

Foundation Session - 2

Agenda

- Recursive Runtimes Practice (also check Master's Theorem)
- Factorial Runtime (Permutations Example)
- Palindrome
- Power a^b Methods
- Count Digits in a Number
- Counts Digits in all Numbers from 1 to N
- Check if a number is Prime
- Sieve of Eratosthenes

Recursive Runtimes Practice:

Steps for recursive tree method for solving recurrences:

- Step-1: Draw a recursive tree
- Step-2: Calculate the work done by each node assuming the recursive calls to be $O(1)$ (say, it is **C**)
- Step-3: Calculate the maximum level of nodes in the recursive tree (say, it is **H**)
- Step-4: Sum up the work done at all the levels in the recursive tree using the previous three steps.

Remember the GP Sum formula $\rightarrow (a*(r^n - 1)) / (r - 1)$

Where,

a = first term of the GP.

r = multiplying factor of the GP.

n = number of terms in the GP.

Example-1:

```
void fun(int n, bool b) {  
    ...  
    return fun(n/2, true) + fun(n/2, false);  
}
```

Example-2: $F(n) = F(n/2) + F(n/4) + c$

```
void fun(int n) {  
    ...  
    return fun(n/2) + fun(n/4);  
}
```

Special Palindrome

Given a string, check if it is a special palindrome.

A string is a palindrome if it remains the same when reversed.

But we define a special palindrome as:

A phrase is a palindrome if, after converting all uppercase letters into lowercase letters and removing all non-alphanumeric characters, it reads the same forward and backward.

Input: "level"

Output: true

Input: "levels"

Output: false

Input: "race a car"

Output: false

Explanation: "raceacar" is not a palindrome

Input: "A man: nama"

Output: true

Explanation: "amannama" is a palindrome

Follow-up: Check if a given integer is a palindrome.

Input: 123
Output: false

Input: 1234321
Output: true

Power a^b

Given two numbers a and b , find a^b (a power b).
Assume $a \geq 1$ and $b \geq 0$

Input: $a = 2, b = 3$
Output: 8

Input: $a = 5, b = 2$
Output: 25

Input: $a = 7653, b = 0$
Output: 1

Count Digits in a Number

Given a number, count its number of digits.

Input: 1234313
Output: 7

Input: 0
Output: 1

Counts Digits in all Numbers from 1 to N

Given a number n , find the sum of the number of digits in all numbers from 1 to n .

Input: 10

Output: 11

Explanation:

All numbers from 1 to 10: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

Number of digits in these:[1, 1, 1, 1, 1, 1, 1, 1, 1, 2]

Input: 25

Output: 41

Solution:

$N = 1500$

Single digit numbers = 9

Two digit numbers = $99 - 9 = 90$

Three digit numbers = $999 - 99 = 900$

Four digit numbers = $1500 - 999 = 501$

$9 + (2 \times 90) + (3 \times 900) + (4 \times 501)$

TODO-1: Find the sum of all digits in a given number.

TODO-2: Find the sum of digits in all the numbers from 1 to N.

Check if a number is Prime

Given a number n, check if it is a prime number.

(Any number n which is divisible by **only** 1 and n is a prime number).

Input: 0

Output: false

Input: 1

Output: false

Input: 17

Output: true

Input: 20

Output: false

Sieve of Eratosthenes

Given a number n , find all the prime numbers less than or equal to n .

Input: 10

Output: [2, 3, 5, 7]

Input: 20

Output: [2, 3, 5, 7, 11, 13, 17, 19]