DETERMINANT

A scalar value that is a certain function to the entries of a square matrix

It is denoted as “det”, and the determinant of matrix A is denoted as **det A**

Determinant for 2x2 matrices

$$A = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

$$\det A = \begin{bmatrix} a & c \\ b & d \end{bmatrix} = ad - bc$$

$$x\begin{bmatrix} a & b \\ c & d \end{bmatrix} = x(ad - bc)$$

Determinant for 3x3 matrices

$$A = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

$$\det A = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

NOTE

you go from LEFT to RIGHT

$$\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} \quad \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} \quad \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

$$g\begin{bmatrix} b & e \\ c & f \end{bmatrix} - d\begin{bmatrix} b & h \\ c & i \end{bmatrix} + a\begin{bmatrix} e & h \\ f & i \end{bmatrix}$$

$$g(bf - ec) - d(bi - hc) + a(ei - hf)$$

middle part is −

AREA OF TRIANGLE WITH MATRICES

Let’s say you have a triangle with points A, B, C
where
A (a, b)
B (c, d)
C (e, f)

The area is equal to

always 1

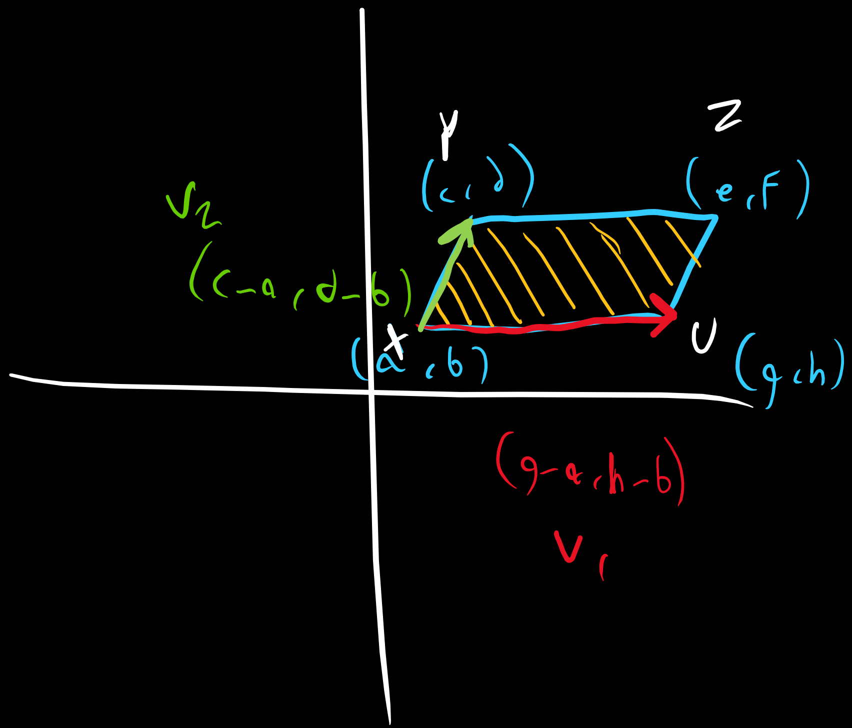
$$\frac{1}{2}\begin{array}{c} \begin{bmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{bmatrix} \end{array} \begin{array}{l} A \\ B \\ C \end{array}$$

The determinant of this matrix multiplied by 1.5

$$\frac{1}{2} \times \left([cf - de] - b(c - e) + a(d - f) \right)$$

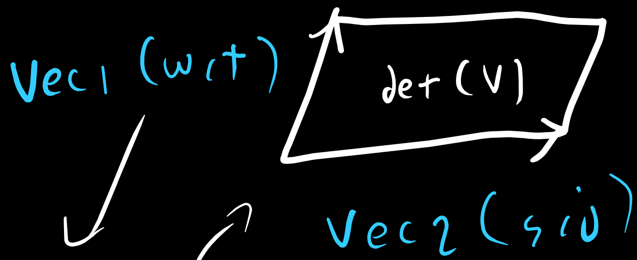
AREA OF PARALLELOGRAM

So let’s say you have this parallelogram with points
X (a, b), Y (c, d), Z (e, f), U (g, h)
Firstly, get the 2 main vectors that are going in the positive direction



Vector 1 = U – X = (g - a , h - b)
Vector 2 = Y – X = (c - a , d - b)

Let’s denote vec1 as (w, t)
Let’s denote vec2 as (s, j)



matrix
$$V \leftarrow \begin{bmatrix} w & t \\ s & j \end{bmatrix}$$

Then put vector 1 on top of vector 2 in a 2x2 matrix
The area of the parallelogram is equal to the determinant of that matrix

$$\text{area} = wj - ts$$

FINDING MULTIPLICATIVE INVERSE OF MATRICES

it is the inverse of a matrix and if a matrix is denoted as **A**, then the inverse is denoted as **A^{-1}**
when multiplying **A** and **A^{-1}** you get an identity matrix with the same size

Identiy matrix is a square matrix in the rref, where there is a diagonal line of ones and all other stuff are 0s

$$I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A \begin{bmatrix} a & c \\ b & d \end{bmatrix} \rightarrow \begin{bmatrix} a & c \\ b & d \end{bmatrix} \rightarrow \begin{bmatrix} d & b \\ c & a \end{bmatrix} \rightarrow \begin{bmatrix} d & b \times -1 \\ c \times -1 & a \end{bmatrix}$$

$$\downarrow$$

$$A^{-1} \begin{bmatrix} \frac{d}{\det A} & \frac{-b}{\det A} \\ \frac{-c}{\det A} & \frac{a}{\det A} \end{bmatrix} \leftarrow \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \times \frac{1}{\det A} \leftarrow \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Any matrix that its value/determinant is 0, will not have any multiplicative inverse because 1/0 is 0

2 VARIABLE EQUATION SOLVING WITH CRAMER'S RULE

Let's say you have these 2 equations

2x + y = -1

3x – 5y = -21

$$2x + 1y = -1$$
$$3x - 5y = -21$$

we need an x, y value

$$x = \frac{\det \begin{vmatrix} -1 & 1 \\ -21 & -5 \end{vmatrix}}{\det \begin{vmatrix} 2 & 1 \\ 3 & -5 \end{vmatrix}}$$

$$y = \frac{\det \begin{vmatrix} 2 & -1 \\ 3 & -21 \end{vmatrix}}{\det \begin{vmatrix} 2 & 1 \\ 3 & -5 \end{vmatrix}}$$

These types are called **Determinant Matrix** or **D** or Δ

The matrices above are denoted by stuff like

X matrix $x\Delta$
Y matrix $y\Delta$
Z matrix $z\Delta$
Etc.. $z\Delta$

$$x = \frac{-21-5}{-10-3} = \frac{-26}{-13} = 2$$

$$y = \frac{-42+3}{-10-3} = \frac{-39}{-13} = 3$$

3 VARIABLE EQUATION SOLVING WITH CRAMER'S RULE

Imagine these equations

5x – 2y + 9z = -7
-2x + y – 4z = 5
3x – 10y – 8z = 0

$$5x - 2y + 9z = -7$$
$$-2x + 1y - 4z = 5$$
$$3x - 10y - 8z = 0$$

$$\Delta = \det\begin{bmatrix} 5 & -2 & 9 \\ -2 & 1 & -4 \\ 3 & -10 & -8 \end{bmatrix} =$$

5	-2	9	=	-7
-2	1	-4	=	5
3	-10	-8	=	0

$$x\Delta = \det\begin{bmatrix} -7 & -2 & 9 \\ 5 & 1 & -4 \\ 0 & -10 & -8 \end{bmatrix} =$$

$$y\Delta = \det\begin{bmatrix} 5 & -7 & 9 \\ -2 & 5 & -4 \\ 3 & 0 & -8 \end{bmatrix} =$$

$$z\Delta = \det\begin{bmatrix} 5 & -2 & -7 \\ -2 & 1 & 5 \\ 3 & -10 & 0 \end{bmatrix} =$$

$$x = \frac{\Delta x}{\Delta} = \frac{42}{9} = -\frac{14}{3}$$

$$y = \frac{\Delta y}{\Delta} = \frac{-221}{9} = \frac{-221}{9}$$

$$z = \frac{\Delta z}{\Delta} = \frac{-36}{9} = -4$$

USING MULTIPLICATIVE INVERSE TO SOLVE EQUATIONS

As we used Cramer's rule in solving equations we can also uses the multiplicative inverse of matrices in solving it, the following law explain this :-

$$ax + by = m$$
$$cx + dy = k$$

$$\overset{A}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} m \\ k \end{bmatrix}$$

$$\begin{bmatrix} m \\ k \end{bmatrix} \times \underbrace{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \times \frac{1}{\det A}}_{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} = \begin{bmatrix} x \\ y \end{bmatrix}$$

3x3 zero triangle determinant

in $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

if $d = 0$, $g = 0$, $h = 0$ or $\begin{bmatrix} a & b & c \\ 0 & e & f \\ 0 & 0 & i \end{bmatrix}$

then $\det A = a \times e \times i$