Math 345 Fall 2025

Homework #2
Due: September 22, 2025, 11:59 p.m.

Show work to justify your answers and write neatly (or type) for full credit. Your work must be uploaded onto Gradescope (accessed through Canvas).

1. Problem 1.5.8. If Laplace's equation is satisfied in three dimensions, show that

$$\oiint_S \nabla u \cdot \hat{\mathbf{n}}\, dS = 0$$

   for any closed surface. (*Hint*: Use the divergence theorem.) Give a physical interpretation of this result (in the context of heat flow).

2. Problem 2.2.5. In this exercise, we derive superposition principles for nonhomogeneous problems.
   (a) Consider $L(u) = f$. If u_p is a particular solution, $L(u_p) = f$, and if u_1 and u_2 re homogeneous solutions, $L(u_i) = 0$, show that $u = u_p + c_1 u_1 + c_2 u_2$ is another particular solution.
   (b) If $L(u) = f_1 + f_2$, and $u_{p,i}$ is a particular solution corresponding to f_i, what is a particular solution for $f_1 + f_2$?

3. Problem 2.3.1. For the following partial differential equation,

$$\frac{\partial u}{\partial t} = \frac{k}{r^2}\frac{\partial u}{\partial r}\left(r^2 \frac{\partial u}{\partial r}\right),$$

   what ordinary differential equations are implied by the method of separation of variables?

4. Problem 2.3.2. Find the general solution of each boundary value problem, or show that no solution exists. Note that there may be no solution, a unique solution, or infinitely many solutions.
   (a) $y'' + 4y = 0$, $y(0) = 3$, $y(1) = 5$.
   (b) $y'' + 4y = 0$, $y(0) = 3$, $y(\pi) = 5$.
   (c) $y'' + 4y = 0$, $y(0) = 3$, $y(\pi) = 3$.

5. Problem 2.3.7. Evaluate (be careful if $n = m$)

$$\int_0^L \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right)\, dx, \quad \text{for } n > 0, m > 0.$$

   Use the trigonometric identity $\sin a \sin b = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$.

6. Problem 2.4.4. Consider the boundary value problem

$$\frac{d^2\phi}{dx^2} + \lambda\phi = 0, \quad 0 < x < L,$$

   subject to $\phi(0) = 0$, and $\frac{d\phi}{dx}(L) = 0$.
   (a) Show the eigenfunctions are of the form

$$\phi_n(x) = \sin\left(\frac{(2n-1)\pi x}{2L}\right), \quad \text{for } n = 1, 2, 3\ldots$$

(b) Use the trigonometric identity $\sin a \sin b = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$ to show that these eigenfunctions satisfy

$$\int_0^L \phi_n(x)\phi_m(x)\,dx = \begin{cases} 0 & n \neq m \\ L/2 & n = m. \end{cases}$$