Testing for the prevalence or transience of a shock effect in the post-shock setting

Abstract

We provide a hypothesis test for the existence or transience of a shock in the post-shock setting of Lin and Eck [2021].

1 Setting

1.1 Model setup

The assumed autoregressive models considered are that in Lin and Eck [2021], we describe details here. Let $I(\cdot)$ be an indicator function, T_i be the time length of the time series i for $i = 1, \ldots, n+1$, and T_i^* be the time point just before the one when the shock is known to occur, with $T_i^* < T_i$. For $t = 1, \ldots, T_i$ and $i = 1, \ldots, n+1$, the model $\mathcal{M}_1$ is defined as

$$\mathcal{M}_1\colon y_{i,t} = \eta_i + \alpha_i D_{i,t} + \phi_i y_{i,t-1} + \theta_i' \mathbf{x}_{i,t} + \varepsilon_{i,t} \tag{1}$$

where $D_{i,t} = I(t = T_i^* + 1)$ and $\mathbf{x}_{i,t} \in \mathbb{R}^p$ with $p \geq 1$. We assume that the $\mathbf{x}_{i,t}$s are fixed. Let $|x|$ denote the absolute value of x for $x \in \mathbb{R}$. For $i = 1, \ldots, n+1$ and $t = 1, \ldots, T_i$, the random effects structure for $\mathcal{M}_1$ is:

$$\begin{aligned}
\eta_i &\overset{iid}{\sim} \mathcal{F}_\eta \text{ with } \mathrm{E}_{\mathcal{F}_\eta}(\eta_i) = 0, \mathrm{Var}_{\mathcal{F}_\eta}(\eta_i) = \sigma_\eta^2 \\
\phi_i &\overset{iid}{\sim} \mathcal{F}_\phi \text{ where } |\mathcal{F}_\phi| < 1, \\
\theta_i &\overset{iid}{\sim} \mathcal{F}_\theta \text{ with } \mathrm{E}_{\mathcal{F}_\theta}(\theta_i) = \mu_\theta, \mathrm{Var}_{\mathcal{F}_\theta}(\theta_i) = \Sigma_\theta^2 \\
\alpha_i &\overset{iid}{\sim} \mathcal{F}_\alpha \text{ with } \mathrm{E}_{\mathcal{F}_\alpha}(\alpha_i) = \mu_\alpha, \mathrm{Var}_{\mathcal{F}_\alpha}(\alpha_i) = \sigma_\alpha^2 \\
\varepsilon_{i,t} &\overset{iid}{\sim} \mathcal{F}_{\varepsilon_i} \text{ with } \mathrm{E}_{\mathcal{F}_{\varepsilon_i}}(\varepsilon_{i,t}) = 0, \mathrm{Var}_{\mathcal{F}_{\varepsilon_i}}(\varepsilon_{i,t}) = \sigma_i^2, \\
\eta_i &\perp\!\!\!\perp \alpha_i \perp\!\!\!\perp \phi_i \perp\!\!\!\perp \theta_i \perp\!\!\!\perp \varepsilon_{i,t},
\end{aligned}$$

where $\perp\!\!\!\perp$ denotes the independence operator. We also consider a modeling framework where the shock effects are linear functions of covariates with an additional additive mean-zero error. For $i = 1, \ldots, n+1$, the random effects structure for this model (model $\mathcal{M}_2$) is:

$$\mathcal{M}_2\colon \begin{array}{l} y_{i,t} = \eta_i + \alpha_i D_{i,t} + \phi_i y_{i,t-1} + \theta_i' \mathbf{x}_{i,t} + \varepsilon_{i,t} \\ \alpha_i = \mu_\alpha + \delta_i' \mathbf{x}_{i,T_i^*+1} + \tilde{\varepsilon}_i, \end{array} \tag{2}$$

where the added random effects are

$$\begin{aligned}
\tilde{\varepsilon}_i &\overset{iid}{\sim} \mathcal{F}_{\tilde{\varepsilon}} \text{ with } \mathrm{E}_{\mathcal{F}_{\tilde{\varepsilon}}}(\tilde{\varepsilon}_i) = 0, \mathrm{Var}_{\mathcal{F}_{\tilde{\varepsilon}}}(\tilde{\varepsilon}_i) = \sigma_\alpha^2 \\
\eta_i &\perp\!\!\!\perp \alpha_i \perp\!\!\!\perp \phi_i \perp\!\!\!\perp \theta_i \perp\!\!\!\perp \varepsilon_{i,t} \perp\!\!\!\perp \tilde{\varepsilon}_i.
\end{aligned}$$

We further define $\tilde{\alpha}_i = \mu_\alpha + \delta_i' \mathbf{x}_{i,T_i^*+1}$. We will investigate the post-shock aggregated estimators in $\mathcal{M}_2$ in settings where δ_i is either fixed or random. We let $\mathcal{M}_{21}$ denote model $\mathcal{M}_2$ with $\delta_i = \delta$ for $i = 1, \ldots, n+1$, where δ is a fixed unknown parameter. We let $\mathcal{M}_{22}$ denote model $\mathcal{M}_2$ with the following random effects :

$$
\begin{aligned}
&\delta_i \overset{iid}{\sim} \mathcal{F}_\delta \text{ with } \mathrm{E}_{\mathcal{F}_\delta}(\delta_i) = \mu_\delta, \mathrm{Var}_{\mathcal{F}_\delta}(\delta_i) = \Sigma_\delta \\
&\delta_i \perp\!\!\!\perp \tilde{\varepsilon}_i.
\end{aligned}
$$

We further define the parameter sets

$$
\begin{aligned}
\Theta &= \{(\eta_i, \phi_i, \theta_i, \alpha_i, \mathbf{x}_{i,t}, y_{i,t-1}, \delta_i) \colon t = 1, \ldots, T_i, i = 2, \ldots, n+1\} \\
\Theta_1 &= \{(\eta_i, \phi_i, \theta_i, \alpha_i, \mathbf{x}_{i,t}, y_{i,t-1}, \delta_i) \colon t = 1, \ldots, T_i, i = 1\}
\end{aligned} \tag{3}
$$

where Θ and Θ_1 can adapt to $\mathcal{M}_1$ by dropping δ_i. We assume this for notational simplicity.

1.2 Forecast

In our post-shock setting we consider the following candidate forecasts:

$$
\begin{aligned}
&\text{Forecast 1} : \hat{y}^1_{1,T_1^*+1} = \hat{\eta}_1 + \hat{\phi}_1 y_{1,T_1^*} + \hat{\theta}_1' \mathbf{x}_{1,T_1^*+1}, \\
&\text{Forecast 2} : \hat{y}^2_{1,T_1^*+1} = \hat{\eta}_1 + \hat{\phi}_1 y_{1,T_1^*} + \hat{\theta}_1' \mathbf{x}_{1,T_1^*+1} + \hat{\alpha},
\end{aligned}
$$

where $\hat{\eta}_1$, $\hat{\phi}_1$, and $\hat{\theta}_1$ are all OLS estimators of η_1, ϕ_1, and θ_1, respectively, and $\hat{\alpha}$ is some form of estimator for the shock effect of time series of interest, i.e., α_1.

Throughout the rest of this article we highlight when the information from the time series donor pool, indexed by $\{y_{i,t} \colon t = 1, \ldots, T_i, i = 2, \ldots, n+1\}$, can be used to construct a shock effect estimator $\hat{\alpha}$ in which Forecast 2 beats Forecast 1 and vice-versa. We will consider the different dynamic panel models $\mathcal{M}_1$, $\mathcal{M}_{21}$, and $\mathcal{M}_{22}$. We want to determine which forecast is appropriate over a horizon while the methods in Lin and Eck [2021] were only appropriate in the nowcasting setting in which prediction was only focused on the response immediately following the shock.

1.3 Multi-horizon forecast evaluation

We now consider an approach for comparing the forecasts over a specified time horizon after a shock has occurred by borrowing information across similar forecasts.

Quaedvlieg [2021] provided a methodology for comparing forecasts jointly across all horizons of a forecast path, $h = 1, \ldots, H$. In our post-shock setting, we want to compare the forecasts

$$
\hat{y}^{1,h}_{1,t} \text{ and } \hat{y}^{2,h}_{1,t}
$$

where $y^{1,h}_{1,t}$ is the forecast for y_t that accounts for the yet-to-be observed structural shock and is based on the information set $\mathcal{F}_{t-h}$, and $\hat{y}^{2,h}_{1,t}$ is defined similarly for the forecast that does not include any shock effect information. We will compare these forecasts in terms of their loss differential

$$
d_t = L_{1,t} - L_{2,t},
$$

where $L_{j,t} = L(y_t, \hat{y}_{j,t})$, $j = 1, 2$, and L is a loss function. Hypothesis tests in Quaedvlieg [2021] are with respect to $E(d_t) = \mu_t$. Conditions for these tests require conditions of Giacomini and White [2006].

Note: We need more formality for constructing $\hat{y}^{1,h}_{1,t}$. We could use the forecasts in Lin and Eck [2021] and then consider h-ahead methods after adjusting for the shock. Or we could consider aggregation approaches which average all post-shock responses of the series in the donor pool.

We will consider the average superior predictive ability (aSPA) to assess whether or not a shock is permanent or transitory. The aSPA investigates forecast comparisons based on their weighted average loss difference

$$\mu^{(AVG)} = \mathbf{w}^T \mu = \sum_{h=1}^{H} w_h \mu^h$$

with weights $\mathbf{w}$ that sum to one. Note that aSPA requires the user to take a stand on the relative importance of under-performance at one horizon against out-performance at another, and note that it is likely that $\mu^h > 0$ for h closer to 1 since the user expects that a structural shock will occur and the structural shock is taken into account by forecast 1.

References

Raffaella Giacomini and Halbert White. Tests of conditional predictive ability. *Econometrica*, 74(6): 1545–1578, 2006.

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