

# Supplemental Materials for “Minimizing post shock forecasting error using disparate information”

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In the supplementary materials, we provide details for some procedures that are not discussed in the manuscript. Section 1 provides statistical evidence for approximate independence between the shock-effects nested in 2008 September time series in the analysis of Conoco Phillips stock. Section 2 details the algorithms of  $\mathcal{B}_f$ . Section 3 lists the tables for simulations under  $\mathcal{M}_1$ , whose results are discussed in Section 4 in the manuscript. Section 4 lists the results for the non niqueness of  $\mathbf{W}^*$  when  $2p < n$ , where  $\mathbf{W}^*$  is very likely to lie in the boundary of parameter space  $\mathcal{W}$ .

## 1 Supplementary materials for data analysis

The independence of the estimated September, 2008 shock-effects are further tested using likelihood ratio test (LRT) based on their estimated covariance matrix. The estimated covariance matrix is

$$\hat{\Sigma} = \begin{pmatrix} 4.012 & 0.362 & -0.062 \\ 0.362 & 3.894 & -0.029 \\ -0.062 & -0.029 & 3.927 \end{pmatrix}.$$

with degrees of freedoms 35. Using the LRT for independence between blocks of random variables [Marden, 2015, Section 10.2], the LRT test statistic is 0.304 with  $p$ -value of 0.581. Therefore, we do not reject the null hypothesis that the three estimated shock-effects are independent.

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## 2 Bootstrap algorithms of fixed donor pool bootstrapping $\mathcal{B}_f$

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**Algorithm 1:** Parametric bootstrap for approximation for mean and variance of shock-effect estimators of  $\alpha_1$ .

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**Input:**  $B$  – the number of parametric bootstraps

$\{(y_{i,t}, \mathbf{x}_{i,t}) : i = 2, \dots, n+1, t = 0, \dots, T_i\}$  – the data

$\{T_i^* : i = 1, \dots, n+1\}$  – the time point just before the shock

$\{\hat{\varepsilon}_{i,t} : t = 1, \dots, T_i\}$  – the collection of residuals for  $t = 1, \dots, T_i$

$\{\hat{\eta}_i, \hat{\alpha}_i, \hat{\phi}_i, \hat{\theta}_i, \hat{\beta}_i : i = 2, \dots, n+1\}$  – the OLS estimates

**Result:** The sample mean, and sample variance of bootstrapped adjustment estimator, inverse-variance weighted estimator, and weighted-adjustment estimator.

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1 for  $b = 1 : B$  do
2   for  $i = 2, \dots, n+1$  do
3     Sample with replacement from  $\{\hat{\varepsilon}_{i,t} : t = 1, \dots, T_i\}$  to obtain  $\{\hat{\varepsilon}_{i,t}^{(b)} : t = 1, \dots, T_i\}$ 
4     Define  $y_{i,0}^{(b)} = y_{i,0}$ 
5     for  $t = 1, \dots, T_i$  do
6       Compute  $y_{i,t}^{(b)} = \hat{\eta}_i + \hat{\alpha}_i 1(t = T_i^* + 1) + \hat{\phi}_i y_{i,t-1}^{(b)} + \theta_i' \mathbf{x}_{i,t} + \beta_i' \mathbf{x}_{i,t-1} + \hat{\varepsilon}_{i,t}^{(b)}$ 
7     end
8     Compute  $\hat{\alpha}_i^{(b)}$  based on OLS estimation with  $\{(y_{i,t}^{(b)}, \mathbf{x}_{i,t}) : t = 0, \dots, T_i\}$ 
9   end
10  Compute the  $b$ th shock-effect estimate  $\hat{\alpha}_{\text{est}}^{(b)}$  for  $\text{est} \in \{\text{adj}, \text{wadj}, \text{IVW}\}$ 
11 end
12 Compute the sample mean, and sample variance of  $\{\hat{\alpha}_{\text{est}}^{(b)} : b = 1, \dots, B\}$  for
    $\text{est} \in \{\text{adj}, \text{wadj}, \text{IVW}\}$ 

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## 3 Tables for $\mathcal{M}_1$

**Table 1:** 30 Monte Carlo simulations of  $\mathcal{M}_1$  for  $\mathcal{B}_u$  with varying  $n$  and  $\sigma_\alpha$

$n$	$\sigma_\alpha$	Guess			LOOCV with $k$ random draws			Distance to $y_{1,T_1^*+1}$			
		$\hat{\delta}_{\text{adj}}$	$\hat{\delta}_{\text{wadj}}$	$\hat{\delta}_{\text{IVW}}$	$\bar{C}^{(k)}(\hat{\delta}_{\text{adj}})$	$\bar{C}^{(k)}(\hat{\delta}_{\text{wadj}})$	$\bar{C}^{(k)}(\hat{\delta}_{\text{IVW}})$	Original	$\hat{\alpha}_{\text{adj}}$	$\hat{\alpha}_{\text{wadj}}$	$\hat{\alpha}_{\text{IVW}}$
5	5	1 (0)	1 (0)	1 (0)	0.95 (0.02)	0.95 (0.02)	0.95 (0.02)	51.11 (2.86)	12.36 (1.99)	14.74 (2.31)	12.19 (1.96)
	10	1 (0)	1 (0)	1 (0)	0.94 (0.02)	0.94 (0.02)	0.94 (0.02)	50.76 (3.51)	14.66 (2.47)	17.49 (2.9)	14.25 (2.47)
	25	1 (0)	1 (0)	1 (0)	0.8 (0.04)	0.83 (0.03)	0.79 (0.04)	53.08 (4.85)	24.1 (4.18)	28.57 (4.9)	23.41 (4.17)
	50	0.9 (0.06)	0.93 (0.05)	0.9 (0.06)	0.65 (0.05)	0.66 (0.04)	0.65 (0.05)	61.41 (7.7)	44.02 (6.88)	52.31 (7.88)	43.11 (6.85)
	100	0.63 (0.09)	0.7 (0.09)	0.63 (0.09)	0.47 (0.05)	0.47 (0.04)	0.47 (0.05)	91.4 (13.21)	84.69 (12.6)	100.26 (14.24)	83.46 (12.5)
10	5	1 (0)	1 (0)	1 (0)	0.99 (0.01)	0.99 (0.01)	0.99 (0.01)	49.1 (2.79)	12.33 (1.96)	12.84 (1.99)	12.54 (1.93)
	10	1 (0)	1 (0)	1 (0)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	47.42 (3.32)	14.44 (2.43)	15.77 (2.48)	14.65 (2.41)
	25	0.97 (0.03)	1 (0)	0.97 (0.03)	0.8 (0.03)	0.83 (0.03)	0.8 (0.03)	44.87 (4.71)	24.76 (3.99)	28.81 (4.05)	25.09 (3.94)
	50	0.9 (0.06)	0.93 (0.05)	0.9 (0.06)	0.65 (0.04)	0.64 (0.04)	0.65 (0.04)	49.9 (6.64)	44.8 (6.89)	52.51 (7.13)	45.06 (6.85)
	100	0.57 (0.09)	0.63 (0.09)	0.57 (0.09)	0.52 (0.05)	0.55 (0.04)	0.49 (0.05)	74.62 (11.8)	85.94 (13.05)	100.73 (13.67)	86.14 (12.99)
15	5	1 (0)	1 (0)	1 (0)	0.96 (0.01)	0.95 (0.02)	0.96 (0.01)	48.64 (2.33)	10.13 (1.41)	11.89 (1.69)	10.18 (1.38)
	10	1 (0)	1 (0)	1 (0)	0.95 (0.02)	0.93 (0.02)	0.95 (0.02)	46.93 (2.83)	12.51 (1.73)	14.36 (1.73)	12.48 (1.74)
	25	0.97 (0.03)	0.97 (0.03)	0.97 (0.03)	0.83 (0.03)	0.81 (0.03)	0.84 (0.03)	43.53 (4.48)	22.81 (3.46)	24.1 (3.18)	22.79 (3.48)
	50	0.9 (0.06)	0.97 (0.03)	0.93 (0.05)	0.63 (0.04)	0.61 (0.04)	0.61 (0.03)	50.91 (5.81)	42.61 (6.82)	44.53 (6.2)	42.71 (6.77)
	100	0.77 (0.08)	0.83 (0.07)	0.73 (0.08)	0.53 (0.04)	0.51 (0.04)	0.54 (0.04)	80.48 (10.37)	87.65 (12.82)	90.21 (11.88)	87.71 (12.71)
25	5	1 (0)	1 (0)	1 (0)	0.97 (0.01)	0.97 (0.01)	0.97 (0.01)	50.91 (2.67)	10.87 (1.57)	12.07 (1.84)	11.01 (1.58)
	10	1 (0)	1 (0)	1 (0)	0.96 (0.01)	0.95 (0.02)	0.96 (0.01)	51.43 (3.03)	12.85 (1.66)	15.01 (2.03)	13.02 (1.67)
	25	1 (0)	1 (0)	1 (0)	0.87 (0.02)	0.85 (0.03)	0.87 (0.02)	53.32 (4.9)	22.43 (2.55)	26.95 (3.06)	22.65 (2.57)
	50	0.87 (0.06)	0.9 (0.06)	0.87 (0.06)	0.66 (0.03)	0.67 (0.04)	0.65 (0.03)	64.22 (7)	41.26 (4.83)	48.43 (5.6)	41.39 (4.87)
	100	0.73 (0.08)	0.73 (0.08)	0.73 (0.08)	0.55 (0.05)	0.51 (0.05)	0.55 (0.05)	95.79 (11.73)	80.67 (9.79)	92.78 (11.05)	80.64 (9.86)

**Table 2:** 30 Monte Carlo simulations of  $\mathcal{M}_1$  for  $\mathcal{B}_u$  with varying  $\sigma$  and  $\sigma_\alpha$

$\sigma$	$\sigma_\alpha$	Guess			LOOCV with $k$ random draws			Distance to $y_{1:T_1^*+1}$			
		$\hat{\delta}_{\hat{\alpha}_{\text{adj}}}$	$\hat{\delta}_{\hat{\alpha}_{\text{wadj}}}$	$\hat{\delta}_{\hat{\alpha}_{\text{IVW}}}$	$\tilde{\mathcal{C}}^{(k)}(\hat{\delta}_{\hat{\alpha}_{\text{adj}}})$	$\tilde{\mathcal{C}}^{(k)}(\hat{\delta}_{\hat{\alpha}_{\text{wadj}}})$	$\tilde{\mathcal{C}}^{(k)}(\hat{\delta}_{\hat{\alpha}_{\text{IVW}}})$	Original	$\hat{\alpha}_{\text{adj}}$	$\hat{\alpha}_{\text{wadj}}$	$\hat{\alpha}_{\text{IVW}}$
5	5	1 (0)	1 (0)	1 (0)	0.99 (0.01)	0.99 (0.01)	0.99 (0.01)	48.77 (1.65)	7.14 (1.22)	7.77 (1.25)	7.26 (1.21)
	10	1 (0)	1 (0)	1 (0)	0.97 (0.01)	0.97 (0.01)	0.97 (0.01)	47.1 (2.32)	10.4 (1.72)	12.1 (1.73)	10.51 (1.72)
	25	1 (0)	1 (0)	1 (0)	0.81 (0.03)	0.84 (0.03)	0.81 (0.03)	44.03 (4.05)	22.38 (3.42)	26.23 (3.54)	22.5 (3.4)
	50	0.9 (0.06)	0.97 (0.03)	0.9 (0.06)	0.64 (0.04)	0.63 (0.04)	0.63 (0.04)	47.72 (6.35)	42.94 (6.5)	50.34 (6.81)	43.03 (6.47)
	100	0.57 (0.09)	0.63 (0.09)	0.57 (0.09)	0.5 (0.05)	0.51 (0.04)	0.5 (0.04)	72.88 (11.48)	84.74 (12.69)	98.56 (13.51)	84.85 (12.62)
10	5	1 (0)	1 (0)	1 (0)	0.99 (0.01)	0.99 (0.01)	0.99 (0.01)	49.1 (2.79)	12.33 (1.96)	12.84 (1.99)	12.54 (1.93)
	10	1 (0)	1 (0)	1 (0)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	47.42 (3.32)	14.44 (2.43)	15.77 (2.48)	14.65 (2.41)
	25	0.97 (0.03)	1 (0)	0.97 (0.03)	0.8 (0.03)	0.83 (0.03)	0.8 (0.03)	44.87 (4.71)	24.76 (3.99)	28.81 (4.05)	25.09 (3.94)
	50	0.9 (0.06)	0.93 (0.05)	0.9 (0.06)	0.65 (0.04)	0.64 (0.04)	0.65 (0.04)	49.9 (6.64)	44.8 (6.89)	52.51 (7.13)	45.06 (6.85)
	100	0.57 (0.09)	0.63 (0.09)	0.57 (0.09)	0.52 (0.05)	0.55 (0.04)	0.49 (0.05)	74.62 (11.8)	85.94 (13.05)	100.73 (13.67)	86.14 (12.99)
25	5	1 (0)	1 (0)	1 (0)	0.82 (0.03)	0.82 (0.03)	0.83 (0.03)	51.82 (5.58)	28.83 (4.36)	29.08 (4.47)	29.2 (4.29)
	10	1 (0)	1 (0)	1 (0)	0.8 (0.03)	0.8 (0.03)	0.78 (0.04)	51.37 (5.7)	29.93 (4.79)	31.03 (4.85)	30.44 (4.71)
	25	0.93 (0.05)	0.97 (0.03)	0.93 (0.05)	0.75 (0.04)	0.74 (0.04)	0.75 (0.04)	51.77 (6.36)	37.51 (5.98)	40.78 (6.1)	37.99 (5.92)
	50	0.8 (0.07)	0.83 (0.07)	0.8 (0.07)	0.6 (0.05)	0.59 (0.04)	0.59 (0.05)	57.39 (8.25)	53.52 (8.71)	62.34 (8.6)	54.02 (8.66)
	100	0.53 (0.09)	0.57 (0.09)	0.57 (0.09)	0.49 (0.05)	0.53 (0.04)	0.5 (0.05)	80.22 (13.49)	92.77 (14.41)	108.3 (14.81)	93.51 (14.29)
50	5	0.67 (0.09)	0.67 (0.09)	0.67 (0.09)	0.55 (0.05)	0.51 (0.04)	0.55 (0.05)	67.29 (9.18)	57.37 (8.43)	57.45 (8.66)	58.08 (8.26)
	10	0.67 (0.09)	0.67 (0.09)	0.67 (0.09)	0.53 (0.05)	0.53 (0.04)	0.53 (0.05)	67.72 (9.24)	58.18 (8.8)	58.99 (8.96)	58.92 (8.65)
	25	0.73 (0.08)	0.73 (0.08)	0.7 (0.09)	0.53 (0.05)	0.55 (0.05)	0.53 (0.05)	69.36 (9.85)	62.1 (10.11)	65.26 (10.16)	63.15 (9.93)
	50	0.63 (0.09)	0.7 (0.09)	0.63 (0.09)	0.53 (0.05)	0.5 (0.04)	0.53 (0.05)	75.13 (11.58)	76.15 (11.97)	82.62 (12.17)	77.1 (11.85)
	100	0.53 (0.09)	0.6 (0.09)	0.53 (0.09)	0.52 (0.05)	0.55 (0.04)	0.5 (0.05)	98.34 (15.85)	108.03 (17.49)	125.76 (17.14)	108.99 (17.39)
100	5	0.37 (0.09)	0.33 (0.09)	0.4 (0.09)	0.42 (0.04)	0.49 (0.04)	0.41 (0.04)	114.58 (15.95)	114.76 (16.6)	114.79 (17.05)	115.96 (16.28)
	10	0.37 (0.09)	0.33 (0.09)	0.37 (0.09)	0.42 (0.04)	0.46 (0.04)	0.42 (0.04)	115.61 (16)	115.49 (16.93)	115.95 (17.35)	116.81 (16.6)
	25	0.4 (0.09)	0.37 (0.09)	0.37 (0.09)	0.39 (0.04)	0.46 (0.04)	0.39 (0.05)	118.76 (16.45)	117.96 (18.14)	120.56 (18.37)	119.47 (17.82)
	50	0.47 (0.09)	0.37 (0.09)	0.43 (0.09)	0.46 (0.05)	0.51 (0.05)	0.45 (0.05)	124.94 (17.83)	125.31 (20.27)	131.64 (20.36)	127.15 (19.96)
	100	0.4 (0.09)	0.43 (0.09)	0.4 (0.09)	0.48 (0.05)	0.53 (0.03)	0.49 (0.05)	146.49 (20.97)	153.41 (24.01)	166.43 (24.35)	155.32 (23.75)

**Table 3:** 30 Monte Carlo simulations of  $\mathcal{M}_1$  for  $\mathcal{B}_f$  with varying  $\sigma$  and  $\sigma_\alpha$

$\sigma$	$\sigma_\alpha$	Guess			LOOCV with $k$ random draws			Distance to $y_{1:T_1^*+1}$			
		$\hat{\delta}_{\hat{\alpha}_{\text{adj}}}$	$\hat{\delta}_{\hat{\alpha}_{\text{wadj}}}$	$\hat{\delta}_{\hat{\alpha}_{\text{IVW}}}$	$\tilde{\mathcal{C}}^{(k)}(\hat{\delta}_{\hat{\alpha}_{\text{adj}}})$	$\tilde{\mathcal{C}}^{(k)}(\hat{\delta}_{\hat{\alpha}_{\text{wadj}}})$	$\tilde{\mathcal{C}}^{(k)}(\hat{\delta}_{\hat{\alpha}_{\text{IVW}}})$	Original	$\hat{\alpha}_{\text{adj}}$	$\hat{\alpha}_{\text{wadj}}$	$\hat{\alpha}_{\text{IVW}}$
5	5	1 (0)	1 (0)	1 (0)	0.94 (0.02)	0.94 (0.02)	0.94 (0.02)	50.11 (2.43)	11.11 (1.85)	13.2 (1.91)	10.97 (1.85)
	10	1 (0)	1 (0)	1 (0)	0.93 (0.02)	0.93 (0.02)	0.93 (0.02)	50.03 (2.81)	13.68 (1.86)	15.89 (1.88)	13.62 (1.86)
	25	1 (0)	1 (0)	1 (0)	0.78 (0.03)	0.77 (0.03)	0.79 (0.03)	49.82 (4.54)	21.75 (2.6)	24.99 (2.53)	21.98 (2.61)
	50	0.93 (0.05)	1 (0)	0.93 (0.05)	0.61 (0.05)	0.61 (0.04)	0.61 (0.05)	57.05 (6.07)	36.31 (4.58)	40.79 (4.79)	36.97 (4.62)
	100	0.73 (0.08)	0.97 (0.03)	0.67 (0.09)	0.45 (0.05)	0.49 (0.05)	0.45 (0.05)	88.65 (7.05)	65.85 (9.15)	76.22 (9.13)	67.41 (9.28)
10	5	1 (0)	1 (0)	1 (0)	0.96 (0.02)	0.96 (0.02)	0.96 (0.02)	50.21 (2.49)	10.87 (1.57)	10.89 (1.62)	10.85 (1.53)
	10	1 (0)	1 (0)	1 (0)	0.94 (0.02)	0.94 (0.02)	0.94 (0.02)	49.02 (2.76)	13.62 (1.6)	14.02 (1.74)	13.67 (1.56)
	25	0.93 (0.05)	1 (0)	0.93 (0.05)	0.75 (0.05)	0.81 (0.03)	0.76 (0.05)	45.91 (4.58)	24.55 (3.3)	24.27 (4.09)	24.43 (3.27)
	50	0.77 (0.08)	0.97 (0.03)	0.77 (0.08)	0.61 (0.05)	0.66 (0.04)	0.62 (0.05)	49.9 (7.12)	44.63 (7.23)	44.98 (8.51)	43.98 (7.2)
	100	0.67 (0.09)	0.87 (0.06)	0.63 (0.09)	0.53 (0.05)	0.48 (0.04)	0.54 (0.05)	74.74 (13.07)	89.35 (14.73)	89.56 (17.32)	87.87 (14.63)
15	5	1 (0)	1 (0)	1 (0)	0.94 (0.02)	0.94 (0.02)	0.94 (0.02)	53.03 (2.15)	9.39 (1.51)	11.1 (1.66)	9.4 (1.47)
	10	1 (0)	1 (0)	1 (0)	0.91 (0.03)	0.91 (0.03)	0.91 (0.03)	52.46 (2.66)	11.65 (1.87)	14 (2.06)	11.79 (1.82)
	25	1 (0)	1 (0)	1 (0)	0.77 (0.04)	0.76 (0.03)	0.76 (0.04)	52.76 (4.47)	23.88 (3.47)	26.97 (3.82)	23.95 (3.47)
	50	0.9 (0.06)	1 (0)	0.9 (0.06)	0.59 (0.04)	0.59 (0.04)	0.59 (0.04)	59.8 (7.66)	46.85 (6.7)	52.97 (6.8)	47.12 (6.69)
	100	0.8 (0.07)	0.9 (0.06)	0.8 (0.07)	0.47 (0.04)	0.47 (0.03)	0.47 (0.04)	95.95 (12.52)	96.35 (12.82)	107.42 (12.67)	97 (12.77)
25	5	1 (0)	1 (0)	1 (0)	0.92 (0.02)	0.93 (0.02)	0.92 (0.02)	55.1 (2.73)	11.66 (1.93)	13.34 (2.08)	11.77 (1.92)
	10	1 (0)	1 (0)	1 (0)	0.89 (0.02)	0.91 (0.02)	0.89 (0.02)	55 (3)	13.63 (1.93)	14.94 (2.04)	13.67 (1.93)
	25	1 (0)	1 (0)	1 (0)	0.79 (0.04)	0.77 (0.04)	0.79 (0.04)	55.02 (4.34)	21.29 (2.6)	22.14 (2.77)	21.17 (2.64)
	50	0.87 (0.06)	1 (0)	0.87 (0.06)	0.59 (0.04)	0.63 (0.04)	0.61 (0.05)	58 (6.77)	35.88 (4.69)	38.22 (4.85)	36.02 (4.69)
	100	0.7 (0.09)	0.97 (0.03)	0.7 (0.09)	0.49 (0.04)	0.51 (0.04)	0.47 (0.04)	71.18 (12.1)	68.53 (9.01)	74.18 (9.38)	69.17 (8.94)

**Table 4:** 30 Monte Carlo simulations of  $\mathcal{M}_1$  for  $\mathcal{B}_f$  with varying  $\sigma$  and  $\sigma_\alpha$

$\sigma$	$\sigma_\alpha$	Guess			LOOCV with $k$ random draws			Distance to $y_{1,T_1^*+1}$			
		$\hat{\delta}_{\text{adj}}$	$\hat{\delta}_{\text{wadj}}$	$\hat{\delta}_{\text{IVW}}$	$\bar{\mathcal{C}}^{(k)}(\hat{\delta}_{\text{adj}})$	$\bar{\mathcal{C}}^{(k)}(\hat{\delta}_{\text{wadj}})$	$\bar{\mathcal{C}}^{(k)}(\hat{\delta}_{\text{IVW}})$	Original	$\hat{\alpha}_{\text{adj}}$	$\hat{\alpha}_{\text{wadj}}$	$\hat{\alpha}_{\text{IVW}}$
5	5	1 (0)	1 (0)	1 (0)	0.99 (0.01)	0.99 (0.01)	0.99 (0.01)	49.62 (1.34)	6.69 (0.77)	6.83 (0.86)	6.72 (0.74)
	10	1 (0)	1 (0)	1 (0)	0.98 (0.01)	0.97 (0.02)	0.98 (0.01)	48.43 (1.96)	10.21 (1.29)	10.17 (1.61)	10.19 (1.28)
	25	0.97 (0.03)	1 (0)	0.97 (0.03)	0.79 (0.03)	0.83 (0.02)	0.79 (0.03)	45.46 (4.33)	22.21 (3.62)	22.48 (4.25)	21.88 (3.6)
	100	0.73 (0.08)	0.97 (0.03)	0.73 (0.08)	0.62 (0.05)	0.67 (0.04)	0.63 (0.05)	48.82 (7.36)	44.59 (7.37)	44.76 (8.66)	43.85 (7.32)
10	5	0.67 (0.09)	0.93 (0.05)	0.63 (0.09)	0.52 (0.05)	0.49 (0.05)	0.53 (0.05)	74.59 (13.36)	89.83 (14.92)	90.85 (17.33)	88.25 (14.81)
	10	1 (0)	1 (0)	1 (0)	0.96 (0.02)	0.96 (0.02)	0.96 (0.02)	50.21 (2.49)	10.87 (1.57)	10.89 (1.62)	10.85 (1.53)
	25	0.93 (0.05)	1 (0)	0.93 (0.05)	0.94 (0.02)	0.94 (0.02)	0.94 (0.02)	49.02 (2.76)	13.62 (1.6)	14.02 (1.74)	13.67 (1.56)
	100	0.77 (0.08)	0.97 (0.03)	0.77 (0.08)	0.75 (0.05)	0.81 (0.03)	0.76 (0.05)	45.91 (4.58)	24.55 (3.3)	24.27 (4.09)	24.43 (3.27)
25	5	0.67 (0.09)	0.87 (0.06)	0.63 (0.09)	0.61 (0.05)	0.66 (0.04)	0.62 (0.05)	49.9 (7.12)	44.63 (7.23)	44.98 (8.51)	43.98 (7.2)
	10	0.93 (0.05)	0.97 (0.03)	0.97 (0.03)	0.73 (0.04)	0.75 (0.03)	0.74 (0.04)	53.85 (5.67)	27.64 (3.65)	26.5 (3.93)	27.35 (3.57)
	25	0.87 (0.06)	0.97 (0.03)	0.9 (0.06)	0.73 (0.04)	0.75 (0.03)	0.72 (0.04)	52.88 (5.55)	27.72 (3.65)	27.22 (3.87)	27.55 (3.58)
	100	0.7 (0.09)	0.77 (0.08)	0.7 (0.09)	0.65 (0.05)	0.69 (0.04)	0.63 (0.05)	51.06 (5.81)	34.43 (3.84)	35.67 (4.15)	34.48 (3.76)
50	5	0.63 (0.09)	0.87 (0.06)	0.6 (0.09)	0.54 (0.05)	0.59 (0.05)	0.55 (0.05)	55.05 (7.13)	52.38 (6.22)	52.41 (7.77)	52.1 (6.2)
	10	0.73 (0.08)	0.77 (0.08)	0.8 (0.07)	0.51 (0.05)	0.49 (0.05)	0.48 (0.05)	75.13 (12.85)	91.17 (13.96)	90.79 (16.82)	90.39 (13.83)
	25	0.7 (0.09)	0.7 (0.09)	0.7 (0.09)	0.49 (0.05)	0.49 (0.04)	0.49 (0.04)	66.08 (10.43)	55.58 (7.37)	54.79 (7.57)	54.65 (7.27)
	100	0.63 (0.09)	0.63 (0.09)	0.63 (0.09)	0.49 (0.05)	0.54 (0.04)	0.48 (0.05)	64.11 (10.44)	55.29 (7.19)	53.48 (7.69)	54.85 (7)
100	5	0.57 (0.09)	0.8 (0.07)	0.6 (0.09)	0.48 (0.05)	0.47 (0.05)	0.49 (0.05)	62.24 (10.1)	55.85 (7.31)	56.56 (7.52)	55.44 (7.24)
	10	0.53 (0.09)	0.43 (0.09)	0.53 (0.09)	0.49 (0.05)	0.54 (0.04)	0.48 (0.05)	65.53 (10.01)	68.53 (7.56)	71.85 (7.98)	68.61 (7.44)
	25	0.5 (0.09)	0.47 (0.09)	0.5 (0.09)	0.46 (0.05)	0.53 (0.05)	0.51 (0.05)	85.95 (12.65)	104.55 (12.28)	105.56 (15.21)	103.99 (12.25)
	100	0.5 (0.09)	0.5 (0.09)	0.53 (0.09)	0.37 (0.04)	0.42 (0.05)	0.37 (0.04)	112.06 (17.96)	111.81 (14.87)	111.78 (14.92)	109.72 (14.68)
100	5	0.5 (0.09)	0.47 (0.09)	0.53 (0.09)	0.38 (0.04)	0.43 (0.05)	0.38 (0.04)	110.61 (17.81)	110.68 (14.77)	110.47 (14.9)	109 (14.52)
	10	0.53 (0.09)	0.47 (0.09)	0.5 (0.09)	0.39 (0.05)	0.46 (0.05)	0.39 (0.05)	106.24 (17.67)	110.56 (14.13)	108.12 (15.04)	109.6 (13.82)
	25	0.5 (0.09)	0.5 (0.09)	0.53 (0.09)	0.46 (0.05)	0.42 (0.05)	0.46 (0.05)	102.84 (17.66)	111.5 (14.54)	114.27 (14.69)	110.64 (14.41)
	100	0.5 (0.09)	0.67 (0.09)	0.5 (0.09)	0.45 (0.05)	0.39 (0.05)	0.46 (0.05)	118.84 (16.75)	136.28 (15.17)	144.32 (15.66)	136.65 (14.88)

## 4 Boundary Case ( $2p < n$ )

**Table 5:** 30 Monte Carlo simulations of  $\mathcal{M}_2$  for  $\mathcal{B}_u$  with varying  $n$  and  $\sigma_\alpha$  ( $p = 2$ , boundary case)

$n$	$\sigma_\alpha$	Guess			LOOCV with $k$ random draws			Distance to $y_{1,T_1^*+1}$			
		$\hat{\delta}_{\text{adj}}$	$\hat{\delta}_{\text{wadj}}$	$\hat{\delta}_{\text{IVW}}$	$\bar{\mathcal{C}}^{(k)}(\hat{\delta}_{\text{adj}})$	$\bar{\mathcal{C}}^{(k)}(\hat{\delta}_{\text{wadj}})$	$\bar{\mathcal{C}}^{(k)}(\hat{\delta}_{\text{IVW}})$	Original	$\hat{\alpha}_{\text{adj}}$	$\hat{\alpha}_{\text{wadj}}$	$\hat{\alpha}_{\text{IVW}}$
5	1	1 (0)	1 (0)	1 (0)	0.99 (0.01)	0.99 (0.01)	0.99 (0.01)	59.12 (2.5)	10.73 (1.34)	13.54 (1.81)	10.59 (1.35)
	5	1 (0)	1 (0)	1 (0)	0.99 (0.01)	0.99 (0.01)	0.99 (0.01)	59.57 (2.63)	11.24 (1.61)	15.42 (2.06)	11.19 (1.61)
	10	0.97 (0.03)	1 (0)	0.97 (0.03)	0.91 (0.03)	0.93 (0.02)	0.91 (0.02)	60.13 (3.08)	13.77 (2.03)	18.82 (2.68)	13.85 (2.01)
	25	0.83 (0.07)	0.9 (0.06)	0.83 (0.07)	0.72 (0.04)	0.75 (0.04)	0.71 (0.04)	62.49 (5.1)	26.53 (3.52)	34.4 (4.71)	26.62 (3.5)
10	50	0.67 (0.09)	0.7 (0.09)	0.7 (0.09)	0.49 (0.06)	0.53 (0.05)	0.5 (0.06)	72.46 (7.97)	50.17 (6.54)	63.61 (8.47)	50.25 (6.55)
	1	1 (0)	1 (0)	1 (0)	0.99 (0.01)	0.99 (0.01)	0.99 (0.01)	61.13 (2.17)	9.58 (1.52)	12.21 (1.89)	9.58 (1.56)
	5	1 (0)	1 (0)	1 (0)	0.99 (0.01)	0.99 (0.01)	0.99 (0.01)	61.3 (2.08)	9.18 (1.44)	11.89 (1.68)	9.25 (1.47)
	10	1 (0)	1 (0)	1 (0)	0.97 (0.01)	0.98 (0.01)	0.97 (0.01)	61.52 (2.28)	10.97 (1.39)	13.32 (1.63)	11.05 (1.41)
25	25	0.97 (0.03)	0.93 (0.05)	0.97 (0.03)	0.77 (0.04)	0.77 (0.04)	0.78 (0.04)	62.17 (4.11)	20.07 (2.64)	21.84 (3.38)	20.17 (2.61)
	50	0.73 (0.08)	0.8 (0.07)	0.77 (0.08)	0.56 (0.04)	0.61 (0.03)	0.55 (0.04)	63.99 (7.96)	38.79 (5.64)	44.19 (6.71)	38.87 (5.57)
50	1	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)	1 (0)	56.41 (1.85)	9.71 (1.24)	9.15 (0.99)	9.77 (1.26)
	5	1 (0)	1 (0)	1 (0)	1 (0)	0.99 (0.01)	1 (0)	56.41 (2.35)	11.91 (1.43)	10.65 (1.32)	11.96 (1.46)
	10	1 (0)	1 (0)	1 (0)	0.98 (0.01)	0.97 (0.01)	0.98 (0.01)	56.42 (3.08)	14.71 (1.85)	14.3 (1.74)	14.77 (1.88)
	25	0.87 (0.06)	0.97 (0.03)	0.87 (0.06)	0.82 (0.04)	0.77 (0.04)	0.83 (0.04)	57.92 (4.93)	24.04 (3.34)	27.4 (3.46)	24.2 (3.36)
100	50	0.7 (0.09)	0.77 (0.08)	0.7 (0.09)	0.63 (0.04)	0.61 (0.04)	0.62 (0.04)	68.27 (6.57)	42.03 (5.63)	50.98 (6.57)	42.24 (5.68)
	1	1 (0)	1 (0)	1 (0)	0.99 (0.01)	0.99 (0.01)	0.99 (0.01)	57.87 (2.42)	10.38 (1.54)	14.29 (1.36)	10.35 (1.55)
	5	1 (0)	1 (0)	1 (0)	0.98 (0.01)	0.98 (0.01)	0.98 (0.01)	58.2 (2.44)	10.55 (1.6)	14.05 (1.62)	10.51 (1.61)
	10	1 (0)	1 (0)	1 (0)	0.95 (0.02)	0.94 (0.02)	0.95 (0.02)	58.6 (2.71)	11.43 (1.92)	15.05 (2.08)	11.41 (1.93)
25	25	0.97 (0.03)	0.93 (0.05)	0.93 (0.05)	0.77 (0.04)	0.79 (0.03)	0.77 (0.04)	59.81 (4.47)	19.09 (3.21)	24.74 (3.61)	19.05 (3.21)
	50	0.8 (0.07)	0.83 (0.07)	0.8 (0.07)	0.61 (0.04)	0.65 (0.04)	0.61 (0.04)	64.73 (7.53)	35 (6.02)	44.55 (6.94)	34.89 (6.02)

## References

John I Marden. Multivariate statistics: Old school. *University of Illinois*, 2015.