Economic Dispatch, Unit Commitment, Long-term Energy Planning

Optimization Problem

Minimize $f(x)=P\cdot c_2\cdot P^T+c_1\cdot P+c_0$ (o)

s.t.

$P_{k,g,min} \leq P_{k,g,} \leq P_{k,g,max}$ (I) $Q_{k,g,min} \leq Q_{k,g,} \leq Q_{k,g,max}$ (II)

$P_{km,min} \leq P_{km} \leq P_{km,max}$ (III) $Q_{km,min} \leq Q_{km} \leq Q_{km,max}$ (IV)

$$P_{km} = V_k^2(g_{skm} + g_{km}) - V_k V_m[g_{km}\cos(\delta_k - \delta_m) + b_{km}\sin(\delta_k - \delta_m)]$$
$$Q_{km} = -V_k^2(b_{skm} + b_{km}) - V_k V_m[g_{km}\sin(\delta_k - \delta_m) - b_{km}\cos(\delta_k - \delta_m)] \quad (V)$$

$$\sum_k P_{km} = P_k = P_{k,g} - P_{k,d} \quad (VI)$$

$$\sum_k Q_{km} = Q_k = Q_{k,g} - Q_{k,d} \quad (VII)$$

$$V_{k,min} \leq V_k \leq V_{k,max} \quad (VIII)$$

Economic dispatch without constraints

- System comprises 10 generators of the following characteristics
- P_{G1}, P_{G2}, P_{G3} : P_n= 0-100MW, Q_n= ±80MVar, $f(P)=0.01\cdot P^2+ 0.04\cdot P+0.001$
- P_{G4}, P_{G5} : P_n= 0-200MW, Q_n= ±100MVar, $f(P)=0.01\cdot P^2+ 0.02\cdot P+0.001$
- P_{G6}, P_{G7} : P_n= 0-50MW, Q_n= ±0MVar, $f(P)=0.02\cdot P^2+ 0.04\cdot P+0.001$
- P_{G8}, P_{G9}, P_{G10} (renewables – not "dispatchable"): $P_{t=1}$= [5,1,3]MW, $P_{t=2}$= [2,1,2]MW, Q_n= ±0MVar, f(P)=0

How should they be scheduled for the next two hours that the demand will be $P_{t=1}$= 550MW, $P_{t=2}$= 650MW, $Q_{t=1}$= 300MVar, $Q_{t=2}$= 400MVar

Costs given in S_b=100MVA

Vector of variables of the Problem

$$X = [P_{g1,t=1}, P_{g2,t=1}, P_{g3,t=1}, P_{g4,t=1}, P_{g5,t=1}, P_{g6,t=1}, P_{g7,t=1}, P_{g8,t=1}, P_{g9,t=1}, P_{g10,t=1},$$
$$Q_{g1,t=1}, Q_{g2,t=1}, Q_{g3,t=1}, Q_{g4,t=1}, Q_{g5,t=1}, Q_{g6,t=1}, Q_{g7,t=1}, Q_{g8,t=1}, Q_{g9,t=1}, Q_{g10,t=1},$$
$$P_{g1,t=2}, P_{g2,t=2}, P_{g3,t=2}, P_{g4,t=2}, P_{g5,t=2}, P_{g6,t=2}, P_{g7,t=2}, P_{g8,t=2}, P_{g9,t=2}, P_{g10,t=2},$$
$$Q_{g1,t=2}, Q_{g2,t=2}, Q_{g3,t=2}, Q_{g4,t=2}, Q_{g5,t=2}, Q_{g6,t=2}, Q_{g7,t=2}, Q_{g8,t=2}, Q_{g9,t=2}, Q_{g10,t=2}]$$

ED_Non_Constrained Code

```
% upper and lower bounds of the variables
% we make the intelligent guess that the renewables will be
% fully dispatched since they are the cheapest and the
% demand is much greater than their availability, hence we
% will bound them exactly at their availability
xmax = [ 1,1,1,2,2,0.5,0.5,0.05,0.01,0.03,0.8,0.8,0.8,1,1,0,0,0,0,0]; xmax=
[xmax,xmax]; xmax(1,28:30) = [0.02,0.01,0.02];
xmin = [zeros(1,7),0.05,0.01,0.03,-0.8,-0.8,-0.8,-1,-1,zeros(1,5)];xmin=
[xmin,xmin];xmin(1,28:30) = [0.02,0.01,0.02];

% the first line is the equality of generation and demand
% for active power for t=1
% the secondline is the equality of generation and demand
% for reactive power for t=1
% the first line is the equality of generation and demand
% for active power for t=2
% the secondline is the equality of generation and demand
% for reactive power for t=2
Aeq = [ones(1,10),zeros(1,30);zeros(1,10),ones(1,10),zeros(1,20);];
Aeq = [Aeq; zeros(1,20),ones(1,10),zeros(1,10);zeros(1,30),ones(1,10)];
beq = [ 5.5, 3, 6.5, 4];

% quadratic cost term expressed as a Hessian matrix
% multiplying the quadratic terms by 2 due to the
% Hessian operation per se
c2 = [0.01, 0.01, 0.01, 0.01, 0.01, 0.02, 0.02, 0, 0, 0, zeros(1,10)];
c2=[c2,c2];
H=2*diag(c2); %min[(1/2)*x^T*H*x +...]

% linear cost term
f = [0.04, 0.04, 0.04, 0.02, 0.02, 0.04, 0.04, zeros(1,13)]; f=[f,f];

% just transposing the line vectors that need to be
% for the proper execution of the quadprog
%f=f'; beq=beq'; xmin=xmin'; xmax=xmax';

% execution of the optimizaion
```

```
x = quadprog(H,f,[],[],Aeq,beq,xmin,xmax)


% some checks to see that the dispatching is righ
%[sum(x(1:10)), sum(x(11:20)), sum(x(21:30)), sum(x(31:40))]

% the output should be the active power demand of hour 1,
% the reactive power demand of hour 2, the active power
% demand of hour 2, the reactive power demand of hour 2,
% i.e. 5.5   3   6.5   4
```

Adding some constraints to the Economic Dispatch Problem

- The active and reactive power of generator 1 cannot ramp up more than 10% of nominal in an hour
- The active and reactive power of generator 4 cannot ramp up more than 5% of nominal in an hour
- Generator 2 cannot generate more than 100 MWh

How should they be scheduled for the next two hours that the demand will be $P_{t=1}$= 550MW, $P_{t=2}$= 650MW, $Q_{t=1}$= 300MVar, $Q_{t=2}$= 400MVar

Ramp Constraints as Equations

- **$P_{g1,t=2} - P_{g1,t=1} \leq 10\% * 100$ MW**
  - $-P_{g1,t=1} + P_{g1,t=2} \leq 0.10$ pu
- **$Q_{g1,t=2} - Q_{g1,t=1} \leq 10\% * 80$ MVar**
  - $-Q_{g1,t=1} + Q_{g1,t=2} \leq 0.08$
- **$P_{g4,t=2} - P_{g4,t=1} \leq 5\% * 200$ MW**
- **$Q_{g4,t=2} - Q_{g4,t=1} \leq 5\% * 100$ MVar**

Ramp constraint - Matlab Formulation

- **$1*P_{g1,t=2} - 1*P_{g1,t=1} <= 0.1$ <=>**

$- 1*P_{g1,t=1} + 0*P_{g2,t=1} + 0*P_{g3,t=1} + 0*P_{g4,t=1} + 0*P_{g5,t=1} + 0*P_{g6,t=1} + 0*P_{g7,t=1} + 0*P_{g8,t=1} + 0*P_{g9,t=1} + 0*P_{g10,t=1} + 0*Q_{g1,t=1} + 0*Q_{g2,t=1} + 0*Q_{g3,t=1} + 0*Q_{g4,t=1} + 0*Q_{g5,t=1} + 0*Q_{g6,t=1} + 0*Q_{g7,t=1} + 0*Q_{g8,t=1} + 0*Q_{g9,t=1} + 0*Q_{g10,t=1} +1*P_{g1,t=2} + 0*P_{g2,t=2} + 0*P_{g3,t=2} + 0*P_{g4,t=2} + 0*P_{g5,t=2} + 0*P_{g6,t=2} + 0*P_{g7,t=2} + 0*P_{g8,t=2}+ 0*P_{g9,t=2} + 0*P_{g10,t=2} + 0*Q_{g1,t=2} + 0*Q_{g2,t=2} + 0*Q_{g3,t=2} + 0*Q_{g4,t=2} + 0*Q_{g5,t=2} + 0*Q_{g6,t=2} + 0*Q_{g7,t=2} + 0*Q_{g8,t=2} + 0*Q_{g9,t=2} + 0*Q_{g10,t=2} <= 0.1$

- Aineq*X <= bineq =>
  [-1,(19 zeros),1,(19 zeros)] <= 0.1

- **$1*Q_{g1,t=2} - 1*Q_{g1,t=1} <= 0.08$**
- $X = [P_{g1,t=1}, P_{g2,t=1}, P_{g3,t=1}, P_{g4,t=1}, P_{g5,t=1}, P_{g6,t=1}, P_{g7,t=1}, P_{g8,t=1}, P_{g9,t=1}, P_{g10,t=1}, Q_{g1,t=1}, Q_{g2,t=1}, Q_{g3,t=1}, Q_{g4,t=1}, Q_{g5,t=1}, Q_{g6,t=1}, Q_{g7,t=1}, Q_{g8,t=1}, Q_{g9,t=1}, Q_{g10,t=1}, P_{g1,t=2}, P_{g2,t=2}, P_{g3,t=2}, P_{g4,t=2}, P_{g5,t=2}, P_{g6,t=2}, P_{g7,t=2}, P_{g8,t=2}, P_{g9,t=2}, P_{g10,t=2}, Q_{g1,t=2}, Q_{g2,t=2}, Q_{g3,t=2}, Q_{g4,t=2}, Q_{g5,t=2}, Q_{g6,t=2}, Q_{g7,t=2}, Q_{g8,t=2}, Q_{g9,t=2}, Q_{g10,t=2}]$
- [(10 zeros),-1,(19 zeros), 1, (9 zeros)] <= 0.08
- And similar for generator 4...

Energy Constraint Formulation

Energy Constraint

- **$1*P_{g2,t=1} + 1*P_{g2,t=2} <= 1$**
- $X = [P_{g1,t=1}, P_{g2,t=1}, P_{g3,t=1}, P_{g4,t=1}, P_{g5,t=1}, P_{g6,t=1}, P_{g7,t=1}, P_{g8,t=1}, P_{g9,t=1}, P_{g10,t=1}, Q_{g1,t=1}, Q_{g2,t=1}, Q_{g3,t=1}, Q_{g4,t=1}, Q_{g5,t=1}, Q_{g6,t=1}, Q_{g7,t=1}, Q_{g8,t=1}, Q_{g9,t=1}, Q_{g10,t=1}, P_{g1,t=2}, P_{g2,t=2}, P_{g3,t=2}, P_{g4,t=2}, P_{g5,t=2}, P_{g6,t=2}, P_{g7,t=2}, P_{g8,t=2}, P_{g9,t=2}, P_{g10,t=2}, Q_{g1,t=2}, Q_{g2,t=2}, Q_{g3,t=2}, Q_{g4,t=2}, Q_{g5,t=2}, Q_{g6,t=2}, Q_{g7,t=2}, Q_{g8,t=2}, Q_{g9,t=2}, Q_{g10,t=2}]$
- [0,1,(19 zeros),1,(18 zeros)] <= 1

ED_Constrained Code

```
% Inequality constraints for ramp-up rates
% first active and reactive power for G1
% then active and reactive power for G4
% rows: # of constraints, columns: # of variables
Aineq=[-1,zeros(1,19),1,zeros(1,19)]; bineq=[0.1];
Aineq=[Aineq;zeros(1,10),-1,zeros(1,19),1,zeros(1,9)]; bineq=[bineq;0.08];
Aineq=[Aineq;0,0,0,-1,zeros(1,19),1,zeros(1,16)]; bineq=[bineq;0.1];
Aineq=[Aineq;zeros(1,13),-1,zeros(1,19),1,zeros(1,6)]; bineq=[bineq;0.05];

Aineq=[Aineq;-Aineq];
bineq=[bineq; bineq];

% execution of the optimization
x2 = quadprog(H_new,f_new,Aineq,bineq,Aeq_new,beq_new,xmin_new,xmax_new);
display(x2)
%sum(x2(11:20))
%sum(x2(31:40))
% addint the energy constraint for G2
Aineq=[Aineq;0,1,zeros(1,19),1,zeros(1,18)];
bineq=[bineq;1];

% execution of the optimization
x3 = quadprog(H_new,f_new,Aineq,bineq,Aeq_new,beq_new,xmin_new,xmax_new);
display(x3);
```

Unit Commitment – Set-Up

- Long-term scheduling of resources
- Resources can be off/on => require binary (integer) variables
- Resources might be off/on for specific times
- Every time a resource starts/stops there is an extra cost

(usually no consideration of grid, i.e. extension of economic dispatch)

Unit Commitment – Mixed-Integer Linear Programming (MILP) formulation

Minimize $f(x)=\Sigma_{t=[1,T]}(c_{1,t}\cdot P_{g,t}+ \xi_t\cdot c_{0,t}) + \Sigma_{t=[1,T]}\Sigma_k(SU+SD)$

s.t.

$$\xi_{k,t}\cdot P_{k,g,min} \le P_{k,g,t} \le \xi_{k,t}\cdot P_{k,g,max} \text{ , } \xi_{k,t}\cdot Q_{k,g,min} \le Q_{k,g,t} \le \xi_{k,t}\cdot Q_{k,g,max}$$

$$SU_{k,max}(\xi_{k,t}- \Sigma_{r=[1,t]}\xi_{k,t-r}) \le SU_{k,t} \text{ , } 0 \le SU_{k,t} \quad \text{for } t=[1,T]$$

$$SD_{k,max}(\xi_{k,t-1} - \xi_{k,t}) \le SD_{k,t} \text{ , } 0 \le SD_{k,t} \quad \text{for } t=[1,T]$$

$$\Sigma_k P_{k,g,t} = P_{demand,t} \text{ , } \Sigma_k Q_{k,g,t} = Q_{demand,t} \quad \text{for } t=[1,T]$$

- The active and reactive power of generator 1 cannot ramp up more than 10% in an hour
- The active and reactive power of generator 4 cannot ramp up more than 5% in an hour
- Generator 2 cannot generate more than 100 MWh

How should they be scheduled for the next two hours that the demand will be $P_{t=1}$= 550MW, $P_{t=2}$= 650MW, $Q_{t=1}$= 300MVar, $Q_{t=2}$= 400MVar? What is the decision on whether the generators will be ON or OFF for the first hour of operation?

Vector of variables of the Problem

$$\begin{aligned} X = [&P_{g1,t=1}, P_{g2,t=1}, P_{g3,t=1}, P_{g4,t=1}, P_{g5,t=1}, P_{g6,t=1}, P_{g7,t=1}, P_{g8,t=1}, P_{g9,t=1}, P_{g10,t=1},\\ &Q_{g1,t=1}, Q_{g2,t=1}, Q_{g3,t=1}, Q_{g4,t=1}, Q_{g5,t=1}, Q_{g6,t=1}, Q_{g7,t=1}, Q_{g8,t=1}, Q_{g9,t=1}, Q_{g10,t=1},\\ &P_{g1,t=2}, P_{g2,t=2}, P_{g3,t=2}, P_{g4,t=2}, P_{g5,t=2}, P_{g6,t=2}, P_{g7,t=2}, P_{g8,t=2}, P_{g9,t=2}, P_{g10,t=2},\\ &Q_{g1,t=2}, Q_{g2,t=2}, Q_{g3,t=2}, Q_{g4,t=2}, Q_{g5,t=2}, Q_{g6,t=2}, Q_{g7,t=2}, Q_{g8,t=2}, Q_{g9,t=2}, Q_{g10,t=2},\\ &\lambda_{g1,t=1}, \lambda_{g2,t=1}, \lambda_{g3,t=1}, \lambda_{g4,t=1}, \lambda_{g5,t=1}, \lambda_{g6,t=1}, \lambda_{g7,t=1}, \lambda_{g8,t=1}, \lambda_{g9,t=1}, \lambda_{g10,t=1},\\ &\lambda_{g1,t=2}, \lambda_{g2,t=2}, \lambda_{g3,t=2}, \lambda_{g4,t=2}, \lambda_{g5,t=2}, \lambda_{g6,t=2}, \lambda_{g7,t=2}, \lambda_{g8,t=2}, \lambda_{g9,t=2}, \lambda_{g10,t=2}] \end{aligned}$$

Where λ are the binary decision variables for ON/OFF status of the generator (second hour λ not required, but added for consistency)

Unit Commitment Code

```
% incorporating the binary variables for the operation
% of generators
% upper/lower bounds are set to 0/1 since the
% integer variables have to be binary
xmin=[xmin;zeros(20,1)]; xmax=[xmax;ones(20,1)];
% updating the inequality and equality constraint
% matrices with zeros for the new integer
% variables
Aineq=[Aineq,zeros(size(Aineq,1),20)];
Aeq=[Aeq,zeros(size(Aeq,1),20)];
% extending the linear cost vector with zeros for the
% integer variables
f=[f;zeros(20,1)];

% executing the integer programming
%x = intlinprog(f,intcon,A,b,Aeq,beq,lb,ub)
%
x4 = intlinprog(f,[41:60],Aineq,bineq,Aeq,beq,xmin,xmax);
disp(x4)
% extending the inequality constraints matrix to
% include the effect of the binary variables on
% whether the generator is ON/OFF
% ON Constraint:-pi + ui * min_outputi <= 0
% OFF Constraint:pi - ui * max_outputi <= 0
%ξk,t⋅Pk,g,min ≤ Pk,g,t ≤ ξk,t⋅Pk,g,max , ξk,t⋅Qk,g,min ≤ Qk,g,t ≤ ξk,t⋅Qk,g,max
Aineq=[Aineq;-diag(ones(10,1)),zeros(10,30),diag(xmin(1:10)),zeros(10,10)];
Aineq=[Aineq;diag(ones(10,1)),zeros(10,30),-diag(xmax(1:10)),zeros(10,10)];
bineq=[bineq;zeros(20,1)];

% executing the integer programming
x5 = intlinprog(f,[41:60],Aineq,bineq,Aeq,beq,xmin,xmax);
display(x5)
```