

Genetics and population analysis

Genome analysis

DEploid: Untangling multiplicity of infection in *Plasmodium falciparum*.

Sha Joe Zhu^{1,*}, Jacob Almagro Garcia¹ and Gil McVean^{1,*}

¹ Wellcome Trust Centre for Human Genetics, University of Oxford, Oxford OX3 7BN, UK

*To whom correspondence should be addressed.

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Abstract

Motivation: Multiplicity of infection in the malarial parasite *Plasmodium falciparum* affects key phenotypic traits, including drug resistance and risk of severe disease. Advances in protocols and sequencing technology have made possible to obtain high-coverage genome-wide sequence data from blood samples taken in the field. However, analyzing and interpreting such data is challenging because of the high rate of multiple infections present in the field.

Results: The software package *DEploid* learns haplotype structure from a reference panel of clonal isolates, and deconvolutes sequences of mixed samples. It reports the number of strains, the mixing proportions and the haplotypes present in an isolate, allowing researchers to study malaria infection history with an unprecedented level of detail.

Availability and implementation: The open source implementation *DEploid* is freely available at <https://github.com/mcveanlab/dEploid> under the conditions of the GPLv3 license.

Contact: joe.zhu@well.ox.ac.uk or mcvean@well.ox.ac.uk

Supplementary information: Supplementary data are available at *Bioinformatics* online.

1 Introduction

Malaria is still one of the top global health problems. Transmitted by mosquitos of the genus *Anopheles*, the majority of malaria related deaths are caused by the *Plasmodium falciparum* parasite (WHO, 2015). Patients are often infected with more than one parasite strain, due to bites from multiple mosquitoes, mosquitoes carrying multiple genetic types or a combination of both. Multiplicity of infection can lead to competitions among co-existing strains and may increase disease development (de Roode et al., 2005), higher transmission rates (Arnot, 1998) and even the spread of drug resistance (de Roode et al., 2004).

The presence of multiple strains of *P. falciparum* makes fine scale analysis of genetic variation very challenging since genetic differences between the genetic types of this haploid organism will render as heterozygous loci. Mixed calls also confounds methods that exploit haplotype data to detect, among other phenomena, the occurrence of natural selection or recent demographic events. In light of these difficulties, researchers usually focus on clonal infections or resort to heuristics

methods for resolving heterozygous genotypes. The former approach discards valuable information regarding genetic diversity and inbreeding whereas the latter tends to create chimeric haplotypes that are not suitable for analysis, unless mixed calls are very sparse.

Phasing or deconvoluting the strains of a mixed infection is a harder problem than phasing diploid organisms because the levels of mixture within isolates (i.e. the abundance of each genetic type) vary greatly and are unknown. Existing tools for phasing diploid organisms, such as Beagle (Browning and Browning, 2007) and IMPUTE2 (Howie et al., 2009), are not designed to cope with this. Galinsky et al. (2015) and O'Brien et al. (2015) have attempted to address the mixed infection problem by solving the mixed proportion from allele frequencies, yet the haplotypes within a mixed isolate remain unclear.

As part of the Pf3k project (Pf3k, 2016), an effort to map the genetic diversity of *P. falciparum* at global scale, we have developed the *DEploid*, a software package for deconvoluting mixed infections. The program provides estimates for the number of different genetic types present in the isolate, the proportion or abundance of each strain and their sequences (i.e. haplotypes). To our knowledge, *DEploid* is the first package able

to deconvolute strain haplotypes and provides a unique opportunity for researchers to study inbreeding and infection history at fine scale.

2 Methods

Overall, we use Markov chain Monte Carlo (MCMC) methods to learn the number of parasite strains and the proportions of allele frequencies, and use sampling method to infer the haplotype of each strain.

2.1 Notations

Let's first introduce some of the notation. Suppose that our data D are the allele counts of sample j at a given site i , denoted as $r_{j,i}$ and $a_{j,i}$ for reference and alternative alleles respectively. The allele frequencies within sample (WSAF) $p_{j,i}$ and at the population level (PLAF) f_i can be calculated by $\frac{a_{j,i}}{a_{j,i}+r_{j,i}}$ and $\frac{\sum_j a_{j,i}}{\sum_j a_{j,i} + \sum_j r_{j,i}}$.

Since all data in this section is subjected to the same sample, we drop the subscript j from now on. Let $\mathbf{w} = [w_1, \dots, w_k]$ and $\mathbf{h}_i = [h_{1,i}, \dots, h_{k,i}]$ denote the proportions and haplotypes of k parasite strain at site i . O'Brien et al. (2015) suggest to express the expected WSAF q_i :

$$q_i = (\mathbf{w} \cdot \mathbf{h}_i) = \sum_{k=1}^K w_k \cdot h_{k,i}. \quad (1)$$

2.1.1 Likelihood of data given the expected WSAF

Suppose unadjusted allele frequency is q_i , given the reads error rate e , the expected allele frequency of 'REF' read as 'ALT' is $(1 - q_i)e$, and the expected allele frequency of 'ALT' read as 'REF' is $q_i e$. Thus, we adjust the WSAF take into account of read error as follows:

$$\pi_i = q_i + (1 - q_i)e - q_i e = q_i + (1 - 2q_i)e. \quad (2)$$

We take into account over-dispersion in read counts, modelling the count distribution as a Beta-binomial distribution. Specifically, the read counts of 'ALT' are identically and independently distributed (i.i.d.) with probability π_i (adjusted), i.e. $a_i \sim \text{Binom}(\pi_i, a_i + r_i)$, and $\pi_i \sim \text{Beta}(\alpha, \beta)$, where $\pi_i = \alpha / (\alpha + \beta)$. From experience, we set $\alpha = 100 \cdot q_i$ and $\beta = 100 \cdot (1 - q_i)$. Hence we can express the likelihood of the data using:

$$L(q_i|D) = P(D|q_i) \propto \frac{\Gamma(a_i + 100 \cdot \pi_i) \Gamma(r_i + 100 \cdot (1 - \pi_i))}{\Gamma(100 \cdot \pi_i) \Gamma(100 \cdot (1 - \pi_i))}, \quad (3)$$

of which expected WSAF q_i is adjusted through Eqn.(2).

i	Marker index
j	Sample index
r	Read count for reference allele
a	Read count for alternative allele
f	Population level allele frequency (PLAF)
k	Number of strains within sample
\mathbf{w}	Proportion of strains
\mathbf{h}_i	haplotypes of k parasite strain at site i
p	Observed within sample allele frequency (WSAF)
q	Unadjusted expected WSAF
π	Adjusted expected WSAF

Table 1. Notation summary

2.2 Technical details

Overall, we generate MCMC samples for the proportions \mathbf{w} and the haplotypes \mathbf{h} for given number of strains. In particular, we assume there are more strains than we actually need, start the MCMC chain with a fixed k . As the values of proportion drops, “zero-out” the “noisy” strain. As for the MCMC moves, we use a Metropolis-Hastings algorithm to sample proportions \mathbf{w} given \mathbf{h} (see section 2.2.1); and use Gib sampler to update \mathbf{h} of given \mathbf{w} , which are further divided into cases when building the reference panel (see section 2.3) and deconvolute mixed samples (see section 2.4.2).

2.2.1 MCMC update for proportions

We use a sparse update on \mathbf{w} . We introduce a multivariate normal variable titre $\mathbf{x} = [x_1, \dots, x_k]$, where each x is i.i.d. normally distributed from $N(0, 3)$, with the density function $d(x)$. We sample x s, then transform to $w = e^x$. We then normalise vector \mathbf{w} by the sum, to obtain a new sample \mathbf{w} . Thus, the density of \mathbf{p} is equivalent to the product of $d(x)$ s, which leads us to The prior ratio is equal to $\frac{\prod_i^k d(x'_i)}{\prod_i^k d(x_i)}$; and the Hastings ratio is 1. Note that the move from x to x' , δx is symmetrical.

2.3 Infer the reference strains

In practice, we assume clonal sample haplotypes capture the diversities of haplotype structures given all samples. We use them as the reference strains for start, and deconvolute the rest mixed samples from them. We start with a set of clonal sample candidate, and run the algorithm to confirm they are in fact clonal. We use Gib sampler to update \mathbf{h} of given \mathbf{w} , randomly select one strain at the time, or a pair of strains to update in order to improve the mixing of the MCMC process.

2.3.1 Update a single strain at one time

Choose haplotype strain s uniformly at random from these K strains, consider both cases of updating the state of strain s at position i to 0 and 1, we compute the WSAF and its associated likelihood as follows: First of all, regardless what state that strain s at position i has, we need to remove it from the current WSAF, i.e. subtract $w_s \cdot h_s$ from Eqn. (1), which gives

$$q_{i,-s} = \sum_{k \neq s} w_k \cdot h_k = \text{Eqn. (1)} - w_s \cdot h_s \quad (4)$$

Therefore, updating strain s of state 0 and 1, so the WSAF is

$$q_{i,g_s=0} = \text{Eqn. (4)} \quad (5)$$

$$q_{i,g_s=1} = \text{Eqn. (4)} + w_s \times 1 \quad (6)$$

Substitute equations (5) and (6) into Eqn. (3) to compute associated likelihood $L(q_{i,g_s=*}|D)$, which is expressed as $L(g_s = *|D)$ in short, for the rest of the paper.

As one of our MCMC step to update the haplotypes, we sample the state (genotype) of strain s at each position according to the posterior probability at site i ,

$$P(g_s = *|D) \propto L(g_s = *|D) \times P(g_s = *). \quad (7)$$

2.3.2 Update two haplotypes at one time

In order to improve the MCMC mixing, we update two haplotypes at one time. Suppose random sampling two strains to update, namely, s_1 and s_2 . Similar to Eqn. (4), we have

$$\begin{aligned} q_{i,-s_1,-s_2} &= \sum_{k \neq s_1, s_2} w_k \cdot h_k \\ &= \text{Eqn. (1)} - w_{s_1} \cdot h_{s_1} - w_{s_2} \cdot h_{s_2} \end{aligned} \quad (8)$$

Further more, we have

$$q_{i,g_{s_1}=0,g_{s_2}=0} = \text{Eqn. (8)} \quad (9)$$

$$q_{i,g_{s_1}=0,g_{s_2}=0} = \text{Eqn. (8)} + \cdot w_{s_1} \times 1 \quad (10)$$

$$q_{i,g_{s_1}=0,g_{s_2}=1} = \text{Eqn. (8)} + \cdot w_{s_2} \times 1 \quad (11)$$

$$q_{i,g_{s_1}=0,g_{s_2}=1} = \text{Eqn. (8)} + \cdot w_{s_1} \times 1 + w_{s_2} \times 1 \quad (12)$$

Substitute expressions. (9) to (12), into Eqn. (3), we then obtain their associated likelihood $L(q_{i,g_{s_1}=*,g_{s_2}=\#}|D)$, which is denoted as $L(g_{s_1}=*,g_{s_2}=\#|D)$ in the rest of the paper.

Similar to Eqn. (7), we sample the state (genotype) of strains s_1 and s_2 simultaneously at each position according to the posterior probability at site i :

$$P(g_{s_1}=*,g_{s_2}=\#|D) \propto L(g_{s_1}=*,g_{s_2}=\#|D) \times P(g_{s_1}=*,g_{s_2}=\#), \quad (13)$$

where $P(g_{s_1}=*,g_{s_2}=\#) = P(g_{s_1}=*) \cdot P(g_{s_2}=\#)$, assume s_1 and s_2 are independent.

2.4 Deconvolute the mixed isolates

We use Li and Stephens (2003)’s hidden Markov model frame work as a starting point. The following modifications are made:

- likelihood of data given the expected WSAF rather than the “product of approximate conditionals” (PAC).
- multiple strains with variable proportion rather than two sequences with equal probability.
- simplifying the mutation model with a fixed miss copying operation.

2.4.1 Update single haplotype with LD

Recombination map model The first case refers to staying on the same path and the second to a recombination event (i.e switch). Let ψ_i is given by $\psi_i = N_e G_i$, with N_e being the effective population size and G_i the genetic distance between loci i and $i + 1$. We assume a uniform recombination map, genetic distances are computed by $G_i = D_i / \text{morgan}$ where D_i denotes the physical distance between loci i and $i + 1$ in nucleotide, morgan is the average morgan distance, which we use 1500000, $N_e = 10$.

Whereas recombination probabilities for a segment are computed by the following function. Note that **we scale the probabilities with the number of haplotypes in the reference panel**. Let RP denote the set of the strains in the reference panel. For position $i > 1$, let ρ'_i denote the probability of **no** recombinations from site $i - 1$ to i , we have $\rho'_i = \exp(-\psi_i)$. Thus, the probability of recombining from any strain in the panel is $\frac{1 - \rho'_i}{|RP|}$, where $|RP|$ is the size of the panel.

A crucial difference between our method and Li and Stephens (2003)’s model is that mixed samples can have more than two strains, with unknown proportions. We randomly choose the strains to update, then apply LS’s algorithm to sample the path using Gibbs sampler given the proportion \mathbf{p} (see example in Fig. 1).

In addition to updating the haplotypes from the panel, we take into account of miss copying (see example shown in Fig. 1), which allow the actual genotype differ from the path, in order to improve the likelihood of data.

1. Consider the likelihood as the emission probabilities at site i . Let’s use g_p and g_s to denote the genotype of the copied path and the updated

Ref. Haplotype 1	0	1	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1
Ref. Haplotype 2	0	1	0	1	0	0	0	0	0	0	0	1	0	0	1	0	1
Ref. Haplotype 3	0	1	0	0	0	0	0	0	1	0	0	1	0	0	1	0	1
Ref. Haplotype 4	0	1	0	0	0	1	1	0	1	0	1	0	0	0	1	0	0
Ref. Haplotype 5	0	0	1	0	0	0	0	1	0	0	0	1	0	0	1	0	1
Strain 1	0	1	0	1	0	1	0	0	0	0	0	1	0	0	1	0	1
Strain 2	0	1	0	0	0	0	0	0	0	0	0	1	0	0	1	0	1

Fig. 1. Illustration of Li and Stephens (2003)’s algorithm. Strain haplotype 1 is made up from reference haplotype segments of 1, 2, and 4; and strain haplotype 3 is made up from reference haplotype segments of 1, 2, 3 and 4. With miss copying, we allow strain states differ from the path: At the end position of strain 3, the path is copied from reference haplotype 4, with the state of “0”.

strain respectively. We have:

$$L(g_p = *|D) = L(g_s = *|D) \times P(g_p = g_s) + L(g_s = 1 - *|D) \times P(g_p \neq g_s) \quad (14)$$

where $* \in \{0, 1\}$, and $1 - *$ indicates the event that g_s takes value that differs from g_p . Let μ denote the probability of miss copying, we have

$$\begin{cases} P(g_p = g_s) = 1 - \mu, \\ P(g_p \neq g_s) = \mu. \end{cases}$$

2. Compute the probability of path at each position using forward algorithm. Therefore, we have the posterior probability of path (reference strain) p at position i as:

$$P_i(g_p|D) \propto L(g_p|D) \times \left(\rho'_i \cdot P_{i-1}(g_p|D) + \frac{1 - \rho'_i}{|RP|} \cdot \sum_{x \in R} P_{i-1}(g_x|D) \right). \quad (15)$$

In the HMM frame work, $L(g_p|D)$ is the emission probability of observing data D given the hidden state of the path, ρ'_i and $\frac{1 - \rho'_i}{|RP|}$ are the transition probabilities from position $i - 1$ to i , of which reflect the recombination event in our context.

3. Sample the path up to position i , i.e. backwards, start from the end of the sequence. At the end position, sample path according $f_{u,end}$. for the $i - 1$ position, first sample if a recombination events had happened with the probabilities proportional to

$$\begin{cases} \rho'_i \cdot f_{u,i-1} & \text{no recombined,} \\ (1 - \rho'_i) \cdot \sum_{x \in R} f_{x,i-1} & \text{recombined.} \end{cases}$$

If it was recombined, sample the path u , according to $f_{u,i-1}$.

4. Ultimately, given the state of the path at each site, we now want to sample the genotype according to the posterior probabilities:

$$P(g_s = *|D) = \begin{cases} P(g_p = *|D) \cdot (1 - \mu), & g_s = g_p; \\ P(g_p = 1 - *|D) \cdot \mu, & g_s \neq g_p. \end{cases} \quad (16)$$

2.4.2 Update pair of haplotypes with LD

Similarly to the previous section, we need to

1. Compute the emission probabilities

$$L(g_{p_1} = *, g_{p_2} = \# | D) = P(g_{p_1} = g_{s_1}, g_{p_2} = g_{s_2}) \cdot L(g_{s_1} = *, g_{s_2} = \# | D) + P(g_{p_1} = g_{s_1}, g_{p_2} \neq g_{s_2}) \cdot L(g_{s_1} = *, g_{s_2} = \# | D) + P(g_{p_1} \neq g_{s_1}, g_{p_2} = g_{s_2}) \cdot L(g_{s_1} = 1 - *, g_{s_2} = \# | D) + P(g_{p_1} \neq g_{s_1}, g_{p_2} \neq g_{s_2}) \cdot L(g_{s_1} = 1 - *, g_{s_2} = \# | D) \quad (17)$$

where

$$\begin{aligned} P(g_{p_1} = g_{s_1}, g_{p_2} = g_{s_2}) &= (1 - \mu) \cdot (1 - \mu), \\ P(g_{p_1} \neq g_{s_1}, g_{p_2} = g_{s_2}) &= \mu \cdot (1 - \mu), \\ P(g_{p_1} = g_{s_1}, g_{p_2} \neq g_{s_2}) &= \mu \cdot (1 - \mu), \\ P(g_{p_1} \neq g_{s_1}, g_{p_2} \neq g_{s_2}) &= \mu \cdot \mu. \end{aligned}$$

2. Compute the probability of path at each position using forward algorithm.

Similar to Equation (15), for all possible pair of the copying strain, we take into account of the possibility of one strain recombines and the other does not with the probability of $\rho'_i \cdot \frac{1 - \rho'_i}{|RP|}$; both recombines, with the probability of $\rho'_i \cdot \rho'_i$; neither recombines, with the probability of $\frac{1 - \rho'_i}{|RP|} \cdot \frac{1 - \rho'_i}{|RP|}$, assuming that recombination events of two copying strains are independent from each other.

$$\begin{aligned} P_i(g_{p_1}, g_{p_2} | D) &\propto [P_{i-1}(g_{p_1}, g_{p_2} | D) \cdot \rho'_i \cdot \rho'_i + \\ &\sum_{x \in R} P_{i-1}(g_{p_1}, g_x | D) \cdot \rho'_i \cdot \frac{1 - \rho'_i}{|RP|} + \\ &\sum_{y \in R} P_{i-1}(g_y, g_{p_2} | D) \cdot \rho'_i \cdot \frac{1 - \rho'_i}{|RP|} + \\ &\sum_{x, y \in R \cdot R} P_{i-1}(g_x, g_y | D) \cdot \frac{1 - \rho'_i}{|RP|} \cdot \frac{1 - \rho'_i}{|RP|}] \cdot L(g_{p_1}, g_{p_2} | D) \quad (18) \end{aligned}$$

3. Sample the path up to position i , i.e. backwards, start from the end of the panel. At the end position, sample path according $P(p_1 = u, p_2 = v) = f_{u,v,end}$. for the $i - 1$ position, first sample if a recombination events had happened given the probabilities of

$$\begin{cases} f_{u,v,i-1} \cdot \rho'_i \cdot \rho'_i, & \text{no recombine} \quad (19) \\ \sum_{* \in RP} f_{u,*,i-1} \cdot \frac{1 - \rho'_i}{|RP|} \cdot \rho'_i, & \text{u recombine} \quad (20) \\ \sum_{* \in RP} f_{*,v,i-1} \cdot \frac{1 - \rho'_i}{|RP|} \cdot \rho'_i, & \text{v recombine} \quad (21) \\ \sum_{*,* \in RP \cdot RP} f_{*,*,i-1} \cdot \frac{1 - \rho'_i}{|RP|} \cdot \frac{1 - \rho'_i}{|RP|}, & \text{both recombine} \quad (22) \end{cases}$$

If it both recombined, sample the path, according $P(p_1 = u, p_2 = v) = f(u, v, i - 1)$. If one of them recombined, sample the path according to the marginal probability of $P(p_1 = u) = f(u, i - 1)$.

4. Ultimately, we consider add miss copies similar to the previous section, and sample the strain state given the path state with probabilities:

$$P(g_{s_1} = *, g_{s_2} = \# | D) = \begin{cases} P(g_{p_1} = *, g_{p_2} = \# | D) \cdot (1 - \mu) \cdot (1 - \mu), & g_{s_1} = g_{p_1} \text{ and } g_{s_2} = g_{p_2}; \\ P(g_{p_1} = *, g_{p_2} = 1 - \# | D) \cdot (1 - \mu) \cdot \mu, & g_{s_1} = g_{p_1} \text{ and } g_{s_2} \neq g_{p_2}; \\ P(g_{p_1} = 1 - *, g_{p_2} = \# | D) \cdot \mu \cdot (1 - \mu), & g_{s_1} \neq g_{p_1} \text{ and } g_{s_2} = g_{p_2}; \\ P(g_{p_1} = 1 - *, g_{p_2} = 1 - \# | D) \cdot \mu \cdot \mu, & g_{s_1} \neq g_{p_1} \text{ and } g_{s_2} \neq g_{p_2}. \end{cases} \quad (23)$$

- Randomly update a single strain or two strains simultaneously.

A crucial difference between our method and LS's model is that mixed samples can have more than two strains, with unknown proportions. We randomly choose the strains to update, then apply LS's algorithm to sample the path (see Fig. ??) using Gibbs sampler with given proportion rather than 50/50 in the cases of diploid samples. Updating the haplotypes from the paths, take into account of miss copying. Our model benefits from combining information from both the reference haplotypes as well as the data. For *de novo* mutations which are not found reference panel, our method will infer mutations based on read count from data.

3 Validation

We apply our program to 27 lab-mixed *in vitro* samples (Wendler, 2015) to validate our methods and program. Our method successfully recovers the proportions with haplotypes of the input. Detailed validation steps are described in the supplement material as well as the Pf3k pilot phase main article. An example is given in Fig. ??.

4 Performance

4.1 Accuracy

4.2 Run-time assessment

5 Discussion

The program DEploid and its analysis pipeline is originally developed for *P. falciparum* studies. With some specific minor parameter changes, DEploid can be used for deconvolute *P. vivax* sequence data (Pearson et al., 2016). The framework is suitable for deconvoluting mixed genomes with unknown proportions. It can thus be extended to a wider range of applications, such as deconvoluting cancer tumour cell genomes or Ebola virus genomes.

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