MATH2

SELF ASSESSMENT QUESTIONS

UNIT 1

(1) Show that the distance btw $P(a, 2a)$ & $Q(-3a, -a)$ is $5a$

Soln

$x_1 \; y_1$

$P(a, 2a)$

$Q(-3a, -a)$

$x_2 \; y_2$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(-3a - a)^2 + (-a - 2a)^2}$$

$$= \sqrt{(-4a)^2 + (-3a)^2} = \sqrt{16a^2 + 9a^2} = \sqrt{25a^2}$$

$$= 5a$$

(2) Find the distance btw $(2, 1)$ and $(2, 5)$

$x_1 \; y_1$ $\quad$ $x_2 \; y_2$

Soln

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(2 - 2)^2 + (5 - 1)^2} = \sqrt{0^2 + 4^2}$$

$$= \sqrt{0 + 4^2}$$

$$= \sqrt{16} = 4$$

③ Find the mid point of line ② xy of x(6,2) & y(4,8)

Soln

x(6,2) y(4,8)

Midpoint of xy = $\left(\frac{6+4}{2}, \frac{8+2}{2}\right) = (5,5)$

from $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

④ Find the slope of line CB for C(4,2) & B(1,6)

(x_1 y_1 x_2 y_2)

Soln

$m = \text{Slope} = \frac{y_2-y_1}{x_2-x_1} = \frac{6-2}{1-4} = \frac{4}{-3} = -4/3$

⑤ (i) Distance btw A(0,5) and B(-3,8)

Soln

$d = \sqrt{(-3)^2+(3)^2} = \sqrt{9+9} = \sqrt{18}$

$= 3\sqrt{2}$

(ii) Distance btw A$\left(-\frac{1}{2}, \frac{5}{8}\right)$ and B$\left(-\frac{3}{2}, \frac{1}{4}\right)$

(x_1 y_1 x_2 y_2)

Soln

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (3)$$

$$= \sqrt{\left[-\frac{3}{2} - \left(-\frac{1}{2}\right)\right]^2 + \left[\frac{1}{4} - \frac{5}{8}\right]^2} = \sqrt{\left(\frac{-2}{2}\right)^2 + \left(\frac{-3}{8}\right)^2}$$

$$= \sqrt{1 + \frac{9}{64}} = \sqrt{\frac{73}{64}}$$

(6) Obtain the point that divide the following ratio 2:3

(i) $A(0, 5)$ and $B(-3, 8)$
$x_1\ y_1$ $x_2\ y_2$

Soln

$m : n = 2 : 3$

$$x = \frac{nx_1 + mx_2}{m+n} \qquad y = \frac{ny_1 + my_2}{m+n}$$

$$= \frac{3(0) + 2(-3)}{5} \qquad = \frac{3(5) + 2(8)}{5}$$

$$x = -6/5 \qquad y = 31/5$$

$$\left(-\frac{6}{5}, \frac{31}{5}\right)$$

(ii) $A\left(-\frac{1}{2}, \frac{5}{8}\right)$ and $B\left(-\frac{3}{2}, \frac{1}{4}\right)$
$x_1\ y_1$ $x_2\ y_2$

Soln

$m:n = 2:3$

$$x = \frac{3\left(\frac{1}{2}\right) + 2\left(-\frac{3}{2}\right)}{5}$$

$$= \frac{\frac{-3}{2} - 3}{5}$$

$$= \frac{-9}{2} \times \frac{1}{5}$$

$$x = \frac{-9}{10}$$

④

$$y = \frac{3\left(\frac{5}{8}\right) + 2\left(\frac{1}{4}\right)}{5}$$

$$= \frac{\frac{15}{8} + \frac{1}{2}}{5}$$

$$= \frac{19}{8} \times \frac{1}{5}$$

$$y = \frac{19}{40}$$

$$(x, y) = \left(\frac{-9}{10}, \frac{19}{40}\right)$$

⑦ Find the slope of the line joining $(-1, -5)$ & $(-3, -2)$

Soln

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - (-5)}{-3 - (-1)} = \frac{-2+5}{-3+1} = \frac{3}{-2} = \frac{-3}{2}$$

Unit 2

(1) Find the eqn of line thru $(-1, 5)$ whose slope is 3.
$x_1 \quad y_1$

Soln

gradient and one point form

$$(y - y_1) = m(x - x_1)$$

$$y - 5 = 3(x - (-1))$$

$$y - 5 = 3x + 3$$

$$y = 3x + 3 + 5 \;, \quad y = 3x + 8$$

(2) Find the eqn of line thru $(3\sqrt{5}, 2\sqrt{7})$ & $(-\sqrt{5}, -3\sqrt{7})$
$x_1 \quad y_1 \qquad x_2 \quad y_2$

Write the eqn in linear, slope intercept & intercept form.

Soln

gradient and two point form

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{y - 2\sqrt{7}}{x - 3\sqrt{5}} = \frac{-3\sqrt{7} - 2\sqrt{7}}{-\sqrt{5} - 3\sqrt{5}}$$

$$\frac{y - 2\sqrt{7}}{x - 3\sqrt{5}} = \frac{-5\sqrt{7}}{-4\sqrt{5}}$$

$$-4\sqrt{5}\left(y-2\sqrt{7}\right)=-5\sqrt{7}\left(x-3\sqrt{5}\right) \quad (6)$$

$$-4\sqrt{5}y+8\sqrt{35}=-5\sqrt{7}x+15\sqrt{35}$$

$$-4\sqrt{5}y=-5\sqrt{7}x+15\sqrt{35}-8\sqrt{35}$$

$$-4\sqrt{5}y=-5\sqrt{7}x+7\sqrt{35}$$

(i) $-5\sqrt{7}x-4\sqrt{5}y+7\sqrt{35}=0$

$ax+bx+c=0$ (linear form)

where $a=-5\sqrt{7}$, $b=-4\sqrt{5}$, $c=7\sqrt{35}$

(ii) $-4\sqrt{5}y=-5\sqrt{7}x+7\sqrt{35}$

$$y=\frac{-5\sqrt{7}x}{-4\sqrt{5}}+\frac{7\sqrt{35}}{-4\sqrt{5}}$$

$$y=\frac{5\left(\sqrt{7}\times\sqrt{5}\right)}{4\sqrt{5}\times\sqrt{5}}x+\frac{7\sqrt{7}\times\sqrt{5}}{-4\sqrt{5}}$$

$$y=\frac{5\sqrt{35}}{20}x-\frac{7\sqrt{7}}{4}$$

$y=mx+c$ (Slope-intercept form)

where $m=\frac{5\sqrt{35}}{20}$ and $c=\frac{-7\sqrt{7}}{4}$

(iii) $-4\sqrt{5}y = -5\sqrt{7}x + 7\sqrt{35}$ (7)

$$\frac{-4\sqrt{5}y}{7\sqrt{35}} + \frac{5\sqrt{7}x}{7\sqrt{35}} = \frac{7\sqrt{35}}{7\sqrt{35}}$$

$$\frac{-4}{7\sqrt{7}}y + \frac{5}{7\sqrt{5}}x = 1$$ ⟵ by rationalizing

$$\frac{-4\sqrt{7}}{7\times 7}y + \frac{5\sqrt{5}}{7\times 5}x = 1$$

$$\frac{-4\sqrt{7}}{49}y + \frac{5\sqrt{5}}{35}x = 1$$

$$\frac{5\sqrt{5}}{35}x - \frac{4\sqrt{7}}{49}y = 1$$

$$\frac{x}{a} + \frac{y}{b} = 1$$ (intercept form)

where $a = \frac{35}{5\sqrt{5}}$, $b = \frac{-49}{4\sqrt{7}}$.

(3) Find the tangent of the angle btw $3y = 2x$ & $y = 7x - 4$ (8)

Soln

$$\tan\theta = \frac{m_2 - m_1}{1 + m_2 m_1}$$

from (1) $3y = 2x$

$y = \frac{2}{3}x + 0$

$m_1 = \frac{2}{3}$

from (2) $y = 7x - 4$

$m_2 = 7$

$$\tan\theta = \frac{7 - \frac{2}{3}}{1 + 7 \times \frac{2}{3}} = \frac{\frac{21-2}{3}}{1 + \frac{14}{3}} = \frac{\frac{19}{3}}{\frac{17}{3}}$$

$$= \frac{19}{3} \times \frac{3}{17} = \frac{19}{17}$$

$\tan\theta = \frac{19}{17}$ //

UNIT 3

(1) Find the slope of a line perpendicular to line whose eqn is $2y + 6x = 24$

Soln

for perpendicularity, $m_1 = -1/m_2$

If $2y + 6x = 24$ is the second line

$$2y = -6x + 24$$

$$y = -3x + 12 \quad , \quad m_2 = -3$$

$$m_1 = -1/m_2 = -1/-3 = \underline{1/3}$$ — slope of the first line

(2) Given point A(-2, 8) and B(4, 4). Find the equation of the perpendicular bisector of line AB.

Soln

The bisector of AB is the midpoint of AB

Midpoint of AB $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{4 + (-2)}{2}, \frac{7 + 3}{2}\right)$

$= (1, 5)$

the line AB has the slope m_1 which is

~~gradient of two point form~~

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad A(-2, 3) \quad B(4, 7)$

$x_1\ y_1 \quad x_2\ y_2$

$= \frac{7-3}{4-(-2)} = \frac{4}{6} = 2/3$

m_1 for line AB is $2/3$

$M_2 = -\frac{1}{m_2}$ (since its perpendicular)

$M_2 = -1/2/3 = -1 \times 3/2 = -3/2$

M_2 is the slope of the perpendicular bisector of AB at the midpoint $(1, 5)$

Hence, the eqn is generated by using gradient and one point form

$y - y_1 = m(x - x_1) \qquad m = -3/2$

$(x_1, y_1) = (1, 5)$

$y - 5 = -3/2(x - 1)$

$2y - 10 = -3x + 3$

$2y = -3x + 13$

$y = -3/2x + 13/2$ //

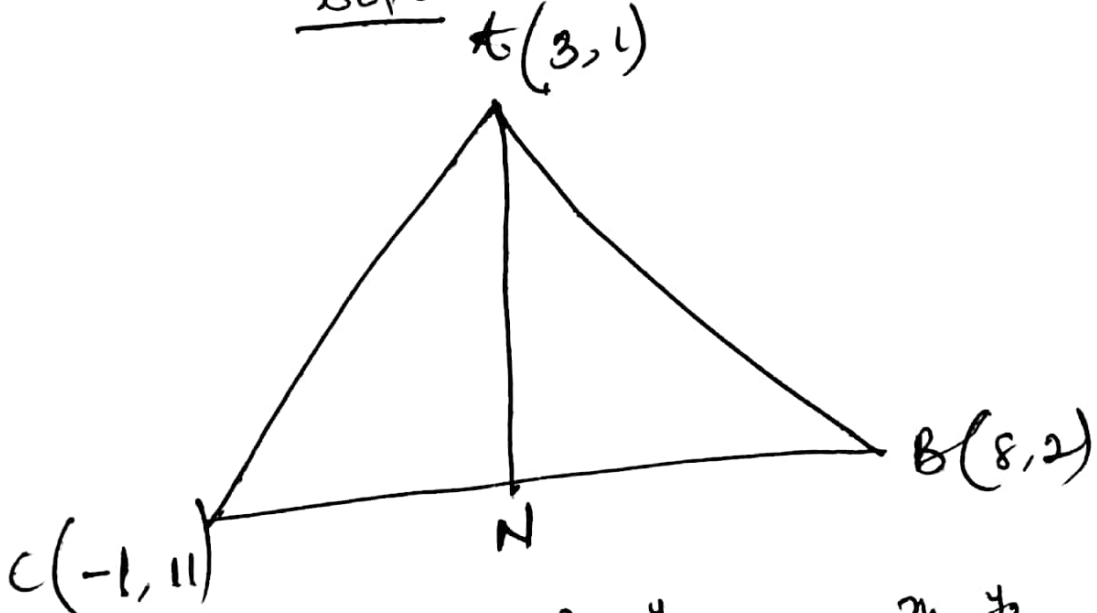
(3) The points A(3,1), B(8,2) and C(-1,11) are the vertices of a triangle. If N is the foot of the perpendicular from A to BC, find the

(i) eqn of BC

(ii) eqn of AN

(iii) length AN

Soln



(i) eqn of BC

x_1 y_1 x_2 y_2

B(8,2) C(-1,11)

Here, two points form

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

$$\frac{y-2}{x-8} = \frac{11-2}{-1-8}$$

$$y-2(-9) = x-8(9)$$

$-9y + 18 = 9x - 72$

$-9y = 9x - 90$

$-y = x - 10 \;;\; y = -x + 10$ //

(ii) Eqn of AN

N is midpoint of BC B(8,2) C(-1,11)

Midpoint of BC $= \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$

$= \frac{8 + (-1)}{2}, \frac{11 + 2}{2}$

$N = \left(\frac{7}{2}, \frac{13}{2}\right)$

A = (3,1), N = (7/2, 13/2)

Eqn of AN, $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{y-1}{x-3} = \frac{13/2 - 1}{7/2 - 3}$

$\frac{y-1}{x-3} = \frac{11/2}{1/2}$

$1/2(y-1) = 11/2(x-3)$

$y - 1 = 11(x-3)$

$y - 1 = 11(x - 3)$

$y - 1 = 11x - 33$

$y = 11x - 32$

(iii) length

$A = (3, 1) \quad M(7/2, 13/2)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(7/2 - 3)^2 + (13/2 - 1)^2}$

$= \sqrt{(1/2)^2 + (11/2)^2} = \sqrt{1/4 + 121/4}$

$= \sqrt{122/4} = \frac{\sqrt{122}}{2}$