Prospective testing for the prevalence or transience of a shock effect before it occurs

Abstract

We develop a hypothesis testing procedure to prospectively test whether an anticipated shock is likely to be transient or permanent over a time horizon. We achieve this by borrowing knowledge from other time series that have undergone similar shocks for which post-shock outcomes are observed. These additional time series form a donor pool. For each of the time series in the donor pool we calculate a p-value corresponding to a hypothesis test on the relevance of the inclusion of shock-effect information in predicting the response over the time horizon. These p-values are then combined to form an aggregated p-value which guides one decision in determining whether the shock effect for the time series under study is expected to be prevalent or transient. This p-value can be computed before the shock-effect is observed in the time series under study provided one can form a suitable donor pool. Several simulated data examples, and two real data examples of forecasting Conoco Phillips stock price and .... are provided for verification and illustration.

1 Introduction

We provide forecasting methodology for assessing the lingering effect of an anticipated structural shock to a time series under study. We focus on the setting in which a structural shock has occurred and one desires a prediction for the post-shock response over a set time horizon H. Specific interest is in determining whether the shock is expected to be permanent or transient over H. Standard forecasting methods may not yield any guidance on the post-shock trajectories [Baumeister and Kilian, 2014b]. This is a general problem that has many real life applications. For example, one may acquire terrible or great news about a company and desire to determine whether that news is bound to impact the stock price of that company over a relevant time period. Companies may be interested in forecasting the demand of their products after they were involved in a brand crisis, but they only have recent sales data from pre-crises times. All is not lost in this forecasting setting, one may be able to supplement the present forecast with past data borrowed from other time series which contain post-shock trajectories arising from materially similar structural shocks.

The core idea of our methodology is to sensibly aggregate similar past realized shock effects which arose from other time series, and then incorporate the aggregated shock effect estimator into the present forecast.

Our testing method embraces ideas from forecast aggregation in the post-shock setting [Lin and Eck, 2021], forecast comparison [Diebold and Mariano, 1995, Quaedvlieg, 2021], p-value combination, conditional forecasting [Baumeister and Kilian, 2014b, Kilian and Lütkepohl, 2017], time series pooling using cross-sectional panel data [Ramaswamy et al., 1993, Pesaran et al., 1999, Hoogstrate et al., 2000, Baltagi, 2008, Koop and Korobilis, 2012, Liu et al., 2020], forecasting with judgement and models [Svensson, 2005, Monti, 2008], synthetic control methodology [Abadie et al., 2010, Agarwal et al., 2020], expectation shocks [Croushore and Evans, 2006, Baumeister and Kilian, 2014a, Clements et al., 2019].

2 Setting

We will suppose that a researcher has multivariate time series data $\mathbf{y}_{i,t}$, $t = 1, \ldots, T_i$ and $i = 1, \ldots, n+1$. We let $\mathbf{y}_{i,t} = (y_{i,t}, \mathbf{x}_{i,t})$ where $y_{i,t}$ is a scalar response and $\mathbf{x}_{i,t}$ is a vector of covariates that are revealed to the analyst prior to the observation of $y_{1,t}$. Suppose that the analyst is interested in forecasting $y_{1,t}$, the first time series in the collection. We will suppose that each time series $\mathbf{y}_{i,t}$ undergoes a shock at time $T_i^* \leq T_i + 1$. To define an interesting setting, we will suppose that $T_1^* = T_1 + 1$, and $1 < T_i^* < T_i + 1$ for $i \geq 2$. We will suppose that $\mathbf{x}_{i,t=T_i^*}$ is observed before the shock takes effect on $y_{i,t=T_i^*}$.

We are interested in point forecasts $y_{i,t}^h$ at multiple horizons, $h = 1, \ldots, H$ with the aim of determining whether the shock has an effect on $y_{i,t}^h$. Quaedvlieg [2021] provided a methodology for comparing forecasts jointly across all horizons of a forecast path, $h = 1, \ldots, H$. In our post-shock setting, we want to compare the forecasts

$$\hat{y}_{1,t}^{i,h} \text{ and } \hat{y}_{i,t}^{2,h}$$

where $y_{i,t}^{1,h}$ is the forecast for $y_{i,t}$ that accounts for the yet-to-be observed structural shock and is based on the information set $\mathcal{F}_{t-h}$, and $\hat{y}_{i,t}^{2,h}$ is defined similarly for the forecast that does not include any shock effect information. We will compare these forecasts in terms of their loss differential

$$\mathbf{d}_{i,t} = \text{Ł}_{i,t,1} - \text{Ł}_{i,t,2},$$

where $L_{i,t,j} \in \mathbb{R}^H$ has elements $L^h(y_{i,t}, \hat{y}_{i,t}^{h,j})$, $j = 1, 2$, and L is a loss function. Hypothesis tests in Quaedvlieg [2021] are with respect to $E(\dot{i}, t) = \mu_{i,t}$. Conditions for these tests require conditions of Giacomini and White [2006].

We will be interested in $\mu_i = \lim_{T\to\infty} \frac{1}{T}\sum_{i=1}^{T} \mu_{i,t}$.

Note: We need more formality for constructing $\hat{y}_{1,t}^{1,h}$. We could use the forecasts in Lin and Eck [2021] and then consider h-ahead methods after adjusting for the shock. Or we could consider aggregation approaches which average all post-shock responses of the series in the donor pool.

We will consider the average superior predictive ability (aSPA) to assess whether or not a shock is permanent or transitory. The aSPA investigates forecast comparisons based on their weighted average loss difference

$$\mu^{(AVG)} = \mathbf{w}^T \mu = \sum_{h=1}^{H} w_h \mu^h$$

with weights $\mathbf{w}$ that sum to one. Note that aSPA requires the user to take a stand on the relative importance of under-performance at one horizon against out-performance at another, and note that it is likely that $\mu^h > 0$ for h closer to 1 since the user expects that a structural shock will occur and the structural shock is taken into account by forecast 1.

2.1 Model setup

The assumed autoregressive models considered are that in Lin and Eck [2021], we describe details here. Let $I(\cdot)$ be an indicator function, T_i be the time length of the time series i for $i = 1, \ldots, n+1$, and T_i^* be the time point just before the one when the shock is known to occur, with $T_i^* < T_i$. For $t = 1, \ldots, T_i$ and $i = 1, \ldots, n+1$, the model $\mathcal{M}_1$ is defined as

$$\mathcal{M}_1\colon y_{i,t} = \eta_i + \alpha_i D_{i,t} + \phi_i y_{i,t-1} + \theta_i' \mathbf{x}_{i,t} + \varepsilon_{i,t} \tag{1}$$

where $D_{i,t} = I(t = T_i^* + 1)$ and $\mathbf{x}_{i,t} \in \mathbb{R}^p$ with $p \geq 1$. We assume that the $\mathbf{x}_{i,t}$s are fixed. Let $|x|$ denote the absolute value of x for $x \in \mathbb{R}$. For $i = 1, \ldots, n+1$ and $t = 1, \ldots, T_i$, the random effects structure for $\mathcal{M}_1$ is:

$$\eta_i \overset{iid}{\sim} \mathcal{F}_\eta \text{ with } \mathrm{E}_{\mathcal{F}_\eta}(\eta_i) = 0, \mathrm{Var}_{\mathcal{F}_\eta}(\eta_i) = \sigma_\eta^2$$

$$
\begin{aligned}
&\phi_i \overset{iid}{\sim} \mathcal{F}_\phi \text{ where } |\mathcal{F}_\phi| < 1,\\
&\theta_i \overset{iid}{\sim} \mathcal{F}_\theta \text{ with } \mathrm{E}_{\mathcal{F}_\theta}(\theta_i) = \mu_\theta, \mathrm{Var}_{\mathcal{F}_\theta}(\theta_i) = \Sigma_\theta^2\\
&\alpha_i \overset{iid}{\sim} \mathcal{F}_\alpha \text{ with } \mathrm{E}_{\mathcal{F}_\alpha}(\alpha_i) = \mu_\alpha, \mathrm{Var}_{\mathcal{F}_\alpha}(\alpha_i) = \sigma_\alpha^2\\
&\varepsilon_{i,t} \overset{iid}{\sim} \mathcal{F}_{\varepsilon_i} \text{ with } \mathrm{E}_{\mathcal{F}_{\varepsilon_i}}(\varepsilon_{i,t}) = 0, \mathrm{Var}_{\mathcal{F}_{\varepsilon_i}}(\varepsilon_{i,t}) = \sigma_i^2,\\
&\eta_i \perp\!\!\!\perp \alpha_i \perp\!\!\!\perp \phi_i \perp\!\!\!\perp \theta_i \perp\!\!\!\perp \varepsilon_{i,t},
\end{aligned}
$$

where $\perp\!\!\!\perp$ denotes the independence operator. We also consider a modeling framework where the shock effects are linear functions of covariates with an additional additive mean-zero error. For $i = 1, \ldots, n+1$, the random effects structure for this model (model $\mathcal{M}_2$) is:

$$
\mathcal{M}_2\colon \begin{array}{l} y_{i,t} = \eta_i + \alpha_i D_{i,t} + \phi_i y_{i,t-1} + \theta_i' \mathbf{x}_{i,t} + \varepsilon_{i,t} \\ \alpha_i = \mu_\alpha + \delta_i' \mathbf{x}_{i,T_i^*+1} + \tilde{\varepsilon}_i, \end{array} \tag{2}
$$

where the added random effects are

$$
\begin{aligned}
&\tilde{\varepsilon}_i \overset{iid}{\sim} \mathcal{F}_{\tilde{\varepsilon}} \text{ with } \mathrm{E}_{\mathcal{F}_{\tilde{\varepsilon}}}(\tilde{\varepsilon}_i) = 0, \mathrm{Var}_{\mathcal{F}_{\tilde{\varepsilon}}}(\tilde{\varepsilon}_i) = \sigma_\alpha^2\\
&\eta_i \perp\!\!\!\perp \alpha_i \perp\!\!\!\perp \phi_i \perp\!\!\!\perp \theta_i \perp\!\!\!\perp \varepsilon_{i,t} \perp\!\!\!\perp \tilde{\varepsilon}_i.
\end{aligned}
$$

We further define $\tilde{\alpha}_i = \mu_\alpha + \delta_i' \mathbf{x}_{i,T_i^*+1}$. We will investigate the post-shock aggregated estimators in $\mathcal{M}_2$ in settings where δ_i is either fixed or random. We let $\mathcal{M}_{21}$ denote model $\mathcal{M}_2$ with $\delta_i = \delta$ for $i = 1, \ldots, n+1$, where δ is a fixed unknown parameter. We let $\mathcal{M}_{22}$ denote model $\mathcal{M}_2$ with the following random effects :

$$
\begin{aligned}
&\delta_i \overset{iid}{\sim} \mathcal{F}_\delta \text{ with } \mathrm{E}_{\mathcal{F}_\delta}(\delta_i) = \mu_\delta, \mathrm{Var}_{\mathcal{F}_\delta}(\delta_i) = \Sigma_\delta\\
&\delta_i \perp\!\!\!\perp \tilde{\varepsilon}_i.
\end{aligned}
$$

We further define the parameter sets

$$
\begin{aligned}
\Theta &= \{(\eta_i, \phi_i, \theta_i, \alpha_i, \mathbf{x}_{i,t}, y_{i,t-1}, \delta_i)\colon t = 1, \ldots, T_i, i = 2, \ldots, n+1\}\\
\Theta_1 &= \{(\eta_i, \phi_i, \theta_i, \alpha_i, \mathbf{x}_{i,t}, y_{i,t-1}, \delta_i)\colon t = 1, \ldots, T_i, i = 1\}
\end{aligned} \tag{3}
$$

where Θ and Θ_1 can adapt to $\mathcal{M}_1$ by dropping δ_i. We assume this for notational simplicity.

2.2 Forecast

In our post-shock setting we consider the following candidate forecasts:

$$
\begin{aligned}
&\text{Forecast 1}: \hat{y}^1_{1,T_1^*+1} = \hat{\eta}_1 + \hat{\phi}_1 y_{1,T_1^*} + \hat{\theta}_1' \mathbf{x}_{1,T_1^*+1},\\
&\text{Forecast 2}: \hat{y}^2_{1,T_1^*+1} = \hat{\eta}_1 + \hat{\phi}_1 y_{1,T_1^*} + \hat{\theta}_1' \mathbf{x}_{1,T_1^*+1} + \hat{\alpha},
\end{aligned}
$$

where $\hat{\eta}_1$, $\hat{\phi}_1$, and $\hat{\theta}_1$ are all OLS estimators of η_1, ϕ_1, and θ_1, respectively, and $\hat{\alpha}$ is some form of estimator for the shock effect of time series of interest, i.e., α_1.

Throughout the rest of this article we highlight when the information from the time series donor pool, indexed by $\{y_{i,t}\colon t = 1, \ldots, T_i, i = 2, \ldots, n+1\}$, can be used to construct a shock effect estimator $\hat{\alpha}$ in which Forecast 2 beats Forecast 1 and vice-versa. We will consider the different dynamic panel models $\mathcal{M}_1$, $\mathcal{M}_{21}$, and $\mathcal{M}_{22}$. We want to determine which forecast is appropriate over a horizon while the methods in Lin and Eck [2021] were only appropriate in the nowcasting setting in which prediction was only focused on the response immediately following the shock.

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