# Minimizing post-shock forecasting error using disparate information

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**Abstract**

We develop a forecasting methodology for providing credible forecasts for time series that have recently undergone a shock. We achieve this by borrowing knowledge from disparate time series that have undergone similar shocks for which post-shock outcome is recorded. Three shock effect estimators are motivated with the aim of minimizing average forecast risk. We propose risk-reduction propositions that provide conditions that establish when our methodology works. Bootstrap procedures are provided to estimate the variability of our shock effect estimators; and these procedures can be used to assess the potential prospective success of post-shock forecasts. Leave-one-out cross validation with $k$ random draws is proposed to estimate correctness of risk-reduction propositions, prospectively informing users of the probabilities that this prospective evaluation is in line with the reality. Several simulated data examples, and a real data example of forecasting Conoco Phillips stock price are provided for verification and illustration.

## 1 Introduction

We provide forecasting adjustment techniques with the goal of lowering overall forecast error when the time series under study has undergone a structural shock. We focus on the specific setting in which a structural shock has occurred and one desires a prediction for the post-shock response at the next time point. Standard forecasting methods may not yield accurate predictions in the presence of such structural shocks [Baumeister and Kilian, 2014b]. This is a general problem that has many real life applications. For example, one may be interested in forecasting the stock price of a company tomorrow after hearing terrible or great news about the company after hours trading. Companies may be interested in forecasting the demand of their products to adjust production after they were involved in a brand crisis, but they only have recent sales data for which the company is operating well. All is not lost in this setting, one may be able to supplement the present forecast with past data borrowed from disparate time series which contain similar structural shocks. The core idea of our methodology is to sensibly aggregate similar past realized shock effects which arose from disparate time series, and then incorporate the aggregated shock effect estimator into the present forecast. Our method of combining disparate shock effects embraces ideas from conditional forecasting [Baumeister and Kilian, 2014b, Kilian and Lütkepohl, 2017], time series pooling using cross-sectional panel data [Ramaswamy et al., 1993, Pesaran et al., 1999, Hoogstrate et al., 2000, Baltagi, 2008, Koop and Korobilis, 2012, Liu et al., 2020], forecasting with judgement and models [Svensson, 2005, Monti, 2008], synthetic control methodology [Abadie et al., 2010, Agarwal et al., 2020], and expectation shocks [Croushore and Evans, 2006, Baumeister and Kilian, 2014a, Clements et al., 2019].

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We study the post-shock forecasting problem in the context of additive shock effects in linear autoregressive models. In this post-shock forecasting setting, the researcher has a time series of interest which is known to have recently undergone a structural shock, and the post-shock response is not observed. In this setting, the additive shock effect is a random effect that is parameterized in the autoregressive model. The shock effect is then estimated using ordinary least squares (OLS). The researcher must move beyond the modeling paradigm that they were previously working under to accommodate this new shock effect [Monti, 2008, Svensson, 2005]. One method for estimating the shock effect is to produce a conditional point forecast where a sequence of non-zero future structural shocks are conditioned upon and estimated [Baumeister and Kilian, 2014b]. Such conditional point forecasts are appropriate when the shock sequence considered is within the range of historical experience [Kilian and Lütkepohl, 2017]. On the other hand, our methodology allows for the inclusion of outside data sources and covariates into this conditional forecasting context provided that the shock effects from outside data sources are all thought to arise from a data generating process similar to that of the shock under study. For further differences of assumptions on shocks, our methodology allows for unprecedented shocks and no observation of past shocks.

In our methodological framework, the researcher creates a synthetic panel of disparate time series which have undergone similar structural shocks in the past. Construction of the donor pool that forms this synthetic panel is similar to that in synthetic control methodology (SCM) [Abadie et al., 2010]. As in SCM, care is needed when forming the donor pool of disparate time series. However, there are key differences between our framework and SCM. We assume that the disparate time series are independent from the time series under study before the timing of the shock. We also assume that the shock effects for each disparate time series are independent realizations from some unknown distribution with existing first and second moments.

We estimate the shock effects that are present in the disparate time series for which post-shock responses are observed. We then aggregate these estimated shock effects and use this aggregated estimate as an estimator for the shock effect in the time series of interest. This estimator is then added to a forecast for the yet to be realized post-shock response corresponding to the time series of interest. Shock effects in our post-shock forecasting framework is similar to "expectation shocks" which are studied in Clements et al. [2019]. The context in Clements et al. [2019] allows for consistent estimation of expectation shocks under a vector autoregressive model, possibly involving an instrumental variable approach as in Croushore and Evans [2006]. In our context, the yet to observed shock effect of interest is a random effect, and we can only partially estimate features of the random effect distribution using the disparate time series.

In this article, we will assume a simple auto regressive data generating process similar to that in Blundell and Bond [1998] with a general random-effect structure. Therefore, our methodology is similar to the "$K$ latent pooling" framework of Ramaswamy et al. [1993]. However, our model formulation is more general than Ramaswamy et al. [1993]. In our model, the donor pool can consist of dependent time series but time series within the donor pool should be independent of the time series of interest. However, mutual independence among time series in the donor pool can aid prospective evaluation of the reliability of our method. We consider three aggregation techniques: simple averaging, inverse-variance weighted averaging, and similarity weighting. The latter technique is similar to the weighting in synthetic control methodology [Abadie et al., 2010]. Our auto regressive model will consider present day covariates to better motivate similarity weighting. The considered adjustment strategies all target the mean of the shock effect distribution. Such an estimation strategy can reduce mean squared error (MSE) when variation in the shock effect distribution is small relative to the mean. We provide risk-reduction propositions that detail the conditions when the adjusted forecasts will work better than the original forecast. The involved parameters in the risk-reduction propositions can be estimated by a residual bootstrap procedure that we develop. We also motivate a simple leave-one-out cross validation with $k$ random draws procedure which can prospectively assess the performance of our shock effect adjustment estimators. This prospective assessment does not require the observation of the post-shock response. Our Monte Carlo simulation results show that the risk-reduction propositions are nearly perfectly correct when the model for the shock effects is identified well with appropriate covariates under a fixed design. We demonstrate the utility of our methodology in a real data analysis in which we forecast the stock price of Conoco Phillips shares that experienced a large structural shock on March 9th, 2020. We will show that our proposed adjustment estimators yield much better results than no adjustment in this setting. We also use this example to demonstrate settings in which the shock effect may be decomposed into separate estimable parts. We now motivate our framework for post-shock forecasting.

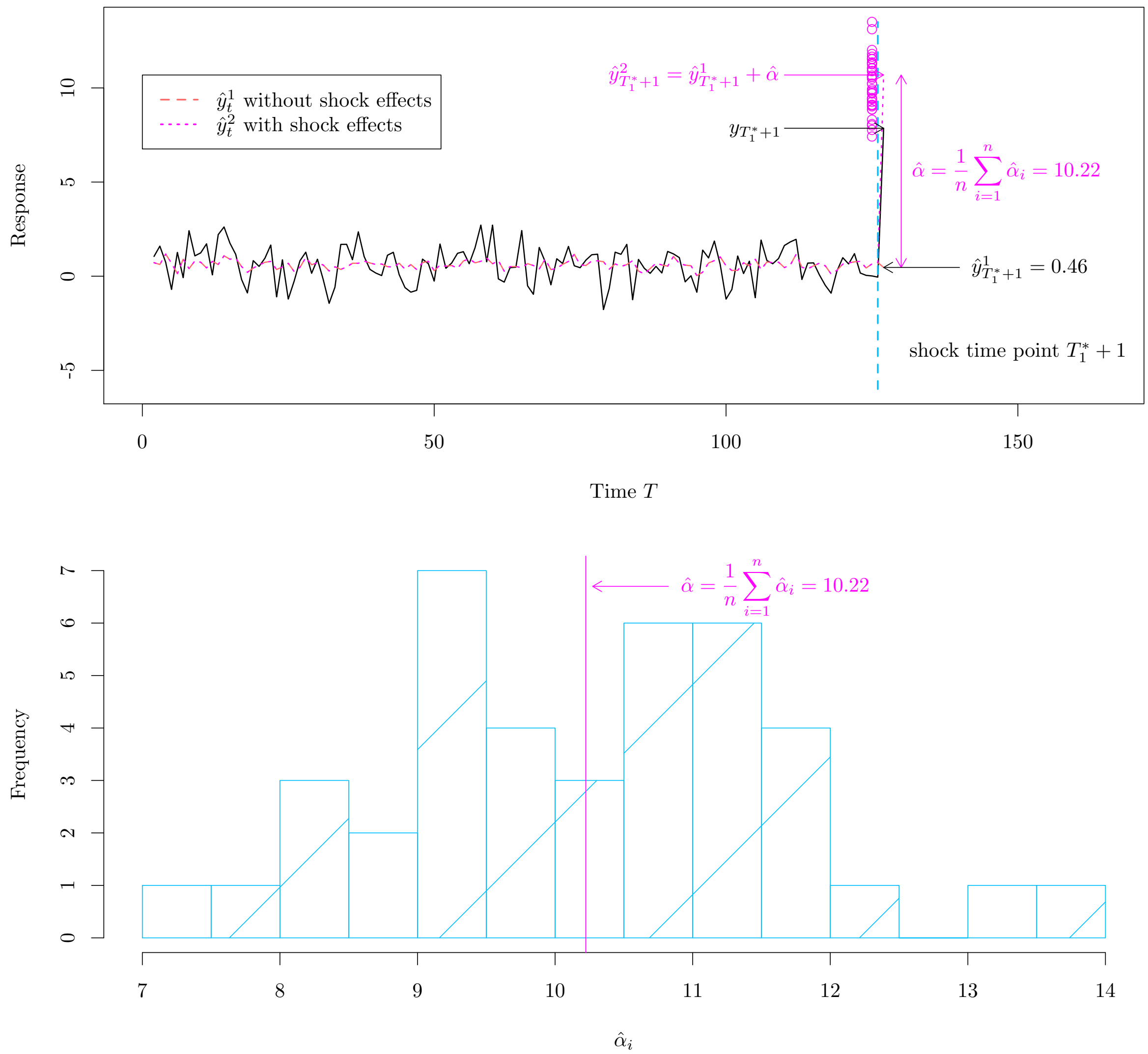


**Figure 1.** The time series experience a shock at $T_1^* + 1 = 126$ with true shock effect $\alpha = 9.21$. (a) presents the comparison of the prediction without adjustment and one that uses simple averaging of estimated shock effects given $n = 40$ disparate time series. (b) shows the histogram for the least square estimates $\hat{\alpha}_i$s used in estimating shock $\alpha_1$ for $i = 2, \ldots, 41$. The magenta dots represent $\hat{\alpha}_i$ from disparate time series. The prediction of $\hat{y}^2_{T_1^*+1}$ and $\hat{y}^1_{T_1^*+1}$ differs only by an adjustment $\hat{\alpha} = 10.22$. It is clear that $\hat{y}^2_{T_1^*+1}$ performs better than $\hat{y}^1_{T_1^*+1}$.

## 2 Setting

We will suppose that a researcher has time series data $(y_{i,t}, \mathbf{x}_{i,t})$, for $t = 1, \ldots, T_i$ and $i = 1, \ldots, n + 1$, where $y_{i,t}$ is a scalar response and $\mathbf{x}_{i,t}$ is a vector of covariates that are revealed to the analyst prior to the observation of $y_{1,t}$. Suppose that the analyst is interested in forecasting $y_{1,t}$, the first time series in the collection. We will suppose that specific interest is in forecasting the response after the occurrence of a structural shock. To gauge the performance of forecasts, we consider forecast risk in the form of MSE,

$$R_T = \frac{1}{T}\sum_{t=1}^{T} \mathrm{E}(\hat{y}_{1,t} - y_{1,t})^2,$$

and root mean squared error (RMSE), given by $\sqrt{R_T}$, in our analyses. In this article, we focus on post-shock prediction where forecasts methods only differ at the next future time point. Thus the MSE reduces to the magnitude $\mathrm{E}(\hat{y}_{1,t} - y_{1,t})^2$.

Our post-shock forecasting methodology will consist of selecting covariates $\mathbf{x}_{i,t}$, constructing a suitable donor pool of candidate time series that have undergone similar structural shocks to the time series under study, and specifying a model for the time series $(y_{i,t}, \mathbf{x}_{i,t})$, for $t = 1, \ldots, T_i$ and $i = 1, \ldots, n+1$. In this article, we consider a dynamic panel data model with autoregressive structure similar to that in Blundell and Bond [1998]. Our dynamic panel model includes an additional shock effect whose presence or absence is given by the binary variable $D_{i,t}$, and we will assume that the donor pool time series are independent of the time series under study. The details of this model are in the next section.

Figure 1 provides a simple intuition of the practical usefulness of our proposed methodology. This figure depicts a time series that experienced a shock at time point $T_1^* + 1 = 126$. It is supposed that the researcher does not have any information beyond $T_1^* + 1$, but does have observations of forty disparate time series that have previously undergone a similar shock for which post-shock responses are recorded. Similarity in this context means that the shock effects are random variables that from a common distribution. In this example, the mean of the estimated shock effects is taken as a shock effect estimator for the time series under study. Forecasts are then made by adding this shock effect estimator to the estimated response values obtained from the estimation procedure that ignores the shock. It is apparent from Figure 1 that adjusting forecasts in this manner 1) leads to a reduction in forecasting risk; 2) does not fully recover the true shock effect. We evaluate the performance of this post-shock forecasting methodology throughout this article; we outline situations for when it is expected to work and when it is not.

## 2.1 Model Setup

In this section, we will describe the assumed dynamic panel models for which post-shock aggregated estimators are provided. The basic structures of these models are the same for all time-series in the analysis, the differences between them lie in the setup of the shock effect distribution.

Let $I(\cdot)$ be an indicator function, $T_i$ be the time length of the time series $i$ for $i = 1, \ldots, n+1$, and $T_i^*$ be the time point just before the one when the shock is known to occur, with $T_i^* < T_i$. For $t = 1, \ldots, T_i$ and $i = 1, \ldots, n+1$, the model $\mathcal{M}_1$ is defined as

$$\mathcal{M}_1 \colon y_{i,t} = \eta_i + \alpha_i D_{i,t} + \phi_i y_{i,t-1} + \theta_i' \mathbf{x}_{i,t} + \varepsilon_{i,t} \tag{1}$$

where $D_{i,t} = I(t = T_i^* + 1)$ and $\mathbf{x}_{i,t} \in \mathbb{R}^p$ with $p \geq 1$. We assume that the $\mathbf{x}_{i,t}$'s are fixed. Let $|x|$ denote the absolute value of $x$ for $x \in \mathbb{R}$. For $i = 1, \ldots, n+1$ and $t = 1, \ldots, T_i$, the random effects structure for $\mathcal{M}_1$ is:

$$\begin{aligned}
\eta_i &\overset{iid}{\sim} \mathrm{E}(\eta_i) = 0, \mathrm{Var}(\eta_i) = \sigma_\eta^2 \\
\phi_i &\overset{iid}{\sim} |\phi_i| < 1, \\
\theta_i &\overset{iid}{\sim} \mathrm{E}(\theta_i) = \mu_\theta, \mathrm{Var}(\theta_i) = \Sigma_\theta^2 \\
\alpha_i &\overset{iid}{\sim} \mathrm{E}(\alpha_i) = \mu_\alpha, \mathrm{Var}(\alpha_i) = \sigma_\alpha^2, \\
\varepsilon_{i,t} &\overset{iid}{\sim} \mathrm{E}(\varepsilon_{i,t}) = 0, \mathrm{Var}(\varepsilon_{i,t}) = \sigma^2 \text{ where } \sigma > 0,
\end{aligned}$$

$$\eta_i \perp\!\!\!\perp \alpha_i \perp\!\!\!\perp \phi_i \perp\!\!\!\perp \theta_i \perp\!\!\!\perp \varepsilon_{i,t}.$$

Notice that $\mathcal{M}_1$ assumes that $\alpha_i$ are iid with $\mathrm{E}(\alpha_i) = \mu_\alpha$ for $i = 1, \ldots, n+1$. We also consider a model where the shock effects are linear functions of covariates with an additional additive mean-zero error. For $i = 1, \ldots, n+1$, the random effects structure for this model (model $\mathcal{M}_2$) is:

$$\mathcal{M}_2\colon \begin{array}{l} y_{i,t} = \eta_i + \alpha_i D_{i,t} + \phi_i y_{i,t-1} + \theta_i' \mathbf{x}_{i,t} + \varepsilon_{i,t} \\ \alpha_i = \mu_\alpha + \delta_i' \mathbf{x}_{i,T_i^*+1} + \tilde{\varepsilon}_i, \end{array} \tag{2}$$

where the added random effects are

$$\begin{array}{l} \tilde{\varepsilon}_i \overset{iid}{\sim} \mathrm{E}(\tilde{\varepsilon}) = 0, \mathrm{Var}(\tilde{\varepsilon}) = \sigma_\alpha^2 \text{ where } \sigma_\alpha > 0 \\ \eta_i \perp\!\!\!\perp \alpha_i \perp\!\!\!\perp \phi_i \perp\!\!\!\perp \theta_i \perp\!\!\!\perp \varepsilon_{i,t} \perp\!\!\!\perp \tilde{\varepsilon}_i. \end{array}$$

We further define $\tilde{\alpha}_i = \mu_\alpha + \delta_i' \mathbf{x}_{i,T_i^*+1}$. We will investigate the post-shock aggregated estimators in $\mathcal{M}_2$ in settings where $\delta_i$ is either fixed or random. We let $\mathcal{M}_{21}$ denote model $\mathcal{M}_2$ with $\delta_i = \delta$ for $i = 1, \ldots, n+1$, where $\delta$ is a fixed unknown parameter. We let $\mathcal{M}_{22}$ denote model $\mathcal{M}_2$ with the following random effects structure for $\delta_i$:

$$\delta_i \overset{iid}{\sim} \mathrm{E}(\delta) = \mu_\delta, \mathrm{Var}(\delta) = \Sigma_\delta \quad \text{with} \quad \delta_i \perp\!\!\!\perp \tilde{\varepsilon}_i.$$

We further define the parameter sets

$$\begin{array}{rcl} \Theta & = & \{(\eta_i, \phi_i, \theta_i, \alpha_i, \mathbf{x}_{i,t}, y_{i,t-1}, \delta_i)\colon t = 1, \ldots, T_i, i = 2, \ldots, n+1\}, \\ \Theta_1 & = & \{(\eta_i, \phi_i, \theta_i, \alpha_i, \mathbf{x}_{i,t}, y_{i,t-1}, \delta_i)\colon t = 1, \ldots, T_i, i = 1\}, \end{array} \tag{3}$$

where $\Theta$ and $\Theta_1$ can adapt to $\mathcal{M}_1$ by dropping $\delta_i$. We assume this for notational simplicity.

## 2.2 Forecast

In this section we show how post-shock aggregate estimators improve upon standard forecasts that do not account for the shock effect. More formally, we will consider the following candidate forecasts:

$$\begin{array}{l} \text{Forecast 1} : \hat{y}^1_{1,T_1^*+1} = \hat{\eta}_1 + \hat{\phi}_1 y_{1,T_1^*} + \hat{\theta}_1' \mathbf{x}_{1,T_1^*+1}, \\ \text{Forecast 2} : \hat{y}^2_{1,T_1^*+1} = \hat{\eta}_1 + \hat{\phi}_1 y_{1,T_1^*} + \hat{\theta}_1' \mathbf{x}_{1,T_1^*+1} + \hat{\alpha}, \end{array}$$

where $\hat{\eta}_1$, $\hat{\phi}_1$, and $\hat{\theta}_1$ are all OLS estimators of $\eta_1$, $\phi_1$, and $\theta_1$, respectively, and $\hat{\alpha}$ is some form of estimator for the shock effect of time series of interest, i.e., $\alpha_1$. The first forecast ignores the presence of $\alpha_1$ while the second forecast incorporates an estimate of $\alpha_1$ that is obtained from the other independent forecasts under study.

Note that the two forecasts do not differ in their predictions for $y_{1,t}$, $t = 1, \ldots T_1^*$. Instead, they only differ in predicting $y_{1,T_1^*+1}$. Throughout the rest of this article we show that the collection of disparate time series $\{y_{i,t}\colon t = 1, \ldots, T_i, i = 2, \ldots, n+1\}$ has the potential to improve the forecasts for $y_{1,T_1^*+1}$ under different circumstances for the dynamic panel model $\mathcal{M}_1$, $\mathcal{M}_{21}$, and $\mathcal{M}_{22}$. Improvement will be measured by assessing the reduction in risk that Forecast 2 offers over Forecast 1. We will return to the theoretical details of risk-reduction in Section 3.

We specifically focus on predictions for $y_{1,T_1^*+1}$, the first post-shock response. It is important to note that in general $\hat{\alpha}$ is not a consistent estimator of the unobserved $\alpha_1$ nor does it converge to $\alpha_1$. Despite these inferential shortcomings, adjustment of the forecast for $y_{1,T_1^*+1}$ through the addition of $\hat{\alpha}$ has the potential to lower forecast risk under several conditions corresponding to different estimators of $\alpha_1$.

### 2.3 Construction of shock effects estimators

We now construct the aggregated estimators of the shock effects that appear in Forecast 2 (see Section 2.2). We use these to forecast response values $y_{1,T_1^*+1}$ assuming that $T_1^*$ is known. First, we introduce the procedures of parameter estimation for $\mathcal{M}_1$, $\mathcal{M}_{21}$, and $\mathcal{M}_{22}$ (see Section 2.1). Conditional on all regression parameters, previous responses, and covariates, the response variable $y_{i,t}$ in $\mathcal{M}_1$, $\mathcal{M}_{21}$, and $\mathcal{M}_{22}$ has distribution

$$y_{i,t} \sim N(\eta_i + \alpha_i D_{i,t} + \phi_i y_{i,t-1} + \theta_i' \mathbf{x}_{i,t}, \sigma^2).$$

For $i = 2, \ldots, n+1$, all parameters in this model will be estimated with ordinary least squares (OLS) using historical data of $t = 1, \ldots, T_i$. For $i = 1$, we estimate all the parameters but $\alpha_1$ using OLS procedures for $t = 1, \ldots, T_1^*$. In particular, let $\hat{\alpha}_i$, $i = 2, \ldots, n+1$ be the OLS estimate of $\alpha_i$. Note that parameter estimation for $\mathcal{M}_1$ is identically the same as that for $\mathcal{M}_{21}$ or $\mathcal{M}_{22}$. We emphasize that $\alpha_i$s are random variables but the OLS estimation is conditioned on the their realizations from some distribution.

Second, we introduce the candidate estimators for $\alpha_1$. Define the adjustment estimator for time series $i = 1$ by

$$\hat{\alpha}_{\text{adj}} = \frac{1}{n} \sum_{i=2}^{n+1} \hat{\alpha}_i, \tag{4}$$

where the $\hat{\alpha}_i$s in (4) are OLS estimators of the $\alpha_i$s for $i = 2, \ldots, n+1$. We can use $\hat{\alpha}_{\text{adj}}$ as an estimator for the unknown $\alpha_1$ term for which no meaningful estimation information otherwise exists. It is intuitive that $\hat{\alpha}_{\text{adj}}$ should perform well under $\mathcal{M}_1$ where we assume that $\alpha_i$'s share the same mean for $i = 1, \ldots, n+1$. However, it can also be shown that $\hat{\alpha}_{\text{adj}}$ may be less favorable in $\mathcal{M}_{21}$ and $\mathcal{M}_{22}$, which will be discussed in detail in Section 3.

We also consider the inverse-variance weighted estimator in practical settings where the $T_i$'s and $T_i^*$'s vary greatly across $i = 2, \ldots, n+1$. The inverse-variance weighted estimator is defined as

$$\hat{\alpha}_{\text{IVW}} = \frac{\sum_{i=2}^{n+1} \hat{\alpha}_i / \hat{\sigma}_{i\alpha}^2}{\sum_{i=2}^{n+1} 1/\hat{\sigma}_{i\alpha}^2}, \qquad \text{where} \qquad \hat{\sigma}_{i\alpha}^2 = \hat{\sigma}_i^2 (\mathbf{U}_i' \mathbf{U}_i)_{22}^{-1},$$

where $\hat{\alpha}_i$ is the OLS estimator of $\alpha_i$, $\hat{\sigma}_i$ is the residual standard error from OLS estimation, and $\mathbf{U}_i$ is the design matrix for OLS with respect to time series for $i = 2, \ldots, n+1$. Note that since $\sigma$ is unknown, estimation is required and the numerator and denominator terms are dependent in general. However, $\hat{\alpha}_{\text{IVW}}$ can be a reasonable estimator in practical settings. We do not provide closed form expressions for $\mathrm{E}(\hat{\alpha}_{\text{IVW}})$ and $\mathrm{Var}(\hat{\alpha}_{\text{IVW}})$ but empirical performance of $\hat{\alpha}_{\text{IVW}}$ is assessed via Monte Carlo simulation (see Section 4).

We now motivate a weighted-adjustment estimator for model $\mathcal{M}_{21}$ and $\mathcal{M}_{22}$. Our weighted-adjustment estimator is inspired by the weighting techniques in synthetic control methodology (SCM) developed in Abadie et al. [2010]. However, our weighted-adjustment estimator is not a causal estimator and our estimation premise is a reversal of that in SCM. Our objective is in predicting a post-shock response $y_{1,T_1^*+1}$ that is not yet observed using disparate time series whose post-shock responses are observed.

We use similar notation as that in Abadie et al. [2010] to motivate our weighted-adjustment estimator. Consider a $\mathbf{W} \in \mathbb{R}^n$ weight vector $\mathbf{W} = (w_2, \ldots, w_{n+1})'$, where $w_i \in [0,1]$ for all $i = 2, \ldots, n+1$. Construct

$$\mathbf{X}_1 = \mathbf{x}_{1,T_1^*+1}', \quad \mathbf{X} = \begin{pmatrix} \mathbf{x}_{2,T_2^*+1}' \\ \vdots \\ \mathbf{x}_{n+1,T_{n+1}^*+1}' \end{pmatrix}, \quad \text{and} \quad \hat{\mathbf{X}}_1(\mathbf{W}) = \mathbf{W}'\mathbf{X},$$

where $\mathbf{X}_1, \hat{\mathbf{X}}_1(\mathbf{W}) \in \mathbb{R}^{1 \times p}$. Define $\mathcal{W} = \{\mathbf{W} \in [0,1]^n \colon 1_n' \mathbf{W} = 1\}$. Suppose there exists $\mathbf{W}^* \in \mathcal{W}$ with

$\mathbf{W}^* = (w_2^*, \ldots, w_{n+1}^*)'$ such that

$$\mathbf{X}_1 = \hat{\mathbf{X}}_1(\mathbf{W}^*), \quad i.e., \quad \mathbf{x}_{1,T_1^*+1} = \sum_{i=2}^{n+1} w_i^* \mathbf{x}_{i,T_i^*+1}. \tag{5}$$

Note that (5) tries to find $\mathbf{W}^*$ such that $\mathbf{x}_{1,T_1^*+1}$ is a convex combination of $\mathbf{x}_{i,T_i^*+1}$ for $i = 2, \ldots, n+1$ with weights $\mathbf{W}^*$. Therefore, $\mathbf{W}^*$ should exist as long as $\mathbf{X}_1$ falls in the convex hull of

$$\left\{ \mathbf{x}'_{2,T_2^*+1}, \ldots, \mathbf{x}'_{n+1,T_{n+1}^*+1} \right\}.$$

Our weighted-adjustment estimator will therefore perform well when the pool of disparate time series posses similar covariates to the time series for which no post-shock responses are observed. We compute $\mathbf{W}^*$ as

$$\mathbf{W}^* = \underset{\mathbf{W} \in \mathcal{W}}{\arg\min} \left\| \mathbf{X}_1 - \hat{\mathbf{X}}_1(\mathbf{W}) \right\|_p. \tag{6}$$

Abadie et al. [2010] commented that we can select $\mathbf{W}^*$ so that (5) holds approximately and that weighted-adjustment estimation techniques of this form are not appropriate when the fit is poor. Note that $\mathbf{W}^*$ is not random since the covariates are assumed to be fixed. Since $\mathcal{W}$ is a closed and bounded subset of $\mathbb{R}^n$, $\mathcal{W}$ is compact. Because the objective function is continuous in $\mathbf{W}$, $\mathbf{W}^*$ will always exist. Our weighted-adjustment estimator for the shock effect $\alpha_1$ is

$$\hat{\alpha}_{\text{wadj}} = \sum_{i=2}^{n+1} w_i^* \hat{\alpha}_i \quad \text{for} \quad \mathbf{W}^* = \begin{pmatrix} w_2^* & \cdots & w_{n+1}^* \end{pmatrix}'.$$

We further define

$$\mathbf{V} = (\mathbf{x}_{2,T_2^*+1}, \ldots, \mathbf{x}_{n+1,T_{n+1}^*+1}).$$

**Proposition 1.** *If $\mathbf{V}$ has full rank and it exists some $\mathbf{W}$ satisfies (5), the solution to (6) is unique.*

Proposition 1 details some conditions when $\mathbf{W}^*$ is unique. Note that $\mathbf{V}$ is $p \times n$. Therefore, if the covariates are of full rank and the true solution lies in the convex and compact $\mathcal{W}$, a sufficient condition for $\mathbf{W}^*$ to be unique is $p \geq n$. However, when $p < n$, $\mathbf{W}^*$ may not be unique. If it exists some $\mathbf{W}^*$ satisfies (5) and $p < n$, there are infinitely many solutions to (5). The issue of non-uniqueness is further discussed in Section 3.2.

**Remark 1.** In Section 2.1 we specify that $\mathbf{x}_{i,t}, \theta_i \in \mathbb{R}^p$. However, it is not necessary that the all $p$ covariates are important for every time series under study. The regression coefficients $\theta_i$ are nuisance parameters that are not of primary importance. It will be understood that structural 0s in $\theta_i$ correspond to variables that are unimportant.

**Remark 2.** Our forecasting premise and estimation construction shares similarities with Bayesian viewpoints. From a Bayesian perspective, if we assign a prior $\pi$ to $\alpha_1$, $\hat{\alpha}_{\text{adj}}$, $\hat{\alpha}_{\text{wadj}}$, and $\hat{\alpha}_{\text{IVW}}$ can be interpreted as the Bayes rules with respect to $\pi$ under different loss functions. If the sampling distribution of the data and $\pi$ are known, it is possible to compute the Bayes risks of $\hat{\alpha}_{\text{adj}}$, $\hat{\alpha}_{\text{wadj}}$, and $\hat{\alpha}_{\text{IVW}}$ with respect to $\pi$, thus enabling comparisons among them. Additionally, from Theorem 2.4 in Chapter 5 of Lehmann and Casella [2006], $\hat{\alpha}_{\text{adj}}$, $\hat{\alpha}_{\text{wadj}}$, and $\hat{\alpha}_{\text{IVW}}$ are admissible if they are unique with probability one.

## 3 Forecast risk and properties of shock effects estimators

In this section, we discuss the properties that are related to forecast-risk reduction. In discussion of risk, it is useful to derive expressions for expectation and variance of the adjustment estimator $\hat{\alpha}_{\text{adj}}$ and weighted-adjustment estimator. The expressions for the expectations are as follow,

(i) Under $\mathcal{M}_1$, $\mathrm{E}(\hat{\alpha}_{\text{adj}}) = \mathrm{E}(\hat{\alpha}_{\text{wadj}}) = \mu_\alpha$.

(ii) Under $\mathcal{M}_{21}$,

$$\mathrm{E}(\hat{\alpha}_{\text{adj}}) = \mu_\alpha + \frac{1}{n}\sum_{i=2}^{n+1} \delta' \mathbf{x}_{i,T_i^*+1} \quad \text{and} \quad \mathrm{E}(\hat{\alpha}_{\text{wadj}}) = \mu_\alpha + \delta' \mathbf{x}_{1,T_1^*+1}.$$

(iii) Under $\mathcal{M}_{22}$,

$$\mathrm{E}(\hat{\alpha}_{\text{adj}}) = \mu_\alpha + \frac{1}{n}\sum_{i=2}^{n+1} \mu_\delta' \mathbf{x}_{i,T_i^*+1} \quad \text{and} \quad \mathrm{E}(\hat{\alpha}_{\text{wadj}}) = \mu_\alpha + \mu_\delta' \mathbf{x}_{1,T_1^*+1}.$$

Formal justification for these results can be found in Appendix. Note that $\hat{\alpha}_{\text{adj}}$, $\hat{\alpha}_{\text{wadj}}$, and $\hat{\alpha}_{\text{IVW}}$ are not unbiased estimators for $\alpha_1$. Notice that under $\mathcal{M}_1$, $\hat{\alpha}_{\text{adj}}$ and $\hat{\alpha}_{\text{adj}}$ are unbiased estimators for $\mathrm{E}(\alpha_1) = \mu_\alpha$ (see distributional details of $\alpha_1$ in Section 2.1). Nevertheless, $\hat{\alpha}_{\text{adj}}$ is a biased estimator for $\mathrm{E}(\alpha_1)$ but $\hat{\alpha}_{\text{wadj}}$ is an unbiased estimator for $\mathrm{E}(\alpha_1)$ under both $\mathcal{M}_{21}$ and $\mathcal{M}_{22}$. Thus, we collect these results as the following proposition.

**Proposition 2.**

*(i) Under $\mathcal{M}_1$, $\hat{\alpha}_{\text{adj}}$ is an unbiased estimator of $\mathrm{E}(\alpha_1)$. Under $\mathcal{M}_{21}$ and $\mathcal{M}_{22}$, $\hat{\alpha}_{\text{adj}}$ is a biased estimator of $\mathrm{E}(\alpha_1)$ in general.*

*(ii) Suppose that $\mathbf{W}^*$ satisfies (5). Under $\mathcal{M}_1$, $\mathcal{M}_{21}$ and $\mathcal{M}_{22}$, $\hat{\alpha}_{\text{wadj}}$ is an unbiased estimator of $\mathrm{E}(\alpha_1)$.*

Unbiasedness properties for $\mathrm{E}(\alpha_1)$ of $\hat{\alpha}_{\text{adj}}$ and $\hat{\alpha}_{\text{wadj}}$ allow for simple conditions for risk-reduction to hold, and more importantly motivates a bootstrap estimation for evaluation of these conditions. These conditions and bootstrap will be discussed in Section 3.1 and 3.2, respectively. Next, we present the variance expressions for $\hat{\alpha}_{\text{adj}}$ and $\hat{\alpha}_{\text{wadj}}$ as below.

(i) Under $\mathcal{M}_1$ and $\mathcal{M}_{21}$,

$$\mathrm{Var}(\hat{\alpha}_{\text{adj}}) = \frac{\sigma^2}{n^2}\sum_{i=2}^{n+1} \mathrm{E}\big\{(\mathbf{U}_i'\mathbf{U}_i)^{-1}_{22}\big\} + \frac{\sigma_\alpha^2}{n^2}$$
$$\mathrm{Var}(\hat{\alpha}_{\text{wadj}}) = \sigma^2 \sum_{i=2}^{n+1} (w_i^*)^2 \mathrm{E}\big\{(\mathbf{U}_i'\mathbf{U}_i)^{-1}_{22}\big\} + \sigma_\alpha^2 \sum_{i=2}^{n+1}(w_i^*)^2$$

(ii) Under $\mathcal{M}_{22}$,

$$\mathrm{Var}(\hat{\alpha}_{\text{adj}}) = \frac{\sigma^2}{n^2}\sum_{i=2}^{n+1} \mathrm{E}\big\{(\mathbf{U}_i'\mathbf{U}_i)^{-1}_{22}\big\} + \frac{1}{n^2}(\mathbf{x}_{i,T_i^*+1}'\Sigma_\delta \mathbf{x}_{i,T_i^*+1} + \sigma_\alpha^2)$$
$$\mathrm{Var}(\hat{\alpha}_{\text{wadj}}) = \sigma^2 \sum_{i=2}^{n+1} (w_i^*)^2 \mathrm{E}\big\{(\mathbf{U}_i'\mathbf{U}_i)^{-1}_{22}\big\} + \sum_{i=2}^{n+1}(w_i^*)^2(\mathbf{x}_{i,T_i^*+1}'\Sigma_\delta \mathbf{x}_{i,T_i^*+1} + \sigma_\alpha^2).$$

Formal justification for these results can be found in Appendix. Note that the variances are not comparable in closed-form because of the term $\mathrm{E}\{(\mathbf{U}_i'\mathbf{U}_i)^{-1}_{22}\}$. This term exists because of the inclusion of the random lagged response in our auto regressive model formulation.

Section 3.1 details conditions needed for risk-reduction and comparisons of adjustment estimators. These conditions involve variances and expectations which may be difficult to compute in practice. To make use of those conditions in practice, estimation is required. Sections 3.2 introduce parametric bootstrap, which estimates the involved parameters in those conditions and thus motivates prospective decision-making about whether $\hat{\alpha}_i$ reduces the risk. Section 3.3 describes leave-one-out cross validation with $k$ random draws procedures, which prospectively estimate the correctness of such decision without observation of the post-shock response for the time series under study. Our simulations verify these procedures.

### 3.1 Risk-reduction conditions for shock effects estimators

In this section we will discuss the conditions for risk reduction for individual shock effects estimators under $\mathcal{M}_1$, $\mathcal{M}_{21}$, and $\mathcal{M}_{22}$. For an adjustment estimator $\hat{\alpha}$, we will write the risk-reduction as $\Delta(\hat{\alpha}) = R_{T_1^*+1,1} - R_{T_1^*+1,2}$ where $R_{T_1^*+1,2}$ is the risk of Forecast 2 calculated using the adjustment estimator $\hat{\alpha}$.

#### 3.1.1 Conditions under $\mathcal{M}_1$

Recall that Proposition 2 implies that the adjustment estimator $\hat{\alpha}_{\mathrm{adj}}$ and weighted-adjustment estimator $\hat{\alpha}_{\mathrm{wadj}}$ are unbiased for $\mathrm{E}(\alpha_1)$ under $\mathcal{M}_1$. With this result, we will have the following propositions that specify the conditions that are necessary for risk reduction.

**Proposition 3.** *Under $\mathcal{M}_1$,*

*(i)* $\Delta(\hat{\alpha}_{\mathrm{adj}}) > 0$ *when* $\mathrm{Var}(\hat{\alpha}_{\mathrm{adj}}) < \mu_\alpha^2$.

*(ii) if $\mathbf{W}^*$ satisfies (5), then* $\Delta(\hat{\alpha}_{\mathrm{wadj}}) > 0$ *when* $\mathrm{Var}(\hat{\alpha}_{\mathrm{wadj}}) < \mu_\alpha^2$.

Proposition 3 says that under $\mathcal{M}_1$ if the variance of the estimator is smaller than the squared mean of $\alpha_1$, those estimators will enjoy the risk reduction properties. In this setting, under $\mathcal{M}_1$, $\Delta(\hat{\alpha}_{\mathrm{adj}}) = \mu_\alpha^2 - \mathrm{Var}(\hat{\alpha}_{\mathrm{adj}})$ and $\Delta(\hat{\alpha}_{\mathrm{wadj}}) = \mu_\alpha^2 - \mathrm{Var}(\hat{\alpha}_{\mathrm{wadj}})$. From Proposition 3, we obtain a risk-reduction condition

$$\mathrm{Var}(\hat{\alpha}_{\mathrm{adj}}) = \frac{\sigma^2}{n^2}\sum_{i=2}^{n+1}\mathrm{E}\{(\mathbf{U}_i'\mathbf{U}_i)^{-1}_{22}\} + \frac{\sigma_\alpha^2}{n^2} < \mu_\alpha^2. \tag{7}$$

Condition (7) implies two facts: (1) adjustment (Forecast 2) is preferable to no adjustment (Forecast 1) asymptotically in $n$ whenever $\mu_\alpha \neq 0$ (see Forecast in Section 2.2); (2) In finite donor pool settings, adjustment is preferable to no adjustment when $\mu_\alpha$ is large relative to its variability and overall regression variability.

If $\mathbf{W}^*$ does not satisfy (5), its unbiased properties for $\mathrm{E}(\alpha_1)$ should hold approximately when the fit in (6) is appropriate as commented in Section 2.3. From Proposition 3 and the variance expression for $\hat{\alpha}_{\mathrm{wadj}}$, the risk-reduction condition for $\hat{\alpha}_{\mathrm{wadj}}$ is

$$\mathrm{Var}(\hat{\alpha}_{\mathrm{wadj}}) = \sigma^2\sum_{i=2}^{n+1}(w_i^*)^2\mathrm{E}\{(\mathbf{U}_i'\mathbf{U}_i)^{-1}_{22}\} + \sigma_\alpha^2\sum_{i=2}^{n+1}(w_i^*)^2 < \mu_\alpha^2. \tag{8}$$

In this case, adjustment is preferable to no adjustment when $\mu_\alpha$ is large relative to the weighted sum of variances for shock effects for other time series and overall regression variability. However, the above criteria are generally difficult to evaluate in practice. Sections 3.2 and 3.3 provide detailed treatments on how to estimate the sign of $\Delta(\hat{\alpha})$ in practice.

### 3.1.2 Conditions under $\mathcal{M}_{21}$ and $\mathcal{M}_{22}$

The shock effects $\alpha_i$s have different means under $\mathcal{M}_{21}$ and $\mathcal{M}_{22}$ unlike under $\mathcal{M}_1$. However, Proposition 2 implies that $\hat{\alpha}_{\text{wadj}}$ is an unbiased estimator of $\mathrm{E}(\alpha_1)$. We now state conditions for risk-reduction.

**Proposition 4.** *If $\mathbf{W}^*$ satisfies (5), then $\Delta(\hat{\alpha}_{\text{wadj}}) > 0$ when $\mathrm{Var}(\hat{\alpha}_{\text{wadj}}) < (\mathrm{E}(\alpha_1))^2$ under $\mathcal{M}_{21}$ and $\mathcal{M}_{22}$.*

Under Proposition 4, we can obtain a risk-reduction inequality that is similar to (8),

$$\mathrm{Var}(\hat{\alpha}_{\text{wadj}}) = \sigma^2 \sum_{i=2}^{n+1} (w_i^*)^2 \mathrm{E}\{(\mathbf{U}_i'\mathbf{U}_i)^{-1}_{22}\} + \sum_{i=2}^{n+1} (w_i^*)^2 (\mathbf{x}'_{i,T_i^*+1}\Sigma_\delta \mathbf{x}_{i,T_i^*+1} + \sigma_\alpha^2) < (\mathrm{E}(\alpha_1))^2,$$

where $\mathbf{x}'_{i,T_i^*+1}\Sigma_\delta \mathbf{x}_{i,T_i^*+1} + \sigma_\alpha^2$ may be replaced with $\sigma_\alpha^2$ in $\mathcal{M}_{21}$. The conclusions and intuitions will be identically the same as what we have in Section 3.1.1. Proposition 2 shows that $\hat{\alpha}_{\text{adj}}$ is a biased estimator of $\mathrm{E}(\alpha_1)$ under $\mathcal{M}_{21}$ and $\mathcal{M}_{22}$ generally. Hence, Proposition 3 no longer holds for $\hat{\alpha}_{\text{adj}}$ under $\mathcal{M}_{21}$ and $\mathcal{M}_{22}$.

As an alternative, we can derive similar risk-reduction conditions that are appropriate for this setting. By Lemma 1 (see Section 7) and risk decomposition, we will achieve risk-reduction as long as

$$\begin{aligned}
\mathrm{E}(\alpha_1^2) = \mathrm{Var}(\alpha_1) + (\mathrm{E}(\alpha_1))^2 &> \mathrm{E}(\hat{\alpha}_{\text{adj}} - \alpha_1)^2 \\
&= \mathrm{Var}(\hat{\alpha}_{\text{adj}}) + (\mathrm{E}(\hat{\alpha}_{\text{adj}}) - \alpha_1)^2 \\
&= \mathrm{Var}(\hat{\alpha}_{\text{adj}}) + \mathrm{Var}(\alpha_1) + (\mathrm{E}(\hat{\alpha}_{\text{adj}}) - \mathrm{E}(\alpha_1))^2.
\end{aligned}$$

The above inequality simplifies to

$$(\mathrm{E}(\alpha_1))^2 > \mathrm{Var}(\hat{\alpha}_{\text{adj}}) + (\mathrm{E}(\hat{\alpha}_{\text{adj}}) - \mathrm{E}(\alpha_1))^2. \tag{9}$$

As mentioned in Section 2.3, it is difficult to evaluate the expectation and variance of $\hat{\alpha}_{\text{IVW}}$. We note that $\hat{\alpha}_{\text{IVW}}$ is generally biased for $\mathrm{E}(\alpha_1)$. That is to say we can adapt the above proof to derive the risk-reduction conditions for $\hat{\alpha}_{\text{IVW}}$: under $\mathcal{M}_1$, $\mathcal{M}_{21}$, and $\mathcal{M}_{22}$, $\Delta(\hat{\alpha}_{\text{IVW}}) > 0$ when $\mathrm{Var}(\hat{\alpha}_{\text{IVW}}) + (\mathrm{E}(\hat{\alpha}_{\text{IVW}}) - \mathrm{E}(\alpha_1))^2 < (\mathrm{E}(\alpha_1))^2$. In fact, more generally, using similar proof of Lemma 1, it can be shown that under $\mathcal{M}_2$, the risk-reduction quantities are

$$\begin{aligned}
\Delta(\hat{\alpha}_{\text{adj}}) &= (\mathrm{E}(\alpha_1))^2 - \mathrm{Var}(\hat{\alpha}_{\text{adj}}) - (\mathrm{E}(\hat{\alpha}_{\text{adj}}) - \mathrm{E}(\alpha_1))^2, \\
\Delta(\hat{\alpha}_{\text{IVW}}) &= (\mathrm{E}(\alpha_1))^2 - \mathrm{Var}(\hat{\alpha}_{\text{IVW}}) - (\mathrm{E}(\hat{\alpha}_{\text{IVW}}) - \mathrm{E}(\alpha_1))^2, \\
\Delta(\hat{\alpha}_{\text{wadj}}) &= (\mathrm{E}(\alpha_1))^2 - \mathrm{Var}(\hat{\alpha}_{\text{adj}}),
\end{aligned}$$

where we estimate $\Delta(\hat{\alpha})$ for estimator $\hat{\alpha}$ using the bootstrap and leave-one-out cross validation with $k$ random draws techniques developed in Sections 3.2 and 3.3.

## 3.2 Bootstrap for risk-reduction evaluation problems

In this section, we present bootstrap procedures that approximate the distribution of our shock effect estimators, checks the underlying conditions of our risk reduction propositions, and estimate risk-reduction quantity using plug-in approach in practice. Our procedure involves the resampling of residuals in the separate OLS fits. This procedure has its origins in Section 6 of Efron and Tibshirani [1986] and Chapter 12 of Kilian and Lütkepohl [2017]. Our procedure involves the resampling of the residuals which are assumed to be the realizations of an iid process.

Our first bootstrap procedure is as follows: Let $B$ be the bootstrap sample size. At iteration $b$, first resample the indices $I = \{2, \ldots, n+1\}$ of the donor pool with replacement to form $I^{(b)}$ with cardinality $n$, where we note that the elements of $I^{(b)}$ may not be unique in terms of their indices in the donor pool.

Initialize $y_{i,0}^{(b)} = y_{i,0}$ for all $i \in I^{(b)}$. Then, resample the residuals under models $\mathcal{M}_1$ or $\mathcal{M}_2$, compute the bootstrapped response $y_{i,t}^{(b)}$ for $t \in \{1, \ldots, T_i\}$ using the model estimated by original data, and obtain shock effect estimators for each of the disparate time series for all $i \in I^{(b)}$. These shock effect estimators are then used to construct any of the adjustment estimators $\hat{\alpha}_{\text{adj}}^{(b)}$, $\hat{\alpha}_{\text{wadj}}^{(b)}$, and $\hat{\alpha}_{\text{IVW}}^{(b)}$, for $b = 1, \ldots, B$. We can then estimate distributional quantities of our shock effect estimators under our considered models with the bootstrap samples $\hat{\alpha}_{\text{adj}}^{(b)}$, $\hat{\alpha}_{\text{wadj}}^{(b)}$, and $\hat{\alpha}_{\text{IVW}}^{(b)}$, for $b = 1, \ldots, B$. We denote this procedure by $\mathcal{B}_u$. We motivate a second bootstrap procedure $\mathcal{B}_f$ which treats the the donor pool as fixed, and not a realization from an infinite super-population. Therefore, there is no resampling of the donor pool in $\mathcal{B}_f$, it is otherwise similar to $\mathcal{B}_u$. An algorithmic formulation of $\mathcal{B}_u$ and $\mathcal{B}_f$ are outlined in Section 2 in the Supplementary Materials.

We will explicitly use these bootstrapped samples of shock effect estimators to check the risk-reduction conditions in Propositions 3 and 4. Recall that $\hat{\alpha}_{\text{adj}}$, $\hat{\alpha}_{\text{wadj}}$ and $\hat{\alpha}_{\text{IVW}}$ are unbiased estimators of their expectations, and $\hat{\alpha}_{\text{wadj}}$ is an unbiased estimator of $\mathrm{E}(\alpha_1)$ under $\mathcal{M}_1$ and $\mathcal{M}_2$ from Proposition 2. Our bootstrap procedure estimates the variance of our adjustment estimators. We can then estimate the risk-reduction propositions and inequalities. For example, we can estimate $\Delta(\hat{\alpha}_{\text{adj}})$ under model $\mathcal{M}_{21}$ or $\mathcal{M}_{22}$ with

$$\hat{\Delta}(\hat{\alpha}_{\text{adj}}) = (\hat{\alpha}_{\text{wadj}})^2 - S^2_{\hat{\alpha}_{\text{adj}}} - (\hat{\alpha}_{\text{adj}} - \hat{\alpha}_{\text{wadj}})^2,$$

where $S^2_{\hat{\alpha}_{\text{adj}}}$ is the bootstrap sample variance estimator for $\mathrm{Var}(\hat{\alpha}_{\text{adj}})$.

We stress that our bootstrap approximations cannot alleviate the inherent bias of using our adjustment estimators as surrogates for $\alpha_1$. We caution that the bootstrapping residuals in OLS estimation may not provide valid inference in moderate or high dimension where $p < T_i$ but $p/T_i$ is not close to zero for $i \in \{2, \ldots, n+1\}$ [El Karoui and Purdom, 2018]; see alternatives for residual bootstrapping in linear models in El Karoui and Purdom [2018].

Recall that $\mathbf{W}^*$ may not be unique if the conditions in Proposition 1 are not satisfied. Non-uniqueness might be a concern theoretically. It is due to the fact that infinitely many different weights can lead to infinitely many non-unique $\hat{\alpha}_{\text{wadj}}$'s all targeting on the same $\alpha_1$. For example, consider the case where the size of donor pool to be 2, $\mathrm{Var}(\hat{\alpha}_2) = 1$, $\mathrm{Var}(\hat{\alpha}_3) = 2$, and there are two solutions to (5), say, $\mathbf{W}_1^* = (1, 0)$ and $\mathbf{W}_2^* = (0, 1)$. In this scenario, the weighted adjustment estimator induced by $\mathbf{W}_1^*$ has variance 1 whereas the one by $\mathbf{W}_2^*$ has variance 2. Nevertheless, even if $\hat{\alpha}_i$ has the same variance across $i = 2, \ldots, n+1$, the same issue would occur if there were infinitely many $\mathbf{W}^*$ with different norms. It is possible to resolve this issue by selecting a unique weight $\mathbf{W}^*$ that optimizes a desirable objective function, prior to which one should find the bases spanning the subspace of $\mathbf{W}^*$ satisfying (5). However, non-uniqueness of $\mathbf{W}^*$ would not be a problem for inferential purposes. This is because all risk-reduction propositions and other properties established will still hold. Simulation examples listed in the Section 4 of the Supplementary Materials verify this point.

### 3.3 Leave-one-out cross validation with $k$ random draws

In this section, we adapt leave-one-out cross validation (LOOCV) to our estimation context in order to provide prospective evaluations of our adjustment techniques. Our proposed LOOCV procedure has its roots in Section 7.10 of Hastie et al. [2009]. Recall in Section 2.1 that we are given the data $\{(\mathbf{x}_{i,t}, y_{i,t})\colon i = 1, \ldots, n+1, t = 1, \ldots, T_i\}$, where $\{(\mathbf{x}_{1,t}, y_{1,t})\colon t = 1, \ldots, T_1\}$ is the data of the time series of interest and the remaining observations form the donor pool. For iteration $m \in \{1, \ldots, n\}$ of our LOOCV procedure, we set aside $\{(\mathbf{x}_{m+1,t}, y_{m+1,t})\colon t = 1, \ldots, T_{m+1}\}$ as the time series of interest, and construct a new donor pool $\{(\mathbf{x}_{i,t}, y_{i,t})\colon i \in \mathcal{I}_m, t = 1, \ldots, T_i\}$, where $\mathcal{I}_m = \{2, \ldots, n+1\} \setminus \{m+1\}$. Since the post-shock response $y_{m+1,T^*_{m+1}+1}$ is observed, we can evaluate the performance of our adjustment estimators and the original forecast made without adjustment (i.e., Forecast 1 in Section 2.2).

LOOCV can be very computationally intensive when $n$ is large, especially when combined with bootstrapping. To alleviate these concerns we can perform LOOCV with a random subset of $k \leq n$ iterations selected without replacement. In this setting, we let $\mathcal{J}$ be the randomly sampled indices. For $m \in \mathcal{J}$, we set aside $\{(\mathbf{x}_{m+1,t}, y_{m+1,t})\colon t = 1, \ldots, T_{m+1}\}$ as the time series of interest, and construct a new donor pool $\{(\mathbf{x}_{i,t}, y_{i,t})\colon i \in \mathcal{I}, t = 1, \ldots, T_i\}$, where $\mathcal{I} = \{2, \ldots, n+1\} \setminus \{m+1\}$. Based on the new donor pool, we estimate relevant parameters using bootstrap procedures outlined in Section 3.2. In other words, $k$ times of bootstrapping are nested in a LOOCV procedure. We find that $k = 5$ or $k = 10$ iterations of LOOCV performs well.

We now outline how LOOCV can be used to prospectively assess the performance of adjustment estimators. Let $\mathcal{A}$ be the set of adjustment estimators. For each $\hat{\alpha} \in \mathcal{A}$, let $\delta_{\hat{\alpha}} = I(\hat{\Delta}(\hat{\alpha}) > 0)$ be a decision rule where $I(\cdot)$ is the indicator function and a 1 corresponds to the decision to use estimator $\hat{\alpha}$. If $\Delta(\hat{\alpha}) > 0$ ($\Delta(\hat{\alpha}) < 0$, respectively) but $\delta_{\hat{\alpha}}$ incorrectly reported 1 (0, respectively) so that it makes the decision not to use $\hat{\alpha}$ (to use $\hat{\alpha}$, respectively), $\delta_{\hat{\alpha}}$ is said to be incorrect. If $\Delta(\hat{\alpha}) < 0$ ($\Delta(\hat{\alpha}) > 0$, respectively) and $\delta_{\hat{\alpha}}$ correctly reported 0 (1, respectively) so that it makes the decision to use $\hat{\alpha}$, $\delta_{\hat{\alpha}}$ is said to be correct. These situations are depicted in the following table:

| | | Decision | |
|---|---|---|---|
| | | $\delta_{\hat{\alpha}} = 1$ | $\delta_{\hat{\alpha}} = 0$ |
| Truth | $\Delta(\hat{\alpha}) > 0$ | Correct | Incorrect |
| | $\Delta(\hat{\alpha}) < 0$ | Incorrect | Correct |

We will use $\mathcal{C}(\delta_{\hat{\alpha}}) = I(\delta_{\hat{\alpha}} \text{ is correct})$ as a metric that evaluates the performance of forecasts made with the adjustment estimator $\hat{\alpha}$. If $\mathrm{E}(\mathcal{C}(\delta_{\hat{\alpha}})) > 0.5$, we claim that $\delta_{\hat{\alpha}}$ is better than random guessing. Note that $\mathcal{C}(\delta_{\hat{\alpha}})$ can generally be computed only when the post-shock response is observed. However, it is possible to estimate $\mathrm{E}(\mathcal{C}(\delta_{\hat{\alpha}}))$ using LOOCV. The LOOCV estimates for $\mathrm{E}(\mathcal{C}(\delta_{\hat{\alpha}}))$ are

$$\bar{\mathcal{C}}(\delta_{\hat{\alpha}}) = \frac{1}{n} \sum_{m=1}^{n} \mathcal{C}^{(-m)}(\delta_{\hat{\alpha}}), \tag{10}$$

where $\mathcal{C}^{(-m)}(\delta_{\hat{\alpha}})$ is computed with respect to donor pool with index set $\mathcal{I}_m$ and the $m+1$ time series is treated as the time series of interest. The LOOCV with $k$ random draws estimates $\mathrm{E}(\mathcal{C}(\delta_{\hat{\alpha}}))$ as

$$\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}}) = \frac{1}{k} \sum_{m \in \mathcal{J}} \mathcal{C}^{(-m)}(\delta_{\hat{\alpha}}), \tag{11}$$

where $\mathcal{J}$ is the set of the $k$ randomly sampled indicies.

**Remark 3.** Note that we allow the time series within the donor pool to be dependent but donor pool should be independent of the time series of interest. However, if we assume the mutual independence structure, $\bar{\mathcal{C}}(\delta_{\hat{\alpha}})$ will be an almost unbiased estimator of $\mathrm{E}(\mathcal{C}(\delta_{\hat{\alpha}}))$ [Marden, 2015, Page 222]. In other words, mutual independence assumption is not related to $\mathrm{E}(\mathcal{C}(\delta_{\hat{\alpha}}))$ (i.e., the intrinsic correctness) but its estimation.

# 4 Numerical Examples

## 4.1 Modeling setup

In this section we provide justification for our methods based on Monte Carlo simulation. We implemented our simulation based on $\mathcal{M}_{22}$ with negligibly small $\Sigma_\delta$ approximating the design of $\mathcal{M}_{21}$. We consider $p = 25$ and $\mu_\alpha = 2$, where $p = 25$ is set to satisfy conditions in Proposition 1. Parameter setup of our simulations is detailed as follows: the $\phi_i$'s are sampled independently from Uniform$(0,1)$. We sampled $T_i$'s independently from Gamma$(15,10)$ that are further rounded to integers, where the minimum allowable value of $T_i$ is fixed to be 90. We will randomly draw $T_i^*$ from $\{p+4,\ldots,T_i-1\}$. The choices of $T_i$ and $T_i^*$ are set up to satisfy a necessary condition for the design matrix of OLS estimation to have full rank. Moreover, it is designed to illustrate the performance of $\hat{\alpha}_{\text{IVW}}$ that may perform well in time series with varying lengths. Additionally, we generated the covariates from Gamma$(1,2)$ to set up a setting when the $\hat{\alpha}_{\text{wadj}}$ may perform well. Last, we set $\delta_i \overset{iid}{\sim} \mathcal{N}(1,0.5)$ and $\theta_i \sim \mathcal{N}(0,1)$. We will consider parameter setup by varying $\sigma$ in the model of $y_{i,t}$, $n$, the donor pool size, and $\sigma_\alpha$ in the model of $\alpha_i$. We choose a Monte Carlo sample size of 30 replications and a bootstrap sample size of $B=200$ for computation. Means and standard errors for estimated quantities will be recorded. Our LOOCV procedure will consider $k=5$ random draws. Recall in Section 3.3 that $B$ times of bootstrap are nested in a LOOCV with $k$ random draws. It implies that $B(k+1)$ times of bootstrap replications are required for each Monte Carlo simulation.

### 4.2 Performance metrics

Our adjustment estimators will be evaluated by multiple criteria. We interpret $\delta_{\hat{\alpha}} = I(\hat{\Delta}(\hat{\alpha})>0)$ for $\hat{\alpha} \in \mathcal{A}$ as the guess, with 1 indicating that $\hat{\alpha}$ provides risk-reduction over the simple no-adjustment forecast, and 0 indicates the converse. We will consider the LOOCV estimators (10) and (11) to assess correct decision making. We will also consider the Euclidean distance between the post-shock forecasts $\hat{y}_{1,T_1^*+1}$, $\hat{y}_{1,T_1^*+1}+\hat{\alpha}_{\text{adj}}$, $\hat{y}_{1,T_1^*+1}+\hat{\alpha}_{\text{wadj}}$, and $\hat{y}_{1,T_1^*+1}+\hat{\alpha}_{\text{IVW}}$ and the realized post-shock response $y_{1,T_1^*+1}$. The first two metrics can combine to assess our forecasting methodology prospectively while the latter requires the realization of the post-shock response $y_{1,T_1^*+1}$.

### 4.3 Monte Carlo results

In this section, we discuss simulation results for the bootstrap procedures used in estimating parameters for risk-reduction propositions and inequalities. We mainly discuss simulations under $\mathcal{M}_2$ (see Section 2.1) for $\mathcal{B}_u$ and $\mathcal{B}_f$ (see Section 3.2) with comparisons to those under $\mathcal{M}_1$ whose results are listed in Section 3 in the Supplementary Materials. Two simulation setups are investigated.

In the first simulation setup, we consider the parameter combination of $n \in \{5,10,15,25\}$ and $\sigma_\alpha \in \{5,10,25,50,100\}$ where we fix $\sigma=10$. Note that $\text{E}(\text{E}(\alpha_1))=52$, where the last expectation is operated under the density of the covariates. In other words, data with $\sigma_\alpha \in \{5,10,25,50,100\}$ should well represent the situations when the signal of the covariates is strong and when it is nearly lost. Results are displayed in Table 1.

In the second simulation setup, we consider the parameter combination of $\sigma,\sigma_\alpha \in \{5,10,25,50,100\}$ where we fix $n=10$. Likewise, $\sigma,\sigma_\alpha \in \{5,10,25,50,100\}$ will produce situations when the signal of the covariates is strong and when it is nearly lost in the model of both $y_{i,t}$ and $\alpha_i$. Results are in Table 2.

First, assuming that $\bar{C}^{(k)}(\delta_{\hat{\alpha}})$ well estimates $\text{E}(\mathcal{C}(\delta_{\hat{\alpha}}))$ and fixing $n$, we observe from Table 1 that the decision making of $\delta_{\hat{\alpha}}$ is nearly correct for $\hat{\alpha} \in \mathcal{A}$ when $\sigma_\alpha$ is small from Table 1. The reasons can be explained as follows. When $\sigma_\alpha$ is small, the signal of the covariates is strong so that $\hat{\alpha}_{\text{wadj}}$ will be expected to capture the signal according to construction of $\hat{\alpha}_{\text{wadj}}$ in Section 2.3. Moreover, when $\sigma_\alpha$ is small, $\mathcal{M}_{22}$ approximates $\mathcal{M}_{21}$ such that estimation of $\text{E}(\alpha_1)$ should be nearly unbiased according to Proposition 2. However, when the signal of the covariates is poor ($\sigma_\alpha$ is large), the decision rule $\delta_{\hat{\alpha}}$ becomes unreliable for $\hat{\alpha} \in \mathcal{A}$. It is to be expected since the bootstrap estimates become more biased. However, users can be warned by $\bar{C}^{(k)}(\delta_{\hat{\alpha}})$ to have an idea of the effectiveness of $\delta_{\hat{\alpha}}$. Second, fixing $\sigma_\alpha$, we can observe that the correctness of $\delta_{\hat{\alpha}}$ increases when $n$ increases. It is due to the robustness gain in estimation when $n$ increases.

Additionally, we observe that in most cases $\delta_{\hat{\alpha}_{\text{wadj}}}$ reports $\hat{\alpha}_{\text{wadj}}$ reduces the risk even when $\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{wadj}}})$ starts to break down, though they follow similar patterns. Recall from Section 3.2 that $\hat{\Delta}(\hat{\alpha})$ contains

**Table 1:** 30 Monte Carlo simulations of $\mathcal{M}_2$ for $\mathcal{B}_u$ with varying $n$ and $\sigma_\alpha$

| | | Guess | | | LOOCV with $k$ random draws | | | Distance to $y_{1,T_1^*+1}$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | $\sigma_\alpha$ | $\delta_{\hat{\alpha}_{\text{adj}}}$ | $\delta_{\hat{\alpha}_{\text{wadj}}}$ | $\delta_{\hat{\alpha}_{\text{IVW}}}$ | $\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{adj}}})$ | $\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{wadj}}})$ | $\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{IVW}}})$ | Original | $\hat{\alpha}_{\text{adj}}$ | $\hat{\alpha}_{\text{wadj}}$ | $\hat{\alpha}_{\text{IVW}}$ |
| | 5 | 1 (0) | 1 (0) | 1 (0) | 0.91 (0.03) | 0.91 (0.02) | 0.9 (0.03) | 53.23 (4.1) | 15.88 (2.1) | 16.78 (2.37) | 15.82 (2.07) |
| | 10 | 0.97 (0.03) | 1 (0) | 0.97 (0.03) | 0.89 (0.03) | 0.9 (0.03) | 0.89 (0.03) | 53.01 (4.47) | 17.83 (2.38) | 19.56 (2.56) | 17.61 (2.36) |
| 5 | 25 | 0.93 (0.05) | 0.97 (0.03) | 0.93 (0.05) | 0.74 (0.04) | 0.81 (0.04) | 0.75 (0.04) | 53.38 (5.92) | 26.44 (3.8) | 29.06 (4) | 26.11 (3.75) |
| | 50 | 0.83 (0.07) | 0.83 (0.07) | 0.8 (0.07) | 0.59 (0.05) | 0.64 (0.05) | 0.59 (0.05) | 61.68 (7.73) | 46 (6.31) | 47.3 (7.14) | 45.25 (6.32) |
| | 100 | 0.7 (0.09) | 0.87 (0.06) | 0.7 (0.09) | 0.53 (0.05) | 0.54 (0.05) | 0.53 (0.06) | 85.68 (12.95) | 87.25 (11.86) | 87.07 (13.63) | 85.65 (12.02) |
| | 5 | 1 (0) | 1 (0) | 1 (0) | 0.91 (0.03) | 0.92 (0.02) | 0.91 (0.03) | 48.18 (4.59) | 20.47 (2.71) | 19.13 (2.97) | 20.53 (2.73) |
| | 10 | 1 (0) | 1 (0) | 1 (0) | 0.87 (0.03) | 0.89 (0.03) | 0.87 (0.03) | 48.93 (4.71) | 21.27 (2.6) | 19.24 (3.03) | 21.31 (2.61) |
| 10 | 25 | 0.93 (0.05) | 0.97 (0.03) | 0.9 (0.06) | 0.74 (0.03) | 0.77 (0.03) | 0.74 (0.03) | 51.18 (5.7) | 26.68 (2.78) | 27.17 (3) | 26.53 (2.77) |
| | 50 | 0.8 (0.07) | 0.8 (0.07) | 0.8 (0.07) | 0.57 (0.04) | 0.61 (0.04) | 0.57 (0.04) | 57.82 (7.81) | 40.51 (4.37) | 46.85 (4.02) | 40.19 (4.27) |
| | 100 | 0.73 (0.08) | 0.93 (0.05) | 0.7 (0.09) | 0.51 (0.04) | 0.51 (0.04) | 0.5 (0.04) | 79.3 (12.44) | 72.33 (9.12) | 88.81 (8.46) | 71.83 (8.85) |
| | 5 | 1 (0) | 1 (0) | 1 (0) | 0.94 (0.02) | 0.95 (0.02) | 0.94 (0.02) | 51.11 (3.05) | 14.94 (2.36) | 14.09 (2.37) | 15.09 (2.35) |
| | 10 | 1 (0) | 1 (0) | 1 (0) | 0.92 (0.02) | 0.91 (0.02) | 0.91 (0.02) | 52.34 (3.19) | 15.64 (2.73) | 15.29 (2.74) | 15.89 (2.68) |
| 15 | 25 | 0.93 (0.05) | 0.97 (0.03) | 0.93 (0.05) | 0.73 (0.04) | 0.76 (0.04) | 0.73 (0.04) | 56.03 (5.2) | 25.49 (3.85) | 27.38 (3.87) | 25.27 (3.84) |
| | 50 | 0.8 (0.07) | 0.83 (0.07) | 0.8 (0.07) | 0.56 (0.04) | 0.6 (0.04) | 0.57 (0.04) | 71.37 (7.76) | 47.25 (6.42) | 52.06 (6.57) | 46.41 (6.43) |
| | 100 | 0.63 (0.09) | 0.67 (0.09) | 0.63 (0.09) | 0.52 (0.04) | 0.42 (0.04) | 0.53 (0.04) | 111.91 (13.83) | 92.95 (12.34) | 103.13 (12.74) | 91.07 (12.37) |
| | 5 | 1 (0) | 1 (0) | 1 (0) | 0.93 (0.02) | 0.94 (0.02) | 0.93 (0.02) | 47.79 (2.93) | 14.83 (1.72) | 14.83 (2.04) | 14.76 (1.72) |
| | 10 | 1 (0) | 1 (0) | 1 (0) | 0.89 (0.03) | 0.91 (0.02) | 0.89 (0.03) | 47.93 (3.25) | 16.55 (1.89) | 17.55 (2.12) | 16.53 (1.88) |
| 25 | 25 | 1 (0) | 1 (0) | 1 (0) | 0.83 (0.03) | 0.82 (0.03) | 0.83 (0.03) | 49.78 (5.01) | 26.42 (3.38) | 29.11 (3.4) | 26.45 (3.35) |
| | 50 | 0.97 (0.03) | 1 (0) | 0.93 (0.05) | 0.64 (0.05) | 0.63 (0.05) | 0.64 (0.05) | 62.64 (7.4) | 48.84 (6.4) | 52.67 (6.28) | 48.8 (6.35) |
| | 100 | 0.83 (0.07) | 0.8 (0.07) | 0.83 (0.07) | 0.57 (0.05) | 0.59 (0.05) | 0.59 (0.05) | 103.37 (12.23) | 97.81 (12.52) | 102.4 (12.49) | 97.63 (12.45) |

the squared bias for estimating $\mathrm{E}(\alpha_1)$. But it is not present for $\hat{\Delta}(\hat{\alpha}_{\text{wadj}})$ since we applied the fact $\hat{\alpha}_{\text{wadj}}$ is unbiased for $\mathrm{E}(\alpha_1)$ from Proposition 2 in plugging it in with replacing $\mathrm{E}(\alpha_1)$. Therefore, when the signal from covariates is poorer, $\delta_{\hat{\alpha}_{\text{wadj}}}$ becomes less conservative. Besides, the averaged $I\big(\hat{\Delta}(\hat{\alpha}) > 0\big)$ times $\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}})$ can provide an approximation for the probability that $\hat{\alpha}$ actually reduces the risk assuming an symmetry of correctness between the cases when $\hat{\Delta}(\hat{\alpha}) > 0$ and when $\hat{\Delta}(\hat{\alpha}) < 0$. For example, when $n = 5$ and $\sigma_\alpha = 50$, the probability that $\hat{\alpha}_{\text{adj}}$ reduces the risk is approximately $0.83 \times 0.59 = 0.490$ from Table 1. In other words, the probability that $\hat{\alpha}$ reduces the risk has the same pattern as $\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}})$ has with $n$ and $\sigma_\alpha$ for $\hat{\alpha} \in \mathcal{A}$.

From columns related to distance to $y_{1,T_1^*+1}$ in Table 1, as $\sigma_\alpha$ increases, the prediction appears to be poorer. When $\sigma_\alpha = 5, 10, 25$, forecasts using $\hat{\alpha}_{\text{adj}}$, $\hat{\alpha}_{\text{wadj}}$, and $\hat{\alpha}_{\text{IVW}}$ are always better than the original forecast significantly. But it does not hold generally for the case when $\sigma_\alpha = 50, 100$. It is reasonable in that when the $\sigma_\alpha$ is large, it is difficult to find a reliable estimate of $\alpha_1$. Nevertheless, no statistical evidence has been found to support the claim that $n$ matters in prediction. In other words, the size of the donor pool matters for producing reliable decision-making of $\delta_{\hat{\alpha}}$ rather than reliable prediction.

**Table 2:** 30 Monte Carlo simulations of $\mathcal{M}_2$ for $\mathcal{B}_u$ with varying $\sigma$ and $\sigma_\alpha$

| | | Guess | | | LOOCV with $k$ random draws | | | Distance to $y_{1,T_1^*+1}$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\sigma$ | $\sigma_\alpha$ | $\delta_{\hat{\alpha}_{\text{adj}}}$ | $\delta_{\hat{\alpha}_{\text{wadj}}}$ | $\delta_{\hat{\alpha}_{\text{IVW}}}$ | $\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{adj}}})$ | $\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{wadj}}})$ | $\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{IVW}}})$ | Original | $\hat{\alpha}_{\text{adj}}$ | $\hat{\alpha}_{\text{wadj}}$ | $\hat{\alpha}_{\text{IVW}}$ |
| | 5 | 1 (0) | 1 (0) | 1 (0) | 0.94 (0.02) | 0.97 (0.01) | 0.95 (0.02) | 48.84 (3.4) | 15.72 (1.93) | 15.32 (1.94) | 15.72 (1.93) |
| | 10 | 1 (0) | 1 (0) | 1 (0) | 0.92 (0.03) | 0.94 (0.02) | 0.92 (0.03) | 49.54 (3.73) | 17.07 (2.01) | 16.08 (2.16) | 17.12 (1.99) |
| 5 | 25 | 0.87 (0.06) | 1 (0) | 0.87 (0.06) | 0.77 (0.02) | 0.81 (0.03) | 0.77 (0.03) | 51.78 (5.22) | 24.78 (2.57) | 26.12 (2.49) | 24.62 (2.54) |
| | 50 | 0.8 (0.07) | 0.83 (0.07) | 0.8 (0.07) | 0.59 (0.05) | 0.61 (0.04) | 0.59 (0.05) | 58.62 (7.74) | 40.09 (4.7) | 47.09 (4.09) | 39.85 (4.58) |
| | 100 | 0.7 (0.09) | 0.93 (0.05) | 0.73 (0.08) | 0.5 (0.04) | 0.49 (0.04) | 0.51 (0.04) | 82.66 (12.19) | 72.83 (9.75) | 89.03 (9.15) | 72.31 (9.5) |
| | 5 | 1 (0) | 1 (0) | 1 (0) | 0.91 (0.03) | 0.92 (0.02) | 0.91 (0.03) | 48.18 (4.59) | 20.47 (2.71) | 19.13 (2.97) | 20.53 (2.73) |
| | 10 | 1 (0) | 1 (0) | 1 (0) | 0.87 (0.03) | 0.89 (0.03) | 0.87 (0.03) | 48.93 (4.71) | 21.27 (2.6) | 19.24 (3.03) | 21.31 (2.61) |
| 10 | 25 | 0.93 (0.05) | 0.97 (0.03) | 0.9 (0.06) | 0.74 (0.03) | 0.77 (0.03) | 0.74 (0.03) | 51.18 (5.7) | 26.68 (2.78) | 27.17 (3) | 26.53 (2.77) |
| | 50 | 0.8 (0.07) | 0.8 (0.07) | 0.8 (0.07) | 0.57 (0.04) | 0.61 (0.04) | 0.57 (0.04) | 57.82 (7.81) | 40.51 (4.37) | 46.85 (4.02) | 40.19 (4.27) |
| | 100 | 0.73 (0.08) | 0.93 (0.05) | 0.7 (0.09) | 0.51 (0.04) | 0.51 (0.04) | 0.5 (0.04) | 79.3 (12.44) | 72.33 (9.12) | 88.81 (8.46) | 71.83 (8.85) |
| | 5 | 0.97 (0.03) | 1 (0) | 0.97 (0.03) | 0.7 (0.04) | 0.76 (0.03) | 0.7 (0.04) | 50.09 (8.27) | 38.1 (5.75) | 37.98 (5.5) | 38.44 (5.76) |
| | 10 | 0.97 (0.03) | 1 (0) | 0.97 (0.03) | 0.69 (0.04) | 0.74 (0.03) | 0.69 (0.04) | 50.82 (8.15) | 37.63 (5.61) | 36.33 (5.64) | 37.99 (5.61) |
| 25 | 25 | 0.9 (0.06) | 0.9 (0.06) | 0.9 (0.06) | 0.62 (0.04) | 0.64 (0.04) | 0.61 (0.04) | 53.01 (8.22) | 38.78 (5.29) | 36.82 (5.76) | 38.88 (5.31) |
| | 50 | 0.8 (0.07) | 0.8 (0.07) | 0.8 (0.07) | 0.53 (0.04) | 0.53 (0.04) | 0.53 (0.04) | 58.9 (9.12) | 46.77 (5.54) | 50.61 (5.71) | 46.6 (5.51) |
| | 100 | 0.7 (0.09) | 0.9 (0.06) | 0.67 (0.09) | 0.51 (0.03) | 0.56 (0.04) | 0.5 (0.03) | 79.64 (12.21) | 72.76 (8.79) | 89.48 (7.98) | 72.17 (8.59) |
| | 5 | 0.77 (0.08) | 0.8 (0.07) | 0.77 (0.08) | 0.6 (0.05) | 0.63 (0.04) | 0.59 (0.04) | 71.22 (13) | 70.31 (10.4) | 72.3 (9.26) | 70.79 (10.45) |
| | 10 | 0.77 (0.08) | 0.77 (0.08) | 0.77 (0.08) | 0.6 (0.05) | 0.63 (0.05) | 0.6 (0.05) | 70.85 (12.91) | 69.43 (10.22) | 70.65 (9.29) | 69.94 (10.26) |
| 50 | 25 | 0.7 (0.09) | 0.73 (0.08) | 0.7 (0.09) | 0.54 (0.05) | 0.56 (0.05) | 0.55 (0.05) | 70.32 (12.81) | 67.61 (9.86) | 67 (9.58) | 68.06 (9.89) |
| | 50 | 0.67 (0.09) | 0.7 (0.09) | 0.67 (0.09) | 0.51 (0.05) | 0.51 (0.04) | 0.51 (0.05) | 74.01 (12.66) | 68.69 (9.63) | 67.9 (10.21) | 68.91 (9.64) |
| | 100 | 0.5 (0.09) | 0.6 (0.09) | 0.47 (0.09) | 0.47 (0.05) | 0.49 (0.04) | 0.45 (0.05) | 92.71 (13.06) | 83.66 (10.79) | 94.53 (11.2) | 83.56 (10.63) |
| | 5 | 0.47 (0.09) | 0.47 (0.09) | 0.47 (0.09) | 0.51 (0.06) | 0.57 (0.05) | 0.49 (0.06) | 130.47 (22.59) | 135.16 (19.73) | 141.42 (16.98) | 136.3 (19.72) |
| | 10 | 0.47 (0.09) | 0.47 (0.09) | 0.47 (0.09) | 0.51 (0.05) | 0.53 (0.05) | 0.51 (0.06) | 129.49 (22.49) | 134.09 (19.52) | 139.69 (16.96) | 135.26 (19.51) |
| 100 | 25 | 0.47 (0.09) | 0.43 (0.09) | 0.5 (0.09) | 0.53 (0.06) | 0.57 (0.05) | 0.51 (0.06) | 127.17 (22.22) | 131.43 (18.97) | 134.47 (17.22) | 132.42 (18.99) |
| | 50 | 0.5 (0.09) | 0.43 (0.09) | 0.5 (0.09) | 0.48 (0.06) | 0.56 (0.04) | 0.48 (0.05) | 125.72 (21.8) | 129.27 (18.16) | 129.59 (17.59) | 130.27 (18.15) |
| | 100 | 0.47 (0.09) | 0.47 (0.09) | 0.43 (0.09) | 0.43 (0.06) | 0.57 (0.04) | 0.47 (0.06) | 128.38 (21.86) | 130.05 (18.05) | 131.83 (19.08) | 130.33 (18.06) |

From Table 2, we observe that as $\sigma_\alpha$ increases fixing $\sigma$, $\bar{\mathcal{C}}(\delta_{\hat{\alpha}})$ decreases, which is a pattern similar to the one shown in the first experiment. Furthermore, as $\sigma$ increases fixing $\sigma_\alpha$, $\bar{\mathcal{C}}(\delta_{\hat{\alpha}})$ decreases as well. Note that the correctness hinges on the estimation of the parameters. Since $\hat{\alpha}_{\text{wadj}}$ is a linear combination of OLS estimates, as $\sigma$ increases, $\text{Var}(\hat{\alpha}_{\text{wadj}})$ increases as well. Therefore, $\hat{\alpha}_{\text{wadj}}$ become more volatile and its estimation of $\text{E}(\alpha_1)$ can be less reliable. Those reasons can explain why an increase of $\sigma_\alpha$ contributes to a decrease of $\bar{\mathcal{C}}(\delta_{\hat{\alpha}})$. We observe similar patterns for distance to $y_{1,T_1^*+1}$ as well. When $\sigma$ increases with fixing $\sigma_\alpha$, it is likely that the degree of variation of $y_{1,t}$ exceeds the extent of adjustment improvement $\hat{\alpha}$ can contribute to for $\hat{\alpha} \in \mathcal{A}$.

With respect to averaged $I(\hat{\Delta}(\hat{\alpha}) > 0)$ (i.e., the guess), it starts to decrease as $\sigma$ increases. This is reasonable if we believe the bootstrap estimate $S^2_{\hat{\alpha}}$ provides a good approximation for $\text{Var}(\hat{\alpha})$ for $\hat{\alpha} \in \mathcal{A}$. The reasons are outlined as follows: Recall in Section 3.1.2, the conditions of risk-reduction propositions involve $(\text{E}(\alpha_1))^2 > \text{Var}(\hat{\alpha}) + (\text{E}(\hat{\alpha}) - \text{E}(\alpha_1))^2$ for $\hat{\alpha} \in \mathcal{A}$. Notice that $\text{Var}(\hat{\alpha})$ is an increasing function of $\sigma$ since $\hat{\alpha}$ is estimated by OLS. Therefore, it explains the reason why the increase of $\sigma$ would result in a decrease of averaged $I(\hat{\Delta}(\hat{\alpha}) > 0)$ since the inequality is not likely to hold when $\text{Var}(\hat{\alpha})$ increases.

Simulation for $\mathcal{B}_f$ with the same parameter setup as that of $\mathcal{B}_u$ are implemented. See Table 3 and Table 4 for results. Comparing Table 1 and Table 2 yields that when $n$ is moderately small ($n = 10$) and $\sigma_\alpha$ is small ($\sigma_\alpha = 5$), $\mathcal{B}_u$ is better than $\mathcal{B}_f$ with statistical evidence. For other situations, $\mathcal{B}_u$ and $\mathcal{B}_f$ are rather similar. It is likely that the extra randomness from sampling with replacement from donor pool compensates for the possible noises from a small donor pool. Concerning Table 2 and Table 4, it appears that when $n = 10$ and $\sigma_\alpha = 5$, $\mathcal{B}_u$ is better than $\mathcal{B}_f$ when $\sigma$ increases. It might be the case that additional layer of bootstrap in the donor pool buffers the negative effects on $\bar{\mathcal{C}}(\delta_{\hat{\alpha}})$ introduced from increasing variation of $y_{i,t}$. However, when $\sigma_\alpha$ increases over 5 and $n = 10$, $\mathcal{B}_f$ and $\mathcal{B}_u$ are quite similar under situations of different $\sigma$ and $\sigma_\alpha$. In conclusion, $\mathcal{B}_u$ is better than $\mathcal{B}_f$ when the signal of the covariates is strong and $n$ is moderately small; otherwise, they are similar.

Simulation results corresponding to model $\mathcal{M}_1$ are listed in Section 3 in Supplementary Materials. Results under model $\mathcal{M}_1$ are very similar to those of $\mathcal{M}_2$, except for the difference among estimators. The results show that (1) the performance of $\hat{\alpha}_{\text{adj}}$ and $\hat{\alpha}_{\text{IVW}}$ are nearly the same and (2) in many situations, $\hat{\alpha}_{\text{adj}}$ and $\hat{\alpha}_{\text{IVW}}$ are better than $\hat{\alpha}_{\text{wadj}}$; in other situations, they are mostly the same. Recall that in $\mathcal{M}_1$, the models for $\alpha_1$ do not involve the covariates. Therefore, similarity weighting may not be informative when the model for $\alpha_i$ is identified wrongly. Under $\mathcal{M}_1$, simple averaging, aimed for a reduction of variance, or inverse-variance weighting, targeting on reducing negative effects from varying time lengths, may work better.

**Table 3:** 30 Monte Carlo simulations of $\mathcal{M}_2$ for $\mathcal{B}_f$ with varying $n$ and $\sigma_\alpha$

| | | Guess | | | LOOCV with $k$ random draws | | | Distance to $y_{1,T_1^*+1}$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | $\sigma_\alpha$ | $\delta_{\hat{\alpha}_{\text{adj}}}$ | $\delta_{\hat{\alpha}_{\text{wadj}}}$ | $\delta_{\hat{\alpha}_{\text{IVW}}}$ | $\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{adj}}})$ | $\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{wadj}}})$ | $\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{IVW}}})$ | Original | $\hat{\alpha}_{\text{adj}}$ | $\hat{\alpha}_{\text{wadj}}$ | $\hat{\alpha}_{\text{IVW}}$ |
| | 5 | 1 (0) | 1 (0) | 1 (0) | 0.89 (0.03) | 0.92 (0.02) | 0.89 (0.03) | 48.52 (3.93) | 15.74 (2.34) | 15.76 (2.34) | 15.16 (2.24) |
| | 10 | 1 (0) | 1 (0) | 1 (0) | 0.89 (0.02) | 0.91 (0.02) | 0.89 (0.02) | 47.7 (4.35) | 18.26 (2.37) | 18.97 (2.42) | 17.68 (2.28) |
| 5 | 25 | 0.97 (0.03) | 1 (0) | 0.93 (0.05) | 0.79 (0.03) | 0.81 (0.03) | 0.77 (0.03) | 46.95 (6.11) | 27.35 (3.81) | 30.88 (3.84) | 26.58 (3.83) |
| | 50 | 0.8 (0.07) | 0.93 (0.05) | 0.8 (0.07) | 0.62 (0.03) | 0.65 (0.03) | 0.63 (0.03) | 56.85 (8.64) | 46.96 (7.02) | 52.92 (7.45) | 45.95 (7.17) |
| | 100 | 0.73 (0.08) | 1 (0) | 0.8 (0.07) | 0.53 (0.04) | 0.53 (0.04) | 0.55 (0.04) | 99.22 (12.84) | 93.4 (12.97) | 103.82 (14.1) | 91.95 (13.34) |
| | 5 | 1 (0) | 1 (0) | 1 (0) | 0.86 (0.03) | 0.88 (0.02) | 0.86 (0.03) | 50.59 (5.24) | 29.19 (5.2) | 31.4 (5.28) | 29.29 (5.22) |
| | 10 | 1 (0) | 1 (0) | 1 (0) | 0.82 (0.03) | 0.84 (0.03) | 0.82 (0.03) | 51.17 (5.49) | 31.55 (5.33) | 33.91 (5.56) | 31.7 (5.35) |
| 10 | 25 | 0.93 (0.05) | 1 (0) | 0.93 (0.05) | 0.72 (0.04) | 0.75 (0.04) | 0.71 (0.04) | 53.53 (6.58) | 40.05 (6.03) | 43.5 (6.66) | 40.43 (6) |
| | 50 | 0.87 (0.06) | 0.97 (0.03) | 0.87 (0.06) | 0.55 (0.04) | 0.58 (0.05) | 0.55 (0.05) | 62.45 (8.25) | 55.56 (8.11) | 62.15 (9.19) | 56.12 (8.04) |
| | 100 | 0.77 (0.08) | 0.97 (0.03) | 0.73 (0.08) | 0.49 (0.05) | 0.44 (0.05) | 0.46 (0.05) | 85.72 (12.73) | 89.5 (13.37) | 103.25 (15.07) | 89.92 (13.29) |
| | 5 | 1 (0) | 1 (0) | 1 (0) | 0.95 (0.02) | 0.92 (0.03) | 0.95 (0.02) | 52.1 (2.96) | 14.04 (1.78) | 13.36 (2.07) | 14.11 (1.76) |
| | 10 | 1 (0) | 1 (0) | 1 (0) | 0.92 (0.02) | 0.9 (0.03) | 0.93 (0.02) | 52.25 (3.3) | 15.12 (1.93) | 14.24 (2.29) | 15.18 (1.9) |
| 15 | 25 | 0.93 (0.05) | 1 (0) | 0.9 (0.06) | 0.8 (0.03) | 0.8 (0.03) | 0.8 (0.03) | 52.71 (5.28) | 22.98 (2.9) | 22.6 (3.42) | 22.95 (2.88) |
| | 50 | 0.7 (0.09) | 0.9 (0.06) | 0.7 (0.09) | 0.65 (0.03) | 0.65 (0.03) | 0.65 (0.04) | 58.65 (8.48) | 39.51 (5.65) | 40.8 (6.3) | 39.35 (5.63) |
| | 100 | 0.6 (0.09) | 0.87 (0.06) | 0.6 (0.09) | 0.47 (0.05) | 0.45 (0.04) | 0.45 (0.05) | 88.76 (13.66) | 75.93 (11.52) | 81.89 (12.19) | 75.94 (11.41) |
| | 5 | 1 (0) | 1 (0) | 1 (0) | 0.94 (0.02) | 0.95 (0.02) | 0.94 (0.02) | 50.55 (2.9) | 12.13 (1.77) | 14.22 (1.96) | 12.09 (1.77) |
| | 10 | 1 (0) | 1 (0) | 1 (0) | 0.93 (0.02) | 0.95 (0.02) | 0.93 (0.02) | 49.13 (3.31) | 14.78 (1.85) | 18.21 (2) | 14.75 (1.85) |
| 25 | 25 | 1 (0) | 1 (0) | 1 (0) | 0.83 (0.02) | 0.85 (0.02) | 0.83 (0.03) | 47.38 (5.13) | 26.85 (3.33) | 32.95 (3.59) | 26.81 (3.32) |
| | 50 | 0.97 (0.03) | 1 (0) | 0.97 (0.03) | 0.61 (0.04) | 0.71 (0.04) | 0.62 (0.04) | 56.73 (7.93) | 50.96 (6.63) | 60.62 (7.21) | 50.88 (6.59) |
| | 100 | 0.8 (0.07) | 0.93 (0.05) | 0.8 (0.07) | 0.49 (0.05) | 0.51 (0.04) | 0.49 (0.05) | 93.79 (14.67) | 102.05 (13.37) | 116.45 (15.19) | 101.55 (13.38) |

**Table 4:** 30 Monte Carlo simulations of $\mathcal{M}_2$ for $\mathcal{B}_f$ with varying $\sigma$ and $\sigma_\alpha$

| | | Guess | | | LOOCV with $k$ random draws | | | Distance to $y_{1,T_1^*+1}$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\sigma$ | $\sigma_\alpha$ | $\delta_{\hat{\alpha}_{\text{adj}}}$ | $\delta_{\hat{\alpha}_{\text{wadj}}}$ | $\delta_{\hat{\alpha}_{\text{IVW}}}$ | $\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{adj}}})$ | $\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{wadj}}})$ | $\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{IVW}}})$ | Original | $\hat{\alpha}_{\text{adj}}$ | $\hat{\alpha}_{\text{wadj}}$ | $\hat{\alpha}_{\text{IVW}}$ |
| | 5 | 1 (0) | 1 (0) | 1 (0) | 0.94 (0.02) | 0.95 (0.02) | 0.94 (0.02) | 50.04 (3.65) | 21.75 (4.34) | 22.68 (4.41) | 21.83 (4.34) |
| | 10 | 1 (0) | 1 (0) | 1 (0) | 0.93 (0.02) | 0.92 (0.02) | 0.93 (0.02) | 49.93 (4.11) | 24.3 (4.44) | 25.72 (4.58) | 24.4 (4.45) |
| 5 | 25 | 0.97 (0.03) | 1 (0) | 0.97 (0.03) | 0.73 (0.04) | 0.8 (0.03) | 0.73 (0.04) | 51.29 (5.39) | 32.8 (5.25) | 35.39 (5.78) | 33.11 (5.21) |
| | 100 | 0.77 (0.08) | 0.97 (0.03) | 0.7 (0.09) | 0.49 (0.04) | 0.44 (0.05) | 0.47 (0.04) | 82.55 (11.61) | 84.26 (12.7) | 98.66 (13.96) | 84.62 (12.59) |
| | 5 | 1 (0) | 1 (0) | 1 (0) | 0.86 (0.03) | 0.88 (0.02) | 0.86 (0.03) | 50.59 (5.24) | 29.19 (5.2) | 31.4 (5.28) | 29.29 (5.22) |
| | 10 | 1 (0) | 1 (0) | 1 (0) | 0.82 (0.03) | 0.84 (0.03) | 0.82 (0.03) | 51.17 (5.49) | 31.55 (5.33) | 33.91 (5.56) | 31.7 (5.35) |
| 10 | 25 | 0.93 (0.05) | 1 (0) | 0.93 (0.05) | 0.72 (0.04) | 0.75 (0.04) | 0.71 (0.04) | 53.53 (6.58) | 40.05 (6.03) | 43.5 (6.66) | 40.43 (6) |
| | 50 | 0.87 (0.06) | 0.97 (0.03) | 0.87 (0.06) | 0.55 (0.04) | 0.58 (0.05) | 0.55 (0.05) | 62.45 (8.25) | 55.56 (8.11) | 62.15 (9.19) | 56.12 (8.04) |
| | 100 | 0.77 (0.08) | 0.97 (0.03) | 0.73 (0.08) | 0.49 (0.05) | 0.44 (0.05) | 0.46 (0.05) | 85.72 (12.73) | 89.5 (13.37) | 103.25 (15.07) | 89.92 (13.29) |
| | 5 | 0.97 (0.03) | 1 (0) | 0.97 (0.03) | 0.7 (0.03) | 0.73 (0.03) | 0.71 (0.04) | 57.87 (8.76) | 50.31 (7.58) | 57.25 (7.75) | 50.53 (7.64) |
| | 10 | 0.97 (0.03) | 1 (0) | 0.97 (0.03) | 0.68 (0.04) | 0.69 (0.04) | 0.69 (0.04) | 58.41 (9.11) | 51.62 (7.91) | 58.61 (8.28) | 51.85 (7.98) |
| 25 | 25 | 0.93 (0.05) | 0.97 (0.03) | 0.9 (0.06) | 0.63 (0.04) | 0.68 (0.04) | 0.63 (0.04) | 62.02 (10.02) | 59.08 (8.53) | 65.55 (9.64) | 59.68 (8.53) |
| | 50 | 0.87 (0.06) | 0.9 (0.06) | 0.87 (0.06) | 0.54 (0.04) | 0.59 (0.04) | 0.52 (0.04) | 71.01 (11.73) | 73.52 (10.18) | 81.81 (11.98) | 74.51 (10.09) |
| | 100 | 0.73 (0.08) | 0.87 (0.06) | 0.73 (0.08) | 0.47 (0.05) | 0.45 (0.04) | 0.5 (0.05) | 95.93 (15.59) | 104.49 (15.12) | 119.38 (17.62) | 105.72 (14.97) |
| | 5 | 0.8 (0.07) | 0.77 (0.08) | 0.8 (0.07) | 0.52 (0.04) | 0.49 (0.05) | 0.52 (0.04) | 85.95 (14.57) | 90.02 (13.45) | 103.03 (13.91) | 90.08 (13.64) |
| | 10 | 0.8 (0.07) | 0.73 (0.08) | 0.8 (0.07) | 0.55 (0.05) | 0.5 (0.04) | 0.53 (0.05) | 86.44 (14.95) | 90.89 (13.79) | 104.62 (14.28) | 91.03 (13.98) |
| 50 | 25 | 0.77 (0.08) | 0.77 (0.08) | 0.77 (0.08) | 0.53 (0.04) | 0.46 (0.04) | 0.53 (0.04) | 90.26 (15.82) | 95.36 (14.69) | 109.53 (15.69) | 95.79 (14.83) |
| | 50 | 0.77 (0.08) | 0.8 (0.07) | 0.77 (0.08) | 0.48 (0.05) | 0.45 (0.05) | 0.45 (0.05) | 99.52 (17.26) | 106.52 (16.17) | 120.99 (18.13) | 107.55 (16.19) |
| | 100 | 0.57 (0.09) | 0.77 (0.08) | 0.63 (0.09) | 0.41 (0.04) | 0.45 (0.03) | 0.41 (0.04) | 123.11 (20.61) | 135.24 (19.74) | 151.62 (23.42) | 137.21 (19.55) |
| | 5 | 0.63 (0.09) | 0.57 (0.09) | 0.63 (0.09) | 0.48 (0.05) | 0.48 (0.03) | 0.47 (0.05) | 156.82 (26.36) | 170.06 (25.92) | 196.4 (26.55) | 170.13 (26.27) |
| | 10 | 0.63 (0.09) | 0.57 (0.09) | 0.67 (0.09) | 0.46 (0.05) | 0.47 (0.03) | 0.47 (0.05) | 157.3 (26.76) | 170.93 (26.21) | 197.96 (26.87) | 171.07 (26.56) |
| 100 | 25 | 0.67 (0.09) | 0.63 (0.09) | 0.67 (0.09) | 0.44 (0.04) | 0.5 (0.03) | 0.45 (0.04) | 160.32 (27.73) | 173.66 (27.22) | 202.61 (28.01) | 174.28 (27.48) |
| | 50 | 0.67 (0.09) | 0.67 (0.09) | 0.67 (0.09) | 0.39 (0.04) | 0.43 (0.03) | 0.38 (0.04) | 166.98 (29.35) | 182.83 (28.37) | 210.38 (30.48) | 183.67 (28.61) |
| | 100 | 0.6 (0.09) | 0.67 (0.09) | 0.53 (0.09) | 0.4 (0.04) | 0.45 (0.04) | 0.41 (0.05) | 188.29 (32.07) | 203.7 (31.63) | 233.44 (35.31) | 205.4 (31.72) |

# 5 Forecasting Conoco Phillips stock in the presence of shocks

We demonstrate our post-shock forecasting methodology on a time series of Conoco Phillips share prices after the occurrence of a structural shock. Conoco Phillips is a large oil and gas resources company [ConocoPhillips, 2020]. The particular post-shock response that we predict happened after trading ended on Friday March 6th, 2020 and before trading began on Monday March 9th, 2020. It is reasonable that the timing of this shock is known, several events occurred over the trading weekend which had an impact on stock markets and the oil markets. For example, Russia and OPEC began a battle for global oil price control on Sunday, March 8th [Sukhankin, 2020], and several US states began declaring state of emergencies in response to the evolving coronavirus pandemic [New York State Government, 2020, Alonso, 2020]. In this analysis we make the following design considerations:

(1) **Selection of model**. We will use an AR(1) model to forecast Conoco Phillips stock price. This model has been shown to beat no-change forecasts when predicting oil prices over time horizons of one and three months [Alquist et al., 2013]. For these reasons, we will consider 30 pre-shock trading days and we will forecast the immediate shock effect. All estimates will be adjusted for inflation. The model setup for AR(1) is exactly the same as what is stated in Section 2.1 with addition of shock effects. All the parameters are estimated using OLS.

(2) **Selection of covariates**. We consider different covariates for the model of $\alpha_i$ and $y_{i,t}$. The model of $\alpha_i$ incorporates daily S&P 500 index prices, West Texas Intermediate (WTI) crude oil prices, dollar index, 13 Week treasury bill rates, and Chicago Board Options Exchange (CBOE) volatility index. In contrast, the model of $y_{i,t}$ disregards CBOE volatility index. It is because CBOE volatity index is a metric for capturing market risk and sentiment, which can be highly related to the shock effect that results from investor behavior. It is not favorable in the model of $y_{i,t}$ since it may hinder the estimation of $\alpha_i$.

(3) **Construction of donor pool**. Our donor pool consists of Conoco Phillips shock effects observed in the past. We consider shock effects which occurred on March 14, 2008, several days in September, 2008, and November 27, 2014. The first sets of shock effects were observed during recessions that possessed similar characteristics to the current recession. In particular, all of these recessions were predicated by an inversion of the yield curve [Bauer and Mertens, 2018]. These 2008 shock effects correspond to the collapse of Bear Stearns, the placement of Fannie May and Freddie Mac in conservatorship on September 7th, the collapse of Lehman Brothers on September 15th, and the closing of Washington Mutual on September 25th [Shorter, 2008, Ewing and Malik, 2013, Dwyer and Tkac, 2009, Longstaff, 2010]. The last shock effect corresponds to an OPEC induced supply side shock effect [Huppmann and Holz, 2015].

We assume that the five shocks are independent of the shock that Conoco Phillips experienced on March 9, 2020. The covariates and response of time series in the donor pool are adjusted for inflation. Note that there are three shock effects nested in the time series 2008 September, we assume that these three shocks are independent, where the assumption checks using likelihood ratio test are provided in the Section 1 in the Supplementary Materials. The estimated shock-effects for $\alpha_i$ are $-0.922, -7.063, -5.777, -6.395, -4.207$ for $i = 2, \ldots, 6$, respectively. Under $\mathcal{M}_2$, we computed $\hat{\alpha}_{\text{adj}}$, weighted adjustment $\hat{\alpha}_{\text{wadj}}$, and $\hat{\alpha}_{\text{IVW}}$. Note that non-uniqueness problems will not occur in this analysis since the conditions of Proposition 1 are satisfied. To avoid the effect of unit differences on weighting, we center and scaled the covariates in weights computation but not in the model of $y_{i,t}$. For $\hat{\alpha}_{\text{wadj}}$, we observe that $\mathbf{W}^* = (0.000, 0.000, 0.000, 0.273, 0.727)$ and $\|\mathbf{X}_1 - \hat{\mathbf{X}}_1(\mathbf{W}^*)\|_2 = 3.440$. Note that the norm is computed using the $k$-dimensional Euclidean metric. The solution $\mathbf{W}^*$ suggests that the shock effect of interest is very similar to the September 25, 2008 shock effect and the November 27, 2014 shock effect.

The resulting shock effect estimates are $\hat{\alpha}_{\text{adj}} = -4.872$, $\hat{\alpha}_{\text{wadj}} = -4.805$, and $\hat{\alpha}_{\text{IVW}} = -4.384$. Using the bootstrap procedure $\mathcal{B}_f$, we estimated parameters for risk-reduction propositions and risk-reduction quantities proposed in Section 3. The estimated bootstrap variances for $\hat{\alpha}_{\text{adj}}$, $\hat{\alpha}_{\text{wadj}}$, and $\hat{\alpha}_{\text{IVW}}$ are 0.419, 0.559, and 0.667, respectively. We verify the consistency of the result yielded by risk-reduction propositions with the reality as below.

We can see from Figure 2 that $\hat{\alpha}_{\text{adj}}$, $\hat{\alpha}_{\text{wadj}}$ and $\hat{\alpha}_{\text{IVW}}$ perform decently well, and they do not recover the magnitude of the shock effect but are much better than unadjusted forecasts that do not account for shock effects. The unadjusted forecast misses the post-shock response by 9.870 dollars whereas the use

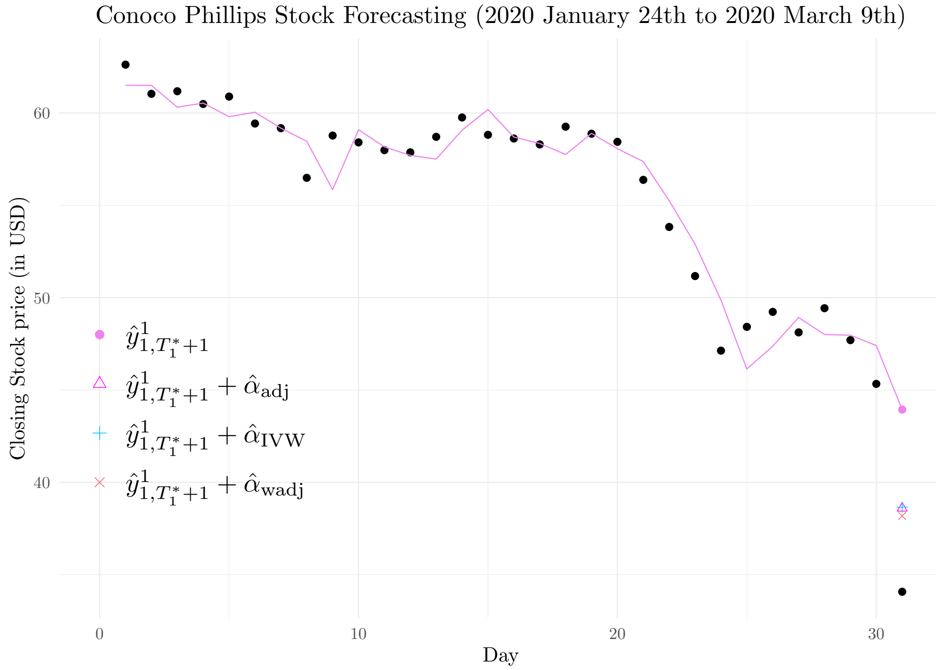


**Figure 2:** March 9th, 2020 post-shock forecasts for Conoco Phillips stock price.

of $\hat{\alpha}_{\mathrm{adj}}$, $\hat{\alpha}_{\mathrm{wadj}}$, and $\hat{\alpha}_{\mathrm{IVW}}$ misses by 5.324, 5.392, and 5.813 dollars, respectively. The true shock effect is not fully recovered by $\hat{\alpha}_{\mathrm{adj}}$, $\hat{\alpha}_{\mathrm{wadj}}$, and $\hat{\alpha}_{\mathrm{IVW}}$. This may be a result of a poorly constructed donor pool. The shock on March 9th, 2020 is in the midst of the COVID-19 pandemic and oil production volatility. It is difficult to find available stock market time series data that were generated under a similar setting. In any event, the shock on March 9th, 2020 was the largest price shock to Conoco Phillips shares by a wide margin, even after adjusting for inflation.

From another perspective, it is possible that the stock of Conoco Phillips actually experienced multiple shocks on 2020 March 9th. For example, Kilian [2009] studied the effect that different supply and demand shocks have on oil prices through a vector auto regressive model. Their model postulates an additive nature of shock effects, although the additivity parameters requires estimation in their context. Motivated by his study, we also studied additive shock effect estimators where the shock effects corresponding to separate supply and demand shocks are added to estimate the unknown shock effect. The supply shock donor pool consists of the November 27, 2014 shock effect; and the demand shock donor pool consists of the remaining shock effects. The additive adjustment estimator computed by adding the $\hat{\alpha}_{\mathrm{adj}}$, $\hat{\alpha}_{\mathrm{wadj}}$, and $\hat{\alpha}_{\mathrm{IVW}}$ estimators for the demand and supply shock effects only, respectively, miss the post-shock Conoco Philips share price by 0.951, 0.405, and 1.460 dollars. These additive adjustment estimators do extremely well in this additive shock setting.

There have been several other recent methods developed for forecasting COVID-19 cases. For example, Lee et al. [2020] constructed a Bayesian hierarchical model embracing data integration to improve predictive precision of COVID-19 infection trajectories for different countries. A similar setup may be appropriate for post-shock forecasting but may be too dependent upon model specification for the shock distribution. Plessen [2020] employed a data-mining approach to combine COVID-19 data from different countries as input to predict global net daily infections and deaths of COVID-19 using a clustering approach. However, there is a tremendous amount of volatility in this form of COVID-19 data, and the fit of this prediction method may be improved with modeling structure or preprocessing of the donor pool. Agarwal et al. [2020] proposed a model-free synthetic intervention method to predict unobserved potential outcomes after different interventions given a donor pool of observed outcomes with given interventions. They also provide useful guidelines for how to estimate the effects of potential interventions by giving recommendations for choosing the metric of interest, the intervention of interest, time horizons, and the donor pool. Although the methodology in Agarwal et al. [2020] is quite general, there is no guarantee for theoretical properties in prediction without assuming any distributional structure.

# 6 Discussion

We developed a methodology for forecasting post-shock response values after the occurrence of a structural shock. Our methodology is as follows: construct a synthetic panel of disparate time series which have undergone similar shocks, estimate the shock effects in those series, aggregate them, and then adjust the original forecast by adding the aggregated shock effect estimator to the original forecast. We provided risk-reduction propositions and empirical tools that can prospectively assess the effectiveness of our adjustment strategies in additive shock effect settings. The model, under which we verify these claims, is a simple AR(1) model. Similar results can be obtained for more general models such as AR($p$), vector autoregression, and generalized autoregressive conditional heteroskedasticity models.

Generally, multiple shock effects can be nested within a time series; and time series in the donor pool can be dependent. As an example, we considered a dependency structure for the September 2008 shock effects in our analysis of Conoco Phillips stock. But we note that consistency estimates from LOOCV with $k$ random draws may not work well if donor pool candidates are not mutually independent since the almost unbiased property hinges on the mutual independence among candidates in the donor pool. Although it is reflected in $\mathcal{M}_2$, we stress that our proposed methods allow $\alpha_i$ to follow arbitrary distributions provided that its first and second moments exist. The covariates in the model for $\alpha_i$ under $\mathcal{M}_2$ can be different from the covariates in the model of $y_{i,t}$. We also note that our post-shock framework can be extended to settings where the shock effect can be decomposed into separable estimable parts. An example of this is the additive shock effect estimators that we studied in our Conoco Phillips analysis. Although our work is developed for time-series or AR($p$) models, in fact, it can be generalized to any similar setting with a model of the response, whose parameters can be estimated unbiasedly, an additive shock-effect structure, and the structure that the time series in the donor pool are independent of the one of interest.

Our bootstrap procedures can be extended to approximate the distribution of shock effect estimators from more general time series. If the data are subject to heteroskedasticity of unknown form, bootstrapping tuples of regressands and regressors proposed by Freedman [1981] is robust in this situation with asymptotic validity in autoregressive models established by Gonçalves and Kilian [2004]. If serial correlation exists in the data, various block bootstrapping procedures [Künsch, 1989, Liu et al., 1992] can be possible reasonable alternatives. Note that the pseudo time series generated by our proposed parametric bootstrap are not stationary. If stationarity is of concern, one can be referred to the stationary bootstrap invented by Politis and Romano [1994] for stationary and weakly dependent time series. Nevertheless, it was shown that approximation accuracy might be a cost for the stationary bootstrap in autoregressive models in finite sample [Berkowitz et al., 2000]. More work related to bootstrapping time series can be referred to Chapters 3 and 4 in Politis et al. [1999], Berkowitz and Kilian [2000], and Chapter 12 in Kilian and Lütkepohl [2017]. It is up to users in terms of selecting which procedure to choose but under different assumptions on the time series.

We have implicitly assumed that $\mathbf{W}^*$ is non-degenerate in the population in the simulation examples. Recall that in Section 2.3 we noted that if there exists some $\mathbf{W}^*$ which satisfies (5) and $p < n$, then there will be infinitely many solutions to $\mathbf{W}^*$. In this scenario, $\mathbf{W}^*$ will take values on the boundary of $\mathcal{W}$, in which case bootstrapping may fail to estimate the distribution of $\hat{\alpha}_{\text{wadj}}$ [Andrews, 2000]. When $p < n$ and there exists some $\mathbf{W}^* \in \mathcal{W}$ satisfies (5), $\mathcal{B}_u$ fails since the non-uniqueness due to $p < n$ will guarantee degeneracy of $\mathbf{W}^*$. However, this issue will not occur under $\mathcal{B}_f$ since it takes $\mathbf{W}^*$ as being fixed and the parameter space is $\Theta$ that does not involve the constrained $\mathcal{W}$. But it does not seriously compromise the inference according to our simulation results in Section 4 in the Supplementary Materials.

Note that there are some philosophical distinctions between $\mathcal{B}_u$ and $\mathcal{B}_f$. $\mathcal{B}_u$ treats the donor pool as realizations from some infinite super-population of potential donors. In contrast, $\mathcal{B}_f$ treats the donor pool as being fixed and known before the analysis is conducted, where the randomness comes from parameters and idiosyncratic error.

A double bootstrap procedure with similar steps to the bootstrap technique in Section 3.2 can estimate the distribution of $\hat{\Delta}(\hat{\alpha})$ for $\hat{\alpha} \in \mathcal{A}$. The double bootstrap, instead of checking whether $\Delta(\hat{\alpha}) > 0$, can check whether a bootstrap percentile interval of resampled estimates of $\Delta(\hat{\alpha})$ contain 0 at a desired error threshold. We investigated such a double bootstrap procedure and found that it produced inferences that were similar to those produced using the bootstrap techniques developed in the main text.

# 7 Appendix

## 7.1 Justification of Expectation of $\hat{\alpha}_{\text{adj}}$ and $\hat{\alpha}_{\text{wadj}}$

The building block for the following proof is the fact that least squares is conditionally unbiased conditioned on $\Theta$.

**Case I: under $\mathcal{M}_1$:** It follows that under $\mathcal{M}_1$ (see Section 2.1),

$$\mathrm{E}(\hat{\alpha}_{\text{adj}}) = \frac{1}{n}\sum_{i=2}^{n+1}\mathrm{E}(\mathrm{E}(\hat{\alpha}_i|\Theta)) = \mu_\alpha \quad \text{and} \quad \mathrm{E}(\hat{\alpha}_{\text{wadj}}) = \sum_{i=2}^{n+1} w_i^* \mathrm{E}(\mathrm{E}(\hat{\alpha}_i|\Theta)) = \sum_{i=2}^{n+1} w_i^* \mu_\alpha = \mu_\alpha.$$

where we used the fact that $\sum_{i=2}^{n+1} w_i^* = 1$.

**Case II: under $\mathcal{M}_{21}$ and $\mathcal{M}_{22}$:** Since $\mathrm{E}(\tilde{\varepsilon}_{i,T_i}) = 0$, $\mathrm{E}(\hat{\tilde{\alpha}}_i) = \mathrm{E}(\tilde{\alpha}_i) = \mathrm{E}(\alpha_i)$, it follows that

$$\begin{aligned}
\mathrm{E}(\hat{\alpha}_{\text{wadj}}) &= \mathrm{E}\left\{\mathrm{E}\left(\sum_{i=2}^{n+1} w_i^* \hat{\alpha}_i|\Theta\right)\right\} = \mathrm{E}\left(\sum_{i=2}^{n+1} w_i^* \alpha_i\right) \\
&= \mathrm{E}\Bigg\{\sum_{i=2}^{n+1} w_i^* \left[\mu_\alpha + \delta_i' \mathbf{x}_{i,T_i^*+1}\right]\Bigg\} \\
&= \mu_\alpha + \mu_\delta' \sum_{i=2}^{n+1} w_i^* \mathbf{x}_{i,T_i^*+1} && (\mathbf{W} \in \mathcal{W}) \\
&= \mu_\alpha + \mu_\delta' \mathbf{x}_{1,T_1^*+1}. && (\text{from } (5))
\end{aligned}$$

Similarly,

$$\mathrm{E}(\hat{\alpha}_{\text{adj}}) = \mu_\alpha + \frac{1}{n}\sum_{i=2}^{n+1} \mu_\delta' \mathbf{x}_{i,T_i^*+1}.$$

## 7.2 Justification of Variance of $\hat{\alpha}_{\text{adj}}$ and $\hat{\alpha}_{\text{wadj}}$

Notice that under the setting of OLS, the design matrix for $\mathcal{M}_2$ is the same as the one for $\mathcal{M}_1$. Therefore, it follows that

$$\begin{aligned}
\mathrm{Var}(\hat{\alpha}_{\text{wadj}}) &= \mathrm{E}(\mathrm{Var}(\hat{\alpha}_{\text{wadj}}|\Theta)) + \mathrm{Var}(\mathrm{E}(\hat{\alpha}_{\text{wadj}}|\Theta)) \\
&= \mathrm{E}\left\{\mathrm{Var}\left(\sum_{i=2}^{n+1} w_i^* \hat{\alpha}_i|\Theta\right)\right\} + \mathrm{Var}\left(\sum_{i=2}^{n+1} w_i^* \alpha_i\right)
\end{aligned}$$

Under $\mathcal{M}_{21}$ where $\delta_i = \delta$ are fixed unknown parameters, we will have

$$\mathrm{Var}(\hat{\alpha}_{\text{wadj}}) = \mathrm{E}\left\{\sum_{i=2}^{n+1}(w_i^*)^2(\sigma^2(\mathbf{U}_i'\mathbf{U}_i)^{-1}_{22})\right\} + \sigma_\alpha^2 \sum_{i=2}^{n+1}(w_i^*)^2$$

$$= \sigma^2 \sum_{i=2}^{n+1} (w_i^*)^2 \mathrm{E}\big\{(\mathbf{U}_i'\mathbf{U}_i)^{-1}_{22}\big\} + \sigma_\alpha^2 \sum_{i=2}^{n+1} (w_i^*)^2. \tag{12}$$

Similarly, under $\mathcal{M}_{22}$ where we assume $\delta_i \perp\!\!\!\perp \varepsilon_{i,t}$, we have

$$\mathrm{Var}(\hat{\alpha}_{\mathrm{wadj}}) = \sigma^2 \sum_{i=2}^{n+1} (w_i^*)^2 \mathrm{E}\big\{(\mathbf{U}_i'\mathbf{U}_i)^{-1}_{22}\big\} + \sum_{i=2}^{n+1} (w_i^*)^2 (\mathbf{x}'_{i,T_i^*+1}\Sigma_\delta \mathbf{x}_{i,T_i^*+1} + \sigma_\alpha^2)$$

For the adjustment estimator, we simply replace $\mathbf{W}^*$ with $1/n\mathbf{1}_n$. Thus, under $\mathcal{M}_{21}$ we have

$$\mathrm{Var}(\hat{\alpha}_{\mathrm{adj}}) = \frac{\sigma^2}{n^2} \sum_{i=2}^{n+1} \mathrm{E}\big\{(\mathbf{U}_i'\mathbf{U}_i)^{-1}_{22}\big\} + \frac{\sigma_\alpha^2}{n^2}$$

Under $\mathcal{M}_{22}$, we shall have

$$\mathrm{Var}(\hat{\alpha}_{\mathrm{adj}}) = \frac{\sigma^2}{n^2} \sum_{i=2}^{n+1} \mathrm{E}\big\{(\mathbf{U}_i'\mathbf{U}_i)^{-1}_{22}\big\} + \frac{1}{n^2} (\mathbf{x}'_{i,T_i^*+1}\Sigma_\delta \mathbf{x}_{i,T_i^*+1} + \sigma_\alpha^2).$$

Notice that $\mathcal{M}_1$ differs from $\mathcal{M}_{21}$ only by its mean parameterization of $\alpha$ (see Section 2.1). In other words, the variances of $\hat{\alpha}_{\mathrm{adj}}$ and $\hat{\alpha}_{\mathrm{wadj}}$ under $\mathcal{M}_1$ are the same for those under $\mathcal{M}_{21}$.

### 7.3 Proofs for lemmas and propositions

**Proof of Proposition 1** The proof of Li [2019] in Appendix A.2 and A.3 adapts easily to Proposition 1. □

**Proof of Proposition 2** The proof for unbiasedness follows immediately from discussions related to expectation in Section 3. For the biasedness of $\hat{\alpha}_{\mathrm{adj}}$ under $\mathcal{M}_{21}$ and $\mathcal{M}_{22}$, we write the bias term for $\hat{\alpha}_{\mathrm{adj}}$ as below.

$$\mathrm{Bias}(\hat{\alpha}_{\mathrm{adj}}) = \begin{cases} \frac{1}{n}\sum_{i=2}^{n+1} \delta'(\mathbf{x}_{i,T_i^*+1} - n\mathbf{x}_{1,T_1^*+1}) & \text{for } \mathcal{M}_{21} \\ \frac{1}{n}\sum_{i=2}^{n+1} \mu_\delta'(\mathbf{x}_{i,T_i^*+1} - n\mathbf{x}_{1,T_1^*+1}) & \text{for } \mathcal{M}_{22} \end{cases}.$$

But it may be unbiased in some special circumstances when the above bias turns out to be 0. □

**Lemma 1.** *The forecast risk reduction is* $R_{T_1^*+1,1} - R_{T_1^*+1,2} = \mathrm{E}(\alpha_1^2) - \mathrm{E}(\hat{\alpha} - \alpha_1)^2$ *for all estimators of* $\alpha_1$ *that are independent of* $\Theta_1$ *(see Section 2.1).*

**Proof of Lemma 1** Define

$$C(\Theta_1) = \hat{\eta}_1 + \hat{\phi}_1 y_{1,T_1^*} + \hat{\theta}_1' \mathbf{x}_{1,T_1^*+1} - (\eta_1 + \phi_1 y_{1,T_1^*} + \theta_1' \mathbf{x}_{1,T_1^*+1}),$$

where $\Theta_1$ is as defined in (3). Notice that

$$R_{T_1^*+1,1} = \mathrm{E}\big\{\big(C(\Theta_1) - \alpha_1\big)^2\big\} \qquad \text{and} \qquad R_{T_1^*+1,2} = \mathrm{E}\big\{\big(C(\Theta_1) + \hat{\alpha} - \alpha_1\big)^2\big\}.$$

It follows that

$$R_{T_1^*+1,1} - R_{T_1^*+1,2} = \mathrm{E}(\alpha_1^2) - 2\mathrm{E}(C(\Theta_1)\hat{\alpha}) - \mathrm{E}(\hat{\alpha} - \alpha_1)^2.$$

Assuming $\mathbf{S} = (\mathbf{1}_n, \mathbf{y}_{1,t-1}, \mathbf{x}_1)$ has full rank, under OLS setting, $\hat{\eta}_1$, $\hat{\phi}_1$, and $\hat{\theta}_1$ are unbiased estimators of $\eta_1$, $\phi_1$, and $\theta_1$, respectively under conditioning of $\Theta_1$. Since we assume $\hat{\alpha}$ is independent of $\Theta_1$, through the method of iterated expectation,

$$\mathrm{E}(C(\Theta_1)\hat{\alpha}) = \mathrm{E}\big\{\hat{\alpha} \cdot \mathrm{E}(C(\Theta_1) \mid \Theta_1)\big\} = 0.$$

It follows that

$$R_{T_1^*+1,1} - R_{T_1^*+1,2} = \mathrm{E}(\alpha_1^2) - \mathrm{E}(\hat{\alpha} - \alpha_1)^2,$$

which finishes the proof. $\square$

**Proof of Proposition 3** The proofs are arranged into two separate parts as below.

**Proof for statement (i):** Under $\mathcal{M}_1$, $\hat{\alpha}_{\mathrm{adj}}$ is an unbiased estimator of $\mathrm{E}(\alpha_1)$ because

$$\begin{aligned}\mathrm{E}\left(\frac{1}{n}\sum_{i=2}^{n+1}\hat{\alpha}_i\right) = \frac{1}{n}\sum_{i=2}^{n+1}\mathrm{E}(\hat{\alpha}_i) &= \frac{1}{n}\sum_{i=2}^{n+1}\mathrm{E}(\mathrm{E}(\hat{\alpha}_i \mid \Theta)) \\ &= \frac{1}{n}\sum_{i=2}^{n+1}\mathrm{E}(\alpha_i) = \mu_\alpha = \mathrm{E}(\alpha_1),\end{aligned}$$

where we used the fact that OLS estimator is unbiased when the design matrix $\mathbf{U}_i$ is of full rank for all $i = 2, \ldots, n+1$. Because $\alpha_1 \perp\!\!\!\perp \varepsilon_{i,t}$, $\mathrm{E}(\hat{\alpha}_{\mathrm{adj}}\alpha_1) = \mathrm{E}(\hat{\alpha}_{\mathrm{adj}})\mathrm{E}(\alpha_1) = (\mathrm{E}(\hat{\alpha}_{\mathrm{adj}}))^2$. By Lemma 1,

$$\begin{aligned}R_{T_1^*+1,1} - R_{T_1^*+1,2} &= \mathrm{E}(\alpha_1^2) - \mathrm{E}(\hat{\alpha}_{\mathrm{adj}} - \alpha_1)^2 \\ &= \mathrm{E}(\alpha_1^2) - \mathrm{E}(\alpha_1^2) - \mathrm{E}(\hat{\alpha}_{\mathrm{adj}}^2) + 2\mathrm{E}(\hat{\alpha}_{\mathrm{adj}}\alpha_1) \\ &= \mu_\alpha^2 - \mathrm{Var}(\hat{\alpha}_{\mathrm{adj}})\end{aligned}$$

Therefore, as long as we have $\mathrm{Var}(\hat{\alpha}_{\mathrm{adj}}) < \mu_\alpha^2$, we will achieve the risk reduction.

**Proof for statement (ii):** By Proposition 2, the property that $\hat{\alpha}_{\mathrm{wadj}}$ is an unbiased estimator of $\mu_\alpha$ holds for $\mathcal{M}_1$. The remainder of the proof follows a similar argument to the proof of statement (i). $\square$

**Proof of Proposition 4** By Proposition 2, the property that $\hat{\alpha}_{\mathrm{wadj}}$ is an unbiased estimator of $\mathrm{E}(\alpha_1)$ holds for $\mathcal{M}_{21}$ and $\mathcal{M}_{22}$. The remainder of the proof follows a similar argument to the proof of Proposition 3. $\square$

## References


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