

Figure 1: *Double bootstrap* Simulation with $B = 500$, $p = 13$, $\mu_\alpha = 10$, $X_{i,t} \stackrel{iid}{\sim} \Gamma(1, 10)$, $\delta_i \sim \mathcal{N}(2\mathbf{1}_p, \sigma_\delta^2 \mathbf{I}_p)$, $\gamma_i \sim \mathcal{N}(2\mathbf{1}_p, \sigma_\gamma^2 \mathbf{I}_p)$ with $\sigma_\delta = \sigma_\gamma = 1$ and $\sigma = 1$

n	σ_α	Bias		Consistency					
		$ \hat{\alpha}_{\text{adj}}^\dagger - \mathbb{E}(\hat{\alpha}_{\text{adj}}) $	$ \hat{\alpha}_{\text{wadj}}^\dagger - \mathbb{E}(\alpha_1) $	Guess			Proposition		
				$\hat{\alpha}_{\text{adj}}$	$\hat{\alpha}_{\text{wadj}}$	$\hat{\alpha}_{\text{IVW}}$	$\hat{\alpha}_{\text{adj}}$	$\hat{\alpha}_{\text{wadj}}$	$\hat{\alpha}_{\text{IVW}}$
5	1	28.742 (2.911)	87.932 (8.292)	1 (0)	1 (0)	1 (0)	0.98 (0.02)	1 (0)	0.98 (0.02)
	5	28.74 (2.9)	87.346 (8.312)	1 (0)	1 (0)	1 (0)	0.98 (0.02)	1 (0)	0.98 (0.02)
	10	28.742 (2.913)	86.615 (8.357)	1 (0)	1 (0)	1 (0)	0.98 (0.02)	1 (0)	0.98 (0.02)
	25	29.323 (3.024)	84.986 (8.495)	1 (0)	1 (0)	1 (0)	0.98 (0.02)	1 (0)	0.98 (0.02)
	100	43.439 (4.457)	87.463 (9.793)	1 (0)	1 (0)	1 (0)	0.98 (0.02)	0.98 (0.02)	0.98 (0.02)
10	1	18.541 (2.258)	77.577 (8.909)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)
	5	18.456 (2.265)	77.457 (8.969)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)
	10	18.463 (2.277)	77.504 (9.026)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)
	25	19.307 (2.306)	77.706 (9.29)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)
	100	31.557 (3.223)	88.11 (11.057)	1 (0)	1 (0)	1 (0)	0.96 (0.028)	0.98 (0.02)	0.96 (0.028)
15	1	15.152 (1.58)	86.296 (9.39)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)
	5	15.411 (1.608)	86.145 (9.396)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)
	10	15.734 (1.66)	85.957 (9.417)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)
	25	17.168 (1.833)	85.911 (9.467)	1 (0)	1 (0)	1 (0)	1 (0)	0.98 (0.02)	1 (0)
	100	29.884 (2.977)	95.744 (9.65)	1 (0)	1 (0)	1 (0)	0.9 (0.043)	0.9 (0.043)	0.9 (0.043)
25	1	11.834 (1.157)	66.679 (8.566)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)
	5	11.708 (1.182)	66.752 (8.542)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)
	10	11.758 (1.189)	66.863 (8.517)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)
	25	12.425 (1.221)	67.218 (8.496)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)
	100	20.426 (2.156)	75.549 (8.53)	1 (0)	1 (0)	1 (0)	0.94 (0.034)	0.98 (0.02)	0.94 (0.034)

Figure 2: Simulation with $B = 200$, $p = 13$, $\mu_\alpha = 2$, $X_{i,t} \stackrel{iid}{\sim} \Gamma(1, 2)$, $\delta_i \sim \mathcal{N}(2\mathbf{1}_p, \sigma_\delta^2 \mathbf{I}_p)$, $\gamma_i \sim \mathcal{N}(\mathbf{1}_p, \sigma_\gamma^2 \mathbf{I}_p)$ with $\sigma_\delta = \sigma_\gamma = 0.5$ and $\sigma = 10$

		Bias		Consistency						
n	σ_α			Guess			Proposition			Best
		$ \hat{\alpha}_{\text{adj}}^\dagger - \text{E}(\hat{\alpha}_{\text{adj}}) $	$ \hat{\alpha}_{\text{wadj}}^\dagger - \text{E}(\alpha_1) $	$\hat{\alpha}_{\text{adj}}$	$\hat{\alpha}_{\text{wadj}}$	$\hat{\alpha}_{\text{IVW}}$	$\hat{\alpha}_{\text{adj}}$	$\hat{\alpha}_{\text{wadj}}$	$\hat{\alpha}_{\text{IVW}}$	
5	5	5.516 (0.379)	10.812 (0.786)	0.99 (0.01)	1 (0)	0.99 (0.01)	0.94 (0.024)	0.96 (0.02)	0.94 (0.024)	0.33 (0.047)
	10	6.421 (0.436)	11.804 (0.876)	0.99 (0.01)	1 (0)	0.99 (0.01)	0.92 (0.027)	0.94 (0.024)	0.92 (0.027)	0.31 (0.046)
	25	10.338 (0.763)	16.839 (1.299)	0.93 (0.026)	0.98 (0.014)	0.93 (0.026)	0.83 (0.038)	0.84 (0.037)	0.83 (0.038)	0.32 (0.047)
	50	18.744 (1.357)	28.068 (2.171)	0.83 (0.038)	0.95 (0.022)	0.83 (0.038)	0.67 (0.047)	0.73 (0.045)	0.66 (0.048)	0.42 (0.05)
	100	36.587 (2.592)	52.071 (4.143)	0.76 (0.043)	0.98 (0.014)	0.76 (0.043)	0.59 (0.049)	0.62 (0.049)	0.58 (0.05)	0.46 (0.05)
10	5	3.596 (0.237)	9.874 (0.717)	1 (0)	1 (0)	1 (0)	0.94 (0.024)	0.95 (0.022)	0.94 (0.024)	0.32 (0.047)
	10	4.099 (0.262)	9.825 (0.722)	1 (0)	1 (0)	1 (0)	0.92 (0.027)	0.94 (0.024)	0.92 (0.027)	0.25 (0.044)
	25	6.681 (0.458)	11.809 (0.902)	0.98 (0.014)	1 (0)	0.99 (0.01)	0.78 (0.042)	0.78 (0.042)	0.82 (0.039)	0.32 (0.047)
	50	12.411 (0.837)	19.12 (1.476)	0.93 (0.026)	1 (0)	0.93 (0.026)	0.59 (0.049)	0.59 (0.049)	0.58 (0.05)	0.31 (0.046)
	100	24.383 (1.67)	36.953 (2.925)	0.81 (0.039)	0.97 (0.017)	0.8 (0.04)	0.45 (0.05)	0.45 (0.05)	0.43 (0.05)	0.32 (0.047)
15	5	2.472 (0.162)	7.94 (0.604)	1 (0)	1 (0)	1 (0)	0.9 (0.03)	0.89 (0.031)	0.9 (0.03)	0.18 (0.039)
	10	2.961 (0.183)	8.9 (0.707)	1 (0)	1 (0)	1 (0)	0.88 (0.033)	0.87 (0.034)	0.87 (0.034)	0.19 (0.039)
	25	5.232 (0.358)	14.068 (1.102)	1 (0)	1 (0)	1 (0)	0.77 (0.042)	0.79 (0.041)	0.77 (0.042)	0.31 (0.046)
	50	9.64 (0.737)	25.147 (1.847)	0.85 (0.036)	1 (0)	0.85 (0.036)	0.58 (0.05)	0.64 (0.048)	0.57 (0.05)	0.4 (0.049)
	100	18.953 (1.509)	48.8 (3.413)	0.69 (0.046)	0.95 (0.022)	0.68 (0.047)	0.53 (0.05)	0.52 (0.05)	0.53 (0.05)	0.41 (0.049)
25	5	2.211 (0.183)	9.181 (0.746)	1 (0)	1 (0)	1 (0)	0.96 (0.02)	0.97 (0.017)	0.95 (0.022)	0.26 (0.044)
	10	2.628 (0.211)	9.579 (0.771)	1 (0)	1 (0)	1 (0)	0.92 (0.027)	0.93 (0.026)	0.92 (0.027)	0.28 (0.045)
	25	4.412 (0.325)	12.446 (0.978)	0.97 (0.017)	1 (0)	0.97 (0.017)	0.84 (0.037)	0.9 (0.03)	0.84 (0.037)	0.39 (0.049)
	50	7.815 (0.561)	20.087 (1.515)	0.88 (0.033)	0.99 (0.01)	0.88 (0.033)	0.65 (0.048)	0.69 (0.046)	0.65 (0.048)	0.42 (0.05)
	100	14.934 (1.059)	37.69 (2.811)	0.74 (0.044)	0.96 (0.02)	0.73 (0.045)	0.54 (0.05)	0.52 (0.05)	0.53 (0.05)	0.35 (0.048)

Figure 3: Simulation with $B = 200$, $p = 13$, $\mu_\alpha = 2$, $X_{i,t} \stackrel{iid}{\sim} \Gamma(1, 2)$, $\delta_i \sim \mathcal{N}(\mathbf{1}_p, \sigma_\delta^2 \mathbf{I}_p)$, $\gamma_i \sim \mathcal{N}(\mathbf{1}_p, \sigma_\gamma^2 \mathbf{I}_p)$ with $\sigma_\delta = \sigma_\gamma = 0.5$ and $\sigma = 10$

n	σ_α	Distance to α_1			Original	Distance to y_{1,T_1^*+1}		
		$\hat{\alpha}_{\text{adj}}$	$\hat{\alpha}_{\text{wadj}}$	$\hat{\alpha}_{\text{IVW}}$		$\hat{\alpha}_{\text{adj}}$	$\hat{\alpha}_{\text{wadj}}$	$\hat{\alpha}_{\text{IVW}}$
5	5	11.725 (0.759)	11.697 (0.77)	11.729 (0.765)	55.665 (1.901)	16.015 (1.355)	16.785 (1.41)	16.14 (1.352)
	10	12.868 (0.926)	13.684 (1.011)	13.066 (0.926)	56.708 (1.971)	17.237 (1.443)	18.393 (1.573)	17.336 (1.453)
	25	21.31 (1.64)	23.992 (1.9)	21.809 (1.627)	59.965 (2.691)	23.874 (2.086)	27.642 (2.245)	24.215 (2.083)
	50	39.875 (3.05)	44.796 (3.512)	40.643 (3.013)	69.245 (4.068)	41.799 (3.318)	47.423 (3.717)	42.346 (3.297)
	100	79.354 (6.01)	88.317 (6.814)	80.151 (5.98)	99.526 (6.566)	80.279 (6.232)	89.234 (7.065)	81.05 (6.192)
10	5	12.171 (0.986)	12.072 (1.056)	12.148 (0.986)	54.956 (1.938)	16.464 (1.263)	17.151 (1.32)	16.414 (1.28)
	10	13.293 (1.129)	14.209 (1.154)	13.16 (1.138)	54.657 (2.099)	17.508 (1.381)	18.897 (1.402)	17.374 (1.405)
	25	22.934 (1.754)	25.814 (1.858)	22.948 (1.759)	54.068 (3.048)	25.704 (1.974)	28.494 (2.101)	25.753 (1.985)
	50	43.573 (3.153)	50.779 (3.219)	43.994 (3.149)	59.051 (4.647)	44.973 (3.356)	51.829 (3.476)	45.452 (3.352)
	100	87.16 (6.186)	101.743 (6.331)	87.816 (6.249)	88.642 (7.381)	88.014 (6.334)	102.414 (6.501)	88.669 (6.401)
15	5	10.085 (0.839)	10.848 (0.804)	10.114 (0.834)	54.124 (2.011)	17.094 (1.557)	18.529 (1.566)	17.228 (1.551)
	10	12.656 (0.992)	13.691 (1.032)	12.702 (0.992)	54.56 (2.208)	18.831 (1.672)	20.791 (1.719)	19.003 (1.67)
	25	23.758 (1.762)	26.584 (1.831)	23.885 (1.769)	56.719 (3.114)	28.487 (2.183)	31.651 (2.359)	28.717 (2.197)
	50	44.848 (3.285)	49.543 (3.482)	45.018 (3.313)	66.12 (4.598)	49.316 (3.368)	53.818 (3.773)	49.558 (3.412)
	100	87.861 (6.529)	96.61 (6.888)	88.434 (6.557)	100.28 (7.164)	92.031 (6.414)	99.966 (7.08)	92.603 (6.469)
25	5	11.969 (0.833)	11.934 (0.887)	11.95 (0.839)	60.475 (2.649)	17.709 (2.08)	18.296 (2.081)	17.778 (2.091)
	10	14.268 (0.996)	14.382 (0.991)	14.279 (0.994)	60.614 (2.767)	19.266 (2.149)	19.877 (2.117)	19.365 (2.156)
	25	23.987 (1.783)	24.782 (1.67)	23.94 (1.78)	61.974 (3.376)	27.789 (2.532)	28.095 (2.484)	27.86 (2.532)
	50	42.392 (3.388)	45.205 (3.081)	42.444 (3.362)	67.704 (4.792)	45.163 (3.752)	47.252 (3.51)	45.109 (3.754)
	100	81.173 (6.679)	87.99 (6.042)	81.223 (6.638)	94.144 (7.449)	83.225 (6.777)	89.548 (6.134)	83.268 (6.747)

Figure 4: Simulation with $B = 500$, $p = 13$, $\mu_\alpha = 10$, $X_{i,t} \stackrel{iid}{\sim} \Gamma(1, 10)$, $\delta_i \sim \mathcal{N}(2\mathbf{1}_p, \sigma_\delta^2 \mathbf{I}_p)$, $\gamma_i \sim \mathcal{N}(2\mathbf{1}_p, \sigma_\gamma^2 \mathbf{I}_p)$ with $\sigma_\delta = \sigma_\gamma = 1$ and $\sigma = 1$

σ	σ_α	Bias		Consistency			
		$ \hat{\alpha}_{\text{adj}}^\dagger - \mathbb{E}(\hat{\alpha}_{\text{adj}}) $	$ \hat{\alpha}_{\text{vadj}}^\dagger - \mathbb{E}(\alpha_1) $	Guess		Proposition	
				$\hat{\alpha}_{\text{adj}}$	$\hat{\alpha}_{\text{vadj}}$	$\hat{\alpha}_{\text{adj}}$	Best
1	1	17.288 (2.073)	79.884 (8.911)	1 (0)	1 (0)	0.995 (0.01)	0.99 (0.014)
	5	17.363 (2.087)	79.975 (8.883)	1 (0)	1 (0)	0.995 (0.01)	0.995 (0.01)
	10	17.533 (2.111)	80.121 (8.857)	1 (0)	1 (0)	0.995 (0.01)	0.995 (0.01)
	25	18.439 (2.239)	80.781 (8.84)	1 (0)	1 (0)	0.995 (0.01)	0.995 (0.01)
	100	29.569 (3.56)	88.474 (10.074)	1 (0)	1 (0)	0.96 (0.028)	0.965 (0.026)
5	1	17.218 (2.069)	80.135 (8.925)	1 (0)	1 (0)	0.995 (0.01)	0.99 (0.014)
	5	17.275 (2.086)	80.215 (8.899)	1 (0)	1 (0)	0.995 (0.01)	0.99 (0.014)
	10	17.444 (2.11)	80.335 (8.878)	1 (0)	1 (0)	0.995 (0.01)	0.99 (0.014)
	25	18.281 (2.249)	81.009 (8.857)	1 (0)	1 (0)	0.995 (0.01)	0.995 (0.01)
	100	29.461 (3.566)	88.805 (10.066)	1 (0)	1 (0)	0.96 (0.028)	0.965 (0.026)
10	1	17.255 (2.074)	80.462 (8.959)	1 (0)	1 (0)	0.995 (0.01)	0.99 (0.014)
	5	17.317 (2.09)	80.541 (8.934)	1 (0)	1 (0)	0.995 (0.01)	0.99 (0.014)
	10	17.435 (2.122)	80.672 (8.91)	1 (0)	1 (0)	0.995 (0.01)	0.99 (0.014)
	25	18.23 (2.266)	81.381 (8.883)	1 (0)	1 (0)	0.995 (0.01)	0.995 (0.01)
	100	29.459 (3.569)	89.257 (10.066)	1 (0)	1 (0)	0.96 (0.028)	0.96 (0.028)
25	1	18.248 (2.137)	81.941 (9.102)	1 (0)	1 (0)	0.99 (0.014)	0.985 (0.017)
	5	18.301 (2.153)	82.028 (9.075)	1 (0)	1 (0)	0.99 (0.014)	0.985 (0.017)
	10	18.398 (2.186)	82.181 (9.047)	1 (0)	1 (0)	0.99 (0.014)	0.985 (0.017)
	25	19.125 (2.329)	82.999 (8.995)	1 (0)	1 (0)	0.985 (0.017)	0.98 (0.02)
	100	29.757 (3.646)	90.991 (10.114)	1 (0)	1 (0)	0.96 (0.028)	0.965 (0.026)
100	1	34.136 (3.372)	98.318 (10.747)	1 (0)	1 (0)	0.955 (0.029)	0.95 (0.031)
	5	34.189 (3.375)	98.279 (10.742)	1 (0)	1 (0)	0.955 (0.029)	0.955 (0.029)
	10	34.275 (3.387)	98.327 (10.731)	1 (0)	1 (0)	0.955 (0.029)	0.95 (0.031)
	25	34.901 (3.428)	98.805 (10.721)	1 (0)	1 (0)	0.95 (0.031)	0.955 (0.029)
	100	41.988 (4.349)	107.054 (11.38)	1 (0)	1 (0)	0.93 (0.036)	0.925 (0.037)

Figure 5: Simulation with $B = 500$, $p = 13$, $\mu_\alpha = 10$, $X_{i,t} \stackrel{iid}{\sim} \Gamma(1, 10)$, $\delta_i \sim \mathcal{N}(2\mathbf{1}_p, \sigma_\delta^2 \mathbf{I}_p)$, $\gamma_i \sim \mathcal{N}(2\mathbf{1}_p, \sigma_\gamma^2 \mathbf{I}_p)$ with $\sigma_\delta = \sigma_\gamma = 1$ and $\sigma = 1$

σ	σ_α	Distance to α_1			Original	Distance to y_{1,T_1^*+1}		
		$\hat{\alpha}_{\text{adj}}$	$\hat{\alpha}_{\text{wadj}}$	$\hat{\alpha}_{\text{IVW}}$		$\hat{\alpha}_{\text{adj}}$	$\hat{\alpha}_{\text{wadj}}$	$\hat{\alpha}_{\text{IVW}}$
1	1	103.994 (11.163)	95.732 (10.912)	103.914 (11.224)	539.648 (17.268)	103.874 (11.162)	95.706 (10.907)	103.829 (11.216)
	5	103.831 (11.109)	95.414 (10.879)	103.628 (11.188)	538.77 (17.218)	103.711 (11.107)	95.343 (10.881)	103.541 (11.18)
	10	103.754 (11.058)	95.276 (10.842)	103.516 (11.139)	537.673 (17.18)	103.63 (11.056)	95.21 (10.844)	103.411 (11.133)
	25	104.595 (10.98)	96.582 (10.765)	104.395 (11.038)	534.382 (17.226)	104.427 (10.984)	96.533 (10.762)	104.254 (11.036)
	100	122.764 (13.583)	125.681 (12.964)	122.515 (13.573)	517.926 (20.725)	122.689 (13.562)	125.608 (12.965)	122.361 (13.568)
5	1	104.043 (11.161)	95.943 (10.94)	104.006 (11.226)	540.334 (17.22)	103.603 (11.156)	95.912 (10.93)	103.704 (11.192)
	5	103.886 (11.106)	95.615 (10.909)	103.739 (11.186)	539.456 (17.17)	103.4 (11.105)	95.575 (10.899)	103.44 (11.149)
	10	103.784 (11.059)	95.501 (10.868)	103.573 (11.147)	538.359 (17.131)	103.293 (11.055)	95.357 (10.875)	103.203 (11.118)
	25	104.579 (10.989)	96.863 (10.78)	104.444 (11.046)	535.068 (17.175)	103.951 (10.998)	96.663 (10.792)	103.869 (11.045)
	100	122.751 (13.587)	125.824 (12.988)	122.59 (13.569)	518.612 (20.676)	122.461 (13.495)	125.455 (13.02)	122.147 (13.511)
10	1	104.119 (11.162)	96.28 (10.979)	104.146 (11.229)	541.206 (17.207)	103.711 (11.143)	96.504 (10.987)	104.01 (11.154)
	5	103.97 (11.104)	96.031 (10.934)	103.878 (11.189)	540.328 (17.156)	103.503 (11.089)	96.237 (10.943)	103.646 (11.126)
	10	103.85 (11.061)	95.939 (10.889)	103.694 (11.153)	539.231 (17.117)	103.408 (11.033)	96.082 (10.907)	103.413 (11.091)
	25	104.601 (10.998)	97.239 (10.81)	104.504 (11.062)	535.94 (17.161)	103.819 (11.009)	97.068 (10.876)	103.688 (11.079)
	100	122.786 (13.587)	126.006 (13.031)	122.71 (13.564)	519.484 (20.66)	122.321 (13.454)	125.566 (13.112)	122.124 (13.467)
25	1	104.586 (11.156)	97.877 (11.098)	104.73 (11.244)	543.483 (17.48)	106.441 (11.157)	100.014 (11.396)	106.714 (11.206)
	5	104.389 (11.107)	97.661 (11.047)	104.463 (11.203)	542.605 (17.431)	106.094 (11.123)	99.839 (11.338)	106.298 (11.183)
	10	104.234 (11.069)	97.632 (10.991)	104.248 (11.171)	541.508 (17.393)	105.872 (11.08)	99.856 (11.272)	105.997 (11.153)
	25	104.814 (11.035)	98.752 (10.939)	104.795 (11.125)	538.217 (17.438)	105.961 (11.084)	100.851 (11.23)	105.855 (11.19)
	100	122.95 (13.606)	126.643 (13.234)	123.147 (13.564)	521.761 (20.901)	122.81 (13.626)	128.209 (13.45)	123.154 (13.59)
100	1	108.954 (11.563)	114.743 (12.27)	110.075 (11.67)	553.71 (24.271)	138.265 (16.674)	148.615 (16.986)	139.104 (16.709)
	5	108.912 (11.482)	114.35 (12.261)	109.845 (11.618)	552.832 (24.238)	138.188 (16.591)	148.839 (16.887)	139.011 (16.63)
	10	108.9 (11.412)	113.987 (12.27)	109.767 (11.553)	551.735 (24.214)	138.12 (16.511)	149.178 (16.785)	138.966 (16.546)
	25	109.429 (11.362)	113.465 (12.489)	110.32 (11.473)	548.444 (24.258)	138.682 (16.326)	150.776 (16.59)	139.417 (16.378)
	100	125.836 (13.979)	135.655 (15.019)	126.9 (13.912)	532.468 (26.711)	155.292 (17.077)	172.65 (17.727)	156.747 (16.972)