Internal Assessment

Monopoly
Property Probabilities

Introduction and Rationale

To me, business is my life. I love to engage on the intellectual end of economics and business theory. My passion is entrepreneurship and I aim to one day be an entrepreneur and to invent something to make the world a better place. I learn as much as possible relating to business, through my classes and through my extracurricular activities. One of the activities I am most involved in is DECA, a useful tool that has allowed me to practice and expand my knowledge of marketing and of business theory. Another activity I have entered is the TYE program based in Portland Oregon. Through their council and mentorship, I have been able to start creating and inventing tools and products I never thought I could.

I love the ideas of business and of modeling success. I believe that business is the framework for societies, and that business has been conducted since the birth of civilization, and even before, in the form of bartering. My view is that our society runs through principles of economics, and I think that modeling elements of our economy can lead an individual to useful and profitable strategies. I perceive that economics and probability can be matched together to create models of economic success that can lead to both short term and long-term success.

I am writing on the topic of Monopoly because I think that it is an excellent model of the elements of real estate business in reality, apart from all of the additional variables that exist in the real world situation, such as sales tax, renovations, appreciation/depreciation, and millions more players and properties. At it's core, real estate success is centered around the principle of buy low, sell high. Monopoly, being centered around this principle, is a great way to experiment and determine potential money-making strategies and probabilities that can transcend the board game and carry over into the real world.

I am trying to figure out if you can predict the total amount of properties an individual will own based on the properties they own after one trip around the board because in Monopoly, a general idea is that the more properties you own, the higher the likelihood of success. This model of success, upon heavy tweaking, might be able to serve as a basis for a more advanced model that can estimate the probable amount of properties an individual will buy in their lifetime.

Aim and Approach

As established in the introduction, the aim of this dive into Monopoly will be to determine how many properties an individual will own immediately after all properties have been bought, based on the amount of properties owned by an individual after one trip around the Monopoly board, and given specific amounts of players and no trading until all properties have been bought.

The dive into the concepts relating to the properties and the determinations that must be made require any testing environment to adhere to strict control of the variables, such as the amount of players playing, the amount of trading of properties and money, the amount an individual collects if they land on or pass go, chance and community chest squares, tax squares, jail, free parking, houses and hotels, and the amount of auctioning that will be done. All the aforementioned variables will be controlled within the data collection period to ensure pure and uncorrupted data.

To answer the query regarding properties I needed to engage in Theoretical Probability and Experimental Probability, Expectations, Discrete Probability Distributions, Mean/Variance/Standard Deviation, and the calculation of Normal Distributions.

In this experiment, I used the shorthand of rounds to represent trips around the board, with one round representing one trip around the board.

I realized early on that it would be next to impossible for me to get the immense amount of data regarding the probabilities involved by simply using my Monopoly game and playing it repeatedly. I knew there had to be some way to get the data I needed in order to develop my mathematical models and in order to model success in Monopoly, so a simulation of Monopoly centered around properties owned after each round was created.

I then utilized the carefully constructed Monopoly simulation in order to gather data regarding the average amount of properties owned after the first round around the board, as well as the average total amount of properties held by players once all properties have been bought given a specific number of properties held after one trip around the board.

After the data collection from the experiment, I was able to create graphs and discern the means which aided in showcasing the results collected, specifically how the more players there were in the game, the less properties they were likely to have after the first go around the board.

I wanted to check my data and ensure that my simulation was indeed a valid representation of a simplified Monopoly, so I decided to evaluate the expected value utilizing the Expectations formula. I engaged myself in making an Expectation model to calculate the expected number of properties an individual owns after the first round around the board, utilizing the average amount of properties you can acquire per roll per property, and the average amount of rolls it takes to get around the Monopoly board and get to or move past go.

Once I had collected my results, I went about formatting them to determine important information about the data, and ultimately discerned the mean, variance and standard deviation.

Data Collection and Results

The Simulation

A computer simulation was created in order to model a Monopoly game in it's most basic form. The simulation operates as follows

- Standard rules of Monopoly as set forth by Hasbro however utilizing the following elements:
    - No going to jail for any reason
    - No monetary gain on free parking
    - No trading
    - No auctioning
    - No adding houses or hotels to a complete set of properties
    - No chance or community chest cards are acted upon
- The Simulation ends after every player completes 200 turns, or when all properties have been bought

Properties Owned After One Trip Around the Board

I utilized the simulation in order to acquire data regarding the probabilities of owning certain amounts of properties after one round. The simulations ran for 50,000 games for each number of players to give an average of the probabilities, and the data was collected to 2 decimal places.

Two Player Game

Number of Properties After Round One	Probability of Occurring for Player (Expressed as a Percentage)	
	1	2
0	0.15	0.41
1	2.56	4.66
2	13.11	17.62
3	29.84	31.67
4	33.15	29.35
5	17.34	12.85
6	3.88	2.75

Table 1: Probabilities of owning different amounts of property after round one in a two-player game

I had only intended to do one simulation; however, I noticed that there seemed to be a slight shift in the data between Player 1 and Player 2, thus I decided to run the simulation with additional players and determine if there could be some significance to being player one in the game of Monopoly.

Five Player Game

Number of Properties After Round One	Probability of Occurring for Player (Expressed as a Percentage)				
	1	2	3	4	5
0	0.60	1.25	1.60	2.83	3.95
1	5.35	8.15	10.94	14.22	17.25
2	18.65	23.42	27.21	29.25	32.11
3	33.75	33.45	32.45	30.86	29.22
4	28.15	24.52	20.35	17.66	14.13
5	11.48	8.10	6.26	4.82	3.43
6	1.78	1.03	0.81	0.45	0.30

Table 2: Probabilities of owning different amounts of property after round one in a five-player game

Eight Player Game

Number of Properties After Round One	Probability of Occurring for Player (Expressed as a Percentage)							
	1	2	3	4	5	6	7	8
0	1.12	2.02	3.1	4.61	6.04	7.51	9.42	11.58
1	8.71	11.9	15.44	18.66	21.85	24.56	27.54	29.83
2	24.27	27.63	30.06	31.75	32.86	33.21	32.77	32.26
3	33.63	32.61	30.76	28.24	25.83	23.64	21.38	19.13
4	23.27	19.68	16.08	13.37	10.97	9.06	7.36	6.03
5	7.66	5.51	4.06	3.06	2.25	1.86	1.41	1.1
6	0.96	0.63	0.48	0.29	0.19	0.15	0.12	0.08

Table 3: Probabilities of owning different amounts of property after round one in an eight-player game

After I had the probabilities of each player holding certain amounts of properties at the end of the first round, I then set about running the simulation to discern the number of properties that each individual would hold after all properties have been bought, based on the number of properties they had at the end of round 1. The results of this simulation can be found in the appendix, as App tables 1-7.

Modelling and Mathematical Manipulation of the Result

Graphing

The next step I did was I graphed the data points into a line graph to evaluate trends and better understand how the data relates to each other in a visual sense. I started with the results from the data sets conveying <u>the number of properties owned after round one.</u>

Two Player

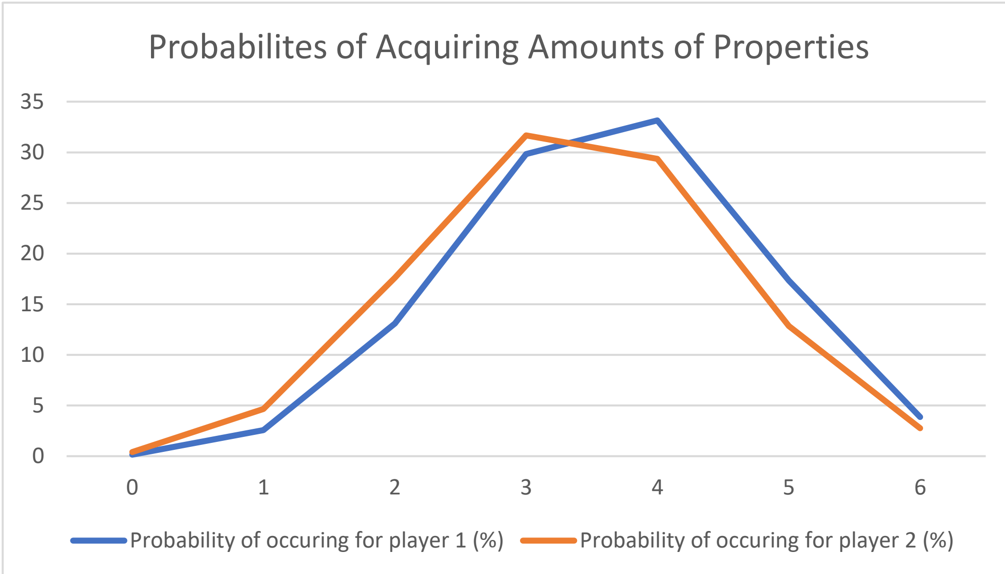

Figure 1: Graph showcasing the probabilities of owning different amounts of property after round one in a two-player game

Five Player

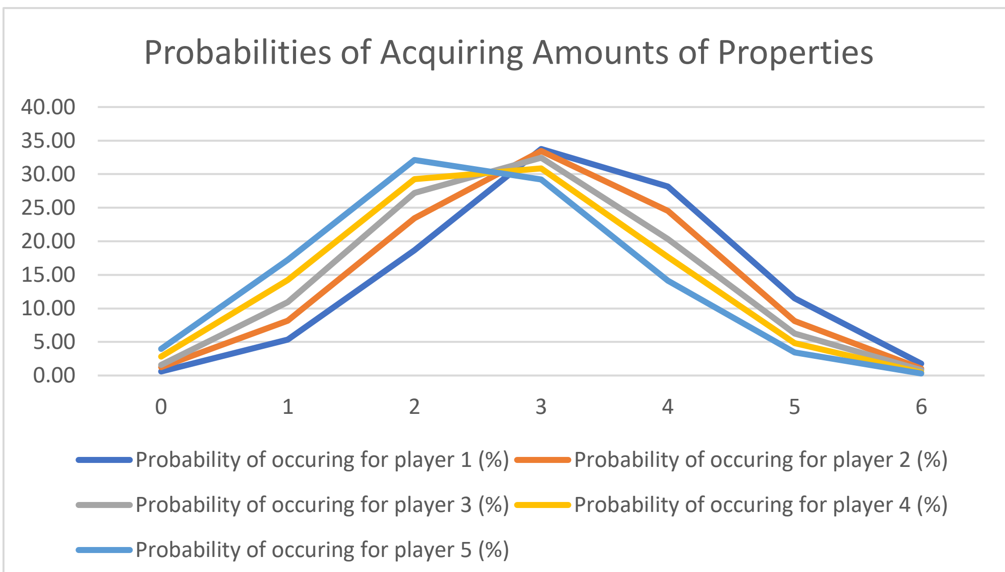

Figure 2: Graph showcasing the probabilities of owning different amounts of property after round one in a five-player game

Eight Player

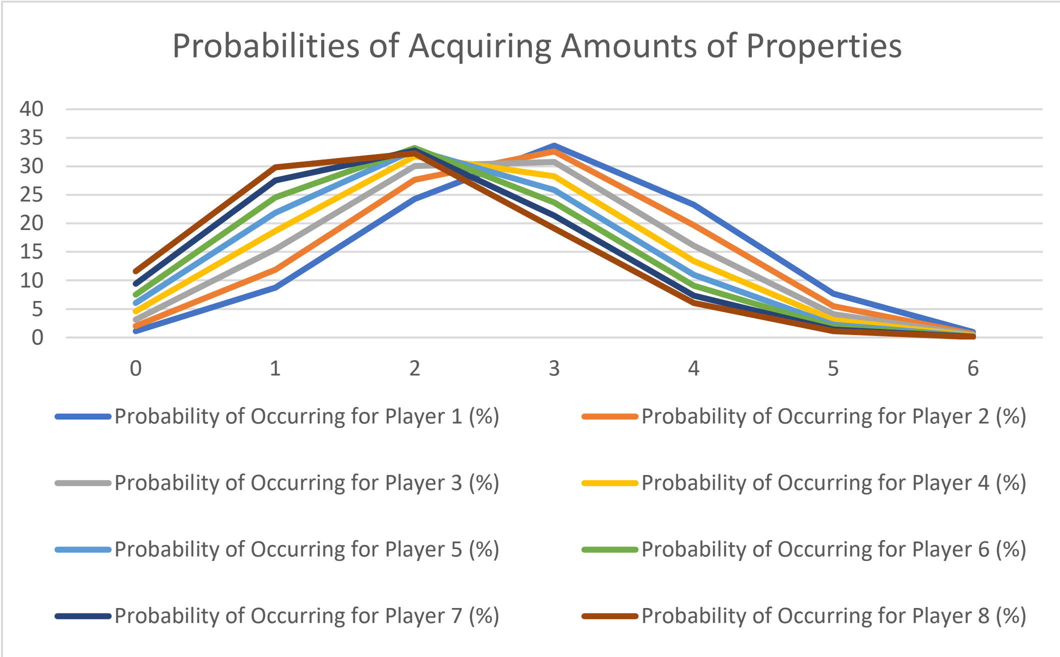

Figure 3: Graph showcasing the probabilities of owning different amounts of property after round one in an eight-player game

From the graphs, we can clearly see that individuals that are positioned farther and farther away from player 1 have a lower average amount of properties owned after round 1. We can also see that as the number of players increases, the average number of properties owned by player one after one trip around the board decreases significantly.

After The graphing of the data sets conveying information regarding the probabilities of owning certain amounts of properties after round one as well as the validity check through expectations, I then graphed the results from the data tables conveying information about the number of properties that each individual would hold after all properties have been bought, based on the number of properties they had at the end of the first trip around the board.

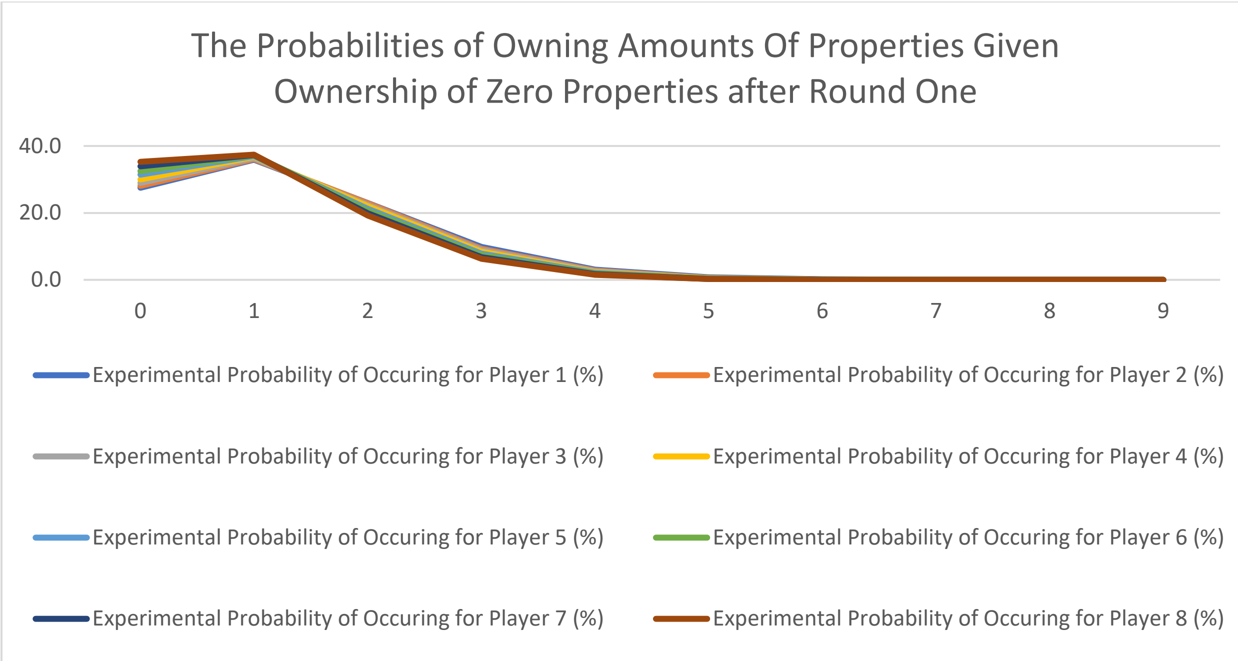

Figure 4: In an eight-player game, each player's probability of holding various amounts of properties given that they held zero properties at the end of round one

I graphed the data collected regarding the number of properties that would be owned after round one; however, I did not think that it would be pertinent to include the rest of graphs. Instead I thought I might convey the same conclusion with a single graph including different data curves from the seven different given property values at the end of one trip around the board. In all the data sets, there is a relatively little overlay in the curves representing no new properties acquired. Then, as the graph showcases the probabilities of acquiring new properties, the curves seem to gradually overlap each other, with the fourth and fifth player being in the middle of the pack. After having noticed this, I pulled out player four's curve from each of the graphs formed from the App Tables 1-7 and formed a graph combining the player 4 curves in order to more easily be able to determine if there was a general theme.

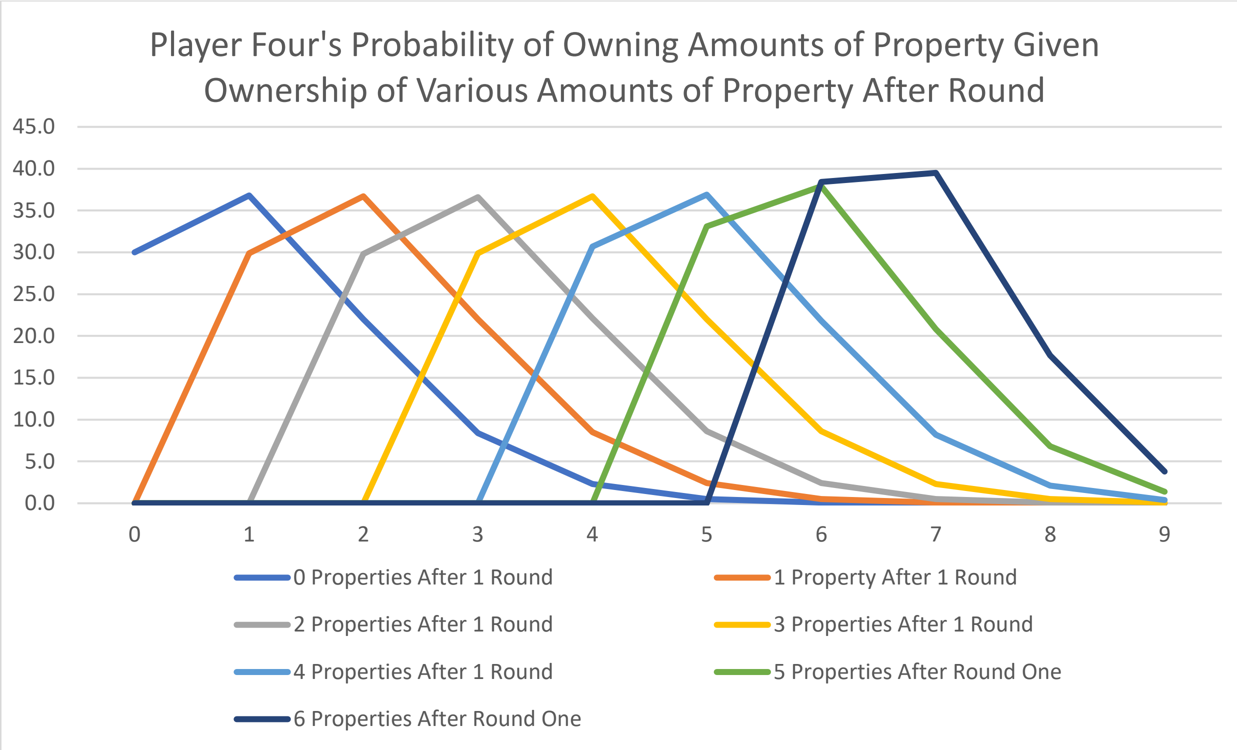

Figure 5: The various player 4s' representing the graphs differing in the amount of properties owned and the corresponding probabilities showcasing the likelihood of the amount of new properties acquired once all properties have been bought.

I then added all the pulled curves into one graph in order to evaluate them side by side. The conclusion that is apparent is that the initial trip around the board is instrumental in shaping the positioning each player has once all properties have been bought. This graph demonstrates that the curves of the graphs do not change significantly, only the positioning of the curves relative to the x-axis. This evaluation suggests that the majority of the strength a player might develop in a Monopoly game is gained in the first round.

Theoretical Expected value

After I had collected my data, I wanted to ensure the validity of the simulation by gathering the expected value based on theoretical thinking and not utilizing the data acquired. We can measure the expected value in terms of total properties an individual might own after one round around the board. To do this, we need to utilize the Theoretical Expectations formula, where $E(x)$ is the expected number of times variable x occurs, P is the probability of event x occurring, and N is the amount of opportunities event x has of occurring.

$$E(x) = PN$$

In our use of the Expectations formula, $E(x)$ will be considered to be the expected total amount of properties bought after round one by player one. We will also consider P to be the probability on any given roll to land on a property. We will then also consider N to be the total amount of rolls required to reach one time around the board.

I was initially going to model the expectation of all players in an eight player game because that would be the closest I can get to the real world scenario while still following the rules of Monopoly; however, I soon realized that the amount of computational power was beyond my mental capabilities, and realized that the only factor that would affect the other player's probability of acquiring a property would be whether or not player one had acquired properties within the other player's roll range, hence meaning the other player's average property count should be somewhat similar to player one's average property count. With these limitations in mind, I proceeded to chart out the probable amount of properties player one would own after one round around the board.

Because the probability of landing on a property shifts depending on where on the board the player is moving from, and also due to the inherent difference in rolling certain numbers within the range of 2 to 12, I have devised a way to calculate the average probability of landing on a property space.

Dice roll	2	3	4	5	6	7	8	9	10	11	12
Probability of Occurring	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Table 4: Using two dice, the probability of rolling any given number between the range of 2 and 12.

Using this dice data, I could then match each dice roll's probability up with the possible properties player one could land on in any given roll. Because to be able to match the corresponding properties from the launching tiles alongside their given roll probabilities would take immense processing power, I used a computer to compute the math. I then recorded the expected probability of landing on a property from each launching tile below.

<u>Calculation:</u>
How the calculation is set up:
- The equivalent of a Monopoly board is constructed although no action was taken
- It is then measured how many properties could be landed on from any given launching position, and the corresponding probability of rolling the number that would send you to each property.
- All the different probabilities of landing on a property from a launch point are added up to get the total probability of landing on a property.
- The calculation then proceeds to repeat itself on every tile after skipping the Mediterranean Avenue tile

- After all launching tile probabilities have been determined, the calculation adds all of the probabilities up and then divides by 39 in order to get the average probability that on any given roll on any given property, a player will land on a property.

Calculation Results:

Launch Tile	Probability of Acquiring a Property (Expressed as a Percentage)	Launch Tile	Probability of Acquiring a Property (Expressed as a Percentage)	Launch Tile	Probability of Acquiring a Property (Expressed as a Percentage)	Launch Tile	Probability of Acquiring a Property (Expressed as a Percentage)
1	63.9	11	72.2	21	88.9	31	55.6
2	-	12	69.4	22	86.1	32	52.8
3	72.2	13	66.7	23	80.6	33	52.8
4	75.0	14	63.9	24	75.0	34	50.0
5	80.6	15	66.7	25	72.2	35	52.8
6	83.3	16	69.4	26	69.4	36	52.8
7	86.1	17	77.8	27	63.9	37	55.6
8	86.1	18	83.3	28	63.9	38	58.3
9	83.3	19	86.1	29	61.1	39	61.1
10	80.6	20	88.9	30	58.3	40	63.9

Table 5: The probabilities of landing on a property given any given each launching tile

Using the data from the computation, I have discerned the average probability of landing on a property to be 0.683, or 68.3%.

Because the average value of a 2 dice roll is 7, and the total amount of tiles on the Monopoly board is 40, I was able to discern that the average amount of dice rolls required to reach or surpass 1 trip around the board was 6.

Based on all the elements listed above, I determined that the expected value of properties owned by player one after one trip around the board is 4 properties (4.098)

This theoretical Expectation, discerned from a calculation of assumptions, is off from the result of the experimental Expectation, discerned from the data set recorded from the experiment. My theory on the discrepancy is that the theoretical calculation does not take into account the scenario, and those like it, where player one falls behind a competitor and loses the leader status, and therefore does not have first dibs on the properties that they land on.

Experimental Expected Value

To find the expected value, or mean, of an already existing data set, we can use the Experimental Expectations formula, which is an expansion on the Theoretical Expectations formula, where $E(x)$ is the expected number of times variable x occurs, and $\sum xP(X = x)$ is the sum of the probabilities that chosen variable X has a value of x, multiplied by the corresponding values of x.

$$E(x) = \sum xP(X = x)$$

In this use of the Expectations formula in an experimental probability setting, $E(x)$ will be the expected value of properties owned after round one, x is the number of properties owned after round one, and $\sum x\, P(X = x)$ will be the sum of the number of properties owned after round one multiplied by the probability of the that a player will get that amount of properties after round one in any given game.

Using figures 1-3, I found the corresponding means of each curve within these figures

	Player Mean							
Number of Players in Game	Player 1	Player 2	Player 3	Player 4	Player 5	Player 6	Player 7	Player 8
2	3.61	3.33						
5	3.25	3.00	2.80	2.63	2.45			
8	2.95	2.75	2.55	2.37	2.21	2.08	1.94	1.82

Table 6: The various means of the curves representing the probability of acquiring various amounts of properties in games of various players

Determination of Variances and Standard Deviations

As previously discussed, I have found the experimental means of each data set regarding properties owned after round one.

I then went about determining the variance and the standard deviation utilizing a combination of the mean and the cosine graph. I made the decision to pursue these factors because I wanted to showcase how tightly the probabilities were clustered around the mean.

The first step in determination of the standard deviation is the discernment of the variance. To this end, I will be utilizing the formula designed to produce the variance from the mean and the corresponding probability, where $Var(x)$ is the variance of outcome x, P is the probability of event x occurring, and N is the amount of opportunities event x has of occurring.

$$Var(x) = NP(1 - P)$$

In my use of the variance formula, $Var(x)$ is the variance of the mean, while NP represents the mean (or expected value), and $(1 - P)$ represents $(1 -$ the corresponding probability in decimal form associated with the mean).

From this formula, I gathered the variance utilizing the determined means as well as the data from each data set. I knew that between the two points that surround the mean, there was a straight line, so I subtracted the smaller y-value from the larger y-value of the surrounding points, and then I took the decimal value of the mean, and I multiplied that decimal value of the mean by the difference of the y-values to get the y-coordinate of the mean. I then took the product and (if the line between the two surrounding points was rising) added it to the lower value, or (if the line between the two surrounding points is decreasing) I subtracted it from the higher y-value in order to get the probability. This probability is all the additional data I needed to acquire the variance, and thus the standard deviation.

Once I gathered the variance, I merely applied a square root to it to convert the variance into the standard deviation.

$$\sigma = \sqrt{Var(x)}$$

After I had the standard deviation, I charted the mean, the variance, and the standard deviation in data tables corresponding to the number of players that participated in the games.

Two-Player Game

Player	1	2
Mean	3.61	3.33
Probability of Mean (%)	31.86	30.90
Variance	1.15	1.02
Standard Deviation	1.07	1.01

Table 7: The mean, the corresponding probability of the mean, the variance, the standard deviation of each player in a two-player game

Five-Player Game

Player	1	2	3	4	5
Mean	3.25	3.00	2.80	2.63	2.45
Probability of Mean (%)	32.425	33.45	31.40	30.26	30.81
Variance	1.05	1.00	0.88	0.80	0.75
Standard Deviation	1.11	1.00	0.94	0.89	0.87

Table 8: The mean, the corresponding probability of the mean, the variance, the standard deviation of each player in a five-player game

Eight-Player Game

Player	1	2	3	4	5	6	7	8
Mean	2.95	2.75	2.55	2.37	2.21	2.08	1.94	1.82
Prob of Mean (%)	33.16	31.37	30.45	30.45	31.38	31.44	32.46	31.82
Variance	0.98	0.86	0.78	0.72	0.69	0.65	0.63	0.58
Standard Deviation	0.99	0.93	0.88	0.85	0.83	0.81	0.79	0.76

Table 9: The mean, the corresponding probability of the mean, the variance, the standard deviation of each player in a eight-player game

One major takeaway from this data table is that as the probability of the mean fluctuates, the variance and standard deviation continue to decrease at a steady rate. One possible explanation for this occurrence is that the players closer to player 1 have a much higher chance of acquiring property amounts farther away from their means, whereas the diversity of amounts of acquisition is inversely reduced for player 8, meaning that player 8 is much more likely to acquire a property count that is much closer to the mean than the players before because they only get to choose from the scraps left over by the players that had their turns before player 8.

Analysis and Conclusion

This experiment initially set out to uncover whether or not one could discern the total amount of properties a player would own based on the number of properties held after round one, and if so, what would that relation look like. The answer to this question was uncovered, alongside many additional observations that are important when considering the strength of a any given player within the game of Monopoly.

The answer to the primary query is yes, one can discern the amount of properties a player is likely to hold once all properties have been sold based on the amount of properties they hold at the end of round one. The relationship demonstrates that regardless of what amount you hold at the end of round one, you will likely pick up an average on 0-1 more properties in an eight player game

On average, players would purchase 0-1 additional properties in the round after the first trip around the board. This showcases the importance of the first round and how a player sets up their positioning compared to their fellow players is largely determined by the dice rolls of round one.

Another major insight was the order of the players does matter when trying to gain a strong position relative to the competitors. The farther away the player is from player one, the lower the average amount of properties they will acquire.

The amount of properties that each player will own comes down to the leader in position on the board. If player one rolls a 2 and player eight rolls the highest roll out of the competitors of an 11, player one loses the leader status and must now fight with the other players for the remaining property scraps left behind by the leader, who is now player eight, whom now has first choice of properties and who now experiences no probability of landing on a property already owned, whereas the trailing players do. The reason player one on average gets more property is because on average, player one remains the leader when compared to the competitors.

Another major finding was that the variance and standard deviation of the amount of properties owned after round one the farther away the players were compared to player one. This may be due to the fact that player one is typically not capable of landing on a property already owned because they are typically the leader, while the farther away the player is from player one the higher the likelihood of landing on an owned property is. This contributes to the inability of players farther away from player one to purchase a wider array of properties, and thus results in a lower standard deviation, meaning the data points are more closely clustered than the standard deviation of the players closer to player one.

What started out as a simple excursion into the realm of probability and statistics turned into a deep dive into understanding markets. The learnings from this experiment weight heavy into the real world, and bring out new questions and new experiments to discern the ultimate truths about business in the real world.

Evaluation and Extensions

This section is highlighting what could have been performed better in the experiment, and what could be done after the steps taken in this probability dive given more time.

- The Theoretical Expectation value was of player 1 was off from the Experimental Expectation value of player 1. I believe this to be the result of not considering that other players may pass player 1 and become the new leader of the group. This factor is unlikely to be able to be nullified, due to the books and books of data tables that would be required to chart the corresponding probabilities of the other players, and determine the astronomical amount of probabilities regarding the likelihood of other players passing each other.
- Given more time, one might be able to acquire the data regarding the average number of times any given player takes the lead in the first round. This might showcase the average competitiveness of each of the players in any given Monopoly game.
- One element that might have added more accuracy to the experiment is the construction of a better simulation. The simulation created was very limited due to the complexity of adding more factors than what is necessary. Given more time, a computer simulation that can consider the variables that were made into constants in the Data Collection and Results section could be constructed, which although would be more complex, may result in more accuracy when compared to a standard game of Monopoly.
- I think that it may be interesting to go into forming the curves of each players probabilities of owning various amounts of properties once all properties are bought into cosine functions, and then determining how the cosine functions relate to each in equation form. This I believe could also reveal more information about the relation between the total amount of properties players end up with given various properties owned after round one.

Appendix

Zero Properties After Round One

Number of properties after all properties have been bought	Probability of Occurring for Player (Expressed as a Percentage)							
	1	2	3	4	5	6	7	8
0	27.5	28.1	29.0	30.0	31.4	32.5	33.9	35.3
1	35.9	36.1	36.4	36.8	37.0	37.2	37.3	37.4
2	22.9	22.8	22.4	22.0	21.2	20.6	19.9	19.2
3	9.8	9.3	8.9	8.4	7.8	7.3	6.8	6.3
4	3.0	2.8	2.6	2.3	2.1	1.9	1.7	1.5
5	0.7	0.6	0.5	0.5	0.4	0.4	0.3	0.3
6	0.2	0.1	0.1	0.1	0.1	0.1	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

App Table 1: The probabilities of holding various amounts of properties after all properties have been bought given players held zero properties after round one.

One Property After Round One

Number of properties after all properties have been bought	Probability of Occurring for Player (Expressed as a Percentage)							
	1	2	3	4	5	6	7	8
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1	26.6	27.5	28.6	29.9	31.3	32.7	34.1	35.5
2	35.6	36.1	36.4	36.7	36.9	37.1	37.2	37.2
3	23.5	23.1	22.6	22.0	21.3	20.5	19.8	19.0
4	10.2	9.7	9.1	8.5	7.9	7.4	6.8	6.4
5	3.2	2.9	2.6	2.4	2.1	1.9	1.7	1.5
6	0.8	0.7	0.6	0.5	0.4	0.4	0.3	0.3
7	0.2	0.1	0.1	0.1	0.1	0.1	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

App Table 2: The probabilities of holding various amounts of properties after all properties have been bought given players held one property after round one.

Two Properties After Round One

Number of properties after all properties have been bought	Probability of Occurring for Player (Expressed as a Percentage)							
	1	2	3	4	5	6	7	8
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	25.8	27.1	28.4	29.8	31.3	32.6	34.0	35.4
3	35.5	36.0	36.3	36.6	36.8	36.9	37.0	37.0
4	24.1	23.4	22.7	22.1	21.3	20.6	19.9	19.2
5	10.5	9.8	9.2	8.6	8.0	7.4	7.0	6.5
6	3.3	2.9	2.7	2.4	2.2	2.0	1.8	1.6
7	0.8	0.7	0.6	0.5	0.4	0.4	0.3	0.3
8	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

App Table 3: The probabilities of holding various amounts of properties after all properties have been bought given players held two properties after round one.

Three Properties After Round One

Number of properties after all properties have been bought	Probability of Occurring for Player (Expressed as a Percentage)							
	1	2	3	4	5	6	7	8
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	25.6	27.0	28.4	29.9	31.3	32.6	33.9	35.1
4	35.6	36.1	36.4	36.7	36.7	36.9	36.9	36.9
5	24.2	23.5	22.8	22.0	21.3	20.6	20.0	29.3
6	10.6	9.8	9.1	8.6	8.0	7.5	7.1	6.7
7	3.2	2.9	2.6	2.3	2.1	2.0	1.8	1.7
8	0.7	0.6	0.5	0.5	0.4	0.4	0.3	0.3
9	0.1	0.1	0.1	0.1	0.1	0.1	0.0	0.0

App Table 4: The probabilities of holding various amounts of properties after all properties have been bought given players held three properties after round one.

Four Properties After Round One

Number of properties after all properties have been bought	Probability of Occurring for Player (Expressed as a Percentage)							
	1	2	3	4	5	6	7	8
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	26.2	27.8	29.2	30.7	32.0	33.3	34.4	35.4
5	36.1	36.5	36.8	36.9	37.0	37.0	37.0	37.0
6	24.1	23.4	22.6	21.8	21.2	20.5	19.9	19.4
7	10.1	9.4	8.7	8.2	7.6	7.2	6.9	6.5
8	2.9	2.5	2.3	2.1	1.9	1.7	1.6	1.5
9	0.5	0.5	0.4	0.4	0.3	0.3	0.3	0.2

App Table 5: The probabilities of holding various amounts of properties after all properties have been bought given players held four properties after round one.

Five Properties After Round One

Number of properties after all properties have been bought	Probability of Occurring for Player (Expressed as a Percentage)							
	1	2	3	4	5	6	7	8
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	28.7	30.2	31.7	33.1	34.3	35.7	36.6	37.7
6	37.6	37.9	38.0	37.9	38.0	37.8	37.8	37.6
7	23.3	22.4	21.5	20.8	20.1	19.4	18.9	18.2
8	8.5	7.8	7.3	6.8	6.4	6.0	5.6	5.4
9	1.9	1.7	1.5	1.4	1.2	1.1	1.1	1.0

App Table 6: The probabilities of holding various amounts of properties after all properties have been bought given players held five properties after round one.

Six Properties After Round One

Number of properties after all properties have been bought	Probability of Occurring for Player (Expressed as a Percentage)							
	1	2	3	4	5	6	7	8
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	33.8	35.6	36.9	38.4	39.7	40.2	42.4	42.7
7	40.2	40.1	40.0	39.5	39.5	39.0	38.7	38.6
8	20.3	19.3	18.6	17.7	16.9	16.9	15.7	15.3
9	5.6	5.0	4.6	4.4	3.9	3.9	3.3	3.5

App Table 7: The probabilities of holding various amounts of properties after all properties have been bought given players held six properties after round one.