THE CENTER OF MASS (CM)

Average location of mass

It's the average of masses factored by their distances from a reference point

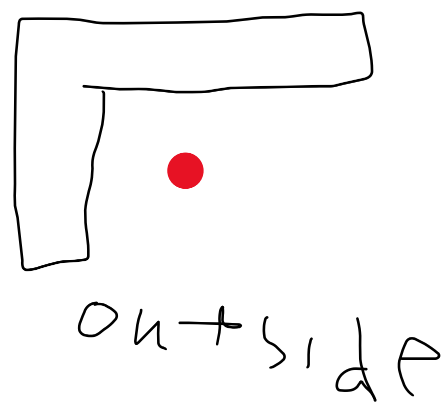
Relation between center of mass and torque

Force at the center of mass -> 0 net torque

DETERMINING THE LOCATION OF THE CENTER OF MASS

SINGLE OBJECT

Location of CM could either be

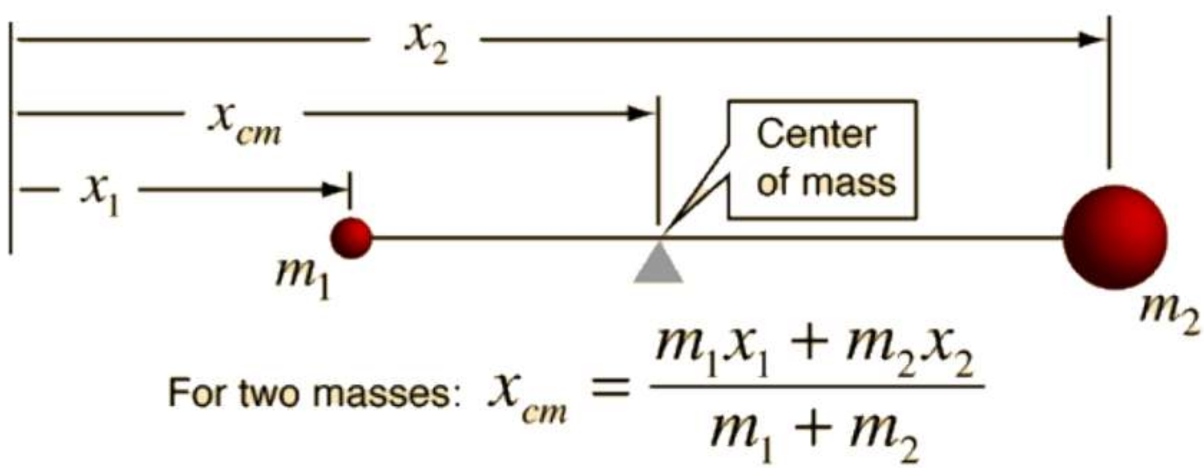


if an object is

- **Uniform** -> CM = geometric center
- **Irregular** -> CM closer to heavier end

MULTIPLE OBJECTS

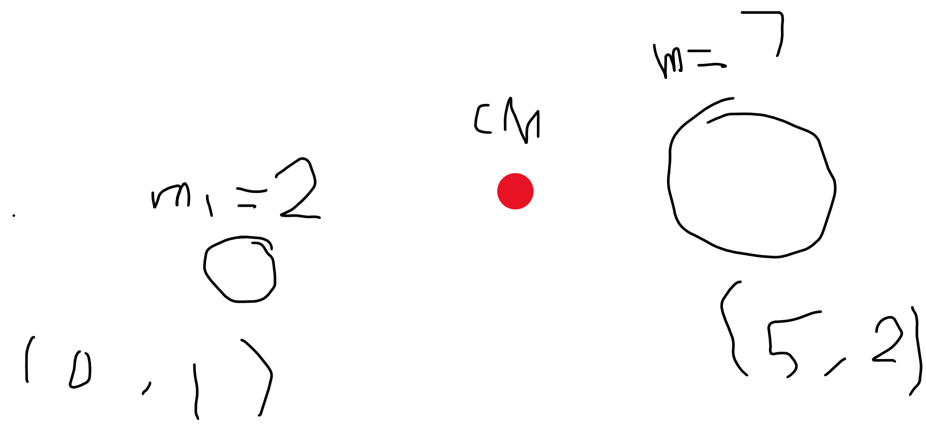
Center of mass for a single axis



Center of mass for more than 1 axis

$$x_{CM} = \frac{\sum_i m_i x_i}{M} \quad y_{CM} = \frac{\sum_i m_i y_i}{M} \quad z_{CM} = \frac{\sum_i m_i z_i}{M}$$

After getting the CM for each axis, we are going to place it in a vector



CM_x = ((2*0)+(5*7))/9 = 3.88

CM_y = ((1*2)+(2*7))/9 = 1.77

CM = (3.88,1.77)

THE CENTER OF GRAVITY (CG)

Average location of weight

The point of the object where the force of gravity is thought to be acting

It depends on the **g** or Gravitational acceleration

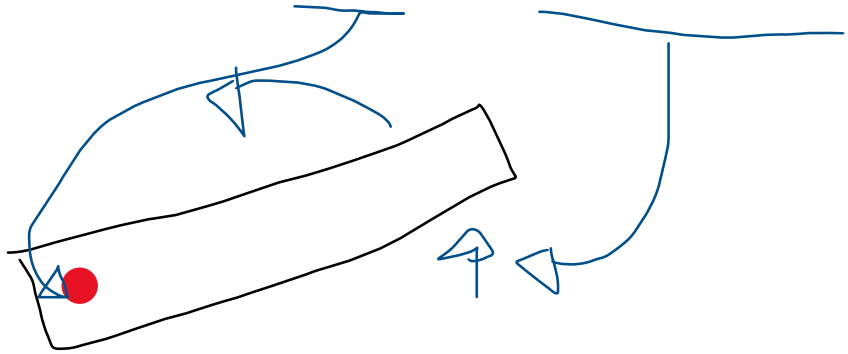
If g is the same throughout the object, **then CG = CM**

You get the center of mass but instead of mass do mass*g

$$x_{cg} = \frac{m_1 g x_1 + m_2 g x_2 + m_3 g x_3}{(m_1 g + m_2 g + m_3 g)}$$

Torque

The rotational or twisting effect of a force from a pivot point when a force applied makes it rotate

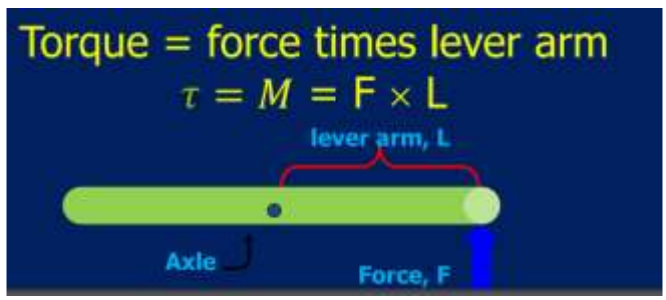


It’s mathematically defined as the cross product of the vector where the anchor point is relative to the point of applied force, and the point of applied force, resulting in an angle or **rotational motion**

Its SI units are **newton-meter or N.m not N/m**

Torque in this case

It’s a force applied at a right angle to a lever, multiplied by the distance from the lever’s fulcrum, pivot, axle, whatever you call it.

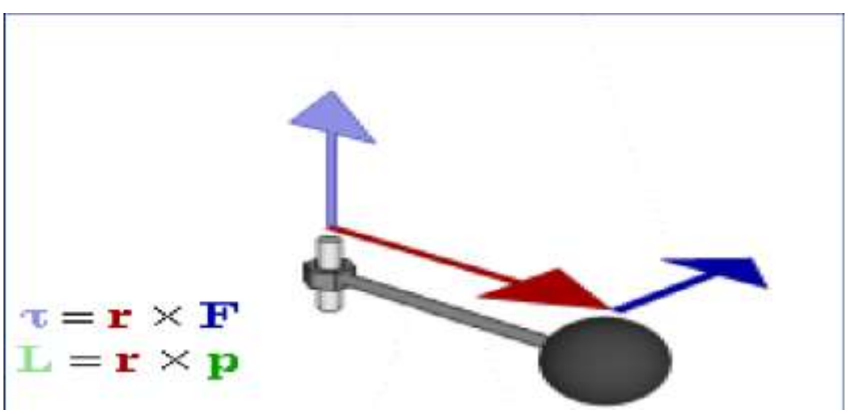


Linear momentum (P)

It’s momentum in a straight line

Angular momentum (L)

It’s momentum in a circle



Torque is **t**, force is **F**, length to pivot is **r**

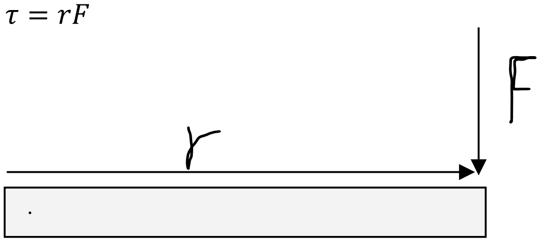
DIRECTION OF TORQUE

Calculating torque

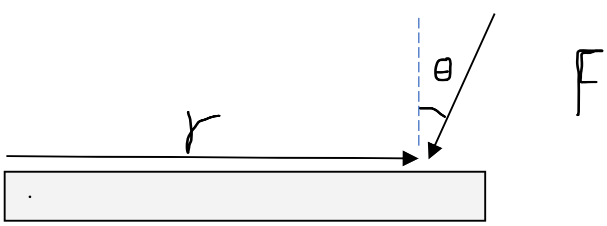
You have 4 scenarios

SENARIO 1 – FORCE IS PERPENDICULAR

$\tau = rF$



SENARIO 2 – FORCE IS NOT PERPENDICULAR



$\tau = rF\,cos(\theta)$

The magnitude of torque depends on three quantities:

1. The force applied
2. The length of the lever arm connecting the axis to the point of force application
3. The angle between the force vector and the lever arm.

When the force is not perpendicular, the perpendicular force can be calculated using

$$\perp F = F\,cos(\theta)$$

So you're just applying the normal torque t = rF but getting the perpendicular force by replacing F with $F\,cos(\theta)$

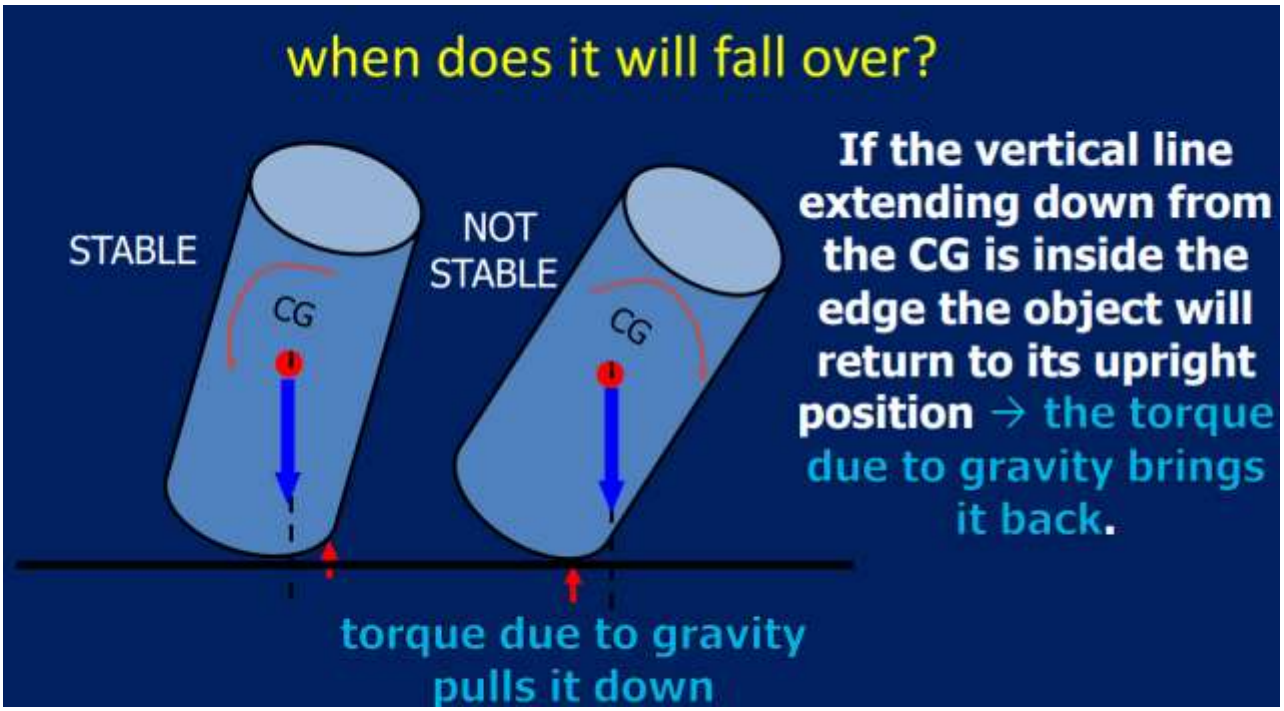
Rigid body equilibrium

It reaches its static equilibrium when the **net force** is **0** and **net torque** is **0**

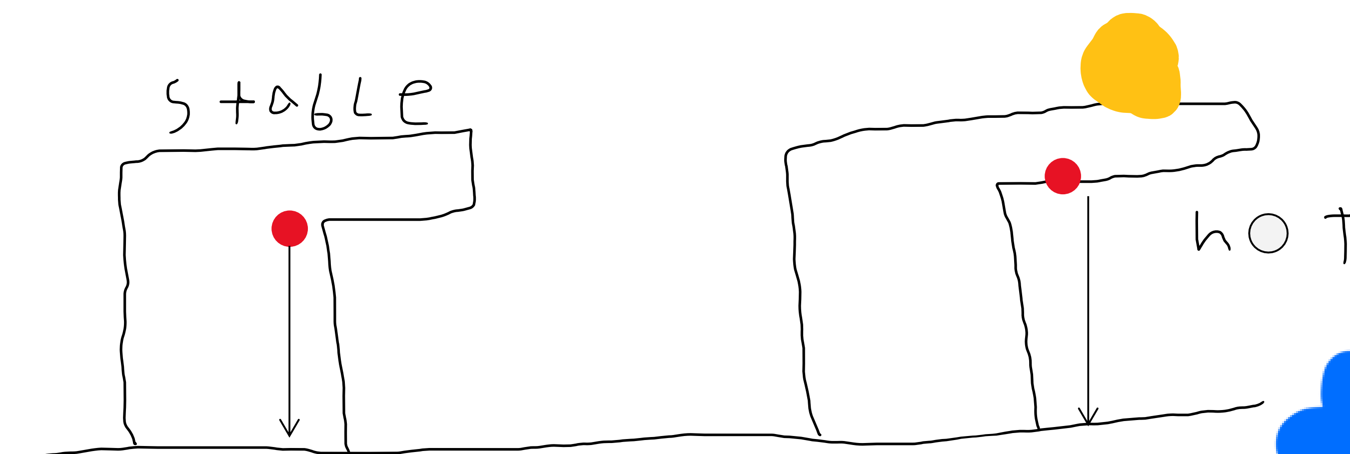
But there are two conditions for equilibrium

- **Condition (1)** is the translational equilibrium, it's when the external force on an object is equal to zero, aka, the acceleration of the center of mass is 0, if an object is a
    - **Point particle** -> then the only condition must be net Force = 0
- **Condition (2)** is the rotational equilibrium, it's when the **net torque** is **zero**, and it applies to extended objects like rods, not point particles

STABILITY



If we extended **CG** downwards, if there was an empty space, the object will fall



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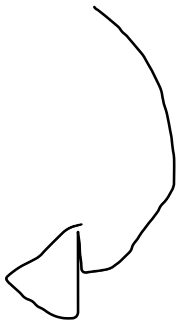
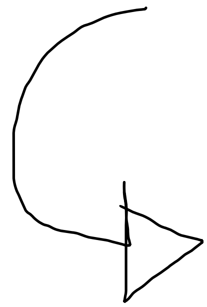
Moment : multiple torques in the same direction, it is calculated using the torque equations and has the same unit



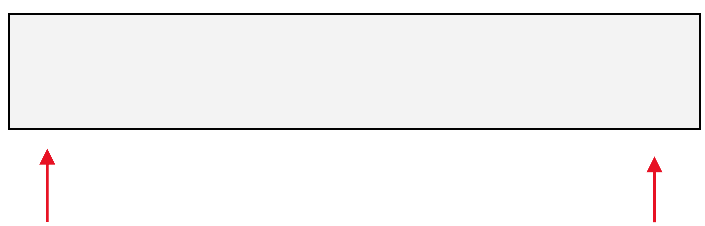
CLOCKWISE MOMENT



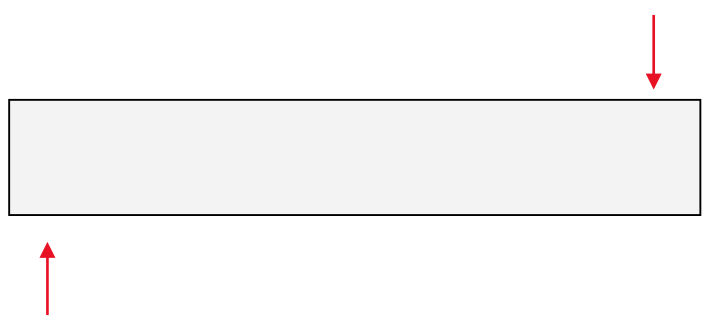
ANTI-CLOCKWISE MOMENT

When a system is in equilibrium

- Clockwise moments = anti-clockwise moments
- Net force = 0
- Net torque = 0



Net torque = 0 , Net force != 0



Net torque != 0, Net force = 0

When the net force on the system is zero, the torque measured from any point in space is the same.

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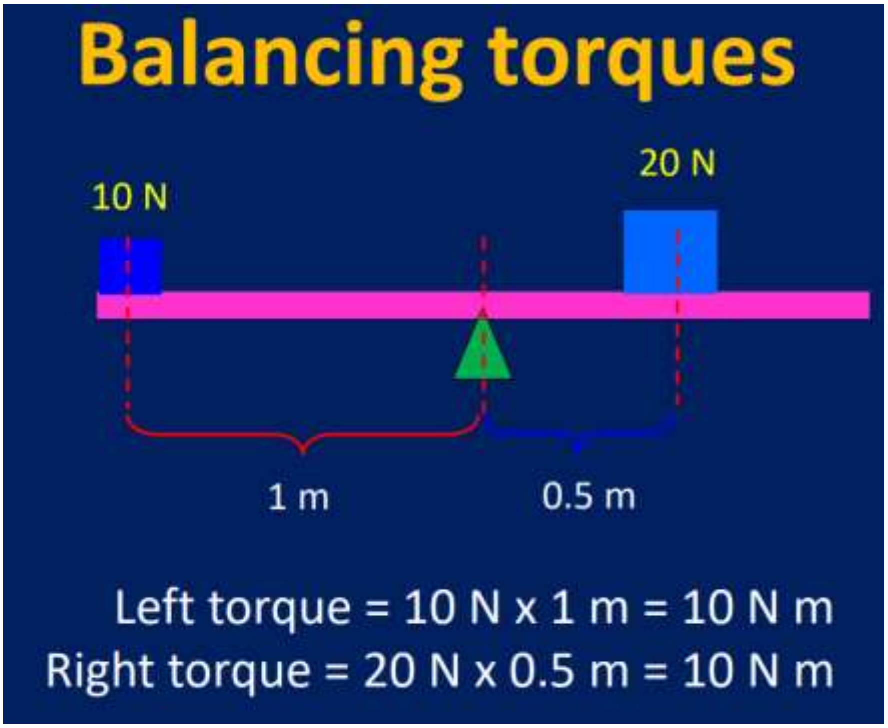
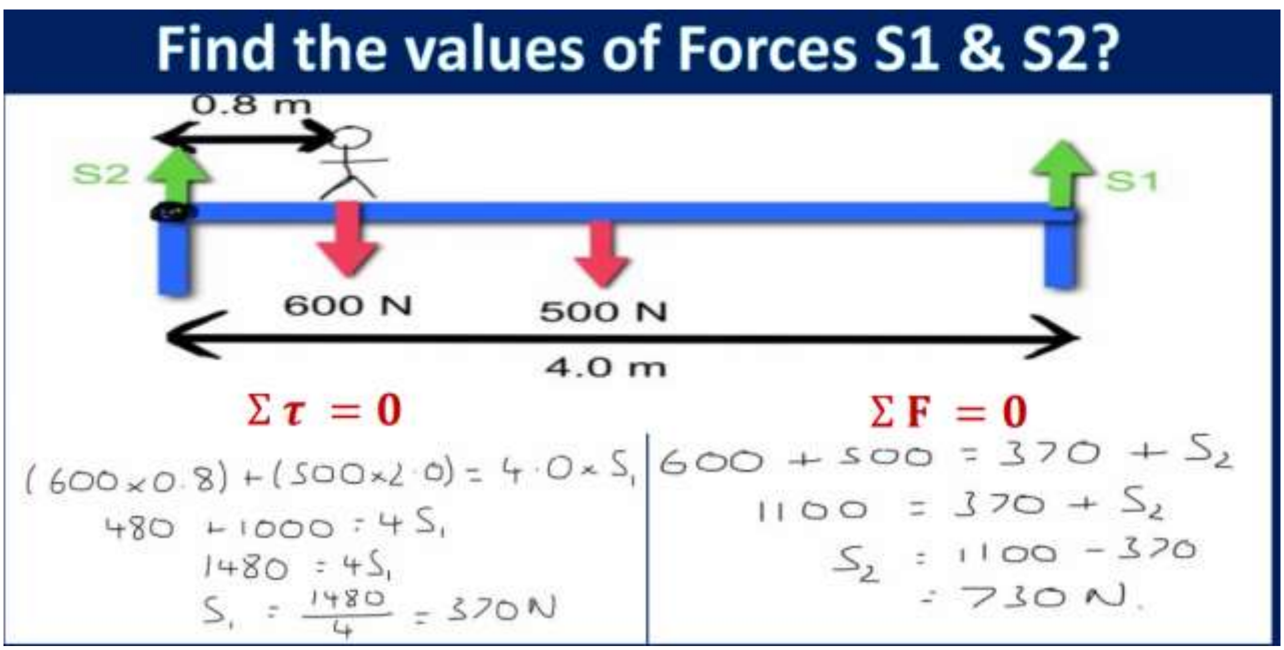


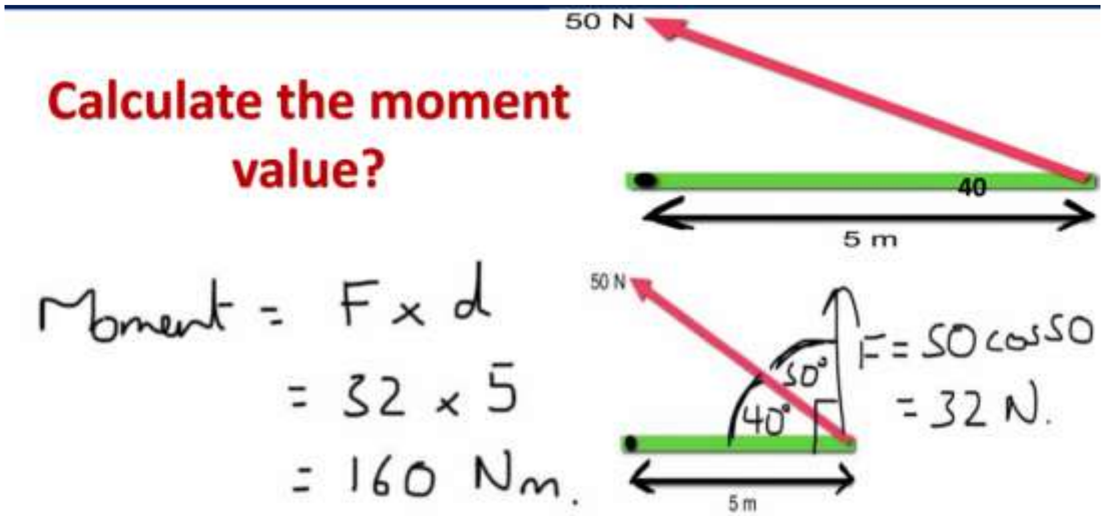
PROBLEMS WITH EXPLAINATIONS



The black dot is the pivot point, here we said if the torque was 0, the torque of S1 would need to equal the opposite torque of the 500 & 600N

Then if we said the net force was 0, the forces S1 & S2 would need to be equal to the 600 & 500 netwons

in the opposite direction



Moment laws = torque laws

A wooden pole is held in an upright position by two wires in tension. These forces are shown in the diagram below Calculate the tension force

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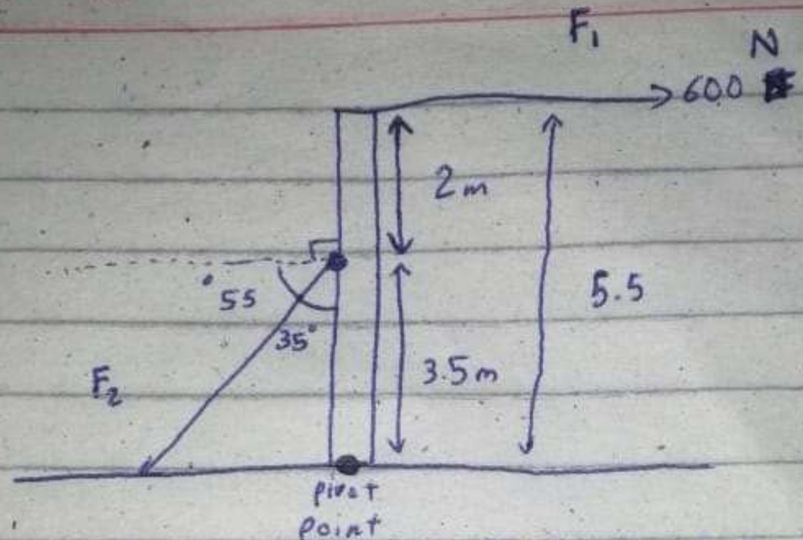


tension force between $\underset{\text{wire1}}{F_1}$ & $\underset{\text{wire2}}{F_2}$

rod is in equilibrium $T_{wire2} = T_{wire1}$

$$\Sigma\tau = 0\ Nm$$

$$\tau_1 = \tau_2$$

$$600 \times 5.5 = \tau_2$$

$$\tau_2 = 3300\ Nm$$

$$F_2 \times 3.5 \times \cos(55) = 3300\ Nm$$

$$F_2 \simeq 1644\ Nm$$

Machines

Are devices that help perform tasks, they are design to achieve at least 1 of these 5 goals

- **Change** energy from one form to another, like how hydroelectric dams convert the kinetic energy of water into electricity
- **Transfer** forces from one object to another, like gears
- **Reduce** the amount of force required for a job, like levers
- **Modify** the speed of something, like a wheel
- **change** the direction of motion

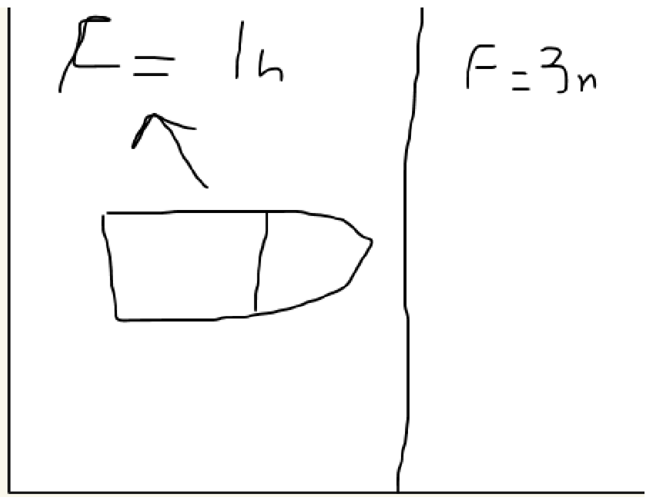
Mechanical advantage (MA)

it's the amplification of a force achieved by using a machine

What if (MA) = 3

If MA = 3
If I apply a force of 1 newton on something
The force applied to that thing will be amplified to 3 newton

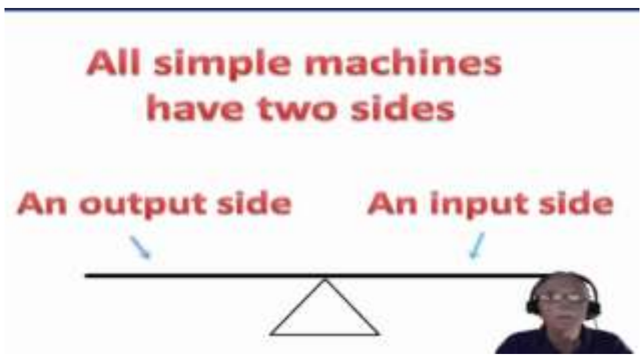
newF = MA * F



$$MA \propto \frac{1}{effort}$$

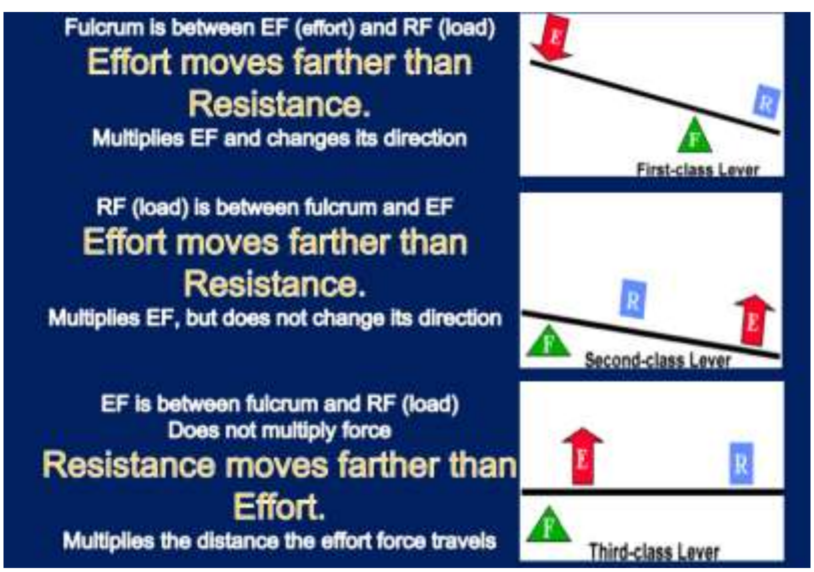
- The more the mechanical advantage **increases**, the **less** effort I have to put
- The more the mechanical advantage **decreases**, the **more** effort I have to put

THE 6 SIMPLE MACHINES



THE LEVER

It consists of a fulcrum “**pivot point**” and a beam, with the Input “**effort**” “**Force**” side and the output “**load**” “**resistance**” side

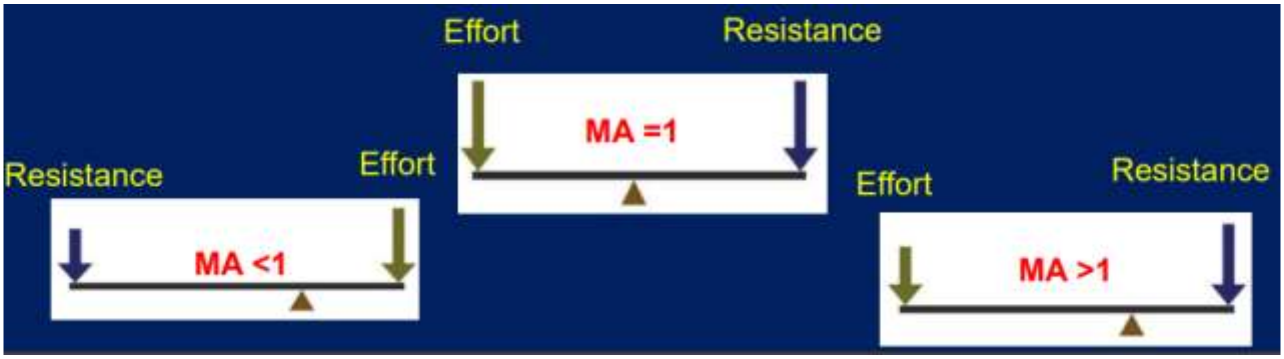


FIRST CLASS LEVER

It's where the **fulcrum** is between the effort and the resistance like a teeter-totter

The forces of **effort** & **resistance** are applying in the same direction which is likely **downward**

Its **MA** depends on who is closer to the fulcrum

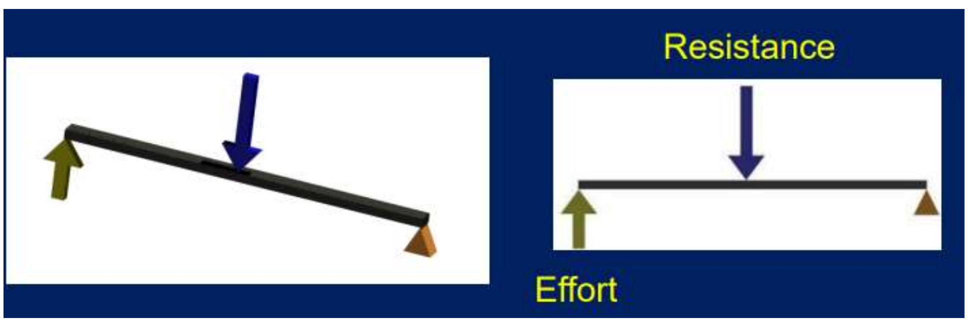


SECOND CLASS LEVER

It's where the **resistance** is between the fulcrum and the effort

Here the effort is pushing against the resistance like a door or wheel-barrel

Its **MA** is always more than 1

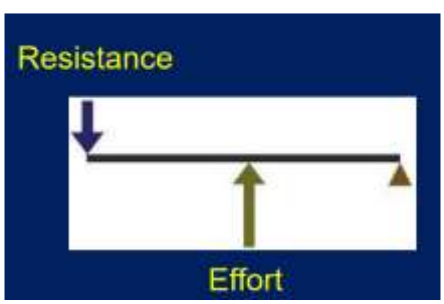


THIRD CLASS LEVERS

It's where the **effort** is between the fulcrum and the resistance

Here the effort is pushing against the resistance like a car jack

Its **MA** is always less than 1



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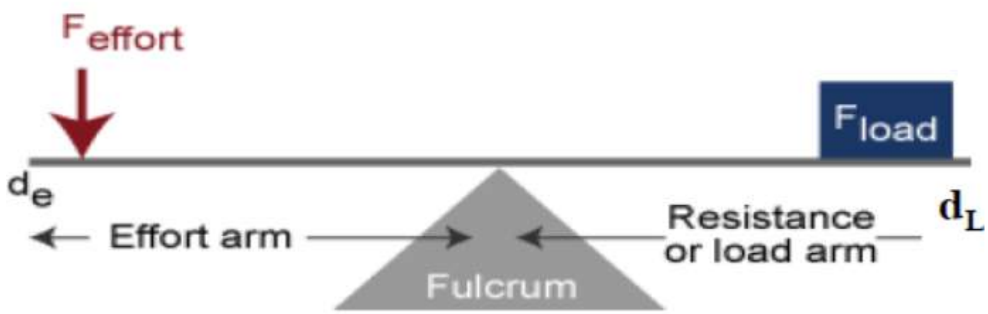
Effort Force (F_E) is the force acting on the effort arm which leads to applying a **load force (F_L)** on the load arm

Effort arm (d_E) is the distance from the effort force to the fulcrum

Same with the **load arm (d_L)**



$$IMA = \frac{\text{Output force}}{\text{Input force}} = \frac{\text{Input arm}}{\text{Output arm}}$$

Torque in a side of a lever = the side’s force x the side’s arm , So

- $T_E = F_E d_E$
- $T_L = F_L d_L$

LAW OF THE LEVER

Magnitude of effort torque = magnitude of load torque

INCLINED PLANE

It’s a flat surface that is tilted to make it easier to move objects upwards or downwards

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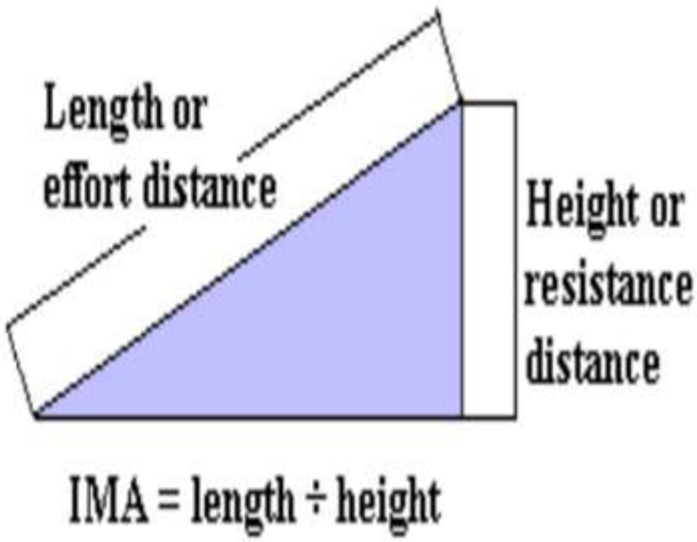


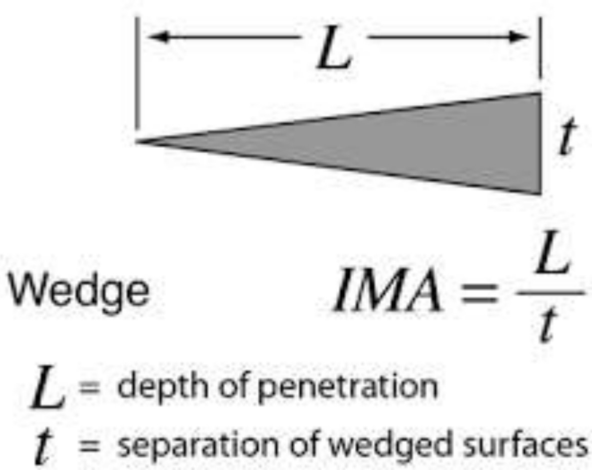
so instead of pushing directly against gravity, you move it over a slope

so if **MA** = 3

you would need just a **1/3** of the force to move the object instead of directly pushing it up

WEDGE

It is two inclined planes put together and it is used to split objects apart or hole them in place, like an **axe**

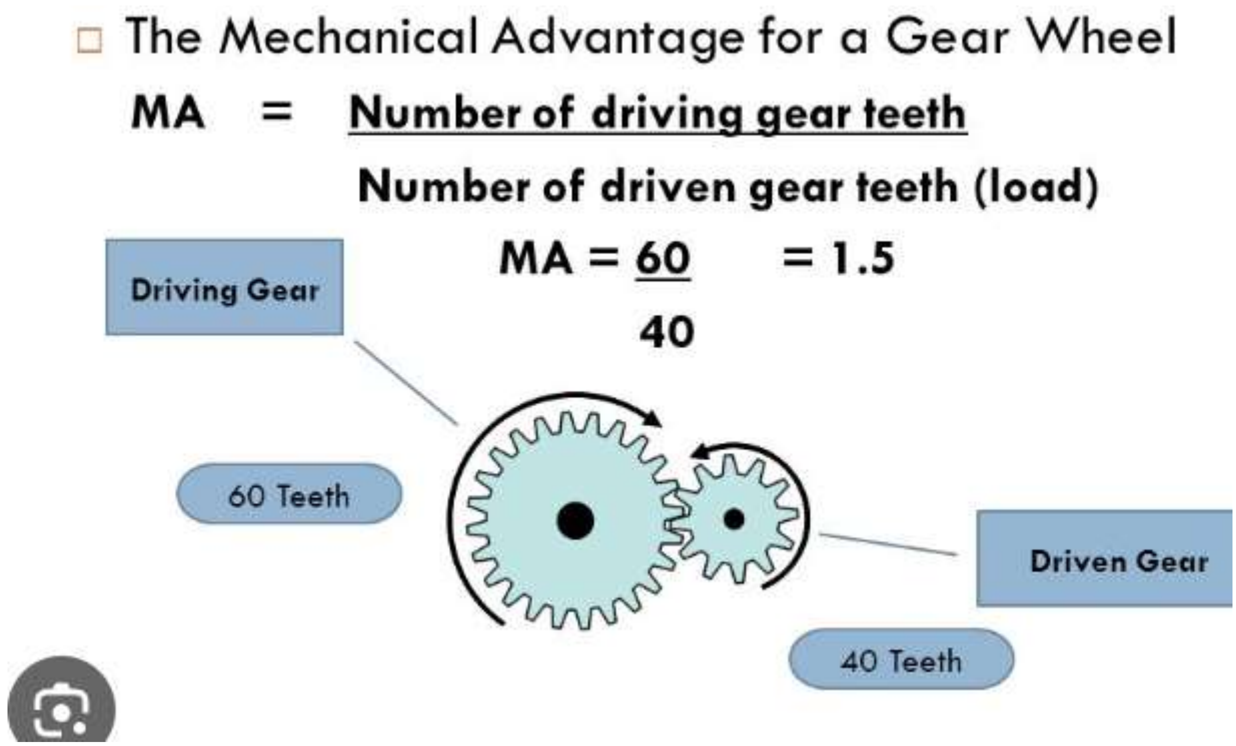


so if **MA = 3**

you would only need 1/3 of the force to separate a material instead of doing it by hand

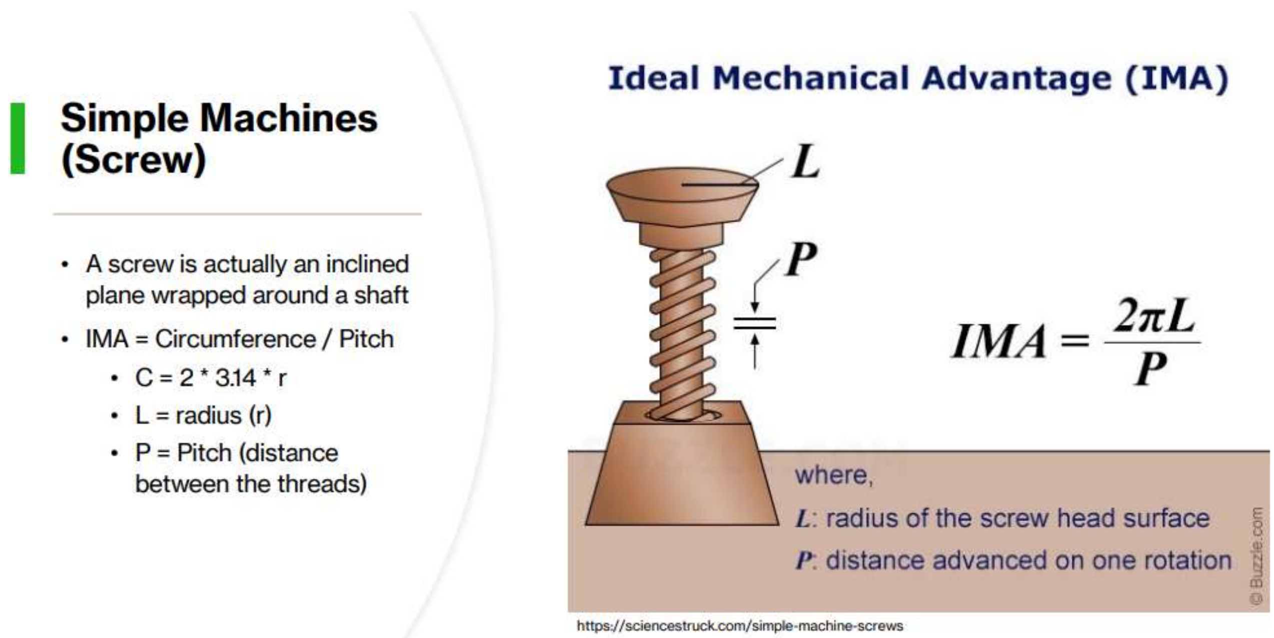
GEARS

They are used to move energy from an area to another



SCREWS

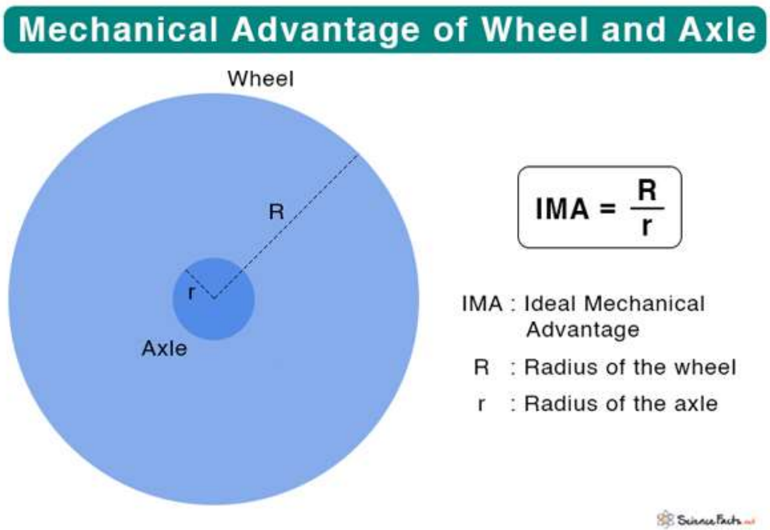
They are an inclined plane wrapped around a cylinder to hold things in place

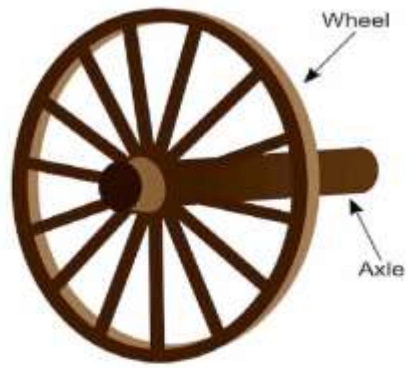


WHEEL & AXLE

It’s just a modified lever

With **MA = force output / force input**





PULLEY

We all know that the tension on one side of a pully is equal to the tension on the other side, so technically

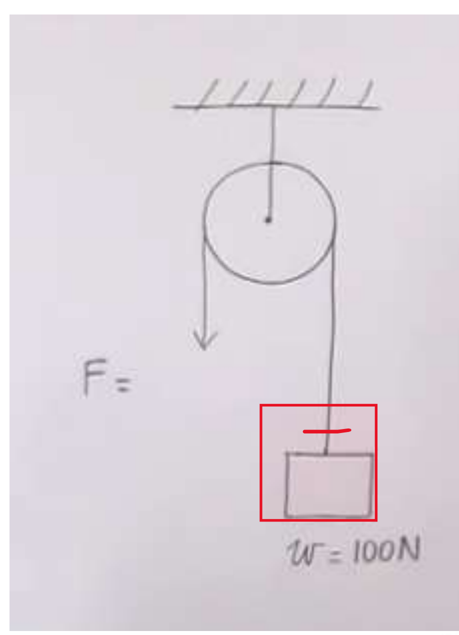
The more ropes we have on the output side, the less the force that we have to add on the input side

MA = number of supporting (OUTPUT) ropes

4 senarios to calculate the **MA**

SENARIO 1

Here because the output rope is only 1, the weight of the whole object would be on that rope, giving us an MA of 1
so you would need 100N of input force



SENARIO 2

here because we have 2 ropes

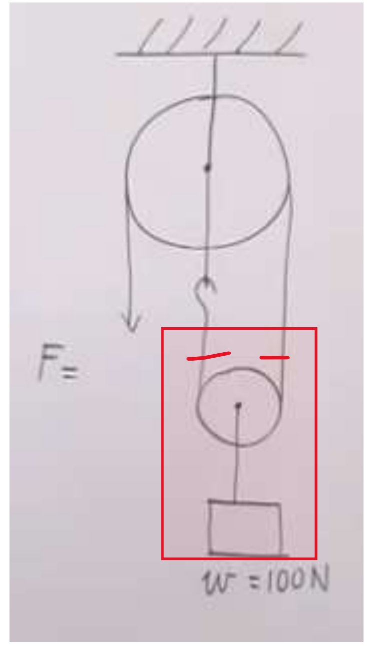
the MA = 2

so you would need 50N to lift the 100N

because the **law of pulleys** says that in a moving pully, the tension on the rope holding it in place and the rope of the attached pully has to be equal, so we would split the 100N into two, one in the rope attached to the ceiling of the moving pulley, and one attached to the actual rope

and because input tension = output tension

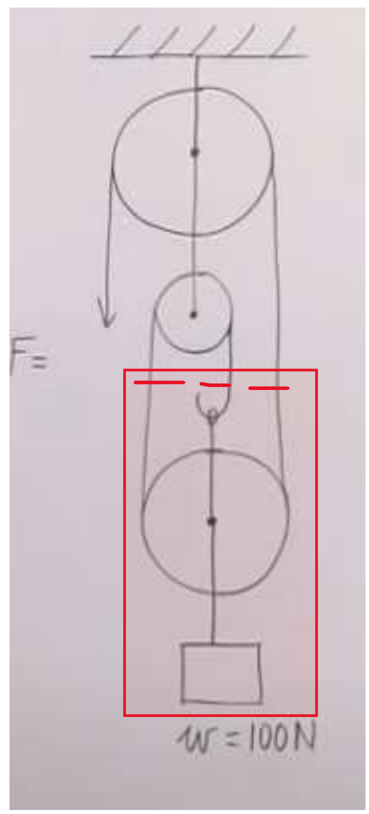
and the output rope tension is 50N
therefore the mechanical advantage is 2



SENARIO 3

Because we have 3 ropes

The **MA = 3**



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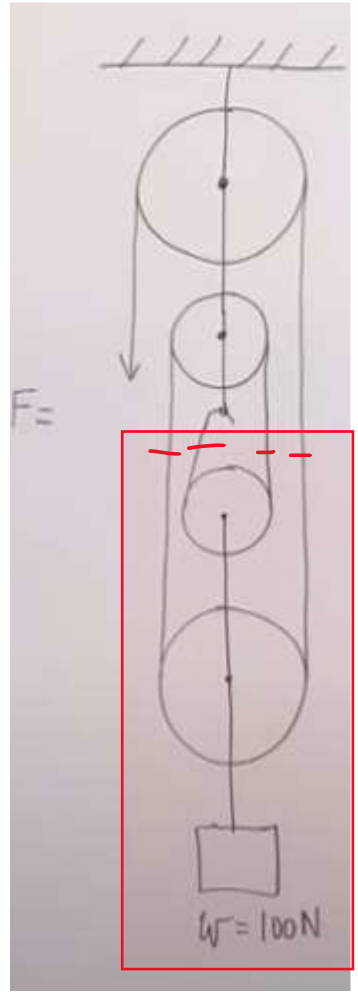
SENARIO 4

Because we have 4 ropes

The **MA = 4**



Angular Acceleration is the rate of change of **angular velocity**, often represented by α

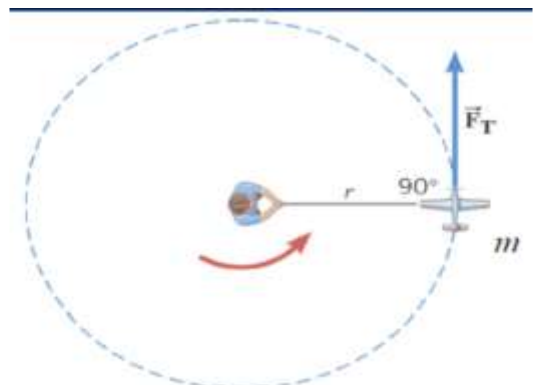
Tangential acceleration (a_T) is the rate of change of **speed** in a curved path where the force is 90°, it is the acceleration times radius or **ar**

Tangential force (F_T) is a force where it's perpendicular on the radius

Rotational inertia (moment of inertia)

Is the tendency of a rotating object to remain in its rotational motion (rest remain rest, rotation in a direction remains rotating in that direction) unless a torque is applied to it, it is represented as **I**

Is a quantity representing the body's tendency to resist angular acceleration that changes its direction of rotation



Is the body's mass multiplied by the square distance to the **axis of rotation**

L = mr²

in this picture, how do we stop or start the spinning process of the plane

well, by adding force

we know that **F = ma**

and **a_T** is tangential force so we can replace it with $r\alpha$

so $F_T = mr\alpha$

and because this is a Tangential force and there is rotation, we can use the law of torques

t = F_T r

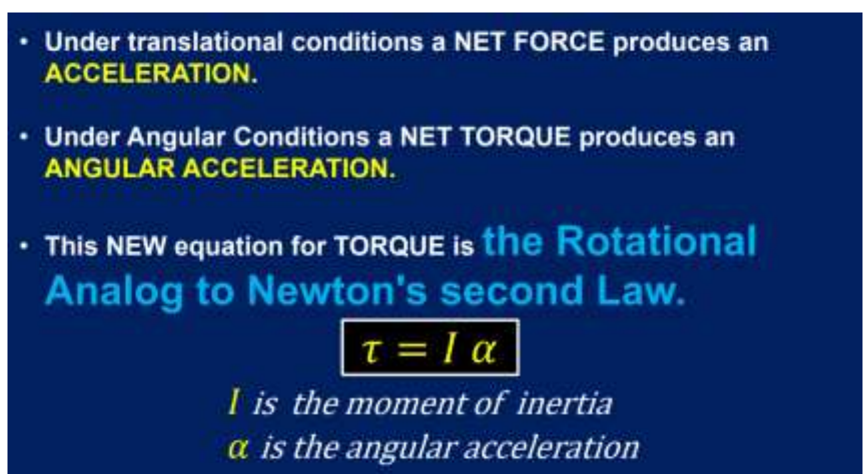
and we replace F_T with mra

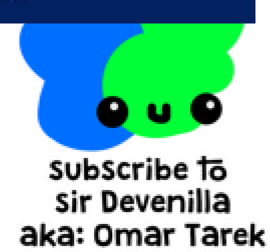
so t = mra * r

or $\tau = (mr^2)\alpha$

and we know that the rotational inertia is L = mr², so we can say that

t = La





Angular motion with newton's first law

When there are no forces or torques acting on an object, it will continue either rotating or being at rest

When a torque is applied to an object, it begins rotating with an acceleration inversely proportional to its moment of inertia

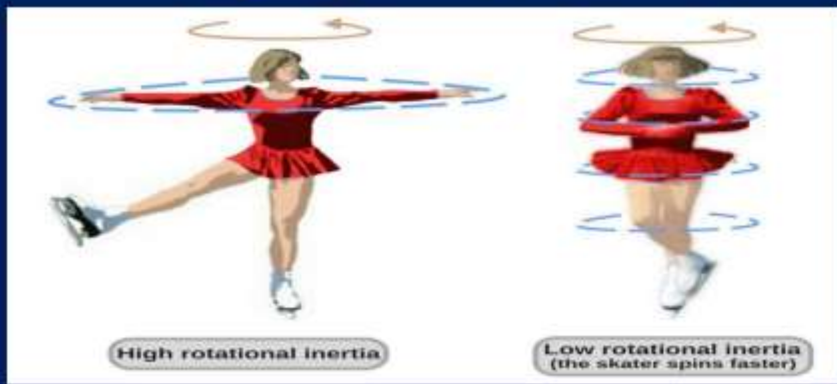
As

- **Moment of inertia** -> rotational mass
- **Torque** -> rotational force

- The moment of inertia depends not only on the mass of the object, but also on how that mass is distributed relative to the rotational axis.
- The moment of inertia of a system tends to be bigger if more mass is located farther away from the rotational axis.
- In simple words, this means that it becomes more difficult to alter the rotational velocity of a system if the object is kept far away from the center of the axis.

Aka, the higher the **r**, the harder it is for the **torque** to take effect

Example 1: **Spinning of a skater**



Example 2: **Why a wire walker may use a pole for balance on may stretch out his arms perpendicular to his trunk in the manner of a pole.**

This technique provides several advantages.

1) It distributes mass away from the pivot point, thereby increasing the moment of inertia.
2) This reduces angular acceleration because a greater force is required to rotate the performer over the wire.

The result is less tipping. In addition the performer can also correct sway by rotating the pole. This will create an equal and opposite torque on the body.

- **So, The horizontal pole has heavy weights on the end, serving two purposes:**

1) Increasing the moment of inertia of the pole
2) Increasing the amount of control over the net torque the tightrope walker has.

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