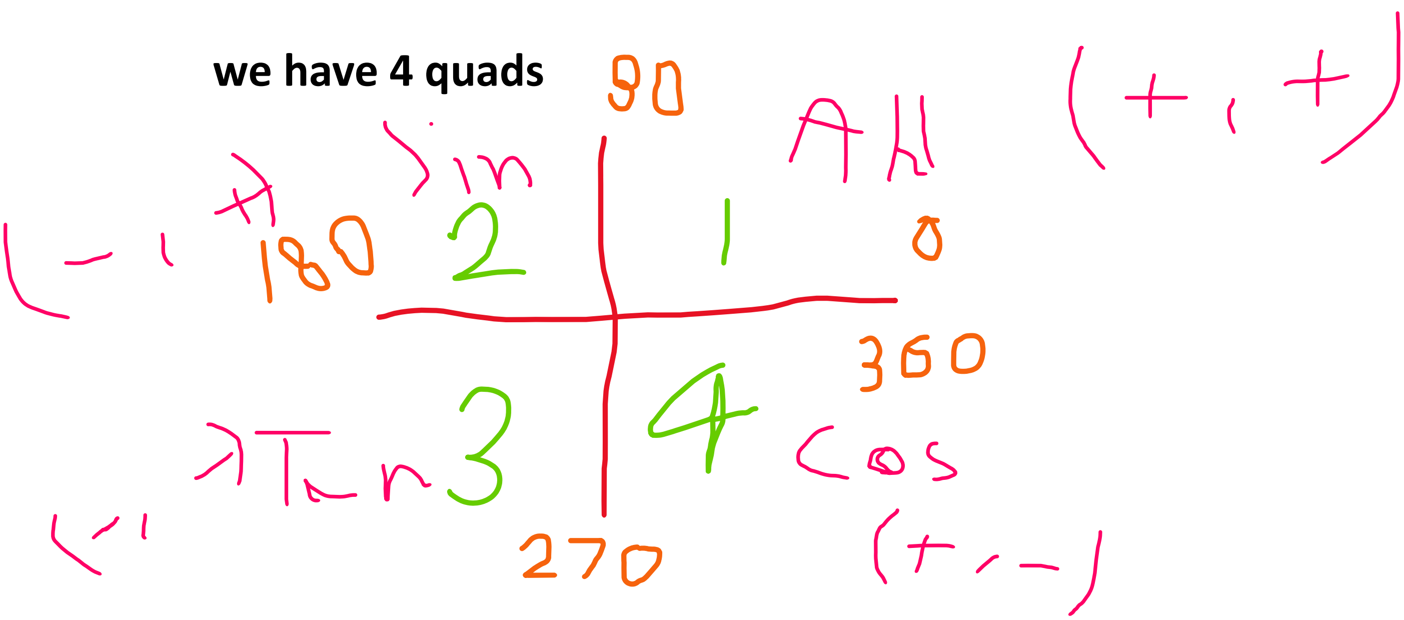
Coterminal angle = angle ± (n x 360)

So 60, 420, -300, 780 are all coterminal angles

Which means that any angle – its coterminal angle = 360

For example -> 60 - -300 = 360

we have 4 quads



When an angle is either 0, 90, 180, 270, 360

It's a quadrantal angle

To get what quadrant we are on, we need the smallest positive angle

So if we are less than 0 we add 360 until we aren't

And if we are bigger than 360 we subtract 360

So -10 = 350 -> 4th quadrant

50 -> 1st quadrant

400 = 40 -> 1st quadrant

we can use these laws to get the ref angle

- **1st** angle = ref
- **2nd** ref = 180 – angle
- **3rd** ref = angle – 180
- **4th** ref = 360 - angle

Quadrants and trig functions relation

The mentioned one for each quad is + while the unmentioned is -

1st quadrant -> x (cosine, cosecant) – y (sin, secant) – (tan, cotan)

2nd quadrant -> y (sin, secant)

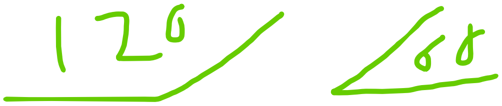
3rd quadrant -> (tan, cotan)

4th quadrant -> x (cosine, secant)

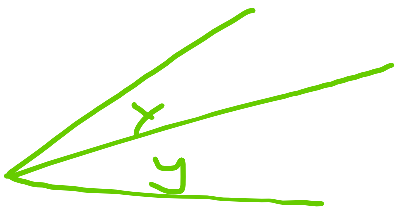
Related Angles

Complementary -> when the sum of two unconnected angles is 90^o

Supplementary -> when the sum of two unconnected angles is 180°

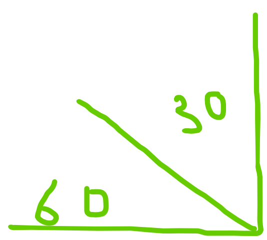


Adjacent -> when two angles are connected

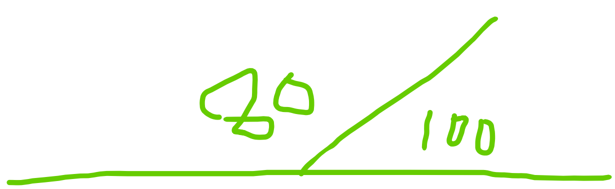


Linear pair ->

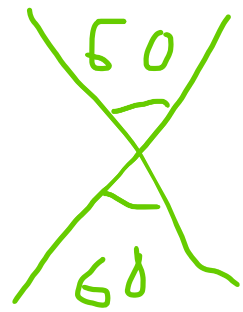
- complementary adjacent -> when the sum of two connected angles is 90°



- supplementary adjacent -> when the sum of two connected angles is 180°



vertically opposite angles -> when two angles are equal and opposite of each other



IF A + B = 180 (SUPPLEMENTARY)

Sin(180 - Ɵ) = sin (Ɵ)

Sin (A) = sin (B)

Cos (A) = cos(B)

Tan (A) = tan (B)

Sin (A) – Sin (B) = 0

IF A + B = 90 (COMPLEMENTARY)

Sin(90 - Ɵ) = cos (Ɵ)

Sin A = Cos B

Cos A = Sin B

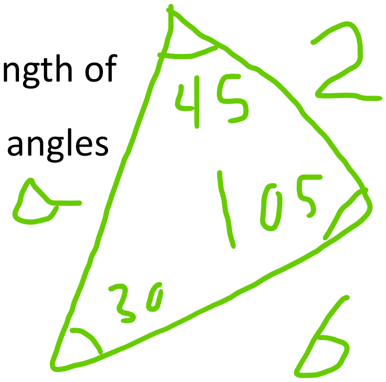
Tan A = Cotan B

Cos (-Ɵ) = cos (Ɵ)

Law of sines

it states that the sine of an angle divided by the length of the opposite side is the same ratio for the other 2 angles

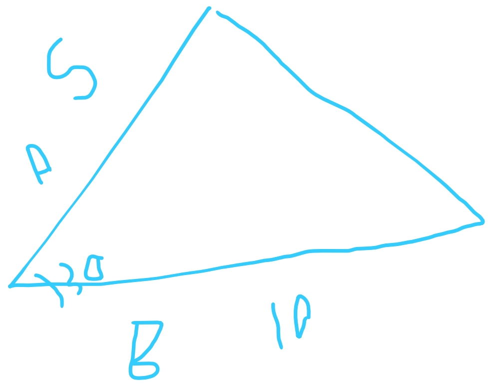
$$\frac{\sin 3o}{2} = \frac{\sin 45}{b} = \frac{\sin 105}{a}$$

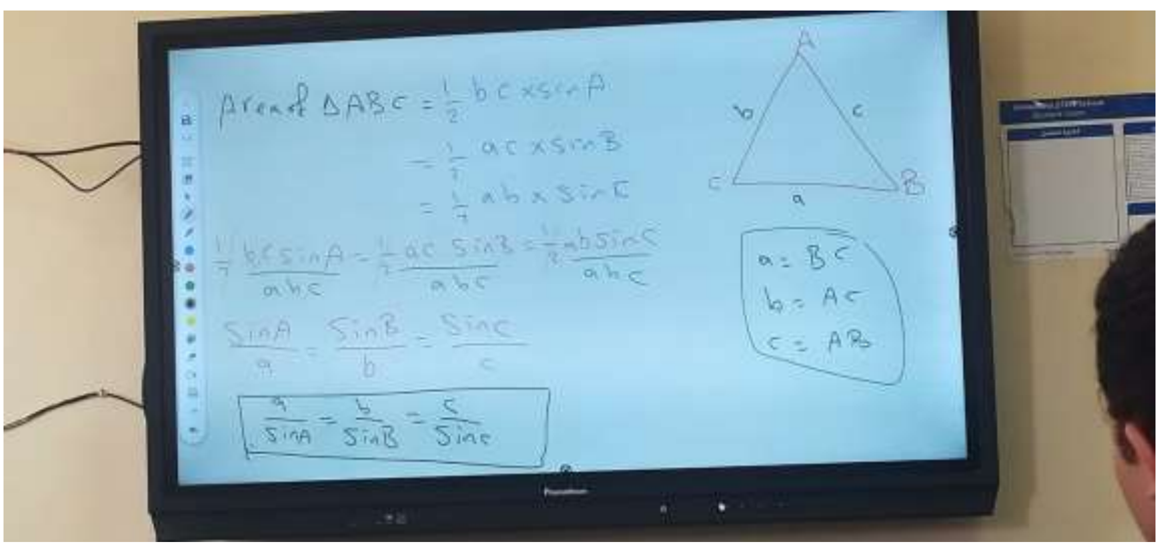


$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

If we want to get the area of a triangle

area = $\frac{1}{2}ab.\sin\,(\theta)$





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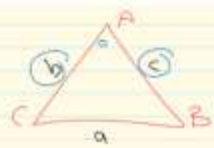
Law of cosines

it states that you can get the length of side when the opposite angle is between two known sides

a = $\sqrt{b^2 + c^2 - 2bc.cos(\theta)}$

$a^2 = b^2 + c^2 - 2bc\cos A$



$b^2 = a^2 + c^2 - 2ac\cos B$

$\triangle XYZ$

$y^2 = x^2 + z^2 - 2xz\cos Y$

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\triangle DEF$

$\cos E = \frac{d^2 + f^2 - e^2}{2df}$

$\boxed{\cos 60 = \frac{1}{2} \quad \cos 120 = -\frac{1}{2}}$

If $m(\angle X) + m(\angle Y) = 180°$

$\cos X = -\cos Y$

$\cos X + \cos Y =$ zero

If $\sin\theta = -\frac{\sqrt{3}}{2}$, where θ is $\boxed{\text{Smallest}}$ positive angle

$\theta = ?$

$\therefore \beta = \sin^{-1}\left(\left|-\frac{\sqrt{3}}{2}\right|\right) = 60$

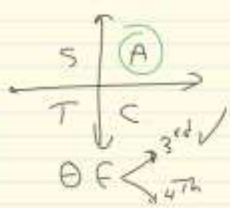
If $\theta \in 1^{st}$ quad $\quad \theta = \beta$

If $\theta \in 2^{nd}$ quad $\quad \theta = 180 - \beta$

$\boxed{\text{If } \theta \in 3^{rd} \text{ quad} \quad \theta = 180 + \beta}$

If $\theta \in 4^{th}$ quad $\quad \theta = 360 - \beta$

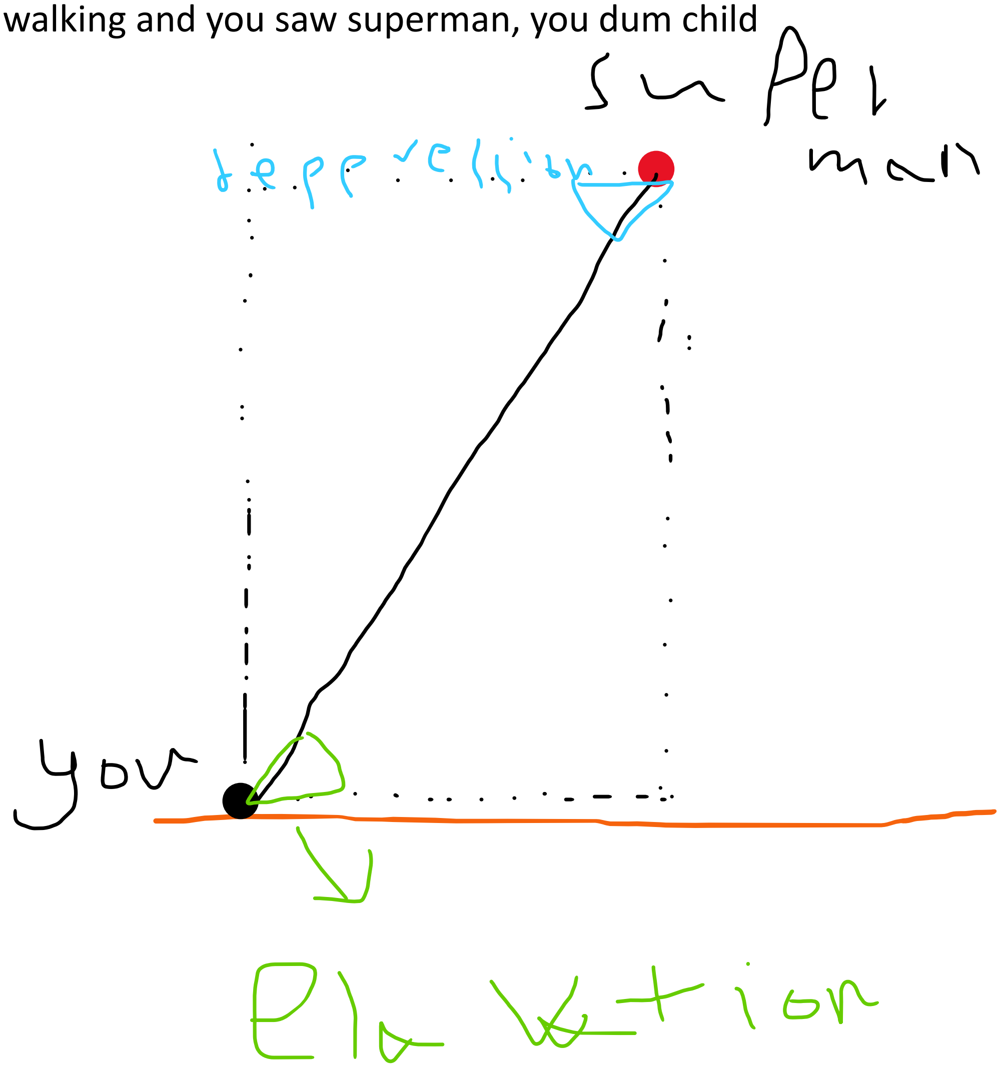
$\therefore \theta = 180 + 60 = 240°$



Angle of elevation and Angle of depression

LET'S DO AN EXAMPLE

So let's say you where walking and you saw superman, you dum child



TYPES OF MEASURING ANGLES

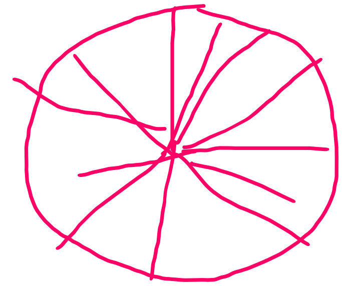
Radian – Degree – Gradian

Degree

We assume a circle is 360^{o}

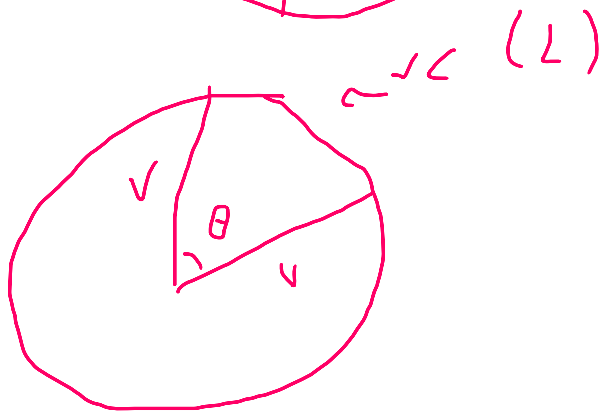
Where $1^o = 60'$

And $1' = 60''$



Radian

$\Theta^{rad} = \frac{l}{r}$



We have two types of angles

Central and inscribed

Central is in the middle

Inscribed is on the edge

Π = 180

How to convert between degree and radian

$$\frac{\boldsymbol{\theta^{rad}}}{\boldsymbol{\pi}} = \frac{\boldsymbol{\theta^{o}}}{\mathbf{180}}$$

Let's say $\Theta^{rad} = \frac{5\pi}{12}$

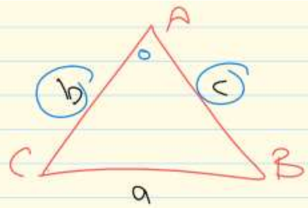
We can replace π with 180 to get the degree angle

So it is 75^o

The unit of degree is $60^o40'23''$

The unit of rad is either 2.4^{rad} or $\frac{\pi}{5}$

$a^2 = b^2 + c^2 - 2bc \cos A$



$b^2 = a^2 + c^2 - 2ac \cos B$

$\triangle XYZ$

$y^2 = x^2 + z^2 - 2xz \cos Y$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}.$$

$\triangle DEF$

$$\cos E = \frac{d^2 + f^2 - e^2}{2df}$$

$\boxed{\begin{array}{l} \cos 60 = \frac{1}{2} \\ \cos 120 = -\frac{1}{2} \end{array}}$

If $m(\angle X) + m(\angle Y) = 180°$

$\cos X = -\cos Y$

$\cos X + \cos Y = \text{zero}$

Q- $\sin\theta = -\frac{\sqrt{3}}{2}$, where θ is $\boxed{\text{Smallest}}$ positive angle

$\theta = ?$

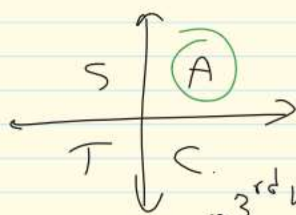
* $\beta = \sin^{-1}\left(\left|-\frac{\sqrt{3}}{2}\right|\right) = 60$

If $\theta \in 1^{st}$ quad. $\theta = \beta$

~~If $\theta \in 2^{nd}$ quad $\theta = 180 - \beta$~~

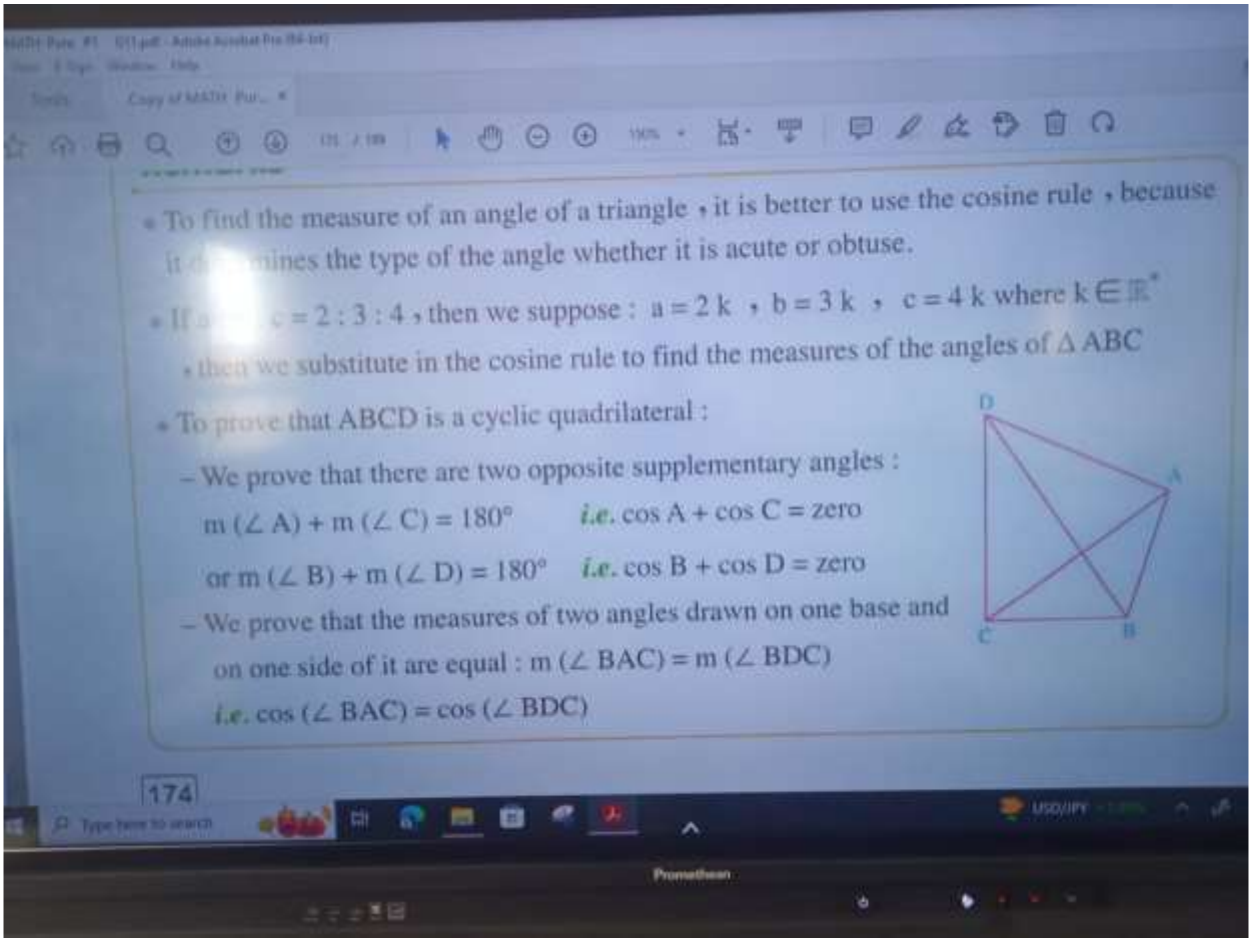
$\boxed{\text{If } \theta \in 3^{rd} \text{ quad} \quad \theta = 180 + \beta}$

If $\theta \in 4^{th}$ quad $\theta = 360 - \beta$



$\theta \in$ 3^{rd} ✓, 4^{th}

$\therefore \theta = 180 + 60 = 240°$

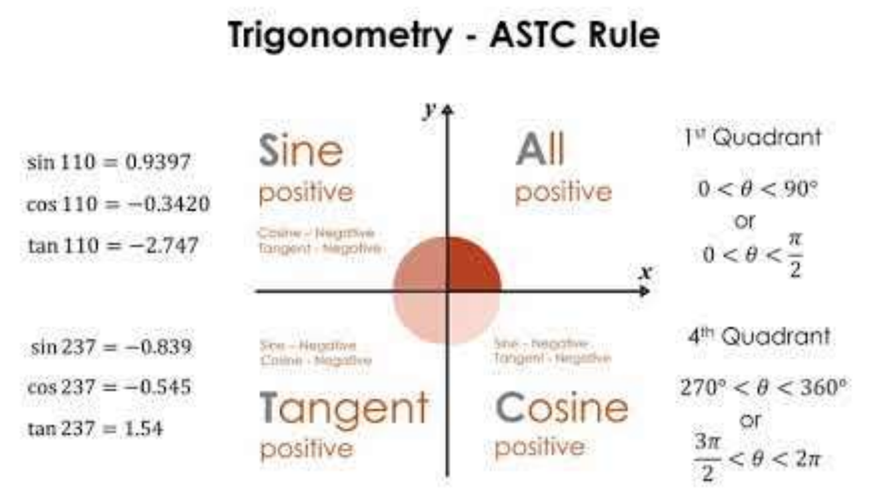


- To find the measure of an angle of a triangle , it is better to use the cosine rule , because it determines the type of the angle whether it is acute or obtuse.
- If $a : b : c = 2 : 3 : 4$, then we suppose : $a = 2k$, $b = 3k$, $c = 4k$ where $k \in \mathbb{R}^*$
  , then we substitute in the cosine rule to find the measures of the angles of ΔABC
- To prove that ABCD is a cyclic quadrilateral :
  - We prove that there are two opposite supplementary angles :
    $m(\angle A) + m(\angle C) = 180°$ *i.e.* $\cos A + \cos C = \text{zero}$
    or $m(\angle B) + m(\angle D) = 180°$ *i.e.* $\cos B + \cos D = \text{zero}$
  - We prove that the measures of two angles drawn on one base and on one side of it are equal : $m(\angle BAC) = m(\angle BDC)$
    i.e. $\cos(\angle BAC) = \cos(\angle BDC)$

NOTES

- HOW TO GET THE INTERIOR ANGLES OF A REGULAR SHAPE
  Angle = $\frac{180(n-2)}{n}$
- Cyclic quadrilateral's opposite angles are supplementary
- The sum of all exterior angles in any shape is 360, so to get a single exterior angle in a regular shape you should just $\frac{360}{n}$

[redacted]



- [redacted]
- When the problem is in radian mode, change the mode in your calculator to radian