Supplemental Materials for "Minimizing post shock forecasting error using disparate information"

Jilei Lin[*], Ziyu Liu[†], and Daniel J. Eck[‡]

Department of Statistics, University of Illinois at Urbana-Champaign

July 15, 2020

Jilei, make sure that the bootstrap algorithm (what is now B_u) is explained correctly. Also add the description for what was the B_c procedure. However, this time refer to it as the fixed donor pool bootstrap. Add the algorithm as well. Make sure to edit the text to make sure that it integrates well with the reported tables. Also, make sure that the predicted response information is included in one table. Remove the best estimator results. Also include an introductory summary of what is in this document and put it here.

1 Supplementary materials for data analysis

The independence of the estimated September, 2008 shock-effects are further tested using likelihood ratio test (LRT) based on their estimated covariance matrix. The estimated covariance matrix is

$$\hat{\boldsymbol{\Sigma}} = \begin{pmatrix} 4.012 & 0.362 & -0.062 \\ 0.362 & 3.894 & -0.029 \\ -0.062 & -0.029 & 3.927 \end{pmatrix}.$$

with degrees of freedoms 35. Using hte LRT for independence between blocks of random variables [Marden, 2015, Section 10.2], the LRT test statistic is 0.304 with p-value of 0.581. Therefore, we do not reject the null hypothesis that the three estimated shock-effects are independent.

[*]jileil2@ilinois.edu

[†]ziyuliu3@illinois.edu

[‡]dje13@illinois.edu

2 Bootstrap algorithms of fixed donor pool bootstrapping $\mathcal{B}_f$

Algorithm 1: Parametric bootstrap for approximation for mean and variance of shock-effect estimators of α_1.

Input: B – the number of parametric bootstraps
$\{(y_{i,t}, \mathbf{x}_{i,t})\colon i = 2, \ldots, n+1, t = 0, \ldots, T_i\}$ – the data
$\{T_i^*\colon i = 1, \ldots, n+1\}$ – the time point just before the shock
$\{\hat{\varepsilon}_{i,t}\colon t = 1, \ldots, T_i\}$ – the collection of residuals for $t = 1, \ldots, T_i$
$\{\hat{\eta}_i, \hat{\alpha}_i, \hat{\phi}_i, \hat{\theta}_i, \hat{\beta}_i\colon i = 2, \ldots, n+1\}$ – the OLS estimates

Result: The sample mean, and sample variance of bootstrapped adjustment estimator, inverse-variance weighted estimator, and weighted-adjustment estimator.

for $b = 1 : B$ **do**
  for $i = 2, \ldots, n+1$ **do**
    Sample with replacement from $\{\hat{\varepsilon}_{i,t}\colon t = 1, \ldots, T_i\}$ to obtain $\{\hat{\varepsilon}_{i,t}^{(b)}\colon t = 1, \ldots, T_i\}$
    Define $y_{i,0}^{(b)} = y_{i,0}$
    for $t = 1, \ldots, T_i$ **do**
      Compute $y_{i,t}^{(b)} = \hat{\eta}_i + \hat{\alpha}_i 1(t = T_i^* + 1) + \hat{\phi}_i y_{i,t-1}^{(b)} + \theta_i' \mathbf{x}_{i,t} + \beta_i' \mathbf{x}_{i,t-1} + \hat{\varepsilon}_{i,t}^{(b)}$
    end
    Compute $\hat{\alpha}_i^{(b)}$ based on OLS estimation with $\{(y_{i,t}^{(b)}, \mathbf{x}_{i,t})\colon t = 0, \ldots, T_i\}$
  end
  Compute the bth shock-effect estimate $\hat{\alpha}_{\text{est}}^{(b)}$ for est $\in \{\text{adj}, \text{wadj}, \text{IVW}\}$
end
Compute the sample mean, and sample variance of $\{\hat{\alpha}_{\text{est}}^{(b)}\colon b = 1, \ldots, B\}$ for est $\in \{\text{adj}, \text{wadj}, \text{IVW}\}$

3 Tables for $\mathcal{M}_1$

Table 1: 30 Monte Carlo simulations of $\mathcal{M}_1$ for $\mathcal{B}_u$ with varying n and σ_α

		Guess			LOOCV with k random draws			Distance to y_{1,T_1^*+1}			
n	σ_α	$\delta_{\hat{\alpha}_{\text{adj}}}$	$\delta_{\hat{\alpha}_{\text{wadj}}}$	$\delta_{\hat{\alpha}_{\text{IVW}}}$	$\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{adj}}})$	$\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{wadj}}})$	$\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{IVW}}})$	Original	$\hat{\alpha}_{\text{adj}}$	$\hat{\alpha}_{\text{wadj}}$	$\hat{\alpha}_{\text{IVW}}$
	5	1 (0)	1 (0)	1 (0)	0.95 (0.02)	0.95 (0.02)	0.95 (0.02)	51.11 (2.86)	12.36 (1.99)	14.74 (2.31)	12.19 (1.96)
	10	1 (0)	1 (0)	1 (0)	0.94 (0.02)	0.94 (0.02)	0.94 (0.02)	50.76 (3.51)	14.66 (2.47)	17.49 (2.9)	14.25 (2.47)
5	25	1 (0)	1 (0)	1 (0)	0.8 (0.04)	0.83 (0.03)	0.79 (0.04)	53.08 (4.85)	24.1 (4.18)	28.57 (4.9)	23.41 (4.17)
	50	0.9 (0.06)	0.93 (0.05)	0.9 (0.06)	0.65 (0.05)	0.66 (0.04)	0.65 (0.05)	61.41 (7.7)	44.02 (6.88)	52.31 (7.88)	43.11 (6.85)
	100	0.63 (0.09)	0.7 (0.09)	0.63 (0.09)	0.47 (0.05)	0.47 (0.04)	0.47 (0.05)	91.4 (13.21)	84.69 (12.6)	100.26 (14.24)	83.46 (12.5)
	5	1 (0)	1 (0)	1 (0)	0.99 (0.01)	0.99 (0.01)	0.99 (0.01)	49.1 (2.79)	12.33 (1.96)	12.84 (1.99)	12.54 (1.93)
	10	1 (0)	1 (0)	1 (0)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	47.42 (3.32)	14.44 (2.43)	15.77 (2.48)	14.65 (2.41)
10	25	0.97 (0.03)	1 (0)	0.97 (0.03)	0.8 (0.03)	0.83 (0.03)	0.8 (0.03)	44.87 (4.71)	24.76 (3.99)	28.81 (4.05)	25.09 (3.94)
	50	0.9 (0.06)	0.93 (0.05)	0.9 (0.06)	0.65 (0.04)	0.64 (0.04)	0.65 (0.04)	49.9 (6.64)	44.8 (6.89)	52.51 (7.13)	45.06 (6.85)
	100	0.57 (0.09)	0.63 (0.09)	0.57 (0.09)	0.52 (0.05)	0.55 (0.04)	0.49 (0.05)	74.62 (11.8)	85.94 (13.05)	100.73 (13.67)	86.14 (12.99)
	5	1 (0)	1 (0)	1 (0)	0.96 (0.01)	0.95 (0.02)	0.96 (0.01)	48.64 (2.33)	10.13 (1.41)	11.89 (1.69)	10.18 (1.38)
	10	1 (0)	1 (0)	1 (0)	0.95 (0.02)	0.93 (0.02)	0.95 (0.02)	46.93 (2.83)	12.51 (1.73)	14.36 (1.73)	12.48 (1.74)
15	25	0.97 (0.03)	0.97 (0.03)	0.97 (0.03)	0.83 (0.03)	0.81 (0.03)	0.84 (0.03)	43.53 (4.48)	22.81 (3.46)	24.1 (3.18)	22.79 (3.48)
	50	0.9 (0.06)	0.97 (0.03)	0.93 (0.05)	0.63 (0.04)	0.61 (0.04)	0.61 (0.03)	50.91 (5.81)	42.61 (6.82)	44.53 (6.2)	42.71 (6.77)
	100	0.77 (0.08)	0.83 (0.07)	0.73 (0.08)	0.53 (0.04)	0.51 (0.04)	0.54 (0.04)	80.48 (10.37)	87.65 (12.82)	90.21 (11.88)	87.71 (12.71)
	5	1 (0)	1 (0)	1 (0)	0.97 (0.01)	0.97 (0.01)	0.97 (0.01)	50.91 (2.67)	10.87 (1.57)	12.07 (1.84)	11.01 (1.58)
	10	1 (0)	1 (0)	1 (0)	0.96 (0.01)	0.95 (0.02)	0.96 (0.01)	51.43 (3.03)	12.85 (1.66)	15.01 (2.03)	13.02 (1.67)
25	25	1 (0)	1 (0)	1 (0)	0.87 (0.02)	0.85 (0.03)	0.87 (0.02)	53.32 (4.9)	22.43 (2.55)	26.95 (3.06)	22.65 (2.57)
	50	0.87 (0.06)	0.9 (0.06)	0.87 (0.06)	0.66 (0.03)	0.67 (0.04)	0.65 (0.03)	64.22 (7)	41.26 (4.83)	48.43 (5.6)	41.39 (4.87)
	100	0.73 (0.08)	0.73 (0.08)	0.73 (0.08)	0.55 (0.05)	0.51 (0.05)	0.55 (0.05)	95.79 (11.73)	80.67 (9.79)	92.78 (11.05)	80.64 (9.86)

Table 2: 30 Monte Carlo simulations of $\mathcal{M}_1$ for $\mathcal{B}_u$ with varying σ and σ_α

		Guess			LOOCV with k random draws			Distance to y_{1,T_1^*+1}			
σ	σ_α	$\delta_{\hat{\alpha}_{\text{adj}}}$	$\delta_{\hat{\alpha}_{\text{wadj}}}$	$\delta_{\hat{\alpha}_{\text{IVW}}}$	$\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{adj}}})$	$\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{wadj}}})$	$\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{IVW}}})$	Original	$\hat{\alpha}_{\text{adj}}$	$\hat{\alpha}_{\text{wadj}}$	$\hat{\alpha}_{\text{IVW}}$
	5	1 (0)	1 (0)	1 (0)	0.99 (0.01)	0.99 (0.01)	0.99 (0.01)	48.77 (1.65)	7.14 (1.22)	7.77 (1.25)	7.26 (1.21)
	10	1 (0)	1 (0)	1 (0)	0.97 (0.01)	0.97 (0.01)	0.97 (0.01)	47.1 (2.32)	10.4 (1.72)	12.1 (1.73)	10.51 (1.72)
5	25	1 (0)	1 (0)	1 (0)	0.81 (0.03)	0.84 (0.03)	0.81 (0.03)	44.03 (4.05)	22.38 (3.42)	26.23 (3.54)	22.5 (3.4)
	50	0.9 (0.06)	0.97 (0.03)	0.9 (0.06)	0.64 (0.04)	0.63 (0.04)	0.63 (0.04)	47.72 (6.35)	42.94 (6.5)	50.34 (6.81)	43.03 (6.47)
	100	0.57 (0.09)	0.63 (0.09)	0.57 (0.09)	0.5 (0.05)	0.51 (0.04)	0.5 (0.04)	72.88 (11.48)	84.74 (12.69)	98.56 (13.51)	84.85 (12.62)
	5	1 (0)	1 (0)	1 (0)	0.99 (0.01)	0.99 (0.01)	0.99 (0.01)	49.1 (2.79)	12.33 (1.96)	12.84 (1.99)	12.54 (1.93)
	10	1 (0)	1 (0)	1 (0)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	47.42 (3.32)	14.44 (2.43)	15.77 (2.48)	14.65 (2.41)
10	25	0.97 (0.03)	1 (0)	0.97 (0.03)	0.8 (0.03)	0.83 (0.03)	0.8 (0.03)	44.87 (4.71)	24.76 (3.99)	28.81 (4.05)	25.09 (3.94)
	50	0.9 (0.06)	0.93 (0.05)	0.9 (0.06)	0.65 (0.04)	0.64 (0.04)	0.65 (0.04)	49.9 (6.64)	44.8 (6.89)	52.51 (7.13)	45.06 (6.85)
	100	0.57 (0.09)	0.63 (0.09)	0.57 (0.09)	0.52 (0.05)	0.55 (0.04)	0.49 (0.05)	74.62 (11.8)	85.94 (13.05)	100.73 (13.67)	86.14 (12.99)
	5	1 (0)	1 (0)	1 (0)	0.82 (0.03)	0.82 (0.03)	0.83 (0.03)	51.82 (5.58)	28.83 (4.36)	29.08 (4.47)	29.2 (4.29)
	10	1 (0)	1 (0)	1 (0)	0.8 (0.03)	0.8 (0.03)	0.78 (0.04)	51.37 (5.7)	29.93 (4.79)	31.03 (4.85)	30.44 (4.71)
25	25	0.93 (0.05)	0.97 (0.03)	0.93 (0.05)	0.75 (0.04)	0.74 (0.04)	0.75 (0.04)	51.77 (6.36)	37.51 (5.98)	40.78 (6.1)	37.99 (5.92)
	50	0.8 (0.07)	0.83 (0.07)	0.8 (0.07)	0.6 (0.05)	0.59 (0.04)	0.59 (0.05)	57.39 (8.25)	53.52 (8.71)	62.34 (8.6)	54.02 (8.66)
	100	0.53 (0.09)	0.57 (0.09)	0.57 (0.09)	0.49 (0.05)	0.53 (0.04)	0.5 (0.05)	80.22 (13.49)	92.77 (14.41)	108.3 (14.81)	93.51 (14.29)
	5	0.67 (0.09)	0.67 (0.09)	0.67 (0.09)	0.55 (0.05)	0.51 (0.04)	0.55 (0.05)	67.29 (9.18)	57.37 (8.43)	57.45 (8.66)	58.08 (8.26)
	10	0.67 (0.09)	0.67 (0.09)	0.67 (0.09)	0.53 (0.05)	0.53 (0.04)	0.53 (0.05)	67.72 (9.24)	58.18 (8.8)	58.99 (8.96)	58.92 (8.65)
50	25	0.73 (0.08)	0.73 (0.08)	0.7 (0.09)	0.53 (0.05)	0.55 (0.05)	0.53 (0.05)	69.36 (9.85)	62.1 (10.11)	65.26 (10.16)	63.15 (9.93)
	50	0.63 (0.09)	0.7 (0.09)	0.63 (0.09)	0.53 (0.05)	0.5 (0.04)	0.53 (0.05)	75.13 (11.58)	76.15 (11.97)	82.62 (12.17)	77.1 (11.85)
	100	0.53 (0.09)	0.6 (0.09)	0.53 (0.09)	0.52 (0.05)	0.55 (0.04)	0.5 (0.05)	98.34 (15.85)	108.03 (17.49)	125.76 (17.14)	108.99 (17.39)
	5	0.37 (0.09)	0.33 (0.09)	0.4 (0.09)	0.42 (0.04)	0.49 (0.04)	0.41 (0.04)	114.58 (15.95)	114.76 (16.6)	114.79 (17.05)	115.96 (16.28)
	10	0.37 (0.09)	0.33 (0.09)	0.37 (0.09)	0.42 (0.04)	0.46 (0.04)	0.42 (0.04)	115.61 (16)	115.49 (16.93)	115.95 (17.35)	116.81 (16.6)
100	25	0.4 (0.09)	0.37 (0.09)	0.37 (0.09)	0.39 (0.04)	0.46 (0.04)	0.39 (0.05)	118.76 (16.45)	117.96 (18.14)	120.56 (18.37)	119.47 (17.82)
	50	0.47 (0.09)	0.37 (0.09)	0.43 (0.09)	0.46 (0.05)	0.51 (0.05)	0.45 (0.05)	124.94 (17.83)	125.31 (20.27)	131.64 (20.36)	127.15 (19.96)
	100	0.4 (0.09)	0.43 (0.09)	0.4 (0.09)	0.48 (0.05)	0.53 (0.03)	0.49 (0.05)	146.49 (20.97)	153.41 (24.01)	166.43 (24.35)	155.32 (23.75)

Table 3: 30 Monte Carlo simulations of $\mathcal{M}_1$ for $\mathcal{B}_f$ with varying σ and σ_α

		Guess			LOOCV with k random draws			Distance to y_{1,T_1^*+1}			
σ	σ_α	$\delta_{\hat{\alpha}_{\text{adj}}}$	$\delta_{\hat{\alpha}_{\text{wadj}}}$	$\delta_{\hat{\alpha}_{\text{IVW}}}$	$\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{adj}}})$	$\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{wadj}}})$	$\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{IVW}}})$	Original	$\hat{\alpha}_{\text{adj}}$	$\hat{\alpha}_{\text{wadj}}$	$\hat{\alpha}_{\text{IVW}}$
	5	1 (0)	1 (0)	1 (0)	0.94 (0.02)	0.94 (0.02)	0.94 (0.02)	50.11 (2.43)	11.11 (1.85)	13.2 (1.91)	10.97 (1.85)
	10	1 (0)	1 (0)	1 (0)	0.93 (0.02)	0.93 (0.02)	0.93 (0.02)	50.03 (2.81)	13.68 (1.86)	15.89 (1.88)	13.62 (1.86)
5	25	1 (0)	1 (0)	1 (0)	0.78 (0.03)	0.77 (0.03)	0.79 (0.03)	49.82 (4.54)	21.75 (2.6)	24.99 (2.53)	21.98 (2.61)
	50	0.93 (0.05)	1 (0)	0.93 (0.05)	0.61 (0.05)	0.61 (0.04)	0.61 (0.05)	57.05 (6.07)	36.31 (4.58)	40.79 (4.79)	36.97 (4.62)
	100	0.73 (0.08)	0.97 (0.03)	0.67 (0.09)	0.45 (0.05)	0.49 (0.05)	0.45 (0.05)	88.65 (7.05)	65.85 (9.15)	76.22 (9.13)	67.41 (9.28)
	5	1 (0)	1 (0)	1 (0)	0.96 (0.02)	0.96 (0.02)	0.96 (0.02)	50.21 (2.49)	10.87 (1.57)	10.89 (1.62)	10.85 (1.53)
	10	1 (0)	1 (0)	1 (0)	0.94 (0.02)	0.94 (0.02)	0.94 (0.02)	49.02 (2.76)	13.62 (1.6)	14.02 (1.74)	13.67 (1.56)
10	25	0.93 (0.05)	1 (0)	0.93 (0.05)	0.75 (0.05)	0.81 (0.03)	0.76 (0.05)	45.91 (4.58)	24.55 (3.3)	24.27 (4.09)	24.43 (3.27)
	50	0.77 (0.08)	0.97 (0.03)	0.77 (0.08)	0.61 (0.05)	0.66 (0.04)	0.62 (0.05)	49.9 (7.12)	44.63 (7.23)	44.98 (8.51)	43.98 (7.2)
	100	0.67 (0.09)	0.87 (0.06)	0.63 (0.09)	0.53 (0.05)	0.48 (0.04)	0.54 (0.05)	74.74 (13.07)	89.35 (14.73)	89.56 (17.32)	87.87 (14.63)
	5	1 (0)	1 (0)	1 (0)	0.94 (0.02)	0.94 (0.02)	0.94 (0.02)	53.03 (2.15)	9.39 (1.51)	11.1 (1.66)	9.4 (1.47)
	10	1 (0)	1 (0)	1 (0)	0.91 (0.03)	0.91 (0.03)	0.91 (0.03)	52.46 (2.66)	11.65 (1.87)	14 (2.06)	11.79 (1.82)
15	25	1 (0)	1 (0)	1 (0)	0.77 (0.04)	0.76 (0.03)	0.76 (0.04)	52.76 (4.47)	23.88 (3.47)	26.97 (3.82)	23.95 (3.47)
	50	0.9 (0.06)	1 (0)	0.9 (0.06)	0.59 (0.04)	0.59 (0.04)	0.59 (0.04)	59.8 (7.66)	46.85 (6.7)	52.97 (6.8)	47.12 (6.69)
	100	0.8 (0.07)	0.9 (0.06)	0.8 (0.07)	0.47 (0.04)	0.47 (0.03)	0.47 (0.04)	95.95 (12.52)	96.35 (12.82)	107.42 (12.67)	97 (12.77)
	5	1 (0)	1 (0)	1 (0)	0.92 (0.02)	0.93 (0.02)	0.92 (0.02)	55.1 (2.73)	11.66 (1.93)	13.34 (2.08)	11.77 (1.92)
	10	1 (0)	1 (0)	1 (0)	0.89 (0.02)	0.91 (0.02)	0.89 (0.02)	55 (3)	13.63 (1.93)	14.94 (2.04)	13.67 (1.93)
25	25	1 (0)	1 (0)	1 (0)	0.79 (0.04)	0.77 (0.04)	0.79 (0.04)	55.02 (4.34)	21.29 (2.6)	22.14 (2.77)	21.17 (2.64)
	50	0.87 (0.06)	1 (0)	0.87 (0.06)	0.59 (0.04)	0.63 (0.04)	0.61 (0.05)	58 (6.77)	35.88 (4.69)	38.22 (4.85)	36.02 (4.69)
	100	0.7 (0.09)	0.97 (0.03)	0.7 (0.09)	0.49 (0.04)	0.51 (0.04)	0.47 (0.04)	71.18 (12.1)	68.53 (9.01)	74.18 (9.38)	69.17 (8.94)

Table 4: 30 Monte Carlo simulations of $\mathcal{M}_1$ for $\mathcal{B}_f$ with varying σ and σ_α

		Guess			LOOCV with k random draws			Distance to y_{1,T_1^*+1}			
σ	σ_α	$\delta_{\hat{\alpha}_{\text{adj}}}$	$\delta_{\hat{\alpha}_{\text{wadj}}}$	$\delta_{\hat{\alpha}_{\text{IVW}}}$	$\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{adj}}})$	$\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{wadj}}})$	$\bar{\mathcal{C}}^{(k)}(\delta_{\hat{\alpha}_{\text{IVW}}})$	Original	$\hat{\alpha}_{\text{adj}}$	$\hat{\alpha}_{\text{wadj}}$	$\hat{\alpha}_{\text{IVW}}$
	5	1 (0)	1 (0)	1 (0)	0.99 (0.01)	0.99 (0.01)	0.99 (0.01)	49.62 (1.34)	6.69 (0.77)	6.83 (0.86)	6.72 (0.74)
	10	1 (0)	1 (0)	1 (0)	0.98 (0.01)	0.97 (0.02)	0.98 (0.01)	48.43 (1.96)	10.21 (1.29)	10.17 (1.61)	10.19 (1.28)
5	25	0.97 (0.03)	1 (0)	0.97 (0.03)	0.79 (0.03)	0.83 (0.02)	0.79 (0.03)	45.46 (4.33)	22.21 (3.62)	22.48 (4.25)	21.88 (3.6)
	50	0.73 (0.08)	0.97 (0.03)	0.73 (0.08)	0.62 (0.05)	0.67 (0.04)	0.63 (0.05)	48.82 (7.36)	44.59 (7.37)	44.76 (8.66)	43.85 (7.32)
	100	0.67 (0.09)	0.93 (0.05)	0.63 (0.09)	0.52 (0.05)	0.49 (0.05)	0.53 (0.05)	74.59 (13.36)	89.83 (14.92)	90.85 (17.33)	88.25 (14.81)
	5	1 (0)	1 (0)	1 (0)	0.96 (0.02)	0.96 (0.02)	0.96 (0.02)	50.21 (2.49)	10.87 (1.57)	10.89 (1.62)	10.85 (1.53)
	10	1 (0)	1 (0)	1 (0)	0.94 (0.02)	0.94 (0.02)	0.94 (0.02)	49.02 (2.76)	13.62 (1.6)	14.02 (1.74)	13.67 (1.56)
10	25	0.93 (0.05)	1 (0)	0.93 (0.05)	0.75 (0.05)	0.81 (0.03)	0.76 (0.05)	45.91 (4.58)	24.55 (3.3)	24.27 (4.09)	24.43 (3.27)
	50	0.77 (0.08)	0.97 (0.03)	0.77 (0.08)	0.61 (0.05)	0.66 (0.04)	0.62 (0.05)	49.9 (7.12)	44.63 (7.23)	44.98 (8.51)	43.98 (7.2)
	100	0.67 (0.09)	0.87 (0.06)	0.63 (0.09)	0.53 (0.05)	0.48 (0.04)	0.54 (0.05)	74.74 (13.07)	89.35 (14.73)	89.56 (17.32)	87.87 (14.63)
	5	0.93 (0.05)	0.97 (0.03)	0.97 (0.03)	0.73 (0.04)	0.75 (0.03)	0.74 (0.04)	53.85 (5.67)	27.64 (3.65)	26.5 (3.93)	27.35 (3.57)
	10	0.97 (0.03)	0.97 (0.03)	0.97 (0.03)	0.73 (0.04)	0.75 (0.03)	0.72 (0.04)	52.88 (5.55)	27.72 (3.65)	27.22 (3.87)	27.55 (3.58)
25	25	0.87 (0.06)	0.97 (0.03)	0.9 (0.06)	0.65 (0.05)	0.69 (0.04)	0.63 (0.05)	51.06 (5.81)	34.43 (3.84)	35.67 (4.15)	34.48 (3.76)
	50	0.7 (0.09)	0.77 (0.08)	0.7 (0.09)	0.54 (0.05)	0.59 (0.05)	0.55 (0.05)	55.05 (7.13)	52.38 (6.22)	52.41 (7.77)	52.1 (6.2)
	100	0.63 (0.09)	0.87 (0.06)	0.6 (0.09)	0.51 (0.05)	0.49 (0.05)	0.48 (0.05)	75.13 (12.85)	91.17 (13.96)	90.79 (16.82)	90.39 (13.83)
	5	0.73 (0.08)	0.77 (0.08)	0.8 (0.07)	0.51 (0.05)	0.47 (0.04)	0.52 (0.05)	66.08 (10.43)	55.58 (7.37)	54.79 (7.57)	54.65 (7.27)
	10	0.7 (0.09)	0.73 (0.08)	0.73 (0.08)	0.49 (0.05)	0.49 (0.04)	0.49 (0.04)	64.11 (10.44)	55.29 (7.19)	53.48 (7.69)	54.85 (7)
50	25	0.7 (0.09)	0.7 (0.09)	0.7 (0.09)	0.48 (0.05)	0.47 (0.05)	0.49 (0.05)	62.24 (10.1)	55.85 (7.31)	56.56 (7.52)	55.44 (7.24)
	50	0.63 (0.09)	0.67 (0.09)	0.63 (0.09)	0.49 (0.05)	0.54 (0.04)	0.48 (0.05)	65.53 (10.01)	68.53 (7.56)	71.85 (7.98)	68.61 (7.44)
	100	0.57 (0.09)	0.8 (0.07)	0.6 (0.09)	0.46 (0.05)	0.53 (0.05)	0.51 (0.05)	85.95 (12.65)	104.55 (12.28)	105.56 (15.21)	103.99 (12.25)
	5	0.53 (0.09)	0.43 (0.09)	0.53 (0.09)	0.37 (0.04)	0.42 (0.05)	0.37 (0.04)	112.06 (17.96)	111.81 (14.87)	111.78 (14.92)	109.72 (14.68)
	10	0.5 (0.09)	0.47 (0.09)	0.53 (0.09)	0.38 (0.04)	0.43 (0.05)	0.38 (0.04)	110.61 (17.81)	110.68 (14.77)	110.47 (14.9)	109 (14.52)
100	25	0.53 (0.09)	0.47 (0.09)	0.5 (0.09)	0.39 (0.05)	0.46 (0.05)	0.39 (0.05)	106.24 (17.67)	110.56 (14.13)	108.12 (15.04)	109.6 (13.82)
	50	0.5 (0.09)	0.5 (0.09)	0.53 (0.09)	0.46 (0.05)	0.42 (0.05)	0.46 (0.05)	102.84 (17.66)	111.5 (14.54)	114.27 (14.69)	110.64 (14.41)
	100	0.5 (0.09)	0.67 (0.09)	0.5 (0.09)	0.45 (0.05)	0.39 (0.05)	0.46 (0.05)	118.84 (16.75)	136.28 (15.17)	144.32 (15.66)	136.65 (14.88)

References

John I Marden. Multivariate statistics: Old school. *University of Illinois*, 2015.