Security Constrained Optimal Power Flow & Grid Expansion Planning

Transmission Expansion Planning (TEP)

Minimize $f(x)=P \cdot c_2 \cdot P^T + c_1 \cdot P + c_0$ *(o)*

s.t.

$P_{k,g,min} \leq P_{k,g,} \leq P_{k,g,max}$ *(I)* $Q_{k,g,min} \leq Q_{k,g,} \leq Q_{k,g,max}$ *(II)*

$P_{km,min} \leq P_{km} \leq P_{km,max}$ *(III)* $Q_{km,min} \leq Q_{km} \leq Q_{km,max}$ *(IV)*

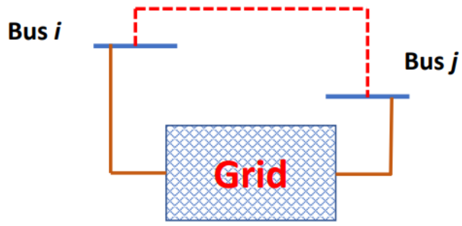
$$P_{km} = V_k^2 (g_{skm} + g_{km}) - V_k V_m [g_{km} \cos(\delta_k - \delta_m) + b_{km} \sin(\delta_k - \delta_m)]$$
$$Q_{km} = -V_k^2 (b_{skm} + b_{km}) - V_k V_m [g_{km} \sin(\delta_k - \delta_m) - b_{km} \cos(\delta_k - \delta_m)]$$ *(V)*

$\sum_k P_{km} = P_k = P_{k,g} - P_{k,d}$ *(VI)*

$\sum_k Q_{km} = Q_k = Q_{k,g} - Q_{k,d}$ *(VII)*

$V_{k,min} \leq V_k \leq V_{k,max}$ *(VIII)*

TEP as conditional decision with binary variables (1/2)



- Using **DC OPF** it is:

min $f(x)=c_1 \cdot P + c_0$ *(o)*

s.t. $P_{k,g,min} \leq P_{k,g,} \leq P_{k,g,max}$ *(I)* $P_{km,min} \leq P_{km} \leq P_{km,max}$ *(III)*

$\sum_k P_{km} = P_k = P_{k,g} - P_{k,d}$ *(VI)*

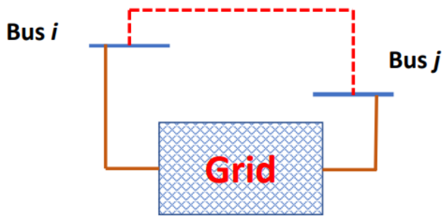
$P_{km} = -b_{km}(\delta_k - \delta_m)$ *(IX)*

- Lines i-Grid & j-Grid already installed, hence expressed through (IX)
- Deciding whether line i-j is built:

$$P_{ij} = -\xi_{ij} b_{ij} (\delta_i - \delta_j), \text{ for } \xi_{ij}\text{: binary var}$$

Not preferable because it makes the set-up non-convex

TEP as conditional decision with binary variables (2/2)



- Reconsider the investment decision as a limit on line's *i-j* capacity 0 before & per nominal characteristics after decided to be installed:

$$P_{ij} = -b_{ij}(\delta_i - \delta_j) \qquad \textit{(no separate number, use IX for it)}$$

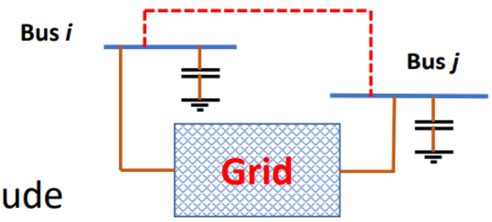
$$\xi_{ij}P_{ij,min} \leq P_{ij} \leq \xi_{ij}P_{ij,max} \qquad \textit{(XII)}$$

for ξ_{ij}: binary var

- Does this suffice? NO! Because if $\xi_{ij} = 0$, then from *(IX)* with *(XII)* => $\boldsymbol{\delta_i = \delta_j}$

We elaborate following, but first…

Effects of Shunt Elements in decoupled OPF (1/2)



- Shunt elements typically affect voltage magnitude
- Hence for **decoupled OPF** it is:

min $\quad f(x)=c_1 \cdot P+c_0 \qquad$ *(o)*

s.t. $\quad$ *(I), (III), (VI), (IX), (XII)*

$$Q_{k,g,min} \leq Q_{k,g,} \leq Q_{k,g,max} \quad \textit{(II)} \qquad Q_{km,min} \leq Q_{km} \leq Q_{km,max} \quad \textit{(IV)}$$

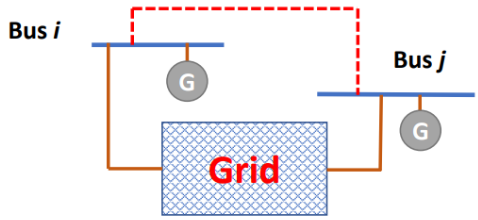
$$\textstyle\sum_k Q_{km} = Q_k = Q_{k,g} - Q_{k,d} \quad \textit{(VII)} \qquad V_{k,min} \leq V_k \leq V_{k,max} \quad \textit{(VIII)}$$

$$Q_{km} = -V_k \cdot b_{skm} - b_{km}(V_k - V_m) \qquad \textit{(XI)}$$

$$\xi_{ij}Q_{ij,min} \leq Q_{ij} \leq \xi_{ij}Q_{ij,max} \qquad \textit{(XIII)}$$

- Similar issue: if $\xi_{ij} = 0$, then from *(XI)* with *(XIII)* => $\boldsymbol{-V_k \cdot b_{sij} = b_{ij}(V_i - V_j)}$

Effects of Shunt Elements in decoupled OPF (2/2)



- Let's "remove" the shunt capacitance effect
- Essentially, shunt capacitance acts as reactive power compensation
- Hence, replace shunt elements with a shadow generator of only reactive power and capacity bounded at the nominal voltage (V_i=1.0 p.u.)

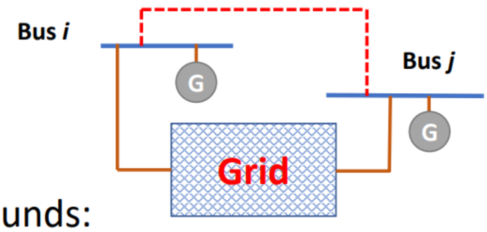
$$Q_{ij} = -b_{ij}(V_i - V_j) \qquad (XI)$$

$$\xi_{ij} Q_{ij,min} \le Q_{ij} \le \xi_{ij} Q_{ij,max} \qquad (XIII)$$

$$-1.0 \cdot \xi_{ij} b_{skm} \le Q_{k,g,} \le -1.0 \cdot \xi_{ij} b_{skm} \qquad (XIV)$$

- Does this suffice? NO! Because if $\xi_{ij} = 0$, then *(XI)* with *(XIII)* => $\boldsymbol{V_i = V_j}$

Eliminating binding of variables pre-investment



- Introduce shadow variables ***w*** with large M bounds:

$$P_{ij} = -b_{ij}(\delta_i - \delta_j - w_{ij,\delta}) \qquad \text{extend } (X) \text{ for } i\text{-}j \text{ line}$$

$$Q_{ij} = -b_{ij}(V_i - V_j - w_{ij,V}) \qquad \text{extend } (XI) \text{ for } i\text{-}j \text{ line}$$

$$\xi_{ij} P_{ij,min} \le P_{ij} \le \xi_{ij} P_{ij,max} \qquad (XII)$$

$$\xi_{ij} Q_{ij,min} \le Q_{ij} \le \xi_{ij} Q_{ij,max} \qquad (XIII)$$

$$-1.0 \cdot \xi_{ij} b_{sij} \le Q_{k,g,} \le -1.0 \cdot \xi_{ij} b_{skm} \qquad (XIV)$$

$$-(1 - \xi_{ij})M \le w_{ij,\delta} \le (1 - \xi_{ij})M \qquad (XV)$$

$$-(1 - \xi_{ij})M \le w_{ij,V} \le (1 - \xi_{ij})M \qquad (XVI)$$

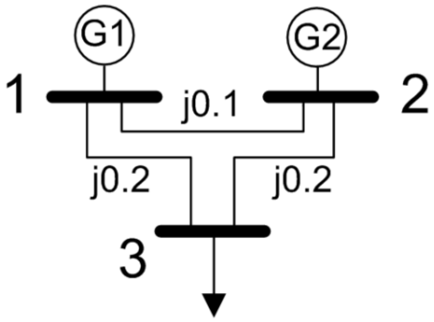
M: in the size of billions

Given data of the problem

- P_{G1}: P_n= 0-200MW, Q_n= ±100MVar, f(P)=0.5·P
- P_{G2}: P_n= 0-100MW, Q_n= ±100MVar, f(P)=2·P

How should they be scheduled for a load of P_{L3}= 250MW and at the expectancy of line 1-2 being disconnected

Reactances in network given in S_b=200MVA



Vector of variables of the DC OPF Problem

$X = [P_{g1,s}, P_{g2,s}, P_{g3,s}, d_{1,s}, d_{2,s}, d_{3,s}, p_{12,s}, p_{13,s}, p_{23,s}, p_{21,s}, p_{31,s}, p_{32,s},$
$P_{g1,c}, P_{g2,c}, P_{g3,c}, d_{1,c}, d_{2,c}, d_{3,c}, p_{12,c}, p_{13,c}, p_{23,c}, p_{21,c}, p_{31,c}, p_{32,c}, w_{12c}, \xi_{12}]$

Where, subscript "s" denotes normal operation, while "c" denotes operation under contingency

Security Constrained OPF Code

```
lear; clc;
f=[0.5,2,0,0,0,0,0,0,0,0,0,0];
% for variables x=[Pg1,Pg2,Pg3,d1,d2,d3,p12,p13,p23,p21,p31,p32]

Aeq=zeros(9,12);
Aeq(1,4)=-10; Aeq(1,5)=10; Aeq(1,7)=1; Aeq(2,4)=-5; Aeq(2,6)=5;
Aeq(2,8)=1; Aeq(3,5)=-5; Aeq(3,6)=5; Aeq(3,9)=1; Aeq(4,4)=10;
Aeq(4,5)=-10; Aeq(4,10)=1; Aeq(5,4)=5; Aeq(5,6)=-5; Aeq(5,11)=1;
Aeq(6,5)=5; Aeq(6,6)=-5; Aeq(6,12)=1; Aeq(7,1)=1; Aeq(7,7)=-1;
Aeq(7,8)=-1; Aeq(8,2)=1; Aeq(8,9)=-1; Aeq(8,10)=-1; Aeq(9,3)=1;
Aeq(9,11)=-1; Aeq(9,12)=-1;
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```
beq=zeros(1,9);
% for the following sequency of constraints: line flows p12, p13, p23, p21, p31,
p32 followed by
% the constraints of summations of line flows to Pg1-Pd1, Pg2-Pd2, Pg3-Pd3

lb=[0,0,-1.25,0,(-pi/2),(-pi/2),-1000,-1000,-1000,-1000,-1000,-1000];
ub=[1,0.5,-1.25,0,(pi/2),(pi/2),1000,1000,1000,1000,1000,1000];
% where 1.25 is the load at bus 3, i.e. Pd3
% lower bounds for generators physically those of zero (could be greater due to
technical minima),
% lower and upper bounds of voltage angles equal to desynchronization angles of
generators,
% line flows allowed to be infinitely large
% there is no Pg3 so by bounding it up and down to zero, it is set as such,
% similarly setting 0 reference angle for one reference bus, in this case bus 1.

x=linprog(f,[],[],Aeq,beq,lb,ub)

% new vector of variables is
% x=[Pg1,Pg2,Pg3,d1,d2,d3,p12,p13,p23,p21,p31,p32,
% Pg1c,Pg2c,Pg3c,d1c,d2c,d3c,p12c,p13c,p23c,p21c,p31c,p32c, w12c, ΞY12]
% where "C" means variable for the contingency case

% cost function
fc=[f,f*0,0,0];
% expand DC OPF with DC OPF under contingency
% fill out zeros (for now) the variables w12c and ΞY12
Aeqc=blkdiag(Aeq,Aeq); Aeqc=[Aeqc,zeros(size(Aeqc,1),2)];
beqc=[beq,beq];

% update active power flows of lines on contingency
% p12c=-b12(d1-d2+w12)=> p12c=0 => d1-d2=-w12
% p21c=-b12(d2-d1+w12) => p21c=0 => d2-d1=-w12
Aeqc(13,end-1:end)=[1,0];
Aeqc(10,end-1:end)=[-1,0];

% upper/lower bounds
lbc=[lb,0,0,-1.25,0,(-pi/2),(-pi/2),-1000,-1000,-1000,-1000,-1000,-1000,-1e9,0];
ubc=[ub,1,0.5,-1.25,0,(pi/2),(pi/2),1000,1000,1000,1000,1000,1000,1e9,1];

% ineq constr for shadow w and flows - use Aeqc to initialize it for ease
% ΞY12*p12min <= p12c <= ΞY12*p12max and ΞY12*p21min <= p21c <= ΞY12*p21max
% -(1-ΞY12)*M<= w12 <=(1-ΞY12)*M
Aineqc = Aeqc(1,:)*0; % Initialize Aineqc as a zero matrix (same size as rows in
Aeqc).

% Rows for ΞY12*p12min ≤ p12c ≤ ΞY12*p12max
Aineqc(1,end) = -1000; % Coefficient for ΞY12 in the lower bound constraint of
p12c.
Aineqc(1,length(f)+7) = -1; % Coefficient for p12c.
Aineqc(2,end) = -1000; % Coefficient for ΞY12 in the upper bound constraint of
p12c.
Aineqc(2,length(f)+7) = 1; % Coefficient for p12c.
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% Rows for ΞY12*p21min ≤ p21c ≤ ΞY12*p21max
Aineqc(3,end) = -1000; % Coefficient for ΞY12 in the lower bound constraint of p21c.
Aineqc(3,length(f)+10) = -1; % Coefficient for p21c.
Aineqc(4,end) = -1000; % Coefficient for ΞY12 in the upper bound constraint of p21c.
Aineqc(4,length(f)+10) = 1; % Coefficient for p21c.

bineqc = [0, 0, 0, 0]; % Right-hand side for the first four inequalities.

Aineqc(5,end) = 1e9; % Coefficient for ΞY12 in the lower bound constraint of w12.
Aineqc(5,end-1) = -1; % Coefficient for w12.
bineqc = [bineqc, 1e9]; % Right-hand side for lower bound.

Aineqc(6,end) = 1e9; % Coefficient for ΞY12 in the upper bound constraint of w12.
Aineqc(6,end-1) = 1; % Coefficient for w12.
bineqc = [bineqc, 1e9]; % Right-hand side for upper bound.

% "deactivate" line 1-2 through the binary variable ΞY
ub(end)=0;

% location of binary variable ΞY at the end of the vector; using length(lbc) for that numbered position
xc=intlinprog(fc,length(lbc),Aineqc,bineqc,Aeqc,beqc,lbc,ubc)
```

Output

xc =

1 0.25 -1.25	Active power dispatch of generators under normal conditions
0 -0.03 -0.14	Voltage angles in radians under normal conditions
0.3 0.7 0.55 -0.3 -0.7 -0.55	Active power flows under normal conditions
1 0.25 -1.25	Active power dispatch of generators under contingency conditions
0 -0.15 -0.2	Voltage angles in radians under contengency conditions
0 1 0.25 0 -1 -0.25	Active power flows under contingency conditions
-1.5 0	Shadow and binary variables