Optimal Power Flow problem, DC and Decoupled OPF

Optimal AC Power Flow

- Economic dispatch with ALL AC network constraints accounted for…

Minimize $f(x)=P \cdot c_2 \cdot P^T+c_1 \cdot P+c_0$ (o)

s.t. $P_{k,g,min} \leq P_{k,g,} \leq P_{k,g,max}$ (I) $Q_{k,g,min} \leq Q_{k,g,} \leq Q_{k,g,max}$ (II)

$P_{km,min} \leq P_{km} \leq P_{km,max}$ (III) $Q_{km,min} \leq Q_{km} \leq Q_{km,max}$ (IV)

$$P_{km} = V_k^2(g_{skm} + g_{km}) - V_k V_m[g_{km}\cos(\delta_k - \delta_m) + b_{km}\sin(\delta_k - \delta_m)]$$
$$Q_{km} = -V_k^2(b_{skm} + b_{km}) - V_k V_m[g_{km}\sin(\delta_k - \delta_m) - b_{km}\cos(\delta_k - \delta_m)] \quad (V)$$

$$\textstyle\sum_k P_{km} = P_k = P_{k,g} - P_{k,d} \quad (VI)$$

$$\textstyle\sum_k Q_{km} = Q_k = Q_{k,g} - Q_{k,d} \quad (VII)$$

$$V_{k,min} \leq V_k \leq V_{k,max} \quad (VIII)$$

Linearized/DC Optimal Power Flow

- It is NOT for DC grids (e.g. more recent HVDC systems – see next slide); it is described as such because it only considers active power

- Linearized economic dispatch over voltage angles…

Minimize $f(x)=c_1 \cdot P+c_0$ (o)

s.t. (I), (III), (VI)

$$P_{km} = -b_{km}(\delta_k - \delta_m) \quad (IX)$$

Linear approximation to Optimal Power Flow for HVDC grids

- Linearized economic dispatch over voltage magnitudes…

Minimize $f(x)=c_1 \cdot P+c_0$ *(o)*

s.t. *(I), (III), (VI), (VIII)*

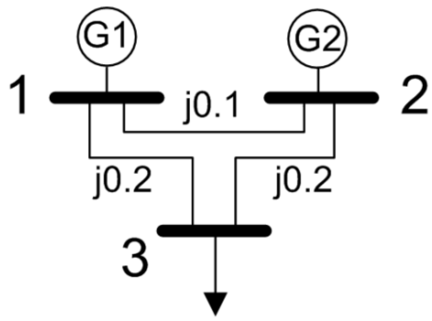
$$P_{km} = -g_{km}(V_k - V_m) \quad (X)$$

Given data of the problem

- P_{G1}: P_n= 0-200MW, Q_n= ±100MVar, f(P)=0.5·P
- P_{G2}: P_n= 0-100MW, Q_n= ±100MVar, f(P)=2·P

How should they be scheduled for a load of P_{L3}= 250MW & Q_{L3}= 120MVar

Reactances in network given in S_b=200MVA



Setting up the DC OPF Problem

$X = [P_1, P_2, P_3, d_1, d_2, d_3, p_{12}, p_{13}, p_{23}, p_{21}, p_{31}, p_{32}]$

$p_{12}=+10(d_1 - d_2)$, $p_{21}=+10(-d_1 + d_2)$

$P_1 = p_{12} + p_{13,}$ $P_2 = p_{23} + p_{21,}$ $P_3 = p_{32} + p_{31,}$

$0 < P_1 < 1$, $-1.25 < P_3 < -1.25$, $0 < P_2 < 0.5$

$p_{13}=+5(d_1 - d_3)$, $p_{31}=+5(-d_1 + d_3)$

$p_{23}=+5(d_2 - d_3)$, $p_{32}=+5(-d_2 + d_3)$

$0 < d_1 < 0$, $-pi/2 < d_2 < +pi/2$, $-pi/2 < d_3 < +pi/2$

- Indicatively 2 of the eq. constraints in the A_{eq} matrix

$p_{12}=+10(d_1 - d_2)$ & $P_3 = p_{32} + p_{31}$

0 0 0 -10 +10 0 1 0 0 0 0 0;

0 0 1 0 0 0 0 0 0 0 -1 -1;

DC OPF code

```
% for variables x=[Pg1,Pg2,Pg3,d1,d2,d3,p12,p13,p23,p21,p31,p32]
f=[0.5,2,0,0,0,0,0,0,0,0,0,0];


Aeq=zeros(9,12); % 12 variables and 9 equality constraints
Aeq(1,4)=-10; Aeq(1,5)=10; Aeq(1,7)=1; Aeq(2,4)=-5; Aeq(2,6)=5;
Aeq(2,8)=1; Aeq(3,5)=-5; Aeq(3,6)=5; Aeq(3,9)=1; Aeq(4,4)=10;
Aeq(4,5)=-10; Aeq(4,10)=1; Aeq(5,4)=5; Aeq(5,6)=-5; Aeq(5,11)=1;
Aeq(6,5)=5; Aeq(6,6)=-5; Aeq(6,12)=1; Aeq(7,1)=1; Aeq(7,7)=-1;
Aeq(7,8)=-1; Aeq(8,2)=1; Aeq(8,9)=-1; Aeq(8,10)=-1; Aeq(9,3)=1;
Aeq(9,11)=-1; Aeq(9,12)=-1;
beq=zeros(1,9);

% for the following sequency of constraints: line flows p12, p13, p23, p21, p31,
p32 followed by
% the constraints of summations of line flows to Pg1-Pd1, Pg2-Pd2, Pg3-Pd3

lb=[0,0,-1.25,0,(-pi/2),(-pi/2),-1000,-1000,-1000,-1000,-1000,-1000];
```

```
ub=[1,0.5,-1.25,0,(pi/2),(pi/2),1000,1000,1000,1000,1000,1000];

% where -1.25 is the load at bus 3, i.e. Pd3
% lower bounds for generators physically those of zero (could be greater due to technical minima),
% lower and upper bounds of voltage angles equal to desynchronization angles of generators,
% line flows allowed to be infinitely large
% there is no Pg3 so by bounding it up and down to zero, it is set as such,
% similarly setting 0 reference angle for one reference bus, in this case bus 1.

x1=linprog(f,[],[],Aeq,beq,lb,ub)
```

Output

```
x =

        1
     0.25
    -1.25
        0
    -0.03
    -0.14
      0.3
      0.7
     0.55
     -0.3
     -0.7
    -0.55
```

$$\left.\begin{matrix} 1 \\ 0.25 \\ -1.25 \end{matrix}\right\} \text{Active power dispatch of generators}$$

$$\left.\begin{matrix} 0 \\ -0.03 \\ -0.14 \end{matrix}\right\} \text{Voltage angles in radians}$$

$$\left.\begin{matrix} 0.3 \\ 0.7 \\ 0.55 \\ -0.3 \\ -0.7 \\ -0.55 \end{matrix}\right\} \text{Active power flows}$$

Optimal Power Flow (Decoupled)

- Linearized economic dispatch over voltage angles & magnitudes...

Minimize $f(x)=c_1 \cdot P+c_0$ *(o)*

s.t. *(I)-(IV), (VI)-(IX)*

$$Q_{km} = -b_{km}(V_k - V_m) \quad (XI)$$

Vector of variables of the decoupled OPF Problem

$$X = [P_1, P_2, P_3, d_1, d_2, d_3, p_{12}, p_{13}, p_{23}, p_{21}, p_{31}, p_{32},$$
$$Q_1, Q_2, Q_3, V_1, V_2, V_3, q_{12}, q_{13}, q_{23}, q_{21}, q_{31}, q_{32}]$$

$$p_{12}=+10(d_1 - d_2) \;\&\; P_3 = p_{32} + p_{31}$$
$$q_{12}=+10(V_1 - V_2) \;\&\; Q_3 = q_{32} + q_{31}$$

Extending code for the Decoupled OPF

```
f=[f,zeros(1,length(f))];
% for variables x=[x,Qg1,Qg2,Qg3,V1,V2,V3,q12,q13,q23,q21,q31,q32]
% assumed that reactive power flow is of zero cost

Aeq=blkdiag(Aeq,Aeq); beq=[beq,beq];
% the decoupled OPF has the exact same description as the DC OPF,
% but for the voltage magnitudes instead of the voltage angles


lb=[lb,-0.5,-0.5,-0.6,1,0.9,0.9,-1000,-1000,-1000,-1000,-1000,-1000];
ub=[ub,0.5,0.5,-0.6,1,1.1,1.1,1000,1000,1000,1000,1000,1000];
% where -0.6 is the reactive power load at bus 3, i.e. Qd3
% slack bus 1 bounded to 1.0 pu voltage magnitude and
% load bus 3 has zero reactive power capacity/generation

x2=linprog(f,[],[],Aeq,beq,lb,ub);
```

Output

x =

1	
0.25	Active power dispatch of generators
-1.25	
0	
-0.03	Voltage angles in radians
-0.14	
0.3	
0.7	
0.55	Active power flows
-0.3	
-0.7	
-0.55	
0.1	
0.5	Reactive power dispatch of generators
-0.6	
1	
1.016	Voltage magnitude in p.u.
0.948	
-0.16	
0.26	
0.34	Reactive power flows
0.16	
-0.26	
-0.34	