# GARCH_part

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## import the dbw and QL loss function

## Auto.garch

## GARCH_part

## Test the SynthVolForecast function

### First round of testing-for each module

### Test Auto.garchx

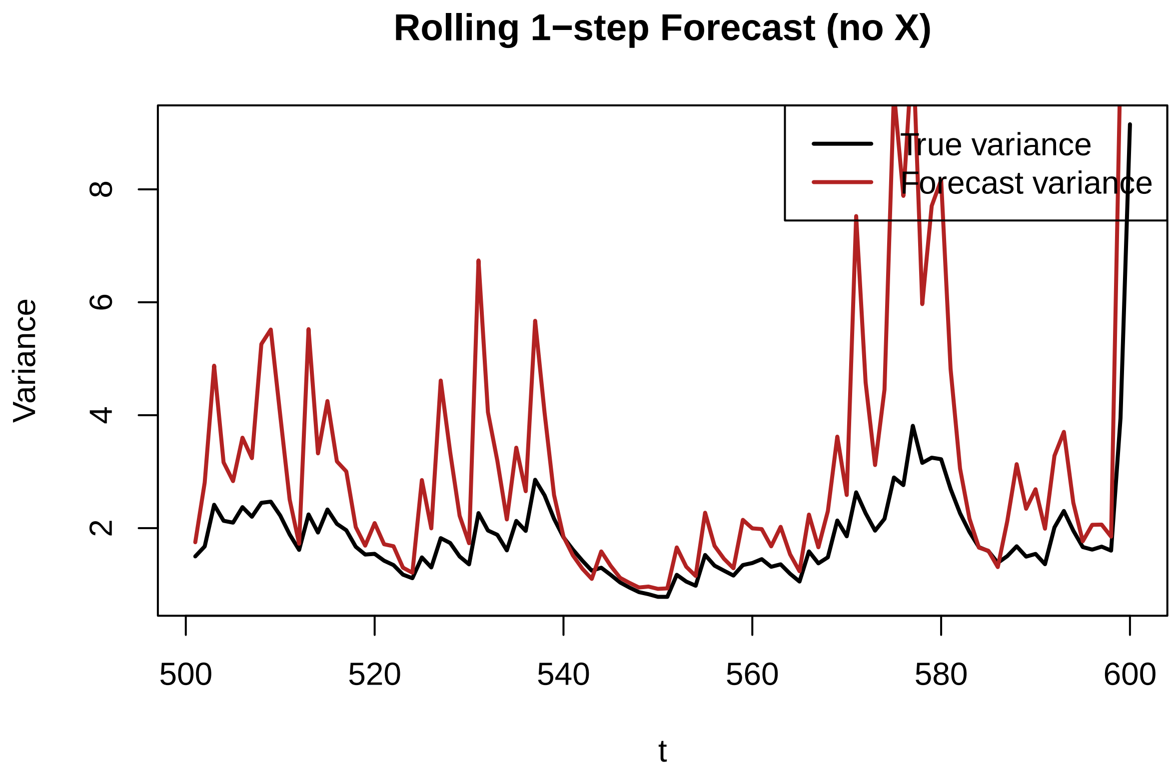
```
## [1] 1
```

```
## [1] 1
```

```
## [1] 2082.705
```

```
##
## Date: Wed Jul  9 04:08:27 2025
## Method: normal ML
## Coefficient covariance: ordinary
## Message (nlminb): relative convergence (4)
## No. of observations (fitted): 599
## Sample: 1 to 600
##
##              intercept    arch1     garch1      asym1     xreg1
## Estimate:   0.30724369 0.152856 0.68843871 0.00000000 0.2751997
## Std. Error: 0.09809368 0.042643 0.06898402 0.06223691 0.0681992
##
## Log-likelihood: -1025.36
```

```
## Rolling 100-step Forecast → MAE=2.1325, RMSE=9.4092, MAPE=64.90%
```




## Project Summary: SynthVolForecast Volatility Modeling Module

### 1. Project Objective

The core goal of this stage is to **develop and validate an automated GARCH/GARCH-X modeling and volatility forecasting pipeline** as a submodule for SynthVolForecast (Synthetic Control Volatility Forecasting). This provides the basis for accurate time-varying variance estimation for synthetic control and causal inference tasks.

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### 2. Completed Work

#### 2.1 GARCH/GARCH-X Model Development & Automation

- Developed the `auto_garchx` function supporting both classic GARCH and GARCH-X (with exogenous variables), including:
  - Automated order (p, q) selection via information criteria (BIC),
  - Unconditional variance backcasting,
  - Two-step refit for improved stability.
- Simulated various scenarios to validate model performance, including:
  - Pure GARCH and GARCH-X processes,
  - Different parameter regimes and covariate structures,
  - Model stability, parameter recovery, and error behavior.

### 2.2 Rolling Forecast Pipeline & Error Metrics

- Built a **rolling 1-step-ahead conditional variance forecast** process:
  - Dynamic refit and prediction at each time step,
  - Evaluation using MAE, RMSE, MAPE,
  - Visualization: real (black) vs. predicted (red) variance series.
- Explored model behavior with and without exogenous regressors.

### 2.3 Code Refactoring & Documentation

- Added detailed English comments throughout core functions (`auto_garchx`, `dbw`, `SynthVolForecast`) for reproducibility and collaboration.
- Organized experiments and outputs for future integration and reporting.

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## 3. Key Insights

- The workflow robustly fits both GARCH and GARCH-X models, with stable order selection and solid forecasting accuracy.
- `auto_garchx` is flexible, automatic, and ready to embed into the larger SynthVolForecast workflow.
- Rolling forecast pipelines provide an empirical foundation for parameter tuning and window length selection.

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## 4. Next Steps

1. **Integrate with SynthVolForecast**
   - Embed the GARCH-based volatility submodule into the overall synthetic control pipeline for both target and donor series.
2. **Expand to High-Dimensional Covariates**
   - Test GARCH-X with higher-dimensional exogenous regressors and more complex settings.
3. **Apply to Real-World Data**
   - Validate on real and more challenging synthetic datasets, and fine-tune methodology.
4. **Documentation and Visualization**
   - Optimize code comments, experiment logging, and result visualization for publication and collaboration.
5. **Academic Preparation**
   - Begin writing methods and results for papers, slides, or thesis documentation.

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## 5. One-Sentence Summary

This phase established an automated, robust GARCH volatility modeling pipeline to serve as the dynamic backbone for SynthVolForecast, enabling reliable, time-varying variance estimation for advanced synthetic control and causal inference tasks.